The analysis of a drug transmission model with family education and public health education

Jun Li [a, *], Mingju Ma [b]

[a] School of Mathematics and Statistics, Xidian University, Xi'an, 710071, PR China
[b] School of Science, Xi'an Polytechnic University, Xi'an, 710048, PR China

A B S T R A C T

In this paper, we formulate a six dimensional drug transmission model to study the effect of family education and public health education. The dynamical behaviors of the model are discussed in terms of the basic reproduction number R_0. By constructing Lyapunov functions, we obtain the drug-free equilibrium is globally asymptotically stable if $R_0 \leq 1$ and the drug addiction equilibrium is globally asymptotically stable if $R_0 > 1$. Sensitivity analyses are performed to seek for effective control measures for drug spread. The analysis show that both the family and public health education can influence the spread of drug transmission. However, the combination of family and public health education is more effective to reduce the prevalence of drugs. Some numerical simulations are given to confirm the obtained theoretical results.

© 2018 The Authors. Production and hosting by Elsevier B.V. on behalf of KeAi Communications Co., Ltd. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).

1. Introduction

Drug abuse is one of the most serious health and social problems around the world and has attracted governments' attentions. More and more people are infected by various drugs. Curiosity, thrill, seeking negative mentality and environmental impact are the main key factors to tempt susceptible individuals to contact with drugs. The U.S. Centers for Disease Control and Prevention in 2015 reported that from 2002 to 2013, the number of Americans over the age of 12 who snort heroin increased 63% and the deaths increased three times in America (News xinhuanet). According to "China's drug situations report in 2015", by the end of 2015, the registered drug addicts are 2.345 million which increased by 14.6% compared to 2014 (Legal people). In addition, the crimes caused by taking drugs are growing rapidly, such as theft, robbery, self-injury, violence and traffic accidents and so on, which severely damage public security and public safety. It was reported by South African police statistics that the drug-related crimes increased from 621 to more than 3000 from 2002 to 2006 in Cape Town (Kapp, 2008). In addition, the drug abuse also aggravates the spread of infectious diseases such as hepatitis and AIDS (Mushanyu, Nyabadza, & Stewart, 2015; Nyabadza, Njagarah, & Smith, 2013). The AIDS patients increased from 13.1% in 2003 to 15.7% in 2005 in the Western Cape Province of South Africa reported by antenatal survey in 2005 (Parry Charles et al., 2009). How to analyze the transmission mechanism of drugs better and present effective prevention measures is a problem that should be solved urgently.

---

* Corresponding author. School of Mathematics and Statistics, Xidian University, Xian, Shaanxi, Taibai South Road No. 2, 710071, PR China.
 E-mail addresses: lijun@xidian.edu.cn (J. Li), mingjuma@163.com (M. Ma).

Mathematical models play very important role in dealing with epidemics and biology, see (Cen, Feng, & Zhao, 2014; Driessche, Watmough, & van den Driessche, 2002; Feng, Cen, Zhao, & Velasco-Hernandez, 2015; Funk, Salathe, & Jansen, 2010; Huo, Chen, & Wang, 2016; Li, Graef, & Wang, 1999; Li, Zhao, & Li, 2013; Li, Zhao, & Zhu, 2015) and references therein. The process of epidemics could be displayed by mathematical models vividly. As a kind of new epidemics, drug abuse and drinking have the similar characteristics with traditional epidemics, and several different mathematical models for drugs had been formulated and studied, see (Liu & Zhang, 2011; Muroya, Li, & Kuniya, 2014; White & Comiskey, 2007). White and Comiskey (2007) first presented a heroin model, in which they divided the total population into susceptible individuals, drug users not in treatment and drug users in treatment. They obtained the basic reproduction number R_0 and gave the sensitivity analysis of R_0. Then they got the conclusion that prevention was better than cure. Based on the work of (White and Comiskey, 2007), Liu and Zhang (2011) took distributed delays into a heroin model. It was shown that the stabilities of equilibria and uniform persistence were related to the basic reproduction number R_0. In view of "light drug" and "heavy drug", Muroya et al. (2014) introduced an SIRS model and discussed the global stability by applying Lyapunov function techniques. The other drug models are referred to (Fang, Li, Martcheva, & Cai, 2014; Huang & Liu, 2013; Kalula & Nyabadza, 2011; Mulone & Straughan, 2009; Samanta, 2011; Wang, Yang, & Li, 2011).

It is well known that the social behaviour plays a vital role when infectious diseases appear and outbreak. The common ways are isolation, vaccination and publicity to the health safety and so on. Some scholars took into account of public health educational campaigns in their models. Bhunu, Mushayabasa, Kojouharov, and Tchuenche (2010) analyzed an HIV/AIDS model with the impact of educational programs. Mukandavire and his partners in (Mukandavire, Garira, & Tchuenche, 2009) studied the effects of public health educational campaigns on the spread of HIV/AIDS. They proved the campaigns could slow down the epidemic. For drinking, Xiang, Song, and Huo (2015) dealt with the global properties of a drinking model with public health education. They concluded that public health education is one of the effective measures to control the drinking problems.

Motivated by above works, we develop a six dimensional drug model with family education and public health education in the paper. According to analyze the model, we attempt to provide an effective suggestion for controlling drug transmission. The organization of this paper is as follows. In the next section, the model is built and some basic properties are derived. In Sec.3, the basic reproduction number R_0 is obtained and the stability of the drug-free equilibrium of system is discussed. In Sec.4, the existence and the stability of the drug-addiction equilibrium are investigated. In Sec.5, some numerical simulations are carried out to illustrate the theoretical results and the sensitivity analysis is performed with respect to model parameters. Some discussions and summaries are given in last section.

2. The model

2.1. The model description

In order to describe the spread process of drugs, we formulate a six dimensional drug model. The total population $N(t)$ is divided into six compartments: susceptible individuals who do not accept the health education(S), susceptible individuals who accept the health education (C), light drug addicts (L), heavy drug addicts (H), drug users in treatment (T) and quit drug-users permanently (R), namely, the total population at time t is given by $N(t) = S(t) + C(t) + L(t) + H(t) + T(t) + R(t)$. The meanings of S, C, L, H, T and R are demonstrated concisely in Table 1. There is evidence that drug abuse phenomenon tends towards younger (Comiskey, 1999), so we focus on the population over the age of 15 old. Generally speaking, the health education includes the family education and the public health education. The individuals before age 15 access to the drug safety education mainly from families, and after the age of 15, the corresponding education comes from society or school, which could be called public health education. We suppose that the new recruits entering the susceptible humans denoted by $d\Lambda$ is a part of the general population. A proportion $1 - q(0 < q < 1)$ of these humans do not accept family education and the complementary proportion q accept family health education. Meanwhile, the proportion μ of S will be educated by public health and enter into class C. β_1 and β_2 are the effective contact rates when susceptible humans (S) contact with light drug addicts (L) and heavy drug addicts (H) respectively. Generally, we assume $\beta_2 > \beta_1$ because a susceptible individual is easier captured by a heavy drug addict. Once someone accepts public health education, he will reject drugs and avoid contacting with addicts consciously, so the effective contacts rates would reduce. Therefore the public health educated susceptible

Table 1
The state variables for model.

Variable	Description
$S(t)$	The number of susceptible drug individuals who do not accept the public health education at time t.
$C(t)$	The number of susceptible drug individuals who accept the public health education at time t.
$L(t)$	The number of light drug addicts at time t.
$H(t)$	The number of heavy drug addicts at time t.
$T(t)$	The number of drug users in treatment at time t.
$R(t)$	The number of permanent quit drug individuals at time t.

individuals (C) turn to light addicts (L) at rates $\beta_1\xi$ and $\beta_2\xi$ where $\xi \in (0,1)$. Based on the aforementioned modeling assumptions, the transfer diagram of the proposed model is shown in Fig. 1.

Corresponding, the proposed model is defined by the following ordinary differential equations:

$$
\begin{cases}
\dfrac{dS}{dt} = (1-q)d\Lambda - \beta_1 SL - \beta_2 SH - (d+\mu)S, \\[2mm]
\dfrac{dC}{dt} = qd\Lambda + \mu S - \beta_1 \xi CL - \beta_2 \xi CH - (d+\delta)C, \\[2mm]
\dfrac{dL}{dt} = \beta_1 SL + \beta_2 SH + \beta_1 \xi CL + \beta_2 \xi CH - (\gamma + \pi + d + d_1)L, \\[2mm]
\dfrac{dH}{dt} = \pi L + \sigma T - (\theta + d + d_2)H, \\[2mm]
\dfrac{dT}{dt} = \theta H + \gamma L - (\sigma + m + d)T, \\[2mm]
\dfrac{dR}{dt} = \delta C + mT - dR.
\end{cases}
\tag{1}
$$

For convenience, we list all the parameters and their meanings in Table 2.

Since the last equation of system (1) does not have any contribution to the other equations of system (1), denote $m_1 = \gamma + \pi + d + d_1$, $m_2 = \theta + d + d_2$, $m_3 = \sigma + m + d$, then from now on we only concentrate on the dynamics analysis of following reduced system:

$$
\begin{cases}
\dfrac{dS}{dt} = (1-q)d\Lambda - \beta_1 SL - \beta_2 SH - (d+\mu)S, \\[2mm]
\dfrac{dC}{dt} = qd\Lambda + \mu S - \beta_1 \xi CL - \beta_2 \xi CH - (d+\delta)C, \\[2mm]
\dfrac{dL}{dt} = \beta_1 SL + \beta_2 SH + \beta_1 \xi CL + \beta_2 \xi CH - m_1 L, \\[2mm]
\dfrac{dH}{dt} = \pi L + \sigma T - m_2 H, \\[2mm]
\dfrac{dT}{dt} = \theta H + \gamma L - m_3 T,
\end{cases}
\tag{2}
$$

with the initial condition

$$
\begin{aligned}
S(0) &= S_0 > 0, \quad C(0) = C_0 > 0, \quad L(0) = L_0 > 0, \\
H(0) &= H_0 > 0, \quad T(0) = T_0 > 0.
\end{aligned}
\tag{3}
$$

From the point of view of biology, we only care about the dynamics of system (2) in the feasible region as follows:
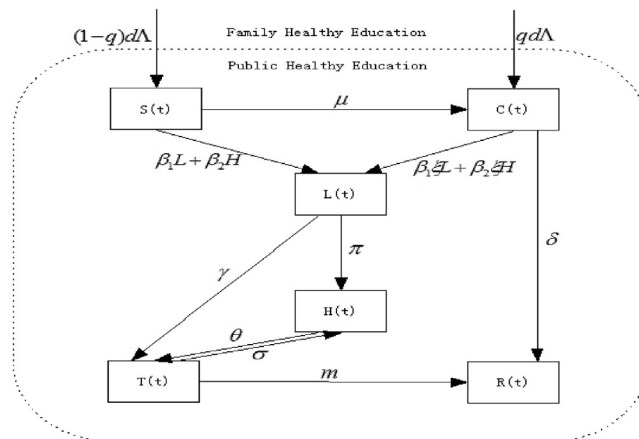

Fig. 1. The transfer diagram of drug model with health education.

Table 2
Description of parameters.

Parameter	Description
d	Natural birth/death rate of humans.
Λ	The total size of new recruits.
q	popularizing rate of family education.
μ	The rate that susceptible individuals accept public health education.
β_1	The effective contact rate that susceptible individuals contact with light drug addicts.
β_2	The effective contact rate that susceptible individuals contact with heavy drug addicts.
ξ	The factor which results the effective contact rates decrease.
d_1	The death rate related the light drug addiction.
d_2	The death rate related the heavy drug addiction.
γ	The treatment rate of light drug addicts.
π	The transformation rate from light addicts to heavy addicts.
δ	The permanent quit drug rate because of public health education.
θ	The treatment rate of heavy addicts.
σ	The relapse rate after treatment.
m	The permanent quit drug rate after treatment.

$$\Omega = \{(S,C,L,H,T)|S \geq 0, C \geq 0, L \geq 0, H \geq 0, T \geq 0, S+C+L+H+T \leq \Lambda\} \tag{4}$$

In the next subsection, it can be showed that the set Ω is positively invariant.

2.2. The basic properties

For system (2), it is necessary to prove all of solutions with initial conditions (3) remain positive for all $t \geq 0$. We first state the following conclusions:

Lemma 2.1. *All the solutions of system* (2) *with initial conditions* (3) *are nonnegative for all $t \geq 0$. For system* (2)*, the set Ω is positively invariant.*

Proof. Suppose (S,C,L,H,T) is any solution of model (2), we have

$$\frac{dS}{dt}\bigg|_{S=0} = (1-q)d\Lambda > 0, \qquad \frac{dC}{dt}\bigg|_{C=0} = qd\Lambda + \mu S \geq 0,$$

$$\frac{dL}{dt}\bigg|_{L=0} = \beta_2 SH + \beta_2 \xi CH \geq 0, \quad \frac{dH}{dt}\bigg|_{H=0} = \pi L + \sigma T \geq 0,$$

$$\frac{dT}{dt}\bigg|_{T=0} = \theta H + \gamma L \geq 0.$$

With the continuity of the solution of system (2), $S(t) > 0, C(t) > 0, L(t) > 0, H(t) > 0, T(t) > 0$ for all $t \geq 0$. Adding all the equations of system (2), it is obtained

$$\frac{d(S+C+L+H+T)}{dt} = d\Lambda - d(S+C+L+H+T) - d_1 L - d_2 H$$

$$\leq d\Lambda - d(S+C+L+H+T).$$

Thus from Birkhoff's and Rota's theorems (Birkhoff & Rota, 1998), it follows $0 \leq S+C+L+H+T \leq \Lambda$ as $t \to \infty$, so the region Ω is a positively invariant set.

3. The drug-free equilibrium E_0

The basic reproduction number R_0 is an important index in epidemiology. It is a threshold value which could represent how many secondary infections result from the introduction of one infected individual into a population of susceptible (Diekmann & Heesterbeek, 2000). In the section, we calculate the basic reproduction number R_0 and give the kinetic properties of the drug-free equilibrium E_0.

3.1. The basic reproduction number

There always exists a drug-free equilibrium $E_0 = (S_0, C_0, 0, 0, 0)$ of system (2), where $S_0 = \frac{(1-q)d\Lambda}{d+\mu}, C_0 = \frac{(\mu+qd)d\Lambda}{(d+\delta)(d+\mu)}$. Next, the basic reproduction number of system (2) will be obtained applying the next generation matrix (Van den Driessche & Watmough, 2002) and the local stability of drug-free equilibrium E_0 will be established.

Rewrite $X = (L, H, T, S, C)$, then it follows from system (2) that $\frac{dX}{dt} = \mathbb{F} - \mathbb{V}$, where

$$
\mathbb{F} = \begin{pmatrix} (S+\xi C)(\beta_1 L + \beta_2 H) \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix},
$$

$$
\mathbb{V} = \begin{pmatrix} m_1 L \\ m_2 H - \pi L - \sigma T \\ m_3 T - \theta H - \gamma L \\ S(\beta_1 L + \beta_2 H) + (d+\mu)S - (1-q)d\Lambda \\ \xi C(\beta_1 L + \beta_2 H) + (d+\delta)C - qd\Lambda - \mu S \end{pmatrix}.
$$

Thus the corresponding linearized matrixes evaluated at drug-free equilibrium E_0 are

$$
F(E_0) = \begin{pmatrix} \beta_1(S_0 + \xi C_0) & \beta_2(S_0 + \xi C_0) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},
$$

and

$$
V(E_0) = \begin{pmatrix} m_1 & 0 & 0 & 0 & 0 \\ -\pi & m_2 & -\sigma & 0 & 0 \\ -\gamma & -\theta & m_3 & 0 & 0 \\ \beta_1 S_0 & \beta_2 S_0 & 0 & d+\mu & 0 \\ \beta_1 \xi C_0 & \beta_2 \xi C_0 & 0 & -\mu & d+\delta \end{pmatrix}.
$$

Obviously, $F(E_0)$ is non-negative. By directly calculate, the basic reproduction number $R_0 = \rho(FV^{-1}) = (S_0 + \xi C_0)\frac{\beta_1 \tilde{A} + \beta_2}{m_1 \tilde{A}}$, where $\tilde{A} = \frac{m_2 m_3 - \theta \sigma}{m_3 \pi + \sigma \gamma}$. Here, we can interpret the dimensionless quantity R_0 is the drug generation number which could measure the average number of new drug users generated by a drug addict in a population of susceptible individuals.

By Theorem 2 in (Van den Driessche & Watmough, 2002), the following conclusion is obvious:

Theorem 3.1. *For system (2), the drug-free equilibrium E_0 is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.*

3.2. The global stability of the drug-free equilibrium E_0

In this subsection, the global stability of the drug-free equilibrium E_0 will be established by applying the Lyapunov functional technique.

Theorem 3.2. *For system (2), the drug-free equilibrium E_0 is globally asymptotically stable when $R_0 \leq 1$.*

Proof. *Define Lyapunov function as follows*:

$$
V = S - S_0 - S_0 \ln \frac{S}{S_0} + C - C_0 - C_0 \ln \frac{C}{C_0} + L
$$

$$
+ \frac{m_3(m_1 - \beta_1(S_0 + \xi C_0))}{\pi m_3 + \sigma \gamma} H + \frac{\sigma(m_1 - \beta_1(S_0 + \xi C_0))}{\pi m_3 + \sigma \gamma} T,
$$

then the derivative of V is given by

$$V' = \left(1 - \frac{S_0}{S}\right)S' + \left(1 - \frac{C_0}{C}\right)C' + L' + \tilde{B}H' + \xi T'$$

$$= (S - S_0)\left((1-q)d\Lambda\left(\frac{1}{S} - \frac{1}{S_0}\right) - \beta_1 L - \beta_2 H\right)$$

$$+ (C - C_0)\left(qd\Lambda\left(\frac{1}{C} - \frac{1}{C_0}\right) + \mu\left(\frac{S}{C} - \frac{S_0}{C_0}\right) - \beta_1\xi L - \beta_2\xi H\right)$$

$$+ (\beta_1 L + \beta_2 H)((S - S_0) + \xi(C - C_0) + S_0 + \xi C_0)$$

$$- m_1 L + \frac{m_3(m_1 - \beta_1(S_0 + \xi C_0))}{\pi m_3 + \sigma\gamma}(\pi L + \sigma T - m_2 H)$$

$$+ \frac{\sigma(m_1 - \beta_1(S_0 + \xi C_0))}{\pi m_3 + \sigma\gamma}(\theta H + \gamma L - m_3 T)$$

$$= (1-q)d\Lambda\left(2 - \frac{S}{S_0} - \frac{S_0}{S}\right) + qd\Lambda\left(2 - \frac{C}{C_0} - \frac{C_0}{C}\right)$$

$$+ \mu S_0\left(1 + \frac{S}{S_0} - \frac{C}{C_0} - \frac{SC_0}{CS_0}\right) + f(H)$$

$$= qd\Lambda\left(2 - \frac{C}{C_0} - \frac{C_0}{C}\right) + dS_0\left(2 - \frac{S}{S_0} - \frac{S_0}{S}\right)$$

$$+ \mu S_0\left(3 - \frac{S_0}{S} - \frac{C}{C_0} - \frac{SC_0}{S_0 C}\right) + f(H),$$

where

$$f(H) = \left(\beta_2(S_0 + \xi C_0) + \tilde{A}(\beta_1(S_0 + \xi C_0) - m_1)\right)H$$
$$= \left(\left(\beta_2 + \beta_1\tilde{A}\right)(S_0 + \xi C_0) - \tilde{A}m_1\right)H$$
$$= m_1\tilde{A}(R_0 - 1)H,$$

and

$$\tilde{B} = \frac{m_3(m_1 - \beta_1(S_0 + \xi C_0))}{\pi m_3 + \sigma\gamma}, \xi = \frac{\sigma(m_1 - \beta_1(S_0 + \xi C_0))}{\pi m_3 + \sigma\gamma}$$

It is easy to see $f(H) \leq 0$ when $R_0 \leq 1$. Furthermore, $f(H) = 0$ if and only if $R_0 = 1$. Meanwhile, by arithmetic geometric average inequality, we have

$$2 - \frac{S}{S_0} - \frac{S_0}{S} \leq 0, \qquad 2 - \frac{C}{C_0} - \frac{C_0}{C} \leq 0,$$

$$3 - \frac{S_0}{S} - \frac{C}{C_0} - \frac{SC_0}{S_0 C} \leq 0.$$

Therefore, $V' \leq 0$ for $S, C > 0$, and $V' = 0$ holds if and only if $C = C_0, S = S_0$ and $R_0 = 1$. The largest invariant set in G is the singleton E_0. By LaSalle (1976), the drug-free equilibrium E_0 is globally asymptotically stable.

4. The drug addiction equilibrium E^* of system (2)

Now, we shall investigate the existence of the drug addiction equilibrium of system (2). The global stability of E^* is considered in detail using Lyapunov function.

4.1. The existence of the drug addiction equilibrium E^*

Let the right of system (2) is equal to zero, that is

$$\begin{cases} (1-q)d\Lambda - \beta_1 SL - \beta_2 SH - (d+\mu)S = 0, \\ qd\Lambda + \mu S - \beta_1 \xi CL - \beta_2 \xi CH - (d+\delta)C = 0, \\ \beta_1 SL + \beta_2 SH + \beta_1 \xi CL + \beta_2 \xi CH - m_1 L = 0, \\ \pi L + \sigma T - m_2 H = 0, \\ \theta H + \gamma L - m_3 T = 0, \end{cases} \tag{5}$$

then the drug addiction equilibrium $E^* = (S^*, C^*, L^*, H^*, T^*)$ is determined by system (5). Solving all equations of system (5), we get

$$L^* = \tilde{A}H^*, \quad T^* = \frac{\pi\theta + m_2\gamma}{m_3\pi + \sigma\gamma}H^*, \quad S^* = \frac{(1-q)d\Lambda}{\left(\beta_1\tilde{A} + \beta_2\right)H^* + d + \mu}. \tag{6}$$

Summing the first three equations of system (5), we get

$$C^* = \frac{d(\Lambda - S^*) - m_1\tilde{A}H^*}{d + \delta}. \tag{7}$$

H^* is the root of following equation:

$$aH^2 + bH + c = 0, \tag{8}$$

where

$$a = \left(\beta_1\tilde{A} + \beta_2\right)^2 \xi m_1 \tilde{A},$$
$$b = \left(\beta_1\tilde{A} + \beta_2\right)\left((d+\mu)\xi m_1\tilde{A} - \left(\beta_1\tilde{A} + \beta_2\right)\xi d\Lambda + m_1\tilde{A}(d+\delta)\right),$$
$$c = (d+\mu)(d+\delta)m_1\tilde{A} - d\Lambda\left(\beta_1\tilde{A} + \beta_2\right)((1-q)(d+\delta) + \xi(\mu + qd))$$
$$= (d+\mu)(d+\delta)m_1\tilde{A}(1 - R_0).$$

It is easy to verify that if $R_0 \leq 1$, then $c \geq 0, b > 0$, there is no positive root of equation (8); if $R_0 > 1$, then $c < 0$, equation (8) exists a unique positive root H^*. The conclusion is described by the following theorem:

Theorem 4.1. *When $R_0 > 1$, the system (2) exist a unique positive equilibrium $E^* = (S^*, C^*, L^*, H^*, T^*)$, where S^*, C^*, L^*, T^* are described as (6) and (7), H^* is the unique positive root of equation (8).*

4.2. The global stability of the drug addiction equilibrium E^*

In the subsection, we show the drug addiction equilibrium E^* is globally asymptotically stable under certain conditions by using Lyapunov function.

Theorem 4.2. *When $R_0 > 1$ and $\frac{(m_3\pi + \sigma\gamma)d\Lambda}{m_3\pi(d+\mu)} < \frac{m_1\tilde{A}}{\beta_1\tilde{A} + \beta_2} < \frac{qd}{\beta_2}$, the equilibrium E^* of system (2) is globally asymptotically stable.*

Proof. Denote $K = \frac{\beta_2(S^* + \xi C^*)}{m_2 m_3 - \theta\sigma}$ and $x = \frac{S^*}{S}, y = \frac{C^*}{C}, z = \frac{L^*}{L}, v = \frac{H^*}{H}, w = \frac{T^*}{T}$. Construct the Lyapunov function to the drug addiction equilibrium E^* of system (2) as follows:

$$V = \left(S - S^* - S^* \ln\frac{S}{S^*}\right) + \left(C - C^* - C^* \ln\frac{C}{C^*}\right) + \left(L - L^* - L^* \ln\frac{L}{L^*}\right)$$
$$+ K\left(m_3\left(H - H^* - H^* \ln\frac{H}{H^*}\right) + \sigma\left(T - T^* - T^* \ln\frac{T}{T^*}\right)\right).$$

Calculating the derivative of V along solution of system (2), we obtain

$$\dot{V} = \left(1 - \frac{S^*}{S}\right)\dot{S} + \left(1 - \frac{C^*}{C}\right)\dot{C} + \left(1 - \frac{L^*}{L}\right)\dot{L} + Km_3\left(1 - \frac{H^*}{H}\right)\dot{H} + K\sigma\left(1 - \frac{T^*}{T}\right)\dot{T}$$

$$= (1-x)\left(\beta_1 S^* L^*\left(1 - \frac{1}{xz}\right) + \beta_2 S^* H^*\left(1 - \frac{1}{xv}\right) + (d+\mu)S^*\left(1 - \frac{1}{x}\right)\right)$$

$$+ (1-y)(\mu S^*\left(\frac{1}{x} - 1\right) + \beta_1 \xi C^* L^*\left(1 - \frac{1}{yz}\right) + \beta_2 \xi C^* H^*\left(1 - \frac{1}{yv}\right)$$

$$+ (d+\delta)C^*\left(1 - \frac{1}{y}\right)) + (1-z)\left(\beta_1 S^* L^*\left(\frac{1}{xz} - 1\right) + \beta_2 S^* H^*\left(\frac{1}{xv} - 1\right)\right.$$

$$+ \beta_1 \xi C^* L^*\left(\frac{1}{yz} - 1\right) + \beta_2 \xi C^* H^*\left(\frac{1}{yv} - 1\right) + m_1 L^*\left(1 - \frac{1}{z}\right))$$

$$+ m_3 K(1-v)\left(\pi L^*\left(\frac{1}{z} - 1\right) + \sigma T^*\left(\frac{1}{w} - 1\right) + m_2 H^*\left(1 - \frac{1}{v}\right)\right)$$

$$+ \sigma K(1-w)\left(\theta H^*\left(\frac{1}{v} - 1\right) + \gamma L^*\left(\frac{1}{z} - 1\right) + m_3 T^*\left(1 - \frac{1}{w}\right)\right)$$

$$= \mu S^* + 2dS^* + 2(d+\delta)C^* + 2m_1 L^* + m_2 m_3 KH^* + m_3 \sigma KT^*$$

$$- (1-q)d\Lambda x - qd\Lambda y - ((d+\delta)C^* + \beta_1 \xi C^* L^*)\frac{1}{y} - (\beta_1 S^* L^* + dS^*)\frac{1}{x}$$

$$- \mu S^*\frac{y}{x} - K\left(m_3 \pi L^*\frac{v}{z} + m_3 \sigma T^*\frac{v}{w} + \sigma\theta H^*\frac{w}{v} + \sigma\gamma L^*\frac{w}{z}\right)$$

$$- \beta_2 S^* H^*\frac{z}{xv} - \beta_2 \xi C^* H^*\frac{z}{yv}$$

$$= \mu S^* + 2dS^* + 2(d+\delta)C^* + 2m_1 L^* + m_3 K(\pi L^* + \sigma T^*) + \sigma K(\theta H^* + \gamma L^*)$$

$$- (\beta_1 S^* L^* + dS^*)\left(x + \frac{1}{x}\right) - \mu S^*\left(x + \frac{y}{x} + \frac{1}{y}\right) - \sigma K\theta H^*\left(\frac{v}{w} + \frac{w}{v}\right)$$

$$- \beta_2 S^* H^*\left(x + \frac{z}{xv} + \frac{v}{z}\right) - \sigma K\gamma L^*\left(\frac{v}{w} + \frac{w}{z} + \frac{z}{yv} + y\right)$$

$$- (m_3 K\pi L^* - \beta_2 S^* H^*)\left(y + \frac{z}{yv} + \frac{v}{z}\right) - (qd\Lambda - \beta_2 \xi C^* H^*)\left(y + \frac{1}{y}\right).$$

Since $\frac{(m_3\pi + \sigma\gamma)d\Lambda}{m_3\pi(d+\mu)} < \frac{m_1\tilde{A}}{\beta_1\tilde{A} + \beta_2}$, then

$$(m_3 K\pi L^* - \beta_2 S^* H^*) = \beta_2 H^*\frac{m_3\pi\xi C^* - \sigma\gamma S^*}{m_3\pi + \sigma\gamma}$$

$$= \frac{\beta_2 H^*}{m_3\pi + \sigma\gamma}\left(\frac{m_3\pi m_1\tilde{A}}{\beta_1\tilde{A} + \beta_2} - (m_3\pi + \sigma\gamma)S^*\right)$$

$$> \frac{\beta_2 H^*}{m_3\pi + \sigma\gamma}\left(\frac{m_3\pi m_1\tilde{A}}{\beta_1\tilde{A} + \beta_2} - (m_3\pi + \sigma\gamma)\frac{(1-q)d\Lambda}{d+\mu}\right)$$

$$> \frac{\beta_2 H^*}{m_3\pi + \sigma\gamma}\left(\frac{m_3\pi m_1\tilde{A}}{\beta_1\tilde{A} + \beta_2} - (m_3\pi + \sigma\gamma)\frac{d\Lambda}{d+\mu}\right)$$

$$> 0,$$

and since $\frac{m_1\tilde{A}}{\beta_1\tilde{A} + \beta_2} < \frac{qd}{\beta_2}$, then

$$qd\Lambda - \beta_2 \xi C^* H^* > qd\Lambda - \beta_2\frac{m_1\tilde{A}}{\beta_1\tilde{A} + \beta_2}H^*$$

$$> \left(qd - \beta_2\frac{m_1\tilde{A}}{\beta_1\tilde{A} + \beta_2}\right)\Lambda$$

$$> 0.$$
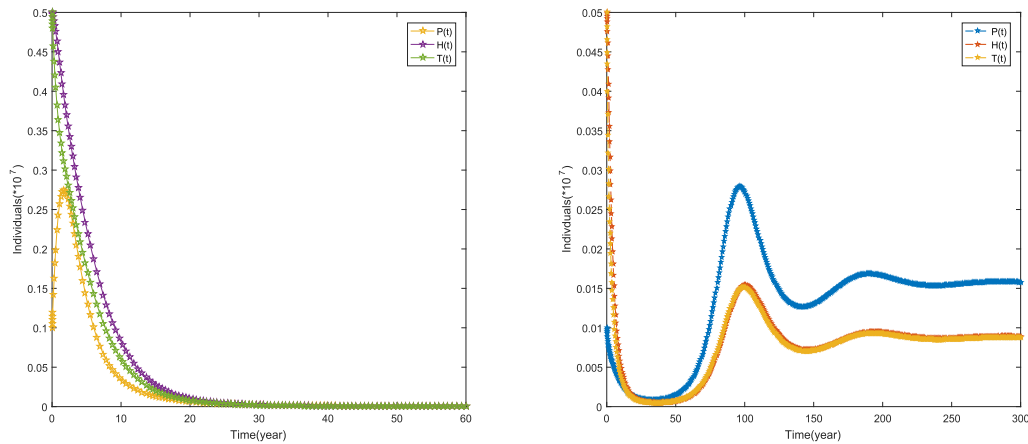
(a) The drug-free equilibrium E_0 is globally asymptotically stable if $R_0 \leq 1$. (b) The drug addiction equilibrium E^* is globally asymptotically stable if $R_0 > 1$.

Fig. 2. The stabilities of equilibria of system (2).

Using arithmetic geometric average inequality, we obtain

$$
\begin{aligned}
\dot{V} \leq &\ \mu S^* + 2dS^* + 2(d+\delta)C^* + 2m_1 L^* + m_3 K(\pi L^* + \sigma T^*) + \sigma K(\theta H^* + \gamma L^*) \\
&- 2(\beta_1 L^* + d)S^* - 3\mu S^* - 3\beta_2 S^* H^* - 3(m_3 K \pi L^* - \beta_2 S^* H^*) \\
&- 2\sigma K \theta H^* - 4\sigma K \gamma L^* - 2((d+\delta)C^* + \beta_1 \xi C^* L^* - \mu S^*) \\
= &\ 2(m_1 L^* - \beta_1 S^* H^* - \beta_2 \xi C^* H^* - \beta_1 \xi C^* L^* - \beta_1 L^* S^*) \\
= &\ 0.
\end{aligned}
$$

The $V' = 0$ holds if and only if $x = y = 1, v = z = w$. Therefore the singleton drug-addiction equilibrium E^* is the maximum invariant set of system (2) on the set $\Omega' = \left\{(S,C,L,H,T) \middle| S = S^*, C = C^*, \frac{L^*}{L} = \frac{H^*}{H} = \frac{T^*}{T}\right\} \subset \Omega$. By LaSalle (1976), the drug addiction equilibrium E^* is globally asymptotically stable.

5. Sensitivity and numerical simulation

In the section, some numerical simulations are present in order to illustrate the theoretical results obtained above. We show the stabilities of equilibria of system (2). Some parameters of the model come from reference (Kalula & Nyabadza, 2011; Nyabadza et al., 2013), $d = 0.02, \beta_1 = 0.7, \beta_2 = 0.8, d_1 = 0.2, d_2 = 0.3, \pi = 0.03, \gamma = 0.3, \theta = 0.421, \xi = 0.9, m = 0.25, \sigma = 0.7$, and the rest of the parametric values are estimated for numerical requirement.

Example 5.1. Fix $\Lambda = 1.2, \delta = 0.04, q = 0.8, \mu = 0.1$, then $R_0 = 0.812019 < 1$, the drug-free equilibrium E_0 is globally asymptotically stable (see Fig. 2(a)).

Example 5.2. Fix $\Lambda = 1.2, \delta = 0.01, q = 0.4, \mu = 0.1$, then $R_0 = 1.60729 > 1$, the drug addiction equilibrium E^* is globally asymptotically stable (see Fig. 2(b)).

For infectious diseases, if the basic reproduction number can decrease below unity, the diseases would be controlled. In order to achieve the goal, it is essential to consider the sensitivity of parameters, especially q and μ. First, we calculate the following partial derivatives.

$$
\frac{\partial R_0}{\partial \sigma} = \frac{\beta_2 (S_0 + \xi C_0)}{m_1 \tilde{A}^2} \frac{m_3 (m_2 \gamma + \theta \pi)\sigma}{(m_3 \pi + \sigma \gamma)^2} > 0,
$$

$$
\frac{\partial R_0}{\partial m} = -\frac{\beta_2 (S_0 + \xi C_0)}{m_1 \tilde{A}^2} \frac{(m_2 \gamma + \theta \pi)\sigma}{(m_3 \pi + \sigma \gamma)^2} < 0.
$$

It is showed that decreasing the relapse rate σ could reduce the basic reproduction number, and increasing the quit drug rate permanently also achieve the same result. Furthermore, we pay attention to the effect of family education and public
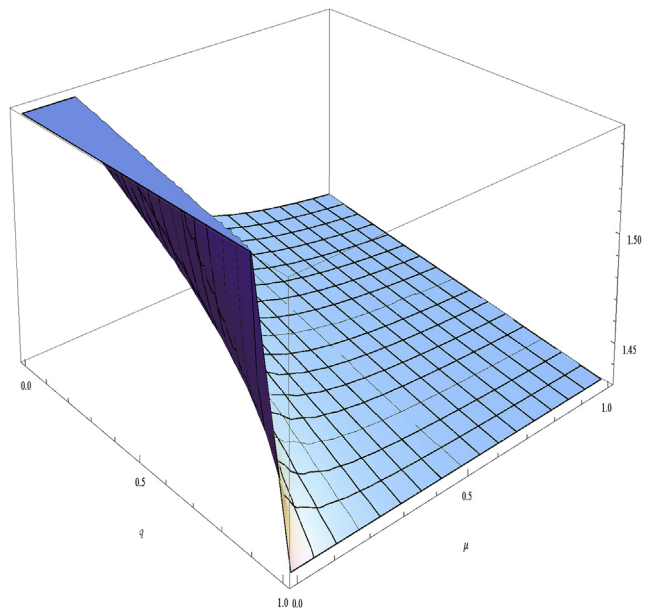
Fig. 3. The relationship among the basic reproduction number R_0, q and μ.

health education on the drug model in the paper, so the partial derivatives of R_0 with respect to q and μ are calculated as follows:

$$\frac{\partial R_0}{\partial q} = \frac{\left(\beta_1 \tilde{A} + \beta_2\right) d\Lambda}{m_1 \tilde{A}(d+\mu)} \frac{d\xi - d - \delta}{d+\delta} < 0,$$

$$\frac{\partial R_0}{\partial \mu} = \frac{\left(\beta_1 \tilde{A} + \beta_2\right) d\Lambda(1-q)}{m_1 \tilde{A}(d+\mu)^2} \left(\frac{d}{d+\delta} - 1\right) < 0.$$

From the results we can see that the basic reproduction number R_0 can be less than unit if reinforce the public health education. At last, we simulate the relationship among the basic reproduction number R_0, the popularizing rate of family education q and the public health education rate μ.

Example 5.3. Fix $\Lambda = 1.2, \delta = 0.02$, then the relationship of R_0, q and μ is showed in Fig. 3. The image shows q and μ have an inversely proportional relationship with R_0, which is consistent with above theoretical calculation.

6. Discussion and summary

In the paper, we formulated a drug model in order to study the effect of the family and public health education on drugs spread. In the model, the drug addicts were divided into light drug addicts and heavy drug addicts and the permanent quit drug was considered. Due to the influence of public health education, the effective contact rates would decrease when susceptible individuals who accept the public health education contact with drug addicts, therefore, the factor $\xi \in (0, 1)$ is brought into β_1 and β_2. Before accepting public health education, some susceptible individuals undergo the family education, so the proportions q and $1 - q$ denote the initial new recruit rate entering susceptible humans. By means of the next generation matrix, we got the basic reproduction number R_0 of system (2). The global stability of system (2) has been proved by using the Lyapunov function. When $R_0 \leq 1$, the drug-free equilibrium E_0 is globally asymptotically stable; when $R_0 > 1$, the drug-addiction equilibrium E^* is globally stable which indicates the drug problem will persist.

We also focus on the effect of family education and public health education on the dynamics of drug model. The sensitivity analysis implies both of them influence the spread of drug transmission. From Fig. 3, it is clear to see that increasing q and μ simultaneously could decrease the basic reproduction number R_0 more rapidly compared to just simply increasing q or μ, which could help government to take effect measures for controlling the drug spread.

7. Conclusion

From a six dimensional drug transmission model, we study the effect of family education and public health education. The analysis show that both the family and public health education can influence the spread of drug transmission. However, the combination of family and public health education is more effective to reduce the prevalence of drugs than one of them simply.

Conflicts of interest

The authors declare that they have no competing interests.

Authors' contributions

All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

Acknowledgements

References

Bhunu, C. P., Mushayabasa, S., Kojouharov, H., & Tchuenche, J. M. (2010). Mathematical analysis of an HIV/AIDS model: Impact of educational programs and abstinence in sub-Saharan Africa. *Journal of Mathematical Modelling and Algorithms, 10*(1), 31—55.

Birkhoff, G., & Rota, G. C. (1998). *Ordinary differential equations*. New York: Wiley.

Cen, X., Feng, Z., & Zhao, Y. (2014). Emerging disease dynamics in a model coupling within-host and between-host systems. *Journal of Theoretical Biology, 361*, 141—151.

Comiskey, C. (1999). *National prevalence of problematic opiate use in Ireland*. EMCDDA Tech. Report.

Diekmann, O., & Heesterbeek, J. A. P. (2000). *Mathematical epidemiology of infectious diseases*. John &Wiley Son, Ltd.

Driessche, P., Watmough, J., & van den Driessche, P. (2002). Reproduction numbers and subthreshold endemic equilibria for compartmental models of disease transmission. *Mathematical Biosciences, 180*, 29—48.

Fang, B., Li, X., Martcheva, M., & Cai, L. (2014). Global stability for a heroin model with two distributed delays. *Discrete & Continuous Dynamical Systems-B, 19*(3), 715—733.

Feng, Z., Cen, X., Zhao, Y., & Velasco-Hernandez, J. X. (2015). Coupled within-host and between-host dynamics and evolution of virulence. *Mathematical Biosciences, 270*, 204—212.

Funk, S., Salathe, M., & Jansen, V. (2010). Modelling the influence of human behaviour on the spread of infectious diseases: A review. *Journal of The Royal Society Interface, 7*, 1247—1256.

Huang, G., & Liu, A. (2013). A note on global stability for a heroin epidemic model with distributed delay. *Applied Mathematics Letters, 26*(7), 687—691.

Huo, H., Chen, R., & Wang, X. (2016). Modelling and stability of HIV/AIDS epidemic model with treatment. *Applied Mathematical Modelling, 40*, 6550—6559.

Kalula, A. S., & Nyabadza, F. (2011). A theoretical model for substance abuse in the presence of treatment. *South African Journal of Science, 108*, 96—107.

Kapp, C. (2008). Crystal meth boom adds to South Africa's health challenges. *Lancet, 37*, 193—194.

LaSalle, J. P. (1976). *The stability of dynamical systems, Regional conference series in applied mathematics*. Philadelphia, PA: SIAM.

http://legal.people.com.cn/n/2015/0325/c42510-26749292.html.

Li, M., Graef, J., & Wang, L. (1999). Global dynamics of a SEIR model with varying total population size. *Mathematical Biosciences, 160*, 191—213.

Liu, J., & Zhang, T. (2011). Global behaviour of a heroin epidemic model with distributed delays. *Applied Mathematics Letters, 24*(10), 1685—1692.

Li, J., Zhao, Y., & Li, S. (2013). Fast and slow dynamics of malaria model with relapse. *Mathematical Biosciences, 246*, 94—104.

Li, J., Zhao, Y., & Zhu, H. (2015). Bifurcation of an SIS model with nonlinear contact rate. *Journal of Mathematical Analysis and Applications, 432*, 1119—1138.

Mukandavire, Z., Garira, W., & Tchuenche, J. M. (2009). Modelling effects of public health educational campaigns on HIV/AIDS transmission dynamics. *Applied Mathematical Modelling, 33*, 2084—2095.

Mulone, G., & Straughan, B. (2009). A note on heroin epidemics. *Mathematical Biosciences, 218*, 138—141.

Muroya, Y., Li, H., & Kuniya, T. (2014). Complete global analysis of an SIRS epidemic model with graded cure and incomplete recovery rates. *Journal of Mathematical Analysis and Applications, 410*, 719—732.

Mushanyu, J., Nyabadza, F., & Stewart, A. G. R. (2015). Modelling the trends of inpatient and outpatient rehabilitation for methamphetamine in the Western Cape province of South Africa. *BMC Research Notes, 8*, 1—13.

http://news.xinhuanet.com/world/2015-07/16/c128025019.htm.

Nyabadza, F., Njagarah, J. B. H., & Smith, R. J. (2013). Modelling the dynamics of crystal meth ('tik') abuse in the presence of drug-supply chain in South Africa. *Bulletin of Mathematical Biology, 75*, 24—48.

Parry Charles, D. H., Dewing, S., Petersen, P., Carney, T., Needle, R., Kroeger, K., & Treger, L. (2009). Rapid assessment of HIV risk behavior in drug using sex workers in three cities in South Africa. *AIDS and Behavior, 13*(5), 849—859.

Samanta, G. P. (2011). Dynamic behaviour for a nonautonomous heroin epidemic model with time delay. *Journal of Applied Mathematics and Computing, 35*, 161—178.

Van den Driessche, P., & Watmough, J. (2002). Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. *Mathematical Biosciences, 180*, 29—48.

Wang, X., Yang, J., & Li, X. (2011). Dynamics of a heroin epidemic model with very population. *Applications of Mathematics, 2*, 732—738.

White, E., & Comiskey, C. (2007). Heroin epidemics, treatment and ODE modelling. *Mathematical Biosciences, 208*, 312—324.

Xiang, H., Song, N., & Huo, H. (2015). Modelling effects of public health educational campaigns on drinking dynamics. *Journal of Biological Dynamics, 10*(1), 164—178.