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Speaking out: A mathematical model of language preservation

Maria Díaz, Jennifer Switkes *

Department of Mathematics and Statistics, California State Polytechnic University, Pomona, CA, USA

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ABSTRACT

Languages evolve as an effect of communal competition while environmental and social dynamics characterize a language. There are thousands of languages spoken around the world but many of them are in danger of going extinct because of language competition and shifts. Modeling language preservation is important because the rise and fall of a language directly impacts the culture attached to it. We present a study of language competition and preservation between a bilingual population and two monolingual populations using a mathematical model involving nonlinear systems of differential equations. Building upon the ideas of previous models in the literature, the model utilizes population proportions with a simple structure that yields noteworthy behavior including a stable spiral with all three language groups preserved. We investigate how bilinguals and monolinguals can coexist as well as how they can affect one another.

Los idiomas evolucionan como efecto de la competencia comunitaria, mientras que las dinámicas ambientales y sociales caracterizan a un idioma. Se hablan miles de idiomas en todo el mundo, pero muchos de ellos están en peligro de extinción debido a la competencia lingüística y los cambios. Modelar la preservación del lenguaje es importante porque el auge y la caída de un idioma impacta directamente en la cultura que se le atribuye. Presentamos un estudio de la competencia y preservación del lenguaje entre una población bilingüe y dos poblaciones monolingües utilizando un modelo matemático que involucra sistemas no lineales de ecuaciones diferenciales. Sobre la base de las ideas de modelos anteriores en la literatura, el modelo utiliza proporciones de población con una estructura simple que produce un comportamiento notable que incluye una espiral estable con los tres grupos lingüísticos preservados. Investigamos cómo los bilingües y los monolingües pueden coexistir y cómo pueden afectarse unos a otros.

1. Introduction

Globalization brings a whole cultural and commercial ecosystem that transforms communities and diversifies the immediate population. As communities throughout the world interconnect, a genuinely co-evolutionary dynamic is being created where bilingualism is increasingly favored [1]. All languages share common characteristics and demonstrate complex network structures at differing degrees including sub-fields of linguistics [2]. As a side effect, these dynamic language systems evolve throughout time as interactions increase in diverse communities. Language can be viewed as a complex adaptive system where all language construction forms are competing for language users [3]. However, as language groups compete for users, language death becomes a possibility. Professing a language death is as bleak as announcing the death of a person for a language is only alive if the language user is [4].

There is a widespread misconception that any decrease in the number of languages is a benefit for society and that language death is nothing about which to mourn [4]. The problem with this misunderstanding is that languages inherently capture the essence, knowledge, and customs of a culture [5]. Society should care about language preservation because language embodies human heritage. Language preservation and interconnectivity of multiple language groups in a community is important because it allows people to build relationships and can lead to the inclusion of language groups such as bilinguals. Therefore, we will be using mathematical methods and differential equations to show that language conservation in a community is possible.

Baggs and Freedman in [6] constructed a model consisting of two differential equations representing the interaction of a monolingual population and a bilingual population. In [7], Baggs and Freedman introduced a model which consists of three differential equations that represent the interaction of two monolingual elements and one bilingual

* Corresponding author.
E-mail address: jmswitkes@cpp.edu (J. Switkes).

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element of a population. The model encompasses language competition where three distinct language groups are present, dominant monolingual, bilingual, and underrepresented monolingual. They engineered situations under which all three elements persist, one dominated monolingual element will become extinct, and the bilingual element will become extinct.

More recent mathematical modeling literature on this topic builds from the work of Abrams and Strogatz [8]. Abrams and Strogatz focused on a community with two language groups in which the changes between groups are caused by the prestige of a language. This one-page article is considered a seminal work in the mathematical modeling of language competition and language death, impacting most of the research literature on this topic published since.

In [9], Mira and Paredes applied the structure in [8] to explore the language competition of Castillian Spanish and Galician, with results suggesting that a stable bilingual population can survive if the competing languages are similar enough. In this model there are three parameters that represent similarity, social status and volatility. In [10], Wyburn and Hayward applied and modified the Baggs-Freedman model [6] in the language competition landscape in modern Wales, Scotland, Ireland, and Brittany. In [11], Mira et al. demonstrated the importance of a stable bilingual group in order for two languages to coexist. In [12], Otero-Espinar et al. focused on obtaining analytic results for models associated with [11]. In [13], Nie et al. built an optimal control problem to protect an endangered language. In these models, the competing languages are both official languages of the region noted. Our model is better suited for a country with a dominant official language and an underrepresented language that is not officially recognized as a language of the country.

An elaborated interpretation of the Abrams–Strogatz model and the Mira and Paredes model was proposed by Colucci et al. [14, 15]. This model has three distinct language groups, language 1, bilingual, and language 2. The model is engineered utilizing bifurcation theory to calculate specific values that describe a stable model where three language groups coexist. Similarly to Abrams and Strogatz's model and the Mira and Paredes model, the change in population proportions is caused by how influential and necessary each of the language groups is. Colucci et al. developed their system by also investigating the model proposed by Baggs and Freedman.

More recently, ideas from the Baggs-Freedman models have been modified for use as a basis for fractional models of language competition and bilingualism (e.g., [16]).

Utilizing ideas from Baggs and Freedman's model, the model of Mira and Paredes, and Colucci et al., we propose a new model that is simple yet captures characteristics that are commonly exhibited in interconnected communities. Utilizing globalization thinking we create a three dimensional system that encompasses language status, immigration, interaction between language groups, and ultimately coexistence of language diversity.

2. The model equations

Incorporating elements from the literature, we engineer a differential system that grasps coexistence yet is simple. We acknowledge the linguistic interplay in a diverse community and express this in our model.

Our system consists of three distinct language groups. Let $X_1(t)$ represent a dominant monolingual population as a function of time t. Let $B(t)$ represent the bilingual population as a function of t. Let $X_2(t)$ represent an underrepresented monolingual population as a function of t. We will denote the time-varying total population size by $N(t) = X_1(t) + B(t) + X_2(t)$.

Note that $x_1(t) = X_1(t)/N(t)$, $b(t) = B(t)/N(t)$, and $x_2(t) = X_2(t)/N(t)$ give the dominant monolingual, bilingual, and underrepresented monolingual population proportions, respectively, with $x_1 + b + x_2 = 1$. The intercommunication between these three language groups is depicted

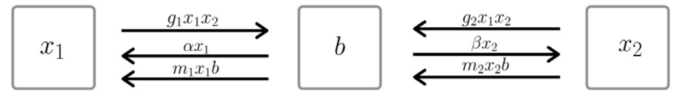


Fig. 1. Compartment diagram for system (1).

in our model. Our model includes elements such as immigration and language status. We incorporate immigration because in real-world communities, that is how a foreign language is typically introduced. Language status is taken into consideration because we want to distinguish between dominant and underrepresented language groups.

Let us now consider the model:

$$\begin{aligned}
\dot{x}_1 &= m_1x_1b - g_1x_1x_2 + \alpha x_1, \\
\dot{b} &= -m_1x_1b + m_2x_2b + g_1x_1x_2 + g_2x_1x_2 - \alpha x_1 - \beta x_2, \\
\dot{x}_2 &= -m_2x_2b - g_2x_1x_2 + \beta x_2.
\end{aligned} \tag{1}$$

The derivatives $\dot{x_1}$, $\dot{b}$, $\dot{x_2}$ are with respect to time. The term $m_i \geq 0$ is the "mass action" interaction parameter between the bilingual population and the monolingual population i. When looking at the differential equation for x_1, the product m_1x_1b is positive and signifies how the interaction between bilinguals and monolinguals in the dominant language group benefits the dominant language group and harms the bilingual group. In the differential equation for x_2, the product $-m_2x_2b$ is negative and signifies how the interaction between bilinguals and monolinguals in the underrepresented language group is a disadvantage for the underrepresented monolingual language group and benefits the bilingual group. The parameter $g_i \geq 0$ characterizes the socializing between both monolingual populations, benefiting the bilingual group at potentially distinct rates for the two monolingual populations. The language status parameter $\alpha \geq 0$ represents the influence that the dominant monolingual language group (represented by x_1) has in the community. This language group can be seen as representing the language that was mainly spoken prior to cultural diversity. The parameter $\beta \geq 0$ symbolizes the immigration rate parameter of individuals that speak a foreign language into the community. We have modeled immigration at a rate proportional to the current underrepresented population proportion, reflecting the idea that as the underrepresented population proportion grows, this will spur additional immigration. This does mean, should the underrepresented population proportion approach zero, that the immigration rate also will approach zero. Future work could model immigration at a constant rate. In the differential equation for b, the product $-\beta x_2$ is negative and signifies how as immigration increases in a community, it negatively affects the bilingual population proportion but benefits the underrepresented monolingual language group. Then, through the mass action terms, this effect negatively impacts the dominant monolingual proportion as well. In our model, the status parameter α only causes the dominant language population proportion to increase at the cost of the bilingual population proportion. Further work could include the status parameter, α, influencing the underrepresented monolingual group to shift toward the bilingual group. In Fig. 1, we present the compartment diagram corresponding to system (1).

We reduce our system to two equations by substituting $b = 1 - x_1 - x_2$ into system (1) and simplifying to obtain

$$\dot{x}_1 = -m_1x_1^2 - m_1x_1x_2 + m_1x_1 - g_1x_1x_2 + \alpha x_1, \tag{2}$$

$$\dot{x}_2 = m_2x_2^2 + m_2x_1x_2 - m_2x_2 - g_2x_1x_2 + \beta x_2. \tag{3}$$

We are interested in exploring language persistence in the model. Mathematically, all three language groups exist when $x_1 + x_2 < 1$, with $x_1 > 0$ and $x_2 > 0$ (resulting in $b > 0$). This region is shown in Fig. 2. At the point $(1, 0)$, the only language that is present is the dominant monolingual language. At the point $(0, 1)$, we have the situation in which individuals only speak the underrepresented language. All the points that lie on the line segment $x_1 + x_2 = 1$, $x_1 \geq 0$, $x_2 \geq 0$, represent the

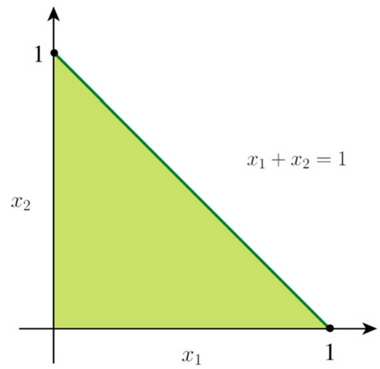


Fig. 2. Phase plane for the monolingual language groups x_1 and x_2. At points in the interior of the triangle, all three language groups have positive population proportions.

scenarios where both monolingual language groups are present and the bilingual language group is absent. We note for later reference that the region shown in Fig. 2 is not positively invariant for our model. Although trajectories do not cross $x_i = 0$, $i = 1, 2$, trajectories can cross $b = 0$. An objective of our analysis is to demonstrate the potential for a stable equilibrium point for which all three language groups persist, that is, to demonstrate that a stable equilibrium point can exist in the interior of the triangle.

3. Equilibrium population proportions

We set $\dot{x}_1$ and $\dot{x}_2$ to zero in order to find potential equilibrium population proportions. Setting $\dot{x}_1$ to zero in (2) we obtain

$$\dot{x}_1 = x_1(m_1 - m_1 x_1 - m_1 x_2 - g_1 x_2 + \alpha) = 0$$

so

$$x_1 = 0 \quad \text{or} \quad x_1 = \frac{\alpha - g_1 x_2}{m_1} - x_2 + 1. \tag{4}$$

Setting $\dot{x}_2$ to zero in (3) we obtain

$$\dot{x}_2 = x_2(m_2 x_2 + m_2 x_1 - g_2 x_1 - m_2 + \beta) = 0$$

so

$$x_2 = 0 \quad \text{or} \quad x_2 = \frac{g_2 x_1 - \beta}{m_2} - x_1 + 1. \tag{5}$$

Thus there are potentially four equilibrium points.

3.1. Equilibrium E_0 with no monolinguals: $x_1 = 0$, $x_2 = 0$

We always have an equilibrium point E_0 at $x_1 = 0$, $x_2 = 0$, $b = 1$, corresponding to the absence of monolingual individuals. Although this model includes immigration of underrepresented monolinguals, the immigration rate is proportional to the number of underrepresented monolinguals already present. Thus if all individuals are bilingual (system at equilibrium E_0) then all individuals will remain bilingual.

3.2. Equilibrium E_1 with no dominant monolinguals: $x_1 = 0$

Setting $x_1 = 0$ in (4) and (5), we obtain $x_2 = 1 - \beta/m_2$. This equilibrium point is only real-world meaningful if $\beta \le m_2$. The parameter β symbolizes the immigration rate parameter of individuals that speak a foreign language into the community while m_2 is the "mass action" interaction parameter between the bilingual population and the underrepresented monolingual population. By real-world meaningful we mean that the model variables take on real-world meaningful values. We note that, when this equilibrium is not real-world meaningful, the equilibrium still influences the dynamics of the system. If $\beta > m_2$ then the coordinate x_2 would be negative. In terms of all three language groups, this equilibrium is located at $x_1 = 0$, $x_2 = 1 - \beta/m_2$, $b = \beta/m_2$. With no dominant monolinguals present, as the immigration rate increases, the equilibrium proportion of bilinguals increases as the underrepresented monolinguals and the bilinguals interact. Similarly, here as the mass action parameter m_2 increases, the equilibrium proportion of underrepresented monolinguals increases.

3.3. Equilibrium E_2 with no underrepresented monolinguals: $x_2 = 0$

Setting $x_2 = 0$ in (4) and (5), we obtain $x_1 = 1 + \alpha/m_1$. This equilibrium point is only real-world meaningful if $\alpha = 0$. If $\alpha > 0$ then the coordinate x_1 is larger than 1, forcing $b < 0$. That is, if the language status parameter α is positive, thus drawing bilinguals to lose their competency in the underrepresented language, then there is no real-world meaningful equilibrium corresponding to the absence of underrepresented monolinguals. We remind the reader that even when the equilibrium is not real-world meaningful, the equilibrium impacts the dynamics of the system. For $\alpha = 0$, in terms of all three language groups, this equilibrium is located at $x_1 = 1$, $x_2 = 0$, $b = 0$, corresponding to all individuals being from the dominant monolingual language group. There is no opportunity for a sustainable bilingual population corresponding to the absence of underrepresented monolingual individuals.

Mathematically, as noted earlier, the region shown in Fig. 2 is not positively invariant for our model. Even when the equilibrium E_2 is not in the region shown in Fig. 2, it is still possible mathematically for trajectories that start inside the region to converge to this equilibrium. When a trajectory hits the boundary $x_1 + x_2 = 1$, $x_1 \in [0, 1]$, analysis of the simulation run is stopped at the corresponding time. In regard to the lack of invariance, we refer the reader to the simplest analogous case, that of the model of a falling object subject to constant acceleration due to gravity. When the object hits the ground the simulation run is stopped and a different model is applied to describe the dynamics upon impact.

3.4. Equilibrium E_3 with both monolingual groups present: $x_1 > 0$, $x_2 > 0$

Using (4) and (5) with $x_1 > 0$ and $x_2 > 0$, we obtain

$$x_1 = \frac{(\beta - m_2)g_1 + (\alpha m_2 + \beta m_1)}{g_1 g_2 - g_1 m_2 + g_2 m_1}, \qquad x_2 = \frac{(\alpha + m_1)g_2 - (\alpha m_2 + \beta m_1)}{g_1 g_2 - g_1 m_2 + g_2 m_1}.$$

The corresponding population proportion for bilinguals is

$$b = 1 - \frac{(\beta - m_2)g_1 + (\alpha + m_1)g_2}{g_1 g_2 - g_1 m_2 + g_2 m_1}.$$

These equilibrium proportions are real-world meaningful provided that the parameters yield $0 \le x_1 \le 1$, $0 \le x_2 \le 1$, and $0 \le b \le 1$. We value the preservation of language diversity and culture. Thus, we are especially interested in scenarios in which equilibrium E_3 is stable with $x_1 > 0$, $x_2 > 0$, $b > 0$. That is, we will explore scenarios in which a stable equilibrium E_3 exists in the interior of the triangle in the x_1 and x_2 phase plane in Fig. 2.

4. Stability analysis

With the intention to examine the stability of the equilibrium points, we compute the Jacobian matrix of system (2)–(3), $J(x_1, x_2)$. For a system $\dot{x}_1 = f_1(x_1, x_2)$, $\dot{x}_2 = f_2(x_1, x_2)$, the (i, j) entry $e_{i,j}$ of the Jacobian matrix is given by the partial derivative $\partial f_i/\partial x_j$. For system (2)–(3), the entries of the Jacobian matrix are

$$e_{11} = \alpha + m_1 - g_1 x_2 - 2m_1 x_1 - m_1 x_2,$$

$$e_{12} = -g_1 x_1 - m_1 x_1,$$

$$e_{21} = m_2 x_2 - g_2 x_2,$$

$$e_{22} = \beta - m_2 - g_2 x_1 + m_2 x_1 + 2m_2 x_2.$$

We proceed with classical stability analysis for the equilibrium population proportions. For each equilibrium point, we evaluate the Jacobian matrix at the point and compute the corresponding eigenvalues, leading to classification of each equilibrium point under various scenarios.

We will assume throughout the remaining analysis that all parameters are positive, with the exception of the parameter α which we assume to be non-negative.

4.1. Equilibrium E_0

For the equilibrium point $E_0(0,0)$, the Jacobian matrix is

$$J_{E_0} = \begin{bmatrix} \alpha + m_1 & 0 \\ 0 & \beta - m_2 \end{bmatrix}.$$

The resulting eigenvalues are:

$$\lambda_1 = \alpha + m_1 \qquad \text{and} \qquad \lambda_2 = \beta - m_2.$$

Under our assumptions of positive parameters (and non-negative α) the first eigenvalue is positive and the second eigenvalue can be positive or negative (assuming $\beta \neq m_2$). This means that $E_0(0,0)$ is an unstable source (if $\beta > m_2$) or a saddle (if $\beta < m_2$). Therefore, in real-world terms for this model, starting with nonzero monolingual proportions, it will never occur that over time only the bilingual population persists.

4.2. Equilibrium E_1

For the equilibrium point $E_1(0, 1-\beta/m_2)$, the Jacobian matrix is

$$J_{E_1} = \begin{bmatrix} \frac{(\alpha m_2+\beta m_1)+(\beta-m_2)g_1}{m_2} & 0 \\ \frac{(\beta-m_2)(g_2-m_2)}{m_2} & -(\beta - m_2) \end{bmatrix}.$$

The resulting eigenvalues are:

$$\lambda_1 = -(\beta - m_2) \qquad \text{and} \qquad \lambda_2 = \frac{(\alpha m_2 + \beta m_1) + (\beta - m_2)g_1}{m_2}.$$

As stated previously, in order for this equilibrium point to be real-world meaningful, we need the restriction $\beta \leq m_2$. Assuming that $\beta < m_2$, the first eigenvalue is positive. The second eigenvalue can be positive or negative (or zero). Assuming the second eigenvalue is not zero, this means that $E_1(0, 1-\beta/m_2)$ when it is real-world meaningful is either an unstable source (if both eigenvalues are positive) or a saddle (if the second eigenvalue is negative). Mathematically, if $\beta > m_2$, then E_1 is a saddle as well. Also note that if E_0 is an unstable source, then E_1 is not real-world meaningful. If E_0 is a saddle, then E_1 is real-world meaningful and unstable. Therefore, in real-world terms for this model, starting with nonzero monolingual proportions, it will never occur that over time only the underrepresented monolingual population persists.

4.3. Equilibrium E_2

For the equilibrium point $E_2(1+\alpha/m_1, 0)$, the Jacobian matrix is

$$J_{E_2} = \begin{bmatrix} -(\alpha + m_1) & \frac{-(\alpha+m_1)(g_1+m_1)}{m_1} \\ 0 & \frac{-(\alpha+m_1)g_2+(\alpha m_2+\beta m_1)}{m_1} \end{bmatrix}.$$

The resulting eigenvalues are:

$$\lambda_1 = -(\alpha + m_1) \qquad \text{and} \qquad \lambda_2 = \frac{-(\alpha + m_1)g_2 + (\alpha m_2 + \beta m_1)}{m_1}.$$

Recall that in order for this equilibrium point to be real-world meaningful, we need the restriction $\alpha = 0$. We have assumed that $m_1 > 0$. The first eigenvalue is negative. Since we have the restriction $\alpha = 0$, we can re-write the second eigenvalue. The second eigenvalue simplifies to $\lambda_2 = \beta - g_2$, and its sign is dependent on the values of the parameters. If $\beta > g_2$ then $E_2(1,0)$ is a saddle. If $\beta < g_2$, then $E_2(1,0)$ is a stable sink. In real-world terms, for $\alpha = 0$, if $\beta < g_2$ then in the long run population proportions starting close enough to this equilibrium will tend towards this equilibrium in which only the dominant monolingual group persists and the other two language groups go extinct.

We summarize the stability results so far in Theorem 1.

Theorem 1. *Consider system (1) under the given conditions of non-negative parameters. (i.) The equilibrium $E_0(0,0)$ is unstable. (ii.) The equilibrium $E_1(0, 1-\beta/m_2)$ is unstable when real-world meaningful. (iii.) The equilibrium $E_2(1+\alpha/m_1, 0)$ is stable when real-world meaningful if and only if $\beta < g_2$.*

We now turn our attention to the equilibrium E_3, the only potential equilibrium for which all three language groups are present.

4.4. Equilibrium E_3

For the equilibrium point

$$E_3\left(\frac{(\beta - m_2)g_1 + (\alpha m_2 + \beta m_1)}{g_1 g_2 - g_1 m_2 + g_2 m_1}, \frac{(\alpha + m_1)g_2 - (\alpha m_2 + \beta m_1)}{g_1 g_2 - g_1 m_2 + g_2 m_1}\right),$$

the Jacobian matrix is

$$J_{E_3} = \begin{bmatrix} -m_1\left(\frac{(\beta-m_2)g_1+(\alpha m_2+\beta m_1)}{g_1g_2-g_1m_2+g_2m_1}\right) & -(g_1+m_1)\left(\frac{(\beta-m_2)g_1+(\alpha m_2+\beta m_1)}{g_1g_2-g_1m_2+g_2m_1}\right) \\ -(g_2-m_2)\left(\frac{(\alpha+m_1)g_2-(\alpha m_2+\beta m_1)}{g_1g_2-g_1m_2+g_2m_1}\right) & m_2\left(\frac{(\alpha+m_1)g_2-(\alpha m_2+\beta m_1)}{g_1g_2-g_1m_2+g_2m_1}\right) \end{bmatrix}.$$

We will now show that if E_3 is real-world meaningful with all three language groups present and is stable, then E_2 is not stable. That is, we will show that for such real-world situations bi-stability does not occur. Suppose that E_3 is real-world meaningful with all three language groups present and is stable, with coordinates (x_1^*, x_2^*). Note that J_{E_3} can be rewritten as

$$J_{E_3} = \begin{bmatrix} -m_1 x_1^* & -(g_1 + m_1)x_1^* \\ -(g_2 - m_2)x_2^* & m_2 x_2^* \end{bmatrix}.$$

Since E_3 is stable, we have that

$$\begin{aligned} \det(J_{E_3}) &= -m_1 m_2 x_1^* x_2^* - (g_1 + m_1)(g_2 - m_2)x_1^* x_2^* \\ &= -(g_1 g_2 - g_1 m_2 + g_2 m_1)x_1^* x_2^* \\ &> 0. \end{aligned}$$

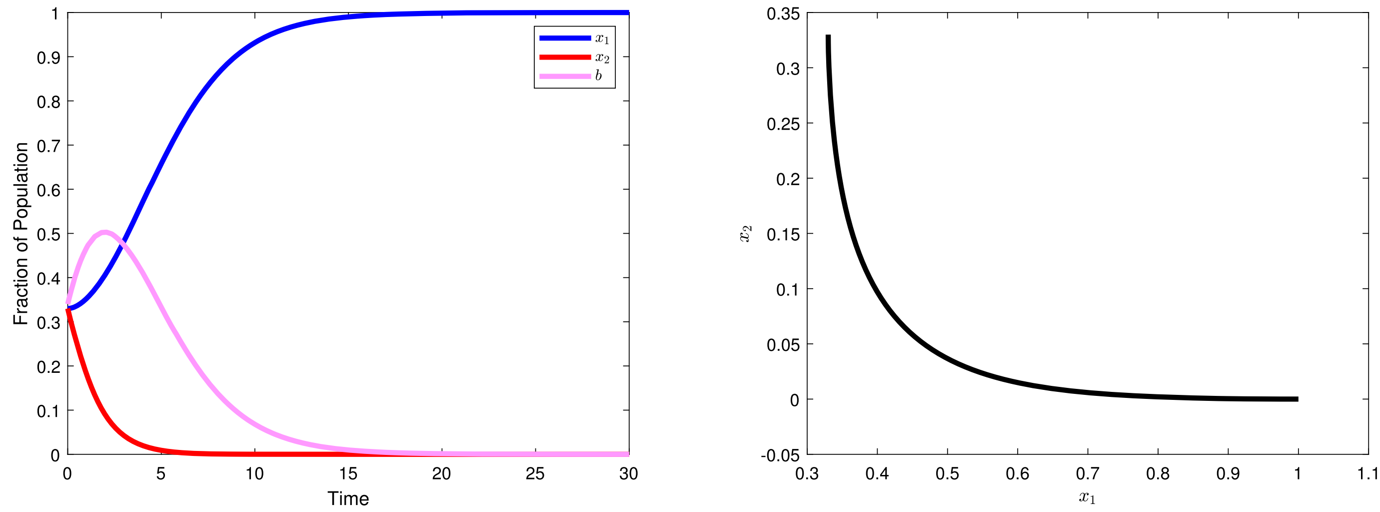
Since E_3 is real-world meaningful with all three population groups present, we have that $x_1^* > 0$ and $x_2^* > 0$. Thus, $g_1 g_2 - g_1 m_2 + g_2 m_1 < 0$. Since $x_2^* > 0$, we then have that $(\alpha + m_1)g_2 - (\alpha m_2 + \beta m_1) < 0$. But then λ_2, the second eigenvalue for E_2, is positive, and therefore E_2 is not stable in this scenario. Thus, for meaningful real-world situations, bi-stability does not occur. Similarly, assuming $m_1 > 0$, if E_3 is real-world meaningful with all three language groups present and is a saddle, then E_2 is a stable sink. We summarize these results in Theorem 2.

Theorem 2. *Consider system (1) under the given conditions of non-negative parameters, with $m_1 > 0$. Suppose equilibrium $E_3(x_1^*, x_2^*)$ is real-world meaningful. Then $E_3(x_1^*, x_2^*)$ is stable if and only if $E_2(1+\alpha/m_1, 0)$ is unstable.*

That is, if $E_3(x_1^*, x_2^*)$ is real-world meaningful and stable, then it is the only stable equilibrium for the system. We note also that

$$\mathrm{Tr}(J_{E_3}) = m_2 x_2^* - m_1 x_1^*$$

and remind the reader that E_3 is a stable sink if and only if $\mathrm{Tr}(J_{E_3}) < 0$, $\det(J_{E_3}) > 0$, and $(\mathrm{Tr}(J_{E_3}))^2 - 4\det(J_{E_3}) > 0$, and E_3 is a stable spiral if and only if $\mathrm{Tr}(J_{E_3}) < 0$ and $(\mathrm{Tr}(J_{E_3}))^2 - 4\det(J_{E_3}) < 0$.



(a) Solution curves as a function of time.

(b) Trajectory in the phase plane.

Fig. 3. Only the dominant language group x_1 persists as the population proportions converge to the equilibrium point E_2. Solution curves as a function of time are shown in (a). The corresponding trajectory in the phase plane is shown in (b).

Using MATLAB, we performed a six-dimensional grid search for sets of parameter values producing eigenvalue pairs with negative real parts, thus resulting in E_3 being stable. In Section 5, we will introduce examples of parameter sets in which the population proportions head in the long run towards equilibrium E_3 as a stable sink (both eigenvalues are negative), as well as examples of parameter sets in which the population proportions head in the long run towards equilibrium E_3 as a stable spiral (eigenvalues are given by a complex conjugate pair with negative real part). In the stable spiral behavior, the population proportions oscillate with decaying amplitude about the equilibrium, converging towards the equilibrium proportions.

From the analysis of this section, if population proportions converge toward an equilibrium, then only two outcomes are possible in the long run: Either only the dominant language group persists or all three language groups persist.

The coexistence of all three language groups is very important because of the application it has to real-world communities. We are especially interested in the stable spiral behavior, since we have not previously seen this behavior in the literature on mathematical modeling of language persistence.

5. Language preservation

Modeling language preservation is important for diverse communities because it proves that a foreign language should not be seen as a threat to the dominant language. The introduction of a secondary language in a community has various benefits especially if it is introduced to children at a young age. Developing children that grow up in a bilingual environment gain a more fruitful understanding of things and obtain a solid foundation of cognitive development [17]. The inclusion of a foreign language strengthens diverse communities which is why we are so adamant about proposing a coexisting environment.

In this section we will look at different instances for system (2)–(3) in which one or more of the language groups is preserved through convergence to an equilibrium. We will analyze the general question of coexistence depending on various sets of parameter values m_i, g_i, α, and β. The time interval varies from example to example because of how the specific scenario behaves. For oscillatory graphs the time interval is increased to see the behavior clearly. All numerical solutions for the systems of differential equations are obtained using MATLAB.

For each example, in the left-hand plot we show solution curves as a function of time for the three language groups. In the right-hand plot, we show trajectories in the x_1 and x_2 phase plane. Example 3 and Example 4 have complex conjugate eigenvalues with negative real part, leading to a stable spiral. For all of the simulations, time t is in years.

5.1. Only the dominant language group persists

In Example 1, we will look at a case in which the only language group that persists is the dominant monolingual. The parameters for this model are given as follows (Table 1):

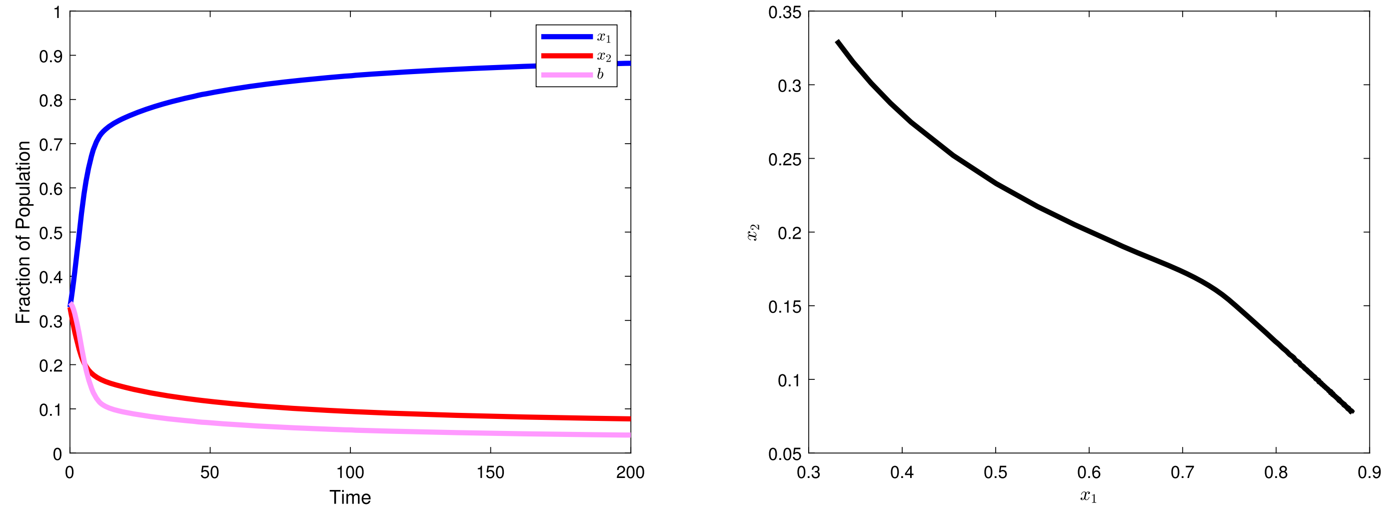
Table 1. Parameters that allow x_1 to persist.

Parameters for Example 1					
Parameter	Value	Parameter	Value	Initial Cond.	Value
m_1	.4	g_2	.8	$x_1(0)$	.33
m_2	.8	α	0	$x_2(0)$	.33
g_1	.4	β	0	$b(0)$	.34

In this example we have the parameters α and β set to zero. This means that there are no language status effects and no immigration effects. Using the values in the table above for system (2)–(3) we obtain the left graph below for population proportion solution curves as functions of time, along with the right graph showing the corresponding trajectory converging to $(1, 0)$ in the x_1 and x_2 phase plane. Equilibria E_0 and E_1 are saddle equilibria; equilibrium E_3 is an unstable source and is not real-world meaningful as it is located outside the region in Fig. 2. Equilibrium E_2 is a stable sink (Fig. 3).

At first the bilingual and dominant monolingual language groups coexist but the relationship is unsustainable with the given parameters. Unfortunately over time the population only speaks one language, the dominant language.

This scenario brings up a very important phenomenon in the real world, language maintenance in children. This synopsis saddens us because it is more common than we would like it to be. A situation like this can occur when toddlers are bilingual and remain bilingual until they start going to school. At around age 5, they start to lose grasp of the underrepresented language. The parents of a bilingual child make all the difference, their influence and consistency when communicating with them can allow the child to continue to be fluent in both languages. Unfortunately, many parents start to enforce the dominant monolingual language at home and before they know it the child only speaks the dominant language and completely forgets the underrepresented one.



(a) Solution curves as a function of time.

(b) Trajectory in the phase plane.

Fig. 4. All language groups persist. The solution curves shown in (a) and phase plane shown in (b) correspond to the two negative eigenvalues, as population proportions converge to the equilibrium point E_3 without cycling.

This scenario is so disheartening because the child had the resources and potential to grow up bilingual but was stripped from that opportunity. A big goal and why we emphasize the importance of coexistence is to show to the world that people with different cultural backgrounds can coincide and create a more diverse community.

The following examples will have coexistence of all three language groups but in different ways.

5.2. All language groups persist (no cycling)

In Example 2, we will look at a case in which, for initial conditions close enough to the equilibrium E_3, all language groups coexist but the dominant monolingual language group makes up most of the population. The parameters for this model are given as follows (Table 2):

Table 2. Parameters that allow all language groups to persist.

Parameters for Example 2					
Parameter	Value	Parameter	Value	Initial Cond.	Value
m_1	.9	g_2	.3	$x_1(0)$	.33
m_2	.9	α	.01	$x_2(0)$	.33
g_1	.6	β	.3	$b(0)$	.34

In contrast to Example 1, the parameters α and β are no longer zero which means this structure takes into account language status and immigration. The terms m_1, m_2, and g_1 all increased while g_2 is reduced by over 50% in comparison to Example 1. Here, equilibria E_0, E_1, and E_2 are saddle equilibria. Equilibrium E_3 is a stable sink, and population proportions starting close enough to it approach it without oscillatory behavior (Fig. 4). This can be seen when plotting trajectories in the phase plane.

As you can see here, we have a model that can exhibit a stable interior equilibrium that is a sink. To the left we have the graph of population proportion solution curves as a function of time, where we exhibit coexistence. However, the dominant monolingual language group makes up 90% of the total population. The other two language groups never converge to zero but the population proportions for both of these groups get very small. Bilinguals make up 3.$\bar{3}$% of the total population while the underrepresented monolingual language group makes up 6.$\bar{6}$%. For initial conditions close enough to the equilibrium, the phase plane in this example simply shows convergence to the equilibrium, with no cycling.

This community resembles a very small rustic town in Southern California in which the Hispanic population makes up a very small percentage of the total population. This community is not diverse in its politics or its dialect. The lack of dialect variety is often seen in small towns like this, which is unfortunate because the educational institutions mirror this in terms of course offerings.

In the next examples we show that our system can exhibit a stable spiral.

5.3. All language groups persist (with cycling)

In Example 3, we will look at a case in which, for initial conditions close enough to the equilibrium E_3, all language groups coexist and the proportional difference between language groups is a lot smaller than it was in Example 2. The parameters for this model are given as follows (Table 3):

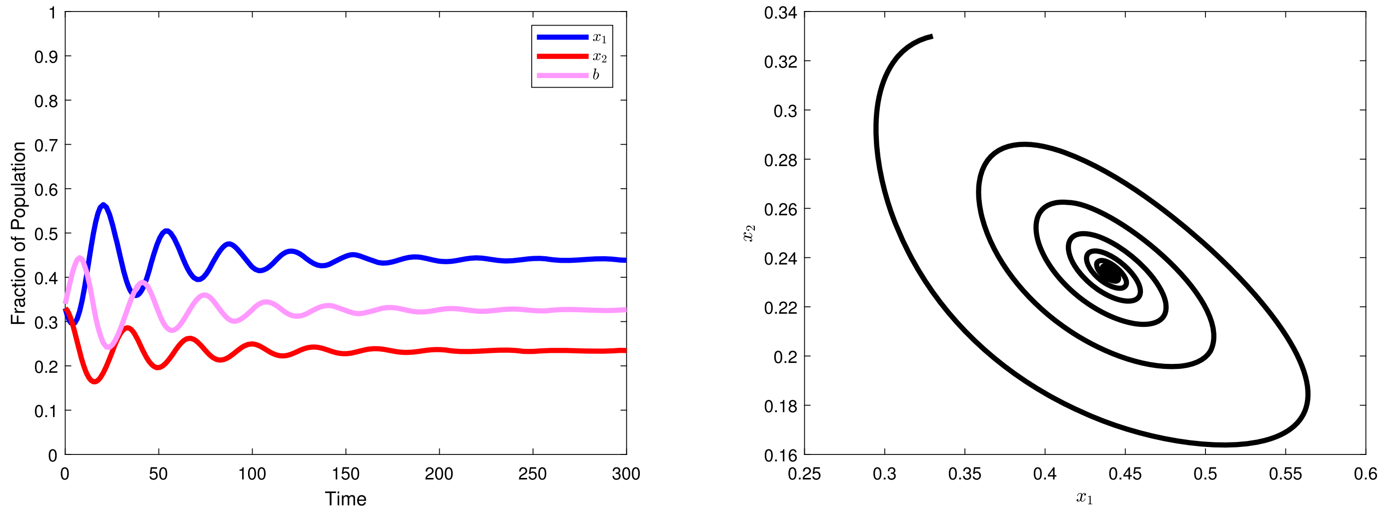
Table 3. Parameters that allow all language groups to persist.

Parameters for Example 3					
Parameter	Value	Parameter	Value	Initial Cond.	Value
m_1	.4	g_2	.01	$x_1(0)$	.33
m_2	.6	α	.01	$x_2(0)$	.33
g_1	.6	β	.2	$b(0)$	.34

The parameters that are different from Example 2 are m_1, m_2, g_2, and β. All of the values for these parameters are decreased in comparison to Example 2. The parameter α is very small. This means that language status for x_1 is relatively low and barely has impact on the outcomes. The impact of interaction between bilinguals and both monolingual populations is very high. Equilibria E_0, E_1, and E_2 are saddle equilibria; equilibrium E_3 is a stable spiral as shown in the phase plane (Fig. 5).

In this example we have all language groups persisting. The dominant monolingual language group makes up 44% of the total population. Bilinguals make up 32.6% while the underrepresented monolingual language group makes up 23.4%.

We find this synopsis to be an accurate representation of a city like Los Angeles where most people only speak English but there are a lot of individuals that are bilingual and immigrants that only speak their foreign language. The graph of population proportion solution curves shows the language population proportions oscillating up and down but



(a) Solution curves as a function of time. (b) Trajectory in the phase plane.

Fig. 5. All language groups persist. Eigenvalues are a complex conjugate pair with negative real part, as population proportions converge to the equilibrium point E_3 with cycling. Solution curves as a function of time are shown in (a). The corresponding trajectory in the phase plane is shown in (b).

decaying in amplitude. For initial conditions close enough to the equilibrium, the phase plane shows a stable spiral. The equilibrium levels in the first plot correspond to the equilibrium point which is in the center of the spiral in the second plot.

5.4. All language groups persist (with cycling and prominent bilinguals)

In Example 4, we will look at a case in which, for initial conditions close enough to the equilibrium E_3, all language groups coexist and the bilingual population is the most prominent. The parameters for this model are given as follows (Table 4):

Table 4. Parameters that allow all language groups to persist.

Parameters for Example 4					
Parameter	Value	Parameter	Value	Initial Cond.	Value
m_1	.2	g_2	.01	$x_1(0)$	.33
m_2	.4	α	.01	$x_2(0)$	.33
g_1	.8	β	.2	$b(0)$	.34

Parameters m_1, m_2, and g_1 are different from Example 3. We decreased the interaction between bilinguals and both monolingual language groups. As in Example 3, equilibria E_0, E_1, and E_2 are saddle equilibria; equilibrium E_3 is a stable spiral as shown in the phase plane (Fig. 6).

In this example we have all language groups persisting but the bilingual group is the presiding language group. The dominant monolingual language group makes up 37.4% of the total population. Bilinguals make up 49.1% while the underrepresented monolingual language group makes up 13.5%. The bilingual population makes up almost half of the total population which is fantastic because it means that more people in the community can speak two languages rather than just one.

As in the previous example, the graph to the left shows the language population proportions oscillating and decaying in amplitude. The oscillations in this example are dramatic but eventually stabilize. When we graph the phase plane of x_1 and x_2, we obtain a counterclockwise spiral into the equilibrium and decaying in amplitude. The equilibrium levels in the first plot correspond to the equilibrium point which is in the center of the spiral in the second plot.

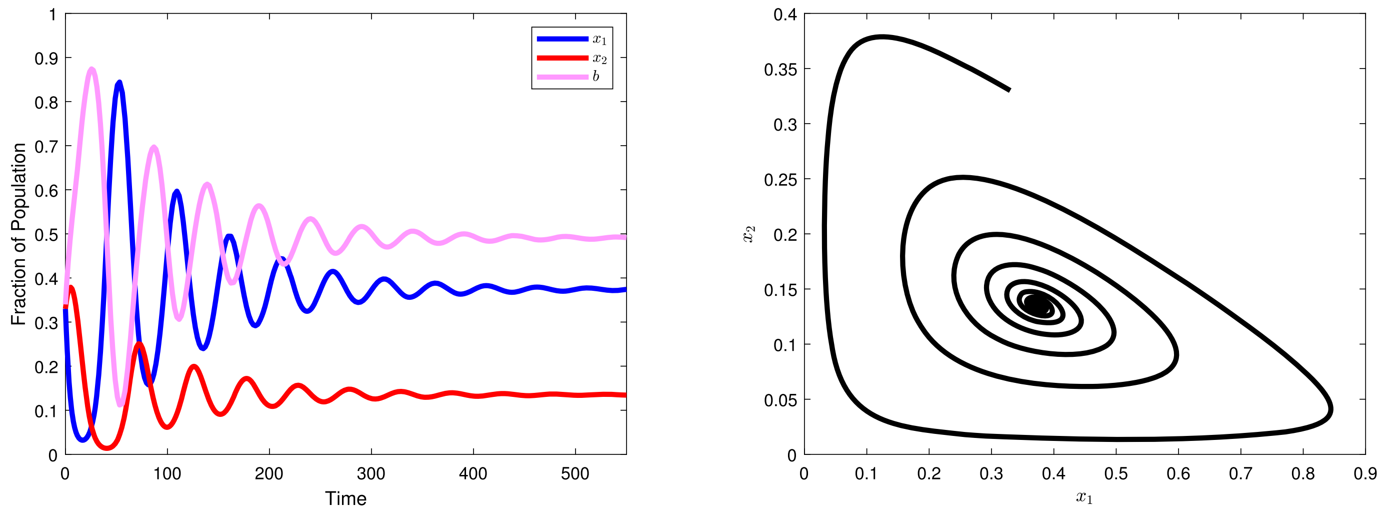
A population with these characteristics is possible with the help of the education system. If schools place a focus on accurately teaching more than one language, we can achieve an environment in which students are fluent in two languages. There are special schools in the United States where language dualism is emphasized and learning in two languages is effective.

6. Discussion

Language preservation upholds cultural preservation. Unfortunately in the United States, cultural assimilation causes many immigrant families to lose linguistic ties to their home countries in a couple generations. Experts approximate that more than half the world's population can speak at least two languages yet according to the U.S. Census only 20 percent in America can [18, 19]. When a language stops being spoken, it is at risk of becoming a dead language [4]. When it comes to contemplating the introduction of a second language, one fallacy is that it may confuse a child during the early learning phase [1]. It is understandable that parents would have these concerns, but researchers have proved that they are not only erroneous, but the exact opposite is true: bilingual children learn better and quicker than monolingual children [17]. We hope educators, parents, and community members understand the importance and benefits of supporting a multilingual agenda. In order to bring awareness to this issue and prevent further generations from losing ties to their home country, we have successfully modeled language preservation. We were able to show that all language groups can coexist, that is, that the extinction of bilingual communities is not inevitable.

We have introduced a new model for language competition in which we model the social interactions amongst bilinguals and monolinguals. In the presence of immigration, language status, and continuous interaction we found that a harmonious community where all three language groups thrive is possible. However, by eliminating immigration and language status along with adjusting the interaction terms, we also exhibited a case in which only the dominant monolingual language group persisted. This situation is unfortunate but likely possible if society keeps forcing foreigners to fully assimilate into the dominant culture. Our model successfully shows that coexistence is possible.

There are many factors present in real communities that we have not considered in our model. Our conclusion is only valid under the assumptions that were introduced in the model formulation. Further work could involve refining our model so that it can capture the real-world idea of multiple languages persisting using real data. Real data will help shape simulation and strengthen our arguments. Lastly, further work could also include incorporating a third language.



(a) Solution curves as a function of time. (b) Trajectory in the phase plane.

Fig. 6. All language groups persist. Eigenvalues are a complex conjugate pair with negative real part, as population proportions converge to the equilibrium point E_3 with cycling and prominent bilinguals. Solution curves as a function of time are shown in (a). The corresponding trajectory in the phase plane is shown in (b).

La preservación del idioma defiende la preservación cultural. Desafortunadamente, en los Estados Unidos, la asimilación cultural hace que muchas familias inmigrantes pierdan lazos lingüísticos con sus países de origen en un par de generaciones. Los expertos calculan que más de la mitad de la población mundial puede hablar al menos dos idiomas; sin embargo, según el censo de los Estados Unidos, solo el 20 por ciento en los Estados Unidos puede [18, 19]. Cuando un idioma deja de hablarse, está en riesgo de convertirse en una lengua muerta [4]. Cuando se trata de contemplar la introducción de un segundo idioma, una falacia es que puede confundir al niño durante la fase de aprendizaje temprano [1]. Es comprensible que los padres tengan estas preocupaciones, pero los investigadores han demostrado que no solo son erróneas, sino que ocurre exactamente lo contrario: los niños bilingües aprenden mejor y más rápido que los niños monolingües [17]. Esperamos que los educadores, padres y miembros de la comunidad comprendan la importancia y los beneficios de apoyar una agenda multilingüe. Para crear conciencia sobre este problema y evitar que las generaciones futuras pierdan los lazos con su país de origen, hemos modelado con éxito la preservación del idioma. Pudimos demostrar que todos los grupos lingüísticos pueden coexistir, es decir, que la extinción de las comunidades bilingües no es inevitable.

Hemos introducido un nuevo modelo de competencia lingüística en el que modelamos las interacciones sociales entre bilingües y monolingües. En presencia de inmigración, estatus lingüístico e interacción continua, encontramos que es posible una comunidad armoniosa donde prosperen los tres grupos lingüísticos. Sin embargo, al eliminar la inmigración y el estatus lingüístico junto con el ajuste de los términos de interacción, también exhibimos un caso en el que solo persistió el grupo lingüístico monolingüe dominante. Esta situación es lamentable, pero probablemente posible si la sociedad sigue obligando a los extranjeros a asimilarse por completo a la cultura dominante. Nuestro modelo muestra con éxito que la coexistencia es posible.

Hay muchos factores presentes en comunidades reales que no hemos considerado en nuestro modelo. Nuestra conclusión solo es válida bajo las suposiciones que se introdujeron en la formulación del modelo. El trabajo adicional podría implicar refinar nuestro modelo para que pueda capturar la idea del mundo real de que varios lenguajes persisten utilizando datos reales. Los datos reales ayudarán a dar forma a la simulación y fortalecer nuestros argumentos. Por último, el trabajo adicional también podría incluir la incorporación de un tercer idioma.

Author contribution statement

M. Díaz, J. Switkes: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

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The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

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