

1 An Agent-Based Model of Metabolic Signaling Oscillations 2 in *Bacillus subtilis* Biofilms

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11 Abstract

12 Microbes of nearly every species can form biofilms, communities of cells bound
13 together by a self-produced matrix. It is not understood how variation at the cellular
14 level impacts putatively beneficial, colony-level behaviors, such as cell-to-cell sig-
15 naling. Here we investigate this problem with an agent-based computational model
16 of metabolically driven electrochemical signaling in *Bacillus subtilis* biofilms. In this
17 process, glutamate-starved interior cells release potassium, triggering a depolar-
18 izing wave that spreads to exterior cells and limits their glutamate uptake. More
19 nutrients diffuse to the interior, temporarily reducing glutamate stress and lead-
20 ing to oscillations. In our model, each cell has a membrane potential coupled to
21 metabolism. As a simulated biofilm grows, collective membrane potential oscilla-
22 tions arise spontaneously as cells deplete nutrients and trigger potassium release,
23 reproducing experimental observations. We further validate our model by compar-
24 ing spatial signaling patterns and cellular signaling rates with those observed ex-
25 perimentally. By oscillating external glutamate and potassium, we find that biofilms
26 synchronize to external potassium more strongly than to glutamate, providing a
27 potential mechanism for previously observed biofilm synchronization. By tracking
28 cellular glutamate concentrations, we find that oscillations evenly distribute nutri-
29 ents in space: non-oscillating biofilms have an external layer of well-fed cells sur-
30 rounding a starved core, whereas oscillating biofilms exhibit a relatively uniform

31 distribution of glutamate. Our work shows the potential of agent-based models to
32 connect cellular properties to collective phenomena and facilitates studies of how
33 inheritance of cellular traits can affect the evolution of group behaviors.

34 Introduction

35 Bacterial biofilms are large communities of cells that exist in nearly every envi-
36 ronment [10]. They are bound together by an extracellular matrix that provides both
37 stability and protection [3, 8, 1]. Biofilms exhibit a variety of emergent behaviors that
38 give biofilm-dwelling microbes advantages unavailable to planktonic cells [16, 31, 42].
39 For example, cells within biofilms differentiate into heterogeneous phenotypes [22, 45,
40 46, 21], divide labor [28, 36], and coordinate behavior via chemical signals [13, 33, 49,
41 23]. These group phenomena have led researchers to assert that biofilms represent a
42 transition between single-celled and multicellular life [38, 7].

43 A striking multicellular behavior is the presence of cell-to-cell electrochemical sig-
44 nals that influence metabolism in *Bacillus subtilis* biofilms [25, 34]. As a biofilm ex-
45 pands, fewer nutrients penetrate to the center; most are consumed by exterior cells
46 [43, 51]. The paucity of nutrients in the interior raises a problem: if interior cells are
47 starved, the integrity of the biofilm is at risk [25]. *In vitro* *B. subtilis* biofilms exhibit a be-
48 havior that seems to allow them to navigate this challenge. When interior cells become
49 starved, they release potassium, depolarizing nearby cells and hampering their ability
50 to absorb glutamate. In turn, nearby cells become distressed, release potassium, and
51 hyperpolarize, eventually leading to a wave of potassium release. This wave propa-
52 gates to the biofilm exterior [34]. It has been hypothesized that glutamate consumption
53 among cells in the exterior slows down enough that glutamate can diffuse to the cen-
54 ter [25]. Once interior cells have enough glutamate, they cease releasing potassium,
55 allowing exterior cells to repolarize and resume consumption, eventually leading to
56 stress and another wave of potassium release.

57 These repeated waves of potassium release have been referred to as a form of
58 microbial “signaling” [34, 29, 11]. Potassium signaling has been proposed to allocate
59 nutrients efficiently at the colony level [25, 24], and it is heterogeneous at the cellular
60 level. Some cells participate in signaling and hyperpolarize during signaling waves,
61 whereas others do not [20]. It is unknown how cellular variation in signaling behavior
62 affects biofilm-level properties, such as distributions of nutrients. In order to answer
63 this question, we need models that can connect cell-level properties, such as signaling
64 state and inheritance of signaling behavior, to colony-level phenomena.

65 Several computational models of *B. subtilis* signaling behavior have been intro-
66 duced to explore hypotheses about the causes and effects of signaling. Zhai and
67 colleagues (2019) proposed an agent-based model to explain observations they had
68 made about signaling. Their *in vitro* experiments revealed that a roughly constant pro-
69 portion of cells signal each oscillation, that the same cells tend to release potassium
70 in repeated signaling waves, and that signaling behavior is weakly heritable—that is,
71 daughter cells of signaling cells are more likely than average to participate in signaling
72 waves. They modeled signaling as a percolation process in which a cell only signals
73 during a depolarization wave if it both has a binary trait that predisposes it to signaling
74 and is adjacent to another signaling cell in the biofilm. Using an agent-based model in
75 which agents represent individual cells allowed them to test whether signaling in this
76 manner would transmit a signal across the biofilm consistently. However, their model
77 focused on small sub-regions of the biofilm to match the limitations of their experi-
78 mental system—a roughly 35x230 rectangle of cells at the edge of the biofilm, where
79 the colony is close to two-dimensional. Their model also did not include nutrient diffu-
80 sion or uptake, preventing its use for studying how individual cell behaviors affect the
81 distribution of nutrients or growth of the biofilm.

82 Other models of *B. subtilis* depolarization waves are based on systems of differen-
83 tial equations. Martinez-Corral et al. (2018) produced a model of a one-dimensional
84 slice of the biofilm, extending from the center to an edge. Ford et al. (2021) extended
85 this to two dimensions, simulating a complete biofilm. Both models aimed to capture
86 signaling and nutrient patterns at the scale of an entire biofilm. These models explicitly
87 simulate nutrient diffusion and metabolism and have signaling operate through mech-
88 anisms that depend on internal glutamate concentration, providing powerful and accu-
89 rate recreation of biofilm-wide signaling dynamics. However, modeling these complex
90 interactions at a larger scale using differential equations comes at the cost of res-
91 olution. These models describe phenomena on the scale of the biofilm but do not
92 distinguish individual cells. Their advantages are thus opposite those of the agent-
93 based model of Zhai and colleagues, but neither can describe the effects of individual
94 cell behaviors on broad patterns of nutrient distribution or signaling.

95 The model we propose strikes a compromise between the flexibility and resolution
96 of the agent-based approach of Zhai and colleagues (2019) and the scalability of ODE
97 models. Our approach is agent-based, but the agent-based elements are overlaid
98 on a simplified version of the ODE model developed by Martinez-Corral et al. (2019).
99 Via this hybrid strategy, our model retains some of the benefits of both previous ap-

100 proaches. Our model enables simulation at the scale of an entire flow-cell biofilm [15,
101 35], comprising approximately 51,000 individual cells, each with unique potassium,
102 glutamate, membrane potential, and signaling dynamics.

103 We validate our model by comparing the behaviors of simulated biofilms with those
104 observed in experiments, including signaling patterns at local and colony-wide scales,
105 response to various stressors, and growth patterns. We show that many of the distinc-
106 tive features of *B. subtilis* signaling, including waves of depolarization and the fraction,
107 identity, and descent of cells that participate in signaling, can emerge naturally from
108 our model. We then demonstrate the application of our model by exploring open ques-
109 tions regarding synchronization of oscillations among neighboring biofilms [24] and the
110 effect of signaling on glutamate distribution.

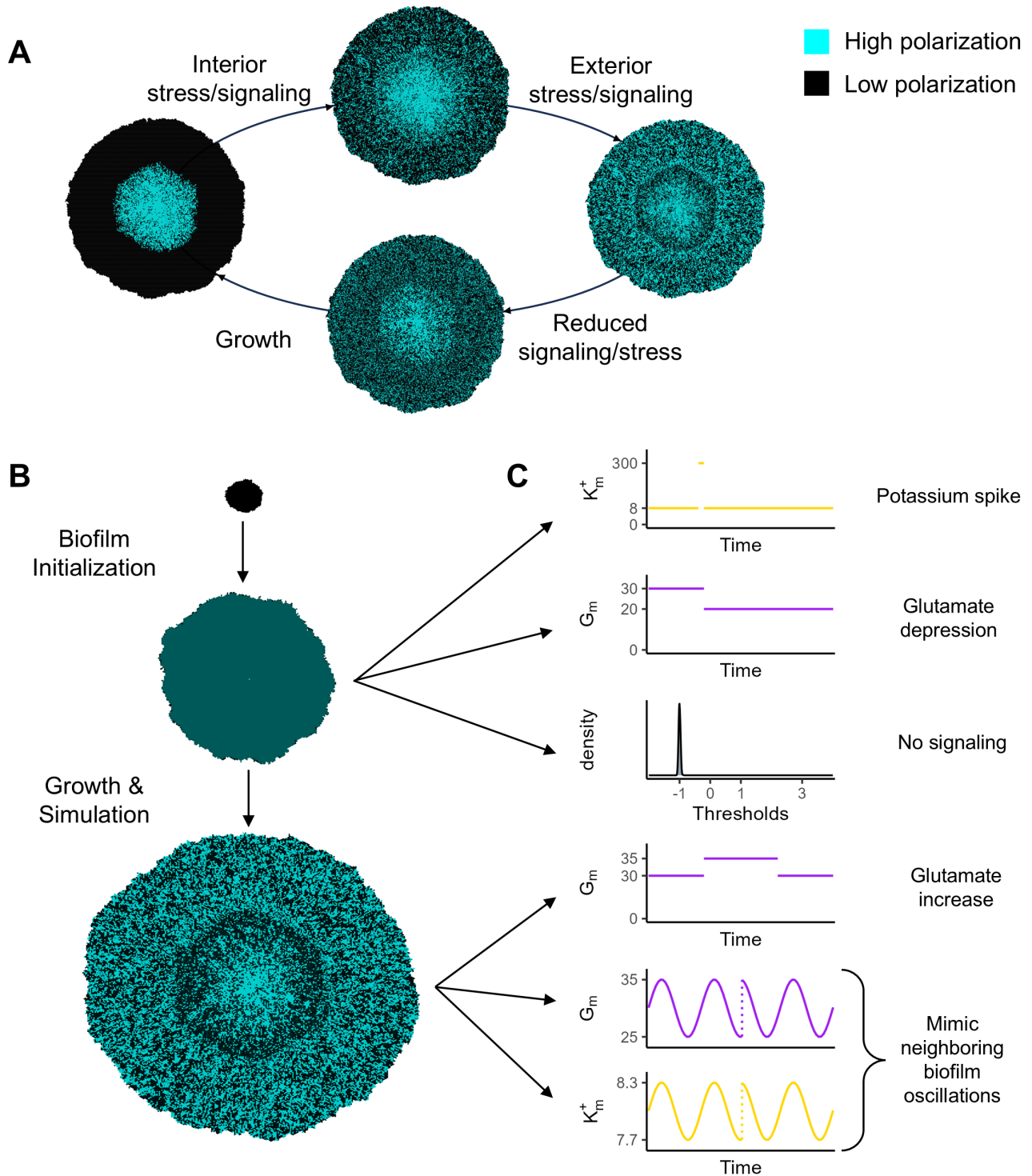
120 Results

121 Model overview

122 Our model aims to describe an oscillating hyperpolarization-depolarization behav-
123 ior observed in *B. subtilis* biofilms grown in flow cells [25, 34]. In such scenarios, when
124 a biofilm grows past a certain size, metabolically stressed interior cells release potas-
125 sium ions. The primary source of nitrogen in flow-cell experiments is glutamate [25],
126 and cells absorb glutamate via a transporter whose activity depends on membrane
127 potential. This transporter is more efficient when the cell is hyperpolarized—that is,
128 when there is a greater charge differential between the interior of the cell and the ex-
129 tracellular media [44]. By releasing charged potassium ions, stressed cells increase
130 their membrane polarization and therefore their ability to absorb nutrients.

131 Releasing potassium ions has an additional effect of depolarizing surrounding cells.
132 Prindle et al. (2015) hypothesized that when interior cells are extremely stressed and
133 release a sufficiently large amount of potassium, they can depolarize surrounding cells
134 enough to slow their nutrient uptake. If enough cells release potassium, a chain reac-
135 tion can be triggered in which nearby cells become depolarized, undergo metabolic
136 stress, and then release ions and hyperpolarize in response. Ion release can be
137 thought of as a form of signaling, albeit one that has direct effects on cell physiol-
138 ogy. If enough cells signal, it can lead neighboring cells to signal, causing a wave of
139 depolarization to spread across the biofilm.

140 As the wave of depolarization crosses the biofilm, nutrient absorption across the



111

112 *Figure 1: A schematic of our model. (A) shows the cycle of oscillations: growth causing*
 113 *interior stress, leading to signaling (indicated by cyan cells) and exterior stress, causing slowed*
 114 *growth and a reduction in stress, and finally back to resumed growth. (B) shows our simulation*
 115 *process, beginning with a very small cluster of cells, growing it for a period of time without*
 116 *simulating nutrients, and then growing to full size and running for many iterations with nutrient*
 117 *and signaling simulation. (C) shows the questions we pursue, including testing the effects of*
 118 *varying levels of glutamate and potassium, and of suppressing signaling.*

141 entire colony slows. This eventually allows nutrients to diffuse to the interior of the
142 biofilm and thereby reduce metabolic stress. A side effect of reduced nutrient uptake
143 is a corresponding reduction in growth [2], particularly in exterior cells where most
144 biofilm expansion occurs [25, 47]. After the wave of depolarization reaches the ex-
145 terior of the biofilm and nutrients diffuse through the biofilm and reduce stress in the
146 interior, growth can resume. Consequently, whereas there is consistently rapid growth
147 when the biofilm is small, once it surpasses a threshold size—determined by nutri-
148 ent concentration in the media and the biofilm’s shape and density—it transitions to
149 periodic growth, with growth pausing when the exterior of the biofilm is depolarized.

150 Our model describes the signaling waves that appear to drive these oscillations
151 in growth (Figure 1A). We developed an agent-based model that explicitly simulates
152 each cell spatially on a two-dimensional plane. Our model is hexagonal (to mimic the
153 approximate 6-neighbor structure of a 2D biofilm [20]), and can be run at the scale of
154 an entire flow-cell biofilm, with a radius of approximately 145 cells. We model gluta-
155 mate as diffusing into the biofilm from outside and being consumed by cells; uptake of
156 glutamate causes a cell’s internal glutamate level to increase. When cells are below an
157 individual-specific threshold level of internal glutamate, they release potassium ions,
158 allowing faster glutamate uptake.

159 Intracellular glutamate (G_i) and potassium (K_e), extracellular glutamate (G_e) and
160 potassium (K_e), and cell membrane potential (V) are regulated by four equations taken
161 from Martinez-Corral et al. (2019) with simplifications (equations S1, S3-S5, see Ta-
162 ble S2 for parameter values). Each cell has a signaling threshold, T_i —when a cell’s
163 internal glutamate drops below T_i , the cell signals. T_i is treated as a property of the
164 cell that remains fixed throughout the cell’s lifespan. Cells pass their signaling thresh-
165 old to their offspring, with a certain amount of noise, causing signaling behavior to be
166 partially heritable, as observed *in vitro* [50]. (We use the term “heritable” to refer to
167 the correlation between mother and daughter cells, without assuming that the source
168 of variation between lineages is genetic, which is unlikely in clonal biofilms.) An illus-
169 tration of the potassium, glutamate, and membrane potential for a single cell during a
170 signaling wave is shown in Figure S1.

171 Although the equations governing biofilm behavior are modeled on those of Martinez-
172 Corral et al. (2019), we made modifications for use in an agent-based model. For com-
173 putational tractability, we discretized coarsely with respect to time, applying the equa-
174 tions every time step (“tick”). A tick represents a period of approximately one minute,
175 an interval with respect to which potassium diffuses rapidly (Supplement S3.4.1, [30]).

176 This coarse time grid allowed us to model potassium diffusion simply by averaging it
177 across the biofilm each tick. Glutamate diffuses more slowly than potassium [37, 34],
178 so we model its diffusion, albeit in a simplified way (described in Supplement S3.1).
179 Each tick, basal glutamate in the media (G_m) diffuses into the biofilm and is absorbed
180 by cells according to equation S1.

181 We initialized biofilms with a small number of cells such that glutamate diffused
182 to the center easily. We then allowed them to grow to a radius of approximately 145
183 cells (a population of $\sim 51,000$), at which point we stopped growth (Figure 1B). At each
184 tick during the growth phase, we selected one-fortieth of the cells on the perimeter
185 of the biofilm network, uniformly at random and with replacement, to reproduce. This
186 produced growth consistent with the doubling time of *B. subtilis* (between 45 minutes
187 and 6 hours [4, 9, 18]). Each daughter cell was placed in one of the empty nodes
188 adjacent to the parent. Its signaling threshold was drawn from a truncated normal
189 distribution with μ equal to the parent's threshold, σ (corresponding to the standard
190 deviation on a non-truncated normal distribution) of 1, and bounds of [0, 3] (further
191 described in Supplement S2). Once growth stopped, we continued the simulation for
192 a total of 3000 ticks. This time period corresponds to approximately 48 hours, longer
193 than *in vitro* biofilms have been observed to maintain oscillatory behavior. To replicate
194 previous studies and make new predictions, we simulated biofilms under a variety of
195 conditions, including reduced and increased basal glutamate, oscillated basal gluta-
196 mate and potassium, and a short flood of potassium to depolarize the biofilm (Figure
197 1C).

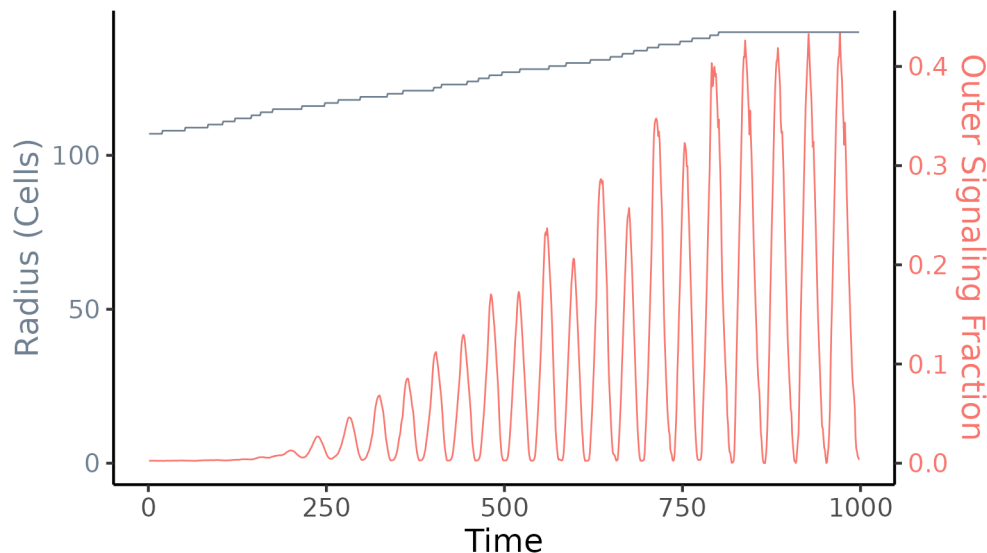
198 **Model validation**

199 **Patterns of signaling**

200 We initially explored our model by replicating behaviors and findings from previous
201 work. We first examined whether our model produced simulated biofilms in which
202 signaling oscillations behave similarly to *in vitro* observations. At a gross level, videos
203 of oscillations in *in vitro* biofilms and in our simulated biofilms and reveal many similar
204 features (videos available as files S5.1 and S5.2).

205 Martinez-Corral et al. (2018) observed that oscillations generally begin at a radius
206 of 200-350 μm under environmental conditions identical to those in our model (30 mM
207 glutamate). We found oscillations to start at a radius of around 110 cells (Figure 2),
208 which corresponds to approximately 220-330 μm [39, 48].

209 Zhai et al. (2019) found that near the edge of the biofilm, approximately 43% of
210 cells are signaling during the peak of each oscillation. This observation motivated
211 their investigation of signaling in terms of percolation theory—43% is near the mini-
212 mum fraction of signaling cells that guarantees a signal moving between adjacent cells
213 can cross the biofilm, given their other assumptions. Our simulated biofilms behave
214 similarly, with approximately 43% of cells in the outer layers of the biofilm signaling at
215 the height of each signaling oscillation (Figure 2).



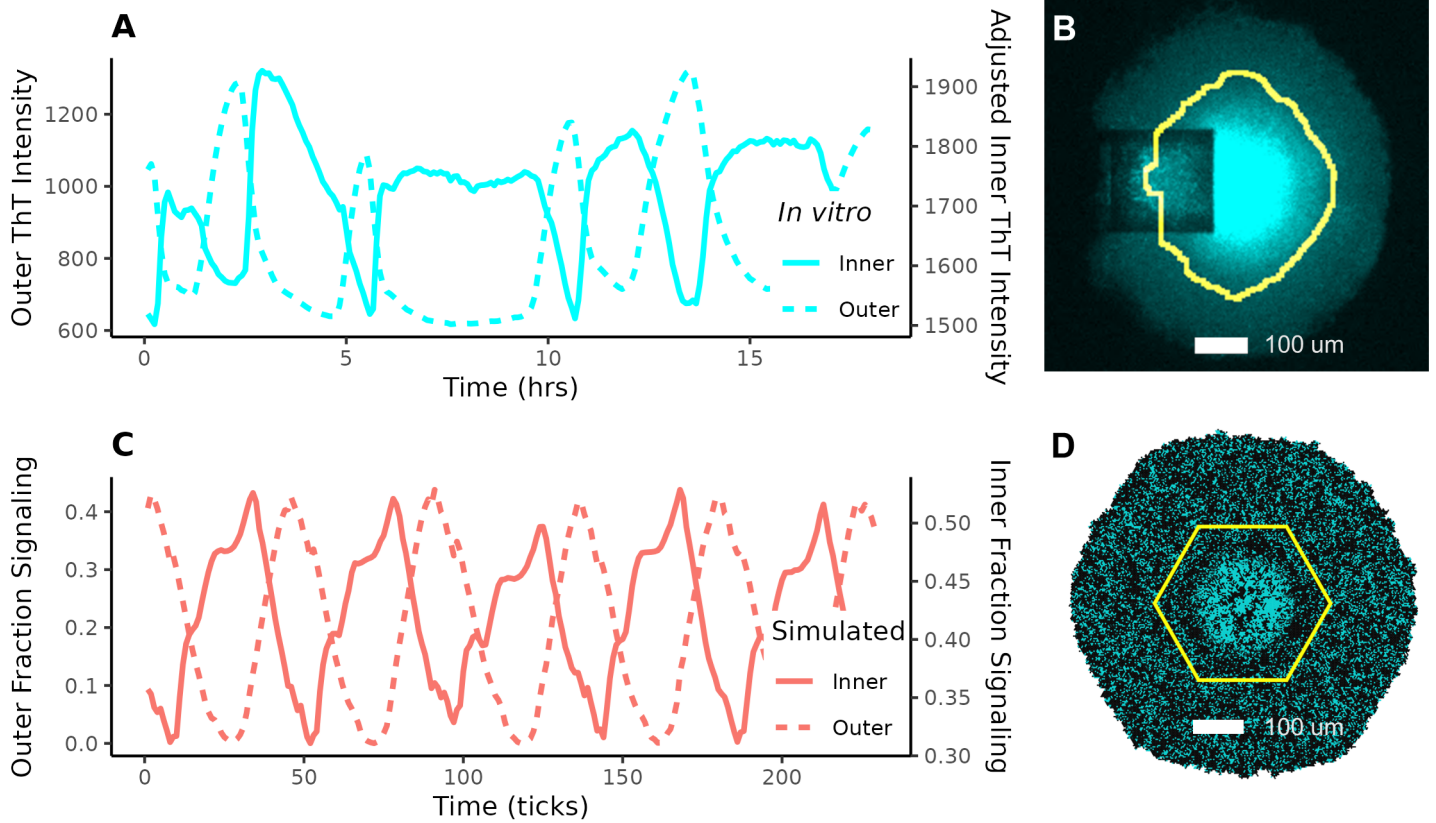
216

217 *Figure 2: Radius (gray) and fraction of signaling cells (red) in the outer region of a simulated*
218 *growing biofilm over time. The radius indicates the distance of the cell farthest from the center.*
219 *Growth is limited to a radius of approximately 145 cells. A version with growth to a much larger*
220 *size is shown in Figure S2, demonstrating the collapse of oscillations when the biofilm grows*
221 *too large.*

223 In experimental time-lapse images of biofilm signaling, the interior and exterior of
224 the biofilm oscillate approximately in antiphase, with the interior exhibiting much higher
225 polarization (Supplement S5.1, Figure 3A). *In vitro*, the division between the interior
226 and exterior (defined by oscillation) appears sharp (Figure 3B). In our simulations,
227 we observed the same boundary (Supplement S5.2 and Figures 3D and S3). The
228 difference in polarization can also be observed by comparing the vertical axes for
229 inner and outer cells in Figure 3, panels A and C.

241 **Single-cell signaling behavior**

242 Larkin et al. (2018) found a bimodal distribution of cell-level membrane potentials
243 during signaling peaks. Cells that had recently signaled had substantially more nega-
244 tive membrane potentials than those that had not. The membrane potential distribution



230

231 *Figure 3: Comparison between in vitro observations of oscillations in the interior and exterior*
232 *of the biofilm (A, B), and our simulations of the same (C, D). In the in vitro observations,*
233 *time is given in hours and the y-axis shows the average Thioflavin-T (ThT) intensity in each*
234 *region. ThT is a stain used to detect membrane polarization; polarized cells absorb it and*
235 *exhibit fluorescence [19, 34]. Note that the interior has much higher ThT intensity than the*
236 *exterior. (B) is an in vitro fluorescence image of a signaling biofilm (cyan represents ThT*
237 *intensity; the square is a cell loading trap) and (D) is snapshot from our model, both with the*
238 *boundary between inner and outer cells highlighted (yellow). In (C) and (D) cyan represents*
239 *cell membrane polarization.*

245 during signaling peaks was also bimodal in our simulations, and we used the bimodal-
246 ity to define signaling vs. non-signaling cells, classifying those on the more highly
247 polarized mode as signaling (Figure S4).

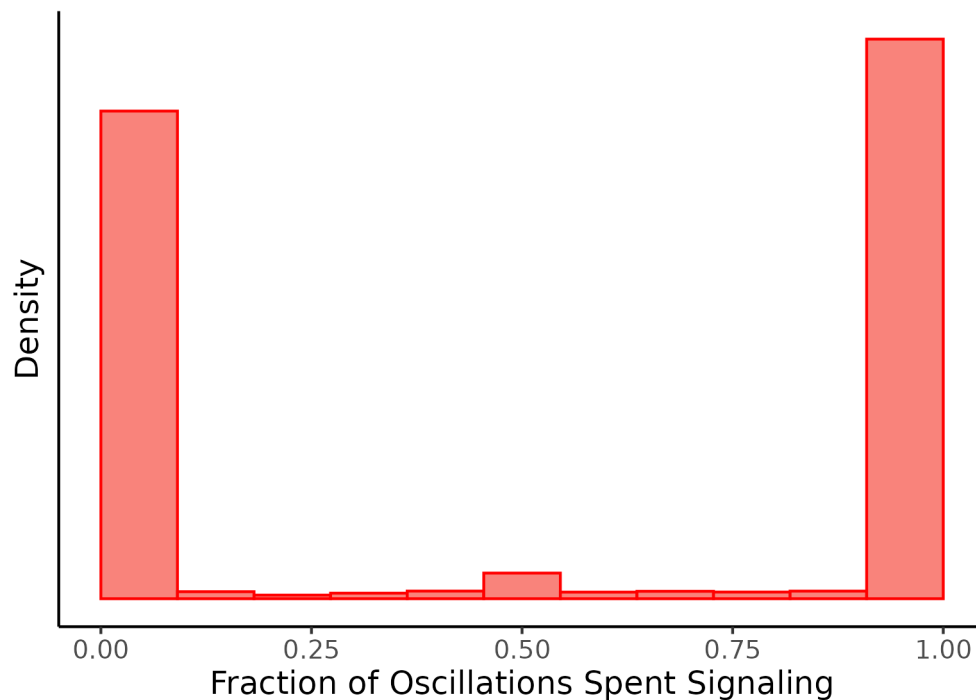
248 At the individual-cell level, signaling behavior is consistent across oscillations: cells
249 that signal in a given wave are more likely to participate in other waves of signaling. To
250 characterize this consistency, we used *in vitro* lineage tracing across two oscillations,
251 again focusing only on exterior cells. We found that across a pair of oscillations, 33% of
252 cells signaled in both waves (compared with $\sim 18\%$ expected if signaling participation
253 is independent between waves), 47% did not participate in either signaling wave, and
254 20% switched their signaling behavior between waves (with roughly half going either
255 direction). These proportions are inconsistent with the null hypothesis that cell-level
256 signaling behavior is independent between waves (Fisher's exact test $p < 10^{-24}$). We
257 then measured pairwise consistency in our simulations to compare with our *in vitro*
258 findings. In our simulations, we observed similar behavior, with 37% consistently sig-
259 naling, 52% consistently not signaling, and 11% switching (Table 1).

	Observed	Simulated
Signaling Fraction	0.43 ± 0.02	0.43 ± 0.012
Signaler Recurrence	0.60 ± 0.1	0.58 ± 0.023
Non-signaler Recurrence	0.78 ± 0.1	0.69 ± 0.023
Consistent Signaling Fraction	0.38 ± 0.03	0.37 ± 0.005
Consistent Non-signaling Fraction	0.50 ± 0.03	0.52 ± 0.008
Inconsistent Fraction	0.12 ± 0.02	0.11 ± 0.004

Table 1: A comparison between individual-cell behaviors observed in vitro and those predicted by our simulations. All simulated values are for exterior cells only. Signaling fraction and recurrence rates are from Zhai et al. (2019). Signaling fraction is the maximum proportion of cells simultaneously signaling during each oscillation. The recurrence rates are the probabilities that a daughter cell will exhibit the same signaling state as its parent in a given oscillation. Errors for observed results are standard errors. Zhai and colleagues do not give error rates for their calculations, so these are estimates. Errors for all simulated results are standard deviations. Consistency fractions are based on data from Larkin et. al (2018), with errors estimated as for a binomially distributed observation. For the signaling fraction and pairwise recurrences, these are across 20 runs. The rest are across five. Table S1 is an extended version of this table with data from inner cells and the total population, additional measures, and a description of the standard error estimation.

260 In our simulations, we also examined consistency across many waves of signaling
261 and across an entire signaling oscillation, not just looking at a snapshot of signaling
262 during the peak. Figure 4 shows cellular signaling consistency across 30 oscillations,
263 with 5 replications. We found that 50% of cells consistently signaled ($> 90\%$ of the

264 time), 44% consistently did not signal ($< 10\%$ of the time), and 6% were inconsistent,
265 with a smaller mode at 50% participation among cells that signaled inconsistently.
266 Note that this adds up to more than the mean of 43% signalers observed at oscillation
267 peaks. This is due to the fact that more than 43% of cells signal each oscillation, but
268 some signal before and some after each peak.



269

270 *Figure 4: Histogram of the average number of signaling peaks during which cells signaled. Ap-*
271 *proximately 250,000 cells were tracked across 30 oscillations, and each bar in the histogram*
272 *represents the number of cells that signaled in a proportion of signaling waves in the corre-*
273 *sponding range. Peaks at one and zero indicate that most cells were consistent in signaling or*
274 *not signaling (respectively).*

276 Finally, Zhai et al. (2019) found that signaling behavior appears heritable—the
277 daughter cells of cells that participate in signaling are more likely to participate in sig-
278 naling themselves. In our model, the signaling thresholds of individual cells are noisily
279 inherited, and this inheritance aligns with the observations of Zhai and colleagues. For
280 example, with our selected values for signaling threshold inheritance, approximately
281 58% of daughter cells of signaling cells signal themselves, and approximately 69%
282 of daughter cells of cells that do not participate in signaling also do not participate,
283 close to the observations of Zhai et al. (Table 1). Further exploration of the effect of
284 cell-level threshold on signaling appears in Supplement S2 and Figure S5. To main-
285 tain comparability to the findings of Zhai and colleagues, we measured concordance

286 of signaling status for each mother-daughter pair during a peak of signaling (though
287 different measures are given in Table S1).

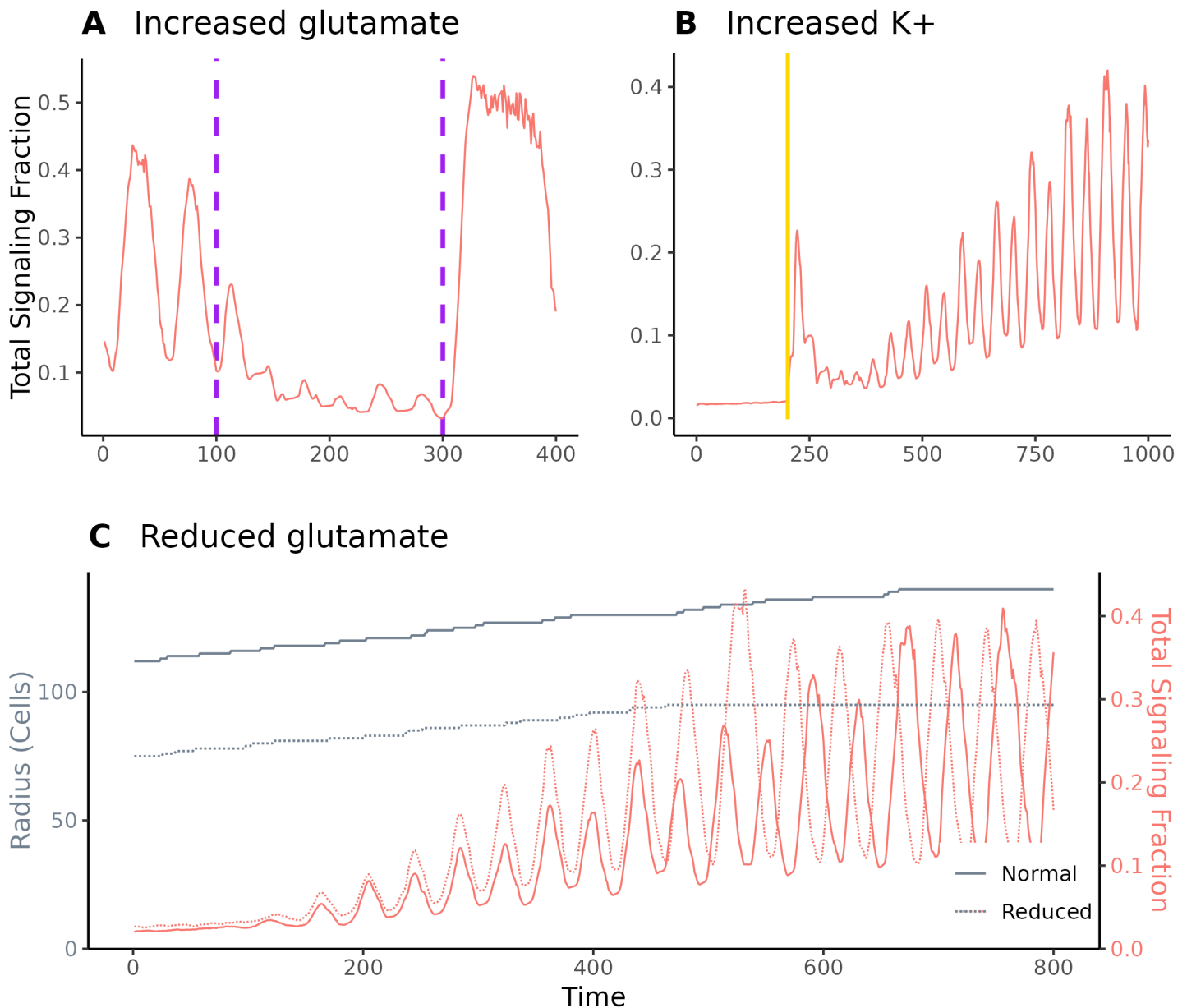
288 **Responses to media perturbations**

289 *B. subtilis* biofilm oscillation experiments have taken place within a strictly con-
290 trolled environment, where glutamate, as the only nitrogen source in the media, acts
291 as a limiting nutrient. Liu et al. (2015) showed that, in such an environment, oscilla-
292 tions can decrease or stop in response to an increase in basal glutamate (the level of
293 glutamate in the media surrounding the biofilm). Martinez-Corral et al. (2018) further
294 found that oscillations would begin at a smaller biofilm size if basal glutamate were
295 reduced, and showed that depolarization during biofilm growth can cause a wave of
296 signaling. Figure 5 shows the results of simulations intended to replicate these findings
297 in our model. By increasing basal glutamate, we weakened oscillations (Figure 5A). By
298 drastically increasing potassium to depolarize the biofilm, we caused an initial peak of
299 signaling (Figure 5B), and by lowering basal glutamate, we triggered early oscillations
300 (Figure 5C).

313 **Applications and predictions**

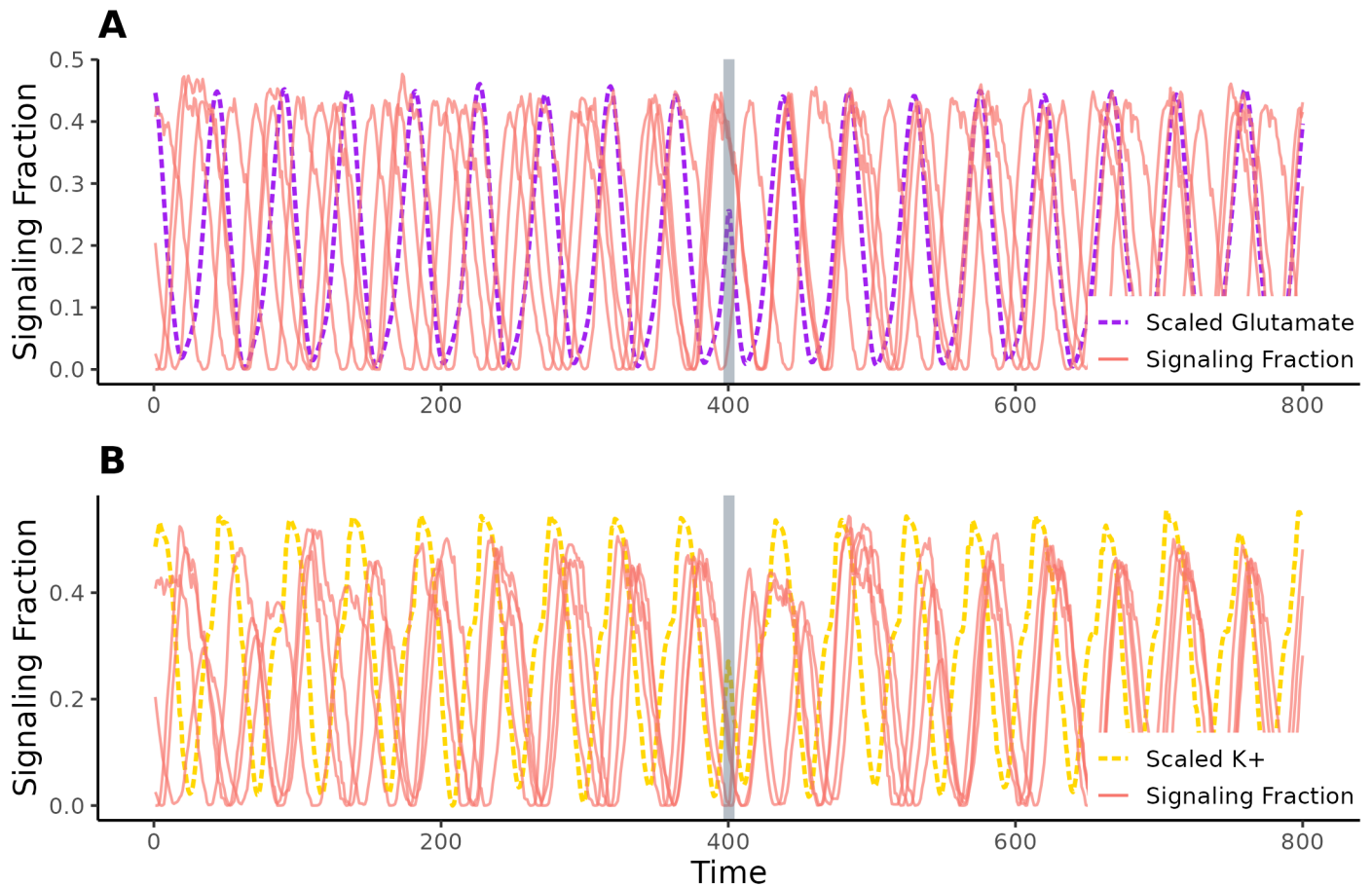
314 **Oscillation synchronization between adjacent biofilms**

315 In addition to reproducing previously observed experimental results, our model can
316 make predictions that motivate new experiments. Liu et al. (2017) found that two
317 biofilms that are adjacent to each other will shift their oscillations to synchronize, but
318 they did not identify a mechanism for this synchronization. Two molecules whose ex-
319 ternal concentrations are likely affected by depolarization waves are glutamate and
320 potassium. To test whether our model could replicate synchronization and explore its
321 explanation, we imposed external oscillations of both glutamate and potassium within
322 our simulations. Our model parameters include basal levels of glutamate and potas-
323 sium, so we simulated the effect of signaling in an adjacent biofilm by oscillating basal
324 glutamate and basal potassium separately (Figure 6). Glutamate oscillations do lead
325 the biofilm to synchronize, but only if the magnitude of glutamate oscillation is substan-
326 tially greater than we would expect to be caused by a neighboring biofilm (Figure S6).
327 In contrast, signaling oscillations change rapidly to be synchronized if basal potassium
328 is oscillated even at relatively low magnitude. Our model therefore replicates the syn-
329 chronizing behavior observed by Liu et al. (2017) and predicts that it is more strongly
330 driven by neighboring biofilms' effects on potassium than those on glutamate.



301

302 *Figure 5: Effects of environmental conditions on signaling. (A) Increasing basal glutamate from*
303 *30 mM to 35 mM from ticks 100 to 300 in a biofilm that has been stably oscillating caused a*
304 *depression in oscillation magnitude. (B) Depolarizing a growing biofilm by increasing basal*
305 *potassium from 8 to 300 mM for five ticks (indicated by the gold band) caused a wave of*
306 *signaling. This mimicked the methodology from Martinez Corral et al. (2018). (C) By growing*
307 *a biofilm in a reduced-glutamate environment ($G_m = 20$ mM) we caused oscillations to begin*
308 *at a much smaller population size. The radius for this biofilm levels off earlier because the*
309 *oscillations will collapse if the biofilm grows to full size (Figure S2). Note that signaling rates*
310 *in this figure are for the entire biofilm, not just outer cells, and are therefore sometimes higher*
312 *than those reported elsewhere.*



331

332 *Figure 6: A comparison to determine whether the synchronization observed between adjacent*
333 *biofilms is affected by (A) glutamate or (B) potassium ions. We oscillate basal glutamate (violet)*
334 *by $(-0.07, 0.1)$ mM and basal potassium (gold) by $(-0.07, 0.06)$ mM following the trajectories*
335 *of glutamate among exterior cells and external potassium respectively, taken from one of our*
336 *simulations. After 400 ticks we accelerated the basal oscillation by a quarter period. Each solid*
337 *red line indicates a different simulation. Glutamate oscillations do not appear to have a strong*
338 *effect. However, when potassium is changed, the biofilm's oscillations rapidly shift in response*
339 *and remain closely synchronized across replicates.*

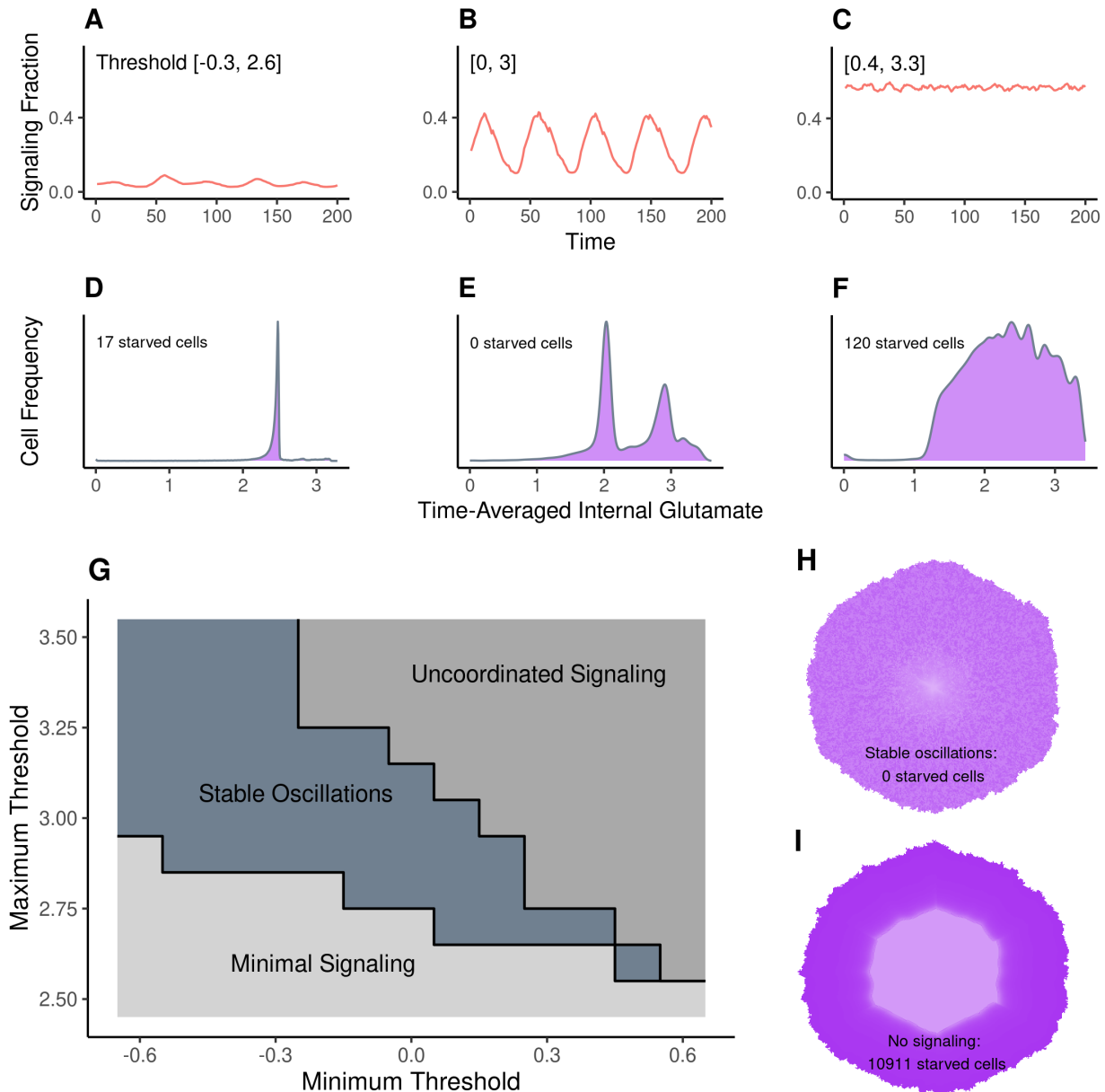
341 **Threshold effects**

342 In our model, the propensity of a cell to signal is determined by its stress threshold.
343 If a cell's internal glutamate falls below its stress threshold, then the cell will signal. The
344 results described above were simulated using thresholds distributed over a truncated
345 normal distribution, with a mode on the parental-cell threshold, lower bound of 0, upper
346 bound of 3, and σ of 1. To explore the effects of this distribution, we tested the signaling
347 patterns and internal glutamate of biofilms across a variety of threshold bounds. We
348 found that the distribution bounds must fall within a certain range in order for signaling
349 to remain stable (Figures 7G, S7). If the maximum bound is too low, then signaling
350 occurs, but only at very low levels (Figure 7A and D). There are never enough signalers
351 to starve the exterior and trigger a wave of signaling, so only the interior cells signal. If
352 the minimum bound is too high, then signaling collapses (Figure 7C and F). Too many
353 cells signal simultaneously, and signaling is uncoordinated. All cells become stressed
354 enough to signal and at any given time half or more are signaling. Between these
355 regimes, the biofilm exhibits stable oscillations (Figure 7B and E).

356 It has been proposed that potassium signaling promotes an even distribution of glu-
357 tamate across the biofilm, plausibly improving the survival rate of interior cells [25, 34].
358 We tested this idea by tracking the distribution of glutamate across cells in simulations
359 that either did or did not include signaling behavior. By comparing the mean internal
360 glutamate of cells across oscillations, we can see the effect of signaling. Without any
361 signaling, exterior cells obtained substantial glutamate, but interior cells did not, with
362 more than 10,000 (approximately 20% of all cells) reaching zero glutamate (Figure 7I).
363 However, in simulated biofilms that signal, glutamate is much more evenly distributed
364 across the biofilm, with zero cells having no glutamate (Figure 7H). Any amount of
365 signaling produced substantially fewer starving cells (Figures 7A-C and S8), but only
366 stable oscillations resulted in no starved cells. This suggests that potassium signaling
367 does promote even distribution of glutamate by slowing growth and allowing glutamate
368 to diffuse to interior cells, potentially increasing the stability of the biofilm during peri-
369 ods of high metabolic stress.

388 **Discussion**

389 We introduced a computational model of metabolic signaling in *B. subtilis* biofilms.
390 Previous models of this behavior have either been small in scope, only able to exam-
391 ine local behaviors of cells and omitting nutrients, or large in scope but unable to study
392 heterogeneity in cell-level behavior [50, 11]. We have developed a model that bridges



370

371 *Figure 7: The effects of signaling threshold range on oscillations patterns and glutamate distri-*
 372 *bution. (A) shows the fraction of signalers over 200 ticks for a biofilm with low thresholds $[-0.3,$*
 373 *2.6]. (Cells with stress thresholds ≤ 0 never signal; more negative values of the lower bound*
 374 *lead to more cells that never signal.) (D) displays the corresponding internal glutamate levels*
 375 *averaged across time for all cells in the biofilm. Seventeen cells starved—had less than 10^{-5}*
 376 *mM internal glutamate on average after the end of biofilm growth. (B) and (E) display the same*
 377 *for a range of $[0, 3]$, and (C) and (F) for $[0.4, 3.3]$. (G) shows a phase diagram of the region*
 378 *of maximum and minimum signaling thresholds in which we observe stable oscillations. The*
 379 *region of stable oscillations produces oscillations with a range of more than 20% between the*
 380 *lowest level of signalers and the highest (eg. (B)). Minimal signaling indicates a low average*
 381 *level of signaling (as seen in (A)), and the region of uncoordinated signaling produces results*
 382 *like in (C). The trajectories for the simulations used to produce this phase plot are in Figure S7.*
 383 *(H) is the time-averaged internal glutamate for the biofilm in (B), dark purple indicating higher*
 384 *internal glutamate. (I) is the same, except for a biofilm with no signaling, leading to the interior*
 385 *10,911 cells starving. Versions of (H) for the other two boundary conditions can be found in*
 386 *Figure S8.*

393 this gap, allowing the examination of the effect of cell-level behaviors on broader sig-
394 naling patterns and the concentration of nutrients across the biofilm. We were able
395 to replicate both individual-cell and biofilm-scale observations from previous work and
396 new experiments, including oscillation and growth patterns, signaling in interior and
397 exterior cells, and synchronization between neighboring biofilms. We also found sup-
398 port for the hypothesis that signaling results in a more even distribution of glutamate,
399 which may extend the lifespan of a biofilm during periods of stress.

400 Previous models of *B. subtilis* signaling have adopted various assumptions about
401 the effects of signaling on individual cells and the biofilm. On one hand, the models of
402 Prindle and colleagues (2015), Martinez-Corral and colleagues (2018, 2019), and Ford
403 and colleagues (2021) encoded assumptions that imply that signaling will increase
404 glutamate uptake for the signaling cell both by directly increasing the cell's ability to
405 absorb glutamate, and suppressing glutamate absorption for neighboring cells.

406 On the other hand, the models in Larkin et al. (2018) and Zhai et al. (2019) priori-
407 tized the observation that hyperpolarized cells experience slower growth [24], although
408 more recent work has suggested that the slow growth of hyperpolarized cells may be
409 an artifact of ThT staining itself inhibiting growth [14]. Larkin and colleagues hypoth-
410 esized signaling to be costly to the individual cell but beneficial to the biofilm as a
411 whole, as it promotes a more even distribution of glutamate. Further, they noticed that
412 the fraction of cells that signal in a given wave was close to the minimum number of
413 cells necessary for the signaling wave to propagate across the exterior of the biofilm as
414 predicted by percolation theory [41] (where signalers are randomly distributed among
415 non-signalers and a signal is propagated by direct contact between two signaling cells).
416 They interpreted this observation as being consistent with the idea that signaling cells
417 act altruistically, sacrificing their own growth to promote the integrity of the biofilm.

418 In our model, we adopt assumptions similar to those of Prindle et al. (2015) and
419 Martinez-Corral et al. (2019) that lead to signaling typically increasing the glutamate
420 uptake of the signaling cell. At the same time, we replicate the heterogeneity in sig-
421 naling behavior, the fraction of signaling cells, and the individual-level consistency of
422 signaling across waves emphasized by Larkin et al. (2018) and Zhai et al. (2019).
423 Thus, the individual-cell-level signaling patterns observed by the latter studies—and
424 particularly a fraction of signaling cells near the percolation-theory threshold for sig-
425 nal transmission—can be attained without an explicit trade-off between individual-level
426 growth and group-level glutamate distribution. However, like Larkin et al. (2018) and
427 Zhai et al. (2019), our results are consistent with the idea that cell-level heterogene-

428 ity is important. In our model, a particular amount of variation in propensity to signal
429 is necessary to achieve synchronized oscillations. In the presence of such variation,
430 the cells with the highest propensity to signal hyperpolarize first. Once enough cells
431 participate, a wave of signaling occurs, relieving glutamate stress and suppressing
432 further signaling. Under this hypothesis, the participating fraction of cells may be near
433 the level predicted by percolation theory because once that level is reached, stress is
434 relieved and further signaling is not required.

435 The observation that a requisite level of variation in signaling propensity is neces-
436 sary to produce coordinated waves of signaling in our model raises further questions.
437 What could be the source of variation in signaling propensity, and how could this vari-
438 ation be maintained? *In vitro* biofilms observed to participate in signaling are typically
439 clonal, so variation in signaling behavior is unlikely to be genetic in well-studied cases.
440 Yet signaling behavior is observed to be heritable, in the sense that daughter cells are
441 more likely to participate in signaling waves if their mother cell signals. One specula-
442 tive possibility is that the regulatory network controlling potassium channel expression
443 [27] results in multi-generational epigenetic inheritance of signaling [45, 32]. What-
444 ever the source of the variation, on the basis of current observations, if the apparent
445 individual-level cost of signaling is in fact an artifact of ThT staining [14], cells with
446 a proclivity to signal might be expected to increase in frequency within the biofilm,
447 taking up more glutamate than their neighbors, dividing more quickly, and potentially
448 transmitting (non-genetically) their elevated propensity to signal to their offspring. De-
449 pending on how propensity to signal is realized and transmitted, such a process could
450 lead to a decline of variation in propensity to signal, or at least to a decline of heritable
451 variation, if continued long enough and if there are no forces generating new heritable
452 mutation (analogous to mutation). (Our model contains such a force, as random de-
453 viations from a parent cell's signaling threshold are partially inherited by offspring.) In
454 our model, if too many cells signal, oscillations cease to be coordinated, and the distri-
455 bution of internal glutamate—while much more even than in the complete absence of
456 signaling—leaves some cells at the interior of the biofilm starved of glutamate. Thus,
457 our model raises a possibility that is almost the reverse of the one raised by Larkin
458 et al. (2018) and Zhai et al. (2019)—if signaling improves glutamate uptake for the
459 signaling cell and reduces glutamate uptake for its neighbors, we might think of the
460 cells that do not signal, rather than the ones that do, as acting altruistically, giving up
461 their access to glutamate so that interior cells are not starved. There remain other
462 possibilities—there may in fact be a cost of signaling to the individual, the increase in
463 glutamate uptake from signaling may be dependent on the signaling state of a cell's

464 neighbors, or any of a number of others. In our current implementation, reproduction
465 is not dependent on internal glutamate, so we do not explore such questions, but they
466 are important for future theoretical and experimental work.

467 Another area of future study involves extending our model to predict how other
468 processes are altered by emergent electrochemical signaling. For example, the ex-
469 pression of some genes has been proposed to be regulated by ion-responsive kinases
470 [12]. By coupling cellular potassium flux to gene expression in our model, we could
471 predict patterns of gene expression heterogeneity that would arise due to signaling. In
472 addition, other cell phenotypes are regulated by nutrient conditions, notably matrix pro-
473 duction and sporulation [26]. By modeling the response of genetic circuits that control
474 the differentiation into these phenotypes [5], we could predict how the altered distribu-
475 tion of nutrients in signaling biofilms in turn alters the distribution of matrix producers
476 and spores [46, 40, 6]. Our model may prove valuable to understanding the feedback
477 between cellular phenomena and emergent nutrient conditions within biofilms, a topic
478 of recent interest [17].

479 Overall, our work shows that combining agent-based and diffusion-based models
480 can account for the emergence of community-level properties from interactions of indi-
481 vidual cells. Doing so allows us to study the effect of signaling behavior on the biofilm
482 as a whole, and on individual cells, taking into account heterogeneity among cells.
483 That so many of the collective and cell-level signatures of *B. subtilis* biofilm signal-
484 ing can be observed in a simple model hints at a relatively simple set of principles
485 governing *in vitro* signaling behavior.

486 **Methods**

487 **Model development**

488 Our model is a network agent-based model, where cells are simulated as individual
489 “agents,” each with their own set of rules for interacting with each other and their envi-
490 ronment. Cells are placed on a network, where each cell is on a node and can interact
491 with its neighbors. In the context of biofilms, neighbors are adjacent cells. During each
492 unit of time (a “tick,” representing 1.2 minutes in this model) every cell performs actions
493 according to their governing equations, and the environment is updated. We model the
494 biofilm as hexagonal, matching observations by Larkin et al. (2018) that cells in these
495 biofilms have a modal value of 6 immediate neighbors.

496 To determine which interactions to include and how cells should behave, we fol-
497 lowed the model from Martinez-Corral et al. (2019). Their model is an ODE system
498 describing a one-dimensional cross-section of a *B. subtilis* biofilm. We simplify their
499 equations to be tractable for an agent-based model, leaving us with 4 equations (S1,
500 S3-S5) that describe potassium uptake and release, glutamate uptake and consump-
501 tion, membrane potential, and the interactions between potassium, glutamate, and
502 membrane potential.

503 Initialization and growth

504 To initialize the model, we “grow” the biofilm, drawing each layer from the previous
505 one. We begin by making a hexagon of 7 cells (6 outer and one center cell). These
506 have signaling thresholds (the level of internal glutamate they can drop to before they
507 will signal) randomly drawn from a uniform between 0 and 3. We then grow the biofilm
508 to a radius of 50 cells while all external variables remain static: we ignore diffusion,
509 metabolism, and signaling during this period. Each tick we randomly select one-fortieth
510 of the cells on the perimeter of the biofilm network, with replacement, to reproduce.
511 Each daughter cell (j) is a clone of its parent (k), except that its signaling threshold is
512 drawn from a truncated normal with bounds of 0 and 3 in most of the work reported
513 here, and with σ of 1 and μ equal to the parent’s signaling threshold. The cell is placed
514 in one of the empty nodes adjacent to the parent, with probability proportional to the
515 number of neighboring cells each empty node has.

516 Once this initial phase of growth is complete, we begin to simulate potassium and
517 glutamate behavior. Each tick, we update potassium via equation S4, simulating ab-
518 sorption, signaling, and diffusion. Simultaneously, we update glutamate via equations
519 S1 and S3, simulating metabolism and absorption and using the algorithm described
520 in Supplement S3.1 to approximate diffusion. We calculate the change in membrane
521 potentials for each cell based on the results from the potassium calculations (equation
522 S5). We continue growth at a rate of one-fortieth of the perimeter per tick until the
523 network occupies 75% of the maximum size of $\sim 68,000$ cells.

524 Model validation

525 We validated our model by replicating previous experiments by other researchers.
526 As a control, we ran the model 20 times under default conditions (using the parameters
527 given in Table S2). Each run recorded a variety of data, with 5 of the runs recording
528 individual signaling and glutamate data for every cell during each tick. These runs

529 were used to gather summary statistics including signaling rate, recurrence rates and
530 growth trajectories.

531 **Perturbations**

532 To test the effect of increased glutamate, basal glutamate was increased to 35 mM
533 from 30 mM for 200 iterations in a biofilm that had already been growing for 2600
534 iterations. To test potassium shock, we increased basal potassium from 8 mM to 300
535 mM for 5 ticks in a growing biofilm, beginning at 750 ticks. We also simulated a biofilm
536 with basal glutamate at 20 mM, limiting its growth to a radius of approximately 90 cells.
537 The results from these perturbations are shown in Figure 5.

538 **Oscillation synchronization**

539 To explore the effect of a neighboring biofilm signaling in proximity to our simulation,
540 we oscillated basal glutamate and potassium. To replicate the magnitude of change
541 a signaling biofilm would cause in the surrounding media, we used the trajectory of
542 external potassium and that of internal glutamate among exterior cells from a normal
543 run of our model. These oscillated around their means by $(-0.35, 0.32)$ mM and
544 $(-0.36, 0.48)$ mM respectively. We then scaled these by 0.2 to represent the effect
545 of distance, for a final oscillation of $(-0.07, 0.06)$ mM for potassium and $(-0.07, 0.1)$
546 for glutamate. We oscillated each for 400 ticks, then skipped the oscillating molecule
547 forward by a quarter period and simulated for another 400 ticks. These results are
548 given in Figure 6. We also replicated these with more extreme scaling. Glutamate
549 was oscillated by 200% scaling $(-0.72, 0.96)$ and potassium by 5% $(-0.018, 0.016)$.
550 These results are reported in Figure S6.

551 **Experiments**

552 Biofilms experiments were performed in a microfluidic device (CellASIC ONIX2
553 B04-F plate, Millipore Sigma, Burlington, MA, USA) as described in previous work
554 [34, 20]. Cells (*Bacillus subtilis* strain NCIB3610, Bacillus Genetic Stock Center) were
555 streaked on LB agar plates, incubated overnight at 37°C, grown in liquid LB medium,
556 resuspended in liquid msgg medium for additional growth, and loaded into the mi-
557 crofluidic plate. The composition of msgg was 5 mM potassium phosphate (pH 7.0),
558 100 mM MOPS (pH 7.0), 2 mM MgCl₂, 700 μM CaCl₂, 50 μM MnCl₂, 100 μM FeCl₃,
559 1 μM ZnCl₂, 2 μM thiamine HCl, 0.5% (v/v) glycerol and 0.125% (w/v) monosodium
560 glutamate. After cell loading into the microfluidic plate, biofilms were grown under flow

561 at 30°C and Thioflavin-T (ThT) was added to the media for imaging cellular membrane
562 potential after 12 hours of growth [34]. Biofilms were imaged in phase contrast and
563 fluorescence with a 4X, 0.13 NA objective on an Olympus IX-83 microscope (Evident
564 Scientific, Waltham, MA, USA).

565 Time traces of ThT were extracted from time-lapse movies using a machine learning-
566 based segmentation approach implemented in Python, which applies a Random For-
567 est classifier, provided by the Scikit-learn library, trained on manually segmented biofilm
568 images to perform segmentation using the ThT fluorescence channel. In the ThT
569 traces of Figure 3, we subtracted slow accumulation of ThT *post hoc* to make oscilla-
570 tion traces stationary.

571 Pairwise signaling consistency calculations given in Tables 1 and S1 were calcu-
572 lated by tracing the signaling states of approximately 300 cells across a 2 hour period
573 that included two oscillations.

574 Code availability

575 Code used to generate the simulations and figures that appear in this manuscript
576 is available at <https://github.com/Muldero/AgentBasedBsubtilis>.

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