Heliyon 9 (2023) e15615

Contents lists available at ScienceDirect

Heliyon

journal homepage: www.cell.com/heliyon

Research article

A reverse mechanism of advance selling driven by information asymmetry and competition

Yi Zou

School of Management and E-Business, Zhejiang Gongshang University, Hangzhou, 310018, PR China

A R T I C L E I N F O

Keywords:
Advance selling
Newsvendor problem
Pricing
Ordering policy

A B S T R A C T

Enlightened by popular online business practices emerging in recent years, we aim to investigate a problem involving a flipped procedure in contrast with traditional advance selling, which is referred to as "reverse advance selling" (RAS) in this paper. We consider competition and information asymmetry in the market and discuss how they affect the decisions in reverse advance selling. We propose two models to evaluate the benefits of RAS and to characterize the conditions that optimize the pricing and ordering policies for retailers under competition. Furthermore, we examine the impact of factors such as market share, online review, and waiting time and provide insights for retailers to make decisions. The results demonstrate the advantage of adopting RAS when retailers or customers face uncertainty and it is beneficial to update review information. This paper also finds that market share positively affects the profit as well as the ordering quantities of the retailer, while online reviews have opposite impacts on its discount and ordering decisions. The results can guide retailers to make flexible ordering plans that better cater to market demand.

1. Introduction

As an effective measure to match demand and supply in markets, advance selling has been adopted by numerous retailers to improve profits in uncertain environments by extending the selling season [1–3]. Generally, two stages are included in traditional advance selling (TAD): an advance selling period followed by a regular period as needed [4]. It has been widely applied in online business to reduce risks for retailers. Meanwhile, customers can suffer from a lack of information when they commit orders in advance selling [5,6]. They also find it difficult to distinguish the product quality from numerous sellers who can easily manipulate presenting themselves online, intensifying the information asymmetry as a result [7]. TAD is characterized by a big time lag between purchase and product delivery. Existing research has shown that as perceived waiting time increases, customer satisfaction decreases [8]. Owing to the convenience of e-commerce, a great number of vendors enter the market readily and the competition is strengthened. The advantage of a monopolistic retailer can be challenged as its competitors sell substitute products with less waiting time, hence it takes the risk of losing customers who are impatient or information sensitive.

These challenges have driven the emergence and popularity of new business practices in recent years. For instance, many retailers launch a spot period before an advance selling period to mitigate their anxiety about inventory risks under uncertainty. Since this mechanism flips the process in contrast to traditional advance selling, it is termed "reverse advance selling" (RAS) in this paper. Launching a pilot spot selling beforehand allows consumers to share product information and helps to mitigate information symmetry.

E-mail address: mail_zou@126.com.

https://doi.org/10.1016/j.heliyon.2023.e15615
Received 26 November 2022; Received in revised form 13 April 2023; Accepted 17 April 2023
Available online 20 April 2023

For instance, during Singles' Day (the annual online shopping festival) in China in 2021, sales revenues of Tmall and Jingdong (famous online shopping websites) through advance selling sum up to 186.3 billion RMB. Since a large number of retailers on the websites offered attractive discounts or coupons, the demand became highly uncertain, and therefore many of them instituted advance selling to handle these challenges [9]. Most products available for advance selling were not newly released and had a spot selling record before this festival, hence this practice could be considered as a RAS problem. In addition, many retailers managed the spot price and the supply quantity deliberately before advance selling to highlight the advantage of the festival policies. This behavior also demonstrated the connection between the preceding spot selling and the followed advance selling during Single's Day. To this end, it is valuable to discuss further how to optimize the decisions by correlating the two periods. The products covered broad categories and many well-known companies, such as Uniqlo (clothing), Nike (shoes), L'OREAL (cosmetics), and SIEMENS(electrical). Another example is the innovation in e-commerce induced by the boom of live streaming and social network. Taking advantage of the great impact of influencers and the interactive nature of live streaming, live e-commerce become eye-catching recently in China [10]. Normally a host introduces products with discounts passionately and creates an atmosphere of impulse purchase. Consumers are free to join a live streaming chosen from thousands of them, thus it is difficult to know the exact demand of one channel. If the spot products are sold out during the event, an advance selling period may follow as needed. This action includes a series of intertemporal decisions on pricing and ordering problems to be better off. Beyond the above examples, the RAS mechanism has also become popular on a lot of occasions when both buyers and sellers need to handle uncertainties.

The RAS mechanism, which is a case of the newsvendor problem, is still a new research area to our best knowledge. Several innovative characteristics of our RAS mechanism distinguish it from other newsvendor models. In the first place, RAS has two ordering opportunities and the quantity in the first period affects the pricing and ordering decisions in the second period, by contrast, the TAD only involves one ordering decision that is not directly related to the following pricing. Therefore, RAS not only flips the process of advance selling, but also changes the decision pattern. Second, RAS is superior in mitigating information asymmetry with online reviews starting from the first period, while TAD only enables information sharing in the second period. RAS is more suitable for retailers targeting customers with higher information requirement and less patience. To this end, it is important to figure out how online reviews during the spot period mitigate product uncertainty for subsequent customers and encourage them to preorder. Third, RAS integrate the impacts of information asymmetry and competition by considering the comprehensive demand shifting from competitors caused by online review, waiting time, and discount. In this paper, we model the decisions of a retailer in the RAS mechanism and seek to answer the following questions. On what conditions can a retailer benefit from applying RAS under competition? How to optimize the ordering quantity of the first period if a retailer has a plan to advance sell in the second period? Given the ordering quantity in the first period, what are the optimal ordering policy as well as the discount in the advance selling period? How do external factors such as product reviews and market share impact the decisions of a RAS retailer?

This paper is organized as follows. Section 2 offers a brief review of the relevant literature. In section 3 we present the base model and two models of reverse advance selling. Section 4 analyzes the properties of discount and order quantity for advance selling so that we can compare the two models to the base model and characterize conditions to optimize expected profits from the models. In addition, we evaluate the impact of factors such as customers' online reviews and retailers' market share. We also provide numerical examples to illustrate our main conclusions. We present conclusions and avenues for future research in section 5.

2. Literature review

Our work is closely related to the literature in three streams: basic decisions of advance selling, advance selling with competition, information asymmetry and waiting time, and backorder problems.

Ordering and pricing policies have been attractive topics for research in advance selling [11,12]. An ordering policy is the fundamental decision of advance selling that coordinates upstream supply vendors and downstream demand to reduce risks [13]. Advance-booking discounts emerge as the optimal pricing policy in a number of economic environments where the aggregate level of demand is uncertain [14]. Wang et al. [15]discuss the pricing and ordering decisions when the firm faces price-dependent stochastic demand and has the option of purchasing option contracts. Tang and Yin [16] examine how a retailer should jointly determine the order quantity and the retail price of two substitutable products under fixed or variable pricing strategies. To handle the uncertainty such as the valuation and behavior heterogeneity from customers, demand forecast is adopted by firms to be better off [17]. Moe and Fader [18] correlate the data from advance selling and post-launch periods and form two customer groups of different behavior to forecast the demand in spot selling. In contrast to these studies, RAS involves two correlated ordering decisions throughout two periods while most previous literature only makes one ordering decision in the second period. Additionally, the ordering policy in the first period affects the subsequent price in the next period in our models but they are independent in Tang et al. [19].

A lot of papers explore the benefits of advance selling under a setting of a single firm [4,20,21]. Dana [22] demonstrates that price-taking firms may offer an advance purchase discount in a perfectively competitive market and can neither benefit nor are harmed by it Tang et al. [19] and Ma et al. [23] adopt an advance booking discount to attract customers from the competitors and find the discount increases with the competitiveness represented by market share. They believe that advance selling can improve a retailer's profit, but Cachon and Feldman [24] argue that this advantage may not hold because competition in the spot period is likely to force firms to lower advance period prices, which is not favorable to profits. Ma et al. [25] demonstrate that the market power of a manufacture and the risk averse consumers affects the benefit of advance selling. Xue et al. [26]analyze the competition and information asymmetry in selling strategies, find that the supplier prefers advance sale when facing competing retailers with private information. These papers discuss the decisions and benefits in various settings of competition combined with factors such as attitudes and valuation of customers and find different conclusions of advance selling. Hence, it is valuable to explore the comprehensive impact of competition with additional elements. This paper sets a competition structure similar to Tang et al. [19] and Ma et al. [23]. But we assume the demand shifting from the competitor is caused by information sharing, waiting time, and discount. By contrast, Tang et al. [19] and Ma et al. [23] believe it is mainly decided by the discount.

Prior literature finds that the benefits of advance selling can be affected by factors such as online reviews and customers' patience to wait. Su [20] demonstrates that the patience to wait affects the price sensitivity of customers and the seller's revenue consequently. Zhao & Zhang [27] believe that online reviews deliver valuable information for subsequent consumers and affect the pricing decisions of sellers. Information shortage in advance selling can damage their interests by inducing more variation in valuation. In other words, information can encourage customers to behave more actively in advance selling. Loginova et al. [28] find that experienced customers are more inclined to preorder because they already collected knowledge about the products. Wu et al. [29]explore whether firms should advertise the advance ordering opportunity when facing strategic consumers. The RAS mechanism allows customers to share experience to mitigate uncertainty for subsequent buyers in contrast with traditional advance selling.

The RAS mechanism conforms to the definition of Fay and Xie [30] as advance selling, but the process of launching the advance selling period after a regular spot sale period is easy to be confused with a backorder problem. However, they are different in the following aspects. First, most of the backorder problems focus on inventory management interacting with upstream suppliers [31,32], whereas RAS put more attention on satisfying the downstream demand. In particular, the advance selling period of RAS normally doesn't induce inventory problems and shortage costs. Second, the backorder mechanism seldom involves intertemporal pricing for customers in different periods [33,34]. Although some research in this line includes pricing offered to retailers, they normally target different stages of a supply chain in contrast with advance selling [35]. Even if Abad [36] sets a discount backorder for customers, he does not connect the price with demand, which is different from our research. Third, our research shows that a retailer should order fewer products than the standard spot newsvendor model if it has the plan of launching advance selling afterward. Therefore, the RAS mechanism is different from a backorder problem in various aspects.

Our research contributes to the literature of advance selling under competition and information asymmetry in the following aspects. Initially, it explores a novel phenomenon that includes a reverse process in contrast to traditional advance selling. This pattern involves two intertemporal ordering decisions that affect the pricing in the next period, while most prior studies focus on one ordering chance. Furthermore, we examine the importance of competition by setting a brand for other retailers with a market share. It is similar to Tang et al. [19] but we assume a retailer's demand is also affected by customer behaviors such as impatience to wait and attitude to product information from previous consumers. They result in changes in market share and thus the variation of competitive force. Finally, the implementation process of RAS enables consumers to share their experiences from the first period, which is not feasible in traditional advance selling. To this end, the information structure in RAS is more balanced than other related models and is worth investigating jointly with other factors in this paper.

3. The analysis framework

As stated in the above sections, waiting time plays an important role in the purchase decisions of customers. However, it depends on external factors in its supply chain, such as the efficiency of upstream vendors and the patience of downstream customers. Therefore, retailers in our model only decide when to begin the order process but the length of the advance selling period is exogenous. Since time substantially affects the decisions of sellers and buyers, we propose two models that begin the advance selling at different times but have similar procedures, and both of them are based on emerging industry practices. Each of them covers two periods where a retailer sells spot goods in the first period and then launches an advance selling period if spot products are sold out. In the early spot period, consumers buy the products and share experiences by providing comments or by rating them. Their decisions are shaped by factors such as waiting time and discount policies, possibly causing a shift from B to A consequently. The first model determines the ordering quantity of spot products before the commencement of the presale period while the second determines it afterward. Both these models belong to the RAS mechanism but have their respective advantages and limitations. The first model saves waiting time but increases inventory risks because the replenishment is launched before consumers preorder, while the second diminishes inventory risks but with increased waiting times. We analyze and compare the similarity as well as the distinction between them to find a better decision pattern for retailers in RAS.

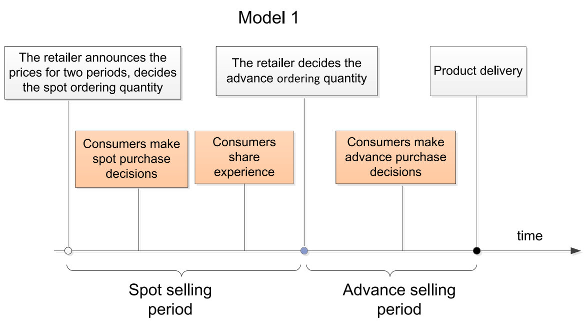

Fig. 1. The sequence of events in model 1.

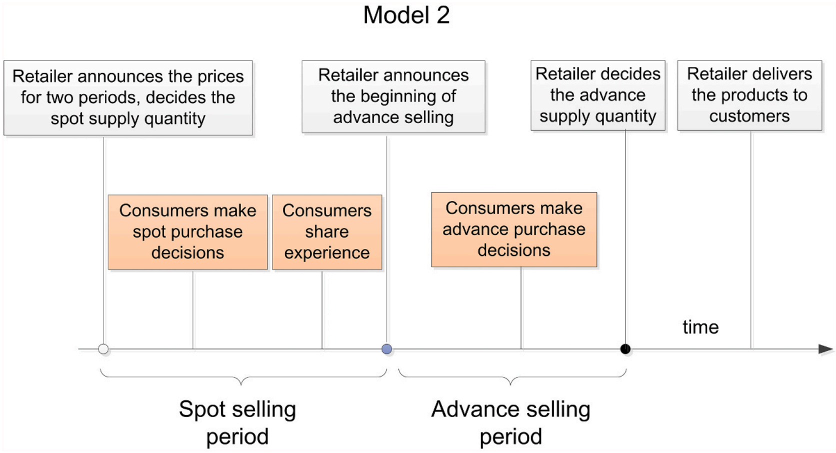

Fig. 2. The sequence of events in model 2.

The sequences of events of the two models are illustrated in Figs. 1 and 2 respectively. Note that we assume the retailer delivers spot products immediately after the purchase and it is not demonstrated in the figures. We analyze the two policies separately in the following sections. We defer the corresponding proofs to the addendum of the present paper.

Given the competition in the market, we assume that a retailer sells product A and that all other competing products are denoted as B. Let the unit cost, spot selling price, and salvage value of product A be c, p, and s respectively, where $s < c < p$. The RAS mechanism includes two periods, a spot period is first launched and then an advance selling period follows if the spot products are sold out. We assume that the joint distributions of demand for Brands A and B are denoted by D_A and D_B are bivariate normal distributions with means of μ_A and μ_B, standard deviations of σ_A and σ_B, as well as the correlation coefficient $\rho \in (-1, 1)$. To simplify the statement of our analysis, we assume that D_A and D_B have the same coefficient of variation θ, where $\theta = \frac{\sigma_A}{\mu_A} = \frac{\sigma_B}{\mu_B}$. This seems reasonable because the products are similar and present similar levels of demand uncertainty. Let μ be the expected total market demand where $\mu = \mu_A + \mu_B$. We summarize the notation used in this paper in Table 1.

Table 1
The notation.

Parameters	Explanations
c	Production cost per unit of brand A
p	Selling price of brand A in spot period
p'	Selling price of brand A in presale
s	Salvage value per unit of brand A
D_A	Demand for brand A, a random variable $\sim N(\mu_A, \sigma_A)$
D_B	Demand for brand B, a random variable $\sim N(\mu_B, \sigma_B)$
ρ	Correlation coefficient between D_A and D_B
μ	Total expected market demand, $\mu = \mu_A + \mu_B$
α	Market share of brand A, $\alpha = \mu_A/\mu$
θ	Coefficient of variation of both brand A and B, $\theta = \sigma_A/\mu_A = \sigma_B/\mu_B$
t	Lead time for the buyers
z	Customer evaluation of brand A
R_B	Fraction of brand B switching to brand An under RAS
Decision variables	**Explanations**
x	Discount coefficient in reverse advance selling, $x = p'/p$
Q	Ordering quantity of brand A in the base model
$\widehat{Q}_0$	Ordering quantity of brand A in the first period in model 1
$\widetilde{Q}_0$	Ordering quantity of brand A in the first period in model 2
$\widehat{Q}$	Ordering quantity in the second period in model 1
$\widetilde{Q}$	Ordering quantity in the second period in model 2
Other notation	**Explanations**
D_0	Demand for brand A in spot selling period in RAS
D_1	Demand for brand A in the first period in model 1
$D_1{}'$	Demand for brand A in the first period in model 2
π	Optimum expected profit in the base model
$\widehat{\pi}$	Expected profit of brand A in model 1
$\widetilde{\pi}$	Expected profit of brand A in model 2

3.1. The base model

Consider the case that the retailer provides regular spot selling. The retailer charges p for every unit with a cost of c and the residual value is s. Q is the ordering quantity of spot goods and we maximize the retailer's profit to determine the optimal quantity Q^*. Let π be the optimal expected profit, where $\pi = \max_{Q\geq 0} E_{D_A}[-cQ + p\min(Q,D_A) + s(Q-D_A)^+]$. It has been proven that the optimal order quantity Q^* and the optimal expected profit π are expressed as:

$$Q^* = \mu_A + k\sigma_A, \pi = (p-c)\mu_A - (p-s)\varphi(k)\sigma_A, \tag{1}$$

where $= \Phi^{-1}((p-c)/(p-s))$, Φ and φ are the distribution and density function of the standard normal distribution, respectively. It is easy to find the optimal expected profit $\pi \geq 0$ if and only if the coefficient of variation $\theta \leq \overline{\theta}$, where $\overline{\theta} = \frac{p-c}{(p-s)\varphi(k)}$.

3.2. The models of reverse advance selling

3.2.1. Model 1: create an ordering plan before advance selling

A long waiting time after the purchase can result in a loss of impatient customers; therefore, we determine the quantity, discount, and length of the presale season before the second period to reduce the waiting time. Demand for spot and presale products is denoted as D_0 and D_1, respectively. Let the residual value be s, let the discount be x, and let the waiting time for the buyers be t. Time t is exogenously given in our models and it equals T_1 in model 1 and T_2 in model 2. To simplify the setup, we don't consider the arrival process of customers, thus t is the same for all customers and it also equals the time length of the advance selling period. The retailer charges $p' = xp$ for every unit in presale, where x is the discount. z denotes the level of product reviews from consumers and we normalize its value within $[-1,1]$, when z is 0 the reviews are neutral or none. The retailer will not institute advance selling after the first period if z is negative. Therefore, z is used to decide whether to launch subsequent advance selling and to evaluate the attraction of this mechanism if z is positive. To simplify the model setup, we assume the reviews truly reflect the product quality and don't discuss the manipulation and distortion of reviews. The advance selling attracts part of the demand of B and the proportion switching from A to B is denoted as $R_B(x,z,t)$ because of discount or good product review. The value of $R_B(x,z,t)$ varies within $[0,1]$. It increases as the discount x or t drops, or as z rises. We assume that $R_B(x,z,t)$ is not negative because the retailer does not institute RAS when $R_B(x,z,t)$ is negative and thus unprofitable.

In the first period we have $D_0 = D_A$ because the spot selling without discount does not attract demand away from B. The spot ordering is denoted as $\widehat{Q}_0$. Consequently, the demand of the second period is equal to the demand for A minus $\widehat{Q}_0$ in addition to the demand switching from B to A, where $D_1 = D_A + R_B(x,z,T_1)D_B - \widehat{Q}_0$. It is easy to determine $D_0 + D_1 = D_A + R_B(x,z,T_1)D_B > D_A$, showing that the total demand for A in model 1 is greater than that of the base model. As both D_0 and D_1 are linear functions of D_A and D_B, which conform to the normal distribution, we can derive the means and standard deviations as follows:

$$\mu_0 = \mu_A \tag{2}$$

$$\mu_1 = \mu_A + R_B(x,z,T_1)\mu_B - \widehat{Q}_0 \tag{3}$$

$$\sigma_0 = \sigma_A \tag{4}$$

$$\sigma_1 = \left[\sigma_A^2 + R_B^2(x,z,T_1)\sigma_B^2 + 2\rho R_B(x,z,T_1)\sigma_A\sigma_B\right]^{1/2} \tag{5}$$

The seller has to determine the optimal ordering amount to maximize the profit $\widehat{\pi}$, if $D_0 < \widehat{Q}_0$, equals $\{-c\widehat{Q}_0 + p\min(\widehat{Q}_0, D_0) + s(\widehat{Q}_0 - D_0)^+\}$ If $D_0 \geq \widehat{Q}_0$, the profit $\widehat{\pi}$ equals $\left\{-c\widehat{Q}_0 + p\min(\widehat{Q}_0, D_0) + \max_{\widehat{Q}\geq 0} E_{D_1}[-c\widehat{Q} + p'\min(\widehat{Q}, D_1) + s(\widehat{Q} - D_1)^+]\right\}$, where $\{-c\widehat{Q} + p'\min(\widehat{Q}, D_1) + s(\widehat{Q} - D_1)^+\}$ represents the profit in the second period. Since the ordering amount is determined before the second period, there may be unsold products with a salvage value of s. Assume that $\widehat{q}_0 = (\widehat{Q}_0 - \mu_A)/\sigma_A$ fits the standard normal distribution. If the probability of $D_0 \geq \widehat{Q}_0$ is $1 - \Phi(\widehat{q}_0)$, the expected profit is as follows: $\widehat{\pi} = \max \int_0^{\widehat{Q}_0} [-c\widehat{Q}_0 + p\min(\widehat{Q}_0, D_0) + s(\widehat{Q}_0 - D_0)^+] f(D_0)d(D_0) + \int_{\widehat{Q}_0}^{+\infty} \left\{-c\widehat{Q}_0 + p\min(\widehat{Q}_0, D_0) + \max_{\widehat{Q}\geq 0} E_{D_1}[-c\widehat{Q} + p'\min(\widehat{Q}, D_1) + s(\widehat{Q} - D_1)^+]\right\} f(D_0)d(D_0)$, where $D_0 = D_A$, $D_1 = D_A + R_B(x,z,T_1)D_B - \widehat{Q}_0$. In terms of equations (2) and (4), we then have the optimal

$$\widehat{Q}^* = \mu_1 + k'\sigma_1 \ , \ \widehat{\pi} = [(p-c) - (p-s)\Phi(\widehat{q}_0)]\widehat{Q}_0 + (p-s)(\Phi(\widehat{q}_0)\mu_A - \varphi(\widehat{q}_0)\sigma_A) + (1 - \Phi(\widehat{q}_0))[(p'-c)\mu_1 - (p'-s)\varphi(k')\sigma_1], \tag{6}$$

where $k' = \Phi^{-1}((p'-c)/(p'-s))$. In equation (6), there are three terms included in $\widehat{\pi}$, where the sum of the first and the second term is the profit from the spot selling, and the third term represents the profit from the advance selling.

3.2.2. Model 2: create an ordering plan after advance selling

In the second model, the retailer creates an ordering plan after the second period. Here, for simplicity, we use $R_B(t)$ to represent $R_B(x,z,t)$, since we don't discuss the change of x and z in this part. The value of x in the two models is equal in this analysis, as is z. The retailer here decides the ordering quantity later than he does in the first model, therefore, it is reasonable to assume the waiting time T_2 is longer than that of the first model for the consumers ($T_2 > T_1$). The proportion of customers switching from B to A is less and is represented as $R_B(T_2)$, where $R_B(T_2) < R_B(T_1)$. Let $\widetilde{Q}_0$ denote the ordering amount in the first period. In the second model, we have $D_0 = D_A$ and $D_1{'} = D_A + R_B(T_2)D_B - \widetilde{Q}_0$. It is easy to show that $\widetilde{Q}_0 + D_1{'} = D_A + R_B(T_2)D_B > D_A$, meaning that the total demand for A is greater than that in the base model. We also derive the following equations:

$$\mu_1{'} = \mu_A + R_B(x,z,T_2)\mu_B - \widetilde{Q}_0 \tag{7}$$

$$\sigma_1{'} = \left[\sigma_A^2 + R_B^2(x,z,T_2)\sigma_B^2 + 2\rho R_B(x,z,T_2)\sigma_A\sigma_B\right]^{1/2}. \tag{8}$$

which model is desirable to the retailer depends on the conditions involved, and we analyze them in the following sections.

The seller has to determine the optimal order quantity to maximize the profit $\widetilde{\pi}$. The profit generated when $D_0 < \widetilde{Q}_0$ is $\{-c\widetilde{Q}_0 + p\min(\widetilde{Q}_0, D_0) + s(\widetilde{Q}_0 - D_0)^+\}$. Since the retailer decides after presale, he will ordering the exact demand amount of $\widetilde{Q} = (p' - c)D_1'$ in this period, thus the profit generated when $D_0 \geq \widetilde{Q}_0$ is $\{-c\widetilde{Q}_0 + p\min(\widetilde{Q}_0, D_0) + E_{D_1'}(p' - c)D_1'\}$. Let $\widetilde{q}_0 = (\widetilde{Q}_0 - \mu_A)/\sigma_A$ to fit the standard normal distribution. Then the probability of $D_0 \geq \widetilde{Q}_0$ is $1 - \Phi(\widetilde{q}_0)$ and the expected profit is as follows:

$$\widetilde{\pi} = \max \int_0^{\widetilde{Q}_0} \left[-c\widetilde{Q}_0 + p\min(\widetilde{Q}_0, D_0) + s(\widetilde{Q}_0 - D_0)^+\right] f(D_0)d(D_0) + \int_{\widetilde{Q}_0}^{+\infty} \left[-c\widetilde{Q}_0 + p\min(\widetilde{Q}_0, D_0) + E_{D_1'}(p' - c)D_1'\right] f(D_0)d(D_0)\ .$$

with analogous calculations in the first model, we can prove that

$$\widetilde{Q}^* = \mu_1{'}, \widetilde{\pi} = \left[(p-c) - (p-s)\Phi\left(\widetilde{q}_0\right)\right]\widetilde{Q}_0 + (p-s)\left(\Phi\left(\widetilde{q}_0\right)\mu_A - \varphi\left(\widetilde{q}_0\right)\sigma_A\right) + \left(1 - \Phi\left(\widetilde{q}_0\right)\right)(p' - c)\mu_1{'}. \tag{9}$$

In equation (9), three terms are included in $\widetilde{\pi}$, where the sum of the first and the second term is the profit generated from the spot selling and the third term is the profit from the advance selling.

We now compare the profits of two models: $\widetilde{\pi}$ given in equation (9) and $\widehat{\pi}$ given in equation (6). For any given $\widetilde{Q}_0 = \widehat{Q}_0$, we have $\widetilde{\pi} - \widehat{\pi} = (1 - \Phi(\widetilde{q}_0))[-(p' - c)(R_B(T_1) - R_B(T_2))\mu_B + (p' - s)\varphi(k')\sigma_1]$. The retailer prefers the second model if $\widetilde{\pi} - \widehat{\pi} > 0$, then we derive the corresponding condition with simple calculations. If $R_B(T_1) - R_B(T_2) \leq \frac{(p'-s)\varphi(k')\sigma_1}{(p'-c)\mu_B}$ we have $\widetilde{\pi} - \widehat{\pi} > 0$. Otherwise, the first is preferred if $R_B(T_1) - R_B(T_2) > \frac{(p'-s)\varphi(k')\sigma_1}{(p'-c)\mu_B}$.

Lemma 1. *For any given $\widetilde{Q}_0 = \widehat{Q}_0$, if $R_B(T_1) - R_B(T_2) \leq \frac{(p'-s)\varphi(k')\sigma_1}{(p'-c)\mu_B}$, then $\widetilde{\pi} \geq \widehat{\pi}$; if $R_B(T_1) - R_B(T_2) > \frac{(p'-s)\varphi(k')\sigma_1}{(p'-c)\mu_B}$, then $\widetilde{\pi} < \widehat{\pi}$.*

Lemma 1 *implies the condition to determine which model is better suited to the seller. It depends on the difference between $R_B(T_1)$ and $R_B(T_2)$, representing the attraction gap of the two models for B. When $R_B(T_1) - R_B(T_2) \leq \frac{(p'-s)\varphi(k')\sigma_1}{(p'-c)\mu_B}$, it is preferable to use the second model, otherwise the first one is better for the retailer.*

Lemma 2. *The optimal expected profit of the second model ($\widetilde{\pi}(\widetilde{x})$) is greater than the optimal expected profit of the base model (π); the optimal expected profit of the first model($\widehat{\pi}(\widehat{x})$) is greater than the optimal expected profit of the base model (π) if $\theta < \widehat{\theta}$ and $R_B(1,0,T_1) > 0$, or if $R_B(1,0,T_1) = 0$.*

Proof. *See the Appendix.*

Lemma 2 *suggests that both RAS models are more profitable than the base model under the above conditions. Since θ represents the variation of demand where $\theta < \overline{\theta}(\widehat{\theta})$ ensures a positive profit for the seller, therefore, the first RAS model performs better in a wider range of θ variation than in the base model because $\widehat{\theta} > \overline{\theta}$. The second model always performs better than the base model because $\widetilde{\pi}(\widetilde{x}) > \pi$. Note that our models assume that $R_B(x,z,t) \geq 0$. If $R_B(x,z,t) < 0$ for reasons such as negative product reviews, the retailer won't institute the RAS mechanism and the conclusions no longer hold. Therefore, the retailer has to set an appropriate discount and waiting time, and evaluate the products to make sure $R_B(x,z,t) \geq 0$ if he decides to adopt RAS.*

Illustrative Example. To better illustrate Lemmas 1 and 2, we present a numerical example with the following parameters: $p = 100$, $c = 40$, $s = 15$ so that $p > c > s$. Additionally, $\alpha = 0.3$, $\mu = 100$, $\theta = 0.3$, $\rho = 0.5$, and $\widetilde{Q}_0 = \widehat{Q}_0 = 15$. Here we assume that $R_B(x,z,t) = y(z,t)(1 - ax^3)$, $y(z,t) = g(t) * z^{1/3}$, and set a=0.9. We set $g(T_1) = 0.6$, $g(T_2) = 0.4$, and z=1. In this example, we don't discuss the impact of variations of z and t. The curves shown in Figure 3 represent the profit gaps between model 1(model 2) and the base model. It is easy to observe that the curves are above the x-axis, showing that the expected profits of our models are greater than those of the base model, conforming to the conclusions of Lemma 2. When x=0.9 and we already know $p' = xp$, we find that $R_B(T_1) - R_B(T_2) = 0.0687$ and $\frac{(p'-s)\varphi(k')\sigma_1}{(p'-c)\mu_B} = 0.0919$ through simple calculations. We compare the values of these two terms and obtain $R_B(T_1) - R_B(T_2) <$

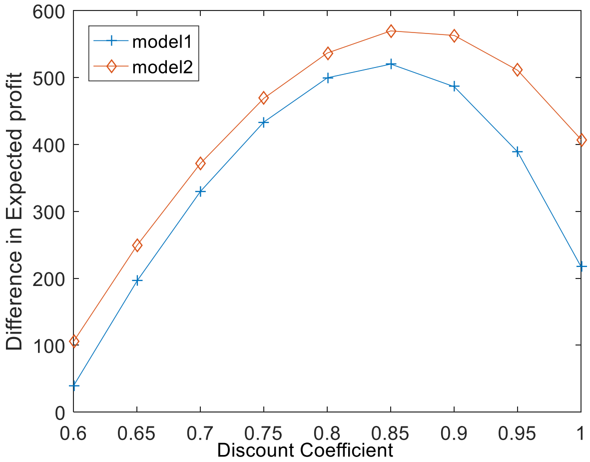

Fig. 3. The difference in expected profits between the two models and the baseline

$\frac{(p'-s)\varphi(k')\sigma_1}{(p'-c)\mu_B}$, indicating $\widetilde{\pi} > \widehat{\pi}$ according to Lemma 1. In Figure 3 we observe that the expected profit of model 2 is greater than that of model 1 ($\widetilde{\pi} > \widehat{\pi}$) when x=0.9. The observations in Figure 3 conform to Lemmas 1 and 2.

Lemma 3. *The expected profits of the two models ($\widehat{\pi}$ and $\widetilde{\pi}$) increase with z.*

Proof. *See the Appendix.*

Since z denotes the product evaluation which affects the attraction of advance selling, Lemma 3 *suggests that if product reviews are better, the profit of the retailer is higher, and this conforms to common sense. Assuming the product reviews truly represent consumer opinions and product quality, products with higher quality should adopt RAS to increase profits.*

After the spot selling, information from customers about product A is released, and the retailer updates the decisions according to z. In our models, the major impact of z is applied on the attraction coefficient $R_B(x,z,T_1)$, thus the demand of A is changed as a result. Therefore we first assume z is neutral with the value of 0 at the beginning and is updated after we collect the feedback information from customers. *To evaluate the impact of updating z, we set a contrast baseline where z is never updated and have the following conclusion through calculations. Let $\widehat{\pi}^{\#}$ denote the expected profit without updating z.*

Proposition 1. *Through updating z, the expected profit of model 1 increases by $\widehat{\pi} - \widehat{\pi}^{\#}$ if $\widehat{\pi}^{\#} > \pi$; otherwise the expected profit of model 1 increases by $\widehat{\pi} - \pi$; the expected profit of model 2 does not change.*

Proof. *See the Appendix.*

Proposition 1 *suggests that the retailer can benefit from updating z and reconsidering the management decisions in model* 1. *Since the decision depends on market information, updating information is proved to be valuable for retailers. Consequently, retailers improve their profit and reduce risks from an uncertain environment. This conclusion doesn't hold in model 2 because the demand information is collected before the retailer decides the ordering quantity and it is not necessary to update z to make decisions.*

Illustrative Example. *Here we use the same example in the above section. The lines in* Fig. 4 *show that the expected profits in the two models increase with the value of z that conforms to* Lemma 3. *It is shown that positive product review increases the profits of the retailers.*

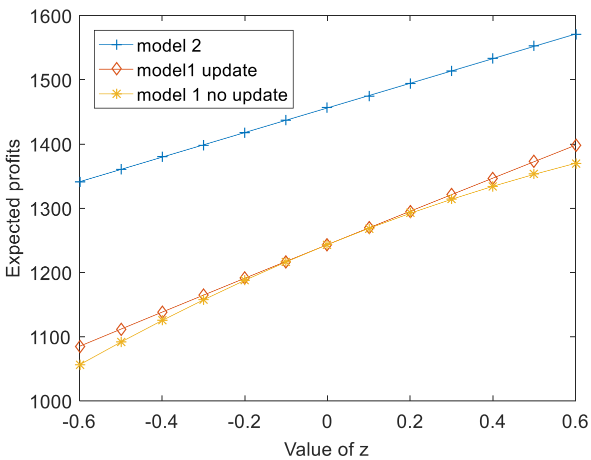

Fig. 4. The expected profits of the two models change with z

Through observing the data without updating z, we find that information update increases the profit of the retailer in model 1 and it doesn't change the profit in model 2. These observations conform to the conclusions in Proposition 1.

4. Decisions of reverse advance selling

4.1. The optimal ordering quantity of spot period

4.1.1. The first model

The amount of spot products is denoted as $\widehat{Q}_0$ and it turns to be a pure advance selling when $\widehat{Q}_0 = 0$. Let $q_0 = (\widehat{Q}_0 - \mu_A)/\sigma_A$.

Proposition 2. *The optimal ordering quantity of spot period* $\widehat{Q}_0^{\,*}$ *is less than* Q^* *in the base model and the condition to optimize* $\widehat{Q}_0$ *is* $p - c - (p-s)\Phi(\widehat{q}_0) - \varphi(\widehat{q}_0)[(p' - c)\mu_1 - (p' - s)\varphi(k')\sigma_1]/\sigma_A - (p' - c)(1 - \Phi(\widehat{q}_0)) = 0$ **.**

Proof. *See the Appendix.*

Corollary 1. *If* $p' > p + s - c$*, the optimal ordering quantity of spot period* $\widehat{Q}_0^{\,*}$ *is decreasing in z and increasing in* α.

Proof. *See the Appendix.*

4.1.2. The second model

The amount of spot products is denoted as $\widetilde{Q}_0$ and it turns to be pure advance selling when $\widetilde{Q}_0 = 0$; let $\widetilde{q}_0 = (\widetilde{Q}_0 - \mu_A)/\sigma_A$.

Proposition 3. *The optimal ordering quantity of spot period* $\widetilde{Q}_0^{\,*}$ *is less than* Q^* *in the base model and the condition to optimize* $\widetilde{Q}_0$ *is* $p - c - (p-s)\Phi\left(\widetilde{q}_0\right) - \varphi\left(\widetilde{q}_0\right)(p' - c)\mu_1'/\mu_A - (p' - c)\left(1 - \Phi\left(\widetilde{q}_0\right)\right) = 0$ **.**

Proof. *See the Appendix.*

Corollary 2. *The optimal ordering quantity of the spot period* $\widetilde{Q}_0^{\,*}$ *is decreasing in z and increasing in* α.

Proof. *See the Appendix.*

Propositions 2 and 3 suggest that the retailer should order fewer spot products in the first period than the base model does. This conclusion demonstrates that the spot selling period in the RAS mechanism is different from the base model of pure spot sales, because the advance selling and discount policy attract part of the demand from spot sales. The demand shifts from the spot period to the presale period, thus the retailer should order less in the spot period. Consequently, the inventory risk for the retailer is reduced without impairing his benefit.

Corollary 1 has the following implications. When the discount is reasonable $x = \frac{p'}{p} > \frac{p+s-c}{p}$*, the retailer should order fewer spot products if the product quality is good, otherwise it should order more. The reason may be that the review information in the first period is sparse and its impact on customers is mild. In contrast, the reviews are abundant in the second period and they can negatively affect the customer if the product quality is not good. The retailers should order more spot products if the market share is larger and vice versa. Since the* $x > \frac{p+s-c}{p}$ *covers a wide range of common discounts, Corollary 1 can be applied in many circumstances. Even if the discount x is not determined, as long as the retailer does not use a very low discount in the future, it can follow the implication. Corollary 2 has similar implications as Corollary 1 but without limitation in the discount setting, because the second model induces lower inventory risk than the first model and its ordering quantity is more robust to price variation.*

Illustrative Example. *First, We use the same example to illustrate Propositions 2 and 3. Based on the given numerical parameters, it can be*

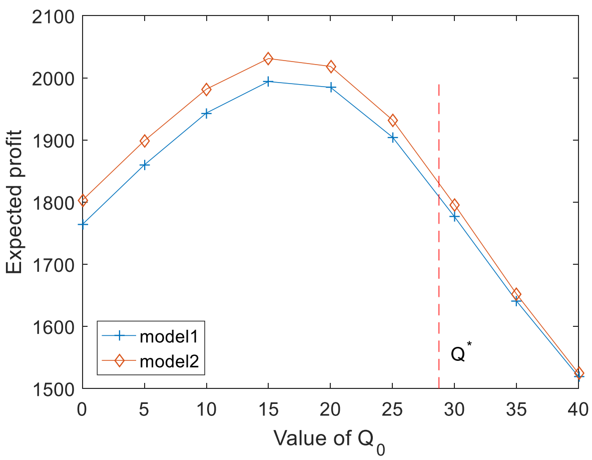

Fig. 5. Optimal quantity of spot products

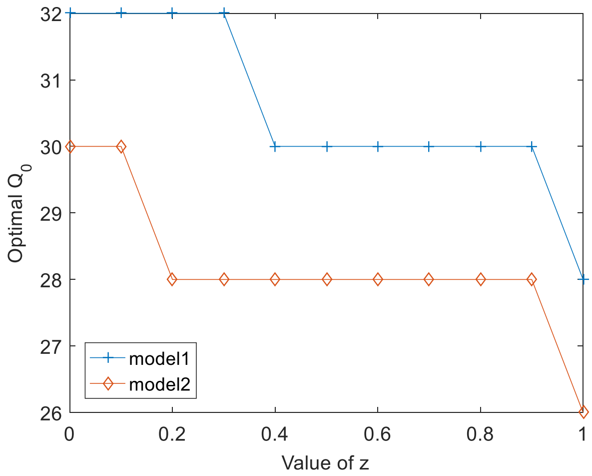

Fig. 6. Optimal spot quantities $\widehat{Q}_0^*$ ($Q^{\sim *}_0$) vs. online review (z).

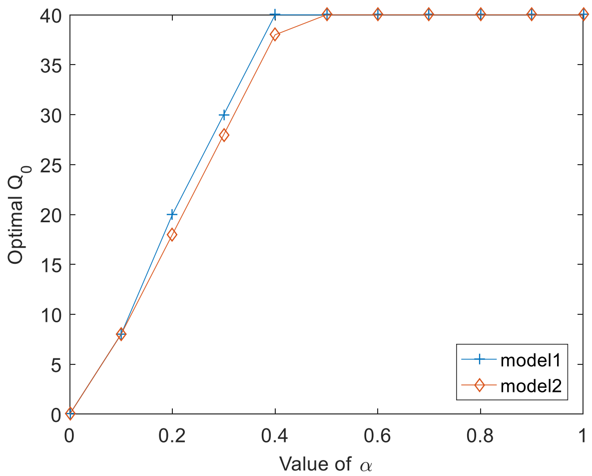

Fig. 7. Optimal spot quantities $\widehat{Q}_0^*$ ($\widetilde{\widetilde{Q}_0}^*$) vs. market share ($\alpha$).

shown that the integer near Q^* *(dash line) is* 29 *and the integer near* $\widehat{Q}_0^*$ *and* $\widetilde{Q}_0^*$ *(top points of the two curves) is* 15 *in* Fig. 5. *It is easy to observe that* $Q^* > \widehat{Q}_0^*$ *and* $Q^* \geq \widetilde{Q}_0^*$, *conforming to the conclusion in Propositions* 2 *and* 3 *respectively.*

To examine the impact of z, *we vary its value between* 0 *and* 1, *and find the corresponding optimal spot quantities* $\widehat{Q}_0^*$ ($\widetilde{Q}_0^*$). *We set* $p' = 80$, p=100, c=40, *and* s=15, *thus we have* $p' > p - c + s$. *It is shown in* Fig. 6 *that* $\widehat{Q}_0^*$ ($\widetilde{Q}_0^*$) *is non-decreasing in* z. *The reason they are not strictly increasing is that we set the quantities as integers and the decimals are ignored. Thus the results in* Fig. 6 *conform to Corollaries* 1 *and* 2.

Similarly, we vary the value of α *between* 0 *and* 1, *and find the corresponding optimal spot quantities* $\widehat{Q}_0^*$ ($\widetilde{Q}_0^*$). Fig. 8 *shows that* $\widehat{Q}_0^*$ ($\widetilde{Q}_0^*$) *increases with* α *at first and reaches the ceiling. Thus the results in* Fig. 7 *conform to Corollaries* 1 *and* 2.

4.2. The optimal discounts

We analyze the optimal discount x of model 2 and examine its impact on the profit of the retailer in this section. We focus on model 2 since we cannot derive an analytical result with management insights from model 1. To simplify the models, we use the function $R_B = y(z,t)(1 - ax^f)$ where $0 \leq y(z,t) \leq 1$, $0 < a < 1$. f represents the response of demand to the discount and a larger f denotes a stronger response. Customers are sensitive to advance selling when f >1, otherwise $f \leq 1$. We assume that z and t are constants when x is considered as a variable, and we let y be the abbreviation of $y(z,t)$. When $x = 1$, there is no discount. $R_B = y(1 - a)$ represents the proportion of demand switching from B to A, where a denotes customers' sensitivity to discounts. R_B drops with x in the range from $y(1 - a)$ to y.

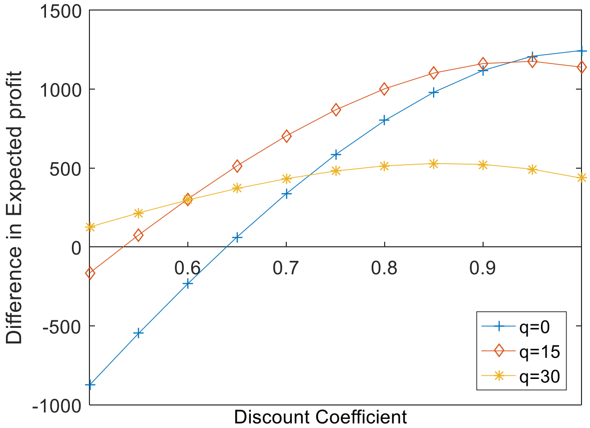

Fig. 8. Optimal discount coefficient

Let $\widetilde{\Delta}(x) = \widetilde{\pi}(x) - \pi$, where $\widetilde{\pi} = \left[(p-c) - (p-s)\Phi\left(\widetilde{q}_0\right)\right]\widetilde{Q}_0 + (p-s)\left(\Phi\left(\widetilde{q}_0\right)\mu_A - \varphi\left(\widetilde{q}_0\right)\sigma_A\right) + (1-\Phi(\widetilde{q}_0))(xp-c)(\mu_A + R_B(x)\mu_B - \widetilde{Q}_0)$ and $\pi = (p-c)\mu_A - (p-s)\varphi(k)\sigma_A$.

Proposition 4. *The condition to adopt the discount policy is denoted as follows: the retailer should use a discount* $\widetilde{x} \in (0,1)$ *when* $\widetilde{Q}_0 > (\mu_A + y\mu_B) - ay\mu_B[p + f(p-c)]/p$*; otherwise, the retailer should not use the discount policy when* $\widetilde{Q}_0 \le (\mu_A + y\mu_B) - ay\mu_B[p + f(p-c)]/p$*.*

Proof. *See the Appendix.*

Proposition 4 *implies that the discount policy depends on the value of spot ordering* $\widetilde{Q}_0$*. When* $\widetilde{Q}_0$ *is greater than the threshold* $(\mu_A + y\mu_B) - ay\mu_B[p + f(p-c)]/p$*, the retailer should use a discount, otherwise it shouldn't. This conclusion is not intuitive to understand and we explain the possible reasons as follows. If the retailer supplies fewer spot products, the profit in the first period will be lower and it has to attract more demand by offering a discount to increase the overall profit of the two periods. On the other hand, if more spot products are sold out and a higher profit is obtained in the first period, it is not necessary for the retailer to offer a discount since the demand that has been attracted cannot offset the loss from a lower discount.*

Illustrative Example. *To better illustrate* Proposition 4*, we apply an example with the following parameters.* $p = 100$; $c=40$; $s=15$; $\alpha = 0.3$, $\mu = 100$*, and* $\theta = 0.3$, $\rho = 0.5$*. As* $R_B = y(z,t)(1 - ax^f)$*, let* $a=0.6$, $f=3$*, and* $y(z,t)=0.6$ *when* $\widetilde{Q}_0 = 0, 15, 30$*, respectively. According to* Proposition 4*, we have* $(\mu_A + y\mu_B) - ay\mu_B[p + f(p-c)]/p = 1.44$, $\widetilde{x} = 1$ *when* $\widetilde{Q}_0 = 0$ *and we have* $\widetilde{x} \in (0,1)$ *when* $\widetilde{Q}_0 = 15, 30$*. The results given in* Fig. 8 *conform to* Proposition 4*.*

Prior to presenting the character of $\widetilde{x}$*, we define a term* $\overline{\alpha}$ *to simplify the following exposition. Let* $\overline{\alpha} = \frac{\widetilde{Q}_0 - y(1+ac/p-2a)\mu}{(1-y(1+ac/p-2a))\mu}$.

Proposition 5. $\widetilde{x}$ *is non-decreasing in* α*. If* $f = 1$*, then*

$$\widetilde{x} = \begin{cases} 1, \text{if } \alpha < \overline{\alpha} \\ \dfrac{p(\mu_A + y\mu_B - \widetilde{Q}_0) + ay\mu_B c}{2ay\mu_B p}, \text{if } \alpha \ge \overline{\alpha} \end{cases}.$$

Proof. *See the Appendix.*

Proposition 5 *suggests that the market share* α *has an important impact on the optimal discount policy in model 2. The retailer should offer a higher value of* $\widetilde{x}$ *when his products have a larger market share and a lower value of* $\widetilde{x}$ *when his market share is smaller. The reason for this conclusion may be that the retailer attracts more demand if his market share is greater and it's not necessary to enhance the profit with a lower discount, otherwise he should offer a discount.*

Illustrative Example. *To better illustrate* Proposition 5*, we apply the same example in* Proposition 3 *except* $f=1$ *and* $\widetilde{Q}_0 = 15$ *and we vary* α *from 0 to 0.7. We find that* $\overline{\alpha} = 0.3275$ *through simple calculations. It is shown in* Fig. 9 *that* $\widetilde{x} < 1$ *when* $\alpha < \overline{\alpha}$ *and* $\widetilde{x} = 1$ *if* $\alpha \ge \overline{\alpha}$*, conforming to the conclusion in* Proposition 5*.*

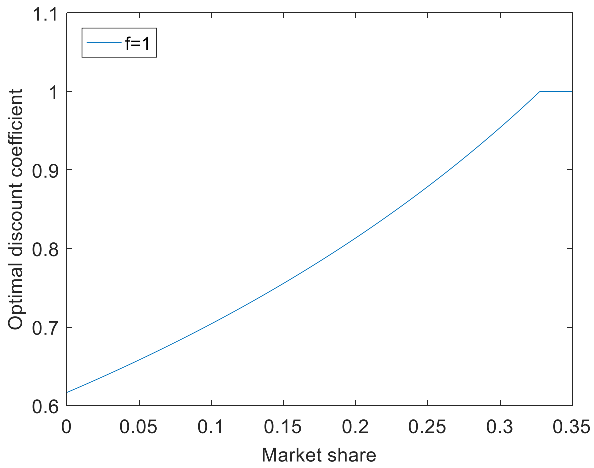

Fig. 9. Optimal discount coefficients vary with the market share

5. Concluding remarks

The major contribution of this paper is to extend the analysis framework of advance selling beyond traditional patterns. Our models enable retailers to quantify the potential benefits and necessary conditions when adopting reverse advance selling. We demonstrate that the RAS mechanisms are more competent than the base model under competition and uncertainty. The retailer successfully increases its profit by updating the review information to obtain a more accurate demand. The conclusions can help guide retailers to make accurate ordering plans that better cater to market demand under uncertainty.

We analyze the optimal ordering quantities of spot and advance selling periods to maximize the expected profit and reduce inventory risks for a retailer. The results suggest that the retailer should order fewer products in the first period than in the base model. This is reasonable because fewer spot products involve lower inventory risk and the retailer has a second chance to sell more products. Additionally, we find that the ordering quantity in the spot period is correlated to the discount policy in the second period. If the retailer has to order more spot products (above the threshold amount), it should offer a discount in the following presale period; otherwise it shouldn't offer a discount. We demonstrates that the market share has a substantial impact on discount policy as well as optimal spot product quantity. A larger market share matches a higher price with a lower discount in the advance selling period. The analysis results also show that a larger market share encourages higher spot ordering quantity in the first period and vice versa. Moreover, the retailer obtains higher revenue when the product reviews are better and vice versa. This result has an implication that if the product quality is low and buyers share reviews online, it may not be suitable to institute the RAS. However, the retailer should order fewer spot products if the product quality is good, otherwise it should order more of them. Although in practice it is difficult to accurately measure some parameters such as correlations and coefficient variations of different products, the optimal discount varies only slightly when these parameters fluctuate over a broad range.

Our conclusions highlight avenues for retailers under competition and information asymmetry to analyze the suitable conditions and possible benefits of adopting advance selling. For instance, if a retailer prepares for a collective promotion event and only depends on personal experience, it may obtain a suboptimal profit or even suffer a loss due to overstock or shortage because the demand within limited time is difficult to forecast in such occasions. Our models can help it to benefit from using prior selling records and customer reviews to mitigate demand risks and setting more reasonable ordering and pricing policies to improve profits. There are two RAS models that retailers can apply, differing in the time to decide the second ordering quantity. The first model sets the order quantity for advance selling immediately after the spot selling and induces less lead time. It is a better fit for retailers with clients who are impatient but loyal to them and applications under intense competition. For example, advance selling in live e-commerce usually adopts this model to encourage customers to commit orders by ensuring shorter deliver time to compete against thousands of vendors. In contrast, the second model is more suitable for retailers with a larger client base who are not very sensitive to waiting time.

Future research may further consider the following issues. First, we don't consider the capacity constraints here, which limit the ordering plan to a certain degree and need to be explored. Second, valuation heterogeneity affects the purchase capacities of consumers and it is valuable to discuss its impacts on the profits and decisions of sellers. Finally, it is important to explore mechanisms involving more than two periods, as such mechanisms are adopted by numerous e-commerce retailers and have been proven effective.

In conclusion, this paper contributes to the literature of newsvendor problems in advance selling. We believe the RAS mechanism examined in this paper serves as a useful tool from which sellers can make appropriate decisions on discounts and ordering plans to improve their profits, reduce inventory risks, and avoid irrational production.

Author contribution statement

Yi Zou: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Wrote the paper.

Data availability statement

Data will be made available on request.

Additional information

No additional information is available for this paper.

Funding statement

This work was supported by the Humanities and Social Sciences Youth Foundation of the Ministry of Education of China (Grant No. 18YJC630278).

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix. Proofs

Proof of Lemma 2. The optimal profit of the first model $\widehat{\pi}$ is equal to the profit $\widehat{\pi}(1)$ or greater than it, where $\widehat{Q}_0 = 0$ and $x = 1$, thus we have $\widehat{\pi}(\widehat{x}) - \pi \geq \widehat{\pi}(1) - \pi$. We prove that $\widehat{\pi}(\widehat{x}) \geq \widehat{\pi}(1) > \pi$ with the following calculations. In terms of equation (1), we compare $\widehat{\pi}(1)$ with π and find that

$\widehat{\pi}(1) - \pi = (p - c)R_B(1,0,T_1)\mu_B + (p - s)\varphi(k)\sigma_A(1 - \sqrt{1 + R_B(1,0,T_1)^2 r^2 + 2\rho r R_B(1,0,T_1)})$ where $\mu_B = r\mu_A$. If $R_B(1,0,T_1) > 0$, we have $\widehat{\pi}(1) - \pi = R_B(1,0,T_1)r\mu_A\left[(p-c) - (p-s)\varphi(k)\theta\left(\frac{\sqrt{1+R_B{}^2(1,0,T_1)r^2+2\rho r R_B(1,0,T_1)}-1}{R_B(1,0,T_1)r}\right)\right]$; if $R_B(1,0,T_1) = 0$, we have $\widehat{\pi}(1) = \pi$. If $R_B(1,0,T_1) > 0$ and $(p-c) - (p-s)\varphi(k)\theta\left(\frac{\sqrt{1+R_B{}^2(1,0,T_1)r^2+2\rho r R_B(1,0,T_1)}-1}{R_B(1,0,T_1)r}\right) \geq 0$, then $\widehat{\pi}(1) - \pi \geq 0$ holds. Since $\sqrt{1 + R_B{}^2(1,0,T_1)r^2 + 2\rho r R_B(1,0,T_1)} \leq R_B(1,0,T_1)r + 1$, it is easy to obtain $\frac{\sqrt{1+R_B{}^2(1,0,T_1)r^2+2\rho r R_B(1,0,T_1)}-1}{R_B(1,0,T_1)r}\Big) \leq 1$. Let $\overline{\theta} = \frac{p-c}{(p-s)\varphi(k)}$ and $\widehat{\theta} = \frac{\overline{\theta}R_B(1,0,T_1)r}{(\sqrt{1+R_B{}^2(1,0,T_1)r^2+2\rho r R_B(1,0,T_1)}-1)}$. It is clear that $\frac{R_B(1,0,T_1)r}{\sqrt{1+R_B{}^2(T_1)r^2+2\rho r R_B(T_1)-1}} > 1$, therefore we have $\widehat{\theta} > \overline{\theta}$. If $\theta < \widehat{\theta}$, we have $\widehat{\pi}(1) - \pi > 0$ and $\widehat{\pi}(\widehat{x}) \geq \widehat{\pi}(1) > \pi$. Similarly, we prove that $\widetilde{\pi}(1) - \pi = (p-c)R_B(1,0,T_2)\mu_B + (p-s)\varphi(k)\sigma_A > 0$, meaning $\widetilde{\pi}(\widetilde{x}) \geq \widetilde{\pi}(1) > \pi$.

Proof of Lemma 3. Assuming $\widehat{q}_0 = (\widehat{Q}_0 - \mu_A)/\sigma_A$ to fit the standard normal distribution, we differentiate the $\widehat{\pi}$ with respect to z and obtain $\partial\widehat{\pi}/\partial z = (1 - \Phi(q_0))\mu_B[(p' - c) - (p' - s)\varphi(k')\rho L]\partial R_B(x,z,T_1)/\partial z$, where $L = \frac{R_B\sigma_B+\rho\sigma_A}{\sqrt{\sigma_A^2+R_B^2\sigma_B^2+2\rho R_B\sigma_A\sigma_B}}$. Since $\rho \leq 1$, it is easy to find that $L = \frac{\sqrt{\rho^2\sigma_A^2+R_B^2\sigma_B^2+2\rho R_B\sigma_A\sigma_B}}{\sqrt{\sigma_A^2+R_B^2\sigma_B^2+2\rho R_B\sigma_A\sigma_B}} \leq 1$. Because $(p' - c) - (p' - s)\varphi(k')\rho L \geq (p' - c) - (p' - s)\varphi(k') \geq 0$, We have $\partial\widehat{\pi}/\partial z \geq 0$ with simple calculations. Since $\partial R_B(z,T_1)/\partial z > 0$, it is shown that $\partial\widehat{\pi}/\partial z \geq 0$. Similarly, we differentiate the $\widetilde{\pi}$ with respect to z and obtain $\partial\widetilde{\pi}/\partial z = (1 - \Phi(q_0))(p' - c)\mu_B * \partial R_B(x,z,T_2)/\partial z$. Since $\partial R_B(x,z,T_2)/\partial z > 0$, it is shown that $\partial\widetilde{\pi}/\partial z \geq 0$.

Proof of Proposition 1. If we don't update z, we calculate the expected profit of the retailer in model 1 as follows. The retailer assumes the review is neutral ($z = 0$) since he obtains no information from the reviews, therefore, he makes decisions based on it. The ordering for the second period is $\widehat{Q}^{\#} = \overline{\mu}_1 + k'\overline{\sigma}_1$, where the mean and standard deviation are $\overline{\mu}_1$ and $\overline{\sigma}_1$ respectively. In terms of equations (3) and (5), we have $\overline{\mu}_1 = \mu_A + R_B(x,0,T_1)\mu_B - \widehat{Q}_0$ and $\overline{\sigma}_1 = [\sigma_A^2 + R_B^2(x,0,T_1)\sigma_B^2 + 2\rho R_B(x,0,T_1)\sigma_A\sigma_B]^{1/2}$.

Let $\widehat{q}^{\#} = (\widehat{Q}^{\#} - \mu_1)/\sigma_1$ to fit the standard normal distribution. Then the expected profit without updating z is $\widehat{\pi}^{\#} = (1 - \Phi(\widehat{q}_0))\{(p' - s)[\Phi(\widehat{q}^{\#})\overline{\mu}_1 - \varphi(\widehat{q}^{\#})\overline{\sigma}_1)] + [(p' - c) - (p' - s)\Phi(\widehat{q}^{\#})]\widehat{Q}^{\#}\}$. Whether the retailer updates the reviews or not, he faces the same actual demand. $\widehat{\pi}$ is the optimal profit when z is known and $\widehat{\pi}^{\#}$ is suboptimal since the retailer assumes a wrong demand without update, thus we derive $\widehat{\pi} - \widehat{\pi}^{\#} > 0$. If $\widehat{\pi}^{\#} > \pi$, the retailer will launch a subsequent advance selling period and the increased profit with update is $\widehat{\pi} - \widehat{\pi}^{\#}$; otherwise, the retailer will not adopt advance selling and the increased profit with update is $\widehat{\pi} - \pi$.

Since the retailer in model 2 decides the ordering quantity after the second period, meaning the demand is already known and up to date by that time. Therefore, the retailer does not have to update z to decide the order.

Proof of Proposition 2. To find the optimal quantity, we differentiate the expected profit $\widehat{\pi}$ with respect to $\widehat{Q}_0$ and get the condition for optimization as $\partial\widehat{\pi}/\partial\widehat{Q}_0 = 0$ where $\partial\widehat{\pi}/\partial\widehat{Q}_0 = [p - c - (p - s)\Phi(\widehat{q}_0)] - \varphi(\widehat{q}_0)[(p' - c)\mu_1 - (p' - s)\varphi(k')\sigma_1]/\sigma_A - (p' - c)(1 - \Phi(\widehat{q}_0))$. (A1)

The first part on the right side $p - c - (p - s)\varphi(\widehat{q}_0)$ in A1 represents the differential result of the profit in the spot period and the rest is for the advance selling period. The profit for the spot period is maximized when $\widehat{q}_0 = \Phi^{-1}((p - c)/(p - s))$ and it declines if $\widehat{q}_0 > \Phi^{-1}((p - c)/(p - s))$. It is obvious that the second part on the right side $-\varphi(\widehat{q}_0)[(p' - c)\mu_1 - (p' - s)\varphi(k')\sigma_1]/\mu_A - (p' - c)(1 -$

$\Phi(\widehat{q}_0))$ is negative. We find that when $\widehat{q}_0 \geq \Phi^{-1}((p-c)/(p-s))$ we have $\partial\widehat{\pi}/\partial\widehat{Q}_0 < 0$, meaning $\widehat{\pi}$ declines with $\widehat{Q}_0$. Therefore, given $\widehat{q}_0 = (\widehat{Q}_0 - \mu_A)/\sigma_A$, the optimal ordering quantity for the first period is $\widehat{Q}_0^* < \mu_A + \Phi^{-1}((p-c)/(p-s))\sigma_A$, where the right side $\mu_A + \Phi^{-1}((p-c)/(p-s))\sigma_A$ is the optimal quantity for the base model Q^*. Therefore, we have $\widehat{Q}_0^* < Q^*$, where $\widehat{Q}_0^*$ is derived from the following equation: $[p-c-(p-s)\Phi(\widehat{q}_0)] - \varphi(\widehat{q}_0)[(p'-c)\mu_1 - (p'-s)\varphi(k')\sigma_1]/\sigma_A - (p'-c)(1-\Phi(\widehat{q}_0)) = 0$.

Proof of Corollary 1. By the Implicit Function Theorem, we differentiate the function $\partial\widehat{\pi}/\partial\widehat{Q}_0$ with respect to z. By substituting A1 and rearranging the result, we can find that $\partial\widehat{Q}_0/\partial z = \frac{-\varphi(\widehat{q}_0)\mu_B[(p'-c)-(p'-s)\varphi(k')\theta L]R_B'}{(p-p'+c-s)\varphi(\widehat{q}_0)\sigma_A^2 + \varphi'(\widehat{q}_0)[(p'-c)\mu_1 - (p'-s)\varphi(k')\sigma_1]\sigma_A}$. (A2)

We have proved $L < 1$ in Lemma 3, thus $-\varphi(\widehat{q}_0)\mu_B[(p'-c)-(p'-s)\varphi(k')\theta L]R_B' < 0$. Since $\partial\widehat{\pi}/\partial\widehat{Q}_0 = 0$, we have $p - p' - (p - p' + c - s)\Phi(\widehat{q}_0) = \varphi(\widehat{q}_0)[(p'-c)\mu_1 - (p'-s)\varphi(k')\sigma_1]/\sigma_A > 0$. After simple calculations we have $\Phi(\widehat{q}_0)\langle\frac{p-p'}{p-p'+c-s}$. If $p' > p + s - c$ we have $\Phi(\widehat{q}_0)\langle\frac{p-p'}{p-p'+c-s} < 1/2$, thus $\varphi'(\widehat{q}_0) > 0$. Therefore, the denominator in (A1) is positive and we obtain $\partial\widehat{Q}_0/\partial z < 0$, indicating $\widehat{Q}_0^*$ is decreasing in z.

Similarly, we can differentiate the function $\partial\widehat{\pi}/\partial\widehat{Q}_0$ with respect to α. Given $\mu_A = \alpha\mu$ and $\mu_B = (1-\alpha)\mu$, by considering the results in the proof of Proposition 2, we can find that $\partial\widehat{Q}_0/\partial\alpha = \frac{\varphi(\widehat{q}_0)\mu(p'-c)R_B}{(p-p'+c-s)\varphi(\widehat{q}_0)\sigma_A^2 + \varphi'(\widehat{q}_0)[(p'-c)\mu_1 - (p'-s)\varphi(k')\sigma_1]\sigma_A}$. (A3)

Since the numerator and denominator in (A3) are positive, we get $\partial\widehat{Q}_0/\partial\alpha > 0$, indicating $\widehat{Q}_0^*$ is increasing in α.

Proof of Proposition 3. To find the optimal quantity we differentiate the expected profit $\widetilde{\pi}$ with respect to $\widetilde{Q}_0$, and get the condition for optimization as $\partial\widetilde{\pi}/\partial Q_0 = 0$, where $\partial\widetilde{\pi}/\partial\widetilde{Q}_0 = p - c - (p-s)\Phi\left(\widetilde{q}_0\right) - \varphi\left(\widetilde{q}_0\right)(p'-c)\mu_1'/\mu_A - (p'-c)\left(1-\Phi\left(\widetilde{q}_0\right)\right)$. (A4)

The first part on the right side $p - c - (p-s)\Phi\left(\widetilde{q}_0\right)$ in A4 represents the differential result of the profit of the spot period and the rest is for the advance selling period. The profit for the spot period is maximized when $\widetilde{q}_0 = \Phi^{-1}((p-s)/(p-c))$ and it declines if $\widetilde{q}_0 > \Phi^{-1}((p-s)/(p-c))$. It is obvious that the second part on the right side $-\varphi\left(\widetilde{q}_0\right)(p'-c)\mu_1'/\mu_A - (p'-c)\left(1-\Phi\left(\widetilde{q}_0\right)\right)\langle 0$, meaning that the profit declines with $\widetilde{Q}_0$. We find that when $\widetilde{q}_0 = \Phi^{-1}((p-s)/(p-c))$, we have $\partial\widetilde{\pi}/\partial\widetilde{Q}_0 < 0$; and $\widetilde{\pi}$ declines with $\widetilde{Q}_0$ when $\widetilde{q}_0 > \Phi^{-1}((p-s)/(p-c))$. Therefore, the optimal ordering quantity for the first period $\widetilde{Q}_0$ is less than the optimal quantity for the base model $\mu_A + \Phi^{-1}((p-c)/(p-s))\sigma_A$.

Proof of Corollary 2. By the Implicit Function Theorem, we can differentiate the function $\partial\widetilde{\pi}/\partial\widetilde{Q}_0$ with respect to z. By substituting A4 and rearranging the result, we can find that $\partial\widetilde{Q}_0/\partial z = \frac{-\varphi(\widetilde{q}_0)\mu_B(p'-c)R_B'}{(p-p'+c-s)\varphi(\widetilde{q}_0)\sigma_A^2 + \varphi'(\widetilde{q}_0)[(p'-c)\mu_1 - (p'-s)\varphi(k')\sigma_1]\sigma_A}$. (A5)

It is easy to show that $\partial\widetilde{Q}_0/\partial z < 0$, indicating $\widetilde{Q}_0^*$ *is decreasing in z*. Similarly, we can differentiate the function $\partial\widetilde{\pi}/\partial\widetilde{Q}_0$ with respect to α. By substituting A4 and rearranging the result, we can find that $\partial\widetilde{Q}_0/\partial\alpha = \frac{\varphi(\widetilde{q}_0)\mu(p'-c)R_B}{(p-p'+c-s)\varphi(\widetilde{q}_0)\sigma_A^2 + \varphi'(\widetilde{q}_0)[(p'-c)\mu_1 - (p'-s)\varphi(k')\sigma_1]\sigma_A}$.

It is easy to show that $\partial\widetilde{Q}_0/\partial\alpha > 0$, indicating $\widetilde{Q}_0^*$ *is increasing in* α.

Proof of Proposition 4. In terms of equations (7) and (8), we calculate the $\widetilde{\Delta}(x)$ by substituting $R_B(x)$ with $y(1-ax^f)$ and differentiate $\widetilde{\Delta}(x)$ with respect to x and obtain the following equation: $\frac{\partial\widetilde{\Delta}(x)}{\partial x} = (1-\Phi(\widetilde{q}_0))\{p(\mu_A + y\mu_B - \widetilde{Q}_0) - ay\mu_B[p(1+f)x - cf]x^{f-1}\}$. Because $xp - c > 0$, we have $p(1+f)x - cf > 0$. It is easy to observe that $\frac{\partial^2\widetilde{\Delta}(x)}{\partial x^2} = -(1-\Phi(\widetilde{q}_0))ayf\mu_B[p(1+f)x - c(f-1)]x^{f-2}$, where $p(1+f)x - c(f-1) > 0$. When the presale time is positive ($y \neq 0$) and $xp - c > 0$, we have $\frac{\partial^2\widetilde{\Delta}(x)}{\partial x^2} < 0$, indicating that $\widetilde{\Delta}(x)$ is concave with respect to x. From simple calculations, it is shown that when $\widetilde{\Delta}'(1) = (1-\Phi(\widetilde{q}_0))\{p(\mu_A + y\mu_B - Q_0) - ay\mu_B[p + f(p-c)]\} \geq 0$, we have $\widetilde{x} = 1$; and when $\widetilde{\Delta}'(1) < 0$, we have $\widetilde{x} \in (0,1)$. The necessary condition for a discount is $\widetilde{Q}_0 > (\mu_A + y\mu_B) - ay\mu_B[p + f(p-c)]/p$, and the retailer should not discount when $\widetilde{Q}_0 \leq (\mu_A + y\mu_B) - ay\mu_B[p + f(p-c)]/p$.

Proof of Proposition 5. By the Implicit Function Theorem, we can differentiate the function $\frac{\partial\widetilde{\Delta}(x)}{\partial x}$. Similar to the proof of Proposition 4, we can find that

$\frac{\partial\widetilde{x}}{\partial\alpha} = \frac{(1-y)\mu + ay\mu[p(1+f)\widetilde{x} - cf]\widetilde{x}^{f-1}}{ayf\mu_B[p(1+f)\widetilde{x} - c(f-1)]\widetilde{x}^{f-2}}$. Since $p(1+f)\widetilde{x} - c(f-1) > 0$ and $p(1+f)\widetilde{x} - cf > 0$, it can be shown that $\frac{\partial\widetilde{x}}{\partial\alpha} > 0$, which means $\widetilde{x}$ is increasing in α. If $f = 1$, $\widetilde{x}$ satisfies the condition that $\frac{\partial\widetilde{x}}{\partial\alpha} = 0$. Through simple calculations we find that when $\alpha < \overline{\alpha}$, we have $\widetilde{x} = \frac{p(\mu_A + y\mu_B - \widetilde{Q}_0) + ay\mu_B c}{2ay\mu_B p}$ and when $\alpha \geq \overline{\alpha}$, we have $\widetilde{x} = 1$.

References

[1] R. Swinney, Selling to strategic consumers when product value is uncertain: the value of matching supply and demand, Manag. Sci. 57 (10) (2011) 1737–1751.

[2] W. Jin, J. Luo, Q. Zhang, Optimal ordering and financing decisions under advance selling and delayed payment for a capital-constrained supply chain, J. Oper. Res. Soc. 69 (12) (2018) 1978–1993.
[3] R. Mohammadivojdan, Y. Merzifonluoglu, J. Geunes, Procurement portfolio planning for a newsvendor with supplier delivery uncertainty, Eur. J. Oper. Res. 297 (3) (2022) 917–929.
[4] L. Xiao, M. Xu, Z. Chen, et al., Optimal pricing for advance selling with uncertain product quality and consumer fitness, J. Oper. Res. Soc. 70 (9) (2019) 1457–1474.
[5] T.-H. Ho, S. Savin, C. Terwiesch, Managing demand and sales dynamics in new product diffusion under supply constraint, Manag. Sci. 48 (2) (2002) 187–206.
[6] Y. He, S. Ray, S. Yin, Group selling, product durability, and consumer behavior, Prod. Oper. Manag. 25 (11) (2016) 1942–1957.
[7] T. Mavlanova, R. Benbunan-Fich, M. Koufaris, Signaling theory and information asymmetry in online commerce, Inf. Manag. 49 (5) (2012) 240–247.
[8] G. Antonides, P.C. Verhoef, M. van Aalst, Consumer perception and evaluation of waiting time: a field experiment, J. Consum. Psychol. 12 (3) (2002) 193–202.
[9] Q. Zhang, X. Cheng, Y.C. Tsao, et al., Advance selling under deposit expansion and consumer's valuation change, Oper. Res. 22 (4) (2021) 3633–3661.
[10] D. YeCA, F. Liu, D. Cho, et al., Investigating switching intention of e-commerce live streaming users, Heliyon 8 (10) (2022), e11145.
[11] B. He, X. Gan, K. Yuan, Entry of online presale of fresh produce: a competitive analysis, Eur. J. Oper. Res. 272 (1) (2019) 339–351.
[12] C. Wang, J. Chen, X. Chen, The impact of customer returns and bidirectional option contract on refund price and order decisions, Eur. J. Oper. Res. (1) (2019) 267–279.
[13] A. Sengupta, S.N. Wiggins, Airline pricing, price dispersion, and ticket characteristics on and off the internet, Am. Econ. J. Econ. Pol. 6 (1) (2014) 272–307.
[14] X. Zhao, K.E. Stecke, Pre-orders for new to-be-released products considering consumer loss aversion, Prod. Oper. Manag. (2) (2010) 198–215.
[15] C. Wang, J. Chen, X. Chen, Pricing and order decisions with option contracts in the presence of customer returns, Int. J. Prod. Econ. (2017) 422–436.
[16] C.S. Tang, R. Yin, Joint ordering and pricing strategies for managing substitutable products, Prod. Oper. Manag. (1) (2007) 138–153.
[17] S. Ma, G. Li, S.P. Sethi, et al., Advance booking discount strategies: competition, information transparency and spot market, Transport. Res. E Logist. Transport. Rev. (2022), 102589.
[18] W. Moe, P. Fader, Using advance purchase orders to forecast new product sales, Market. Sci. 22 (1) (2003) 146, 146.
[19] C.S. Tang, K. Rajaram, A. Alptekinoğlu, et al., The benefits of advance booking discount programs: model and analysis, Manag. Sci. 50 (4) (2004) 465–478.
[20] X. Su, Intertemporal pricing with strategic customer behavior, Manag. Sci. 53 (5) (2007) 726–741.
[21] Y. Papanastasiou, N. Savva, Dynamic pricing in the presence of social learning and strategic consumers, Manag. Sci. 63 (4) (2017) 919–939.
[22] J. Dana, D. James, Advance-purchase discounts and price discrimination in competitive markets, J. Polit. Econ. 106 (2) (1998) 395–422.
[23] S. Ma, J. Lin, W. Xing, et al., Advance booking discount in the presence of spot market, Int. J. Prod. Res. 53 (10) (2015) 2921–2936.
[24] G.P. Cachon, P. Feldman, Is advance selling desirable with competition? Market. Sci. 36 (2) (2017) 214–231.
[25] S. Ma, G. Li, S. Sethi, et al., Advance selling in the presence of market power and risk-averse consumers, Decis. Sci. J. (1) (2019) 142–169.
[26] W. Xue, J. Zuo, X. Xu, Analysis of market competition and information asymmetry on selling strategies, Ann. Oper. Res. 257 (1) (2017) 395–421.
[27] C. Zhao, Y. Zhang, Dynamic quality and pricing decisions in customer-intensive service systems with online reviews, Int. J. Prod. Res. 57 (18) (2019) 5725–5748.
[28] O. Loginova, Pricing strategies in advance selling: should a retailer offer a pre-order price guarantee? Rev. Ind. Organ. 49 (3) (2016) 465–489.
[29] M. Wu, S.X. Zhu, R.H. Teunter, Advance selling and advertising: a newsvendor framework, Decis. Sci. J. (No.1) (2021) 182–215.
[30] S. Fay, J. Xie, The economics of buyer uncertainty: advance selling vs. probabilistic selling, Market. Sci. 29 (6) (2010) 1040–1057.
[31] L. Bao, Z. Liu, Y. Yu, et al., On the decomposition property for a dynamic inventory rationing problem with multiple demand classes and backorder, Eur. J. Oper. Res. 265 (1) (2018) 99–106.
[32] X. Xu, H. Wang, C. Dang, et al., The loss-averse newsvendor model with backordering, Int. J. Prod. Econ. 188 (2017) 1–10.
[33] V. Pando, L.A. San-José, J. García-Laguna, et al., A newsboy problem with an emergency order under a general backorder rate function, Omega 41 (6) (2013) 1020–1028.
[34] Y.-W. Zhou, S.-D. Wang, Manufacturer-buyer coordination for newsvendor-type-products with two ordering opportunities and partial backorders, Eur. J. Oper. Res. 198 (3) (2009) 958–974.
[35] Y.J. Lin, Minimax distribution free procedure with backorder price discount, Int. J. Prod. Econ. 111 (1) (2008) 118–128.
[36] P.L. Abad, Optimal price and order size under partial backordering incorporating shortage, backorder and lost sale costs, Int. J. Prod. Econ. 114 (1) (2008) 179–186.