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Research article

Forecasting education expenditure with a generalized conformable fractional-order nonlinear grey system model

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A R T I C L E I N F O

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A B S T R A C T

As an important human capital investment, education is an effective means to improve the comprehensive quality of people. Education expenditure is an important material guarantee for the development of educational undertakings. Education expenditure data is highly susceptible to numerous economic and social factors that complicate its nonlinear structure. In order to model the complex nonlinear problems of the system, this paper proposes a generalized conformable fractional-order nonlinear grey prediction model for the first time by analyzing the traditional time series-based modeling method in a nonlinear grey domain. The proposed model expands on the classical grey Bernoulli model by introducing the generalized conformable fractional accumulation as a new accumulation generator and utilizes error minimization principles in the modeling process. By altering the optimal order of the model and the cumulative generation operator, this model can adapt to various time series and reduce errors. Finally, the model is applied to education expenditure forecasting, and it is proved that the proposed model achieved good results and has higher accuracy than other models.

1. Introduction

Educational expenditure mainly encompasses spending on developing educational undertakings at all levels, primarily financed by the state. In recent years, the Chinese government has attached great importance to the investment in education and has significantly increased the investment in education. As reported by the Ministry of Education of China, the Announcement of Statistics on the Implementation of National Educational Funds in 2021 highlights that the overall investment in national educational funds has reached 5887.367 billion yuan, displaying a 9.13% increase from the previous year. Educational expenditure proves to be a significant financial element in maintaining educational institutions.

Forecasting education expenditure is fundamental to developing sound education plans and promoting sustainable and equitable development in education. Many scholars have focused on the study. For example, By sorting out the historical data from 2005 to 2016, Hu et al. [1] selected several indicators such as the scale of investment, per-student expenditure, and the proportion of teachers' salaries for comparative analysis, and predicted the development trend of China and OECD countries in 2020–2035 by using univariate time series method. Based on the study of education funds from 1991 to 2018, Shang et al. [2] forecasted the education funds investment in China from 2019 to 2025 with three single-item prediction models, namely, the multiple regression model, Holt-Winter

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non-seasonal exponential smoothing model, and piecewise-linear model, as well as the combined prediction model of the generalized induced ordered weighted average (GIOWA). Combining the dimensions of supply and demand, Yue et al. [3] employed the statistical and econometric regression models to predict the scale and structure of higher education and public education funding in China from 2021 to 2035. In Ref. [4], through regression analysis, the regression equation between the funds of the state financial preschool education, the expenditure of the public finance budget, and the number of kindergartens is established. And then using the equation to calculate the demand for preschool education funds in China and the provinces and cities. Zeng et al. [5] put forward the relevant forecasting methods to predict the demand of preschool education appropriations in China in 2020. Zhang et al. [6] selected the relevant data between 1992 and 2013 and used SPSS19 software to analyze and predict education financial input .

Education expenditure is a complex small data set that is influenced by various factors. For instance, education spending is constrained by factors such as economic growth and enrollment rates. Small sample sizes can reduce the validity of estimators and subsequently lead to less accurate results. In 1982, Deng [7] established the grey system theory, which is a novel approach that can be employed to study problems with little data and poor information. Grey system theory is specifically designed to address complex "small sample" and "information-poor" uncertain systems that only have partial information known and unknown. By extracting valuable information from the known data, grey system theory can be used to accurately describe and effectively monitor the system's behavior and evolution. This approach has now been applied to various fields, including energy, agriculture, education, and so on [8]. In Ref. [9], two grey prediction models, Even GM (1, 1) and Non-homogeneous discrete grey model (NDGM), and ARIMA models were deployed to forecast cocoa bean production of the six major cocoa-producing countries. Yang and Li [10] forecasted and analyzed the demand for main grain varieties by the grey interval forecast, and forecasted the grain production with the GM (1, N) model based on the grey incidence analysis of more influence factors. Liu and Zhang [11] analyzed and predicted hazard risks caused by tropical cyclones in Southern China with fuzzy mathematical and grey models. Bezuglov and Comert [12] predicted short-term freeway traffic parameters with first-order single variable Grey model, GM(1,1) with Fourier error corrections (EFGM), and the Grey Verhulst model with Fourier error corrections. Intharathirat et al. [13] used grey models to forecast municipal solid waste quantity in a developing country. Islam et al. [14] established a particle swarm optimization-based GM(1,1) model to estimate the warehouse's key performance indicators with limited historical information but not absolute. Duman et al. [15] presented an optimized multivariate grey model for electronic equipment waste predictions with multiple inputs in the presence of limited historical data in the United States. Kumar et al. [16] applied three-time series models, namely, Grey-Markov model, Grey-Model with rolling mechanism, and singular spectrum analysis (SSA) to forecast the consumption of conventional energy. Hamzacebi et al. [17] forecasted the annual electricity consumption of Turkey using an optimized grey model implemented both in direct and iterative manners.

In order to enhance the precision of the approach and expand its range of applications, fractional grey modeling-based forecasting techniques have been proposed throughout the past years. For example, Xie et al. [18] used a continuous fractional nonlinear grey Bernoulli model to forecast fuel combustion-related CO_2 emissions. Zeng and Liu [19] proposed a self-adaptive intelligence grey prediction model with the optimal fractional order accumulating operator and applied it to simulate China's electricity consumption from 2001 to 2008 and forecast it from 2009 to 2015. Ma et al. [20] presented a fractional time-delayed grey model with a grey wolf optimizer and used it to forecast the natural gas and coal consumption in Chongqing, China. Wu et al. [21] put forward a time power-based grey model with conformable fractional derivative and used it to predict energy consumption. Huang et al. [22] employed a variable-order fractional discrete grey mode and applied it to predict electricity consumption. Liu et al. [23] used an optimized fractional grey model based on weighted least squares to forecast water consumption. Wang et al. [24] proposed a fractional structural adaptive grey Chebyshev polynomial Bernoulli model and applied it to forecast the renewable energy production of China. He et al. [25] presented a structure adaptive new information priority discrete grey prediction model and used it to forecast renewable energy generation. Xie et al. [26] proposed a grey model based on a conformable fractional derivative, where the 1-order differential equation was replaced with an r-order differential equation with a conformable fractional derivative.

The fractional-order model is introduced as a solution to the significant drawback of the classical integer-order model, which fails to align with experimental findings. Although Xie's model utilizes a generalized conformable accumulation, the order remains confined to the range of [0,1]. Furthermore, the fixed generation function in the existing fractional-order model significantly restricts its adaptability. To address this predicament, we propose a generalized conformable fractional-order nonlinear grey prediction model capable of extending the fractional order to any positive real number, while having a flexible generation function. This generalized fractional-order accumulation not only increases the model's adaptability, but also enhances the space of the optimal solution. For completeness and clarity, the abbreviations and parameters used in the study are listed in Table A.

2. Methods

2.1. Preliminaries

In this section, we will review the classical fractional cumulative operators and conformable fractional derivatives, as well as their discrete forms.

Definition 1. Let there be a non-negative sequence $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n))^T$, $X^{(r)} = \left(x^{(r)}(1), x^{(r)}(2), \ldots x^{(r)}(n)\right)^T$ is the r-order $(0 < r \leq 1)$ accumulated generating of a sequence of $X^{(0)}$, and is defined as Eq. (1):

$$x^{(r)}(k) = \sum_{i=1}^{k} C_{k-i+r-1}^{k-i} x^{(0)}(i), k = 1, 2, \cdots, n \tag{1}$$

where $C_{r-1}^{0} = 1, C_{k-1}^{k} = 0, C_{k-i+r-1}^{k-i} = \frac{(k-i+r-1)(k-i+r-2)\cdots(r+1)r}{(k-i)!} k = 1, 2, \cdots, n.$.

In addition, fractional summation can also be expressed in matrix form shown in Eq. (2):

$$A^r = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ r & 1 & 0 & \cdots & 0 \\ \frac{r(r+1)}{2!} & r & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{r(r+1)\cdots(r+n-2)}{(n-1)!} & \frac{r(r+1)\cdots(r+n-3)}{(n-2)!} & \frac{r(r+1)\cdots(r+n-4)}{(n-3)!} & \cdots & 1 \end{pmatrix} \tag{2}$$

Further ,

$$A^r = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ C_r^1 & 1 & 0 & \cdots & 0 \\ C_{r+1}^2 & C_r^1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C_{r+n-2}^{n-1} & C_{r+n-3}^{n-2} & C_{r+n-4}^{n-3} & \cdots & 1 \end{pmatrix} \tag{3}$$

Eq. (3) is the cumulative matrix of r-order, and $X^{(r)}$ is expressed as Eq. (4):

$$X^{(r)} = A^r X^{(0)} \tag{4}$$

Definition 2. For a given differentiable function $f : [0, \infty) \to R$, the r-order conformable fractional derivative of f is defined as Eq. (5):

$$T_r(f)(t) = \lim_{\varepsilon \to 0} \frac{f(t + \varepsilon t^{1-r}) - f(t)}{\varepsilon} \tag{5}$$

Based on the above definition, Zhao et al. [27] gave the generalized conformable fractional derivative.

Definition 3. For a given differentiable function $f : [0, \infty) \to R$, the generalized r-order conformable fractional derivative of f is expressed as Eq. (6):

$$D_{\psi}^{r} f(u) = \lim_{\varepsilon \to 0} \frac{f(u + \varepsilon \psi(u, r)) - f(u)}{\varepsilon} \tag{6}$$

Based on this definition, Xie et al. [26] proposed a generalized conformable fractional-order accumulation operator to simplify the computational complexity of classical fractional-order accumulation operators and expand their application range.

Definition 4. The generalized conformable fractional-order accumulation of f is defined as Eq. (7):

$$\nabla^{-r} f(k) : = \int_a^b f(k) \nabla k = \sum_{k=a+1}^{b} \frac{f(k)}{\psi(k, r)} \tag{7}$$

where ${}_a^a f(k) \nabla k = \sum_{k=a+1}^{a} \frac{f(k)}{\psi(k, \alpha)} := 0$.

2.2. Generalized conformable fractional-order nonlinear grey Bernoulli model

In this section, we extend Xie's generalized accumulation and difference to a more general form, so that its order can take any positive real number.

Definition 5. According to the definition of general conformable fractional derivative, we can define the fractional difference as Eq. (8):

$$\nabla^r f(k) = \psi(k, r, \lceil r \rceil) \nabla^n f(k), r \in (n, n+1] \tag{8}$$

Definition 6. Let there be a non-negative sequence $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n))^T, X^{(r)} = (x^{(r)}(1), x^{(r)}(2), \cdots, x^{(r)}(n))^T$ is called the

r-order generalized conformable accumulation generates sequences of $X^{(0)}$, where $x^{(r)}(k)$ is defined as Eq. (9):

$$x^{(r)}(k) = \sum_{i=1}^{k} \binom{k-i+\lceil r\rceil-1}{k-i} \frac{x(i)}{\psi(i,r,\lceil r\rceil)}, k=1,2,\cdots,n \tag{9}$$

Proof : when $r \in (0,1]$, we have $\lceil r\rceil = 1$, and $x^{(r)}(k)$ is shown in Eq. (10):

$$\begin{aligned} x^{(r)}(k) := & \begin{pmatrix} x^{(0)}(1) \\ x^{(0)}(2) \\ x^{(0)}(3) \\ \cdots \\ x^{(0)}(n) \end{pmatrix}^T \begin{pmatrix} \frac{1}{\psi(1,r)} & \frac{1}{\psi(1,r,\lceil r\rceil)} & \cdots & \frac{1}{\psi(1,r,\lceil r\rceil)} & \frac{1}{\psi(1,r,\lceil r\rceil)} \\ 0 & \frac{1}{\psi(2,r,\lceil r\rceil)} & \cdots & \frac{1}{\psi(2,r,\lceil r\rceil)} & \frac{1}{\psi(2,r,\lceil r\rceil)} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \frac{1}{\psi(n-1,r,\lceil r\rceil)} & \frac{1}{\psi(n-1,r,\lceil r\rceil)} \\ 0 & 0 & \cdots & 0 & \frac{1}{\psi(n,r,\lceil r\rceil)} \end{pmatrix} \begin{pmatrix} 1 & 1 & \cdots & 1 & 1 \\ 0 & 1 & \cdots & 1 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 1 \\ 0 & 0 & \cdots & 0 & 1 \end{pmatrix} \\ & = \sum_{i=1}^{k} \binom{k-i}{k-i} \frac{f(i)}{\psi(1,r,\lceil r\rceil)}, r>0 \end{aligned} \tag{10}$$

when $r \in (0,2]$, we have $\lceil r\rceil = 2$, and $x^{(r)}(k)$ is expressed as Eq. (11):

$$\begin{aligned} x^{(r)}(k) := & \begin{pmatrix} x^{(0)}(1) \\ x^{(0)}(2) \\ x^{(0)}(3) \\ \cdots \\ x^{(0)}(n) \end{pmatrix}^T \begin{pmatrix} \frac{1}{\psi(1,r)} & \frac{1}{\psi(1,r,\lceil r\rceil)} & \cdots & \frac{1}{\psi(1,r,\lceil r\rceil)} & \frac{1}{\psi(1,r,\lceil r\rceil)} \\ 0 & \frac{1}{\psi(2,r,\lceil r\rceil)} & \cdots & \frac{1}{\psi(2,r,\lceil r\rceil)} & \frac{1}{\psi(2,r,\lceil r\rceil)} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \frac{1}{\psi(n-1,r,\lceil r\rceil)} & \frac{1}{\psi(n-1,r,\lceil r\rceil)} \\ 0 & 0 & \cdots & 0 & \frac{1}{\psi(n,r,\lceil r\rceil)} \end{pmatrix} \begin{pmatrix} 1 & C_2^1 & \cdots & C_{n-1}^{n-2} & C_{n-1}^{n} \\ 0 & 1 & \cdots & C_{n-2}^{n-3} & C_{n-1}^{n-2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & C_2^1 \\ 0 & 0 & \cdots & 0 & 1 \end{pmatrix} \\ = & \sum_{i=1}^{k} \binom{k-i+2-1}{k-i} \frac{f(i)}{\psi(1,r,\lceil r\rceil)}, r>0 \end{aligned} \tag{11}$$

when $r \in (0,n]$, $\lceil r\rceil = m$, $\nabla^{-r} f(k)$ is defined as Eq. (12):

$$\begin{aligned} \nabla^{-r} f(k) = & \begin{pmatrix} x^{(0)}(1) \\ x^{(0)}(2) \\ x^{(0)}(3) \\ \cdots \\ x^{(0)}(n) \end{pmatrix}^T \begin{pmatrix} \frac{1}{\psi(1,r)} & \frac{1}{\psi(1,r,\lceil r\rceil)} & \cdots & \frac{1}{\psi(1,r,\lceil r\rceil)} & \frac{1}{\psi(1,r,\lceil r\rceil)} \\ 0 & \frac{1}{\psi(2,r,\lceil r\rceil)} & \cdots & \frac{1}{\psi(2,r,\lceil r\rceil)} & \frac{1}{\psi(2,r,\lceil r\rceil)} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \frac{1}{\psi(n-1,r,\lceil r\rceil)} & \frac{1}{\psi(n-1,r,\lceil r\rceil)} \\ 0 & 0 & \cdots & 0 & \frac{1}{\psi(n,r,\lceil r\rceil)} \end{pmatrix} \begin{pmatrix} 1 & 1 & \cdots & 1 & 1 \\ 0 & 1 & \cdots & 1 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 1 \\ 0 & 0 & \cdots & 0 & 1 \end{pmatrix}^m \\ & = \sum_{i=1}^{k} \binom{k-i+m-1}{k-i} \frac{f(i)}{\psi(1,r,\lceil r\rceil)}, r>0 \end{aligned} \tag{12}$$

when $r \in (0,n+1]$, $\lceil r\rceil = n+1$, $x^{(r)}(k)$ is expressed as Eq. (13):

$$x^{(r)}(k) = \begin{pmatrix} x^{(0)}(1) \\ x^{(0)}(2) \\ x^{(0)}(3) \\ \cdots \\ x^{(0)}(n) \end{pmatrix}^T \begin{pmatrix} \frac{1}{\psi(1,r)} & \frac{1}{\psi(1,r,\lceil r\rceil)} & \cdots & \frac{1}{\psi(1,r,\lceil r\rceil)} & \frac{1}{\psi(1,r,\lceil r\rceil)} \\ 0 & \frac{1}{\psi(2,r,\lceil r\rceil)} & \cdots & \frac{1}{\psi(2,r,\lceil r\rceil)} & \frac{1}{\psi(2,r,\lceil r\rceil)} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \frac{1}{\psi(n-1,r,\lceil r\rceil)} & \frac{1}{\psi(n-1,r,\lceil r\rceil)} \\ 0 & 0 & \cdots & 0 & \frac{1}{\psi(n,r,\lceil r\rceil)} \end{pmatrix} \begin{pmatrix} 1 & C_m^1 + C_m^0 & \cdots & \sum_{i=0}^{n-3} C_{m+i}^{i+1} & \sum_{i=0}^{n-2} C_{m+i}^{i+1} \\ 0 & 1 & \cdots & \sum_{i=0}^{n-4} C_{m+i}^{i+1} & \sum_{i=0}^{n-3} C_{m+i}^{i+1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & C_m^1 + C_m^0 \\ 0 & 0 & \cdots & 0 & 1 \end{pmatrix}$$
$$= \sum_{i=1}^{k} \binom{k-i+m}{k-i} \frac{f(i)}{\psi(1,r,\lceil r\rceil)}, r > 0 \tag{13}$$

Definition 7. The first-order differential equation of $x^{(r)}$ can be defined as Eq. (14):

$$\frac{dx^{(r)}(t)}{dt} + ax^{(r)}(t) = b\left(x^{(r)}(t)\right)^{\beta}, r > 0 \tag{14}$$

The exact solution of the above equation is shown in Eq. (15):

$$x^{(r)}(t) = \left[\left(\left(x^{(r)}(1)\right)^{1-\beta} - \frac{b}{a}\right)e^{-a(1-\beta)(t-1)} + \frac{b}{a}\right]^{\frac{1}{1-\beta}} \tag{15}$$

The time function of the nonlinear grey prediction model can be obtained by discretization, and the calculation is defined as Eq. (16) :

$$\widehat{x}^{(r)}(k) = \left[\left(\left(x^{(r)}(1)\right)^{1-\beta} - \frac{b}{a}\right)e^{-a(1-\beta)(k-1)} + \frac{b}{a}\right]^{\frac{1}{1-\beta}}, k = 2, 3, \ldots, n \tag{16}$$

The prediction result can be obtained by the inverse operator of the generalized fractional cumulative operator, and the calculation is expressed as Eq. (17) :

$$. \nabla^r f(k) = \psi(k, r, \lceil r\rceil)\nabla^n f(k), r \in (n, n+1] \tag{17}$$

Next, we will consider how to solve the parameters of the model. The parameters of the model can be obtained by the least square operator shown in Eq. (18):

$$(\widehat{a}, \widehat{b})^T = \left(B^T B\right)^{-1} B^T Y \tag{18}$$

where B and Y are calculated according to Eq. (19):

$$B = \begin{pmatrix} -\left(z^{(r)}(2)\right) & \left(z^{(r)}(2)\right)^{\beta} \\ -\left(z^{(r)}(3)\right) & \left(z^{(r)}(3)\right)^{\beta} \\ \vdots & \vdots \\ -\left(z^{(r)}(n)\right) & \left(z^{(r)}(n)\right)^{\beta} \end{pmatrix}, Y = \begin{pmatrix} x^{(r)}(2) - x^r(1) \\ x^{(r)}(3) - x^r(2) \\ \vdots \\ x^{(r)}(n) - x^r(n-1) \end{pmatrix} \tag{19}$$

where $z^{(r)}(k) = 0.5x^{(r)}(k-1) + 0.5x^{(r)}(k)$.

The effectiveness of simulation predictions in the GCFNGBM is dependent on the fractional-order r. To optimize predictions, this study utilizes the minimum mean absolute error percentage (MAPE) as the objective function and constrains the functional relationship between fractional-order r and least square parameters in order to establish a nonlinear programming model. The model is expressed as Eq. (20) and Eq. (21):

$$\min(MAPE) = \frac{1}{n-1}\sum_{k=2}^{n}\left|\frac{\widehat{x}^{(0)}(k) - x^{(0)}(k)}{x^{(0)}(k)}\right| \times 100\% \tag{20}$$

$$s.t.\begin{cases}\nabla^{-r}f(k)=\sum_{i=1}^{k}\begin{pmatrix}k-i+[r]-1\\k-i\end{pmatrix}\dfrac{f(i)}{\psi(1,r,\lceil r\rceil)},r\in R^{+},k\in Z^{+},\\ B=\begin{pmatrix}-\left(z^{(r)}(2)\right) & \left(z^{(r)}(2)\right)^{\gamma}\\ -\left(z^{(r)}(3)\right) & \left(z^{(r)}(3)\right)^{\gamma}\\ \vdots & \vdots\\ -\left(z^{(r)}(n)\right) & \left(z^{(r)}(n)\right)^{\gamma}\end{pmatrix},Y=\begin{pmatrix}x^{(r)}(2)-x^{r}(1)\\x^{(r)}(3)-x^{r}(2)\\ \vdots\\ x^{(r)}(n)-x^{r}(n-1)\end{pmatrix},\\ \widehat{x}^{(r)}(k)=\left[\left(\left(x^{(r)}(1)\right)^{1-\gamma}-\dfrac{b}{a}\right)e^{-a(1-\beta)(k-1)}+\dfrac{b}{a}\right]^{\frac{1}{1-\beta}},k=2,3,\ldots,n,\\ \nabla^{r}f(k)=\psi(k,r,\lceil r\rceil)\nabla^{n}f(k),r\in(n,n+1]\\ \widehat{x}^{(0)}(k)=x^{(0)}(k).\end{cases}\tag{21}$$

The MAPE solution involved absolute value calculation. Since the MAPE is not derivable for r, it is difficult to give the analytical formula of the model. In this paper, the least square method is used to solve parameters to be estimated, and the particle swarm optimizer (PSO) is used to search the hyperparameters of the optimal model.

The modeling details of the GCFNGBM are shown in Fig. 1.

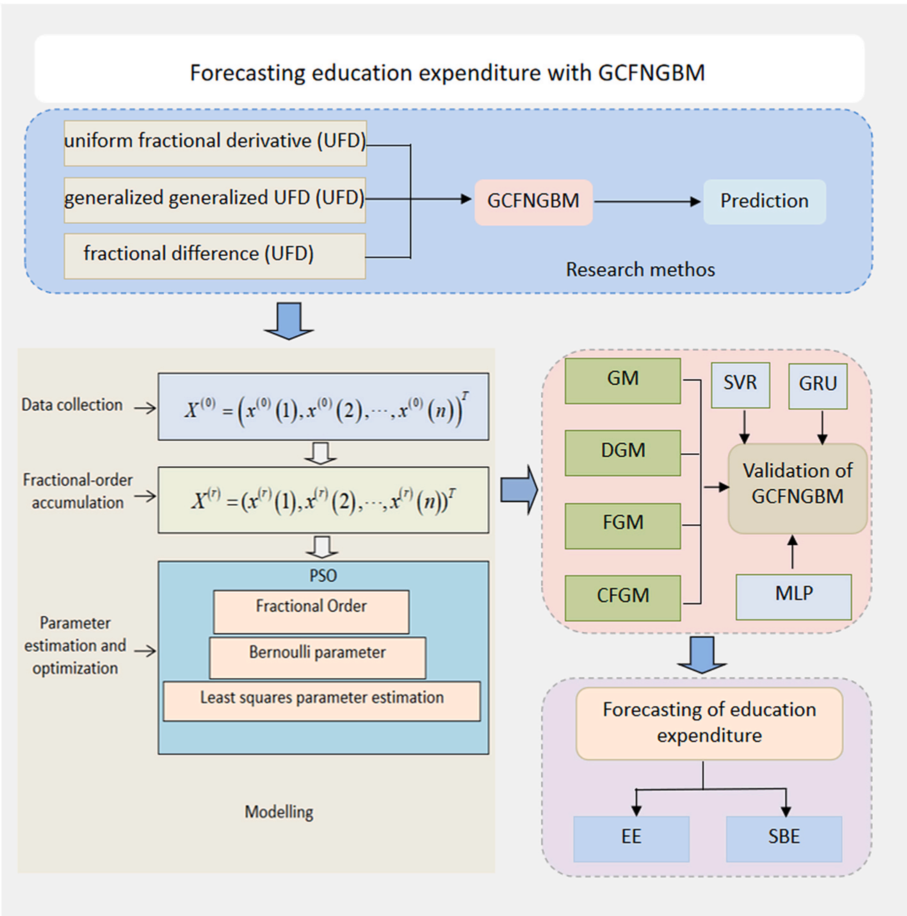


Fig. 1. Modeling details of GCFNGBM.

3. Education expenditure forecasting

3.1. Data set and metrics

In order to verify the validity of the model, we apply the model to educational expenditure prediction. The data source is China's education expenditures, and the raw data come from China Statistics Bureau (http://www.stats.gov.cn). The absolute percentage error (APE), mean absolute percentage error (MAPE), mean square error (MSE), Mean Absolute Error (MAE), and root-mean-square error (RMSE) are employed to measure the accuracy of prediction models. The smaller the MAPE and APE, the better the fitting effect of the model. Their definitions are Eq. (22), Eq. (23), Eq. (24), Eq. (25), and Eq. (26), respectively.

$$\mathrm{APE} = \left| \frac{x^{(0)}(k) - \widehat{x}^{(0)}(k)}{x^{(0)}(k)} \right| \times 100\% \tag{22}$$

$$\mathrm{MAPE} = \frac{1}{n} \sum_{k=1}^{n} \left| \frac{x^{(0)}(k) - \widehat{x}^{(0)}(k)}{x^{(0)}(k)} \right| \times 100\% \tag{23}$$

$$\mathrm{MSE} = \frac{1}{n} \sum_{k=1}^{n} \left(x^{(0)}(k) - \widehat{x}^{(0)}(k) \right)^2 \tag{24}$$

$$\mathrm{MAE} = \frac{1}{n} \sum_{k=1}^{n} \left| x^{(0)}(k) - \widehat{x}^{(0)}(k) \right| \tag{25}$$

$$\mathrm{RMSE} = \sqrt{\frac{1}{n} \sum_{k=1}^{n} \left(x^{(0)}(k) - \widehat{x}^{(0)}(k) \right)^2} \tag{26}$$

where $X^{(0)}(k)$ is the original series, and $\widehat{x}^{(0)}(k)$ is the fitted or predicted data series. Lewis' [28] benchmark of accuracy evaluation is applied in this study, which is listed in Table 1 [28].

3.2. Validation of the GCFNGBM validity

In this section, we compare the GCFNGBM model with four classic models, GM (grey model), DGM (discrete grey model), FGM (fractional grey model), and CFGM (conformable fractional grey model) in forecasting the education expenditure of China. GM is the first grey prediction model, and it is also the most influential and widely used form at present. DGM model is the improvement of GM from continuous form to discrete form, which can realize the unbiased simulation of aligned sub-exponential sequences. FGM generalizes the integer order summation grey model to fractional order summation. Compared with the general fractional cumulative FGM, the form of CFGM is simpler, convenient for calculation and theoretical derivation, and it improves the adaptability and prediction ability of the model.

In the experiments, the optimal order r of the GCFNGBM is 1.0026 produced by the PSO algorithm shown in Fig. 2. The data from 2013 to 2017 was used as fitting, and the data from 2018 to 2019 was used as testing.

Table 2 shows the comparison result of our GCFNGBM model and the other four models. The index of MAPE recorded a fitting error of 0.2435, and a testing error of 0.2294. Comparatively, the fitting errors of GM, DGM, FGM, and CFGM were observed to be 0.3254, 0.3237, 0.2811, and 0.2862, respectively. Among these models, our GCFNGBM achieved the best results, followed by FGM and CFGM. Furthermore, the testing errors of GM, DGM, FGM, and CFGM were found to be 0.3362, 0.3562, 0.3684, and 0.4486, respectively. This highlights that our GCFNGBM excels other models in terms of testing error. Overall, comparing the MAPE of the GCFNGBM model with the other four models reveals that our GCFNGBM produces higher accuracy both in fitting and testing. Fig. 3 visually presents the testing fitting MAPE (F_MAPE) and MAPE (T_MAPE) results.

We conduct a precision comparison of five prediction models for educational expenditures across different regions. The data of the first sixteen years (2013–2017) for fitting, and the data of the next four years (2018–2019) for forecasting. Table 3 presents the precision errors in the test set (2018–2019).

To further confirm the validity of our model, we compare our GCFNGBM with a range of artificial intelligent methods, including Support Vector Regression (SVR) [29], Gated Recurrent Unit (GRU) [30], and Multilayer Perceptron (MLP) [31]. The kernel of the SVR is the radial basis. In GRU, the activation function is ReLU, the optimizer is Adam, and the number of units in the first hidden layer is 50. Both the fitting and testing data utilized in this analysis are from Table 2, with the testing errors in terms of APE displayed in Fig. 4. As shown, our model exhibited lower APEs than the other compared models.

Table 1
Benchmark of modelling accuracy evaluation.

MAPE	≤10%	10%–20%	0%–50%	≥50%
Evaluation	Accurate	Good	Reasonable	Inaccurate

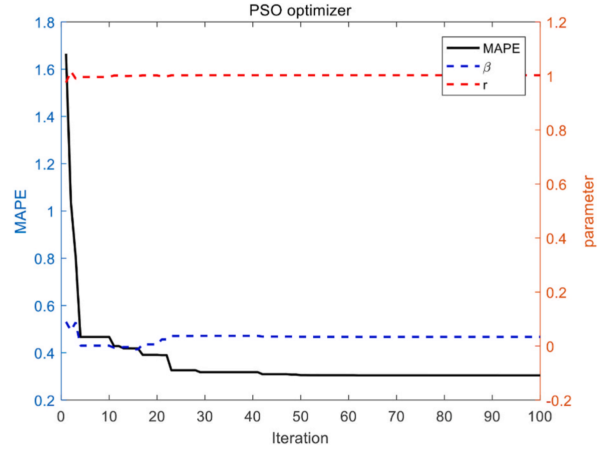


Fig. 2. The optimization process with PSO (β = 0.03386, r = 1.0026).

Table 2
Forecasting the education expenditure (ten thousand yuan) of China.

Model		GM		DGM		FGM		CFGM		GCFNGBM	
Year	Raw data/10^8	Result	APE	Result	APE	Result	APE	Result	APE	Result	APE
2013	3.0365	3.0365	0.0000	3.0365	0.0000	3.0365	0.0000	3.0365	0.0000	3.0365	0.0000
2014	3.2806	3.2912	0.3217	3.2933	0.3857	3.2806	0.0014	3.2806	0.0014	3.2806	0.0014
2015	3.6129	3.5840	0.8004	3.5865	0.7312	3.5870	0.7174	3.5958	0.4738	3.5957	0.4766
2016	3.8888	3.9028	0.3590	3.9057	0.4336	3.9150	0.6727	3.9133	0.6290	3.9176	0.7396
2017	4.2562	4.2500	0.1457	4.2533	0.0682	4.2556	0.0141	4.2423	0.3266	4.2562	0.0000
	MAPE		0.3254		0.3237		0.2811		0.2862		0.2435
2018	4.6143	4.6281	0.2991	4.6319	0.3814	4.6057	0.1864	4.5877	0.5765	4.6163	0.0433
2019	5.0178	5.0398	0.4382	5.0442	0.5259	4.9639	1.0744	4.9528	1.2956	5.0012	0.3311
	MAPE		0.3362		0.3562		0.3684		0.4486		0.2294

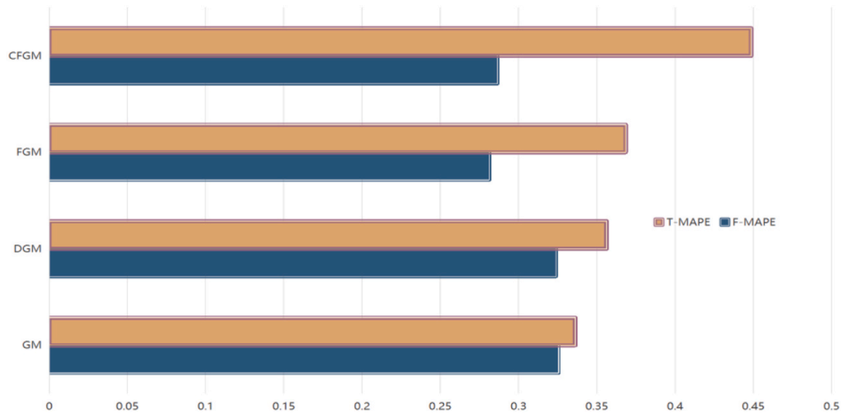


Fig. 3. The testing MAPE(T_MAPE) and fitting MAPE(F_MAPE).

Table 3 illustrates the testing MAPE, MSE, MAE, and RMSE of 11 regions in China. Compared to other models such as GM, DGM, FGM, and CFGM, our GCFNGBM exhibited higher precision. For instance, Beijing City achieved a MAPE of 2.5979, which is lower than the values obtained through GM, DGM, FGM, and CFGM. Moreover, the MSE of GCFNGBM proved to be 1.72E+11, which is also lower when compared to the performance of other models. In summary, through the outcome of the 11 groups' analysis, it can be found that our GCFNGBM model is superior to other models. The fitting errors of the GCFNGBM model, in terms of MAPE, MSE, MAE, and RMSE, are presented in Fig. 5(a)~(d) for the 11 regions in China, respectively.

Table 3
Precision comparison of five prediction models.

Region		GM	DGM	FGM	CFGM	GCFNGBM
Beijing	MAPE	5.2754	5.2632	5.2754	5.2754	2.5979
	MSE	6.78E+11	6.75E+11	6.78E+11	6.78E+11	1.72E+11
	MAE	7.60E+05	7.58E+05	7.60E+05	7.60E+05	3.75E+05
	RMSE	8.23E+05	7.58E+05	8.23E+05	8.23E+05	4.15E+05
Tianjin	MAPE	16.263	16.542	14.344	16.263	5.1082
	MSE	1.05E+12	1.09E+12	8.19E+11	1.05E+12	1.63E+11
	MAE	1.03E+06	1.04E+06	9.05E+05	1.03E+06	3.21E+05
	RMSE	1.03E+06	1.04E+06	9.05E+05	1.03E+06	4.04E+05
Heibai	MAPE	3.2975	3.4789	2.4639	4.3946	2.3681
	MSE	3.76E+11	4.17E+11	3.84E+11	1.15E+12	3.19E+11
	MAE	6.06E+05	6.40E+05	4.84E+05	8.59E+05	4.62E+05
	RMSE	6.13E+05	6.46E+05	6.20E+05	1.07E+06	5.65E+05
Shanxi	MAPE	2.6461	2.8992	9.4287	18.787	0.74468
	MSE	8.40E+10	9.84E+10	9.53E+11	3.64E+12	9.00E+09
	MAE	2.56E+05	2.80E+05	9.08E+05	1.81E+06	72925
	RMSE	2.90E+05	3.14E+05	9.76E+05	1.91E+06	94858
Jiangsu	MAPE	2.4522	2.3933	2.5721	2.5825	2.3147
	MSE	6.74E+11	6.48E+11	7.38E+11	7.44E+11	6.18E+11
	MAE	7.43E+05	7.25E+05	7.79E+05	7.82E+05	7.02E+05
	RMSE	8.21E+05	8.05E+05	8.59E+05	8.63E+05	7.86E+05
Zhejiang	MAPE	5.2221	5.1347	5.2221	5.2221	2.5006
	MSE	2.16E+12	2.10E+12	2.16E+12	2.16E+12	5.48E+11
	MAE	1.37E+06	1.35E+06	1.37E+06	1.37E+06	6.61E+05
	RMSE	1.47E+06	1.45E+06	1.47E+06	1.47E+06	7.41E+05
Anhui	MAPE	0.3566	0.27106	0.93626	1.0247	6.12E-03
	MSE	3.21E+09	1.92E+09	2.23E+10	2.69E+10	1.08E+06
	MAE	55327	41894	1.48E+05	1.62E+05	975.16
	RMSE	56663	43870	1.49E+05	1.64E+05	1041.5
Henan	MAPE	4.2816	4.2171	5.4393	4.2816	2.473
	MSE	1.20E+12	1.16E+12	3.27E+12	1.20E+12	7.00E+11
	MAE	1.09E+06	1.08E+06	1.44E+06	1.09E+06	6.53E+05
	RMSE	1.09E+06	1.08E+06	1.81E+06	1.09E+06	8.37E+05
Hubei	MAPE	8.4054	8.5104	2.0642	2.1071	1.8943
	MSE	1.70E+12	1.74E+12	1.01E+11	1.05E+11	8.94E+10
	MAE	1.29E+06	1.31E+06	3.17E+05	3.23E+05	2.93E+05
	RMSE	1.30E+06	1.32E+06	3.17E+05	3.25E+05	2.99E+05
Sichuan	MAPE	2.354	2.4435	2.8788	2.9707	0.88155
	MSE	2.78E+11	2.99E+11	5.02E+11	5.19E+11	6.40E+10
	MAE	5.14E+05	5.33E+05	6.35E+05	6.55E+05	1.97E+05
	RMSE	5.28E+05	5.47E+05	7.08E+05	7.21E+05	2.53E+05
Qinghai	MAPE	8.6627	8.6422	8.6627	8.6627	6.1084
	MSE	6.17E+10	6.14E+10	6.17E+10	6.17E+10	3.04E+10
	MAE	2.42E+05	2.41E+05	2.42E+05	2.42E+05	1.70E+05
	RMSE	2.48E+05	2.48E+05	2.48E+05	2.48E+05	1.74E+05

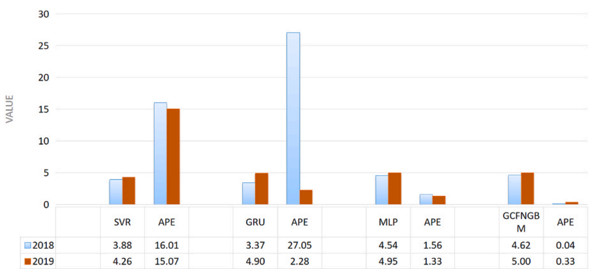


Fig. 4. The comparison of GCFNGB with artificial intelligent methods in testing data set.

3.3. Forecasting the education expenditure of China with GCFNGBM

The country's educational expenditure is funded by various sources which include state budgetary expenditure for education (SBE), investment by the founders of privately-run schools, public donations, income from undertakings, and other educational funds. SBE, in particular, is the allocation of educational appropriations arranged and transferred by central and local financial departments at various levels or the competent departments at higher levels to schools, educational administrations, and educational institutions.

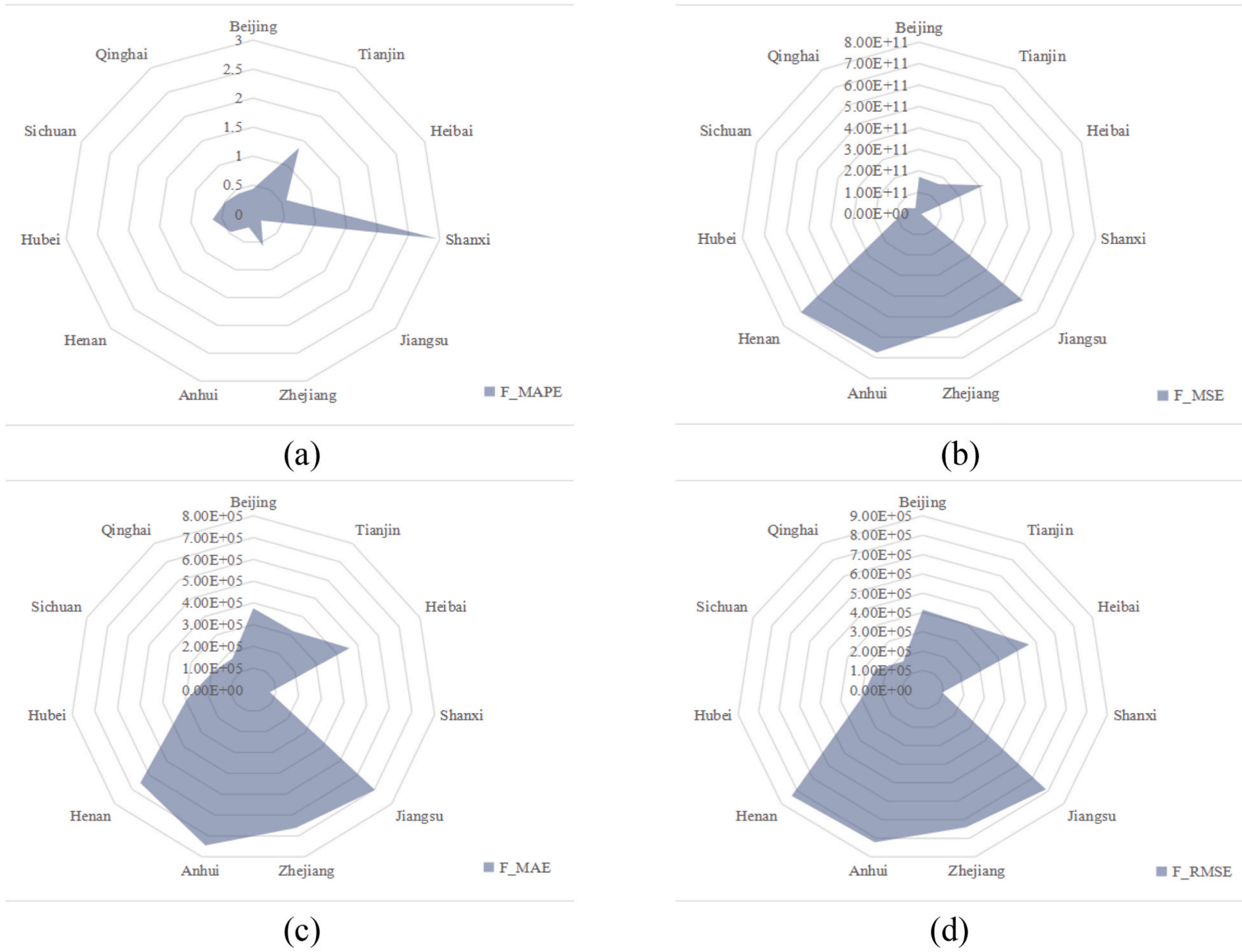


Fig. 5. The fitting errors of the GCFNGBM model in terms of MAPE, MSE, MAE, and RMSE.

These appropriations fall under the subjects of state budget expenditure and account for the majority of national education expenditure. As a result, SBE remains a crucial guarantee for the development of education in a country.

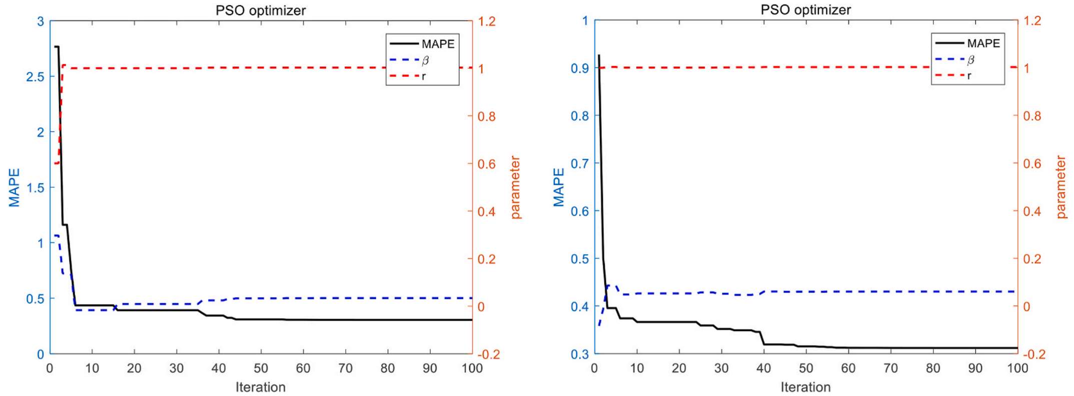
We utilized the GCFNGBM to forecast the education expenditure (EE) and state budgetary expenditure (SBE) of China. Obtained from PSO, the β are 0.03386 and 0.060536, and r are 1.0026 and 1.0024, respectively. Their visualizations are presented in Fig. 6(a) and (b), while Fig. 7 displays the forecasting results of both EE and SBE. As depicted, both categories are expected to show an annual increase.

4. Discussion

4.1. The proposed grey model for forecasting the education expenditure

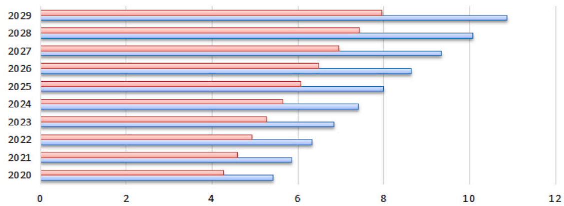
Grey modeling approaches are widely adopted in diverse fields due to their ability to generate meaningful insights using small or limited data. The fractional-order grey model offers a global correlation and reflects the historically dependent process of system function development. Conformable accumulation, which is generally a generalization of integer accumulation, is used in current grey models. Prior to this work, generalized conformable summation was restricted only to the order of [0,1]. In this paper, from the perspective of matrix theory and mathematical induction, we deduce that the fractional order can be any positive real number that exceeds zero. The accumulation of GCFNGBM is highly general, and this generalized fractional-order accumulation enhances model flexibility. It breaks the 0–1 limit, thereby extending the favorable solution space to any positive real number. Additionally, optimization technique, such as particle swarm optimization, is applied to enhance the models' adaptability by choosing the best parameters.

In this study, we compared the GCFNGBM with a group of classic grey models, including GM, DGM, FGM, and CFGM, to fit and test the EE of China. Table 2 shows that the GCFNGBM produced higher precision than the other models in both the fitting and testing phases. We also employed the GCFNGBM to forecast the EE of various regions in China, and Table 3 lists the errors in terms of MAPE, MSE, MAE, and RMSE. These results demonstrate the superior accuracy of our model over other models. Fig. 3 displays the fitting error



(a)EE(β=0.03386, r =1.0026) (b)SBE (β=0.060536, r =1.0024)

Fig. 6. The optimization process for searching parameters.



	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029
SBE/10-8	4.2716	4.5872	4.9219	5.2775	5.6557	6.0582	6.4869	6.9437	7.4306	7.9497
EE/10-8	5.414	5.8575	6.3344	6.8477	7.4004	7.9958	8.6375	9.329	10.074	10.878

Fig. 7. The prediction of EE and SBE (ten thousand yuan).

of the GCFNGBM. Lastly, we used the GCFNGBM to forecast the EE and SBE for the coming years.

The GCFNGBM model has yielded promising results; however, there are limitations for further improvement. One limitation is the unfixed cumulative generating function. Resolving this requires determining the optimal function, a problem that requires further attention. A second limitation arises due to the expansion of the model's solving space, which increases the likelihood of over-fitting. For instance, while training set accuracy is high, test set accuracy is low. Thus, overcoming over-fitting is another challenge that should be addressed.

4.2. *The rapid growth in education expenditure*

Education is one of the main initiatives of the national government. The number of educated people, the level of development in education, and the level of investment in education have become the main objective indicators to assess the cultural quality and civilization of the residents of a country and a region. A significant portion of fiscal expenditure is known as educational expenditure, which includes all costs incurred by the state for educational endeavors. Education expenditure is a key indicator of a nation's degree of educational advancement and has a significant impact on its social and economic growth. Rational and efficient utilization and allocation of these resources will play an important role in promoting science and education. From Fig. 7, it can be seen that the EE and SBE will grow and reach 10.878E+8 and 7.9497E+8, respectively. EE and SEB will expand during the next 10 years. With the development of the national economy, education expenditure will continue to keep increasing, showing a fast growth rate. There are two primary causes:

First, the State places a high priority on education development and makes significant investments in it. Investment in education is an investment in the future workforce and a gauge of a nation's level of intelligence. Today's intelligent products, represented by information and software, call for high-tech expertise, thus senior staff with advanced degrees can significantly enhance the function of the labor force in production.

Second, policy support encourages the expansion of monetary investment in education. The Chinese government has put in place a number of rules to ensure the investment of educational funds and promote the expansion of education. For instance, the Outline of the National Medium- and Long-Term Plan for Education and Development (2010–2020) was published in July 2010, and it included a special chapter on the topic of ensuring educational expenditure over the next 10 years, as well as a new goal for education development that called for the national fiscal expenditure on education to account for 4% of GDP. In 2022, a circular was released on the budget for subsidies for compulsory education in urban and rural areas to ensure the healthy development of compulsory education in urban and rural areas.

The continuous rise in education spending will significantly enhance the consolidation and improvement of the education funding system, which is primarily reliant on government funding through multiple channels. This will provide strong support to the largest education system in the world.

5. Conclusion

A necessary financial requirement for operating schools is the payment of educational expenses in the form of money. In order to forecast education spending, we proposed a generalized conformable fractional-order nonlinear grey system model, abbreviated as GCFNGBM. The merits consist of (1) We extend the classical grey Bernoulli model and introduce the generalized conformable fractional accumulation as a new accumulation generator. (2) The suggested model's flexibility is increased by using the particle swarm optimization technique to choose its parameters. (3) The novel model is used to estimate education spending, and it has better fitting and forecasting capabilities than previous models, which aid in educational decision-making and support the improvement of educational quality.

The grey model is suitable for the prediction with a small data set. Although the forecasting accuracy was further increased by our model, there is still room for improvement. In the subsequent research, we will first consider overcoming some of the shortcomings of the model, such as adding regularization terms during modeling to avoid over-fitting of the model. Second, we'll broaden and deepen our forecasts even more. There are many types of education expenditure. So we will perform a more thorough forecast and analysis of each category in the following stage to assist the government in making decisions.

Data availability statement

The datasets used in this paper are available at http://www.stats.gov.cn.

Author contribution statement

Caixia Liu: Conceived and designed the experiments; Analyzed and interpreted the data; Wrote the paper.
Zhenguo Xu; Keyun Zhao: Contributed reagents, materials, analysis tools or data.
Wanli Xie: Performed the experiments.

Additional information

No additional information is available for this paper.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix

Table A
List of primary abbreviations and parameters.

Abbreviations Nomenclatures		Parameters Nomenclatures	
GM	Grey prediction model	$X^{(0)}$	Original sequence
DGM	Discrete grey model	$X^{(1)}$	First-order cumulative generation sequence
FGM	Fractional grey model	$X^{(r)}$	r-order cumulative generating operator
CFGM	Conformable fractional prediction model	B	Parameter matrix
PSO	Particle swarm optimizer	r	Order of differential equations
APE	Absolute percentage error	β	Bernoulli parameter
MAPE	Mean absolute percentage error	a, b	Least squares parameter estimation
MSE	Mean squared error	Y	Parameter vector
MAE	Mean Absolute Erro	R	Set of real numbers
RMSE	Root mean square error	$\hat{x}^{(r)}$	Prediction results of the proposed model

References

[1] Y. Hu, Y. Tang, Forecast of China's pre-school spending and allocation from 2020 to 2035: based on a comparative study with OECD countries with high returns from pre-school spending, J. Cap. Normal Univ. (4) (2022) 139–152.
[2] K. Shang, D. Liu, G. Yang, Prediction of educational expenditure in the period of the 14th five-year plan based on GIOWA operator, J. Pingdingshan Univ. 37 (2) (2022) 83–94.
[3] C. Yue, W. Qiu, The scale and structure of higher education and public educational funding in China: an Empirical Prediction till 2035, J. East China Normal Univ. (Educ. Sci. Ed.) 39 (6) (2021) 1–16.
[4] M. Xia, Forecast of the Demand for Preschool Education Funds under the Background of the "Universal Two-Child" Policy, Jiangxi Normal University, 2018.
[5] F. Zeng, Prediction of preschool educational appropriations demand in 2020, Jiangsu Sci. Technol. Inform. (10) (2017) 62–63+70.
[6] H. Zhang, Research forecast of China's education financial input in the future ten years, J. Chengdu Normal Univ. 31 (11) (2015) 19–24.
[7] J. Deng, Control problems of grey systems, Syst. Control Lett. 1 (1982) 288–294.
[8] Y. Wang, Z. Yang, L. Wang, Forecasting China's energy production and consumption based on a novel structural adaptive Caputo fractional grey prediction model, Energy 259 (2022), 124935.
[9] T. Quartey-Papafio, S. Liu, S. Javed, Forecasting cocoa production of six major producers through ARIMA and grey models, Grey Syst. Theor. Appl. (2020), https://doi.org/10.1108/GS-04-2020-0050 ahead-of-print(ahead-of-print).
[10] W. Yang, B. Li, Prediction of grain supply and demand structural balance in China based on grey models, Grey Syst. Theor. Appl. 11 (2) (2021) 253–264.
[11] H. Liu, D. Zhang, Analysis and prediction of hazard risks caused by tropical cyclones in Southern China with fuzzy mathematical and grey models, Appl. Math. Model. 36 (2) (2012) 626–637.
[12] A. Bezuglov, G. Comert, Short-term freeway traffic parameter prediction: application of grey system theory models, Expert Syst. Appl. 62 (2016) 284–292.
[13] R. Intharathirat, P. Salam, S. Kumar, et al., Forecasting of municipal solid waste quantity in a developing country using multivariate grey models, Waste Manag. 39 (may) (2015) 3–14.
[14] M. Islam, S. Ali, A. Fathollahi-Fard, G. Kabir, A novel particle swarm optimization-based grey model for the prediction of warehouse performance, J. Comput. Design Eng. 8 (2) (2021) 705–727.
[15] G. Duman, E. Kongar, S. Gupta, Estimation of electronic waste using optimized multivariate grey models, Waste Manag. 95 (2019) 241–249.
[16] U. Kumar, V. Jain, Time series models (Grey-Markov, Grey Model with rolling mechanism and singular spectrum analysis) to forecast energy consumption in India, Energy 35 (4) (2010) 1709–1716.
[17] C. Hamzacebi, H. Es, Forecasting the annual electricity consumption of Turkey using an optimized grey model, Energy 70 (2014) 165–171.
[18] W. Xie, W. Wu, C. Liu, et al., Forecasting fuel combustion-related CO2 emissions by a novel continuous fractional nonlinear grey Bernoulli model with grey wolf optimizer, Environ. Sci. Pollut. Control Ser. 28 (28) (2021) 38128–38144。.
[19] B. Zeng, S. Liu, A self-adaptive intelligence gray prediction model with the optimal fractional order accumulating operator and its application, Math Meth Appl 40 (2017) 7843–7857.
[20] X. Ma, X. Mei, W. Wu, et al., A novel fractional time delayed grey model with Grey Wolf Optimizer and its applications in forecasting the natural gas and coal consumption in Chongqing China, Energy 178 (2019) 487–507.
[21] W. Wu, L. Zeng, C. Liu, et al., A time power-based grey model with conformable fractional derivative and its applications, Chaos, Solit. Fractals 155 (2022), 111657.
[22] M. Huang, C. Liu, A variable-order fractional discrete grey model and its application, J. Intell. Fuzzy Syst. 41 (2) (2021) 3509–3522.
[23] C. Liu, W. Xie, An optimized fractional grey model based on weighted least squares and its application, AIMS Mathematics 8 (2) (2022) 3949–3968.
[24] Y. Wang, R. Nie, P. Chi, et al., A novel fractional structural adaptive grey Chebyshev polynomial Bernoulli model and its application in forecasting renewable energy production of China, Expert Syst. Appl. 210 (2022), 118500.
[25] X. He, Y. Wang, Y. Zhang, et al., A novel structure adaptive new information priority discrete grey prediction model and its application in renewable energy generation forecasting, Appl. Energy 325 (2022), 119854.
[26] W. Xie, C. Liu, W. Wu, W. Li, C. Liu, Continuous grey model with conformable fractional derivative, Chaos, Solit. Fractals 139 (2020), 110285.
[27] D. Zhao, M. Luo, General conformable fractional derivative and its physical interpretation, Calcolo 54 (2017) 903–917.
[28] C. Lewis, Industrial and Business Forecasting Method, butter-worth-Heinemann, London, 1982.
[29] M. Awad, R. Khanna, Support vector regression, in: Efficient Learning Machines, Apress, Berkeley, CA, 2015, https://doi.org/10.1007/978-1-4302-5990-9_4.
[30] R. Dey, F.M. Salem, Gate-variants of Gated Recurrent Unit (GRU) Neural Networks, 2017 IEEE 60th International Midwest Symposium on Circuits and Systems (MWSCAS), IEEE, 2017, pp. 1597–1600.
[31] K.Y. Almansi, A. Shariff, B. Kalantar, et al., Performance evaluation of hospital site suitability using multilayer perceptron (MLP) and analytical hierarchy process (AHP) models in Malacca, Malaysia, Sustainability 14 (7) (2022) 1–36.