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Risk measurement of China's green financial market based on B-spline quantile regression

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ABSTRACT

To accurately measure the spillover effect of China's green financial carbon emission market, a new measurement of conditional value at risk (CoVaR) based on the B-spline quantile methods is proposed. Firstly, the variable coefficient CoVaR model is constructed, and the model coefficients are estimated by the B-spline quantile method. Then, the relationship between Δconditional value at risk (ΔCoVaR) and value at risk (VaR) is considered. In the empirical analysis, we investigate five carbon trading quota risk measurements of the carbon emission projects in China from 2014 to 2022, and verify the B-spline superiority by Monte Carlo simulation. The empirical results show that B-spline method has the highest risk fitting success rate and the smallest error.

1. Introduction

Human industrial production has led to large emissions of environmental pollutants and greenhouse gases, causing systemic global climate warming, glaciers melting and sea level rising, and causing immeasurable losses to the social economy and ecological environment [1]. The Global Climate Conditions 2020 reports that the global average temperature in 2020 is 1.2° Celsius higher than the pre-industrial temperatures [2]. The United Nations Intergovernmental Panel on Climate Change (IPCC) set a key threshold for temperature rise of 1.5° Celsius, above that threshold and would increase the risks of socioeconomic development, food security and extreme weather [3]. At present, greenhouse gases emission dominated by CO_2 has become an international social problem [4]. A study by Stanford University shows that economic activity in all regions is related to climate change, the impact of temperature on economic productivity is nonlinear. If the climate is allowed to warm up, the global per capita GDP fell by 23% by year 2100 compared with no climate change [5]. The United Nations Framework Convention on Climate Change stipulates six greenhouse gases (CH_4, SF_6, PFC_s, HFC_s, N_2O, CO_2) with carbon dioxide equivalent as the unified trading measurement unit, so the carbon emission trading is also referred to as "carbon trading", the carbon trading market has emerged accordingly [6–8]. In 2013, the Chinese government firstly launched the carbon emission project pilot (the demonstration area for carbon emission) in Shenzhen, successively extending it to Chongqing, Guangdong, Beijing, Shanghai, Hubei, Tianjin, etc. [9,10]. China's carbon emission trading market was officially launched in 2021. According to the data of the People's Bank of China, the loans for indirect and direct carbon emission reduction benefit projects in the first quarter of 2022 are 4.22 and 7.79 trillions respectively, accounting for 66.5% of the green loans in total [11,12]. Generally speaking, the regulatory authorities will allocate carbon emission quotas to key emission units on a regularly basis every year according to the demand [13]. If the enterprise's carbon emission quota is rich, it can sell the remaining carbon emission quota in

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the carbon market. If the enterprise exceeds the quota, it needs to purchase the quota from the carbon market [14]. While carrying out green financial carbon emission quota trading business, we must also pay attention to the prevention of market risks [15]. The causes of market risks may be different, but they all have a common feature: the lack of an effective risk management system, resulting in incalculable consequences [16]. Apart from the global financial crisis in 2007 [17], the turmoil in the financial market never stopped in less than 100 years ago, large financial events and crises have become more and more intense. The direct result was the market risk caused by the Nikkei crash [18], which cost the bank more than $1.3 billion in less than 60 days. In similar cases of huge losses due to financial risk mismanagement, Orange County of California lost $1.6 billion in interest rate fluctuations [19]. The multi-billion dollar dilemma faced by Metal Futures Company in Germany [20] and Yamato Bank of Japan [21]. In China, there are also many companies and enterprises, which have been successively dissolved, such as Xinhua Securities, Southern Securities, Anshan Securities, Galaxy Securities and so on [22]. Once a crisis occurs in the carbon market, it will affect the quota limit of carbon emissions of enterprises, increase carbon emissions without consequences, further cause global warming. China's green financial carbon market risk management tools are not yet perfect, the trading subjects passively bear the market price fluctuations risk [23]. The green financial carbon market overall risk caused by this situation will reduce the market participants' enthusiasm [24,25]. It can be predicted that the emergency measures that trigger the government to intervene in the market after the event will be reduced or avoided as far as possible in the future, this problem will be solved mainly through the spontaneous risk management tools of the market.

With the relevant experiments and exploration development, green finance carbon market risk management has also attracted investors' attention [26]. Financial institutions must analyze the relevant risks and hidden dangers through the green finance business operation characteristics, then formulate targeted risk supervision plans and establish a complete regulatory mechanism [27,28]. The basis and core of green finance carbon market risk management are to make a reasonable risk assessment and measure, VaR is an important tool to measure the risk of green financial carbon market risk [29]. Morgan Bank [30] first proposed the VaR, which was widely used in the financial sector. Colucci et al. [31] compared the ShVolVaR model under student distribution with the results of six benchmark industry VaR models, then evaluated the VaR model effect under four different confidence levels. Suarez [32] estimated the value at risk of open market debt and conducted a comparative analysis. Liu et al. [33] presented the risk tail value concept in uncertain random risk analysis and gave its calculation theorem. Yu et al. [34] used simulation technology and data envelope analysis to enrich and expand the risk types and evaluation models in previous green investment research.

As a market risk measure method, CoVaR was first proposed by Adrian et al. [35] to express a financial institution risk spillover effect on the whole market as the tail covariance between the financial institutions and the market. The CoVaR based on the principle of "condition" was developed from the VaR, unlike the VaR that measures an individual institution's risk level, a CoVaR indicates the extent to which a whole market or system is affected when a financial product or institution is troubled at a specific confidence level. Ji et al. [36] used time-varying Copula modeling technology measures CoVaR and ΔCoVaR to verify the energy products' significant risk spillover to agricultural products. Braiek et al. [37] used statistic models to generate income distribution, and proposes a CoVaR portfolio method, using the best Vine Copula fit to calculate upper and downside risks. The results show that the presence of market risk will affect the optimal investment allocation. Wen et al. [38] assessed the market risk extent within different institutions by extending the asymmetric CoVaR method, and performed advantage and significance tests using the Bootstrap Kolmogov-Smirnov statistic, showing the whole market was sensitive to upward gains during the crisis. Xie [39] used quantile regression theory and CoVaR measure to analyze the risk spillover effect between China's bond market and stock market, the CoVaR method not only measured the individual financial institutions (financial markets) risk spillover, but also captured the system risk changes.

Regarding the measure of CoVaR, an effective indicator of spillover effect, the literature shows that the three typical measurement methods are quantile regression (QR) and Copula function method. Keilbar et al. [40] modelled interbank risk spillover effects considering the marginal effect of the quantile regression method. Shou [41] empirically studied on eight different types of sample commercial banks by CoVaR model and quantile regression techniques, from the liquidity perspective, the larger asset banks faced higher liquidity risk, which was more likely led to spillover aggregation of systemic risk. Xu [42] merged copula approach and CoVaR method, revealing asymmetric risk spillovers from the international and China's domestic energy market uncertainties to Hubei and Shenzhen carbon emission pilots, which were the most representative carbon markets in China. On the other hand, regarding quantile estimation methods, Tang et al. [43] proposed a one-step estimation method using B-spline function to estimate the unknown parameters and functions in the spatial semi-parametric variable coefficient partial linear quantile regression. Theoretically, the polynomial spline estimation is a generalization of the local polynomial method, which overcomes the smooth conditions that the local polynomials require the regression function to be derivable everywhere. Common used polynomial spline methods include *P*-spline and B-spline, which are all composed of spline basis functions, while *P*-spline basis function is prone to produce strange design matrix and B-spline is relatively stable. Ting et al. [44] proposed a flexible bistable mechanism composed of switching force B-spline winding beam in the front and rear direction, which has been applied to accurate geometric activation and quantifiable carrying equipment. Xu et al. [45] combined uniform multiphase material interpolation (UMMI) and recurrent multiphase material interpolation (RMMI), proposed a B-spline multi-parameterization method for the topological optimization of thermal elastic structure, which solved the multiple materials thermal elasticity problem. Currently, the B-spline technology plays a greatly important role in machine learning and artificial intelligence.

To our best knowledge, there is little linkage application of green financial carbon market risk measurement and spontaneous supervision, most studies on green finance risks are only conducted from the perspective of risk management and risk assessment [46–48]. Therefore, this paper investigates the CoVaR model and its measurement method. The main contributions are as follows: (1) Building a green financial carbon asset index system, including the yield and relative trading volume of five carbon emission provinces. (2) For the case of green finance carbon market, the risk measurement with quantile model is constructed, and the parameter estimation based on the B-spline method is established. (3) Using Monte Carlo simulation to compare the measurement effect of different parameter estimation methods. Finally, the risk feelings and probability range losses of asset owners, investors and investment institutions are quantified.

The remainder of the paper is organized as follows. Section 2 gives the methodology of this article. Section 3 presents the indicators selection and data source, and presents the risk spillover between carbon emissions of different provinces under B-spline quantile regression. Section 4 discusses the numerical results. Finally, Section 5 concludes the research.

2. Methodology

We first give the definitions of VaR and CoVaR, then take the relative trading volume of assets as the state variable to construct the quantile regression risk measurement model. Thirdly, the variable coefficient model is introduced, the state variables in quantile regression are taken as the covariates of the variable coefficient function. Then, we use B-spline basis function method to estimate the coefficients of variable coefficient function. Finally, the loss function is used to obtain the ΔCoVaR value.

2.1. Model of risk measurement

VaR is the maximum possible loss situation at the corresponding confidence level for a given duration. The fomula is as follows (see, e.g. Ref. [49]):

$$P(\Delta P \geq -\mathrm{VaR}) = 1 - \alpha \tag{1}$$

Where ΔP denotes the loss of the average transaction price of carbon emission pilot within a certain period of Δt, $1-\alpha$ is used to indicate the degree of risk aversion in carbon emission, the greater the aversion, the higher the confidence level. Risk aversion refers to that investors are more inclined to choose safer transactions with lower expected returns when facing transactions with uncertain returns. VaR as a risk measure index, many risk regulatory agencies around the world have used VaR to measure the situation under risk conditions.

Because the VaR method does not consider the mutual influence between provinces when measuring the risk, the contribution of the carbon emission pilot to the market risk cannot be captured. Adrian et al. [35] measured the contribution degree of market risk in financial institutions with the CoVaR method as follows:

$$P\left(X^j \leq \mathrm{CoVaR}^{j|D^i_\alpha} \middle| D^i_\alpha\right) = q \tag{2}$$

Where X^j represents the return rate of the affected pilot, $D^i_\alpha = \{X^i \leq \mathrm{VaR}^i_\alpha\}$, X^i is the return rate of the carbon emission pilot i, $\mathrm{CoVaR}^{j|D^i_\alpha}$ is the conditional risk value of the market in the q quantile. The median status indicates that the carbon emission pilot i is normal, the asset income is expressed as $\mathrm{CoVaR}^{j|D^i_{0.5}}$, the risk contribution degree $\Delta\mathrm{CoVaR}^{j|i}$ of the carbon emission pilot i to other pilots under extreme risk is expressed as (see, e.g. Ref. [50]):

$$\Delta\mathrm{CoVaR}^{j|i} = \mathrm{CoVaR}^{j|D^i_\alpha} - \mathrm{CoVaR}^{j|D^i_{0.5}} \tag{3}$$

2.2. Method of quantile regression

Quantile regression theory was first proposed by Koenker and Bassett [51]. We now give the quantile regression model [51]:

$$R^i_\tau = \alpha^i + \beta^i M_t + \varepsilon^i_t \tag{4}$$

where R^i_τ denotes the revenue sequence of carbon emission pilot i, τ represents the quantile. α^i、β^i is the quantile regression coefficient, M_t represents the state variable (according to previous studies, the change of volume affects the fluctuation of assets, so here we take the volume as the state variable), ε^i_t represents the error term.

Write $F(x) = P(X \leq x)$, for any $0 < \tau < 1$ the quantile of the distribution τ is

$$F^{-1}(\tau) = \inf\{x, F(x) \geq \tau\} \tag{5}$$

Then, the quantile estimation of the parameter τ is

$$\min_{\beta \in R}\left[\sum_{E_i} \tau\left|R^i_\tau - \left(\alpha^i + \beta^i M_t\right)\right| + \sum_{E^c_i}(1-\tau)\left|R^i_\tau - \left(\alpha^i + \beta^i M_t\right)\right|\right] \tag{6}$$

where $E_i = \{R^i_\tau > \alpha^i + \beta^i M_t\}$, and E^c_i is the opposing event. For formulas (5) and (6), one can refer to Ref. [52]. The VaR^i of each pilot is replaced by the quantile regression model to obtain the CoVaR of other pilots in the case of extreme loss (see, e.g. Ref. [53]):

$$\mathrm{CoVaR}^{j|D^i_\tau} = \widehat{\alpha}^{j|i} + \widehat{\beta}^{j|i} M^{j|i}_t + \mathrm{VaR}^i_{\tau,t} \tag{7}$$

In formula (7), $\widehat{\alpha}^{j|i}, \widehat{\beta}^{j|i}$ are the coefficient estimates under the quantile method, $M_t^{j|i}$ is the relative trading volume of pilot j when pilot i is at risk.

2.3. Variable coefficient model

Hastie and Tibshirani [54] proposed variable coefficient model, variable coefficient model belongs to the promotion of the classical linear regression model, in survival analysis, longitudinal data and financial time series are widely used, is one of the greatly popular non-parametric models in recent years. Kader et al. [55] studied and obtained an exact solution for a time-fractional biological population with a nonlinear coefficient of variation. Its general expression is the following:

$$y = \alpha_0(v_0) + x_1\alpha_1(v_1) + \cdots + x_p\alpha_p(v_p) + u \tag{8}$$

In equation (8), y is the response variable and $\alpha_k(v_k)$ is the coefficient corresponding to x_k in the model, whose value is not a fixed constant and changes with the covariate v_k. The $x = (x_1, \cdots, x_p)^T$ is the p dimensional explanatory variable. u is independent random error with mean zero and variance σ^2. In order to simplify the calculation and avoid the "dimension curse" problem, this paper takes the covariate v_k as the state variable with dimension one, let $v_k = M_t$, we obtain the following variable coefficient CoVaR model

$$\text{CoVaR}^{j|D_\tau^i} = \alpha_0(M_t) + \alpha_1(M_t)\text{VaR}_{\tau,t}^i + u \tag{9}$$

2.4. B-spline method

The B-spline method is characterized by local, continuous, and uniform curvature variation [56,57]. For the value of the variable coefficient vector $\alpha(v_t) = (\alpha_0(M_t), \alpha_1(M_t))^T$ in $\text{CoVaR}^{j|D_\tau^i}$, we use the B-spline method based on the B-spline basis function proposed by Bohmer et al. [58]. The B-spline basis function constitutes a B-spline whose N freedom degree recursive formula is

$$\begin{cases} B_{C,0}(t) = \begin{cases} 1, t_C \le t \le t_{C+1}, t_C < t_{C+1} \\ 0, \textit{otherwise} \end{cases} \\ B_{C,L}(t) = \dfrac{t - t_C}{t_{C+L} - t_C} B_{C,L-1}(t) + \dfrac{t_{C+L+1} - t}{t_{C+L+1} - t_{C+1}} B_{C+1,L-1}(t) \end{cases} \tag{10}$$

where $C = 1, \cdots, M + N + 1$ and $L = 1, \cdots, N$.

Thus, $\alpha(M_t)$ can be approximated by the following B-spline basis function

$$\alpha_k(M_t) \approx \sum_{C=1}^{M+N+1} \varphi_{k,C} B_{C,L}(M_t) = B_L^T(M_t)\varphi_k, k = 0, 1, \cdots, q \tag{11}$$

where $B_L^T(M_t)$ represents the B-spline basis function, M is the splines nodes number, N is the B-spline freedom degree and $\varphi_k = (\varphi_{k,1}, \cdots, \varphi_{k,L+N+1})^T$ is the B-spline coefficient. We employ equal spatially homogeneous knots so that nodes are evenly distributed within the domain interval, where the domain $[a, b]$ is equally divided by M knots and the internal knots $t_1, \cdots, t_M$ of the spline function on the domain interval $[a, b]$ satisfies $t_i - t_{i-1} = t_{i+1} - t_i, i = 2, \cdots, M - 1$. Specifically, the spline knots M number selection is crucial, which can balance the smoothness degree between the regression function estimates and the fitted data. We use the *SIC* criterion proposed by Schwarz [59] as follows:

$$SIC(M) = \ln\left\{\sum_{i=1}^{n}\left(Y_i - \Gamma_N^T(X_i, t_i)\widehat{\beta}_M\right)^2\right\} + \frac{\ln(n)}{2n}\{(k+1)(M+N+1)\} \tag{12}$$

A quantile loss function [51] is constructed for the variable coefficient variance equation

$$\min_{\varphi_k(\tau)} \sum_{t=1}^{n} \rho_\tau\left(R_t^i - B_L^T(M_t)\varphi_0(\tau) - B_L^T(M_t)\varphi_1(\tau)R_t^j\right) \tag{13}$$

By formula (13), we obtain the parameter estimation $\varphi_k(\tau)$, then we substitute $\varphi_k(\tau)$ into formula (9), it follows the estimation of CoVaR at t-quantile:

$$\text{CoVaR}_t^{j|D_\tau^i}(\tau) = -B_L^T(M_t)\varphi_0(\tau) - B_L^T(M_t)\varphi_1(\tau)\text{VaR}_{\tau,t}^i \tag{14}$$

Our measurement method has the following advantages: on the one hand, the statistics obtained by quantile regression method have many excellent properties, such as quantile regression having no requirements for the model error distribution, having a data comprehensive description, and the parameter estimates being robust. On the other hand, using B-spline quantile regression method to estimate the variable coefficient will make the result more accurate. This method is not only easy to explain, simple in structure, but also has the robustness of non-parametric model.

Table 1
Income fluctuations.

Province	Mean	Min	Max	Range	S. D.	J-B value
Shanghai (SH)	0.0152	−16.5367	59.5087	76.0454	5.38	42,310
Tianjin (TJ)	−0.0580	−24.7403	26.5628	51.3031	4.21	2974
Beijing (BJ)	0.0528	−36.7977	28.9669	65.7646	5.32	2240
Guangdong (GD)	−0.0528	−41.2157	21.1137	62.3294	4.78	6350
Hubei (HB)	0.5929	−9.2795	117.1600	126.4395	8.79	189,127

Table 2
Relative trading volume fluctuation.

Province	Mean	Min	Max	Range	S. D.	J-B value
Shanghai (SHCJ)	1893.1620	−0.9999	403,159.0000	403160.0000	27,120.39	436,938
Tianjin (TJCJ)	4.7366	−0.9984	499.0000	499.9984	35.61	263,884
Beijing (BJCJ)	16.8297	−0.9989	1135.0000	1135.9989	95.06	83,847
Guangdong (GDCJ)	425.6481	−0.9999	31,340.6700	31,341.6600	3413.02	46,728
Hubei (HBCJ)	15.3564	−0.9974	2215.5800	2216.5744	152.71	353,895

3. Indicators selection and data source

For the research on the green finance market risk, there are not complete materials in the academic circle. The research objects involved in the existing literature are mainly risk spillover effects between commercial banks and stock markets. The mainstream research method is to select multiple representative market risk indicators and uses scientific and effective CoVaR measures to analyze the fluctuations transmission effect between various market risk indicators. Uddin et al. [60] used standard Vine Copula and CoVaR to model the energy commodities multivariate tail dependence structure and spillover effect, found that carbon assets provide diversified benefits for energy commodity investment and promote the energy market to adjust the carbon emissions and trading licensing regulation. Thus, we in accordance with data availability, scientific and objectivity principle, from China seven carbon emissions trading pilot screening data enough and meet the research conditions of five pilots, according to the same trading open time principle for the pilot sample data 222, total data of 2220, time span from April 2014 to January 2022, data from iFinD (www.51ifind.com), the main tools used in this article are R (mainly involve packages: splines, quantreg, tseries etc). Each pilot carbon emission rights average daily transaction price is P_t and the trading volume is V_t. The return rate of $r_t = 100 \times \ln(p_t / p_{t-1})$ is calculated according to the formula, the relative trading volume is $v_t = (V_t - V_{t-1}) \cdot V_{t-1}^{-1}$, each pilot return rate and relative trading volume fluctuation is shown in Tables 1 and 2 respectively. It can be found that the standard deviation (S.D.), average value and each pilot relative trading volume fluctuation range are large, indicating that the relative trading volume does affect the pilot assets' fluctuation, which also means the risks faced by the pilot assets. The larger the Jarque Bera value (J-B value), the higher the sequence's stability, which can be used for relevant quantitative research.

4. Numerical results

4.1. Empirical analysis

To research the green finance risk spillover effect in China, firstly, the measurement indexes and quantification methods are scientifically selected for carbon emission under a dual-carbon development environment. Secondly, quantile regression and B-spline estimation can be used to measure the market risk spillovers among China's carbon emission provinces. Finally, verify the rationality and accuracy of the method used through the failure rate test. Therefore, we construct the following three models to compare the effect of the risk measurement model [51].

$$\text{Model 1}: R_t^i(\tau) = -B_L^T(M_t)_{\varphi_0} B_L^T(M_t)_{\varphi_1}(\tau) \bullet R_{\tau,t}^j \tag{15}$$

$$\text{Model 2}: R_t^i(\tau) = -\widehat{\alpha}_0^{j|i} - \widehat{\beta}^{j|i} M_t^{j|i} - \widehat{\gamma}_0 R_{\tau,t}^j \tag{16}$$

$$\text{Model 3}: R_t^i(\tau) = -\widehat{\alpha}_1^{j|i} - \widehat{\gamma}_1 R_{\tau,t}^j \tag{17}$$

The $\varphi_0(\tau)$、$\varphi_1(\tau)$、$\widehat{\gamma}_0$、$\widehat{\gamma}_1$、$\widehat{\alpha}_0^{j|i}$ and $\widehat{\alpha}_1^{j|i}$ in the above three models are all the parameters to be estimated. Model 1 is the CoVaR value calculated with the state variable as a covariate of the B-spline basis function. For contrast, quantile regression CoVaR model 2 with relative volume as a state variable and quantile CoVaR model 3 without a state variable are constructed.

Limited to space, the article only takes Tianjin as an example to measure Shanghai's carbon emissions risk spillover effect in Tianjin. After fitting the CoVaR model with B-spline quantiles, the CoVaR accuracy was assessed using the failure rate test proposed by Kupiec [61]. Let the study total sample days be T and the days beyond the measure criteria be N, then the failure frequency be $p = N/$

Table 3
Results of fitting success rate at different significance levels of three models.

Model	Sample capacity (*T*)	Failed number (*N*)	Fail frequency (%)	α	Fit success rate (%)
Model 1	221	3	1.4	0.01	71.40
	221	12	5.4	0.05	94.00
Model 2	221	3	1.4	0.01	71.40
	221	13	5.8	0.05	86.21
Model 3	221	4	1.8	0.01	55.56
	221	14	6.3	0.05	79.37

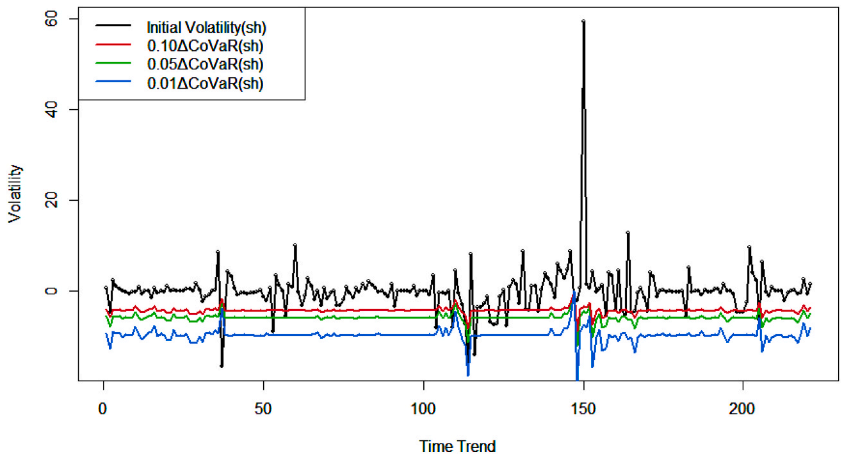

Fig. 1. CoVaR volatility of risk spillover from Shanghai to Beijing under different quantiles.

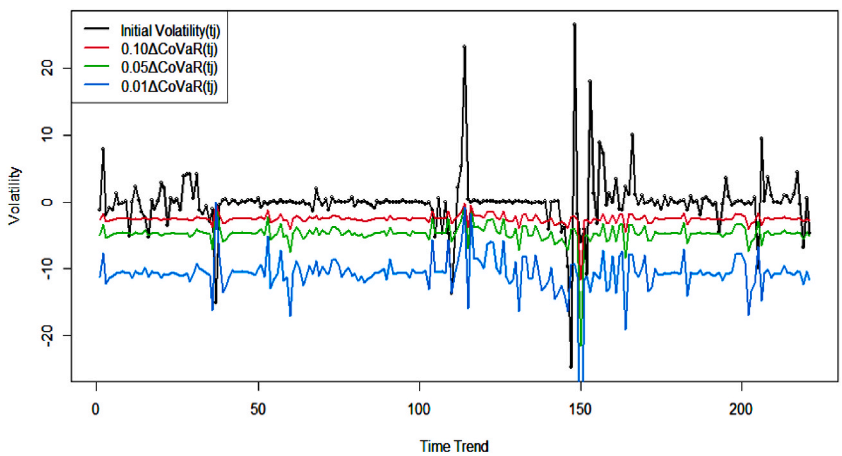

Fig. 2. Δ CoVaR volatility of risk spillover from Tianjin to Beijing under different quantiles.

T. Assuming that the α significance level is established, in the ideal state, the expected failure rate is $p^{'} = \alpha$. If $p < p^{'}$, the risk value is too high and the established model is too conservative. If $p > p^{'}$, the risk value estimate is too low. The model fitting success rate can be recorded as $q = p/\alpha \times 100\%$, and the three models are shown in Table 3.

The sample capacity in Table 3 is 221, the failed number refers to the number of days *N* that each model exceeds the measurement standard. Failure frequency refers to the ratio between failed number and sample capacity, fitting success rate refers to the ratio between the real failure rate and the ideal state (significance level). The closer the fitting success rate is to 100%, the better the model fitting effect is. On the contrary, the worse. It can be obtained from the fitting success rate results in Table 3: at the significance levels of 1% and 5%, the risk overflow of B-spline quantile CoVaR is the most accurate, followed by the quantile CoVaR model 2, and the worst accuracy is the model without state variable 3.

Further, by calculating the Δ CoVaR of the B-spline quantile model, the Δ CoVaR statuses of Shanghai and Tianjin at 1% and 5% quantiles are shown in Figs. 1 and 2:

Table 4
Results of fit success rates at different significance levels of three models.

Pilot	Variable	Coefficient	S. D.	*t* value	*p* value	R^2
SH	VaR (SH)	0.8890	0.0148	59.9000	0.0000	0.9422
TJ	VaR (TJ)	1.0150	0.0164	61.8860	0.0000	0.9456

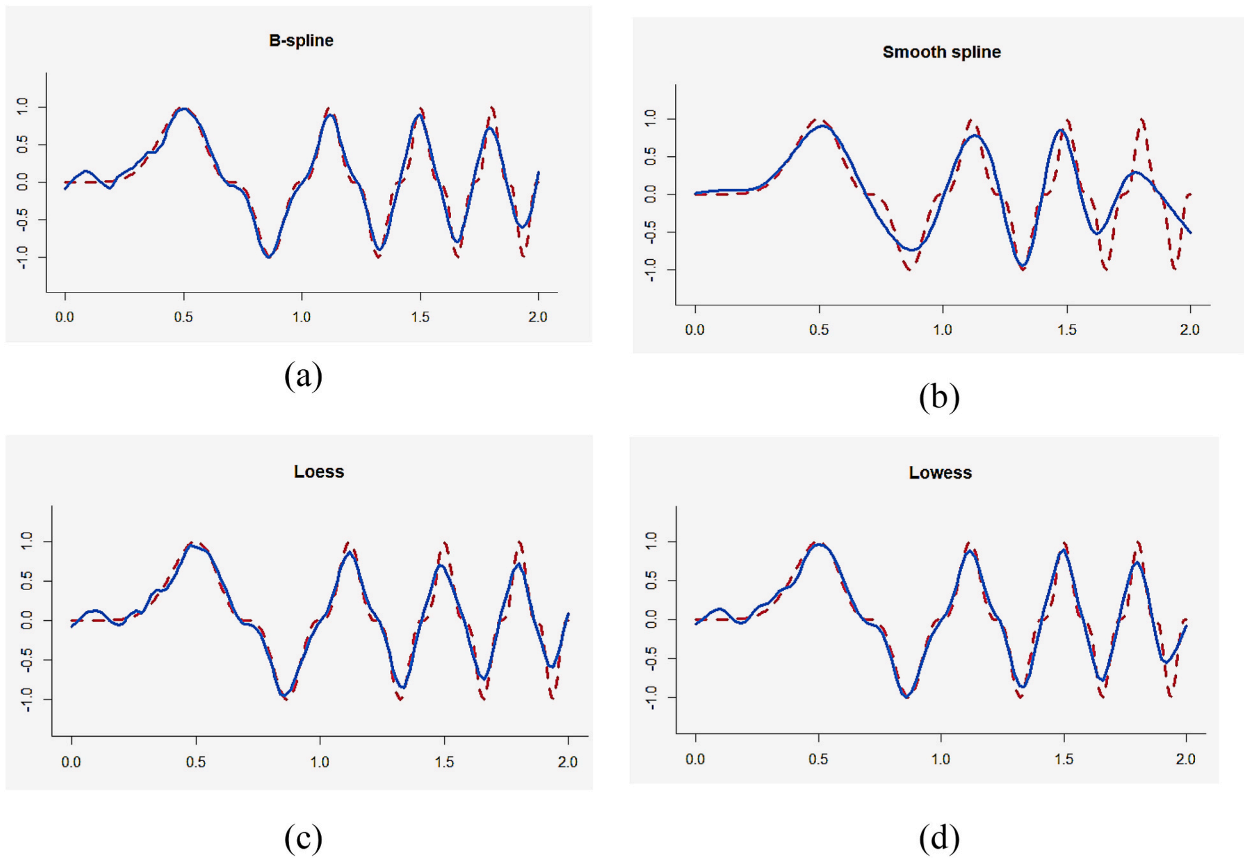

Fig. 3. (a) Fitting curve under B-spline; (b) Fitting curve under Smooth spline; (c) Fitting curve under Loess; (d) Fitting curve under Lowess.

To research the correlation between the risk spillover and the own tail risk of the Shanghai and Tianjin carbon emission pilots, we do a simple regression analysis between VaR and Δ CoVaR under the 1% quantile of the pilots. As shown in Table 4:

According to Table 4, there is a strong positive correlation between the VaR of Shanghai or Tianjin and the Δ CoVaR of Beijing. The analysis results are significant at the 99% confidence level, indicating that the larger the VaR of the pilot itself, the stronger the risk spillover effect. Comparing Shanghai. On the other hand, there is a stronger linear relationship for carbon emission pilot of Tianjin, compared with Shanghai, the coefficient for Tianjin is bigger than 1.

4.2. Monte Carlo simulation

To test the fitting effect of the model, we designed the following scenario Monte Carlo random number trial: considering the random number generation process (DGP) of one-dimensional variables, set $Y = \sin^3(2\pi X^2) + \varepsilon$, the variable X is evenly distributed in the interval [0, 2], the error term ε follows the normal distribution. To illustrate the effectiveness of the B-spline model, we compare it with locally weighted regression (Loess), Smooth splines and locally weighted linear regression (Lowess) models.

The random number sample size is $n = 201$, and the fitting results are shown as follows:

In Fig. 3, dotted lines (the red line) in the figure represent the true values, the solid lines (the blue line) are the various model-fit values. It can be intuitively seen that the smooth spline fit is the worst, and the other three methods can better depict the shape and basic characteristics of each function. Next, we conducted 300, 600, 900 random simulation trials with a sample size of $n = 201$, constructing the root mean square error (*RMSE*) statistics [62] as the evaluation criterion, *RMSE* as defined below

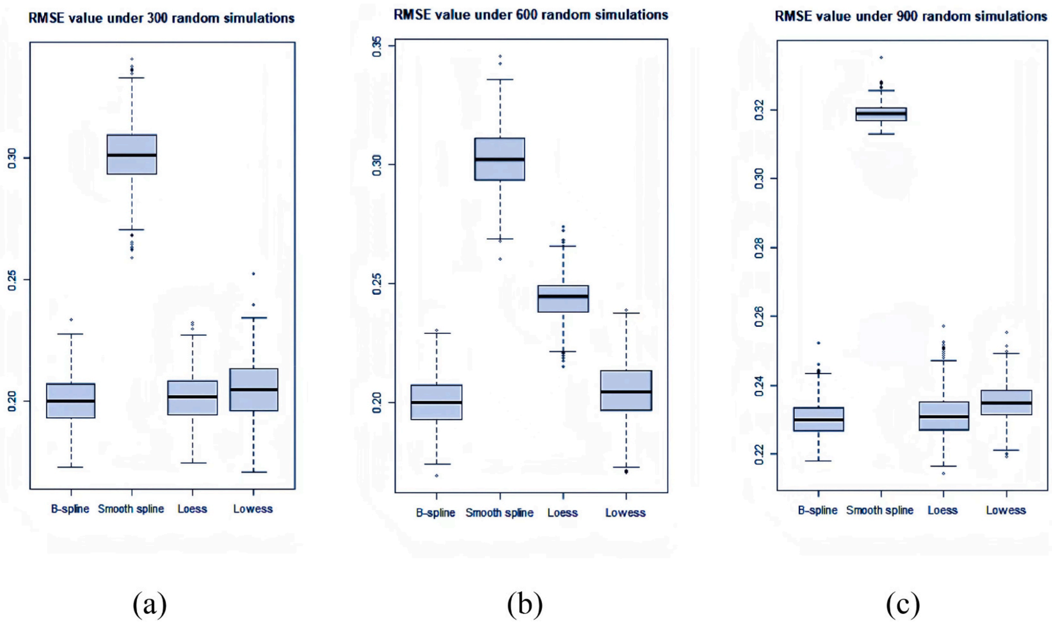

Fig. 4. Boxplots of (a) 300, (b) 600 and (c) 900 random simulations under four models.

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\widehat{Y}_i - Y_i)^2} \tag{18}$$

Its *RMSE* statistic under different models was calculated for each random trial. The calculation results can be shown in the box plot. By comparing the *RMSE* values of 300, 600 and 900 random trials under different models, the B-spline method has the smallest error value and the best fitting effect, while the smooth spline has the worst fitting effect. From the perspective of *RMSE* value, the *RMSE* values of the four models are less than 0.5, indicating that the four model estimation methods are greatly close to the true value, while the B-spline method has the smallest error value, indicating that it is more frequently close to the true value. The simulation effect is shown as follows.

From Fig. 4, it can be seen that the B-spline method has the smallest *RMSE*, while the smooth spline has the largest *RMSE*, which indicates that fitting effect of B-spline method is the best.

5. Conclusion and prospect

Based on China's dual-carbon strategic planning, this paper studies the risk value and spillover effect of green assets. The B-splines model is applied to the quantification of market risk value of carbon emission pilot assets. First, there is a significant positive correlation between the tail risk and the risk state, that is, the larger the VaR of the carbon emission pilot, the greater the Δ CoVaR value for other pilots. Second, compared with the static quantile Δ CoVaR, dynamic Δ CoVaR has a good performance in timely spontaneous risk monitoring and market tracking, which overcomes the serious defects of "ghost effect" in the traditional static Δ CoVaR, and has strong practicability. Third, in the empirical analysis, the *RMSE* values are less than 0.5 in all cases, indicating that the estimates have small error values. At the same time, the test shows that the *RMSE* value of the B-spline method is the minimum by comparing the *RMSE* values of the four estimation methods, which indicates that the estimation method has a good approximation effect. The model has the highest fitting success rate for different models, which is rare in the previous literature. This research is helpful to expand the research perspective of asset risk measurement and provides a theoretical basis for asset risk supervision.

It should be emphasized that the results obtained by using different VaR calculation methods may vary widely, different methods have different assumptions and conditions, leading to the results obtained by studying the same problem may vary widely. That is, a general calculation method has not yet emerged. Secondly, due to the lack of comprehensive evaluation criteria for VaR calculation results, VaR values calculated by different methods need to be comprehensively evaluated from all aspects to judge the advantages and disadvantages of each method. However, these evaluation criteria is relatively rare or only for one aspect of the evaluation, can not give an overall evaluation conclusion. Finally, study the relationship between trading volume and volatility to measure its impact on VaR and its role. This new direction also leaves many unfinished areas. For example, what is the relationship between volatility and trading volume, and what kind of functional relationship between them all needs to be further studied. At the same time, the role and impact of volume indicators in VaR need to be further explored and their modeling and calculation methods need to be further developed.

In addition, the idea of spline quantile constructed in this paper can also be extended to other models and fields, including hedge funds, futures hedging and corporate credit risk, among which the B-spline fitting method can also be replaced by other fitting methods (such as polynomial regression, smooth spline).

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Yuexu Zhao, Weiqi Xu: wrote the paper; analyzed and interpreted the data; conceived and designed the experiments; contributed reagents, materials, analysis tools or data; performed the experiments.

Data availability statement

Data associated with this study has been deposited at iFinD (www.51ifind.com).

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] D. Degroot, K.J. Anchukaitis, J.E. Tierney, et al., The history of climate and society: a review of the influence of climate change on the human past, Environ. Res. 7 (6) (2022) 9–11.
[2] J. Hijazi, UN environment rights bolster case for global climate litigation, Environ. Energy Report 10 (9) (2021) 32–39.
[3] J. Varley, Time is fast running out for climate change solutions–IPCC, Modern Power Systems, Communic. Power Technol. Worldwide 2 (10) (2022) 42–50.
[4] S.T. Shah, A. Basit, H.I. Mohamed, Use of mulches in various tillage conditions reduces the greenhouse gas emission–an overview, Gesunde Pflanz. 5 (6) (2022) 1–23.
[5] B. Marshall, S.M. Hsiang, M. Edward, et al., Global non-linear effect of temperature on economic production, Nature 527 (7) (2015) 75–77.
[6] S.N. Seo, Climate negotiations: the science of a big deal? Climate Change and Economics 514 (3) (2021) 203–222.
[7] J. Hnni, T. Ma, Swiss climate change law, Swiss Energy Gov. 27 (11) (2022) 17–47.
[8] J.K. Mcwhorter, P.R. Halloran, W.J. Skirving, et al., The importance of 1.5°C warming for the great barrier reef, Global Change Biol. 28 (4) (2022) 1332–1341.
[9] M. Standaert, Shanghai launches China's second pilot program for carbon emissions trading, Int. Environ. Reporter 36 (5) (2013) 1701–1702.
[10] H. You, L. Wei, Q. Yang, Current status and suggestions for carbon emissions trading pilots in China, Sino-Global Energy. 14 (8) (2017) 349–361.
[11] C. Duan, Y. Mi, Z. Lan, Analysis models for China's carbon trading market: comparison and outlook, Earth and Environmental Science 8 (12) (2022) 458–474.
[12] S.U. Jian, Y. Liang, L. Ding, et al., Research on China's energy development strategy under carbon neutrality, Bull. Chin. Acad. Sci. 36 (9) (2022) 1001–1009.
[13] C.Q. Ma, Y.S. Ren, Y.J. Zhang, et al., The allocation of carbon emission quotas to five major power generation corporations in China, J. Clean. Prod. 189 (7) (2018) 1–12.
[14] W. Wang, Y.J. Zhang, R. Tol, et al., Does China's carbon emissions trading scheme affect the market power of high-carbon enterprises, Energy Econ. 108 (2) (2022) 105906–105915.
[15] L. Li, A study of International carbon financial market development and risks in the context of low-carbon economy, Contemp. Finance Econ. 18 (3) (2012) 133–149.
[16] V. Chiara, V. Karen, Managing risks in SMEs: a literature review and research agenda, J. Technol. Manag. Innovat. 8 (12) (2013) 1345–1363.
[17] J.D. Nielsen, P. Feldhutter, D. Lando, et al., Corporate bond liquidity before and after the onset of the subprime crisis, J. Financ. Econ. 37 (3) (2012) 691–710.
[18] R.J. Shiller, F. Kon-Ya, Y. Tsutsui, Why did the Nikkei crash? Expanding the scope of expectations data collection, Rev. Econ. Stat. 78 (2) (1996) 881–892.
[19] J.J. Mcandrews, K. Sill, The economic benefits and risks of derivative securities,, Bus. Rev. 1 (2) (2014) 15–26.
[20] R. Willis, Metal futures: metals rise amid trade optimism, Argus metals international: global non-ferrous market prices, News and Analysis 4 (7) (2020).
[21] P. Jorion, Value at Risk: the New Benchmark for Managing Financial Risk, Risk Management, 2000.
[22] W. Zhang, Y. Liu, F. Zhang, et al., Green credit policy and corporate stock price crash risk: evidence from China, Front. Psychol. 13 (10) (2022).
[23] Y. Wang, W. Wang, Risk identification and regulatory system design for the carbon market, Chinese J. Popul. Res. and Environ. 2 (1) (2016) 9–16.
[24] Z. Xiao, S. Ma, H. Sun, et al., Time-varying spillovers among pilot carbon emission trading markets in China, Environ. Sci. Pollut. Control Ser. 29 (5) (2022) 57421–57436.
[25] S. Zhang, H. Ji, M. Tian, et al., High-dimensional nonlinear dependence and risk spillovers analysis between China's carbon market and its major influence factors, Ann. Oper. Res. 28 (7) (2022) 1–30.
[26] B. Aliouat, F. Saci, Green Finance, Corporate Performance and the Role of Financial Institutions, Post-Print, 2022.
[27] K. Berensmann, N. Lindenberg, Green finance: actors, challenges and policy recommendations, Briefing Papers 11 (15) (2016) 112–124.
[28] Z.A. Shaikh, N. Makarichev, The stability of financial institutions and counterparties, Sustaina. Energy Bus. Finance 20 (2) (2022) 159–170.
[29] H. Mzoughi, C. Urom, K. Guesmi, Downside and upside risk spillovers between green finance and energy markets, Finance Res. Lett. 47 (7) (2022) 289–301.
[30] J.P. Morgan, Risk Metrics-Technical Document, fourth ed., 1996.
[31] S. Colucci, Shrunk volatility value at risk: an application on US balanced portfolios, J. Risk Model Valid. 12 (2018) 51–62.
[32] M. Suarez, Comparative analysis of the estimation of value at risk in public market debt, Revista De Econ. Del Caribe. 24 (2) (2020) 49–94.
[33] Y. Liu, D.A. Ralescu, C. Xiao, et al., Tail value at risk in uncertain random environment, Soft Comput. 24 (2) (2020) 2495–2502.
[34] C.H. Yu, X. Wu, D. Zhang, et al., Demand for green finance: resolving financing constraints on green innovation in China, Energy Pol. 153 (1) (2021) 11–22.
[35] T. Adrian, M.K. Brunnermeier, Bank lending in times of time of large bank reserves, Federal Res. Bank of New York Staff Reports 55 (4) (2011) 301–348.
[36] Q. Ji, E. Bouri, D. Roubaud, et al., Risk spillover between energy and agricultural commodity markets: a dependence-switching CoVaR-copula model, Energy Econ. 75 (9) (2018) 14–27.
[37] S. Braiek, R. Bedoui, L. Belkacem, et al., Islamic portfolio optimization under systemic risk: Vine Copula-CoVaR based model, J. Finance Econ. 27 (10) (2022) 1321–1339.
[38] F. Wen, K. Weng, W.X. Zhou, Measuring the contribution of Chinese financial institutions to systemic risk: an extended asymmetric CoVaR approach, Risk Manag. 22 (4) (2020) 1–28.
[39] F. Xie, Analysis on risk spillover effect based on CoVaR method, J. Finan. Dev. Res. 342 (2010) 59–63.

[40] G. Keilbar, W. Wang, Modelling systemic risk using neural network quantile regression, Empir. Econ. 11 (7) (2021) 93–118.
[41] H. Shou, Research on the Risk Spillover Effect of China's Commercial Banks Based on Quantile Regression, the International Conference, ACM, 2017.
[42] Y. Xu, Risk spillover from energy market uncertainties to the Chinese carbon market, Pac. Basin Finance J. 67 (2) (2021) 1435–1451.
[43] Q.G. Tang, P. Jin, B-spline estimation method in spatial semi parametric variable coefficient partial linear quantile regression, Statistics and Information Forum 33 (11) (2018) 9–13.
[44] C. Ting, H.N. Tien, P.L. Chang, et al., Design of a bistable mechanism with B-spline profiled beam for versatile switching forces-science direct, Sensor Actuator Phys. 294 (2019) 173–184.
[45] Z. Xu, W. Zhang, T. Gao, et al., A B-spline multi-parameterization method for multi-material topology optimization of thermoelastic structures, Struct. Multidiscip. Optim. 61 (9) (2020) 923–942.
[46] H.G. Green, R.D. Martin, M.A. Pearson, Robust estimation analytics for financial risk management, in: Conference on Computational Intelligence for Financial Engineering, IEEE, 2002.
[47] J.A. Bingler, C.C. Senni, Taming the Green Swan: How to Improve Climate-Related Financial Risk Assessments, CER-ETH Economics Working Paper Series, 2020.
[48] X.N. Wang, Analysis of the mixed governance model of green financial risk management, J. Changchun Finan. College 12 (2) (2017) 5–9.
[49] D. Duffie, J. Pan, An overview of value at risk, J. Deriv. 4 (3) (1997) 7–49.
[50] J. Cao, L.H. Lei, B. School, Risk spillover effect between financial and real sectors via a dynamic generalized delta CoVaR approach, Syst. Eng. 37 (3) (2019) 122–131.
[51] R. Koenker, Regression Quantile, Econom. 46 (1) (1978) 33–50.
[52] N. Nolde, C. Zhou, M. Zhou, An extreme value approach to CoVaR estimation, Mathematics 3 (1) (2022).
[53] Q.K. Zou, Estimation of mean and standard deviation of Weibull distribution based on sample quantile, Statistics and Decision Making 38 (19) (2022) 16–20.
[54] H. Trevor, T. Robert, Varying coefficient models, J. Roy. Stat. Soc. B 55 (11) (1993) 757–796.
[55] A. Kader, M. Latif, D. Baleanu, Some exact solutions of a variable coefficients fractional biological population model, Math. Methods Appl. Sci. 44 (5) (2021) 4701–4714.
[56] J.Z. Huang, C.O. Wu, Z. Lan, Polynomial spline estimation and inference for varying coefficient models with longitudinal data, Stat. Sin. 14 (3) (2004) 763–788.
[57] S. Okaniwa, A. Nasri, H. Lin, et al., Uniform B-spline curve interpolation with prescribed tangent and curvature vectors, IEEE Trans. Visual. Comput. Graph. 18 (9) (2012) 1474–1487.
[58] K. Bohmer, M. Schempp, Spline Functions, Springer, Berlin. Heidelberg, 1976.
[59] R. Koenker, Z. Quanshui, Conditional quantile estimation and inference for ARCH models, Econom. Theor. 12 (5) (1996) 793–813.
[60] G.S. Uddin, J.A. Hernandez, S.J.H. Shahzad, et al., Multivariate dependence and spillover effects across energy commodities and diversification potentials of carbon assets, Energy Econ. 71 (3) (2018) 35–46.
[61] P.H. Kupiec, Techniques for verifying the accuracy of risk measurement models, J. Deriv. 3 (2) (1995) 73–84.
[62] H. Ueyama, Radiative cooling scale method for correcting hourly surface air temperature error in numerical weather prediction models, Meteorol. Appl. 30 (1) (2023) 2113–2121.