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The Heider balance and the looking-glass self: modelling dynamics of social relations

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We consider the dynamics of interpersonal relations which leads to balanced states in a fully connected network. Here this approach is applied to directed networks with asymmetric relations, and it is generalized to include self-evaluation of actors, according to the 'looking-glass self' theory. A new index of self-acceptance is proposed: the relation of an actor to him/herself is positive, if the majority of his/her positive relations to others are reciprocated. Sets of stable configurations of relations are obtained under the dynamics, where the self-evaluation of some actors is negative. Within each set all configurations have the same structure.

Graphs are known as convenient mathematical representation of social networks for long time[1–3]. In most applications, unweighted links have been used, with the only option that a link is present or not between each pair of nodes. Weak and strong links have been famously distinguished by Mark Granovetter[4,5]; the difference was motivated by their roles in diffusion of information. Directed networks seem to be a natural next step in construction of sociological models, in particular when dealing with conflicts, social inequality and violence[6–9]. Yet asymmetric social relations have been modeled relatively rarely[10–14].

Here we are interested in computational modeling of structural balance, which is a canonical example of an application of graphs in social science[15]. Briefly, the balance theory has been formulated by Fritz Heider in terms of positive and negative relations in triads[16,17]. Soon, a network of balanced triads has been shown to be equivalent to a balanced network[18]. This step has enabled applications of concepts of statistical mechanics, where signed links were equivalent to spin-like variables, and the number of unbalanced triads to energy. Stochastic algorithms designed to locate energy minima have been applied to social networks[19,20]. Besides global minima, equivalent to the structural balance, unbalanced stationary states have been identified[19], hereafter called 'jammed' or 'glassy' states[19,21–23].

On the other hand, the concept of balanced triads has found a psychological support in terms of cognitive dissonance[24]. While most of works have been concentrated on the case of symmetric relations, where negative and positive relations towards each other are always reciprocated, some papers have dealt also with the asymmetric case[21,25]. Although asymmetric relations are established in social modeling[26,27], their consequences for the structural balance are not fully explored. Here we intend to classify jammed states according to the network structure. Instead of stochastic algorithms[19,20], we intend to apply a set of differential equations[28,29] which is supposed to drive the system to a stationary state most close to the initial state. In most cases, jammed states are obtained[21]; this is the main difference with respect to algorithms of simulated annealing, a standard method formulated as to reach a global minimum of energy[30]. Although some jammed states are possible also for symmetric case[19], for asymmetric relations they are generic[21].

Our aim here is twofold. First, we are looking for generic jammed states of networks; such states are asymmetric counterparts of the structurally balanced state[15]. This task is far from trivial; it can be compared to a classification of stationary states of a neural network, where the mutual influence of neurons can be asymmetric[31,32], or to a classification of stationary states of a Boolean network[33] with asymmetric actions of genes. This comparison highlights that our problem belongs to a general family defined as: given the network evolution, find a classification of the fixed points. In our case, the solution is obtained by means of an original algorithm of classification of states of complex systems, worked out in[34,35]. Up to now, the algorithm has been applied successfully to Ising

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and Potts systems of spins, traffic systems, polymer chains, elementary cellular automata, Hubbard rings and Boolean networks; for details we refer to[35]. Here the algorithm is applied for the first time to signed and directed, fully connected networks. Our second aim is more specific and it is related to social relations. Namely, we are able to identify nodes in the network where outgoing positive links are accompanied with negative links from other nodes. Inferring from the Cooley theory of self-looking glass[36], we claim that such a state of a node is equivalent to low self-evaluation.

In the next section we highlight sociological roots of the self-evaluation, based on social relations. In section 3 we clarify the role of the directions of links in the network triads. In two subsequent sections the scheme of calculation is described and the main results are given. The last but one section is devoted to discussion, where we demonstrate that our generic results are reproduced when sociometric data from literature are used. Short remarks on the applied numerical method close the text.

Self-Evaluation by Cooley

The issue of 'social relations' is, naturally, one of foundations of sociology (including microsociology) and social psychology. Broadly understood 'interpretative social sciences' concentrate on proving that human attitudes (including emotions) and behaviors are reactions to the self-perceptions and self-images of the actors and their perceptions and images of their partners (human and non-human) rather than reactions to 'objective' human and non-human objects, processes and situations. Charles H. Cooley's contribution (see, e.g.[36]) is particularly significant in this context. His work is very influential until today in the broadly understood 'symbolic interactionism' (see, e.g.[37–40]), in social psychology (see, e.g.[41,42]), in 'sociology of individuals' (see, e.g.[43,44]), even if practitioners of the latter do not quote Cooley directly. The influence of American classic's version of symbolic interactionism on today's 'sociology of emotions' (see, e.g.[45]) is highly visible. In this article, Cooley, as the founding father of original concepts and hypotheses still applied in various fields of social sciences, is a major source of inspiration.

Famous studies authored by Cooley were based on his analysis and interpretation of belles-lettres (particularly William Shakespeare's dramas) and his observations of social (including emotional) relations within his immediate family, mostly between his children. *If a boy […] has any success, […] he gloats over it […]. He is eager to call in his friends […], saying to them, 'See what I am doing! Is it not remarkable?' feeling elated when it is praised, and resentful or humiliated when fault is found with it* ([36], p. 178).

Let us quote a significant piece of Cooley's book 'Human Nature and the Social Order': *A self-idea […] seems to have three principal elements: the imagination of our appearance to the other person; the imagination of his judgment of that appearance, and some sort of self-feeling, such as pride or mortification. […] The thing that moves us to pride or shame is not the mere mechanical reflection of ourselves, but an imputed sentiment, the imagined effect of this reflection upon another's mind. This is evident from the fact that the character and weight of that other, in whose mind we see ourselves, makes all the difference with our feeling. We are ashamed to seem evasive in the presence of a straightforward man, cowardly in the presence of a brave one, gross in the eyes of a refined one, and so on. We always imagine, and in imagining share, the judgments of the other mind. A man will boast to one person of an action - say some sharp transaction in trade - which he would be ashamed to own to another* ([36], pp. 184-5).

From a sociological perspective, our results indicate that negative self-evaluations come from unreciprocated relations. We dare say that this statement links the concept of Heider balance with the idea of 'looking-glass self' of C. H. Cooley ([36], p.152). Sometimes reported in the form of an aphorism: *each to each a looking glass/ reflects the other that doth pass*[36], the idea highlights the role of society in formation of human identity. Namely, opinion of an individual on herself/himself is formed by perceived opinions of other people.

Equations of Motion

Consider a fully connected network of N nodes and $K = N(N-1)$ links between them, $x_{ij} = \pm 1$. If links are set to be symmetric ($x_{ij} = x_{ji}$), the condition of structural balance is that for each triad (i, j, k) of nodes the links between them obey the rule $x_{ij}x_{jk}x_{ki} = +1$. It is known[18], that in this case the whole network is split in two parts, where $x_{ij} = +1$ if nodes i, j belong to the same part, otherwise $x_{ij} = -1$. This split is termed as 'structural balance' or 'Heider balance' (HB). Putting this in terms of friendly and hostile relations, all relations within each group are friendly, and all relations between groups are hostile. This means in particular, that the hostile relation is not transitive: an enemy of my enemy is my friend.

If the link variables x_{ij} are allowed to be real, the following set of differential equations of time evolution leads generically to HB[28,29]:

$$\frac{dx_{ij}}{dt} = \left(1 - x_{ij}^2\right)\sum_{k}^{N-2} x_{ik}x_{kj}$$

(1)

where $k \neq i$, $k \neq j$ in the sum. The rationale of Eq. (1) is as follows. Once k is either a common friend of i and j or their common enemy, the product $x_{ik}x_{kj}$ is positive and contributes to an increase of x_{ij} in time. On the other hand, if k is a friend of i but an enemy of j (or the opposite), the product $x_{ik}x_{kj}$ is negative and contributes to a decrease of x_{ij}. The factor $1 - x_{ij}^2$ keeps the relation x_{ij} in the finite range $[-1, +1]$; this is the price we pay for the lack of analytical solution of Eq. (1)[29]. We note that discrete algorithms leading to HB have been formulated also for the symmetric case[19,20].

When the condition of symmetry is released, the order of indices does matter. Basically, the inference of actor i about her/his relation to j could be modeled in several ways of ordering; several possible types of directed and signed triads have been considered[10,11,46–48]. Our choice to keep the ordering as in Eq. (1)[21,25] is driven by the lack of transitivity, mentioned above. In terms of social relations, the question on transitivity[2] is: once actor A wonders about his feeling about B, how important is that A dislikes C and C dislikes B? We can imagine that the issue is less important when B is a product[11], on the contrary to the case when B is an enemy or a friend. We note also
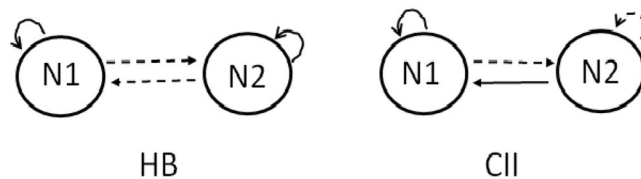
Figure 1. The generic graphs of two classes. The graph noted as HB shows the usual form of the Heider balance: the classes contain $N1$ and $N2$ nodes (actors), with all relations within the same class friendly, all relations between the classes hostile, and all links of this graph symmetric. The graph noted as CII contains class 1 with friendly internal relations and class 2 with internal relations hostile. The relations of those from $N1$ towards those from $N2$ are hostile (dashed arrow), and the relations of those from $N2$ towards those from $N1$ are friendly (continuous arrow).

that another ordering $x_{ik}x_{jk}$, symmetric vs an exchange i and j, would drive the system to symmetric relations, and therefore is out of interest here. Finally, we should add that we treat the character of all relations as known to all actors, what is appropriate only when dealing with small groups; more general picture has been discussed by Carley and Krackhardt[10].

The index F_i of self-evaluation of each actor ($i = 1, 2, ..., N$) is calculated as

$$F_i = \frac{1}{2}\sum_k (1 + x_{ik})x_{ki}. \tag{2}$$

This form of F_i is appropriate as long as we are interested only in the opinions of actors k liked by i ($x_{ik} = +1$); those whom i dislikes ($x_{ik} = -1$) do not contribute to F_i. Here we are motivated by the concept od significant others[49]. Basically, $-N < F_i < N$.

Calculations

Some initial configurations lead to balanced states which are necessarily symmetric. In a symmetric state $F_i \geq 0$ for each actor i; F_i is just the number of friends of i, and this friendship is reciprocated. Some others lead to asymmetric states, which cannot be balanced; here we are interested in those states where $F_i < 0$ for some actors i. Among those liked by her/him, such an actor has more enemies than friends; F_i is just the difference between friends and enemies of this kind.

At this stage, the outcome of the calculation is a set of $K(N)$ matrices x_{ij}. We prefer to keep N odd to evade the cases where the right side of Eq. (1) is equal to zero in a stationary state. We have got results for $K = 10^4$ for each $N = 7, 9, 11, 41, 55$ and 77; for $N = 99$, the dynamic system is found to be less stable numerically. For each N, the matrices are classified as equivalent to unlabeled graphs of classes of nodes. In short, the algorithm is as follows[34,35]:

1. for each node i, the number $M(i)$ of nodes is found such that $x_{im} = +1$, and the number $L(i)$ of nodes is found such that $x_{li} = +1$;
2. nodes of the same M, L are provisionally classified as belonging to the same class;
3. neighbours of nodes in the same class are checked if they belong to the same classes;
4. if not, more fine classes are introduced;
5. steps 3 and 4 are repeated until the condition 3 is true.

The resulting graph is encoded as the matrix of relations between the classes plus information on the numbers of nodes in each class. We note that all nodes in the same class have the same numbers of neighbors in the same classes; the same applies to neighbors of neighbors etc[34,35]. Obviously, all nodes i in the same class have the same value of F_i.

Results

The results on the matrices of relations between classes can be written in the form of graph of classes. To make it more clear, we provide two examples in Fig. 1. There, a continuous (dashed) arrow from class A to class B means a positive (negative) relation of members of A towards class B. The same applies to arrows from A to A; in this case the relation is of the group members towards other members of the same group; the relation F_i of an actor to him/herself is not marked in the figures. On the left there (HB), a classical case of the Heider balance is shown; all links are symmetric, then there are no nodes with F_i negative, and the graph is shown for completeness only. On the right side of the figure, the graph CII contains two classes: class 1 with internal relations friendly, and class 2 with internal relations hostile. The stability conditions for a friendly relation x_{ij} is that the sum $\sum x_{ik}x_{kj}$ (as in Eq. (1)) should be positive. For the internal links within the class 1 this sum is equal $N1 - 2 - N2$, hence the stability in this case yields $N1 > N2 + 2$. For the internal links within the class 2, the appropriate sum should be negative, then their stability is assured if $N2 - 2 - N1 < 0$ - a less demanding condition. Accordingly, the negative link from class 1 to class 2 is stable if $-(N1 - 1) + N2 < 0$, and the positive link from class 2 to class 1 is stable if $-(N2 - 1) + (N1 - 1) > 0$. Summarizing this thread, the stability condition for the graph CII is that $N1 > N2 + 2$. The self-evaluation index $F_i = N1 - 1$ for the class 1, and $F_i = -N1$ for the class 2. Yet we note that the graphs CII are more rare, than the others.
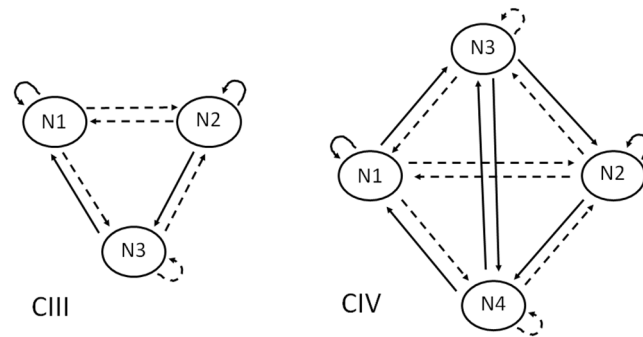
Figure 2. The generic graphs of three (CIII) and four (CIV) classes. N1–N4 mean the numbers of nodes (actors) in a given class. Continuous arrows from N1 to N2 mean friendly relations of $N1$ towards $N2$ (members of 1 like those in 2), and dashed arrows mean hostile relations. The same applies to self-directed arrows: a dashed self-directed arrow means that all members of a given class dislike each other.

N1	N2	N3	N4	#
3	3	1	0	1465
4	2	1	0	1125
2	4	1	0	1111
3	2	2	0	474
2	3	2	0	462
1	5	1	0	443
5	1	1	0	422
1	4	2	0	226
4	1	2	0	224
2	3	1	1	929
4	1	1	1	442

Table 1. Most frequent configurations of graphs of classes for CIII ($N4 = 0$) and CIV ($N4 > 0$) for $N = 7$. $N1 \div N4$ mean the numbers of nodes (actors) in a given class $1 \div 4$, and # counts how many times the graph appeared at the end of evolution in the sample of 10^4 networks. The structure of graphs is shown in Fig. 2. For CIII, configurations are shown for # > 100, and for CIV - for # > 5.

Numerical results indicate that two kinds of configurations of graphs of classes appear much more frequently than others. These two kinds can be classified as CIII and CIV, as they are composed of three and four classes, respectively. Both CIII and CIV appear for different numbers N of actors in particular classes. These graphs are shown in Fig. 2, with the same meaning of continuous and dashed arrows.

For the graph CIII, the stability condition $x_{ij} = sign(\sum_k x_{ik}x_{kj})$ is equivalent to the following: $N1 + N2 > N3 + 2$ for links within class 1, within 2, from 1 to 2 and back, and within class 3. For other links (from 1 to 3 and back, and from 2 to 3 and back), the condition is weaker: $N1 + N2 > N3$. Similarly, for CIV the most demanding condition is $N1 + N2 > N3 + N4 + 2$. As we see in Table 1, all the obtained links are stable. In the last column of Table 1 we show the obtained statistics of CIII and CIV for $N = 7$. The results indicate that asymmetric solutions appear in about 73 percent of our sample of 10^4 networks.

For larger values of N, the number of configurations is larger, but again CIII and CIV appear more frequently. For $N = 9$, the number of actors in the graphs of classes of leading frequencies are listed in Table 2. The sum of the right column covers more than 78 percent of the sample. For $N = 11$, we got 3686 graphs CIII and 5120 graphs CIV, what makes 88 percent of the sample. For $N = 41$ the population changes: 6974 graphs CIV (only those with frequency n not less than 10) and only 10 single graphs CIII. For $N = 55$ and $N = 71$ there are no graphs CIII at all. This effect is presumably a consequence of the fact that the number of partitions of N into four non-zero sets increases with N much quicker, than the number of partitions of N into three non-zero sets. Basically, these numbers are known as the Stirling numbers of second kind, $S(N, 4)$ and $S(N, 3)$. The ratio $S(N, 4)/S(N, 3)$ is 1.2 for $N = 7$, 2.6 for $N = 9$, 5.1 for $N = 11$ but about 3×10^4 for $N = 41$[50]. This calculation is not exact because the stability conditions are ignored there, but even a very rough evaluation explains the observed lack of CIII for $N > 40$. In Fig. 3 the frequencies of various graphs CIV (with different numbers $N1 \div N4$) are shown. We can conclude that the percentage of asymmetric graphs is meaningful.

Another question is the sign of F_i in the obtained classes. Here we are interested in the cases where $F_i < 0$. For CIII, this condition is equivalent to: $N1 < 1$, $N2 < N3 + 1$ and $N1 > 0$ for the actors in the class 1, 2 and 3, respectively. As we can infer from the data in Table 1 for $N = 7$, F_i is always negative only for actors in class 3. The self-feeling F_i of the remaining nodes is sometimes zero, sometimes positive, sometimes it can be negative as well. Similarly, for CIV the values of F_i are equal to $N1 - N3 - 1$, $N2 - N4 - 1$, $N4 - N2$ and $N3 - N1$ for actors in classes

N1	N2	N3	N4	#
4	4	1	0	721
5	3	1	0	610
3	5	1	0	597
4	3	2	0	547
3	4	2	0	545
5	2	2	0	347
2	5	2	0	327
2	6	1	0	294
6	2	1	0	276
3	3	3	0	172
3	4	1	1	1084
5	2	1	1	664
3	3	1	2	435
4	2	1	2	375
4	2	2	1	353
1	6	1	1	196
5	1	1	2	139
5	1	2	1	138

Table 2. Most frequent configurations of graphs of classes for CIII ($N4 = 0$) and CIV ($N4 > 0$) for $N = 9$. The notation is as in Table 1. For CIII, configurations are shown for $\# > 150$, and for CIV - for $\# > 3$.
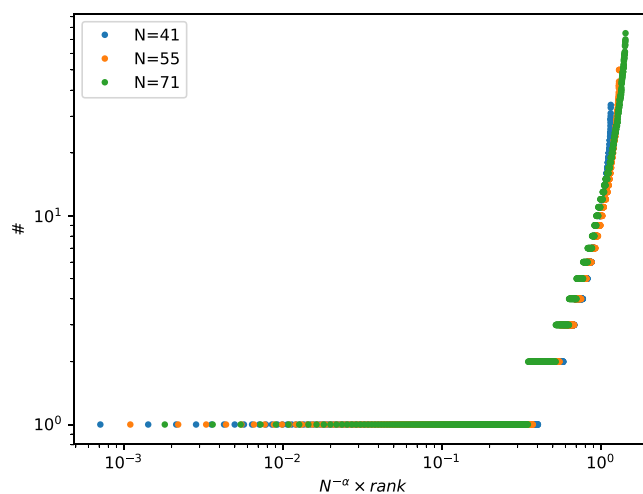
Figure 3. Frequency # of different graphs CIV for $N = 41$, $N = 55$, and $N = 71$. By different graphs we mean that the numbers $N1 \div N4$ are different. The graphs are ordered with increasing frequency; higher rank is assigned to those which appear more frequently. When the rank is multiplied by $N^{-\alpha}$, an approximate scaling yields $\alpha \approx 1.7$.

1, 2, 3 and 4, respectively. As we can infer from the stability condition given above ($N1 + N2 > N3 + N4 + 2$), F_i is negative for most actors in classes 3 and 4.

Discussion
Our numerical results indicate that:

- the evolution Eq. (1) frequently drives the system to asymmetric states with structure CIII or CIV as shown in Fig. 2,
- in small systems ($N \approx 10$) graphs of classes CIII and CIV are abundant, while in larger systems ($N > 40$) graphs CIII disappear,
- the stability conditions demand that the classes 1 and 2 (with positive internal relations) contain more actors than the classes 3 and 4,
- for actors in the classes 3 and 4, negative values of F_i are quite typical.

As it was demonstrated in refs[28,29] in the case of symmetric relations ($x_{ij} = x_{ji}$) the generic stable solution of the Eq. (1) is balanced in the Heider sense. An example of such solution is shown in Fig. 2 (HB) in the form of a graph of classes. Recall that there are two classes, one with $N1$ actors and one with $N2$. Here we show that when

the condition of symmetry is released, the graphs of classes CIII and CIV (shown in Fig. 2) are most probable solutions of Eq. (2). When the number of actors N increases, the frequency of CIV prevails. While the numbers of actors $N1$, $N2$, $N3$, $N4$ vary, the structure of the graph of classes remains unchanged. We dare say that the structure of CIII and foremost CIV with its specific symmetry are counterparts of the balanced state for asymmetric relations. What we do not know is, if there is no more solutions of this kind. Recall that for symmetric relations and $N = 9$, a stable state of three triads has been found, which is different from a classical balanced partition into two groups[19]. At this stage we cannot state that there is no other asymmetric and stable states, different from CIII and CIV.

To comment the structure of the graphs, the graph CII could represent peer rejection[51] of those in class 2 by those in class 1 and by other members of the class 2. The class 3 in CIII can be assigned to a person (or persons) at intermediate position between two mutually hostile groups. She/he aspires to belong to one of them, but is not accepted there; the aspiration makes her/him unwilling to join the other group, where she/he would be accepted. This scheme reminds some fictitious characters known in belles-lettres, as Werther[52] or Julian Sorel[53]. In general, the structure of CIII should be visible when characterizing social positions of mobile individuals, aspiring to move between mutually hostile groups. A similar scheme of relations has been identified in ethnic conflicts, where higher educated immigrants perceive more discrimination that their less educated compatriots. The phenomenon is known as 'paradox of integration' (see[54–57] and references therein). The structure of the graph CIV is the most complex of those found here. There we see two mutually hostile groups 1 and 2 (as usually in the balanced state) plus two relatively small groups 3 and 4.

As it can be inferred from our numerical results, the graphs CIII and CIV appear to be generic. Therefore we can expect that they should be present in some sociometric data. This expectation has been verified by using the Sampson data[58]. Namely, four pairs of non-symmetric, valued matrices $(X, Y) = $ (SAMPLK, SAMPDLK), (SAMPES, SAMPDES), (SAMPIN, SAMPNIN), (SAMPPR, SAMPNPR)[58] were used to form four signed and valued non-symmetric matrices $(X − Y)/5$. The difference measures the positive minus the negative relations, and the factor $1/5$ was used to keep the relations in the range $(−1, 1)$. These four matrices have been used as the initial values of x_{ij}. We have found that only one pair (SAMPLK-SAMPDLK)/5 was dense enough to produce a fully connected graph by means of Eq. (1). The structure of this graph has been found to be identical with CIII. There, the class 1 contains ten actors (1, 2, 4, 5, 6, 7, 11, 12, 14, 16), the class 2: four actors (13, 16, 17, 18), and the class 3 also four (3, 8, 9, 10), with enumeration as the rows and columns of the matrices given in[58]. Two other matrices (SAMPES-SAMPDES)/5 and (SAMPIN-SAMPNIN)/5 appeared to be dense enough, i.e. their evolution lead to fully connected graphs, only if the first row and column was removed; the first actor was apparently less connected than the others. With this cut, the pair (SAMPIN-SAMPNIN)/5 has been found to lead to CIII, with the composition $N1 = 7$, $N2 = 9$, $N3 = 1$. The matrix (SAMPES-SAMPDES)/5 gave CIV with the composition $N1 = 5$, $N2 = 7$, $N3 = 3$, $N4 = 2$. The matrix (SAMPES-SAMPDES)/5 gave CIV with the composition $N1 = 5$, $N2 = 7$, $N3 = 3$, $N4 = 2$, and the same composition has been produced by (SAMPLK-SAMPDLK)/5 as the initial state. The fact that both CIII and CIV are reproduced from the well-known set[58] suggests that these graphs are generic also in other sociometric data.

To conclude, the former theory of removal of cognitive dissonance[28] has been extended here to include asymmetric relations. Our results indicate, that the time evolution starting from generic initial conditions leads in most cases to the graphs of classes CIII and CIV. In other words, for the dynamics given by Eq. 1 these two classes appear to be universal. This computational result is obtained by an application of the classifying algorithm[34,35] to the states of signed and directed fully connected networks. Further, according to the Cooley theory, the structure of these graphs is particularly harmful for actors at some positions in these graphs, indicated above. On the other hand, this structure can be identified in field experiments by means of standard sociometric methods. These results can be of interest for scientists working on conflicts in groups, and for teachers in classes where conflicts appear.

Methods

For each sample out of a set of K fully connected networks of size N, initial conditions for $N(N − 1)$ links $x_{ij}(t = 0)$ have been chosen randomly from a homogeneous distribution $\rho(x) = 1/2$ for $−1 < x < 1$, zero otherwise. Next, the time evolution of x_{ij} has been initialized according to Eq. (1). The resulting set of differential equations has been solved numerically using the *Mathematica* software for a sample of $K = 10^4$ networks for each N. Typically, the system has ended at one of hypercube corners where $x_{ij} = \pm 1$ (with the relative accuracy $10^{−8}$ serving as the stop condition of the simulation) for each pair i, j. The convergence to $x_{ij} = \pm 1$ has been reached in 97% of the cases for $N = 7, 9, 11$, in 96% cases for $N = 41$, in 95% cases for $N = 55$, and in 93% cases for $N = 77$. For $N = 99$, the numerical solution of the system of differential equations has proved to be unstable, with at least 55% of the cases not converging.

Data Availability

The data used here are publicly available at the last position of the list of References (Bagatelj V. and Mrvar A. UCINET IV Datasets (2006)).

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Author Contributions

K.K. designed the research, M.J.K., M.W. and P.G. performed the simulations, K.K. analysed the results, K.K., J.M. and M.W. wrote the text. All authors reviewed the manuscript.

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