

## Radial oscillation and translational motion of a gas bubble in a micro-cavity

Xianmei Zhang, Fan Li, Chenghui Wang<sup>\*</sup>, Jianzhong Guo<sup>\*</sup>, Runyang Mo, Jing Hu, Shi Chen, Jiaxin He, Honghan Liu

*Institute of Shaanxi Key Laboratory of Ultrasonics, Shaanxi Normal University, Xi'an 710119, China*

### ARTICLE INFO

#### Keywords:

Bubble nucleus  
Spherical liquid cavity  
Cavitation bubble  
Radial oscillation  
Translational motion

### ABSTRACT

According to classical nucleation theory, a gas nucleus can grow into a cavitation bubble when the ambient pressure is negative. Here, the growth process of a gas nucleus in a micro-cavity was simplified to two "events", and the full confinement effect of the surrounding medium of the cavity was considered by including the bulk modulus in the equation of state. The Rayleigh–Plesset-like equation of the cavitation bubble in the cavity was derived to model the radial oscillation and translational motion of the cavitation bubble in the local acoustic field. The numerical results show that the nucleation time of the cavitation bubble is sensitive to the initial position of the gas nucleus. The cavity size affects the duration of the radial oscillation of the cavitation bubble, where the duration is shorter for smaller cavities. The equilibrium radius of a cavitation bubble grown from a gas nucleus increases with increasing size of the cavity. There are two possible types of translational motion: reciprocal motion around the center of the cavity and motion toward the cavity wall. The growth process of gas nuclei into cavitation bubbles is also dependent on the compressibility of the surrounding medium and the magnitude of the negative pressure. Therefore, gas nuclei in a liquid cavity can be excited by acoustic waves to form cavitation bubbles, and the translational motion of the cavitation bubbles can be easily observed owing to the confining influence of the medium outside the cavity.

### 1. Introduction

Gas nuclei extensively exist in liquid. When the liquid pressure decreases to negative pressure, the gas nuclei will grow into bubbles, and cavitation might be excited [1]. Cavitation has important applications in many fields [2–5], such as water conservancy [2], chemical engineering [3], and medicine [4,5]. There is increasing interest in biomedical application of ultrasound. A promising noninvasive surgery method has been developed using high-intensity focused ultrasound (HIFU), in which HIFU is applied to treat tumors or lesion tissue at a target site without damaging the intervening tissues [6]. Cavitation cloud histotripsy could be generated at the target site in tissue, where cavitation bubbles are confined to oscillate nonlinearly, and the tissue will heat up [7–9]. This could cause adverse effects, such as protein denaturation, an increase of tissue stiffness, and acoustic attenuation. The kinetic characteristics might be more complex because the oscillating bubbles are stress-confined by the surrounding tissues. The associated potentially important effects need to be further explored.

Bubble dynamics has been investigated for many years, and various models have been developed to describe the motion of bubbles in

aqueous media [10–12], near a rigid/elastic surface [13], and in a tube [14,15]. In recent years, cavitation phenomena that occur in fully confined liquids has received more attention [16–20], and models have been developed to understand the motion of fully confined bubbles in a liquid cavity, and the medium surrounding the cavity is considered to be an elastic/viscoelastic solid [21–27]. It is motivated mainly by applications in biology. Sap cavitation occurs when the tree's pumping system is operating under a negative pressure less than the threshold pressure [19,20]. For this interesting phenomenon, theoretical models of a bubble locate at the center of a spherical liquid cavity surrounded by solid medium were developed to investigate the behaviors of the bubble, such as the vibration frequency and attenuation of acoustic emissions related to cavitation bubbles inside trees [22–24]. In these researches, the breathing mode of cavitation bubbles is considered be the dominant mode, and it was found that the natural frequency of the bubble oscillation increases with the bubble radius, which is different from the case of unbounded liquid. To better understand the cavitation behavior inside cavities, water-filled hydrogel cavities have been experimentally investigated [28–30]. These experiments demonstrated the bubble motion was influenced not only by the elasticity of the gas inside the bubble, but also by the compressibility of the liquid and the medium

<sup>\*</sup> Corresponding authors.

*E-mail addresses:* [wangld001@snnu.edu.cn](mailto:wangld001@snnu.edu.cn) (C. Wang), [guojz@snnu.edu.cn](mailto:guojz@snnu.edu.cn) (J. Guo).

<https://doi.org/10.1016/j.ultsonch.2022.105957>

Received 14 November 2021; Received in revised form 30 January 2022; Accepted 15 February 2022

Available online 18 February 2022

1350-4177/© 2022 The Author(s).

Published by Elsevier B.V. This is an open access article under the CC BY-NC-ND license

(<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

### Nomenclature

#### Roman letters (alphabetical order)

$R_{b0}$ ( $\mu\text{m}$ )	radius of the bubble nucleus
$P_v$ (Pa)	saturated vapor of pressure
$z$ ( $\mu\text{m}$ )	instantaneous position of the bubble
$z_0$ ( $\mu\text{m}$ )	initial position of the bubble nucleus
$R_b$ ( $\mu\text{m}$ )	instantaneous radius of the bubble
$R_c$ ( $\mu\text{m}$ )	instantaneous radius of the cavity
$R_{c0}$ ( $\mu\text{m}$ )	initial radius of the cavity
$V_{c0}$ ( $\mu\text{m}^3$ )	initial liquid cavity of the volume
$P_0$ (kPa)	atmospheric pressure
$P_{ex}$ (MPa)	driven pressure
$V_l$ ( $\mu\text{m}^3$ )	instantaneous volume of the liquid
$P_l$ (Pa)	instantaneous liquid pressure
$K_l$ (GPa)	bulk modulus of liquid
$L$ (J)	Lagrangian
$V_{l0}$ ( $\mu\text{m}^3$ )	initial volume of the liquid

$T$ (J)	kinetic energy
$U$ (J)	potential energy
$P_g$ (Pa)	gas pressure inside the bubble
$F_v$ (N)	viscous drag force
$\eta_l$ (Pa.s)	viscosity of the liquid
$K_c$ (GPa)	bulk modulus of the elastic micro-cavity
$G$ (MPa)	shear modulus of the elastic micro-cavity
$P_b$ (Pa)	liquid pressure on the surface of the bubble
$c_l$ (m/s)	speed of sound in the liquid
$f$ (MHz)	frequency
$K$ (GPa)	effective modulus
$F$ (J)	total free energy
$F^*$ (J)	typical free energy

#### Greek letters (alphabetical order)

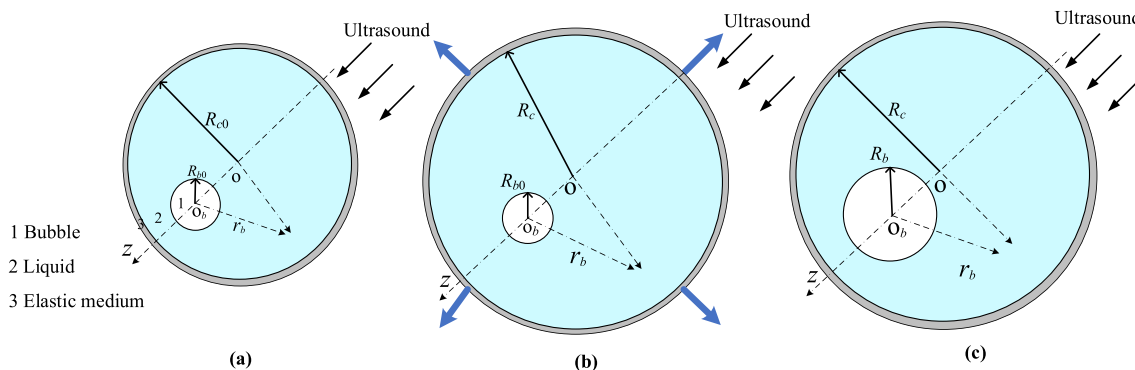
$\rho_l$ ( $\text{kg}/\text{m}^3$ )	density of the liquid
$\sigma_l$ (N/m)	surface tension of the liquid

outside the cavity. The ultra-fast bubble radial oscillations were observed and the violent collapse of cavitation bubbles could generate secondary effects. Therefore, investigation of nonlinear oscillation of cavitation bubbles in cavities is of great importance. Wang [25] derived a Rayleigh–Plesset-like equation to investigate the dynamics of a confined bubble in a liquid cavity surrounded by an elastic solid. The equation predicted reasonable results for the natural frequency of bubble oscillation and transient response to the ambient pressure. The bulk modulus of the surrounding medium could increase the confining effect. Doinikov et al. [31] investigated the coupled nonlinear oscillations of two bubbles located on one diameter of a spherical cavity. They found that the behaviors of the two bubbles were different from that in an unbounded liquid.

The phenomenon of nucleation in a water micro-cavity has shown that cavitation bubbles can be generated near the cavity wall and the bubbles move away from the wall in a short timescale [28]. It was also observed that the largest bubble deformed against the wall because of the strong interaction between the bubble and the wall. Therefore, investigation of bubble nucleation is important for understanding the problem of cavitation in liquid cavities [32]. However, classical nucleation theory cannot be directly applied to these cases. Leonov et al. [33] modified the Blake model of a single cavitation bubble in a rigid micro-confinement. They found that the confinement strongly affects the manifestation of the classical cavitation Blake threshold, and when the cell is exposed to a negative pressure exceeding the Blake threshold, the bubble nucleus abruptly expands to a finite radius related to the size of the liquid cell. Similarly, cavitation in tissue could be simplified by a

model in which bubble nuclei exist in liquid micro-cavities when the tissue is radiated by HIFU [5]. If the tissue surrounding the liquid micro-cavities can be considered to be a viscoelastic medium, the viscoelastic effect on the growth process of bubble nuclei in micro-cavities should be calculated. Vincent et al. [34] developed a model including the contributions of the surface tension, gas and liquid compressibility, and elastic deformation of the solid to the Blake threshold. They found that compressibility of the liquid and elasticity of the surrounding medium can result in ultra-fast bubble radial oscillations. According to previous studies, the behavior of a confined bubble surrounded by an elastic medium is very complicated. The radial oscillation of the bubble could be affected by many factors. The coupled mechanism between bubbles or a bubble and the cavity wall should be further studied.

With the development of the ultrasonic therapy technology, the modulation of the tissue cavitation is particular important to promote precision therapy and avoid secondary damage to biological tissue or organs. The cavitation onset process is closely related to the pressure drop in the tissue fluid caused by ultrasound. The evolution of the cavitation nuclei in biological tissue liquid is very complex and it is necessary to explore the growth mechanism of cavitation nuclei, which to some extent lacks theoretical studies. As far as we know, most of the existing theoretical studies on confined bubble nucleation are devoted to bubbles in the center of the spherical liquid cavity. Here, we propose a general theory to simulate the cavitation process by combining modified nucleation theory and the dynamic behavior of the cavitation bubble. The effects of the relative position of the confined bubble inside the liquid cavity on the general translational motion and pulsation were



**Fig. 1.** Quasi-static state sequence of gas-nucleus growth. When the cavity is exposed to tension  $P_{ex}$ , the gas nucleus grows. (a) Initial state (unperturbed), (b) the cavity stretched under tension, and (c) formation of a cavitation bubble.

investigated. It was found that the acoustic response time is affected by the position of the bubble nucleus, and the oscillations of the bubbles are not in phase. The cavitation bubbles could move away from the wall of the micro-cavity, but they are not trapped in the center of the cavity. These predictions are in good agreement with experimental observations [28].

## 2. Theoretical model

A model of tissue cavitation will be introduced to simulate the growth of bubble nuclei in liquid micro-cavities by considering the effect of the surrounding elastic medium/shell. Suppose that there is a gas nucleus of radius  $R_{b0}$  inside a spherical cavity filled with liquid at saturated vapor pressure  $P_v$ , as shown in Fig. 1(a). The system is not spherically symmetrical because the gas nucleus is not located at the center of the spherical cavity. To consider the coupling effect of the system, two spherical coordinates are defined. The first coordinate  $(r, \theta, \varphi)$  originates from the center of the cavity, and the second local coordinate  $(r_b, \theta_b, \varphi_b)$  originates from the center of the gas nucleus. The z axis passes through the centers of the gas nucleus and cavity, its origin is located at the center of the cavity, and the initial coordinate of the bubble is  $z_0$ . The instantaneous radii of the gas nucleus and cavity are  $R_b$  and  $R_c$  with initial values of  $R_{b0}$  and  $R_{c0}$ , respectively. The gas nucleus is driven to grow by ultrasound wave propagating along the direction parallel to the z-axis.

### 2.1. Growth model: a gas nucleus in a cavity filled with liquid

Initially, in the reference state, for the system with a stable gas nucleus suspended in a liquid cavity with volume  $V_{c0} = (4\pi/3)R_{c0}^3$  (Fig. 1(a)), the pressure inside the undisturbed liquid-filled cavity is equal to the atmospheric pressure  $P_0$ . If the spherical liquid-filled cavity is exposed to negative pressure generated by ultrasound, a sequence of "events" occurs as the gas nucleus grows into a cavitation bubble, as shown in Fig. 1.

First, when the cavity is exposed to negative pressure  $P_{ex}$ , the cavity uniformly expands to radius  $R_c > R_{c0} > R_{b0}$  (Fig. 1(b)). It is assumed that  $R_{b0}/R_c \ll 1$  and the pressure disturbance at the confinement wall is proportional to the change of the liquid volume [28]

$$P_l(V_l) = P_0 - K_l \frac{V_l - V_{l0}}{V_{l0}}, \quad (1)$$

where  $V_{l0}$  and  $V_l$  are the initial and transient volumes of the liquid, respectively,  $P_l(V_l)$  is the transient liquid pressure, and  $K_l$  is the bulk modulus of the liquid.

Second, as the bubble nucleus grows under tension and reaches a new equilibrium state with radius  $R_b$  (Fig. 1(c)), the liquid pressure eventually decreases to [33]

$$P_l = P_0 - K_l \frac{R_c^3 - R_b^3 - R_{c0}^3 + R_{b0}^3}{R_{c0}^3 - R_{b0}^3}. \quad (2)$$

According to Eq. (2), the growth process of the gas nucleus is determined by the ambient pressure of the liquid and the size of the cavity. Therefore, for a fully confined gas nucleus, the cavitation threshold in the closed cavity should be higher than that in an unbounded liquid [32].

### 2.2. Dynamics of the bubble

By applying the Lagrangian formalism, the equations of the pulsation and translational motion of the bubble inside the cavity can be derived. The Lagrangian function  $L = T - U$  is determined by the kinetic energy ( $T$ ) and potential energy ( $U$ ) of the system. The kinetic energy can be approximately expressed as [31]

$$T = 2\pi\rho_l \left[ R_{c0}^3 \dot{R}_c^2 + R_b^3 \left( \dot{R}_b^2 + \frac{\dot{z}^2}{6} \right) \right] + \frac{\pi\rho_l}{R_{c0}^3} \left( 4R_b^4 \dot{R}_b^2 z^2 + 4R_b^5 \dot{R}_b z \dot{z} + R_b^6 \dot{z}^2 \right), \quad (3)$$

where  $\rho_l$  is the density of the liquid.

In a liquid cavity, the buoyancy force on the bubble is much smaller than the viscosity drag. Therefore, the effect of the buoyancy force is ignored. During the cavitation inception process in Fig. 1, the increment of the potential energy stored in the liquid and gas can be estimated by [32]

$$U = - \int_{V_{l0}}^{V_l} P_l dV_l - \int_{R_{b0}}^{R_b} \left( P_g + P_v - \frac{2\sigma_l}{R_b} \right) 4\pi R_b^2 dR_b - z F_v, \quad (4)$$

where  $P_g = \left( P_0 - P_v + \frac{2\sigma_l}{R_{b0}} \right) \frac{R_{b0}^3}{R_b^3}$  is the gas pressure inside the bubble and  $\sigma_l$  is the surface tension. The first term corresponds to the energy stored in the liquid owing to the work done by the external acoustic field in the bubble-cavity system, the second term represents the work done by the gas/vapor inside the cavitation bubble and the additional energy related to the surface tension of the liquid, and the third term describes the work done by the viscous drag force  $F_v$ . If viscous effects are restricted to the thin boundary layer on the bubble surface, the viscous drag force  $F_v$  is given by [35]

$$F_v = -12\pi\eta_l R_b \dot{z}, \quad (5)$$

where  $\eta_l$  is the dynamic viscosity of the liquid. Here, the dependence of the liquid-cavity radius on the bubble radius and external driving  $P_{ex}$  is given by [36]

$$\frac{R_c^3 - R_{c0}^3}{R_{c0}^3} = -\frac{1}{K_c} (P_{ex} - P_0) + \frac{3}{4G} (P_b - P_{ex}), \quad (6)$$

where  $K_c$  and  $G$  are the bulk and shear moduli of elastic micro-confinement, respectively.  $P_b$  is the liquid pressure on the surface of the bubble. After considering liquid compressibility correction [31],  $P_b$  is given by

$$P_b = \left( P_0 - P_v + \frac{2\sigma_l}{R_{b0}} \right) \left( \frac{R_{b0}}{R_b} \right)^3 \left( 1 - \frac{3R_b}{c_l} \right) - \frac{4\eta_l \dot{R}_b}{R_b} - \frac{2\sigma_l}{R_b} - P_0, \quad (7)$$

where  $c_l$  is the speed of sound in the liquid. Substituting Eq. (7) into Eq. (6) gives the expression of  $R_c$ . Substituting  $R_c$  into Eq. (2) then gives the exact expression of the pressure  $P_l$ , which helps to determine the total potential energy in Eq. (4). The Lagrangian function is obtained, and by substituting it into the Lagrangian equations

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{R}_b} - \frac{\partial L}{\partial R_b} = 0, \quad \frac{d}{dt} \frac{\partial L}{\partial \dot{z}} - \frac{\partial L}{\partial z} = 0, \quad (8)$$

the dynamics equations of the cavitation bubble inside the cavity are given by

$$\begin{aligned} & \left( R_b + \frac{R_b^2}{R_{c0}} + \frac{R_b z^2}{R_{c0}^3} \right) \ddot{R}_b + \left( \frac{3}{2} + \frac{4z^2 R_b}{R_{c0}^3} + \frac{2R_b}{R_{c0}} \right) \dot{R}_b^2 + \left( \frac{R_b^3 z}{R_{c0}^3} \right) \ddot{z} \\ & = \frac{\dot{z}^2}{4} + \frac{P}{\rho_l} - \frac{3\eta_l z \dot{z}}{\rho_l R_b^2} - \frac{4R_b^2 \dot{R}_b z \dot{z}}{R_{c0}^3} + \frac{R_b^3 \dot{z}^2}{2R_{c0}^3}, \end{aligned} \quad (9)$$

$$\begin{aligned} & \left( \frac{2R_b^3}{3} \left( 1 + \frac{3R_b^3}{R_{c0}^3} \right) \right) \ddot{z} + 2\dot{R}_b R_b^2 \left( 1 + \frac{6R_b^3}{R_{c0}^3} \right) \dot{z} + 4 \left( \frac{3R_b^4 \dot{z}^2}{R_{c0}^3} + \frac{R_b^5}{R_{c0}^3} - \frac{3\dot{R}_b \eta_l}{\rho_l} \right) z \\ & = 0, \end{aligned} \quad (10)$$

where the pressure  $P$  is expressed as

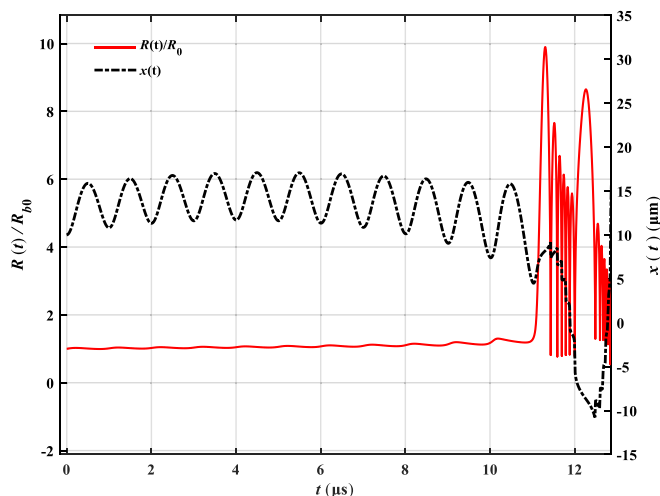


Fig. 2. Translational motion and radial oscillation of a bubble nucleus inside a cavity ( $R_{b0} = 0.5 \mu\text{m}$ ,  $z_0 = 10 \mu\text{m}$ ,  $R_{c0} = 40 \mu\text{m}$ ).

$$P = P_g + P_v - \frac{2\sigma}{R_b} - P_0 + \frac{K_l}{R_{c0}^3 - R_{b0}^3} \left[ R_{c0}^3 \left( \frac{3}{4G} (P_b - P_{ex}) - \frac{1}{K_c} (P_{ex} - P_0) \right) - R_b^3 + R_{b0}^3 \right]. \quad (11)$$

The growth of a confined gas nucleus and its subsequent behavior are of great importance to understand the cavitation effect. Eqs. (9) and (10) describe the general pulsation and translational movement of a fully confined gas nucleus in a micro-cavity. It is clear that the compressibility of the liquid and solid medium affect the kinetic behavior of the gas nucleus. Our model incorporates the growth process of gas nuclei into cavitation bubbles, and it can predict the nonlinear behavior of cavitation bubbles. Therefore, this study provides a new approach to deal with the problem in which a bubble is fully confined inside a liquid cavity.

### 3. Numerical analysis

Vincent et al. [28] observed that bubbles move away from the wall of the micro-cavity (i.e., toward the center of the micro-cavity in the spherical geometry) in a short timescale ( $\sim 10 \mu\text{s}$ ). Therefore, the behavior of a confined bubble in a liquid cavity is very complicated. A nonradial flow field could be generated by a translating bubble in a cavity [34], and the volume pulsation of the bubble would be affected by the translational motion of the bubble. In this section, we will analyze

the acoustic response of a gas nucleus inside a liquid-filled cavity surrounded by an elastic solid medium using Eqs. (9) and (10). The parameters were set to [28]  $\rho_l = 1000 \text{ kg/m}^3$ ,  $c_l = 1500 \text{ m/s}$ ,  $\sigma_l = 0.072 \text{ N/m}$ ,  $\eta_l = 0.001 \text{ Pa s}$ ,  $P_0 = 101.3 \text{ kPa}$ ,  $K_c = 4.5 \text{ GPa}$ ,  $G = 0.075 \text{ MPa}$ ,  $K_l = 2.2 \text{ GPa}$ ,  $z_0 = 10 \mu\text{m}$ ,  $R_{c0} = 40 \mu\text{m}$ , and  $R_{b0} = 0.5 \mu\text{m}$ . The acoustic pressure function is given by

$$P_{ex}(t) = P_a \sin(2\pi ft), \quad (12)$$

where  $P_a$  is the amplitude and  $f$  is the frequency. In the calculations, we set  $P_a = -1 \text{ MPa}$  and  $f = 1 \text{ MHz}$ . The pulsation and translational motion of the gas nucleus are shown in Fig. 2. The time taken for the gas nucleus to grow into a cavitation bubble was about  $10 \mu\text{s}$ . Initially, the gas nucleus slightly moved back and forth around its initial position driven by ultrasound. Subsequently, the gas nucleus rapidly increased to the critical size in a very short time and a cavitation bubble was generated. The cavitation bubble nonlinearly pulsated at its new equilibrium radius. The equilibrium radius of a gas nucleus with a radius of  $0.5 \mu\text{m}$  is about  $2 \mu\text{m}$  when it is excited by ultrasound with a frequency of  $1 \text{ MHz}$  at a given negative pressure of  $-1 \text{ MPa}$ . It is obvious that the grown cavitation bubble is nonlinearly vibrating and it may collapse after a few periods. Based on our numerical results, it can be seen that it is relatively easy to excite cavitation in tissue liquid by comparing with the cavitation initiation process with that in water-filled hydrogel cavities. Therefore, for a fully confined gas nucleus, cavitation can be excited by a continuous ultrasound wave at a relative low pressure amplitude. The translational motion in the evolution process of the cavitation cannot be ignored. It is observed that the nucleus translates toward the center of the cavity and the grown cavitation bubble has a more pronounced translational movement around the center of the cavity. Due to the ultrasonic action and the interaction between the radial pulsation and translational motion, the translational motion has oscillatory characteristics.

Gas nuclei are sensitive to the pressure perturbations from the acoustic wave, especially for gas nuclei located in the region near the center of the cavity. Numerical analysis of the acoustic response of a  $0.5\text{-}\mu\text{m}$  gas nucleus located in a  $40\text{-}\mu\text{m}$  cavity with relative positions of  $z_0 = 1, 10, \text{ and } 30 \mu\text{m}$  is shown in Fig. 3. The gas nucleus near the center of the spherical cavity ( $z_0 = 1 \mu\text{m}$ ) rapidly grew into a cavitation bubble. Subsequently, the bubble nonlinearly oscillated and translated driven by ultrasound. The features of oscillation and translational motion of a cavitation bubbles in confined liquids are more complex and distinctly different from the motions of bubbles in unbounded liquids. Cavitation bubbles in unbounded liquids oscillate periodically driving by acoustic wave, while under the condition that the size of the cavity was small, the

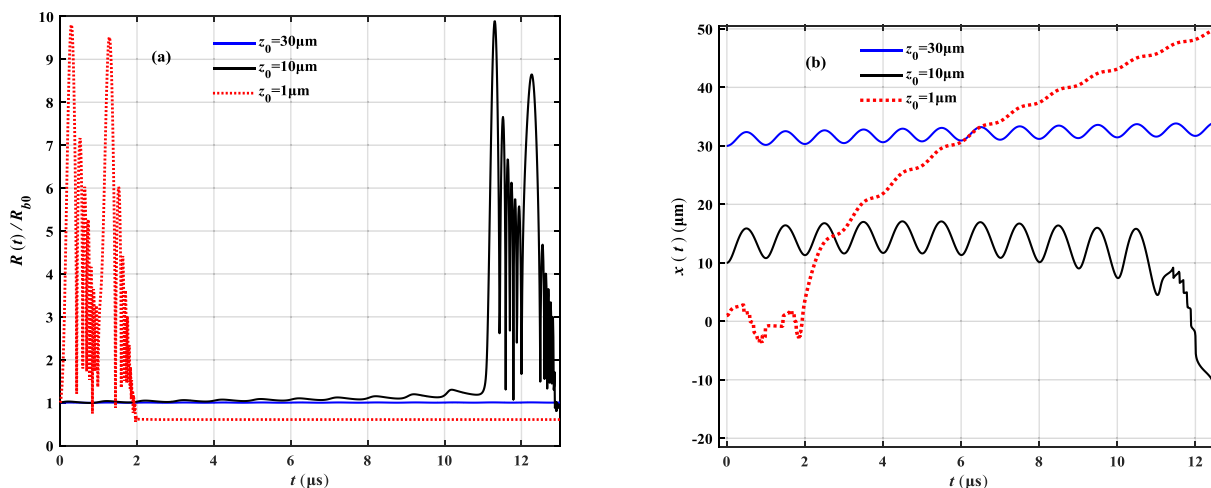
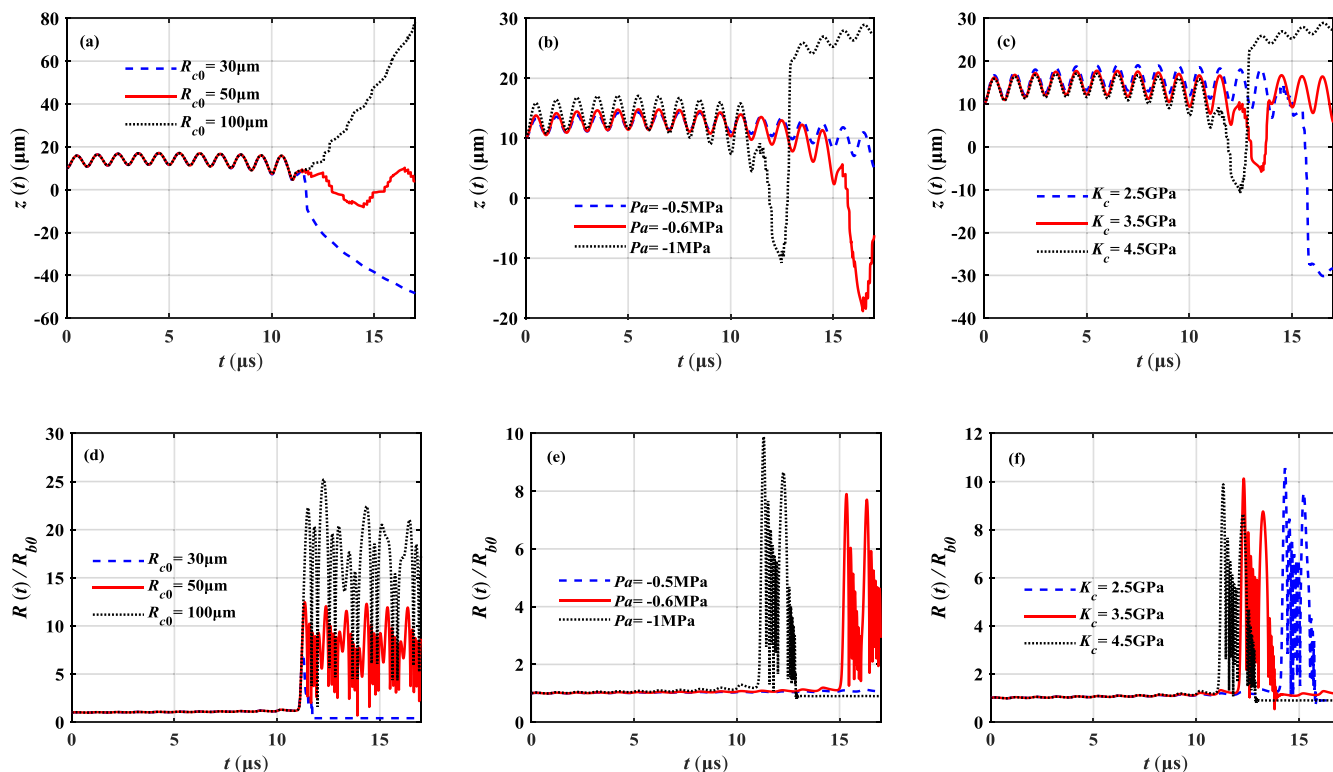


Fig. 3. Acoustic response of a confined bubble nucleus in a cavity for different relative positions of the bubble nucleus. (a) Pulsation motion. (b) Translational motion.



**Fig. 4.** Pulsation and translational motion of a confined bubble in a cavity: (a) and (d) cavity radius of 30, 40, and 100  $\mu\text{m}$  ( $P_a = -1$  MPa,  $K_c = 4.5$  GPa), (b) and (e) acoustic pressure amplitude of  $-0.1$ ,  $-1$ , and  $-1.5$  MPa ( $K_c = 4.5$  GPa,  $R_{c0} = 40$   $\mu\text{m}$ ), and (c) and (f) bulk modulus of 2.5, 3.5, and 4.5 GPa ( $P_a = -1$  MPa,  $R_{c0} = 40$   $\mu\text{m}$ ).

cavitation bubble collapsed after two acoustic cycles of unstable oscillation, and it then moved toward the cavity wall. This phenomenon is also different from the results predicted by Wang [25], which presented that a bubble with an initial radius of 5  $\mu\text{m}$  in a confinement was driven to oscillate in a stable way around its equilibrium radius by acoustic wave when  $P_a = 10$  kPa and  $f = 100$  kHz. Comparing the two curves for the case the initial position  $z_0 = 10$  and 30  $\mu\text{m}$  in Fig. 3(a), it is found that the maximum radius to which the nucleus can grow is almost independent of the initial position. However, the equilibrium radius of the grown cavitation bubble is not same. The nucleus near the center has a relative small equilibrium radius of its grown cavitation bubble. The equilibrium radius decreases periodically with time as it moves towards the center of cavity. The very small collapsed bubble might bounce back into the cavity after colliding with the inner wall, as shown in Fig. 3(b). We found that the closer the gas nucleus was to the cavity wall, the longer it took for it to grow into a cavitation bubble under the same acoustic field conditions, which might be related to the strong interaction between the bubble and the confined medium. When the gas nucleus or cavitation bubble was close to the inner cavity wall, the asymmetry response of the system was stimulated, and local deformation of the surrounding medium created additional pressure on the gas nucleus.

For cavitation in muscle tissue, the mechanical properties vary with the tissue type and composition [37]. To verify the confining effect of the surrounding medium on the cavitation nuclei, the bubble motion was numerically simulated for different cavity sizes, bulk modulus values, and driving pressure amplitudes (Fig. 4). By comparing the three cases of  $R_{c0} = 30$ , 50, and 100  $\mu\text{m}$ , it was found that for a gas nucleus at a certain distance from the center of a spherical cavity, the translational motion of the gas nucleus during growth to the cavitation size was independent of the size of the liquid cavity. When  $R_{c0} = 30$   $\mu\text{m}$ , the formed cavitation bubble collapsed after one or two acoustic cycles, whereas when  $R_{c0} = 50$  or 100  $\mu\text{m}$ , oscillation of the cavitation bubble lasted for

more acoustic cycles (Fig. 4(a) and (d)). There were two types of translational motion: reciprocal motion around the center of the cavity and motion toward the cavity wall. When a liquid cavity is exposed to negative pressure generated by ultrasound, the gas nuclei in the cavity can be excited as cavitation bubbles and behave differently owing to the different locations of the gas nuclei in the cavity. The cavitation bubbles in the small liquid cavity were mainly oscillating and translating around the center of the spherical cavity, while the cavitation bubbles in the large liquid cavity moved toward the cavity wall. In contrast, the size of the cavitation bubbles was larger in the large liquid cavity, which is in good agreement with the observations of Vincent et al. [28]. By increasing the driving pressure of the acoustic wave, the growth process of the gas nucleus to the cavitation size shortened and the amplitude of the oscillation is increased. (Fig. 4(b) and (e)). It should be mentioned that the bubble nucleus expands to cavitation size only when driving pressure exceeds the critical threshold [33]. When  $P_a > -0.5$  MPa, the bubble nucleus can grow to cavitation size and its equilibrium radius increases with the driving pressure amplitude of the acoustic wave, whereas when  $P_a < -0.5$  MPa, the growth of the bubble nucleus is completely suppressed by the confinement, as shown in Fig. 4 (e). The critical pressure is about  $-0.5$  MPa for a 0.5- $\mu\text{m}$  gas nucleus located in a 40- $\mu\text{m}$  cavity, and it is in good agreement with the predictions of Leonov et al. [36]. Similarly, for larger bulk modulus, the interaction between the gas nucleus and the surrounding medium was stronger and the translational displacement of the cavitation bubble during its nonlinear pulsation was larger (Fig. 4(c) and (f)).

The typical free energy corresponds to the energy barrier for nucleation. Vincent [34] modified the free energy of a fully confined bubble by using classical nucleation theory to investigate the effect of the liquid compressibility and solid elasticity of the confined medium. The total free energy  $F$  was given by



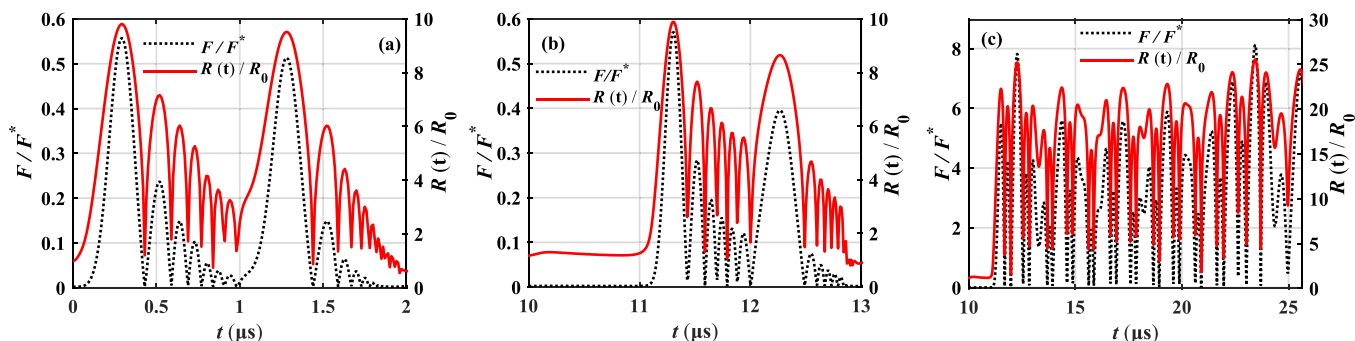


Fig. 5. Response of the total free energy to pulsation of the bubble: (a)  $z_0 = 1 \mu\text{m}$  and  $R_{c0} = 40 \mu\text{m}$ , (b)  $z_0 = 10 \mu\text{m}$  and  $R_{c0} = 40 \mu\text{m}$ , and (c)  $z_0 = 10 \mu\text{m}$  and  $R_{c0} = 100 \mu\text{m}$ .

$$F = \frac{4}{3}\pi R_b^3 \left( P_0 - P_v + \frac{K}{2} \left( \frac{R_b}{R_c} \right)^3 \right) + 4\pi R_b^2 \sigma_l - 4\pi R_{b0}^3 (P_0 - P_v) + \frac{2\sigma_l}{R_{b0}} \ln \left( \frac{R_b}{R_{b0}} \right), \quad (13)$$

where  $\frac{1}{\bar{K}} = \frac{1}{K_c} + \frac{1}{K_l}$  is the effective modulus. The potential energies of the liquid (and its vapor) and confinement elasticity, the bubble interface, and trapped gas contribute to the total energy. By ignoring the liquid compressibility,  $K \approx K_c$ . The total energy  $F$  varies with oscillation of the cavitation bubbles, and the acoustic energy is converted to other forms of energy. The dimensionless form of the free energy of the confined bubble ( $F/F^*$ ) is shown in Fig. 5, where  $F^* = 16\pi\sigma_l/[3(P_0 - P_v)^2]$  is the typical free energy [38].

Bubbles are the energy converter in the cavitation field. The energy-conversion ability of a bubble is determined by its oscillating amplitude, and the free energy reaches the maximum value when the bubble expands to the maximum radius. Fig. 5(a) and (b) show that the effect of the position of bubble nucleus on the free energy is weak, and the free energy in the liquid-cavity varies considerably during nonlinear pulsation. However, a change of the cavity radius has a great influence on the free energy. For smaller cavities, the constraint is stronger, bubble oscillation is weaker and smaller change in free energy. (Fig. 5(b) and (c)). In the largest micro-cavity, the bubbles gathered in the neighborhood of the inner wall [28], where a large bubble was accompanied by some small bubbles. The secondary radiation of the cavitation bubbles might provide new energy to intensify the cavitation inception process.

#### 4. Conclusion

A theoretical model has been proposed to describe the growth of gas nuclei into cavitation bubbles and their nonlinear behavior inside an elastic spherical liquid cavity. Two dynamics equations for the radial oscillation and translational movement of the bubble were derived by considering the cavitation nucleation process. The behavior of the confined gas nucleus driven by an acoustic wave was numerically analyzed. It was found that the onset of cavitation is significantly affected by the relative position of the gas nucleus in the cavity. The gas nucleus near the center of the spherical cavity reaches its new equilibrium radius earliest, and then the cavitation bubble nonlinearly pulsates and translates. The motion characteristics of the cavitation bubble are closely related to the initial position of the bubble nucleus. The equilibrium radius of the cavitation bubble is larger for larger cavities. However, the size of the cavity has little influence on the nucleation time. The nucleation process and movement are also dependent on the bulk modulus of the surrounding medium and the magnitude of the negative pressure. This study reveals that a gas nucleus can be excited to form a cavitation bubble by ultrasound, which provides a theoretical basis for elucidation of the mechanism of the cavitation effect in ultrasound therapy.

#### CRediT authorship contribution statement

**Xianmei Zhang:** Investigation, Conceptualization, Formal analysis, Software, Validation, Writing – original draft. **Fan Li:** Software, Supervision, Formal analysis, Methodology. **Chenghui Wang:** Investigation, Conceptualization, Formal analysis, Methodology, Supervision, Writing – review & editing. **Jianzhong Guo:** Methodology, Writing – review & editing, Resources. **Runyang Mo:** Methodology, Supervision, Writing – review & editing. **Jing Hu:** Formal analysis, Writing – review & editing. **Shi Chen:** Methodology, Writing – review & editing. **Jiaxin He:** Software, Writing – original draft. **Honghan Liu:** Software, Writing – original draft.

#### Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

#### Acknowledgement

This work was supported by the National Natural Science Foundation of China (Grant Nos. 11974232, 11727813 and 12074238, 11974232).

#### References

- [1] E. Herbert, S. Balibar, F. Caupin, Cavitation pressure in water, *Phys. Rev. E* 74 (2006) 1–22, <https://doi.org/10.1103/PhysRevE.74.041603>.
- [2] T. Temesgen, T.T. Bui, M. Han, T.-i. Kim, H. Park, Micro and nanobubble technologies as a new horizon for water-treatment techniques: A review, *Adv. Colloid Interface Sci.* 246 (2017) 40–51, <https://doi.org/10.1016/j.cis.2017.06.011>.
- [3] J. Rooze, E.V. Rebrov, J.C. Schouten, J.T.F. Keurentjes, Dissolved gas and ultrasonic cavitation – A review, *Ultrason. Sonochem.* 20 (1) (2013) 1–11, <https://doi.org/10.1016/j.ultsonch.2012.04.013>.
- [4] C.E. Brennen, Cavitation in medicine, *Interface Focus* 5 (2015) 1–12, <https://doi.org/10.1098/rsfs.2015.0022>.
- [5] M.O. de Andrade, S.R. Haqshenas, K.J. Pakh, N. Saffari, The effects of ultrasound pressure and temperature fields in millisecond bubble nucleation, *Ultrason. Sonochem.* 55 (2019) 262–272, <https://doi.org/10.1016/j.ultsonch.2019.01.019>.
- [6] V.A. Khokhlova, J.B. Fowlkes, W.W. Roberts, G.R. Schade, Z. Xu, T.D. Khokhlova, T.L. Hall, A.D. Maxwell, Y.-N. Wang, C.A. Cain, Histotripsy methods in mechanical disintegration of tissue: Towards clinical applications, *Int. J. Hypertherm.* 31 (2) (2015) 145–162, <https://doi.org/10.3109/02656736.2015.1007538>.
- [7] W.W. Roberts, Development and translation of histotripsy: Current status and future directions, *Curr. Opin. Urol.* 24 (2014) 104–110, <https://doi.org/10.1097/MOU.0000000000000001>.
- [8] T.A. Bigelow, C.L. Thomas, H. Wu, K.M.F. Itani, Histotripsy Treatment of S. Aureus Biofilms on Surgical Mesh Samples under Varying Scan Parameters, *IEEE Trans. Ultrason. Ferroelectr. Freq. Control.* 65 (2018) 1017–1024, <https://doi.org/10.1109/TUFFC.2018.2819363>.
- [9] > C.C. Coussios, C.H. Farny, G. Ter Haar, R.A. Roy, Role of acoustic cavitation in the delivery and monitoring of cancer treatment by high-intensity focused ultrasound (HIFU), *Int. J. Hypertherm.* 23 (2) (2007) 105–120, <https://doi.org/10.1080/02656730701194131>.

- [10] M. Dular, T. Požar, J. Zevnik, R. Petkovšek, High speed observation of damage created by a collapse of a single cavitation bubble, *Wear*. 418–419 (2019) 13–23, <https://doi.org/10.1016/j.wear.2018.11.004>.
- [11] W. Lauterborn, T. Kurz, Physics of bubble oscillations, *Rep. Prog. Phys.* 73 (10) (2010) 106501, <https://doi.org/10.1088/0034-4885/73/10/106501>.
- [12] J. Rodríguez-Rodríguez, A. Sevilla, C. Martínez-Bazán, J.M. Gordillo, Generation of microbubbles with applications to industry and medicine, *Annu. Rev. Fluid Mech.* 47 (1) (2015) 405–429, <https://doi.org/10.1146/annurev-fluid-010814-014658>.
- [13] R. Manasseh, A. Ooi, Frequencies of acoustically interacting bubbles, *Bubble Sci. Eng. Technol.* 1 (1–2) (2009) 58–74, <https://doi.org/10.1179/175889709X446552>.
- [14] S. Martynov, E. Stride, N. Saffari, The natural frequencies of microbubble oscillation in elastic vessels, *J. Acoust. Soc. Am.* 126 (6) (2009) 2963–2972, <https://doi.org/10.1121/1.3243292>.
- [15] C.H. Wang, J.C. Cheng, Nonlinear forced oscillations of gaseous bubbles in elastic microtubules, *Wuli Xuebao/Acta Phys. Sin.* 62 (2013) 1–9, <https://doi.org/10.7498/aps.62.114301>.
- [16] D. Marti, Y. Krüger, D. Fleitmann, M. Frenz, J. Riecka, The effect of surface tension on liquid-gas equilibria in isochoric systems and its application to fluid inclusions, *Fluid Phase Equilib.* 314 (2012) 13–21, <https://doi.org/10.1016/j.fluid.2011.08.010>.
- [17] D. Or, M. Tuller, Cavitation during desaturation of porous media under tension, *Water Resour. Res.* 38 (5) (2002) 19–19–14.
- [18] X. Noblin, N.O. Rojas, J. Westbrook, C. Llorens, M. Argentina, J. Dumais, The fern sporangium: A unique catapult, *Science* (80-) 335 (2012) 1322, <https://doi.org/10.1126/science.1215985>.
- [19] Cochard, Cavitation in trees, *Comptes Rendus Phys.* 7 (2006) 1018–1026, <https://doi.org/10.1016/j.crhy.2006.10.012>.
- [20] M.T. Tyree, M.A. Dixon, Cavitation Events in Thuja occidentalis L.? *Plant Physiol.* 72 (4) (1983) 1094–1099, <https://doi.org/10.1104/pp.72.4.1094>.
- [21] X. Yang, C.C. Church, A model for the dynamics of gas bubbles in soft tissue, *J. Acoust. Soc. Am.* 118 (6) (2005) 3595–3606, <https://doi.org/10.1121/1.2118307>.
- [22] C. Drysdale, A.A. Doinikov, P. Marmottant, Radiation dynamics of a cavitation bubble in a liquid-filled cavity surrounded by an elastic solid, *Phys. Rev. E* 95 (2017) 1–7, <https://doi.org/10.1103/PhysRevE.95.053104>.
- [23] A.A. Doinikov, P. Marmottant, Natural oscillations of a gas bubble in a liquid-filled cavity located in a viscoelastic medium, *J. Sound Vib.* 420 (2018) 61–72, <https://doi.org/10.1016/j.jsv.2018.01.034>.
- [24] A.A. Doinikov, B. Dollet, P. Marmottant, Model for the growth and the oscillation of a cavitation bubble in a spherical liquid-filled cavity enclosed in an elastic medium, *Phys. Rev. E* 97 (2018) 1–12, <https://doi.org/10.1103/PhysRevE.97.013108>.
- [25] Q.X. Wang, Oscillation of a bubble in a liquid confined in an elastic solid, *Phys. Fluids* 29 (7) (2017) 072101, <https://doi.org/10.1063/1.4990837>.
- [26] Z. Tao-Ran, M. Run-Yang, H. Jing, C. Shi, W. Cheng-Hui, G. Jian-Zhong, Dynamic analysis of bubble in liquid cavity wrapped by viscoelastic medium, *Acta Phys. Sin.* 70 (2021) 124301–124301, <https://doi.org/10.7498/aps.70.20201876>.
- [27] M.T. Warnez, E. Johnsen, Numerical modeling of bubble dynamics in viscoelastic media with relaxation, *Phys. Fluids* 27 (6) (2015) 063103, <https://doi.org/10.1063/1.4922598>.
- [28] O. Vincent, P. Marmottant, S.R. Gonzalez-Avila, K. Ando, C.D. Ohl, The fast dynamics of cavitation bubbles within water confined in elastic solids, *Soft Matter*. 10 (2014) 1455–1461, <https://doi.org/10.1039/c3sm52697f>.
- [29] O. Vincent, P. Marmottant, P.A. Quinto-Su, C.D. Ohl, Birth and growth of cavitation bubbles within water under tension confined in a simple synthetic tree, *Phys. Rev. Lett.* 108 (2012) 1–5, <https://doi.org/10.1103/PhysRevLett.108.184502>.
- [30] O. Vincent, D.A. Sessoms, E.J. Huber, J. Guioth, A.D. Stroock, Drying by cavitation and poroelastic relaxations in porous media with macroscopic pores connected by nanoscale throats, *Phys. Rev. Lett.* 113 (2014) 1–5, <https://doi.org/10.1103/PhysRevLett.113.134501>.
- [31] A.A. Doinikov, D. Bienaimé, S.R. Gonzalez-Avila, C.-D. Ohl, P. Marmottant, Nonlinear dynamics of two coupled bubbles oscillating inside a liquid-filled cavity surrounded by an elastic medium, *Phys. Rev. E* 99 (5) (2019), <https://doi.org/10.1103/PhysRevE.99.053106>.
- [32] X.M. Zhang, C.H. Wang, J.Z. Guo, R.Y. Mo, J. Hu, S. Chen, Dynamics of bubbles in a spherical liquid cavity wrapped by an elastic medium, *Acta Phys. Sin.* 70 (2021) 205–213, <https://doi.org/10.7498/aps.70.20210869>.
- [33] K. Leonov, I. Akhatov, Towards a theory of dynamics of a single cavitation bubble in a rigid micro-confinement, *Int. J. Multiph. Flow.* 130 (2020) 103369, <https://doi.org/10.1016/j.ijmultiphaseflow.2020.103369>.
- [34] O. Vincent, P. Marmottant, On the statics and dynamics of fully confined bubbles, *J. Fluid Mech.* 827 (2017) 194–224, <https://doi.org/10.1017/jfm.2017.487>.
- [35] J. Magnaudet, D. Legendre, The viscous drag force on a spherical bubble with a time-dependent radius, *Phys. Fluids* 10 (3) (1998) 550–554, <https://doi.org/10.1063/1.869582>.
- [36] K. Leonov, I. Akhatov, Dynamics of an externally driven cavitation bubble in an elastic microconfinement, *Phys. Rev. E* 104 (2021) 1–15, <https://doi.org/10.1103/PhysRevE.104.015105>.
- [37] S. Qin, K.W. Ferrara, A model for the dynamics of ultrasound contrast agents in soft tissue, *J. Acoust. Soc. Am.* 128 (2010) 1511, <https://doi.org/10.1121/1.3409476>.
- [38] C.E. Brennen, Cavitation and bubble dynamics, 2013. <https://doi.org/10.1017/CBO9781107338760>.