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Research article

Bayesian-based probabilistic decline curve analysis study in unconventional reservoirs

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1. Introduction

The capacity of oil and gas producers to produce from low-permeability shale formations has been greatly expanded due to the rapid development of horizontal drilling and hydraulic fracturing technology [1]. Thus, estimating the reserves and predicting the production trends of these unconventional reservoirs is of paramount importance. To do so, decline curve analysis is the easiest method from a practical point of view. It is as simple as extrapolating past production trends to estimate the reserves and forecast the production [2]. However, the DCA models in the literature are purely deterministic and don't provide any quantification to the uncertainties associated with the reserves estimates. The uncertainties in reserves estimates arise from many reasons [3] including but not limited to the absence of long-term production data, high noise in production data, complexity of depletion because of the interaction of both natural and hydraulically induced fractures, and mostly transient-time flow regime (low permeability hinders reservoirs from reaching boundary-dominated flow regime).

Two approaches exist to estimate production forecasts and reserves [3–5]: the deterministic approach and the probabilistic one.

For the deterministic approach, there is a single value for each parameter in the mathematical model used to fit the production trend – a DCA model. Usually, the values of these parameters are the ones that minimize the error function between the actual and fitted data. The deterministic approach doesn't quantify any uncertainty in its estimates of production rates or reserves.

Unlike the deterministic approach, a probabilistic approach includes uncertainty evaluation as an integral part. By assigning probability distributions to the model parameters, probabilistic decline curves can be generated using one of the statistical techniques. Several authors have applied a probabilistic approach to DCA. Cheng et al. [6] utilized bootstrapping and modified bootstrapping to produce probabilistic decline forecasts and reserve estimates. Also, Gonzalez et al. [3], Gong et al. [5], and Adaveni [7] used Bayesian methodology which relies on Markov Chain Monte Carlo (MCMC) sampling algorithm to generate posterior distributions for the parameters of interest.

The Bayesian methodology came to the mainstream of statistics with the introduction of the Markov Chain Monte Carlo (MCMC) sampling algorithm. MCMC was applied to DCA in unconventional reservoirs by Gong [4]. The Bayesian methodology has been chosen over other stochastic methods like bootstrapping to produce PDCA because it is faster and yields narrower uncertainty ranges as Gong [4] noticed.

Bayesian methodology has been used in various applications to tackle reservoir engineering problems such as data assimilation [8],

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well testing [9], and reservoir characterization [10].

In this study, individual-well DCA will be conducted on 50 oil wells and 54 gas wells from publicly available data. The Bayesian probabilistic methodology will be applied to four DCA methods (Arps, Power Law Exponential (PLE), Duong, and Stretched Exponential Decline (SEPD)), which are widely used in the literature for tight shales. This, in turn, will allow for the estimation of the reserves and their associated uncertainties. Typically, this is presented in terms of three confidence levels (P10, P50, and P90) and a corresponding 80% confidence interval (C.I.) [6].

A question about the reliability (or calibration) of these confidence intervals is then natural. This research will utilize data from oil and gas wells to verify the calibration of these C.I.s or in other words to check if the true value of reserves is bracketed by this C.I. 80% of the time [4,5,11].

We will investigate the effect of the flow regime on calibration and the coverage rate of the C.I.s generated by PDCA. The novel thing about this study is that the findings that we will present later indicate that the calibration of a C.I. is highly dependent on the selection of an appropriate DCA model. This coincides with findings from other researchers including Wahba et al. [12].

JAGS package in R®, an open-source statistics software, has been used to implement the MCMC algorithm and incorporate it with DCA models (e.g., Arps, PLE, SEPD, and Duong) to generate probabilistic production forecasts and reserves estimates.

2. DCA models

Four DCA models that are widely accepted in the industry to model the decline of oil and gas wells are used together with the Bayesian methodology to produce probabilistic estimates of the production forecasts and subsequently the reserve estimates. The four models are as follows.

2.1. Arps' model

Arps [13] proposed a set of empirical mathematical formulas to capture the behavior of production rate versus time. The general form of Arps' model (if $b > 0$) is:

$$q(t) = \frac{q_i}{(1 + b\,D_i t)^{\left(\frac{1}{b}\right)}} \tag{1}$$

where.

$q(t)$: the production rate, volume units/time;
q_i: initial production rate, volume units/time;
D_i: initial decline rate, 1/time;

b: exponent.

When applied to wells from unconventional reservoirs, Arps' model yields a b-exponent greater than 1. Practically, the hyperbolic model can still fit the data, yet the equation when integrated (from zero to infinity) to calculate the cumulative production will result in unbounded reserve estimates [14] – for sure the practical upper limit for the integration is not infinity. Table 1 summarizes the cases corresponding to different b values in Arps equation.

2.2. Power law exponential (PLE) model

Ilk et al. [15] developed this model which incorporates both transient and boundary-dominated flow by introducing a formula for the loss ratio (decline rate).

The formula representing the PLE model is:

$$q(t) = \widehat{q}_i\ exp\,(-D_\infty t - \widehat{D}_i\ t^n) \tag{2}$$

where:

Table 1
Arps' model three forms.

Case	b	Rate-time Relationship
Exponential	$b = 0$	$q(t) = q_i\ exp\,(-D_i t)$
Hyperbolic	$0 < b < 1$	$q(t) = \frac{q_i}{(1 + b\,D_i t)^{\left(\frac{1}{b}\right)}}$
Harmonic	$b = 1$	$q(t) = \frac{q_i}{1 + D_i t}$

$q(t)$: the flow rate at t;
$\widehat{q}_i$: the initial flow rate;
D_∞: loss ratio at $t \rightarrow \infty$;
$\widehat{D_i}$: is a constant.

n: time exponent.From equation (2), at early times the term $\widehat{D_i}\, t^n$ dominates making the decline a sort of hyperbolic decline but at late times, when $t \rightarrow \infty$, the term $D_\infty t$ dominates resulting in an exponential decline similar to that of Arps.

2.3. Stretched exponential production decline (SEPD)

Valkó [16] postulated a DCA model based on his analysis of all publicly available data on gas wells from the Barnett Shale play. The SEPD model is basically the integral (sum) of a large number of contributing volumes individually decaying exponentially.

The SEPD rate-time formula is as follows:

$$q(t) = q_i \; exp\left[-\left(\frac{t}{\tau}\right)^n\right] \tag{3}$$

where:
$q(t)$: production rate at t;
q_i: initial production rate at $t = 0$;
τ: median characteristic time constant;
n: exponent parameter.

2.4. Duong model

To account for fracture-dominated flow in tight shale formations, Duong [17] developed an empirical model.

The Duong rate-time formula is as follows:

$$q(t) = q_1 \, t^{-m} \exp\left[\frac{a}{1-m}\left(t^{1-m} - 1\right)\right] + q_\infty \tag{4}$$

where:
$q(t)$: the flow rate at t;
q_1: the flow rate at $t = 1$;
q_∞: the flow rate at $t = \infty$, it can be zero, positive, or negative. The abandonment rate should be decided on solely economic factors;
a: the intercept of the straight line obtained from a log-log plot of q/G_p or q/N_p vs time;
m: the slope of the straight line obtained from a log-log plot of q/G_p or q/N_p vs time, dimensionless.

3. Bayesian Methodology

A Bayesian model is a statistical model which is basically a mathematical model that is used to imitate and approximate data-generating process [18]. A statistical model would help identify structures in the noisy data. Suppose some historical production data vs time is fitted using Arps' model. The output of this fitting would result in estimates for q_i, D_i and b. Say $q_i = 10\ MMSCF/\ d, D_i = 0.3668\ year^{-1}$ and $b = 0.51$, based on the data. It is natural to ask questions like how am I confident in these figures? How these figures would change if I took another segment of the available data? Using Bayesian theory, we can account for the uncertainty surrounding these estimates. The basics of the Bayesian methodology are outlined as follows:

3.1. Bayes' theory

The mathematical formula for the continuous Bayes' theorem is as follows [18]:

$$p(\theta_j|y) = \frac{p(y|\theta_j)\; p(\theta_j)}{p(y)} = \frac{p(y|\theta_j)\; p(\theta_j)}{\int p(y|\theta_j)\; p(\theta_j) d\theta} \tag{5}$$

where:
$p(\theta_j|y)$: the posterior probability distribution of the parameter θ_j given the data, y.
$p(\theta_j)$: the prior probability distribution of the parameters θ_j.
$p(y)$: the marginalized probability distribution of the data, y, over the range of θ_j.

Gong et al. [5] assigned θ_j to DCA parameters (for example, he used q_i, D_i, b from Arps' model as $\theta_1, \theta_2, \theta_3$ in the Bayes' theorem – the same applies to the other DCA models stated here-above). y is the recorded production data. In the context of Bayes' theorem: historical production data, y, are already determined while the decline curve parameters, θ_j, have a degree of uncertainty to be quantified using Bayes' theorem which in turn incorporates the historical data. A Bayesian model consists of three components 1) the prior distribution which captures the initial uncertainty believed to describe the parameters θ_j without any consideration to the data at hand 2) the likelihood, $p(y|\theta_j)$ which is the probability of the data given the unknown parameter(s) 3) the posterior distribution of the parameters which incorporates both the prior and the likelihood. The prior distribution, $p(\theta_j)$, is the probability distribution of the parameters θ_j. This distribution encapsulates our belief about the parameter based on previous studies or our own knowledge. The likelihood function, $p(y|\theta_j)$, represents the probability distribution of our production data, y, given the model parameters $\theta_1, \theta_2, \ldots, \theta_k$ where k is the total number of DCA model parameters. In this study, the calculated production data is assumed to be normally distributed around the true production data (the historical data, represents the mean of the normal distribution) with a variance σ^2, representing the randomness of error between the historical production data and the calculated one. Namely, the calculated production data follows a normal distribution of a mean μ, this mean is the ACTUAL production rate(s), and a variance σ^2 representing the fitting error. The likelihood function is distributed normally as follows:

$$p(y|\theta_1, \theta_2, \ldots, \theta_k) \sim N(\mu, \sigma^2) \tag{6}$$

The posterior distribution of the DCA parameters is calculated using equation (5) which combines our prior knowledge with the actual data collected.

3.2. MCMC algorithms

MCMC algorithms have been developed to allow sampling from posterior probability distributions (i.e., $p(\theta_1, \theta_2, \ldots, \theta_k|y)$). In this study, Gibb's sampler is used to execute MCMC. Gibbs sampler which adapts Metropolis–Hastings algorithm to perform MCMC over more than just one parameter [19].

Example: basic Gibbs sampler algorithm for three parameters:

The *three* parameters could be thought of as Arps' model parameters and the data, y, is always conditioned on

The Gibbs sampler steps are:

For iteration i in 1 to m, repeat:

a) Set initial starting conditions $(\theta_1{}^{(0)}, \theta_2{}^{(0)}, \theta_3{}^{(0)})$
b) Draw $\theta_1{}^*$ from a proposal distribution q_1 (proposal distribution can be any distribution – for example, normal distribution-that has the same domain as the target distribution of θ_1)
c) Calculate the acceptance ratio, α, as per equation (7) by plugging in the previous iteration's values for $\theta_2{}^{(i-1)}$ and $\theta_3{}^{(i-1)}$

$$\alpha = \frac{\left[\dfrac{p\left(\theta_1{}^*|\theta_2{}^{(i-1)}, \theta_3{}^{(i-1)}, y\right)}{q\left(\theta_1{}^*|\theta_1{}^{(i-1)}\right)}\right]}{\left[\dfrac{p\left(\theta_1{}^{(i-1)}|\theta_2{}^{(i-1)}, \theta_3{}^{(i-1)}, y\right)}{q\left(\theta_1{}^{(i-1)}|\theta_1{}^*\right)}\right]} \tag{7}$$

d) Accept the candidate with a probability *min* $\{1, \alpha\}$ and set $\theta_1{}^{(i)}$ accordingly
e) Draw $\theta_2{}^*$ from a proposal distribution q_2
f) Calculate the acceptance ratio, α, as it has previously done by plugging in the values for $\theta_1 = \theta_1{}^{(i)}$ and $\theta_2 = \theta_2{}^{(i-1)}$

$$\alpha = \frac{\left[\dfrac{p\left(\theta_2{}^*|\theta_1{}^{(i)}, \theta_2{}^{(i-1)}, y\right)}{q\left(\theta_2{}^*|\theta_2{}^{(i-1)}\right)}\right]}{\left[\dfrac{p\left(\theta_2{}^{(i-1)}|\theta_i, \theta_3{}^{(i-1)}, y\right)}{q\left(\theta_2{}^{(i-1)}|\theta_2{}^*\right)}\right]} \tag{8}$$

g) Accept the candidate with a probability *min* $\{1, \alpha\}$ and set $\theta_2{}^{(i)}$ accordingly
h) Draw $\theta_3{}^*$ from a proposal distribution q_3
i) Calculate the acceptance ratio, α, as it has previously done by plugging in the values for $\theta_1 = \theta_1{}^{(i)}$ and $\theta_2 = \theta_2{}^{(i)}$

$$\alpha = \frac{\left[\dfrac{p\left(\theta_3{}^*|\theta_1{}^{(i)}, \theta_2{}^{(i)}, y\right)}{q\left(\theta_3{}^*|\theta_3{}^{(i-1)}\right)}\right]}{\left[\dfrac{p\left(\theta_3{}^{(i-1)}|\theta_1{}^{(i)}, \theta_2{}^{(i)}, y\right)}{q\left(\theta_3{}^{(i-1)}|\theta_3{}^*\right)}\right]} \tag{9}$$

j) Accept the candidate with a probability *min* $\{1, \alpha\}$ and set $\theta_3{}^{(i)}$ accordingly

One iteration of the above algorithm is $(\theta_1{}^{(i)}, \theta_2{}^{(i)}, \theta_3{}^{(i)}))$ constituting one row in the MCMC sampler matrix. Every Gibbs cycle includes an update for each parameter.

The details of the Gibbs sampler can be easily handled by a number of commercially available software packages.

In this research, a software package called JAGS (Just Another Gibbs Sampler) by Martyn Plummer is utilized. The program is free software that can run on all machines and operating systems. Here, a statistical programming language R is used as a platform to run the program using *rjags* package.

In JAGS, just a few lines of code can be used to run very powerful MCMC samplers. The main concern shifts towards the statistical modeling concept rather than writing and implementing the code. A wide range of statical models is fit easily using *rjags*. For each model of the four DCA addressed in this study, two codes are written one for the oil case and one for the gas case. Each code defines the non-informative (non-biased) priors for the parameters based on previous studies' findings (section 5.1) and also defines the likelihood function as discussed in equation (6). MCMC of 3 chains and 5000 iterations are used to produce a posterior distribution of the DCA parameters. The software outputs the posteriori for the parameters and from which the production rates are calculated as we will discuss in section 5.6.

4. Flow regimes in unconventional reservoirs

Unconventional horizontal wells completed with multistage hydraulic fracturing experience multiple flow regimes throughout their lifetime [20]. Formation transient linear flow regime may be the dominant flow regime during most of the lifetimes of the wells [21]. For other cases that have close natural fracture spacing, a liner flow precedes Boundary Dominated Flow (BDF). Moreover, a bilinear flow regime may develop before the liner flow. In a typical bilinear flow, the fluid flow starts from the rock matrix towards the natural fractures then to the hydraulic fractures, and finally to the well [21,22]. Fig. 1 shows a typical well with the three flow regimes over its producing lifetime.

Therefore, four possible decline scenarios may develop [21].

a) Linear
b) Linear-BDF
c) Bilinear-Linear-BDF
d) Bilinear-Linear.

5. Application and results of Bayesian Methodology using MCMC sampling technique on field-wide data

The purpose of this research is to assess the performance of Bayesian Methodology in a sample of 50 oil wells and 54 gas wells. Moreover, the research investigates how the 80% credible interval is calibrated.

The parameters of four decline curve analysis models (Arps, PLE, Duong, and SEPD) and their associated uncertainties are estimated using the Bayesian methodology. Actual publicly available production data of 50 oil wells from in the Permian basin and 54 gas wells in the Marcellus Shale play are utilized.

The data of every individual well is split into two equal sets: one set is for training (the early half of the data) and the other (the late half of the data) is for forecasting (to test the predictivity of the model while evaluating uncertainty at the same time). The 50% training set was used by other authors like Adaveni [7].

Fig. 2 shows "Start of Hindcast" which marks the split of the production data into 2 sets. The vertical line indicates the end of the training set and the start of the prediction set.

For the oil wells, 50 wells with production history starting as early as January 2014 and continuing 22–88 months later (11–44 data points have been used to fit the models) were used. For the gas wells, 54 wells that had been produced to September 2022 have been selected. These wells had been producing for 18–72 months before September 2022 (9–36 data points have been used to fit the models).

The following process explains the steps on which the Bayesian modeling is built.

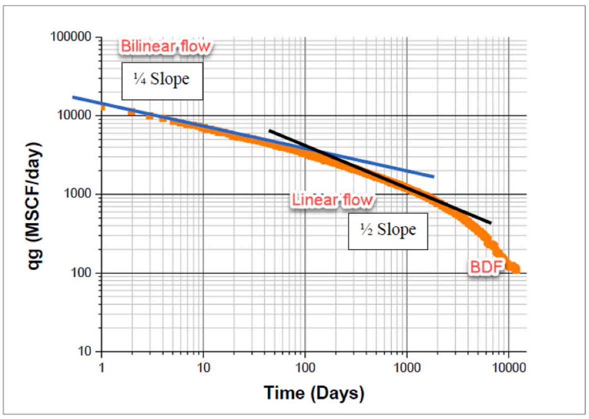

Fig. 1. a log-log plot of production rate vs time shows how to identify the flow regime based on the slope of the line [22].

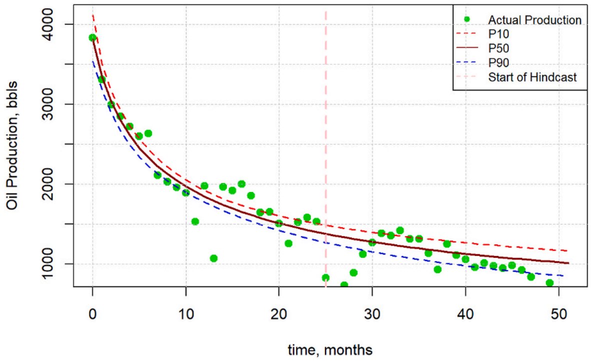

Fig. 2. Actual production data and P10, P50, and P90 estimates for production rates using Arps' model (an oil well example).

5.1. *Prior selection for the models*

In Bayesian analysis, the choice of prior distributions is crucial. In this research, noninformative priors for the parameters, also known as vague priors, are used. The usage of noninformative priors implies that all potential biases in the priors (i.e. priors have information related to researcher beliefs and previous experiences) are excluded and that, in return, makes models primarily data-driven. The knowledge about the range of the parameters obtained from previous studies will be incorporated but values within that range will be equally likely to be chosen. Table 2 summarizes the range for the DCA parameters.

5.2. *Refining of the data*

For data refining, techniques from Duong [17] and Alqattan [24] have been adopted. Early-production data is most of the time noisy due to cleaning of the well, choking, or other operational reasons. All four models used in this research are very sensitive to early-production data. Throughout this research, this data is removed *manually* from the production history to get reliable DCA results. It should be laid out that noisy data during other stages of production is left without any smoothing even if they show high noise as long as they follow the general trend of decline as shown in Fig. 3.

5.3. *Identifying flow regimes*

For the oil and gas wells tackled in this study, the flow regimes are identified using the slope rule outlined by authors like [20–22]. Fig. 1 shows an example of a well undergoing three flow regimes throughout its producing life: bilinear followed by linear and finally BDF. These flow regimes can be identified by the slope of the line when plotting production rates vs time on a log-log scale. Bilinear flow is characterized by ¼ slope, linear flow exhibits ½ slope while BDF shows a progressive curve. The four scenarios, in terms of flow regimes, that an unconventional well can undergo are linear, linear-BDF, bilinear-linear-BDF, and bilinear-liner as outlined in the flow regime in the unconventional reservoir section.

5.4. *Error analysis*

Error analysis is done on the data using four DCA models. The data is fitted using the mean parameters of the Bayesian Analysis coupled with the MCMC algorithm. Then, the error is analyzed. If the error is not randomly distributed around zero when plotted against the fitted values, and has a systematic trend, the model is deemed inconvenient. All wells analyzed here are of acceptable error analysis.

5.5. *MCMC parameters convergence*

In this research, MCMC of 3 chains and 5000 iterations are used to produce a posterior distribution of the DCA parameters. Three metrics have been used to assess convergence on every well/model combination separately.

a) Gelman Diagnosis: Gelman–Rubin [25] developed a technique to assess the convergence between multiple MCMC chains. This metric should be one to indicate convergence.
b) Auto-correlation metric: this metric helps assess convergence because as autocorrelation nears zero MCMC is more reliable.
c) Effective sample size: the higher this number the better. Its maximum is 15,000 because we use 3 chains and 5000 iterations for each.

It should be pointed out that all wells tackled here show good convergence across all four models.

Table 2
Summarizes the range for the DCA parameters of the four models.

Bounds for Decline Curve Model Equations			
DCA Parameter	Lower bound	Upper bound	References
Arps' Model			
q_i, *volume*/ *time*	0.001	100,000	[3,23]
D_i, 1/ *time*	0.01	2	
b	0	6	
Power Law Exponential			
$\widehat{q_i}$	0.001	100,000	[3,15]
D_∞, 1/ *time*	10^{-8}	10^{-2}	
$\widehat{D_i}$, 1/ *time*	0.001	10	
n	0.001	2	
Stretched Exponential Production Decline			
q_i, *volume*/ *time*	0.001	100,000	[3,16]
τ	0	60	
n	0	6	
Duong Model			
q_1	0.001	100,000	[3,17]
m	0.001	2	
a	0.01	5	
q_∞	−1	1	

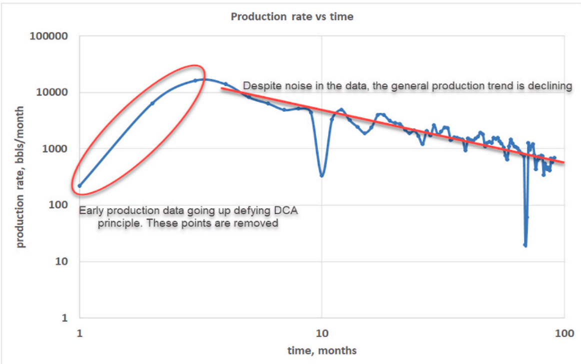

Fig. 3. An oil well undergoing a natural decline.

5.6. *Posterior analysis and reserves confidence intervals*

The culmination of this study is to produce confidence/credible intervals that quantify our uncertainty about the reserves. Bayesian methodology along with the MCMC algorithm has been implemented to estimate the parameters of Arps, PLE, Duong, and SEPD for oil and gas wells.

As laid out previously, 50% of the production data is used as a training set for all of the 4 models. For each parameter (i.e., Arps: q_i,b, D_i; PLE: $\widehat{q_i}$, D_∞ , $\widehat{D_i}$, n; Duong: q_1, m, a, q_∞, SEPD: q_i, τ, n), there are 15,000 estimates (3 chains and 5000 iterations for each). Each decline curve model would produce 15,000 estimates for production rate at each time point. P10, P50, and P90 can be constructed out of this distribution.[1] The actual cumulative production of the second half of the data (prediction data set) is treated as "actual reserve" while Bayesian P90–P50–P10 reserves are the 10th, 50th' and 90th percentile of the cumulative production. The essence of this search is to evaluate how well these percentiles can encapsulate the "true reserves" throughout the 4 DCA methods.

The following example (see Fig. 4) shows how posterior analyses are done on a gas well modeled by the PLE model. The same principle applies to the other three models.

"Start of Hindcast" indicates the end of the training set and the start of the prediction set.

The probability distribution of the cumulative production in the second half of the data using the PLE model is (see Fig. 5)

Table 3 shows the actual cumulative production "true reserve" is successfully bracketed by the 80% credible interval as shown above.where:

N_t: the number of data points in the training set.

R_t^2: the R-squared of the training set.

[1] P90, P50 and P10 are 10th, 50th and 90th percentile of production distribution at a point of time respectively.

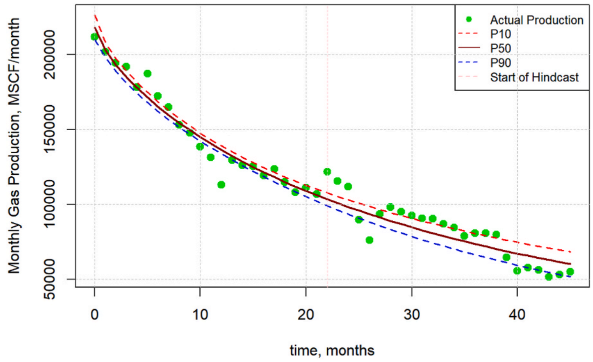

Fig. 4. Actual production data and P10, P50, and P90 estimates for production rates using the PLE model.

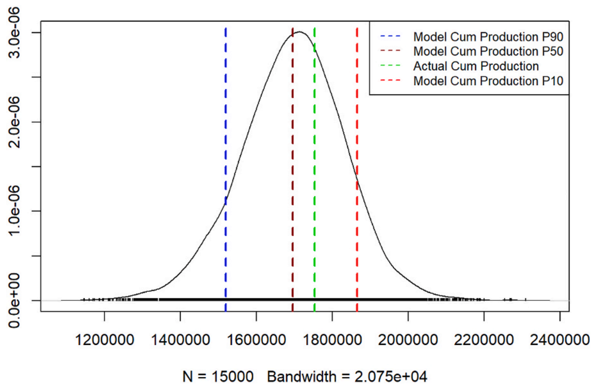

Fig. 5. The probability distribution for cumulative production P90, P50, and P10 estimates using the PLE model. The true reserve is indicated in green color. (For interpretation of the references to color in this figure legend, the reader is referred to the Web version of this article.)

$RMSE_t$: the root mean squared of the training set Q at P50, Q at P90, and Q at P10: the 50th, 10th, and 90th percentiles of the cumulative production respectively.

R_f^2: the R-squared of the forecast set.

$RMSE_f$: the root mean squared of the forecast set.

Flow regime: indicates the flow regime through which the well passes over its entire production period.

5.7. Large-scale results

Probabilistic decline curve analysis using Bayesian methodology coupled with MCMC sampling algorithm has been applied to four decline curve analysis methods to generate probabilistic production forecasts and quantify single-well reserves uncertainty. As mentioned before, 50% of the production data is used as a training dataset to produce the posterior distribution for the DCA models. Then, the rest of the data is used as a prediction dataset. The actual cumulative production of the second half of the data is dubbed as “actual reserve” while Bayesian P90–P50–P10 reserves are the 10th, 50th' and 90th percentile of the cumulative production. In this section, the coverage rate, average relative error, average absolute error, and average $(P10 - P90)/Actual$ for 50 oil wells in the Permian Basin are presented for each decline curve method.

Some definitions are presented below:

Coverage rate, CR: represents the fraction of the wells that fall in the P90–P10 range resulting from the Bayesian methodology. In theory, if the Bayesian methodology is well calibrated, 80% of well reserves should be within P90–P10 estimates. If this is not the case, then either the DCA model is inconvenient for that case or the Bayesian methodology is deficient.

Relative error: The smaller this number the better. It will be used to compare different DCA models on a scale of many wells.

$$\frac{P50 - Actual}{P50} \times 100\% \tag{10}$$

where,

$P50$: is the 50th percentile estimate of the reserve,

Actual: is the actual value of the reserve obtained from the production data of a well.

Table 3
The summary of the posterior analysis for a gas well using the PLE model.

Well Name	N_t	D_{inf}	D_{ihat}	N	q_{ihat}	R_{2t}	$RMSE_t$	Q at P50	Q at P90	Actual Q	Q at P10	R_f^2	$RMSE_f$	Flow Regime
		Mean	Mean	Mean	Mean									
37-115-21640	23	1.01E-05	6.73E-02	7.96E-01	2.17E+05	0.94	8677.7	1,695,698	1,519,376	1,753,875	1,865,079	0.8	11785.8	Bilinear-linear-BDF

Relative absolute error: The smaller this number the better. It is the same as relative error but the numerator is always positive. It will be used to compare different DCA models on a scale of many wells.

$$\frac{|P50 - Actual|}{P50} \times 100\% \tag{11}$$

Average P10–P90/Actual: is the average value of the confidence interval width relative to the actual production. The smaller this number the better as long as the coverage rate is high.

5.7.1. Oil wells

5.7.1.1. Arps. Table 4 shows a summary of the 50 oil wells and the coverage rate percentage. The overall coverage rate, CR, is 58%. This indicates a poorly-calibrated Bayesian P10–P90 range. Theoretically, CR should be 80% (i.e. for the Bayesian methodology to be well-calibrated the CR should be at least 80%). The poor calibration can be attributed to the fact that Arps' model itself doesn't adequately fit the production data, a small number of training data points (the range of the number of training points is between 11 and 44 months), or poor quality of data that Arps cannot tackle. It can be observed that the model did fairly well with bilinear-linear-BDF and bilinear-linear flow regimes (75% and 73.9% CR respectively). It could have reached 80% CR if there were more wells to analyze. However, for both linear-BDF and linear flow regimes it did very poorly (23.1% and 50% CR respectively). Also, the model over-estimates the reserves for the cases where it fails to capture their true reserves.

5.7.1.2. PLE. A summary of 50 oil wells and their associated coverage rate percentage is presented in Table 5. CR of the Bayesian P10–P90 interval is 70% which is a big improvement over the CR of the Arps model. It can be noted that the Bayesian confidence interval (CI) is very well calibrated in the case of bilinear-linear-BDF and bilinear-linear flow regimes (87.5% and 82.6% CR respectively). The revealing performance of PLE in bilinear-linear-BDF and bilinear-linear flow regimes coincides with results from other researchers like Wahba [12]. This might be attributed to the fact that the model was originally developed to tackle the flow regimes starting with the bilinear flow regime. It did fairly well with up to 69% CR for the linear-BDF flow regime. Nonetheless, it had performed very badly at 0% CR for the linear flow regime indicating that the PLE method must not be used to evaluate wells undergoing linear flow throughout their lifetime. For investment analysis applying the PLE model and obtaining estimates of reserves with Bayesian analysis, caution should be exercised if the well has started with a linear flow regime (½ slope for rate vs time on a log-log scale).

5.7.1.3. Duong. Table 6 summarizes the results obtained from applying Bayesian methodology on 50 oil wells and the coverage rate percentage. CR of the Bayesian P10–P90 interval is merely 32% which is very poor (the Bayesian methodology coupled with Duong only captures 32% of the actual reserves). It is worth noting that Bayesian CI is only well calibrated in the case of linear flow regime (½ slope for rate vs time on a log-log scale). This is primarily because the model itself was developed specifically for linear flow regimes [12,17]. For the other flow regimes: bilinear-linear-BDF, bilinear-linear, and linear-BDF, the Duong model performs very poorly (37.5%, 21.7 and 23.1 % CR respectively). For any business planner applying the Duong model and obtaining estimates of reserves with Bayesian analysis, only the case of a linear flow regime throughout the life of the well should be considered. This of course is very challenging to know because if a well starts with a linear flow regime it is not guaranteed that it will continue with this behavior for the rest of its lifetime.

5.7.1.4. SEPD. The breakdown of 50 oil wells and the coverage rate percentage associated with each flow regime are presented in Table 7. CR of the Bayesian P10–P90 interval for all wells tackled in the study is 76% which is the highest among the four DCA methods. Significantly, it can be observed that the Bayesian CI is very well calibrated in the case of bilinear-linear-BDF, bilinear-linear, and linear-BDF flow regimes (87.5%, 82.6%, and 84.6% CR respectively). Nonetheless, it performed very badly at 16% CR for the linear flow regime despite this deficiency, the model successfully underestimated all the reserve estimates. Underestimation of reserves is considered safe in the case of high-risk projects. For an analysis, applying the SEPD model and obtaining the estimates of oil reserves with Bayesian analysis is the best option based on this study. This is because SEPD coupled with Bayesian methodology has been able to capture true reserves correctly and even when it fails, it underestimates the reserves making the economic decision less risk-prone.

Table 4
The coverage rate resulting from Bayesian analysis for 50 oil wells utilizing the Arps DCA model.

Flow Regime	Number of Wells	Captured				CR, %
		Yes	No			
		29	21			58.0
		Total	Total	Overestimated	Underestimated	
bilinear-linear-BDF	8	6	2	0	2	75.0
bilinear-linear	23	17	6	4	2	73.9
linear-BDF	13	3	10	10	0	23.1
linear	6	3	3	2	1	50.0

Table 5
The coverage rate resulting from Bayesian analysis for 50 oil wells utilizing the PLE DCA model.

Flow Regime	Number of Wells	Captured				CR, %
		Yes	No			
		35	15			70.0
		Total	Total	Overestimated	Underestimated	
bilinear-linear-BDF	8	7	1	0	1	87.5
bilinear-linear	23	19	4	2	2	82.6
linear-BDF	13	9	4	1	3	69.2
linear	6	0	6	0	6	0.0

Table 6
The coverage rate resulting from Bayesian analysis for 50 oil wells utilizing the Duong DCA model.

Flow Regime	Number of Wells	Captured				CR, %
		Yes	No			
		16	34			32.0
		Total	Total	Overestimated	Underestimated	
bilinear-linear-BDF	8	3	5	5	0	37.5
bilinear-linear	23	5	18	17	1	21.7
linear-BDF	13	3	10	10	0	23.1
linear	6	5	1	0	1	83.3

Table 7
The coverage rate resulting from Bayesian analysis for 50 oil wells utilizing the SEPD DCA model.

Flow Regime	Number of Wells	Captured				CR, %
		Yes	No			
		38	12			76.0
		Total	Total	Overestimated	Underestimated	
bilinear-linear-BDF	8	7	1	0	1	87.5
bilinear-linear	23	19	4	0	4	82.6
linear-BDF	13	11	2	0	2	84.6
linear	6	1	5	0	5	16.7

5.7.1.5. The distribution of (true-P50)/(P10-90) and the error analysis. The quantity $(True - P50)/(P10 - P90)$ is calculated each well. The density plot of this quantity for the 50 oil wells should be normally distributed around 0.5 if the probabilistic reserves are unbiased [4].

Fig. 6 illustrates how each model performs on the field-wide data when coupled with the Bayesian methodology. A summary of the conclusions drawn from Fig. 6 is as follows:

Arps: the distribution is skewed to the left (the distribution is centered around ¼) indicating a general overestimation of the model.

PLE: the distribution is centered at ½ indicating the Bayesian method coupled with PLE is generally unbiased. Also, the distribution is right-skewed indicating a general *underestimation* of the model.

Duong: the distribution is almost uniform (not showing a clear peak) indicating that the model is *not* reliable at all in predicting the CI of the reserves of the wells at hand.

SEPD: the distribution centered at 0.5 indicating the Bayesian method coupled with SEPD is overall unbiased. Also, the distribution is right-skewed indicating a general *underestimation* of the model.

The error analysis for the 50 oil wells using the Arps, PLE, Duong, and SEPD models has been calculated according to the criteria outlined before as follows.

Table 8 can be summarized as follows:

Arps case: the average relative error is positive indicating that the model *overestimates* the reserves for the well. The average value represented in the last row should be compared with other models to see how wide the confidence interval is. The smaller this quantity the less uncertainty we have in the reserve estimates given that other error metrics are low.

PLE case: the average relative error is negative indicating that the model *underestimated* the reserves for a well. The average quantity (last row) is less than the value for the Arps model so is the average relative error and average absolute relative error.

Duong case: the average relative error is positive indicating that the model *overestimates* the reserves for wells at hand. The average quantity (last row) should be compared with other models to see how wide the confidence interval is. It is less than the value for Arps and PLE models but it is worthless to consider solely since CR here is lower than the previous two models this increase in the figure is

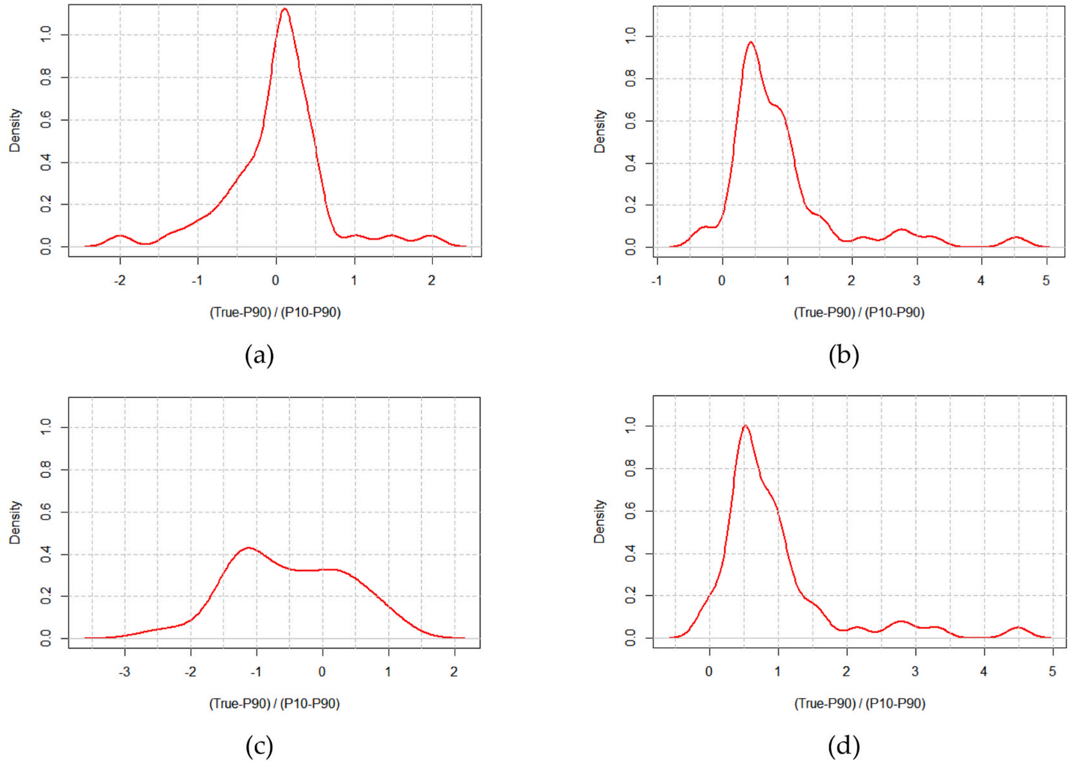

Fig. 6. (True-P90)/(P10–P90) distribution for 50 oil wells using a) the Arps model b) the PLE model c) the Duong model d) the SEPD model.

Table 8
The error analysis for the 50 oil wells using the Arps, PLE, Duong, and SEPD models.

Error metric	Arps	PLE	Duong	SEPD
Average relative error, %	11.9	−2.3	20.1	−9.2
Average absolute relative error, %	22.4	10.1	16.5	27.4
Average (P10–P90)/Actual	0.31	0.29	0.22	0.29

worthless.

SEPD case: the average relative error is negative indicating that the model *underestimated* the reserves for a well.

5.8. Gas wells

As mentioned in the case of oil wells, probabilistic decline curve analysis using Bayesian methodology coupled with MCMC sampling algorithm has been applied to four decline curve analysis methods to generate probabilistic production forecasts and quantify single-well reserves uncertainty. As has been done with the previous section, the coverage rate, average relative error, average absolute error, and average $(P10 - P90)/Actual$ for 54 gas wells in Marcellus Shale play are presented for each decline curve method.

Table 9
The coverage rate resulted from Bayesian analysis for 54 gas wells utilizing the Arps DCA model.

Flow Regime	Number of Wells	Captured				CR, %
		Yes	No			
		32	21			60.4
		Total	Total	Overestimated	Underestimated	
bilinear-linear-BDF	8	4	4	4	0	50.0
bilinear-linear	28	24	3	1	2	85.7
linear-BDF	13	3	10	0	10	23.1
linear	5	1	4	0	4	20.0

5.8.1. *Arps*

The results in Table 9 summarize 54 gas wells and the coverage rate percentage when Bayesian methodology is coupled with the Arps model. The total coverage rate, CR, is 60.4%. This indicates fairly acceptable calibration for the Bayesian P10–P90 range. The less-theoretical calibration may be attributed to the fact that the Arps model itself doesn't adequately fit the production data, a small number of training data points (the range of the number of training points is between 9 and 36 months), a small number of wells have been considered (only 54 wells from the same region) or poor quality of data that the Arps model cannot tackle. It can be observed that the model performs very well with bilinear-linear (85.7% CR). It performed poorly with a bilinear-linear-BDF flow regime (50% CR). However, for both linear-BDF and linear flow regimes it did very poorly (23.1% and 20% CR respectively). So, when applying the Arps model with Bayesian analysis, caution should be exercised with these two flow regimes (linear-BDF and linear). Also, the model overestimates the reserves for all cases in bilinear-linear-BDF it fails to capture. Yet, it underestimates all of the reserves for linear-BDF and linear flow regimes.

5.8.1.1. PLE. Following a similar approach, Table 10 shows a summary of the 54 gas wells and the coverage rate percentage with the PLE model coupled with Bayesian methodology. The coverage rate for all the wells in the study, CR, is 58%. A low percentage like this indicates a poorly-calibrated Bayesian P10–P90 range. The way-less-theoretical calibration may be due to the fact that most of the cases of the wells at hand are of bilinear-linear flow regimes. The model coupled with Bayesian methodology doesn't adequately capture the production data belonging to this flow regime. Other factors may include a small number of training data points (the range of the number of training points is between 9 and 36 months), a small number of wells have been considered (only 54 wells from relatively the same region), or poor quality of data that PLE cannot tackle. It can be observed that the model performs very well with bilinear-linear-BDF (87.5 % CR). It has done a good job capturing the reserves for linear-BDF with 76.9% CR. This CR could have been above 80% if more wells were available for analysis. It performed poorly with a bilinear-linear flow regime (50% CR). However, for both linear-BDF and linear flow regimes it did very poorly (42.9%). Yet, the model safely underestimated all of the cases under investigation. The model has not been able to capture the "true" reserves for any of the linear flow regimes. However, it also underestimated all of the cases. To conclude the results in accordance with the data investigated here, PLE coupled with Bayesian methodology performs well with bilinear-linear-BDF and linear-BDF. However, it fails completely with a linear flow regime.

5.8.1.2. Duong. A brief analysis and statistics related to the performance of Bayesian methodology and the Duong model for 54 gas wells are addressed in Table 11. The most important quantity, the overall coverage rate, CR, is only 46%. This indicates a very poorly-calibrated Bayesian P10–P90 range. The poor calibration may be due to the fact that most of the cases of the wells at hand are of bilinear-linear flow regimes. As discussed earlier, the Duong model has been developed primarily to tackle linear flow regimes. Therefore, the model coupled with Bayesian methodology performs badly with other flow regimes. Other factors concerning low CR may include a small number of training data points (the range of the number of training points is between 9 and 36 months) or a small number of wells that have been considered (only 54 wells from relatively the same region). It can be observed that the model performs ultimately well with a linear flow regime (100 % CR). Other than that, it performed very poorly with linear-BDF, bilinear-linear-BDF, and bilinear-linear flow regimes (23.1%, 37.5%, and 42.9% CR respectively). The model also overestimates most of the cases it fails to capture it is true reserve rendering its application risky because the estimates are very optimistic.

5.8.1.3. SEPD. The coverage rate and the flow regimes breakdown of 54 gas wells are presented in Table 12. The overall coverage rate, CR, is just 40.7%. The poor calibration of the P10–P90 range may be because most of the cases of the wells at hand are of bilinear-linear flow regimes. As outlined before, SEPD works well when BDF is present at the end of the production. The model coupled with Bayesian methodology doesn't adequately represent the production data belonging to flow regimes that don't include BDF. Other factors concerning low CR are related to the limited training data for the model as discussed with other models. It can be observed that the model performs well with bilinear-linear-BDF and linear-BDF flow regimes (87.5% and 76.9% CR). Other than that, it performed very poorly with bilinear-linear and linear flow regimes (17.9% and 0% CR respectively). The model *underestimates all* of the cases it fails to capture. This behavior makes the model coupled with Bayesian methodology a safe option for an engineer predicting the reserves.

5.8.1.4. The distribution of (true-P50)/(P10-90) and the error analysis. The quantity $(True - P50)/(P10 - P90)$ is calculated each well. The density plot of this quantity for the 54 gas wells should be normally distributed around 0.5 if the probabilistic reserves are unbiased [4].

Fig. 7 illustrates how each model performs on the field-wide data when coupled with the Bayesian methodology. A summary of the conclusions drawn from Fig. 7 is as follows:

Arps: the distribution is fairly normal around ½ indicating good predictability for the model. predictability in this context is the model's ability to output CI that captures the "true" reserves. The distribution is somewhat skewed to the left (the area under the curve is more from the left to 0.5) indicating a general *overestimation* of the model.

PLE: the distribution peaks around ¾ which means that the model is biased with this set of data. It is right-skewed indicating that the model is generally *underestimating* the reserves. As indicated before, this distribution should be centered around ½ which is not the case here. This indicates poor predictivity (for the wells in general) for the CI capturing the true reserves.

Duong: the distribution peaks around ½ and is extremely left-skewed indicating that the model is generally *overestimating* the reserves.

Table 10
The coverage rate resulting from Bayesian analysis for 54 gas wells utilizing the PLE DCA model.

Flow Regime	Number of Wells	Captured				CR, %
		Yes	No			
		29	21			58.0
		Total	Total	Overestimated	Underestimated	
bilinear-linear-BDF	8	7	1	0	1	87.5
bilinear-linear	28	12	12	0	12	42.9
linear-BDF	13	10	3	0	3	76.9
linear	5	0	5	0	5	0.0

Table 11
The coverage rate resulting from Bayesian analysis for 54 gas wells utilizing the Duong DCA model.

Flow Regime	Number of Wells	Captured				CR, %
		Yes	No			
		23	27			46.0
		Total	Total	Overestimated	Underestimated	
bilinear-linear-BDF	8	3	5	5	0	37.5
bilinear-linear	28	12	12	11	1	42.9
linear-BDF	13	3	10	9	1	23.1
linear	5	5	0	0	0	100.0

Table 12
The coverage rate resulting from Bayesian analysis for 54 gas wells utilizing the Duong DCA model.

Flow Regime	Number of Wells	Captured				CR, %
		Yes	No			
		22	32			40.7
		Total	Total	Overestimated	Underestimated	
bilinear-linear-BDF	8	7	1	0	1	87.5
bilinear-linear	28	5	23	0	23	17.9
linear-BDF	13	10	3	0	3	76.9
linear	5	0	5	0	5	0.0

PLE: the distribution peaks around 1 and is right-skewed indicating that the model generally *underestimates* the reserves. As indicated before, this distribution should be centered around ½ which is not the case here. This indicates poor predictivity (for the wells in general) for the CI capturing the true reserve.

The error analysis for the 54 gas wells using the Arps, PLE, Duong, and SEPD models has been calculated according to the criteria outlined before as follows.

Table 13 can be summarized as follows:

Arps case: The average relative error is positive indicating that the model *overestimates* the reserves for the wells at hand. The quantity in the last row should be compared with other models to see how wide the confidence interval is. The smaller this quantity the less uncertainty we have in the reserve estimates given that other error metrics are low.

PLE case: The average relative error is negative indicating that the model *underestimates* the reserves for the wells at hand.

Duong case: The average relative error is positive indicating that the model generally *overestimates* the reserves for the wells at hand.

SEPD case: The average relative error is negative indicating that the model generally *underestimates* the reserves for the wells at hand.

6. Conclusions

Oil and gas wells can equally be modeled using the well-known DCA models coupled with Bayesian analysis. Regardless of the nature of the fluid being produced, DCA basically models reflect the flow regime under which a well is flowing.

The following points can be drawn out from the Bayesian analysis with regard to the oil and gas wells tackled in the study.

- Arps' model performs the lowest amongst other models even when incorporated with Bayesian methodology. This is mostly because the model is intrinsically developed for BDF or because the time frame for the production was not enough to exhibit BDF in

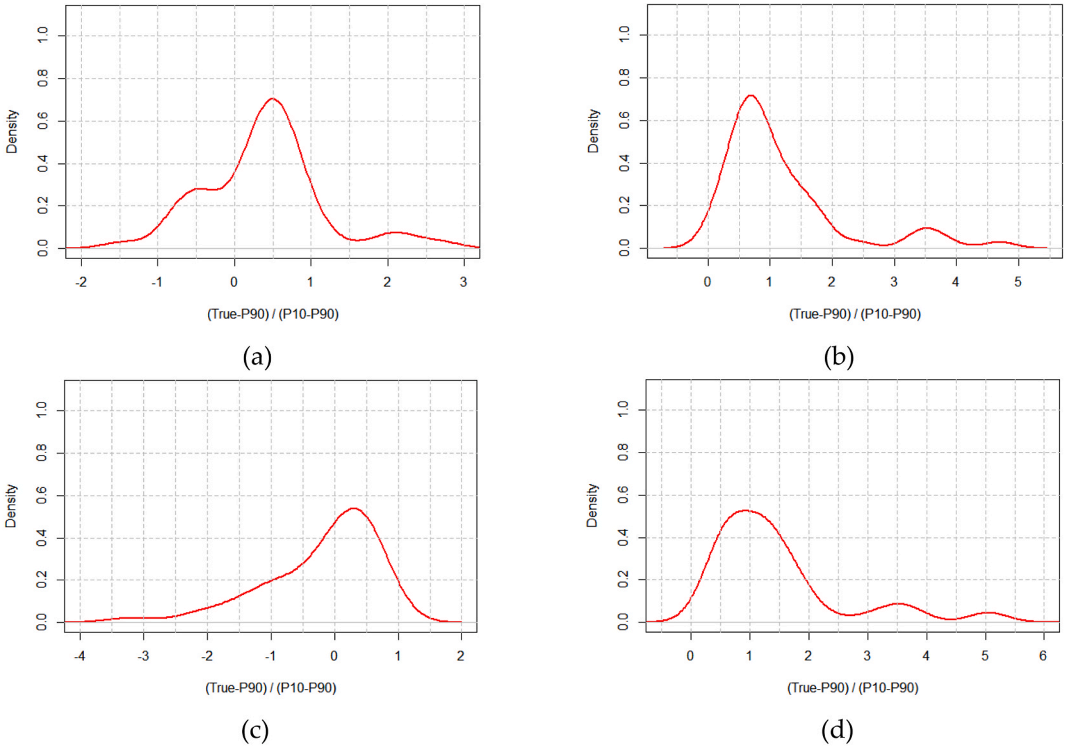

Fig. 7. (True-P90)/(P10–P90) distribution for 54 gas wells using a) the Arps model b) the PLE model c) the Duong model d) the SEPD model.

Table 13
The error analysis for the 54 gas wells using the Arps, PLE, Duong, and SEPD.

Error metric	Arps	PLE	Duong	SEPD
Average relative error, %	2.7	−0.1	8.8	−10.4
Average absolute relative error, %	13.2	0.2	9.1	14.9
Average (P10–P90)/Actual	0.15	0.16	0.11	0.15

many cases. However, in cases where a well starts with a bilinear flow regime Arps model incorporated with Bayesian methodology can be used to yield fairly good results. This is because the bilinear-flow period seems to pitch down the rest of the Arps' decline curve.

- The PLE model can be applied to all cases with a good prediction of the reserves even if a well starts with a linear flow regime that's because the PLE model underestimates the reserves for most of the cases where the reserves are not included in the CI. The model performs best when the well life is long enough to allow BDF to develop.
- Duong model is *only* valid for wells exhibiting linear flow all over their production life which is very hard to predict – as per this study's findings. Thus, researchers should refrain from using Duong in other cases because of its tendency to overestimate the reserves.
- SEPD is the best-performing model amongst the other models because it was able to give an excellent calibration for the coverage rate. The model performs best when the well life is long enough to allow BDF to develop. The only case where it fails is when the linear model is the dominant model throughout the lifetime of a well. Nonetheless, it underestimated all the reserves it failed to capture in the CI. This makes economic decisions less risk-prone if the model is used in economic studies.

Data availability

Sharing research data helps other researchers evaluate your findings, build on your work and to increase trust in your article. We encourage all our authors to make as much of their data publicly available as reasonably possible. Please note that your response to the following questions regarding the public data availability and the reasons for potentially not making data available will be available alongside your article upon publication.

CRediT authorship contribution statement

Ahmed Attia: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Resources, Methodology, Formal analysis, Data curation, Conceptualization. **Mohamed Mostafa:** Supervision, Investigation. **Adel M. Salem:** Supervision, Resources.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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