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Mathematical Modelling of Lockdown Policy for COVID-19

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Abstract

In this paper, we extend the classic SIR model to find an optimal lockdown policy to balance between the economy and people's health during the outbreak of COVID-19. In our model, we intend to solve a two phases optimisation problem: policymakers control the lockdown rate to maximise the overall welfare of the society; people in different health statuses take different decisions on their working hours and consumption to maximise their utility. We develop a novel method to estimate parameters for the model through various additional sources of data. We use the Cournot equilibrium to model people's behaviour. The analysis of simulation results provides scientific suggestions for policymakers to make critical decisions on when to start the lockdown and how strong it should be during the whole period of the outbreak.

Keywords: COVID-19; Equilibrium; SIR; Lockdown; Optimal Control

1. Introduction

As one of the most devastating pandemics in human history, the current outbreak of COVID-19 has already caused millions of deaths around the globe. Numerous prevention measures have been studied by Flaxman et al. (2020) and Tian et al. (2020) in order to control the spread of the virus by the governments. Essentially the most effective prevention of COVID-19 is the lockdown measure which completely ceases the movement of the human being and thus slows down the spread of disease. However, the lockdown measure is incredibly controversial as it imposes a tremendous impact on our society and economy. Hence it might be the most difficult decision to be made by the governments. Especially when and how to impose the lockdown measure is one of the most challenging questions for both politicians and scientists. To address this question, there is a need to develop a mathematical model combining both epidemiology and economics.

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Epidemiological models have been widely studied to analyse the dynamics of the pandemic, e.g. Kucharski et al. (2020), Liu et al. (2020),Wang et al. (2020), Leung (2020). However, there is less discussion on how the lockdown policies can influence the economic decisions of people and the spread of disease and how can policymakers make optimal policy in the epidemic. Ferguson et al. (2020) and Ferguson et al. (2005) analyse the government intervention using epidemiological models with exogenous parameters and evaluate the effect of the intervention by simulation results. Some recent papers focus on analysis of optimal policy and policy effect in the framework of the SIR model or its variants. They studied the effect of different measures including fiscal policy (Trabandt (2020), Faria-e Castro (2020), Guerrieri et al. (2020)), testing and quarantine (Shi (2020), Mongey (2020), Jenny et al. (2020), Aleta et al. (2020)), intervention policy on multi-aged groups (Brotherhood et al. (2020), Acemoglu et al. (2020), Gollier (2020)), social distancing (Venkateswaran (2020), Shimer (2020), Gourinchas (2020)) and lockdown control (Lippi (2020), Niepelt (2020), Acemoglu et al. (2020)). In previous works, Lippi (2020) studied the optimal lockdown policy that minimises the value of fatalities and the output costs of the lockdown policy by locking down part of the susceptible and Infectious population, Acemoglu et al. (2020) researched the optimal lockdown policies on people of different age groups, and Niepelt (2020) maximise the economic activity level with the burden of the health-care system.

We extend the classic SIR model proposed by McKendrick (1927) and Mak (2014) and incorporate an equilibrium framework to study the optimal lockdown policy during the pandemic period. What we innovate from previous works is that they all only took the governments' perspective but did not take people's own reaction to the pandemic and the government policy into consideration, while we adopt the extension to the SIR model from Trabandt (2020) by involving people's economic decision making (consumption and working hours) and embed the SIR model in a simple Cournot equilibrium framework to model people's reaction to each other. In contract to Trabandt (2020) that studied the optimal containment policy by controlling the tax rate, we control the level of lockdown, which is more direct and effective for the governments, especially in the early stage of the pandemic. Using this method, we can enable the lockdown policy to identify a balance between the impact of the epidemics on the economy and people's health.

This paper continues as follows, in section 2, we describe the SIR-Lockdown Model and analyse its properties. In section 3, we discuss the parameter estimation of the model. We present the numerical results of the SIR-Lockdown model in section 4. Section 5 makes conclusions.

2. Model

2.1. Extension of SIR

As shown in the classic SIR model (McKendrick (1927) Hethcote (1989)), we classify people into three categories according to Mak (2014):

- Infectious (I) are those who are tested positive to the virus;
- Recovered (R) are those who have been tested positive to the virus and now recovered;
- Susceptible (S) are those who have not been tested positive to the virus.

We assume that all susceptible people are subjects to be infected with some possibility in direct contact with infectious people, and infectious people will recover with a constant probability of π_r or become dead with another constant probability π_d. Our extension is on the infection. All infection happens via direct contact between susceptible people and infected ones into three types of activities: purchasing and/or consumption of goods and services, working with other people, and other daily activities. A Lockdown policy can be applied to control the working contact, hence change the income flow, which indirectly imposes constraints on the purchasing and consumption.

We use the following equation (15) to describe our extended SIR model for the transition among Susceptible, Infected, Recover, and the Death outcome.

$$T_t = \pi_{s1}\left(S_t C_t^s\right)\left(I_t C_t^i\right) + \pi_{s2}\left(S_t N_t^s\right)\left(I_t N_t^i\right) + \pi_{s3} S_t I_t, \tag{1}$$

$$S_{t+1} = S_t - T_t, \tag{2}$$

$$I_{t+1} = I_t + T_t - (\pi_r + \pi_d)I_t, \tag{3}$$

$$R_{t+1} = R_t + \pi_r I_t, \tag{4}$$

$$D_{t+1} = D_t + \pi_d I_t. \tag{5}$$

In this system of equations, S_t, I_t, R_t and D_t represents the number of people in categories of Susceptible, Infectious, Recovery and Death respectively at time t. We use (C_t^s, N_t^s) to model the (average) consumption behaviour and working hours of susceptible people, (C_t^i, N_t^i) to model the (average) consumption behaviour and working hours of infectious people, and (C_t^r, N_t^r) to model the (average) consumption behaviour and working hours of a recovered person. T_t in equation (1)is the number of newly infectious people in the time period t to $t + 1$ and the three terms in the right-hand side of this equation are used to describe the infection by the three different contact between susceptible people and infectious people via consumption, working, and other types of contact.

We use several constant parameters to describe the transition rate between different categories. π_{s1} reflects the transition rate for a susceptible people get infected by infectious people from direct contact via purchasing/consuming. Similarly, π_{s2} reflects the transition rate from direct contact via working, and π_{s3} reflects the transition rate from other contacts.

Denote $\Delta Y_t = Y_{t+1} - Y_t$ for $Y = S, I, R$, then the dynamics of the SIR model is

$$\Delta S_t = -T_t,$$
$$\Delta I_t = T_t - (\pi_r + \pi_d)I_t$$
$$\Delta R_t = \pi_d I_t.$$

We use vectors and matrices to simplify our presentation. Denote $X_t = (S_t, I_t, R_t)^\top$, $C_t = (C_t^s, C_t^i, C_t^r)^\top$, $n_t = (n_t^s, n_t^i, n_t^r)^\top$, and for any $x = (x_1, x_2, x_3)^\top$, $c = (c_1, c_2, c_3)^\top$, $n = (n_1, n_2, n_3)^\top$, define

$$T(x, c, n) = x_1 x_2 \left(\pi_{s1} c_1 c_2 + \pi_{s2} n_1 n_2 + \pi_{s3} \right) \tag{6}$$

$$F(x, c, n) = (-T(x, c, n), T(x, c, n) - (\pi_r + \pi_d)x_2, \pi_d x_2)^\top \tag{7}$$

then the system can be described as

$$\Delta X_t = F(X_t, C_t, n_t). \tag{8}$$

2.2. Behaviour of individuals in different categories

We study the rational behaviour of all people who maximise their own welfare by choosing proper consumption and working hours like in a normal time, i.e., the virus does not change people's rationality and preference. Also, we use the following utility function to model the utility from consumption and working of an individual,

$$u(c, n) = \ln c - \frac{\theta}{2} n^2 \tag{9}$$

where c is the consumption, and n is the working hours. The first term measures the utility from consumption, and the second term measures the utility from working. Denote by A the average wage per hour of a person, hence the labor income of an individual, with working hour n is $A * n$, which will be the upper bound of the consumption, i.e. $An \geq c$.

Denote by n_0 the full working hours in a unit time before the spread of the virus, which is officially guided by the government. It is natural that n_0 is set optimally for the society, and the optimality brings some information of the parameter θ_0. If a person follows the full working hours n_0 optimally, then her labor income will be An_0. Since the utility function is strictly increasing in the consumption, all labor income should be consumed up, hence the optimal consumption c_0 should be Then by the optimality of n_0, we have $\frac{\partial u(c_0, n_0)}{\partial n} = \frac{1}{n_0} - \theta n_0 = 0$, by which we will choose θ by $\theta = 1/n_0^2$.

The total utility of a flow of consumption and working hours $\{(c_\tau, n_\tau)\}_{\tau=t,\cdots,T}$ is defined by

$$U(c, n.) = \sum_{t=\tau}^{T} \beta^\tau u(c_\tau, n_\tau) \tag{10}$$

To contain the spreading of the virus, governments need to apply a lockdown policy to reduce direct contacts between people, which will impose stricter constraints on their behaviour. In this paper, we study the lockdown policy

by a constraint on the ratio $L \in [0, 1]$ of the working hour in the full working capacity, i.e., given the full working hours n_0, the maximal working hour cannot exceed $n_0 * L$. We suppose the government cannot easily identify individuals into their categories so that the lockdown constraint on the working hours is the same for all people. We formulate the decision making problem for each category with a given lockdown policy $L_{\cdot}$, and then study the lockdown policy-making problem for the government.

2.2.1. Optimal decision of recovered people

Suppose the lockdown measure $L_t \in [0, 1]$ is given for any time t.

A recovered people aims at maximising the total utility

$$J^r(c^r_{\cdot}, n^r_{\cdot}; t) = \sum_{\tau=t}^{T} \beta^{\tau-t} u(c^r_\tau, n^r_\tau) \tag{11}$$

with the constraint $c^r_\tau \leq A n^r_\tau$ and $n^r_\tau \leq n_0 L_\tau$.

Notice that the behaviour of recovered people $(c^r_{\cdot}, n^r_{\cdot})$ plays no role in the spread of the virus, hence the behaviour of recovered people will not affect people in other categories. This is why we start to form this easy-to-handle category.

2.2.2. Optimal behaviour of infectious people

Similar to the case of recovered people, infectious people also need to choose their optimal consumption and working hours $\{(c^s_t, n^s_t)\}_{t=0,1,\cdots,T}$ to maximise their total utility from consumption and working hour, subject to the constraint that the consumption c^s_t cannot exceed the labour income for the working hour n_t, and n_t must be no more than the lockdown policy $n_0 * L_t$.

The labor income of an infectious people is different from other categories. Because they are infected, their health condition is usually worse than other people. So we introduce a constant ϕ to discount their working efficiency, and the labor income from n_t working hour will be $A * \phi * n$. Furthermore, since an infectious people will have a constant probability π_r to recover and suffer a possibility π_d of death, we need to calculate the distribution over all categories at a future time. For an infectious people at time t, he has the probability π_r to recover in the next unit time, π_d to die, and the rest probability $1 - \pi_r - \pi_d$ to stay in the infected category. By this evolution, we can get the conditional probabilities for his health state at a future time $\tau > t$. Denote by $p^{i,i}(t, \tau)$ the probability for him being still infected, $p^{i,r}(t, \tau)$ the probability being recovered, and $p^{i,d}(t, \tau)$ the probability of being dead. Then we can deduce that

$$p^{i,i}(t, \tau) = (1 - \pi_r - \pi_d)^{\tau-t}, \tag{12}$$

$$p^{i,r}(t, \tau) = \pi_r \frac{(1 - (1 - \pi_r - \pi_d)^{\tau-t})}{\pi_r + \pi_d}, \tag{13}$$

$$p^{i,d}(t, \tau) = \pi_d \frac{(1 - (1 - \pi_r - \pi_d)^{\tau-t})}{\pi_r + \pi_d}. \tag{14}$$

If he recovered, he should behave optimally as a recovered people, while if death has happened unfortunately, we cease the accumulation of any utility. So, for a given flow $(c^i_{\cdot}, n^i_{\cdot})$ of consumption and working hours taken by the infectious people from time t, the accumulated utility he can get will be

$$J^i(c^i_{\cdot}, n^i_{\cdot}; t) = \sum_{\tau=t}^{T} \beta^{\tau-t} \left[p^{i,i}(t, \tau) u(c^i_\tau, n^i_\tau) - p^{i,r}(t, \tau) u(c^{r*}_\tau, n^{r*}_\tau) \right]. \tag{15}$$

where (c^{r*}, n^{r*}) is the optimal behaviour of a recovered people determined in the previous case, and $p^{i,i}, p^{i,r}$ are as defined in equation (12, 13).

2.2.3. Behaviour of susceptible people

The decision planning for a susceptible people from time t is much more complicated if we consider the possibilities for this people to turn into infectious, recovered, and death at different future time spots. We avoid the complexity by taking advantage of the optimal value function for an infected, and model the objective function of a susceptible people recursively.

As for the previous two categories, we start from time t and pick up a susceptible person. Denote the state of the SIR model at the starting time as X_t, and the lockdown policy is fully given as $L_.$).

Suppose he will follow a given flow of consumption and working hours $(c_\tau^s, n_\tau^s)_{\tau=t,t+1,\cdots,T}$ before being infected, and then follow the optimal behaviour after been infected, i.e., his consumption and working hours after infected will switch to the optimal control for an infected person from the infection time. We denote his objective value as

$$J^s(c_.^s, n_.^s; t, X_t, L_.) = u(c_t^s, n_t^s) + \beta\tau_t J^{i*}(t+1, L_.)$$
$$+\beta(1-\tau_t)J^s(c_.^s, n_.^s; t+1, X_{t+1}, L_.), \tag{16}$$
$$J^s(c_.^s, n_.^s; T, X_T, L_.) = u(c_T^s, n_T^s), \tag{17}$$

where $\tau_t = \pi_{s1}n_0 A\phi I_t L_t c_t^s + \pi_{s2}n_0 I_t L_t n_t^s + \pi_{s3}I_t$ is the probability of a susceptible person to be infected in the next unit time, $J^{i*}(t+1, L_.)$ is the optimal objective value achievable for an infected person starting from time $t+1$, and X_{t+1} is the SIR state at time $t+1$ resulted by people's behaviour $(c_t^s, n_t^s, c_t^{i*}, n_t^{i*}, c_t^{r*}, n_t^{r*})$ and the time t state X_t.

Now it is natural that we aim at maximising the objective $J^s(c_.^s, n_.^s; t, X_t, L_.)$ over feasible control flow $(c_.^s, n_.^s)$), ,i.e., the optimal behaviour of a susceptible people will be the solution for the optimisation

$$\begin{aligned}\max\ &J^s(c_.^s, n_.^s; t, X_t, L_.)\\ s.t.\ \ &c_\tau^s \le An_\tau^s, \quad n_\tau^s \le n_0 L_\tau, \qquad \forall \tau \in \{t, t+1, \cdots, T\}.\end{aligned} \tag{18}$$

2.3. Optimal Control of the Policymaker

With the optimal behaviour in each category under a given lockdown policy $L_.$, we can easily formulate the optimal policy-making problem into an optimal control problem.

Suppose we start the lockdown problem from some time t_0 with the contamination state X_{t_0} being given by $S_{t_0} = s, I_{t_0} = i$ and $R_{t_0} = r$, then the optimal lockdown policy should be the optimal control problem

$$\max_{L_.}\ J^0(L_.; t, X_t) = \sum_{t=t_0}^T \beta^{t-t_0}\left[S_t u(c_t^{s*}, n_t^{s*}) + I_t u(c_t^{i*}, n_t^{i*}) + R_t u(c_t^{r*}, n_t^{r*})\right], \tag{19}$$

where (c_t^{ca*}, n_t^{ca*}) are the optimal consumption and working hours for people in category ca (ca can be s, i or r), which are all determined in previous optimisation problems.

With this objective, the problem for a regulator is to solve

$$\begin{aligned}\max_{L_.}\ &J^0(L_.; t, X_t),\\ s.t.\ &L_t \in [0,1]\ \ \forall t \in [0, T].\end{aligned} \tag{20}$$

2.4. Solving Scheme

In Problem (20), or its reduced version (19), the optimal decisions of individuals in all three categories are involved. Fortunately, the optimal decisions of recovered and infectious people are trivial due to our good structure of the model, which leaves us to tackle the optimal decision problem (18) for susceptible people before the Problem (20).

We start our solving scheme by tackling the Problem (18) with a given lockdown policy $L_.$. Because of the lockdown constraint, it is almost hopeless for us to get an explicit solution. We solve this optimal control problem numerically in the same was as in Trabandt (2020). In this approach, the optimal control at each time step is regarded as the static optimisation with two constraints from the consumption budget and the lockdown policy on the working hours, and solutions are obtained by solving the corresponding KKT condition[1].

With the optimal control $(c_.^{s*}, n_.^{s*})$ as functions of the lockdown policy $L_.$, we deal with the optimal control problem (20) as an optimisation over the high dimension space $[0, 1]^T$ by the gradient-based interior-point method used in the Matlab function `fmincon`. Although we have no theoretical proof on the convergence of our scheme, our numerical results show the convergence of our scheme.

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[1] In fact, when we use the numerical scheme proposed in Trabandt (2020) to our problem, the derivative used in the KKT condition is not correct due to the absence of a complicated term from the term in equation (16). We decide to ignore this absence due to the following two reasons: (1) if we recover this complicated term, the calculation will be extremely complicated; (2) from real data in the COVID-19 pandemic, we know the coefficient in the third term $\beta\tau_t$) is very close to 0, which is also observed in our numerical results.

3. Model Parameters

In this section, we study how to estimate those parameters in our model from real data, and apply it in an example with COVID-19 data in the UK to get the numerical results for optimal lockdown control.

In our model, we have quite a lot of parameters, and some of them are well-estimated and available from different sources. Let us start from easily accessible ones. For the extended SIR model, without loss of generality, we standardise the total population to $N = 1$, which makes S_t, I_t, R_t and D_t be the proportions of the population of each category in the total population. The unit of a time step is not an essential parameter, we can simply count the time by weeks.

π_r and π_d in the extended SIR model can be easily estimated from historical data, which have been done in several data sources like HPCC covid19 data cluster [2]. In our example, we will use the estimation from Trabandt (2020). π_{s1}, π_{s2} and π_{s3} are complicated to estimate, and we defer the discussion to after all easy ones.

For the characterisation of the decision making for individuals, we still need parameters n_0, θ, β, A, and ϕ. Most of them are quite flexible, and in our examples, we do not estimate them from real data but specify their values in the same way as in different literature. We will do it in our detailed example.

Finally, let us focus on the estimation of $\pi_{s_1}, \pi_{s_2}, \pi_{s_3}$. At any time t, we have $\pi_{s1} c_t^s c_t^i + \pi_{s2} n_t^s n_t^i + \pi_{s3} = \pi_t$, where π_t is the transmission rate in classic SIR model. Similar to π_r and π_d, the quantity π_t is also available in different data source like HPCC covid-19 data cluster[2]. To estimate $\pi_{s1}, \pi_{s2}, \pi_{s3}$, we choose two different time spots t_1 and t_2. The first time spot t_1 can be any time between the onset of the spreading of the virus and the first lockdown measure, and the second time spot t_2 must be in a period where a lockdown measure was applied. With the observation of π_{t_1} and π_{t_2}, we have:

$$\pi_{s1}A^2 n_0^2 + \pi_{s2} n_0^2 + \pi_{s3} = \pi_{t_1}, \quad \pi_{s1}A^2 n_0^2 L_{t_2}^2 + \pi_{s2} n_0^2 L_{t_2}^2 + \pi_{s3} = \pi_{t_2},$$

where L_t is an estimation of actual lockdown rate at time t. These two equations are not enough to give us the values of three parameters, we still need one more equation for the purpose. In the case (as happened in the UK) that no different (non-null) lockdown measures have been applied, the third equation is officially not available. So we assume that

$$\pi_{s2} n_0^2 \times \frac{1}{3} \times \frac{1}{6} = \pi_{s3}.$$

This equation is from the assumption that susceptible people spend about 1/3 of their working hours for other activities related to other types of direct contact, and infectious people spend about half the time of susceptible ones in this type of activity due to the poor health condition. The two proportions 1/3 and 1/6 can be adjusted based on personal experience.

These three equations can give us a good estimation of π_{s1}, π_{s2} and π_{s3}.

3.1. Parameter estimation: an example

We take the COVID-19 in the UK as our example, which started in the year 2019. The only lockdown took place on 23 March 2020 and lifted up in July 2020.

For the estimation of π_{s1}, π_{s2} and π_{s3}, we need to specify some other parameters.

According to the starting of the epidemic and lockdown, we take t_1 to be a time in Jan 2020 and t_2 to be some time in April 2020.

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[2] HPCC systems covid19: https://covid19.hpccsystems.com/

The government released Experimental results of the pilot Office for National Statistics (ONS) online time-use study (collected 28 March to 26 April 2020 across Great Britain) [3] compared with the 2014 to 2015 UK time-use study, which reported the working-not-from-home time. According to the study, the average daily time (in minutes) of working not from home is 97.6 in March/April 2020 and 150.0 in 2014/2015, thus we estimate $L_{t2} = 97.6/150 \approx 0.65$.

Also according to ONS, the average actual weekly hours of work for full-time workers from Dec 2019 to Feb 2020 was 36.9 [4], thereby we set $n_0 = 36.9$. According to the equation $n_0^2 \theta = 1$, we set $\theta = 0.00073$.

We follows the setting of some parameters in literature. The mortality rate is set to be 0.6% from Trabandt (2020). As in Trabandt (2020), we assume that each infected case takes 18 days on average to either recover or die. Since our model is weekly, we have $\pi_d = 0.006 \times 7/18, \pi_r = 7/18 - \pi_d$. The reproduction number R_0 at time t_1 in Jan 2020 is around 1.95 without control measures [5], and between 0.7 to 1.0 in April 2020 after the lockdown [67], we use the middle point 0.85 of this range of R_0 for the calcuation of π_{t_2}. Since in classic SIR model, $R_0 = \beta/\gamma$ where β and γ the infected and recovery transmission rate

$$\pi_{t_1} = 1.95 \times 7/18, \pi_{t_2} = 0.85 \times 7/18.$$

Given a published average annual income [8] 30350 for 52 weeks, we set $A = 15.8172$.

With all quantities involved in the three equations for $(\pi_{s1}, \pi_{s2}, \pi_{s3})$, we get solution

$$\pi_{s1} = 1.244887 \times 10^{-6}, \pi_{s2} = 1.0336 \times 10^{-4}, \pi_{s3} = 0.01759.$$

By the value $n_0 = 36.9$, we take $\theta = 1/(36.9)^2$.
Finally, we copy the value $\phi = 0.8$ from Trabandt (2020).

4. Numerical Results

In this section, we present the result of our numerical experiments under the parameter setting in section 3.2. We do experiments to analyze the impact of the optimal lockdown control policy and smart containment. For every experiment, the initial state is $(S, I, R) = (0.9998, 0.0002, 0)$ and the time horizon is 100 weeks.

4.1. Optimal Lockdown Control

As Figure 1 (a), (d) (page 9) shows, if there is no lockdown control, i.e. the lockdown rate is constant 1 for all time, then under our parameter setting, around 15% of the population will be infected, 0.3% of the population will die and the peak of infection will be above 0.6% at week 50. Under the optimal lockdown control, the proportion of Infectious people decrease to 5.22×10^{-5} at week 50, then raises to 2.5×10^{-4} at week 100. 0.37% of the population will become infected and 0.0068% of the population will die by week 100. The optimal lockdown policy reduces the peak of infection by 95.8% and reduces the number of deaths by 97.7%. The significant life-saving is associated with a recession. Figure 1 (e) (page 9) shows the aggregate consumption under optimal lockdown policy decreases 20%

[3] ONS Dataset https://www.ons.gov.uk/economy/nationalaccounts/satelliteaccounts/datasets/coronavirusandhowpeoplespenttheirtimeunderlockdown

[4] ONS, Average actual weekly hours of work for full-time workers:https://www.ons.gov.uk/employmentandlabourmarket/peopleinwork/earningsandworkinghours/timeseries/ybuy/lms

[5] Coronavirus wikipedia: https://en.wikipedia.org/wiki/Coronavirus_disease_2019

[6] BBC report on R number: https://www.bbc.co.uk/news/health-52677194

[7] The R number in the UK:https://www.gov.uk/guidance/the-r-number-in-the-uk

[8] statista: average full time annual earnings in the uk: https://www.statista.com/statistics/1002964/average-full-time-annual-earnings-in-the-uk/

compares to the no control case at the beginning, but then constantly increases. The average aggregated consumption fall by 6.6% with the optimal lockdown measure. In Figure 1 (f) (page 9), the optimal lockdown rate starts from around 80%, then gradually release to above 95%, the speed of the increase of the lockdown rate first decreases until around week 50, then increase until week 100.

The increase of the infected proportion is because our model has a finite time horizon, and does not take the consequences after a time horizon of 100 weeks into consideration. In the beginning, the aggregated consumption under optimal lockdown control is 20% less but becomes 8.2% more than that of no control in the end. The reason that the optimal lockdown control policy did not cause a severe recession might be that in the no control case, susceptible people will cut back their working hours, as well as their consumption as the infected population increases, and in the optimal lockdown control restricted the infected population so that susceptible people won't cut back their consumption as much. We proved in section 2 that the recovered and Infectious people will work as much time as possible in order to maximize their own utility, but the behaviour of susceptible people is not certain. In the parameter setting of our experiments, the susceptible people almost work as much as possible just as the infected and recovered people do, but slightly reduce their working hours from the upper bound of lockdown constrain near the end of the time horizon, this behaviour may due to the increase of infected proportion, which raises the risk of getting infected for susceptible people.

In general, the optimal lockdown policy saves lives and is more robust in economic recovery, it brings long-term health benefits and economic growth with the cost of a short-term recession.

4.2. Smart Lockdown Control Policy

In the lockdown control policies we studied so far, the government chooses the same lockdown rate for all three kinds of people (susceptible, infectious, and recovered). In this subsection, we consider the smart containment, by which means the policymaker directly chooses working hours for all three kinds of people with the same objective function as previous models. There is no need to apply any lockdown on recovered people because their utility reaches the maximum as their working hour is at the maximum and they do not affect the utility or the transition of susceptible and Infectious people. Our results show that in the smart lockdown control policy, Infectious people almost do not work at the beginning, but then the planner gradually increases their working hours as the infected population decreases rapidly, and susceptible people work almost without fear of becoming infected. Figure 2 shows that compare to the previous optimal lockdown control policy, the smart lockdown policy is much better, since it reduces the number of deaths to a great extent, and almost avoids the recession because the proportion of Infectious people is extremely small. The implement of a smart lockdown control policy requires the planners to know the status of all people and have control over their working hours. In reality, the knowledge of people's status needs measures such as medical testing and rely on the accuracy of testing. Our results suggest that these measures and information that are helpful for taking smart lockdown policy are beneficial for social welfare.

5. Conclusion

In this paper, we extend the canonical epidemiological model SIR to find an optimal decision making with the aim to balance between economy and people's health. In our model, people in different health statuses take different decisions on their working hours and consumption to maximise their own utility, while policymakers control the lockdown rate to maximise the overall welfare, which leads to a two phases optimisation problem. Several parameters in our model are not straightforward to specify using the common epidemic data for modelling. We develop a novel method of parameter estimation through various additional sources of data. Our results show that lockdown measures could effectively reduce the deaths and infections caused by the COVID-19. In the analysis of the smart containment policy, the results suggest that additional information about the health status of people is beneficial, as the optimal lockdown control policy will reach much better outcomes if it could be implemented on people with different health status separately.
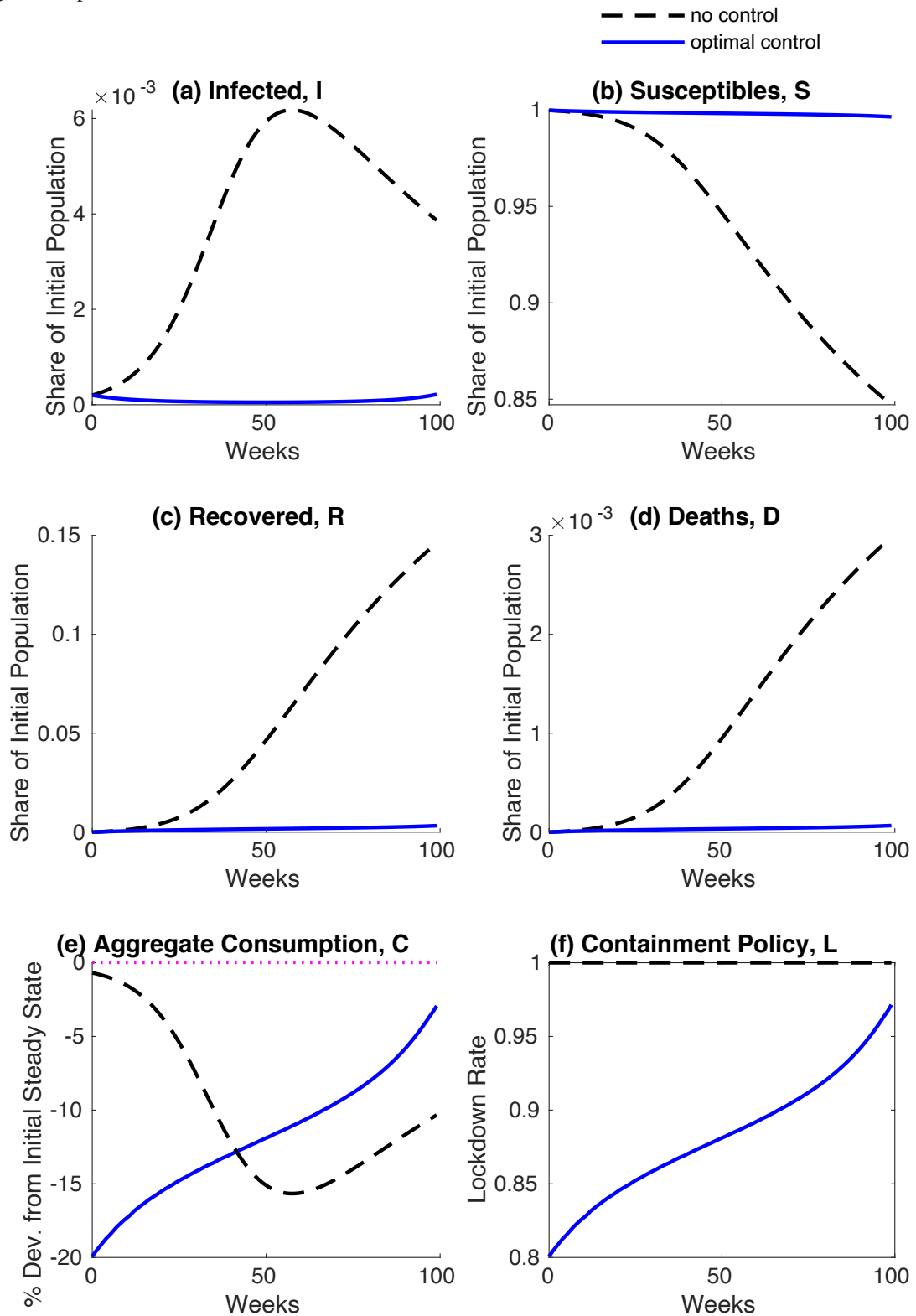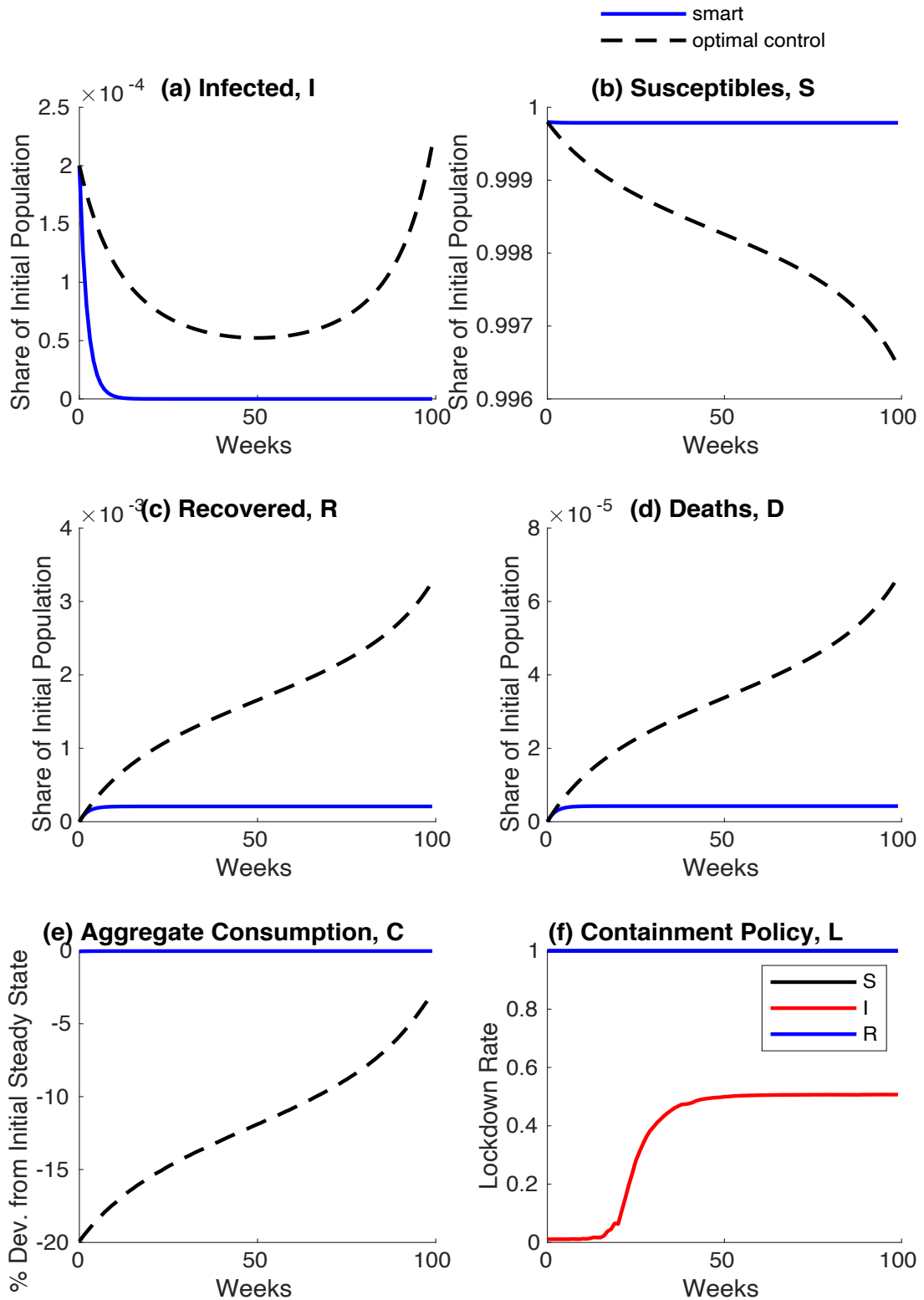
Acknowledgements

Figure 1. Optimal vs. no control

Figure 2.Smart Containment Policy

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