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Effect of selection bias on two sample summary data based Mendelian randomization

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Mendelian randomization (MR) is becoming more and more popular for inferring causal relationship between an exposure and a trait. Typically, instrument SNPs are selected from an exposure GWAS based on their summary statistics and the same summary statistics on the selected SNPs are used for subsequent analyses. However, this practice suffers from selection bias and can invalidate MR methods, as showcased via two popular methods: the summary data-based MR (SMR) method and the two-sample MR Steiger method. The SMR method is conservative while the MR Steiger method can be either conservative or liberal. A simple and yet more powerful alternative to SMR is proposed.

As a feasible alternative to expensive and sometimes impossible randomized clinical trials, Mendelian randomization (MR) is becoming more and more popular for inferring causal relationship between an exposure and a trait^{1–3}. Summary data-based two-sample MR methods often take the following two steps:

- Step 1 Obtain instruments (typically SNPs) from exposure GWAS (Genome-Wide Association Study) that are significant at genome-wide level (typically $p < 5 \times 10^{-8}$);
- Step 2 Investigate the causal relationship between the exposure and the trait, using the summary exposure GWAS statistics at the selected SNPs and a trait GWAS. The summary exposure GWAS statistics are those used in Step 1 for SNP selection.

One appealing feature of these methods is that they only rely on summary statistics on the exposure GWAS and the trait GWAS. Individual-level data are not needed.

The inference validity of this two-step approach is affected by selection bias. When conducting causal inference in Step 2 with respect to the SNPs selected in Step 1, the summary statistics from the exposure GWAS can not be regarded as random samples for the true population association strength^{4–6}. Treating them as random samples leads to over-estimation of the effect size of these SNPs on the exposure. Association strength in a random sample is often much weaker, a phenomenon commonly seen in studies aimed at replicating previous findings. This selection bias has been noted in the literature^{6,7}. But its effect on hypothesis testing related to two sample summary data based Mendelian randomization is largely unknown.

Two popular MR methods, the summary data-based MR method² and the two-sample MR Steiger method¹, are considered. For the summary data-based MR method, the most significant SNP (instead of several SNPs) from a gene is selected as the instrument from the exposure GWAS. For the two-sample MR Steiger method, a SNP significantly associated with both the exposure GWAS and the trait GWAS is selected. The genotype score (0, 1, or 2) at this SNP is denoted by g . The exposure level is denoted by x and the trait value is denoted by y . The Wald statistic on chi-square scale for testing the association between the SNP and the exposure is denoted by W_{gx} . Its value is supposed to be large because it satisfies the selection criterion used in Step 1. For instance, when the selection criterion is $p < 5 \times 10^{-8}$, there must be $W_{gx} > 29.71679$. The Wald statistic for testing the association between the SNP and the trait is denoted by W_{gy} , which is independent of W_{gx} .

Results

Summary data-based MR. Summary data-based MR² (SMR) is a popular MR method for inferring causality between x and y . Its null is $H_0 : b_{xy} = 0$, where b_{xy} is the true regression coefficient for x with y the response. The two-stage least square (2SLS) estimate of b_{xy} is

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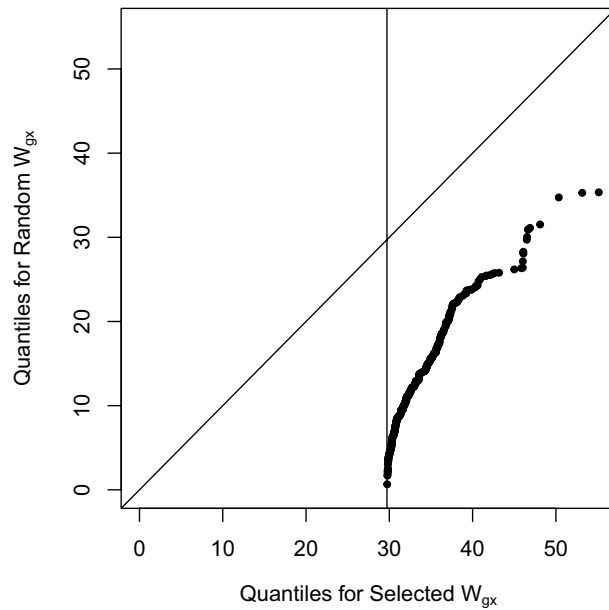


Figure 1. Quantile-quantile plot for selected $\{W_{gx}\}$ (322 out of 10,000) and 322 random $\{W_{gy}\}$. The distribution of selected $\{W_{gx}\}$ is different from the distribution of random $\{W_{gy}\}$ as shown by the deviation of the points from the 45° line. The vertical line indicates the selection threshold $W_{gx} \geq 29.71679$ which corresponds to genome-wide significance level 5×10^{-8} .

$$\hat{b}_{xy} = \frac{\hat{b}_{gy}}{\hat{b}_{gx}}, \quad (1)$$

where \hat{b}_{gx} is the least square estimate of b_{gx} , the regression coefficient for g with x the response, and \hat{b}_{gy} is the least square estimate of b_{gy} , the regression coefficient for g with y the response. \hat{b}_{xy} is also known as the Wald ratio⁵. Causal relationship between exposure x and y exists if the following test statistic is significant²:

$$T_{SMR} = \frac{W_{gx}W_{gy}}{W_{gx} + W_{gy}},$$

where $W_{gx} = [\hat{b}_{gx}/SE(\hat{b}_{gx})]^2$ and $W_{gy} = [\hat{b}_{gy}/SE(\hat{b}_{gy})]^2$ are Wald statistics on chi-square scale. The null distribution of T_{SMR} is approximated by 1-df chi-square using the Delta method².

There are several issues with statistic T_{SMR} . The derivation of its null distribution assumes that \hat{b}_{gx} is a consistent estimator of b_{gx} and (asymptotically) follows a normal distribution², (Online Methods). However, these two conditions do not hold. If the significance level used in Step 1 is 5×10^{-8} , there must be $W_{gx} \geq 29.71679$ which implies $|\hat{b}_{gx}| \geq \sqrt{29.71679} \times SE(\hat{b}_{gx})$. As a result, the distribution of \hat{b}_{gx} is not (asymptotically) normal and \hat{b}_{gx} is not a consistent estimator of b_{gx} . To numerically demonstrate this point, 10,000 random samples of W_{gx} are generated from a 1-df chi-square with a large non-centrality 13 (to make sure there are reasonable number of $\{W_{gx}\}$). Among them, 322 are significant at genome-wide significant level 5×10^{-8} . The quantile-quantile plot of these selected $\{W_{gx}\}$ against 322 random samples $\{W_{gy}\}$ from 1-df chi-square with non-centrality 13 is shown in Fig. 1. The distribution of $\{W_{gx} : W_{gx} \geq 29.71679\}$ is clearly different from the distribution of $\{W_{gx}\}$.

The applicability of the Delta method to approximating the distribution of T_{SMR} is in doubt even in the absence of the selection process imposed on W_{gx} . Approximating the null distribution of T_{SMR} by a 1-df chi-square is equivalent to approximating the null distribution of \hat{b}_{xy} by a normal distribution. However, according to Eq. (1), \hat{b}_{xy} is a ratio of two normals. In general, the distribution of the ratio of two normal variables can not be approximated by a normal as it can take a variety of shapes such as bimodal, unimodal, or asymmetric⁸. It is known that if b_{gx} and b_{gy} are both equal to 0, the distribution of \hat{b}_{xy} would be a Cauchy, a fat-tailed distribution whose mean and variance do not exist. For the case $b_{gx} \neq 0$ and $b_{gy} \neq 0$, the distribution of \hat{b}_{xy} can be approximated by a normal only in certain intervals⁸.

For the case $b_{gy} = 0$, to our best knowledge, there are no known theoretical results regarding whether the distribution of \hat{b}_{xy} can be approximated by a normal. The only thing we are sure about is that the distribution of \hat{b}_{xy} is symmetric because the distribution of $-\hat{b}_{xy} = (-\hat{b}_{gy})/\hat{b}_{gx}$ is the same as the distribution of \hat{b}_{xy} . A numerical example is used to examine the distribution of \hat{b}_{gx} . Ten thousand random \hat{b}_{gx} 's are generated from $N(\sqrt{13}, 1)$ and

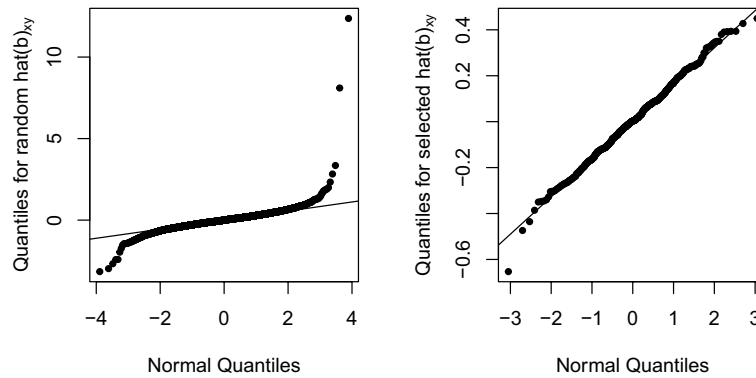


Figure 2. Normal quantile plot for 10,000 $\hat{b}_{xy} = \hat{b}_{gy}/\hat{b}_{gx}$ generated under $b_{gx} = \sqrt{13}$ and $b_{gy} = 0$. The distribution of $\{\hat{b}_{xy}\}$ appears to be fat-tailed compared to a normal (left panel). The distribution of $\{\hat{b}_{xy} : \hat{b}_{gx} \text{ is significant}\}$ (436 out of 10,000) seems to be a normal (right panel) due to selection imposed on \hat{b}_{gx} . See the text for explanation.

10,000 random \hat{b}_{gy} are generated from $N(0, 1)$. A normal quantile plot of $\hat{b}_{gy}/\hat{b}_{gx}$ is generated using the `qqnorm` and `qqline` functions in R with their default settings and is shown in Fig. 2 (left panel). Similar to a Cauchy distribution, the distribution of $\hat{b}_{xy} = \hat{b}_{gy}/\hat{b}_{gx}$ is apparently fat-tailed compared to a normal: the lower end is more negative while the upper end is more positive.

A normal quantile plot is also generated for $\{\hat{b}_{xy} : \hat{W}_{gx} \geq 29.71679\}$ and is shown in the right panel of Fig. 2. It may be surprising that the distribution of $\{\hat{b}_{xy} : \hat{W}_{gx} \geq 29.71679\}$ appears to be a normal. The reason of this phenomenon is that the range of \hat{b}_{gx} is greatly reduced under the selection criterion. According to Eq. (1), \hat{b}_{xy} is roughly proportional to \hat{b}_{gy} with high probability.

A more general argument that the approximating distribution of T_{SMR} is not 1-df chi-square is the following. Since

$$T_{SMR} = W_{gy} \cdot \frac{1}{1 + W_{gy}/W_{gx}},$$

there is $T_{SMR} < W_{gy}$ regardless of the distribution of W_{gx} . That is, T_{SMR} is always dominated by W_{gy} . Similarly, T_{SMR} is always dominated by W_{gx} . Therefore, $T_{SMR} < \min\{W_{gx}, W_{gy}\}$. Since W_{gx} and W_{gy} approximately follow independent 1-df chi-square distributions, the approximating distribution of $\min\{W_{gx}, W_{gy}\}$ can not be 1-df chi-square. Neither the approximate distribution of T_{SMR} . Using a 1-df chi-square distribution for T_{SMR} results in a conservative test.

We performed extensive simulations to investigate the null distribution of the SMR statistic in a more realistic setting by using imputed GWAS genotype data from the Atherosclerosis Risk in Communities (ARIC) study of European-ancestry samples⁹. Specifically, we simulated gene expression levels for each Ensemble gene on autosomes at varying numbers of causal eQTLs ($n = 1, 5, \text{ and } 10$), (narrow sense) heritability levels ($h^2 = 0.1, 0.2, 0.4, 0.8$), and sample sizes ($N = 250, 500, 1000, \text{ and } 2000$). We tested association between all SNPs within each gene and expression levels of the gene, and only genes whose top associated SNP met the selection criteria ($p < 5 \times 10^{-8}$) were subjected to SMR test. GWAS association signals were randomly assigned from a standard normal distribution. Figure 3 shows the QQ plot for the SMR statistics when instrumental eQTLs were selected from genes with 5 causal eQTLs and a level of heritability = 0.4 at all four sample sizes. Clearly, the SMR statistics were lower than expected null values at the tail of distribution, though the distribution became closer to the null at larger sample size, which may be explained by the stronger eQTL signals as shown in our numerical example above. The complete set of QQ plots for the SMR test statistic are shown in Supplementary Figs. S1–S12 online. Overall, our simulations showed that the SMR statistics were conservative and did not strictly follow the 1-df chi-square distribution, especially when the effect size of each individual eQTL was small on average. These results are consistent with our theoretical insights.

More on SMR and a conditional test. One may want to use an estimate of b_{gx} that takes into account the selection. However, such an estimate is not expected to be simple given the complexity of the selection (e.g., the SNP is the most significant one among a number of SNPs). Another alternative is to use another exposure GWAS independent of the exposure GWAS used in Step 1 to estimate b_{gx} and then compute T_{SMR} . However, this is not recommended because T_{SMR} is inherently conservative. T_{SMR} is equal to the half of the harmonic mean of W_{gx} and W_{gy} . Fixing one of W_{gx} and W_{gy} , say W_{gx} , and change W_{gy} , T_{SMR} reaches its smallest value $W_{gx}/2$ when $W_{gy} = W_{gx}$ and converges to W_{gx} when $W_{gy} \rightarrow \infty$. The conservativeness of T_{SMR} is also observed in simulation studies by Veturi and Ritchie¹⁰.

The null hypothesis for T_{SMR} was not specifically defined in Zhu et al.² It is unlikely to be the intended null $H_0 : b_{xy} = 0$. Actually, similar to the Sobel's statistic popular in mediation analysis, the null corresponding to

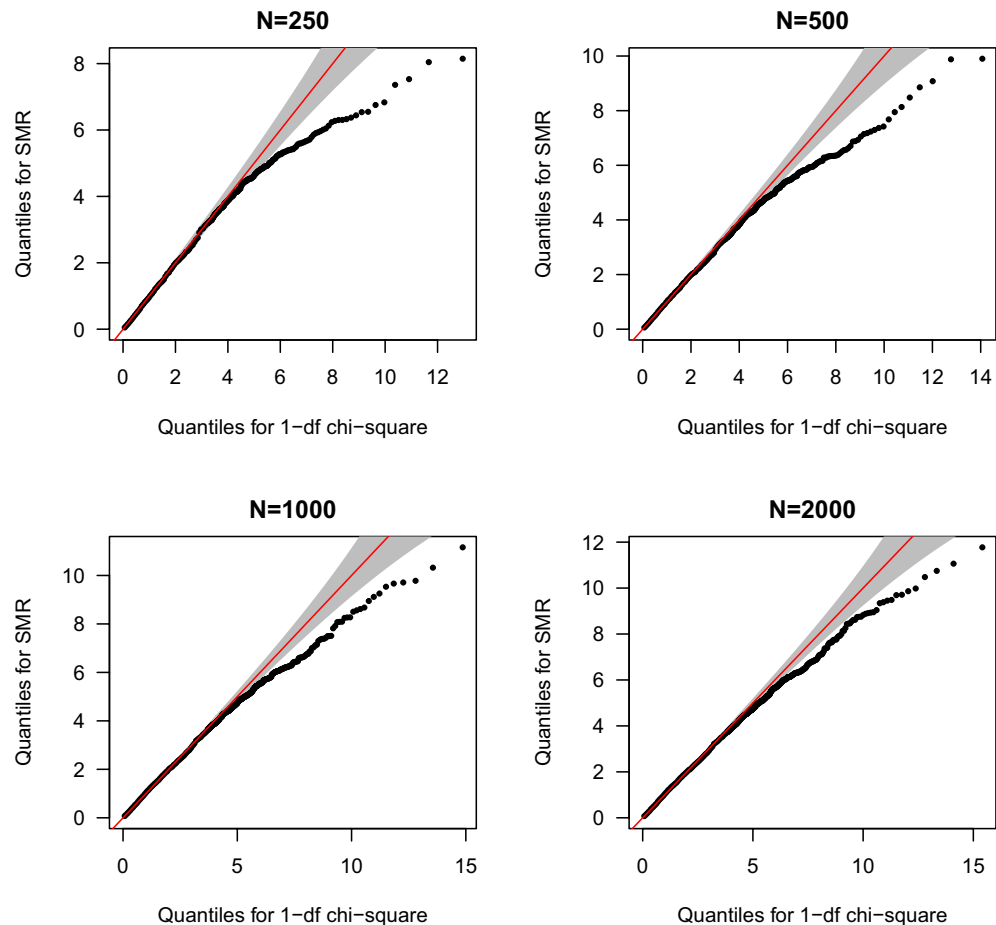


Figure 3. Quantile-quantile plot for simulated SMR statistics against statistics of 1-df chi-square distribution. Instrumental eQTLs for SMR test were top associated eQTL ($p < 5 \times 10^{-8}$) selected from genes whose expression levels were simulated under a genetic model of 5 causal eQTLs and heritability of 0.4 at four different sample sizes ($N = 250, 500, 1000,$ and 2000). The grey areas represent the 95% confidence band around 1-df chi-square statistics.

T_{SMR} is $H_0 : b_{gx} = 0$ or $b_{gy} = 0$. For this null, a statistic more powerful than T_{SMR} is $\min\{W_{gx}, W_{gy}\}$. The statistic $\min\{W_{gx}, W_{gy}\}$ rejects the null $H_0 : b_{gx} = 0$ or $b_{gy} = 0$ if and only if both W_{gx} and W_{gy} are significant. Therefore, whenever $\min\{W_{gx}, W_{gy}\}$ rejects the null, T_{SMR} will but not vice versa. This is because $T_{\text{SMR}} < \min\{W_{gx}, W_{gy}\}$.

A test more powerful than $\min\{W_{gx}, W_{gy}\}$ (hence also more powerful than T_{SMR}) in the current situation is a conditional test. Because the SNP is selected for its significant association with the exposure, the situation $b_{gx} = 0$ can be excluded. Given this information, a meaningful null would be $H_0 : b_{gy} = 0, b_{gx} \neq 0$ for which a test statistic is W_{gy} . The null is rejected when W_{gy} is significant. This test, conditional on a significant W_{gx} statistic, assumes that there is no pleiotropy. That is, the selected SNP affects the trait only through the exposure and there are no other paths. In other words, the selected SNP is a valid instrument. In light of Eq. (1), $b_{gy} = 0$ if and only if $b_{xy} = 0$ when the possibility of $b_{gx} = 0$ is excluded. Hence the null for this conditional test is equivalent to $H_0 : b_{xy} = 0$. This test is asymptotically valid because W_{gy} asymptotically follows a 1-df chi-square distribution. The threshold for significance for this test is not at the genome level. Rather, it is at the gene level and only needs to be corrected for the number of genes for which SNPs are selected for instruments. This results in a more powerful testing procedure than using a genome-wide threshold.

An empirical study. We compared the performance of conditional test we proposed and the SMR test on an empirical study of schizophrenia. We used to-date the largest GWAS summary statistics for schizophrenia¹¹ and the eQTL results from analysis of 1387 brain samples (prefrontal cortex) by the PsychENCODE¹² (downloaded from the SMR data resource website). In total, 9639 genes were tested for SMR at a top associated *cis*-eQTL ($p < 5 \times 10^{-8}$) and 65 genes were significant after Bonferroni correction. In contrast, the conditional test, whose test statistic is W_{gy} and considers only those instrumental eQTLs, discovered 127 Bonferroni-significant genes, including 62 genes not detected by SMR ($p < 0.05/9639 = 5.18726 \times 10^{-6}$, Supplementary Table S1 online). Among those genes missed by SMR, there were several strong candidates for schizophrenia, such as *AKT3*^{13–15}, *RGS6*^{16,17}, and *KCNN3*. It may not be surprising that *AKT3* and *RGS6* were identified as these two

harbored genome-wide significant variants ($p < 5 \times 10^{-8}$) in original GWAS¹¹, but the discovery of *KCNN3* was novel and the strongest SNP-level association evidence for this gene was only at $p = 9 \times 10^{-7}$ (rs10796933). Of note, our previous study also showed evidence for the association of *KCNN3* with schizophrenia through integrated analysis of GWAS with methylation QTL¹⁸.

Two-sample MR Steiger method. The two-sample MR Steiger method^{1,19} assumes that there is a causal relationship between the exposure and the trait and that the selected SNP is a valid instrument for one of them (but it is unknown for which one)¹. A SNP is selected not only for its association with the exposure but also for its association with the trait^{1,19}. The null for the two-sample MR Steiger test is $H_0 : \rho_{gx} = \rho_{gy}$ where $\rho_{gx} = \text{Corr}(g, x)$ and $\rho_{gy} = \text{Corr}(g, y)$ are the (population) Pearson correlation coefficients. Let $\hat{\rho}_{gx}$ and $\hat{\rho}_{gy}$ be the sample correlation coefficients corresponding to ρ_{gx} and ρ_{gy} , respectively. Using Fisher's Z transformation, there are

$$z_{gx} := \frac{1}{2} \ln \frac{1 + |\hat{\rho}_{gx}|}{1 - |\hat{\rho}_{gx}|} \sim N\left(\frac{1}{2} \ln \frac{1 + |\rho_{gx}|}{1 - |\rho_{gx}|}, \frac{1}{n_x - 3}\right), \text{ and} \quad (2)$$

$$z_{gy} := \frac{1}{2} \ln \frac{1 + |\hat{\rho}_{gy}|}{1 - |\hat{\rho}_{gy}|} \sim N\left(\frac{1}{2} \ln \frac{1 + |\rho_{gy}|}{1 - |\rho_{gy}|}, \frac{1}{n_y - 3}\right), \quad (3)$$

where n_x and n_y are sample sizes. The null $H_0 : \rho_{gx} = \rho_{gy}$ is equivalent to saying that the mean of z_{gx} is equal to the mean of z_{gy} . The two-sample MR Steiger method uses the following statistic^{1,19}:

$$T_{\text{Steiger}} = \frac{z_{gx} - z_{gy}}{\sqrt{1/(n_x - 3) + 1/(n_y - 3)}} \sim N(0, 1).$$

If T_{Steiger} is significant and positive, the causal direction is from x to y . If T_{Steiger} is significant and negative, the causal direction is from y to x .

However, the statistic T_{Steiger} does not approximately follow a standard normal distribution because the SNP is selected for its significant p -values. Using a selection criterion $p < 5 \times 10^{-8}$, or W_{gx} and W_{gy} greater than 29.71679 on 1-df chi-square scale, the sample correlation coefficients $|\hat{\rho}_{gx}|$ and $|\hat{\rho}_{gy}|$ would be at least 0.4823663, 0.1700451, or 0.05443772 for $n_x = 100, 1000, \text{ or } 10,000$ given the relationship $|\hat{\rho}_{gx}| = 1/\sqrt{1 + (n_x - 2)/W_{gx}}$. Although this selection procedure is useful for selecting the instrument SNP, it imposes a lower limit on $|\hat{\rho}_{gx}|$ and $|\hat{\rho}_{gy}|$. $|\hat{\rho}_{gx}|$ ($|\hat{\rho}_{gy}|$) over-estimates $|\rho_{gx}|$ ($|\rho_{gy}|$) and is not consistent. The mean of the statistic T_{Steiger} is not around 0 even when $H_0 : \rho_{gx} = \rho_{gy}$ holds if $n_x \neq n_y$. The distribution of z_{gx} is truncated and is not normal. So is the distribution of z_{gy} . The variance of z_{gx} is smaller than $1/(n_x - 3)$ due to selection. Similarly, the variance of z_{gy} is smaller than $1/(n_y - 3)$. When $n_x = n_y$, the numerator of T_{Steiger} is around 0 and T_{Steiger} is conservative. When $n_x \neq n_y$, the numerator of T_{Steiger} is no longer around 0 and T_{Steiger} is liberal. Overall, the distributions of z_{gx} and z_{gy} are truncated normal instead of normal. The argument that the statistic T_{Steiger} follows asymptotically a standard normal does not hold. The two-sample MR Steiger method can be either liberal or conservative.

Numerical examples are constructed. First we consider the case $n_x = 1000, n_y = 10,000$ and demonstrate the effect of selection severity. Ten thousand random samples of z_{gx} and z_{gy} are independently generated from the normal distributions shown in Eqs. (2) and (3). These z_{gx} and z_{gy} form a $10,000 \times 2$ matrix. The first column contains values for z_{gx} and the second for z_{gy} . Only the rows satisfying $z_{gx} \geq 0.5 \ln[(1 + 0.1700451)/(1 - 0.1700451)] = 0.17171315$ and $z_{gy} \geq 0.5 \ln[(1 + 0.05443772)/(1 - 0.05443772)] = 0.05449159$ are kept. This selection criterion corresponds to 5×10^{-8} on the p -value scale. When $\rho_{gx} = \rho_{gy} = 0.15$, there are 2557 (z_{gx}, z_{gy}) selected on which the statistic T_{Steiger} is computed. The sample mean of selected $\{z_{gx}\}$ is 0.1903508 while the sample mean of selected $\{z_{gy}\}$ (= 0.1512602) is lower, as expected. A normal quantile-quantile plot of T_{Steiger} is shown in the left panel of Fig. 4. Clearly the distribution of T_{Steiger} is different from normal. Type I error rates are inflated. At significance level 0.05 and 0.01, the type I error rates (i.e., the proportion of significant T_{Steiger} statistics) are 0.08916699 and 0.01486117, respectively. If $\rho_{gx} = \rho_{gy} = 0.19$, the selection is less severe. Almost 75% (7434 out of 10,000) (z_{gx}, z_{gy})s are selected. Even so, the distribution of T_{Steiger} shows apparent departure from normal as shown in the right panel of Fig. 4. At significance level 0.05 and 0.01, the type I error rates are 0.0306699 and 0.005111649, respectively. In this case, T_{Steiger} appears to be conservative.

We also considered larger sample sizes. When $n_x = 100,000, n_y = 300,000$, and $\rho_{gx} = \rho_{gy} = 0.015$, 2,375 (z_{gx}, z_{gy})s are selected. When $n_x = 150,000, n_y = 400,000$ and $\rho_{gx} = \rho_{gy} = 0.015$, 6,410 (z_{gx}, z_{gy})s are selected. As shown in Fig. 5, there is apparent departure of the distribution of T_{Steiger} from a normal. At significance level 0.05, the type I error rate is 0.09515789 for the case $n_x = 100,000, n_y = 300,000$ and is 0.03728549 for $n_x = 100, 100, n_y = 400,000$. At significance level 0.01, the type I error rates are 0.01515789 and 0.006084243, respectively. The type I error rates can be either inflated or deflated.

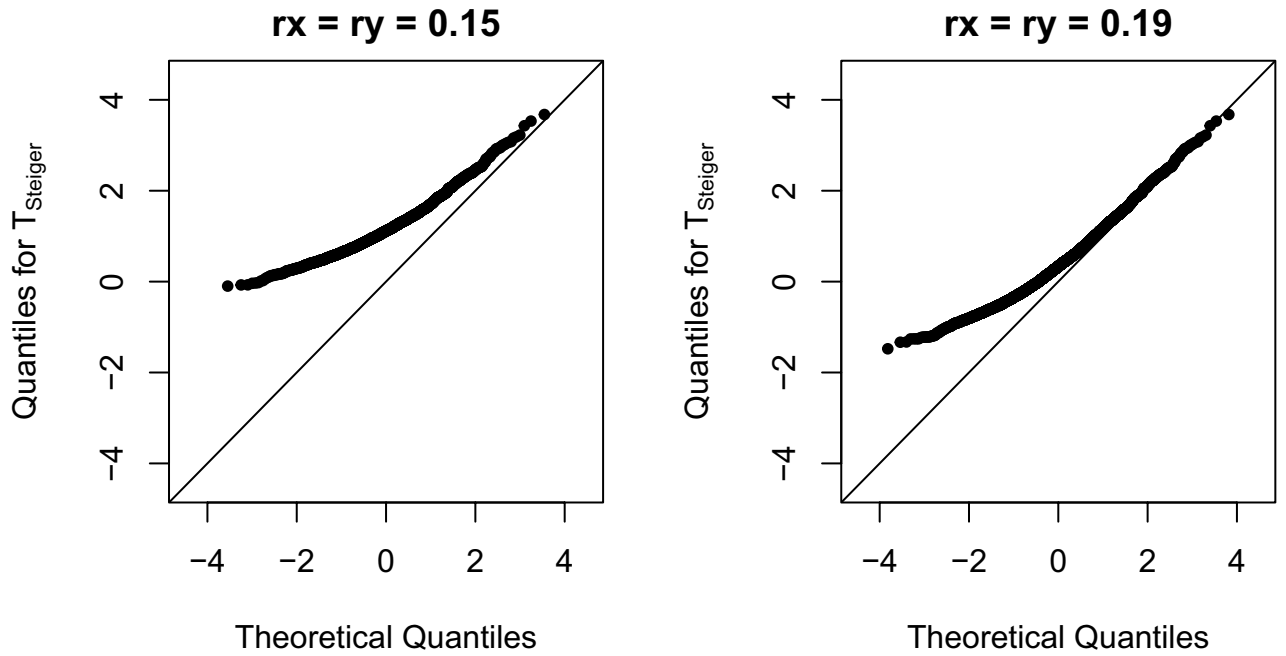


Figure 4. Normal Q-Q plot of simulated $T_{Steiger}$ with $n_x = 1000, n_y = 10,000$. $\{(z_{gx}, z_{gy})\}$ are selected from 10,000 replicates at genome-wide significance level 5×10^{-8} .

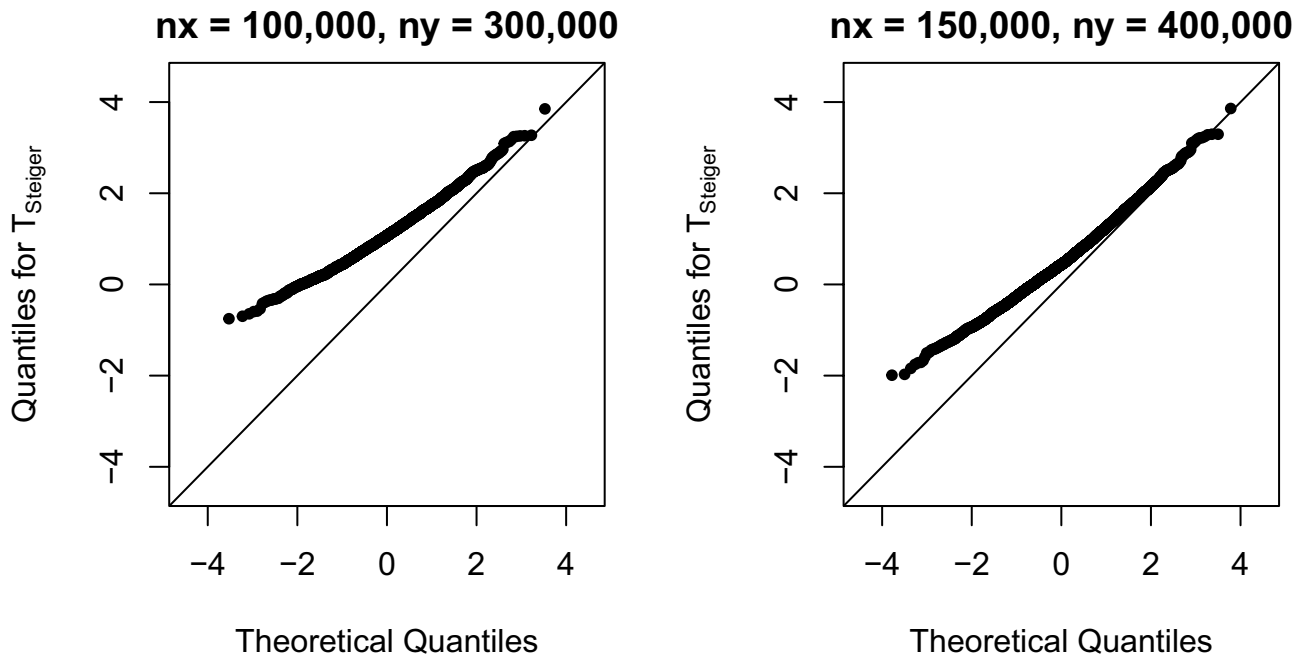


Figure 5. Normal Q-Q plot of simulated $T_{Steiger}$ with $\rho_{gx} = \rho_{gy} = 0.015$. $\{(z_{gx}, z_{gy})\}$ are selected from 10,000 replicates at genome-wide significance level 5×10^{-8} .

One remedy would be to estimate ρ_{gx} and ρ_{gy} by maximizing the conditional likelihood given the SNP selection criteria. Let $\phi(\cdot)$ and $\Phi(\cdot)$ denote the density function and the distribution function of the standard normal, respectively. The likelihood ratio statistic for testing H_0 is $2 \log(L_1/L_0)$ where

$$L_1 = \left[\max_{\mu_{gx}} \frac{\phi(\sqrt{n_x - 3}(z_{gx} - \mu_{gx}))}{1 - \Phi(\sqrt{n_x - 3}(c_{gx} - \mu_{gx}))} \right] \cdot \left[\max_{\mu_{gy}} \frac{\phi(\sqrt{n_y - 3}(z_{gy} - \mu_{gy}))}{1 - \Phi(\sqrt{n_y - 3}(c_{gy} - \mu_{gy}))} \right],$$

$$L_0 = \max_{\mu_g} \left[\frac{\phi(\sqrt{n_x - 3}(z_{gx} - \mu_g))}{1 - \Phi(\sqrt{n_x - 3}(c_{gx} - \mu_g))} \cdot \frac{\phi(\sqrt{n_y - 3}(z_{gy} - \mu_g))}{1 - \Phi(\sqrt{n_y - 3}(c_{gy} - \mu_g))} \right]$$

with c_{gx} and c_{gy} selection thresholds corresponding to z_{gx} and z_{gy} , respectively. However, due to selection, computation of L_1 and L_0 can be challenging. One alternative method is to use an exposure GWAS and a trait GWAS that are independent of those used to select the SNP. However, such studies may be impractical to obtain⁶.

Discussion

Summary statistics MR is subject to selection bias, resulting in excessive false positives (for instance, the MR Steiger method) or missed discoveries (for instance, the SMR method). This bias is a form of winner's curse. Selection bias has been discussed in the literature in the context of the choice of the instrument SNPs⁷, colocalisation test²⁰, and estimation of exposure effect^{5,6}.

Our work complements previous studies on selection bias due to selection of SNPs. While previous work focused on the effect of this bias on the Wald ratio^{5,6} (i.e., estimation), ours focuses on testing whether the exposure causally affects the outcome (i.e., inference). Selection bias leads to underestimation of the Wald ratio⁵ but its effect on type I error rate can be either liberal or conservative depending on the MR method used. Most importantly, the SMR method is conservative even in the absence of selection bias where \hat{b}_{gx} is approximately normal.

Correcting for selection bias is a challenging task. Zhao et al.⁶ get around this issue by using an independent exposure GWAS. On the other hand, our conditional test, an alternative to the SMR method, uses only the trait GWAS. It may be expanded to accommodate multiple instrumental SNPs and the presence of pleiotropy.

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Author contributions

K.W. conceived the experiment and conducted some simulations, S.H. conducted the simulation study and the empirical study. Both authors drafted and reviewed the manuscript.

Competing interests

The authors declare no competing interests.

Additional information

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