



## Research article

# Design, optimisation and reliability allocation for energy systems based on equipment function and operating capacity



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## ABSTRACT

Designers of energy systems often face challenges in balancing the trade-off between cost and reliability. In literature, several papers have presented mathematical models for optimizing the reliability and cost of energy systems. However, the previous models only addressed reliability implicitly, i.e., based on availability and maintenance planning. Others focused on allocation of reliability based on individual equipment requirements via non-linear models that require high computational effort. This work proposes a novel mixed-integer linear programming (MILP) model that combines the use of both input-output (I-O) modelling and linearized parallel system reliability expressions. The proposed MILP model can optimize the design and reliability of energy systems based on equipment function and operating capacity. The model allocates equipment with sufficient reliability to meet system functional requirements and determines the required capacity. A simple pedagogical example is presented in this work to illustrate the features of proposed MILP model. The MILP model is then applied to a polygeneration case study consisting of two scenarios. In the first scenario, the polygeneration system was optimized based on specified reliability requirements. The technologies chosen for Scenario 1 were the CHP module, reverse osmosis unit and vapour compression chiller. The total annualized cost (TAC) for Scenario 1 was 53.3 US\$ million/year. In the second scenario, the minimum reliability level for heat production was increased. The corresponding results indicated that an additional auxiliary boiler must be operated to meet the new requirements. The resulting TAC for the Scenario 2 was 5.3% higher than in the first scenario.

## 1. Introduction

Energy systems such as combined heat power (CHP) and poly-generation systems are touted to possess high efficiencies due to the integrated use of energy sources. However, what prevents such integrated systems from being implemented on a wider scale is the lack of confidence in the reliability of energy systems to perform a function for a given period of time. Reliability is defined as the probability that energy supply is uninterrupted and is expressed as a percentage of time that it is expected to function. Interruptions and variations in energy supply can happen at any time. Although most outages are momentary occurrences and are generally brief, they do not adversely impact anyone other than the most sensitive operations. Nevertheless, an average facility can expect to experience an extended outage every other year. A reason for this could be due to the way the energy system was designed or configured. The design of an energy system can influence how cascading

failures ripple through a network of interdependent process units. This can be explained using Fig. 1. As shown in Fig. 1(i), an energy system is expected to perform its function i.e., to produce process heat and is expected to operate at a minimum reliability ( $R^{\min}$ ). However, the reliability of the CHP unit (given by  $R_2$ ) in Fig. 1(i) is insufficient to meet the expected  $R^{\min}$  and may lead to frequent system outages. To address this, other equipment such as a boiler (with reliability,  $R_1$ ), can be allocated to ensure that the energy system is able to produce heat and operate at an overall reliability ( $R_1$  and  $R_2$ ) beyond  $R^{\min}$  (as shown in Fig. 1(ii)). Fig. 1 evidently implies that equipment reliability is a crucial factor when designing an energy system to perform a given function.

In the past, several approaches have been presented to design energy systems [1]. For instance, Andiappan et al. [2] developed a mathematical model that optimizes the selection and sizing of equipment in a tri-generation system based on seasonal variations in feedstock and energy demand. Meanwhile, Sy et al. [3] presented a target-oriented robust

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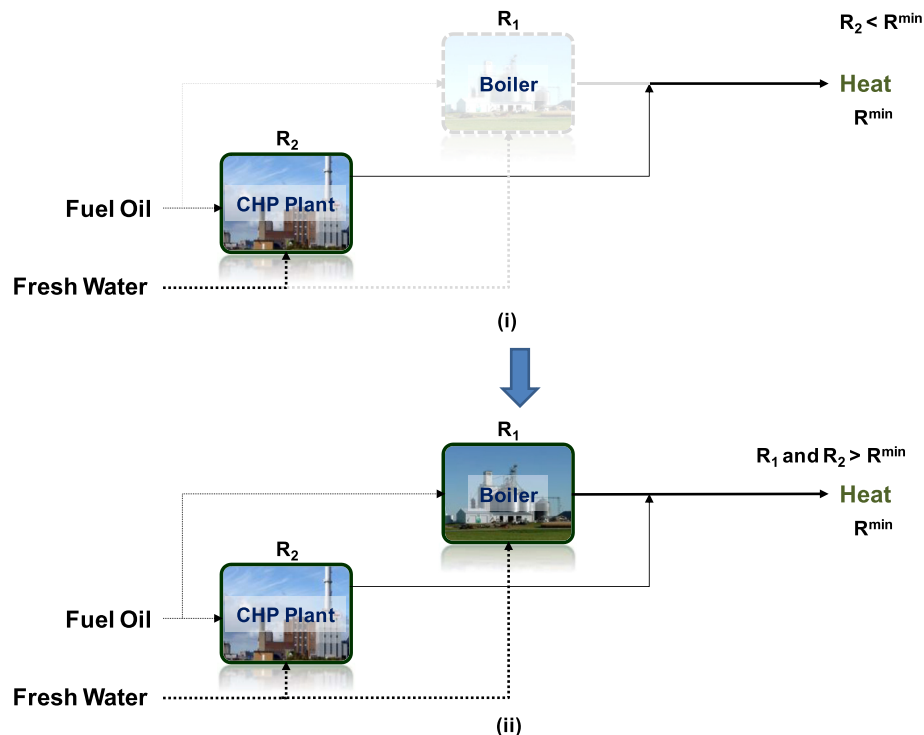


Fig. 1. Allocation of equipment reliability based on function to produce heat.

optimisation (TORO) approach to design a polygeneration system to maximize its robustness against uncertainties. Andiappan et al. [4] developed a *design operability and retrofit analysis* (DORA) framework to analyze economically feasible retrofitting options for energy system designs. Ling et al. [5] presented a minimax regret criterion framework to design biomass CHP systems based on uncertainties in energy demand. Ubando et al. [6] recently proposed an approach that combines fuzzy linear programming and global sensitivity analysis for the optimal design of a tri-generation system. Based on the aforementioned approaches, it is evident that there is limited focus on the consideration of reliability during the design of energy systems.

In view of this gap, Andiappan et al. [7] proposed a grassroots design approach to allocate equipment redundancy. This approach used combined chance-constrained programming and *k-out-of-n* system modelling. Meanwhile, Sun and Liu [8] developed a mixed-integer linear programming (MILP) model for designing steam and power systems considering reliability and uncertainty. The proposed MILP model [8] incorporated the configuration of the system (e.g., equipment capacity, efficiency, failure) as well as operational scheduling when computing the optimized cost (i.e., minimum). Based on [8], the results show that redundancy based on spare capacity and auxiliary equipment will provide more reliability and flexibility in utility systems. Later, a multi-objective optimization model was developed by Rad et al. [9] for the integration of utility systems in process industries. The model [9] used total site analysis, site utility grand composite curves, cogeneration targeting, and exergetic and economic analysis. The optimized grassroots-based design in [9] was then subjected to reliability analysis. In addition, the system availability, which is a function of failure and repair rate, was considered in the analysis as it affects the cost of production due to the expected repair cost [9]. Kim [10] proposed reliability optimization model based on a continuous-time Markov chain approach. This optimization model [10] addressed redundancy allocation problems and reliability-redundancy allocation problems. The model [10] provided the lifetime characteristics of the system consisting of active and standby redundant systems. Next, Pombo et al. [11] developed a

multi-objective model that utilized energy storage systems to increase the network reliability of distribution facilities. The multi-objective model [11] determines the optimal capacity and location of batteries, optimum number and location of switching devices by minimizing equipment cost. Andiappan et al. [12] then developed an integrated framework to improve the reliability of a tri-generation system. The framework determines the most critical equipment within the system and subsequently allocates redundancy. Manesh et al. [13] used Markov chain-based method to analyse the reliability and availability of site utility (or cogeneration) systems. The study [13] is to reduce the number of state spaces and complexity of the utility system (i.e., number of components) while achieving accuracy in the probability calculation needed in the reliability analysis. The method has less computational time and thus can be extended to less complex systems. Ade et al. [14] showed that inherent safety plays a vital role in the reliability of processes at various parts of the design stage. It was observed that inherent safety design principles lead to simplifications (i.e., use of fewer equipment), longer corrective downtime maintenance, and to an increase in overall risk.

Reliability models have been developed for other areas as well. For example, Numminen and Lund [15] developed a qualitative framework to assess the reliability of micro-grids. In Numminen and Lund [15], the important factors that decrease reliability were identified. These factors include the design, installation, and operation and maintenance of local grids. Helyen et al. [16] proposed a fairness index to measure the perceived acceptance of the distribution reliability levels among stakeholders. This society-based indicator considered both equity (i.e., providing exactly what a sector needs) and equality (i.e., providing equally in all sectors) in the reliability modelling framework. Adefarati and Bansal [17] proposed the use of reliability indices based on the cost of system failure or inability to supply power when using renewable energy resources. The indices are as follows: loss of load expectation, loss of load probability, and annual cost of load loss. The optimum configuration in [17] resulted in the reduction of costs related to its life-cycle energy use and greenhouse gas emissions.

A study by Honarmand et al. [18] proposed a supply chain-based

method to determine reliability. This method [18] uses a Markov model with measurements such as loss of load expectation, system average interruption frequency index, and energy not supplied indices to determine the reliability of processes. Next, Penttinen et al. [19] proposed the Open Modelling approach for Availability and Reliability of Systems or OpenMARS, a method to assess performance and risk in complex and dynamic systems. The model [19] is a hybrid of fault tree, Markov, and function-based approaches, which are traditional risk assessment tools. The results of the model are comparable and can be used for risk-informed decision-making. Similarly, Wang et al. [20] developed a model that combined Markov chain-based approach and reliability block diagram to analyse reliability and availability for a hybrid cooling system. The model [20] incorporated both operational availability and functional availability, and maximum allowable downtime to determine acceptable repair rates. Ye et al. [21] proposed a general mixed-integer non-linear programming (MINLP) model that determines optimal selection of parallel units while considering reliability, availability, and cost in chemical process systems. The model in [21] considers fixed probability of single units in terms of availability and provides flexibility on the characteristics of the parallel units (e.g., available capacity, cost of equipment). The resulting rigorous model is applied to ensure the reliability of methanol synthesis and hydrodealkylation processes. Ye et al. [22] extended the MINLP model in [21] to optimize cost of an air separation process based on reliability and frequency of maintenance. Essentially, this model [22] is a Markov chain-based model that considers stochastic failures and repair processes. The strategies employed in the model [22] include installing parallel units for critical components and performing condition-based maintenance. Recently, Hollermann et al. [23] proposed an optimization approach to identify  $(n - 1)$ -reliable energy supply system designs. This approach accounts for the failure of an equipment when another is undergoing maintenance. Other reliability-based approaches have been developed based on process graphs (P-graphs) [24]. For instance, Voll et al. [25] developed an automated superstructure framework using P-graphs to allocate redundancy for distributed energy supply systems. Süle et al. [26] then proposed a method to allocate redundant process units by mapping reliability block diagrams (RBDs) in P-graphs. Meanwhile, Kovacs et al. [27] proposed a P-graph synthesis algorithm to determine reliability of networks based on generic (combined series and parallel) configurations.

The aforementioned works provide substantial basis for models that design based on reliability. However, there are several areas that can be explored further;

- The previous works that have addressed reliability issues focus on equipment reliability but do not explicitly allocate equipment reliability based on specific functions a system should deliver, i.e., based on availability and maintenance planning.
- Previous papers present non-linear models that are generally complex to solve (i.e., either labour intensive or resource intensive or both) and challenging to guarantee a global optimum.
- Previous works [7, 12] focus exclusively on specifying minimum reliability expectations for each individual unit. In reality, these minimum reliability specifications for each unit may not be available due to lack of historical data. In this context, minimum reliability is typically specified for the entire system. Based on this, designers should allocate sufficiently reliable equipment to meet the minimum expectations for the entire system. However, this issue becomes more complex when the system is expected to perform several functions (i.e., produce heat and power).

To address the areas mentioned above, this work proposes a novel MILP model framework to optimize the design of energy systems, considering the reliability of the system to perform a function. In addition, the MILP model proposed in this work could determine the capacity of the equipment in the synthesized energy system. This is achieved via the combined use of a modified parallel system reliability approach and

input-output (I-O) modelling. The modified parallel system reliability approach determines the optimum allocation of equipment of a given function to meet an energy system's minimum required reliability. I-O modelling is used to establish the capacity/sizing of the equipment chosen by the modified parallel system reliability approach.

## 2. Methodology

The problem addressed in this work can be represented in Fig. 2. It is described as follows; given a system with a set of technologies  $j \in J$ , each with unique minimum capacity ( $x_j^{\min}$ ), maximum capacity ( $x_j^{\max}$ ), reliability ( $R_j$ ), variable cost ( $VC_j$ ) and fixed cost ( $FC_j$ ). The system is required to perform its function (i.e., produce heat and power) to meet demands. The system is expected to meet a minimum reliability level to perform a function ( $R_i^l$ ). In this respect, the objective of this work is to synthesize an optimum energy system with allocation of technologies  $j$  with unique  $R_j$  to meet the overall minimum system reliability level for each function while minimizing cost.

A mathematical model can be formulated to solve the above-mentioned design problem. The model for this work is formulated using the approach presented in this section. Firstly, the material and energy balances of the design problem are expressed in the form of an input-output (I-O) relationship as shown in Eq. (1);

$$\sum_j A_{ij}x_j = y_i \quad \forall i \quad (1)$$

As shown in Eq. (1),  $A_{ij}$  is the process matrix of input or output flows of stream  $i$  to or from a given equipment  $j$ .  $A_{ij}$  represents the coefficient values for inputs/output streams  $i$ . Negative values for  $A_{ij}$  represent input streams, while positive values indicate product streams. For example, if an equipment  $j$  takes in 650 kW of heat,  $A_{ij}$  would be defined as  $-650$  for its input heat stream ( $i = \text{heat}$ ). On the other hand, if that same equipment  $j$  produces power at 500 kW,  $A_{ij}$  would be  $+500$  for its product stream ( $i = \text{power}$ ). Meanwhile, the variable  $x_j$  is the operating capacity of equipment  $j$ .  $x_j$  is expressed in terms of percentage (%), whereby if  $x_j = 1$ , it would mean that technology  $j$  is operating at 100% capacity. The variable  $y_i$  represents the net output stream  $i$  from the plant. Note that the value of  $y_i$  can be either positive, negative or zero. Positive values for  $y_i$  indicate that stream  $i$  is a product from the plant. In contrast, a negative  $y_i$  means that stream  $i$  is an input into the plant. In the case where  $y_i$  is zero, that would mean stream  $i$  is purely an intermediate stream.

Note also that  $x_j$  in Eq. (1) is limited by the range shown in Eq. (2). As shown,  $x_j^{\min}$  and  $x_j^{\max}$  are the lower and upper operating capacity limits respectively. This represents the potential range of capacity in which vendors can provide for equipment  $j$ . Alternatively, if there are no constraints on the operating capacity for equipment  $j$  (e.g., reactors),  $x_j^{\max}$  can be replaced with an arbitrary large number while  $x_j^{\min}$  can be set as low as zero. This way, the capacity of equipment  $j$  can be determined without the constraint of size availability in the market. Alongside this,  $b_j$  is a binary variable that indicates whether equipment  $j$  is selected. As shown in Eq. (3),  $b_j$  takes the value of 1 if equipment  $j$  is needed and takes the value of 0 if it is not required.

$$x_j^{\min} b_j \leq x_j \leq x_j^{\max} b_j \quad \forall j \quad (2)$$

$$b_j \in \{0, 1\} \quad \forall j \quad (3)$$

Next, the limits for the net output supply of stream  $i$  are given by Eq. (4). With Eq. (4), material and/or energy demands from the market can be specified in the form of lower and upper limits  $Y_i^l$  and  $Y_i^u$  respectively. This constraint is exclusively aimed at the outputs from the plant.

$$Y_i^l \leq y_i \leq Y_i^u \quad \forall i \quad (4)$$

However, it is important to allocate equipment reliability based on

the required function. Function in this context, refers to the reliability to produce stream  $i$  (e.g., steam, power, etc.). As several equipment may produce the same supply of stream  $i$ , it is assumed that the allocation of equipment  $j$  here is analogous to parallel system reliability. For instance, if two or more equipment can perform the same function, these equipment are assumed to be operating in parallel. Based on this assumption, the allocation of equipment  $j$  based on reliability begins from the expression shown in Eq. (5).

$$R_i^L \leq 1 - \prod_j (1 - R_j^{(T_{ij} b_j)}) \quad \forall i \tag{5}$$

$R_j$  is the reliability for equipment  $j$ . Note that reliability represents the probability of equipment  $j$  operating in a given period. In other words, if  $R_j$  takes the value of 95%, it would mean that equipment  $j$  would operate at least 95% of the time. Meanwhile,  $T_{ij}$  is a user-defined binary topological parameter indicating existence of a path from equipment  $j$  to stream  $i$ . Note that a given system may have several output streams  $i$  but not all of those streams would be required to meet a minimum requirement  $R_i^L$ . Hence,  $T_{ij}$  is introduced to ensure that outputs that must meet  $R_i^L$ , would take the value of 1, while those output streams that do not have to meet  $R_i^L$  are assigned  $T_{ij} = 0$ . For example,  $T_{ij} = 1$  indicates that equipment  $j$  is capable of producing stream  $i$ , and  $T_{ij} = 0$  would indicate otherwise.  $R_i^L$  is the minimum reliability target for equipment in the energy system to produce stream  $i$ . From expression in Eq. (5), it can be seen that the expression is non-linear in nature. Non-linear expressions can cause models to take long durations to solve. In the case where a solution is obtained from non-linear expressions, it would be a challenge to determine whether the solution obtained is globally optimal.

To address this, a linear expression for reliability of supply for stream  $i$  is derived as demonstrated in Eqs. (6), (7), (8). Firstly,  $(1 - R_j)$  from Eq. (5) is the failure rate or unreliability of equipment  $j$ , which is the opposite of reliability  $R_j$  and is given by  $Q_j$ .  $Q_j$  is then substituted into Eq. (5) to give Eq. (6).

$$R_i^L \leq 1 - \prod_j (Q_j^{(T_{ij} b_j)}) \quad \forall i \tag{6}$$

$$R_i^L - 1 \leq - \prod_j (Q_j^{(T_{ij} b_j)}) \quad \forall i \tag{7}$$

$$1 - R_i^L \geq \prod_j (Q_j^{(T_{ij} b_j)}) \quad \forall i \tag{8}$$

Finally, from Eq. (8), a linear expression is formed by introducing logarithmic expressions as shown in Eq. (9);

$$\log(1 - R_i^L) \geq \sum_j (T_{ij} b_j) \log Q_j \quad \forall i \tag{9}$$

The primary objective of this work is to determine the minimized

total annualized cost (TAC) while allocating sufficient equipment reliability to produce stream  $i$ . Thus, the objective function for this model is shown in Eq. (10);

$$\text{minimize TAC} = CF \sum_i PU_i y_i + AF \sum_j (VC_j x_j + FC_j b_j) \tag{10}$$

In Eq. (10),  $CF$  is a conversion factor to ensure that net flowrates are annualized. In addition,  $PU_i$  is the unit price/cost of input stream  $i$ .  $AF$  is an annualising factor for the equipment costs, while  $VC_j$  and  $FC_j$  are variable cost coefficient and fixed cost coefficient of equipment  $j$  cost function respectively.

Note that the proposed MILP model focuses on allocating equipment based on minimum reliability required for the system to perform its functions. However, such reliability does not account for the reliability to produce an exact amount of output of each product. Such consideration is beyond the scope of this work and remains an area for future study.

To demonstrate the proposed methodology, two case studies are presented in this work. The first case study consists of a pedagogical example to illustrate the use of the methodology proposed. Following this, a polygeneration system was solved as a second case study.

### 3. Results & discussion

#### 3.1. Case study 1: pedagogical example

In this case study, a simple pedagogical example is used to demonstrate the proposed approach in the previous section and is aimed at illustrating the methodology in a simplified form. The pedagogical example is a source-sink problem to design an energy system for a hotel resort (Fig. 3). As shown, the hotel resort would require utilities (sinks) such as heat, clean water, power and ice for its day-to-day operations. To meet these demands, there are four potential technologies (sources), each could provide at least one of the utilities mentioned. The standard sizes or capacity for each technology is presented in Table 1. The objective here is to determine the optimal reliability and operating capacity allocation of technologies required to meet the defined utility demands based on cost and minimum reliability required to produce utilities.

To simplify the cost estimation, this example assumes that the annualised variable costs is a linear function of the operating capacity for each technology and is given in Table 2.

To explore the pedagogical example, the following two scenarios were solved;

- Scenario 1: Base Case
- Scenario 2: Increase in Minimum Required Reliability of System to Produce Heat

Scenario 1 represents the base case for this example. The base case in

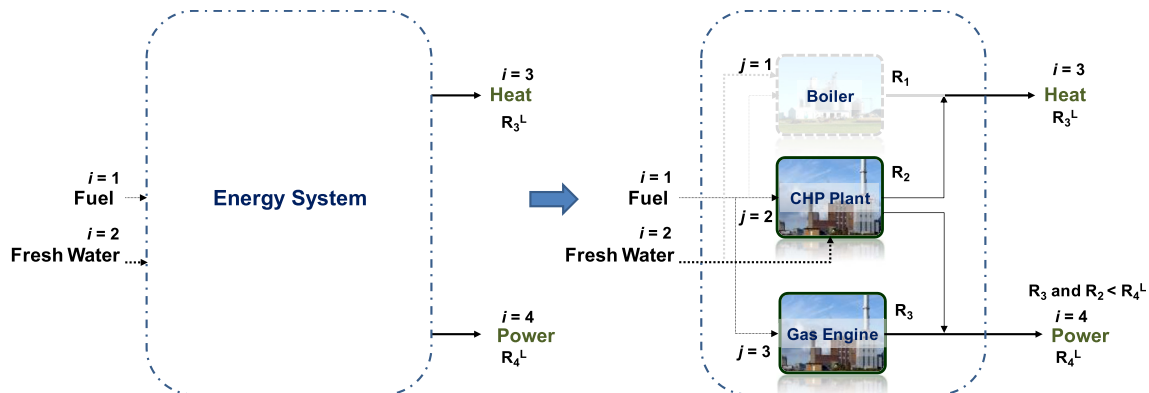


Fig. 2. Problem statement representation.

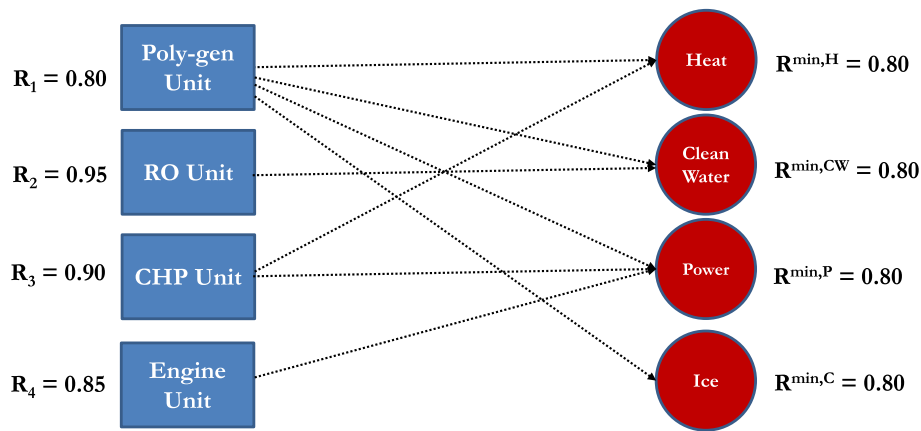


Fig. 3. Source-sink example.

**Table 1**  
Available sizes for each technology.

$A_{ij}$	Poly-gen Unit	RO Unit	CHP Unit	Engine Unit
Heat (kW)	688		500	
Clean Water (L/s)	10	8		
Power (kW)	1,207		800	500
Ice (kW)	800			

**Table 2**  
Annualized capital (variable and fixed) costs.

Unit/Module	Annualized Variable Cost (VC <sub>i</sub> ) (US\$/Operating Capacity)	Annualized Fixed Cost (FC <sub>i</sub> ) (US\$)
Poly-gen Unit	25,000	15,000
RO Unit	130.31	1,000
CHP Unit	16,530	8,000
Engine Unit	5,000	5,000

this sense, refers to a reference design. The reference design in Scenario 1 is used as a basis for comparison in Scenario 2 to analyse changes in decision-making. The minimum required reliability of supply and the minimum energy demands are shown in Table 3. The model developed for this example is coded in LINGO version 14, and is run on a LENOVO P700 with 4 GB RAM and i7 Core 2.59 GHz Processor. The model contains 18 continuous variables, 4 integer variables and 27 constraints. Note the number of variables and constraints are derived from codes inserted in LINGO. For instance, since there are four technologies considered in this example,  $x_j$  would be expanded to variables  $x_1, x_2, x_3$  and  $x_4$  in LINGO. The optimal solutions for both scenarios were obtained within negligible time.

3.1.1. Scenario 1

In Scenario 1, the energy system is expected to meet the minimum required energy demands and required reliability of equipment to

**Table 3**  
Minimum required utility demands.

Required System Function (To produce, $y_i$ )	Minimum Utility Demand ( $Y_i^L$ )	Minimum Required Reliability of System ( $R_i^L$ ), %	
		Scenario 1	Scenario 2
Heat (kW)	688	80	85
Clean Water (L/s)	8	80	80
Power (kW)	800	80	80
Ice (kW)	700	80	80

produce utilities specified in Table 3. Scenario 1 represents the base case for this example. The base case in this sense, refers to a reference design. The reference design in Scenario 1 will then be used to compared in other scenarios to analyse changes in decision-making. The MILP model for this case study is formulated based on the generic equations proposed in Eqs. (2), (3), (4), (5), (6), (7), (8), and (9). The model was then solved by minimising Eq. (10). The optimal solution for Scenario 1 is shown in Fig. 4. As shown, the energy system was able to meet the minimum energy demands and required reliability for the system to produce the aforementioned utilities with just a polygeneration unit of 100% capacity. The corresponding TAC and reliability for the baseline scenario were 40,000 US\$/y and 80% respectively.

3.1.2. Scenario 2

As for Scenario 2, the minimum reliability required for the system to produce heat is now increased to 85%. The same model in Scenario 1 was then solved with the same computational resources and effort. The optimal solution for Scenario 2 is shown in Fig. 5. For Scenario 2, the CHP unit was required for operation alongside the polygeneration unit. This is because the minimum reliability required of the system to produce heat has now been increased and the polygeneration operation on its own, would not be able to offer reliable heat supply. As a result, the CHP unit was selected with operating capacity 35%, while the polygeneration unit dropped in size from 100% in Scenario 1 to 88% in Scenario 2. This means that CHP unit with operating capacity of 35% load is required to supply a fraction of the required heat supply, allowing the poly-gen unit to reduce its capacity. In this respect, the change in design decision (sizing) was evident after increasing the target or minimum reliability. This shows that reliability plays a significant role when sizing and allocating equipment based on reliability to produce utilities. With the operation of the CHP unit, the TAC and system reliability for Scenario 2 were increased to 50,660 US\$/y and 98% respectively.

Note that solutions obtained in Scenarios 1 and 2 may seem obvious, but they demonstrate the validity of the methodology developed in this paper.

3.2. Case study 2: polygeneration system

The proposed methodology is then applied to a modified poly-generation case study from Kasivisvanathan et al. [28]. Fig. 6 shows the flowsheet of the polygeneration system which consists of an auxiliary boiler unit, a combined heat and power (CHP) module, a vapour compression chiller, and a reverse osmosis (RO) unit. Aside from this, it can be seen that the polygeneration system is able to generate various forms of product streams. These include heat, power, cooling, and treated water. The process matrix for the polygeneration system is adapted from Kasivisvanathan et al. [28] and is shown in Table 4. It is assumed that the

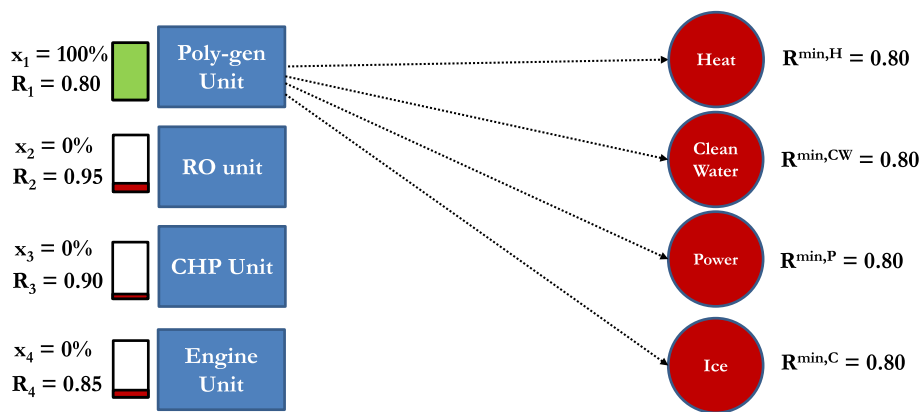


Fig. 4. Optimal solution for Scenario 1 - base case.

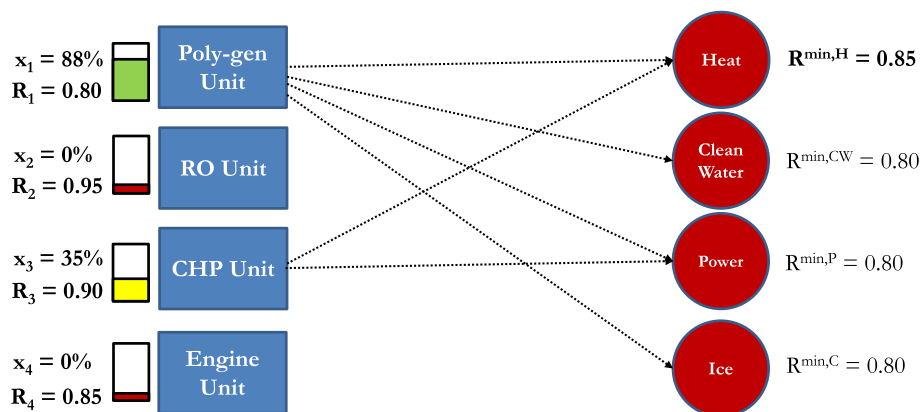


Fig. 5. Optimal solution for Scenario 2 - minimum reliability to produce heat increased to 85%.

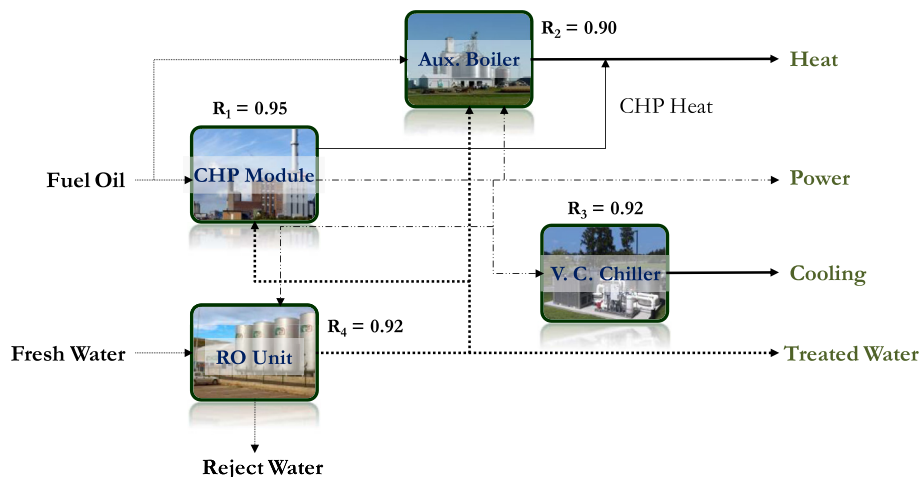


Fig. 6. Polygeneration system.

process matrix from Kasivisvanathan et al. [28] is determined prior at the synthesis stage. It is imperative to note that the synthesis stage determines the optimal selection of technologies and preliminary allocation of flows. The preliminary flows are used as input for the process matrix. In this case study, the aim is to optimize the design of the polygeneration based on its operating capacity and its reliability to produce outputs. Design optimisation focuses on more accurately defining and refining the capacity allocations for technologies determined at the synthesis stage. In this sense, there are possibilities where the size of a unit in a system could

be much smaller or larger than what was determined prior at the synthesis stage.

The feasible partial load operating range and reliability for each unit in the polygeneration is given in Table 5. Note that it is assumed that the reliability,  $R_i$  assigned to each unit, is the resultant reliability from series connections prior to that given unit. For instance, the reliability of the auxiliary boiler ( $R_2 = 0.90$ ) is set, taking into account of its connection with the RO unit and CHP module. Hence, its reliability is a much lower reliability since it is operating at the end of a series connection with the

**Table 4**  
Process matrix for polygeneration system [28].

$A_{ij}$	CHP Module	Auxiliary Boiler (Aux. Boiler)	Vapour Compression Chiller (V.C. Chiller)	RO Unit
Heat (kW)	18,199	6,881		
Power (kW)	12,079	-69	-1,600	-410
Cooling (kW)			8,000	
Treated Water (L/s)	-33	-4		137
Fuel (L/s)	-1.8	-0.23		
Fresh Water (L/s)				-342
Rejected Water (L/s)				205

**Table 5**  
Feasible partial load operating range and reliability for each unit in poly-generation system.

Unit/Module	$x_j^{\min}$ (%)	$x_j^{\max}$ (%)	$R_j$ (%)
CHP Module	30	125	95
Auxiliary Boiler	35	125	90
Vapour Compression Chiller	35	125	92
RO unit	10	125	92

RO unit and the CHP module.

To simplify the cost estimation, this case study assumes piecewise linear cost functions with variable and fixed components. Meanwhile, the annualising factor for the capital cost is assumed as 0.13/y. The annualized capital cost coefficients for each of the units in the polygeneration differ from each other based on the actual rating given in Table 6.

The costs of the materials streams are shown in Table 7. Since the total cost is expressed annually, cost of the material streams must also be expressed on an annual basis by using the annual operating time of 8000 h/y.

To optimize the design of units in the polygeneration system, the following two scenarios were considered;

- Scenario 1: Base Case
- Scenario 2: Increase in Minimum Required Reliability to Produce Heat

Similar to Case Study 1, the base case in Scenario 1 provides a reference design. The reference design here is then used in Scenario 2 for analysing the changes in the design configuration. The minimum

**Table 6**  
Capital (fixed and variable) costs for each unit in polygeneration system.

Unit/Module	Variable Cost Coefficient (VC <sub>j</sub> )	Fixed Cost Coefficient (FC <sub>j</sub> )
CHP Module	1,653 US\$/kW	75.50 million US\$
Auxiliary Boiler	305 US\$/kW	3.95 million US\$
Vapour Compression Chiller	467 US\$/kW	0.24 million US\$
RO Unit	13.31 US\$/L	0.01 million US\$

**Table 7**  
Cost of materials in polygeneration system.

Material ( $y_i$ )	Operating Cost Coefficient (PU <sub>i</sub> )
Fuel Oil	0.90 US\$/L
Fresh Water	0.001 US\$/L

**Table 8**  
Minimum required energy demands.

Required System Function (To produce, $y_i$ )	Minimum Energy Demand ( $Y_i^L$ )	Minimum Reliability of System ( $R_i^L$ ,%)	
		Scenario 1	Scenario 2
Heat (kW)	5,000	90	95
Power (kW)	7,000	95	95
Cooling (kW)	7,000	90	90
Treated Water (L/s)	80	90	90

required reliability and the minimum energy demands are shown in Table 8. The MILP model developed for both scenarios consists of 22 continuous variables, 4 integer variables and 31 constraints. Similarly to Case Study 1, the optimal solutions for both scenarios in Case Study 2 were obtained within few seconds.

3.2.1. Scenario 1

For the first scenario, the polygeneration system is expected to meet the minimum required energy demands, while adhering to the minimum required reliability to produce each supply specified in Table 8. For instance, the required reliability to produce heat is set to 90%. This means that the system needs to produce heat at least 90% of the time. Following this, the I-O model for this case study is formulated based on the generic equations proposed in Eqs. (1), (2), (3), (4), (5), (6), (7), (8), and (9). The model is then solved by minimising Eq. (10) using the same computational resources mentioned in the pedagogical example. Fig. 7 illustrates the optimal solution for Scenario 1. From Fig. 7, it can be seen that both the minimum energy demands and required reliability were

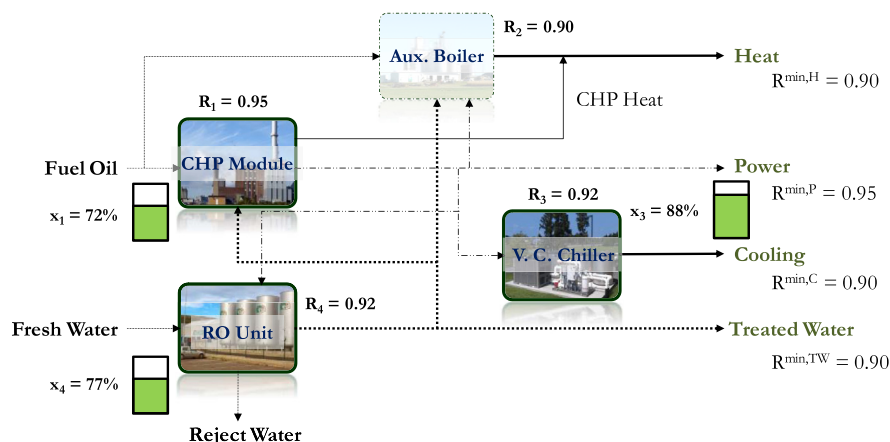


Fig. 7. Scenario 1 - base case.

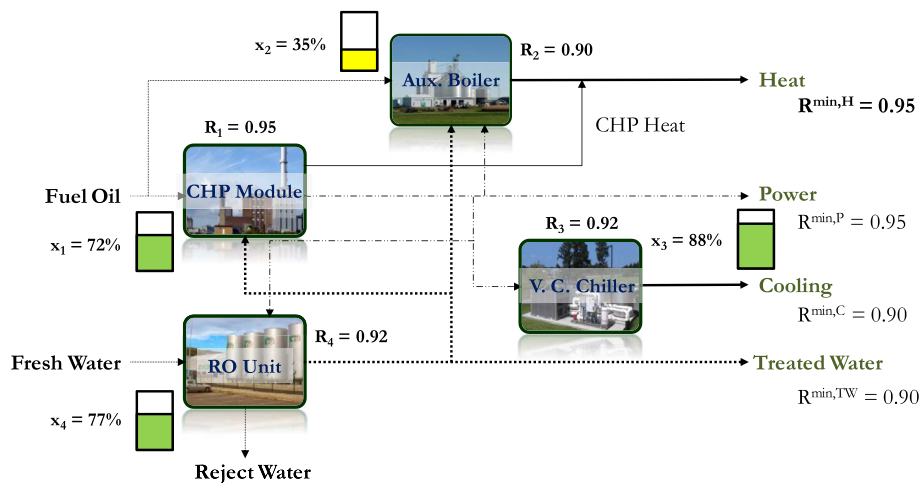


Fig. 8. Scenario 2 - minimum reliability to produce heat increased to 95%.

met without the use of an auxiliary boiler. This is evident as the binary ( $l_2$ ) and operating capacity ( $x_2$ ) variables for the auxiliary boiler were computed as zero, to show non-existence and zero operation respectively. Meanwhile, the optimal solution suggests that the CHP module, V. C. Chiller and RO unit operates at 72%, 88% and 77% capacity respectively. It is worth noting that the CHP plant was chosen over the boiler due to the existence of other utility requirements such as power. Although the boiler is a cheaper option, it is only capable of supplying the heat demand at its expected reliability and not the power requirements specified. On the other hand, the CHP plant can support both heat and power demands as well as their respective reliability requirements. The corresponding TAC for Scenario 1 was computed as 53.3 million US\$/y.

### 3.2.2. Scenario 2

In Scenario 2, the polygeneration system is required to meet the same energy demands in Table 8 but the minimum required reliability to produce heat is now increased to 95%. The model was solved, and the corresponding optimal solution is presented in Fig. 8. Fig. 8 suggests that the auxiliary boiler was required for operation alongside the CHP module. The reason for this is because the CHP is unable to produce heat supply at higher reliability if operated alone. Hence, the auxiliary boiler was selected with an additional operating capacity 35% to support the increase reliability requirements for heat production. The TAC for this scenario was determined as 56.1 million US\$/y, which is a 5.3% increase from Scenario 1.

## 4. Conclusion

A novel MILP model is developed in this work to optimize the design and reliability of energy systems based on equipment function and operating capacity. The proposed MILP model utilises I-O modelling to determine the actual capacity required for each process unit in an energy system. Then, the MILP model uses linearized parallel system reliability expressions to allocate equipment with sufficient reliability to meet system functional requirements of the entire system. To demonstrate the proposed MILP model, two examples were solved. Future work will be directed towards formulating a mathematical model that is able to address reliability expressions for equipment in hybrid (combined series and in parallel) configurations simultaneously. In addition, other conflicting variables such as flexibility and start-up costs can be considered via multi-objective optimization.

## Declarations

### Author contribution statement

Viknesh Andiappan & Michael Francis D. Benjamin: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

Raymond R. Tan & Denny K. S. Ng: Conceived and designed the experiments; Analyzed and interpreted the data; Wrote the paper.

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### Competing interest statement

The authors declare no conflict of interest.

### Additional information

No additional information is available for this paper.

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