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Research article

Compromise optimum allocation in neutrosophic multi-character survey under stratified random sampling using neutrosophic fuzzy programming

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ABSTRACT

Survey sampling has wide range of applications in social and scientific investigation to draw inference about the unknown parameter of interest. In complex surveys, the sample information about the study variable cannot be expressed by a precise number under uncertain environment due fuzziness and indeterminacy. Therefore, this information is expressed by neutrosophic numbers rather than the classical numbers. The neutrosophic statistics, which is generalization of classical statistics, deals with the neutrosophic data that has some degree of indeterminacy and fuzziness. In this study, we investigate the compromise optimum allocation problem for estimating the population means of the neutrosophic study variables in a multi-character stratified random sampling under uncertain per unit measurement cost. We proposed the intuitionistic fuzzy cost function, modeling the fuzzy uncertainty in stratum per unit measurement cost. The compromise optimum allocation problem is formulated as a multi-objective intuitionistic fuzzy optimization problem. The solution methodology is suggested using neutrosophic fuzzy programming and intuitionistic fuzzy programming approaches. A numerical study includes the means estimation of atmospheric variables is presented to explore the real-life application, explain the mathematical formulation, and efficiency comparison with some existing methods. The results show that the suggested methods produce more precise estimates with less utilization of survey resources as compared to some existing methods. The Python is used for statistical analysis, graphical designing and numerical optimization problems are solved using GAMS.

1. Introduction

A stratified sampling design is commonly used to conduct sample surveys related to agriculture, health, markets, demographics, meteorological research, etc. and produce the most efficient results for the heterogeneous statistical population under study. But the sample allocation problem needs to be addressed properly before the implementation of this sampling design because it has a

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significant impact on the utilization of resources and the efficiency of the estimates. In stratified sampling design, the population is divided into homogeneous subgroups, called strata, and then the sample is selected from each stratum such that it maximizes the precision of the estimate under a cost/resource constraint or minimizes the utilization of the cost/resource of the survey to achieve predefined precision of the estimates. First, Neyman [22] proposed the solution to this problem by introducing the "optimum allocation" in which he minimized the objective function, the mean square error of the population mean, under the linear cost function using the Lagrange multiplier optimization technique. Sethi [27] developed an optimum stratification procedure for estimating population mean under optimum allocation. Clark and Steel [8] proposed optimum allocation for a two-stage stratified sampling design, introducing an additional constraint that may produce equal and proportional allocation as special cases of optimum allocation. The rounding rule is used to obtain an integer value of stratum sample size that violates the optimality criteria and sometimes produces an infeasible allocation that does not satisfy the cost constraint. Bretthauer et al. [6] presented two branch and bound procedures modeling the allocation problem as a mathematical programming problem with a convex objective function and solving it using dynamic programming techniques. When multi-variables are under study, an allocation that may be optimal for one characteristic will not be optimal for others until the characteristics are highly correlated and have the same variability. Sukhatme [29] suggested a compromise optimum allocation in multi-character sample surveys by minimizing the trace of mean square errors. Chatterjee [7] extended the optimum allocation procedure to propose the compromise allocation, which minimized the total relative loss in precision due to non-optimum allocation. Kokan and Khan [20] suggested the analytical procedure for compromise sample allocation in a multi-character survey. Cochran [9] suggested that the average of individual characteristics optimum allocation should be used as a compromise sample allocation in this case. Khan et al. [18] proposed weighted multi-objective optimization methods for compromise allocation under a linear cost constraint. Formulating the allocation problem as an integer multi-objective programming problem and matrix optimization problem, Dıaz-Garcıa and Cortez [10] and Díaz-García and Cortez [11] suggested the sample allocation using the value function method, lexicographic method, and distance methods, considering the available preference of each characteristic. Under probabilistic nonlinear cost function, Ghufran et al. [12] proposed the compromise allocation for estimating population mean by minimizing the sum of variance function of estimator for all characteristics under study. Ali et al. [3] used Chebyshev approximation, D_1 distance method, and goal programming method modeling the problem as an integer multi-objective optimization problem. Ullah et al. [32] proposed the compromise allocation by minimizing the mean square errors of ratio estimators under non-linear cost function using goal programming and weighted method.

Although classical optimization methods and procedures have been applied for years, as cited above, to find optimum or compromise allocation with well-defined coefficients of decision variables in objective functions and resource constraints. These deterministic techniques produce crisp values of precision of estimates or total cost, which is difficult and hard objective to achieve by sample surveys in an uncertain environment. Moreover, in most socio-economic, health, agricultural, and demographic surveys, the true values of stratum variance cannot be estimated precisely by sample data because of imprecision and ambiguity of data due to measurement error, incomplete information, subjective judgment, and fuzzy uncertainty in the response. This fuzzy uncertainty is usually described by the membership function, which reflects expert knowledge and preferences in decision-making about the precision of estimates. Fuzzy mathematical programming is used to model the sample allocation problems, incorporating the expert satisfaction degree related to precision by using the membership function. In recent years, some authors have used fuzzy optimization approaches to solve sample allocation problems in stratified sampling, taking into account subjectivity and considering the precision of estimates as a fuzzy number. Ullah et al. [34] used fuzzy geometric programming to find optimum allocation under the quadratic classical cost function for estimating the finite population mean in the presence of non-response. Considering the stratum variability, per unit measurement cost, travel cost, and survey budget as parabolic fuzzy numbers, Gupta and Bari [13] modeled the sample allocation problem as a multi-objective fuzzy optimization problem and found a compromise solution using fuzzy programming for a specified value of membership degree. The results showed that fuzzy programming produced efficient results as compared to a deterministic approach. Tariq et al. [31] used fuzzy geometric programming to solve the optimal allocation problem in two-phase multivariate stratified sampling under the classical linear cost function. Haq et al. [15], Ahmadini et al. [2], Khanam et al. [19] and Jalil et al. [16] applied fuzzy optimization methods to find integer compromise allocation for estimating population mean under classical linear and non linear cost functions, considering measurement unit cost, labor cost, and traveling cost in multivariate stratified sampling. Gupta et al. [14] and Raghav et al. [24] proposed intuitionistic fuzzy programming methods to solve the multi-objective integer optimum allocation problem, estimating the population mean of multiple variables under the study.

Some recent research work on compromise optimum allocation problem is summarized in Table 1 according to estimated parameters, cost functions, and fuzzy optimization methods suggested by different authors

1.1. Research gap

In previous research, the optimum allocation problem in multivariate stratified sampling was formulated as a multi-objective optimization problem. Different fuzzy optimization methods, considering the fuzzy uncertainty in the estimated classical parameter of interest, are used to find compromise allocation, as summarized in Table 1. The classical parameter summarizes the characteristic of the variable whose information is presented by a classical number measured on bivalent or classical propositional logic; that is, there is no uncertainty in the collected information from sample surveys. But in many real-life decision-making problems, the set of information about study variables, like market share prices, daily air temperature, humidity, wind speed, 24-hour blood pressure of a person, etc., is presented in interval form rather than a single determinate value, with some degree of uncertainty due to fussiness, imprecision, and indeterminacy. The neutrosophic statistics are used to summarize the characteristics of such variables by neutrosophic parameters. The significant research gaps are identified as follows:

Table 1
Research review summary.

Reference	Estimated Mean(s)		CF			Compromise Allocation	Remarks
	Classical	Neutrosophic	CL	F	IF		
Ullah et al. [34]	✓		✓			Fuzzy geometric programming	Nonresponse, Nonlinear CF
Gupta and Bari [13]	✓			✓		Fuzzy programming	Complete response, Nonlinear CF
Tariq et al. [31]	✓		✓			Fuzzy geometric programming	Double Sampling, Nonresponse, Nonlinear CF
Haq et al. [15]	✓		✓			Fuzzy goal programming	Complete response, Nonlinear CF
Ahmadini et al. [2]	✓		✓			Fuzzy goal programming, Lagrange multiplier method	Complete response, Probabilistic nonlinear CF
Khanam et al. [19]	✓		✓			Fuzzy programming	Complete response, Linear CF
Gupta et al. [14]	✓					Intuitionistic fuzzy programming	Complete response, Optimize MSE under fixed sample size
Raghav et al. [24]	✓		✓			Intuitionistic fuzzy programming	Complete response, Nonresponse, Nonlinear CF
Jalil et al. [16]	✓		✓			Hierarchical multi-level fuzzy programming	Nonresponse, Nonlinear CF
Proposed		✓			✓	Intuitionistic fuzzy programming, Neutrosophic fuzzy programming	Complete response, Triangular IF cost function

CF = cost function; CL = classical linear; F = Fuzzy; IF = intuitionistic fuzzy.

1. The compromise allocation for estimating neutrosophic population means in multi-character stratified random sampling needs to be addressed due to the emerging development and application of neutrosophic inferential statistics in different research domains.
2. Only a few authors like Gupta and Bari [13] considered the stratum per unit measurement cost as a fuzzy number and modeled it as a fuzzy cost function. The fuzzy cost function does not take into account expert's dissatisfaction degree, which can play an important role in the fuzzy decision-making approach for determining the per unit measurement cost in an uncertain environment.
3. The existing research methodologies for optimum allocation under fuzzy cost constraints are based on $\alpha - cut$ methods to produce solutions with crisp value. The accuracy function or rank function approach is unexplored in understanding and application to compromise allocation problems in applied surveys sampling.
4. Recent research has employed fuzzy programming or intuitionistic fuzzy programming models to find compromise allocations. The existence of an indeterminacy degree encounters some practical aspects of decision-making problems. There is a notable lack of research utilizing the neutrosophic fuzzy programming approach for optimum compromise allocation.

1.2. Aims and scope of the study

Neutrosophic parameter estimation is an important area of inferential statistics in which we estimate indeterminate or imprecise parameters of interest using neutrosophic data collected through sample surveys. The accuracy and precision of neutrosophic estimates are directly proportional to sample size and inversely proportional to the per-unit measurement cost in a sample survey. Therefore, flexible and correct modeling of the cost of a survey utilizing expert knowledge is important in terms of sample size determination. The best sample allocation procedure is economical, efficient, and increases the precision of estimates by utilizing the available resources at the optimum level. For estimation of the neutrosophic population mean, we have formulated the compromise allocation problem in a multivariate stratified sampling design as a multi-objective optimization problem under the intuitionistic fuzzy cost constraint in this article. Our proposed model has *2K* objective functions for the *K* neutrosophic study variable in a multi-character sample survey, while existing models deals with *K* classical variables. The major features of this study are listed as follows:

1. The objective of compromise allocation is to minimize the variance of the neutrosophic sample estimator of the mean in multivariate stratified sampling.
2. The fuzzy uncertainty in stratum per unit measurement cost decision-making is modeled as an intuitionistic fuzzy cost function. The ranking function is proposed to achieve an equivalent classical cost function.
3. The neutrosophic fuzzy programming and intuitionistic fuzzy programming-based solution methodologies are also discussed for integer values of sample size.
4. A numerical study is presented to explore the real-life application and feasibility of the proposed model for solving sample allocation problems in practical multi-character sample surveys for estimating imprecise parameter of interest.
5. The relative efficiency comparisons of the proposed models to existing techniques are given.

The fuzzy parameter estimation methods and sample size determination procedure in survey sampling have advantages over classical techniques due to modeling fuzzy uncertainty, imprecision, and the utilization of expert knowledge. Pandey et al. [23] applied

Table 2
Nomenclature.

Notations	Description
h	Index for strata; $\forall h = 1, 2, 3, \ldots H$.
M_h	Index for h^{th} stratum size
k	Index for characteristics; $\forall k = 1, 2, 3, \ldots K$.
m	Index for estimates; $\forall m = L, U$.
L	Index for lower.
U	Index for upper.
N	Index for neutrosophic.
i	Index for i^{th} unit information.

the fuzzy inference system for estimating potato crop parameters like plant height, area of leaf, etc. Rajabi and Ataie-Ashtiani [25] applied the fuzzy Bayesian inference algorithm for estimating the groundwater flow using expert knowledge. Aslam [5] used neutrosophic statistical techniques for analyzing radar data in the presence of fuzzy uncertainty. Tahir et al. [30] suggested the neutrosophic estimator of mean in simple random sampling and applied it to estimate the mean air temperature of Lahore city, Pakistan. Vishwakarma and Singh [35] investigated the neutrosophic estimator of a finite population mean in rank set sampling. Ullah et al. [33] developed the ratio-type estimator for estimating the imprecise population mean and suggested the sample size estimation procedure in simple random sampling under fuzzy uncertainty. The stratified sampling design produces more efficient results than simple random sampling for heterogeneous study population and is widely used in agricultural, marketing, spatial, biomedical, and meteorological studies. According to the authors best knowledge, this sampling design is not implemented to estimate imprecise parameters of neutrosophic study variables due to a lack of optimum allocation procedures for solving sample allocation problems and estimation methods. The current study will cover this gap. Additionally, expert knowledge is incorporated in modeling the cost of a survey and estimating its parameters. In Section 2, we discussed the neutrosophic estimator of mean, formulation of cost function under fuzzy uncertainty, mathematical modeling of the optimum allocation problem as an intuitionistic fuzzy multi-objective optimization problem and transformed it into an equivalent classical cost constraint multi-objective optimization problem. The solution methodology based on neutrosophic fuzzy programming, intuitionistic fuzzy programming and classical approach is explained in Section 3. Section 4 explains the computational procedure and application to estimate the average (mean) daily air temperature, daily humidity level, daily visibility, and daily wind speed. The comparative analysis of the proposed procedures with existing techniques is performed in Section 5. This study is concluded in Section 6.

2. Problem formulation

Let a multi-character sample survey be planned to collect information about variables of interest like temperature, humidity, wind speed, blood pressure, hemoglobin, cholesterol level, share prices, etc., linked with linguistic variables like low, normal/average, high, etc. The response to such variables is presented by a neutrosophic number because there is fuzzy uncertainty due to imprecision and indeterminacy in linguistic variables. As a result, the estimated parameter of the study variable will be a neutrosophic number presented in interval form. Smandrache introduced the neutrosophical sample statistic, which is used as an estimator to estimate indeterminate parameters of interest in survey sampling. To select a representative sample of size n from a heterogeneous population consisting of M units, we divide the population into non-overlapping homogeneous groups, called strata. The pre-determined sample size n is distributed among H strata to obtain maximum precision of estimates or minimize the mean square error of the neutrosophic estimator by utilizing resources under the given budget of the survey. We can model and solve this problem using multi-objective mathematical programming. The objective is to achieve maximum precision for each estimate of an indeterminate parameter under the cost constraint. The indices given in Table 2 are used in the mathematical formulation of the sample allocation problem in multivariate stratified sampling. Let $Y_{k_N} (k = 1, 2, 3, ..K)$ be the neutrosophic study variables observed from a population consists on M elements. The population is divided into H strata such that $\sum_{h=1}^{H} M_h = 1$. Let $y_{k_{hN_i}} \in [y_{k_{hL_i}}, y_{k_{hU_i}}]$ be the value of Y_{k_N} observed from i^{th} units in h^{th} stratum. Let $\bar{y}_{k_{hN}} \in [\bar{y}_{k_{hL}}, \bar{y}_{k_{hU}}]$ denote the sample mean of k^{th} neutrosophic characteristic in h^{th} stratum which is an estimate of $\bar{Y}_{k_{hN}} \in [\bar{Y}_{k_{hL}}, \bar{Y}_{k_{hU}}]$, the h^{th} stratum population mean of k^{th} characteristic. Let $S^2_{Y_{k_{hN}}} \in [S^2_{Y_{k_{hL}}}, S^2_{Y_{k_{hN}}}]$ be the h^{th} stratum population variance of k^{th} characteristic.

2.1. Objective function

The objective of the planning and execution of a multi-character sample survey under stratified sampling design is to estimate the unknown parameters of interest: population mean, variance, proportion, etc. The sample allocation to various strata and estimation methods significantly affect the accuracy and precision of sample estimates. Smarandache [28] suggested that the neutrosophic sample statistic that can be used as an estimator to estimate the neutrosophic population mean of the k^{th} characteristic in the h^{th} stratum is defined as:

$$\bar{y}_{k_{hN}} = \sum_{i=1}^{M_h} y_{k_{hN_i}}, \bar{y}_{k_{hN}} \in [\bar{y}_{k_{hL}}, \bar{y}_{k_{hU}}] \tag{1}$$

Now, the stratified sample neutrosophic $\bar{y}_{kstN}$ estimator of the population mean of k^{th} neutrosophic characteristics is defined as:

$$\bar{y}_{k_{stN}} = \sum_{h=1}^{H} W_h \bar{y}_{k_{hN}},$$

where $W_h = \frac{M_h}{M}$ and $\bar{y}_{k_{stN}} \in \left[\bar{y}_{k_{stL}}, \bar{y}_{k_{stU}}\right]$.
The $\bar{y}_{k_{stN}}$ is an unbiased estimator of $\bar{Y}_{k_{stN}}$ with variance $V_k\left(\bar{y}_{stN}\right)$, defined as:

$$V_k\left(\bar{y}_{stN}\right) = \sum_{h=1}^{H} \frac{W_h^2 S_{Y_{k_{hN}}}^2}{n_h} - \sum_{h=1}^{H} \frac{W_h^2 S_{Y_{k_{hN}}}^2}{N_h}, \tag{2}$$

$$V_k\left(\bar{y}_{stN}\right) \in \left[V_k\left(\bar{y}_{stL}\right), V_k\left(\bar{y}_{stU}\right)\right].$$

Ignoring the term independent form decision variable n_h in Eq. (2), $V_k\left(\bar{y}_{stN}\right)$ can be written as follows:

$$V_{k,N}\left(\bar{y}_{st}\right) = \sum_{h=1}^{H} \frac{W_h^2 S_{Y_{k_{hN}}}^2}{n_h}, \tag{3}$$

$$V_{k,N}\left(\bar{y}_{st}\right) \in \left[V_{k,L}\left(\bar{y}_{st}\right), V_{k,U}\left(\bar{y}_{st}\right)\right].$$

From Eq. (3), it is clear that the variability of the stratified sample estimates of neutrosophic population means is inversely proportional to the value of stratum sample size n_h. The best sample allocation methodology in multi-character surveys produces an optimum value of n_h that maximizes the precision of each neutrosophic estimate by minimizing the $V_{k,N}\left(\bar{y}_{st}\right)$ $(k = 1, 2, 3, ..., K)$ satisfying the survey budget and other restrictions. Each k^{th} objective contains two sub-objectives: $V_{k,L}\left(\bar{y}_{st}\right)$ and $V_{k,U}\left(\bar{y}_{st}\right)$. So, the total number of objectives functions that are to be minimized are $2K$ to estimate the neutrosophic population mean of K characteristics under the study.

2.2. Constraints

Two types of constraints are considered in this study. First, the utilization of the survey budget available for per-unit measurement cost. Second, the restrictions on stratum sample size.

Cost constraint

Considering the per-unit stratum measurement cost c_h, the fixed cost c_0, and the budget available for sample survey B, the classical cost function in stratified random sampling is defined as:

$$c_0 + \sum_{h=1}^{H} c_h n_h = B. \tag{4}$$

The value of the parameter c_h is decided based on the information available in past surveys, pilot studies, estimated values, etc. Mathematically, the value of c_h is selected from the set E_h that is defined on all possible values of c_h. E_h is defined as:

$$E_h = \left\{a_{h1}, a_{h2}, ..., a_{hi}, ..., a_{ho})\right\}.$$

The value of $c_h \in E_h$ decided by using the indicator function $\phi_{E_h(c_h)}$ is used in the classical cost function. The indicator function is defined as:

$$\phi_{E_h(c_h)} = \begin{cases} 1 & \text{if} \quad c_h = a_h \\ 0 & \quad \text{otherwise,} \end{cases}$$

where $a_h \in E_h$, and it can be an average value or any particular value from the set E_h. In the classical approach, the average value is used as an estimate of parameter c_h in an uncertain environment. The selected value of c_h by this indicator function may be either true or false, i.e., $\phi_{E(c_h)} = \{0, 1\}$. The classical approach to deciding the value of c_h is not more practical in a non-random uncertain environment because the expert's knowledge and the degree of true value are more important than the existence of true value. The fuzzy decision-making approach used to select the value of the per-unit measurement cost is more flexible and is a generalization of the classical approach. Under this approach, the per element h^{th} stratum measurement cost is determined by the true membership function $\mu_{\tilde{c}}^{i}(a_h)$: $E_h \to [0, 1]$, instead of the indicator function. Although the fuzzy approach utilizes the expert's knowledge with a satisfaction degree, it ignores the dissatisfaction or false membership degree, which is equally important in survey sampling. Therefore, intuitionistic fuzzy logic is used to decide the value of c_h. In this approach, the c_h is an intuitionistic fuzzy number denoted by $\tilde{c}_h^{IF}$, defined as follows:

$$\tilde{c}_h^{IF} = \left\{ \left(a_h, \mu_{\tilde{c}}^i(a_h), \nu_{\tilde{c}}^i(a_h) \right) : a_h \in E_h \right\},$$

where $\mu_{\tilde{c}}^i(a_h)$ is the true membership degree, and $\nu_{\tilde{c}}^i(a_h)$ is the false membership degree such that $\mu_{\tilde{c}}^i(a_h) + \nu_{\tilde{c}}^i(a_h) = 1$.

The intuitionistic fuzzy cost function in stratified random sampling is defined as follows:

$$c_0 + \sum_{h=1}^{H} \tilde{c}_h^{IF} n_h = B. \tag{5}$$

This cost function is linear subject to the decision variable n_h.

Bounded decision variable

The decision variable n_h is bounded by a minimum value and a maximum limit. The stratum size M_h is the maximum limit to avoid the possibility of over-sampling. A minimum of two units from each stratum must be selected for estimating stratum variability and comparative analysis among strata. Mathematically, $2 \le n_h \le M_h$.

2.3. Mathematical model

The compromise allocation for estimating the neutrosophic population mean in a multivariate stratified random sampling scheme can be formulated as a multi-objective optimization problem with an intuitionistic fuzzy cost constraint as follows:

$$\left.\begin{aligned} \text{Minimize } V_{k,m}\left(\bar{y}_{st}\right) \\ \text{Subject to} \\ c_0 + \sum_{h=1}^{H} \tilde{c}_h^{IF} n_h = B \\ 2 \le n_h \le M_h \\ n_h (h = 1, 2, ..., H) \text{ are an integers} \\ k = 1, 2, 3, .., K \text{ and } m = L, U. \end{aligned}\right\} \tag{6}$$

Let $\tilde{c}_h^{IF} = (a_1, a_2, a_3)$ be the linear triangular intuitionistic fuzzy per-unit measurement cost and, is defined by $\mu_{\tilde{c}_h^{IF}}(c_h) : \mathbb{R}^+ \to [0.5, 1]$ and $\nu_{\tilde{c}_h^{IF}}(c_h) : \mathbb{R}^+ \to [0.5, 1]$, given as follows:

$$\mu_{\tilde{c}_h^{IF}}(c_h) = \begin{cases} \mu_l = \frac{c_h - a_1}{a_2 - a_1}, & a_1 \le c_h < a_2 \\ 1, & c_h = a_2 \\ \mu_u = \frac{a_3 - c_h}{a_3 - a_2}, & a_2 < c_h \le a_3 \\ 0, & \text{otherwise.} \end{cases}$$

$$\nu_{\tilde{c}_h^{IF}}(c_h) = \begin{cases} \nu_l = \frac{a_2 - c_h}{a_2 - a_1}, & a_1 \le c_h < a_2 \\ 0, & c_h = a_2 \\ \nu_u = \frac{c_h - a_2}{a_3 - a_2}, & a_2 < c_h \le a_3 \\ 0, & \text{otherwise.} \end{cases}$$

Where $\mu_l : \mathbb{R}^+ \to [0.5, 1]$, $\mu_u : \mathbb{R}^+ \to [0.5, 1]$, $\nu_l : \mathbb{R}^+ \to [0.5, 1]$ and $\nu_u : \mathbb{R}^+ \to [0.5, 1]$. Let $\mu^{-1}_{\tilde{c}_h^{IF}}(z)$ and $\nu^{-1}_{\tilde{c}_h^{IF}}(z)$ be the inverse of $\mu_{\tilde{c}_h^{IF}}(c)$ and $\nu_{\tilde{c}_h^{IF}}(c)$, respectively; defined as:

$$\mu^{-1}_{\tilde{c}_h^{IF}}(z_h) = \begin{cases} \mu_l^{-1} = a_1 + z_h(a_2 - a_1), & 0.5 \le z_h \le 1 \\ \mu_u^{-1} = a_3 + z_h(a_2 - a_3), & 0.5 \le z_h \le 1 \end{cases}$$

$$\nu^{-1}_{\tilde{c}_h^{IF}}(z_h) = \begin{cases} \nu_l^{-1} = a_2 + z_h(a_1 - a_2), & 0.5 \le z_h \le 1 \\ \nu_u^{-1} = a_2 + z_h(a_3 - a_2), & 0.5 \le z_h \le 1 \end{cases}$$

Now, the centroid point $(A(\tilde{c}_h^{IF}), Z(\tilde{c}_h^{IF}))$ of triangular intuitionistic fuzzy per-unit measurement cost $\tilde{c}_h^{IF}$ is calculated as follows:

$$A_\mu(\tilde{c}_h^{IF}) = \frac{\int_{a_1}^{a_2} c_h \mu_l dc_h + \int_{a_2}^{a_3} c_h \mu_u dc_h}{\int_{a_1}^{a_2} \mu_l dc_h + \int_{a_2}^{a_3} \mu_u dc},$$

$$A_\mu(\tilde{c}_h^{IF}) = \frac{\int_{a_1}^{a_2} \frac{c_h^2 - a_1 c_h}{a_2 - a_1} dc_h + \int_{a_2}^{a_3} \frac{a_3 c_h - c_h^2}{a_3 - a_2} dc_h}{\int_{a_1}^{a_2} \frac{c_h - a_1}{a_2 - a_1} dc_h + \int_{a_2}^{a_3} \frac{a_3 - c_h}{a_3 - a_2} dc_h},$$

$$A_{\mu}(\tilde{c}_h^{IF}) = \frac{a_1 + a_2 + a_3}{3}. \tag{7}$$

$$A_{\nu}(\tilde{c}_h^{IF}) = \frac{\int_{a_1}^{a_2} c_h \nu_l dc_h + \int_{a_2}^{a_3} c_h \nu_u dc_h}{\int_{a_1}^{a_2} \nu_l dc_h + \int_{a_2}^{a_3} \nu_u dc_h},$$

$$A_{\nu}(\tilde{c}_h^{IF}) = \frac{\int_{a_1}^{a_2} \frac{a_2 c_h - c_h^2}{a_2 - a_1} dc_h + \int_{a_2}^{a_3} \frac{c_h^2 - a_2 c_h}{a_3 - a_2} dc_h}{\int_{a_1}^{a_2} \frac{a_2 - c_h}{a_2 - a_1} dc_h + \int_{a_2}^{a_3} \frac{c_h - a_2}{a_3 - a_2} dc_h},$$

$$A_{\nu}(\tilde{c}_h^{IF}) = \frac{2a_1 - a_2 + 2a_3}{3}. \tag{8}$$

$$Z_{\mu}(\tilde{c}_h^{IF}) = \frac{\int_{0.5}^{1} z_h \mu_u^{-1} dz_h - \int_{0.5}^{1} z_h \mu_l^{-1} dz_h}{\int_{0.5}^{1} \mu_u^{-1} dz_h - \int_{0.5}^{1} \mu_l^{-1} dz_h},$$

$$Z_{\mu}(\tilde{c}_h^{IF}) = \frac{\int_{0.5}^{1} (a_3 z_h + (a_2 - a_3) z_h^2) dz_h - \int_{0.5}^{1} (a_1 z_h + (a_2 - a_1) z_h^2) dz_h}{\int_{0.5}^{1} (a_3 + (a_2 - a_3) z_h) dz_h - \int_{0.5}^{1} (a_1 + (a_2 - a_1) z_h) dz_h},$$

$$Z_{\mu}(\tilde{c}_h^{IF}) = \frac{2}{3}. \tag{9}$$

$$Z_{\nu}(\tilde{c}_h^{IF}) = \frac{\int_{0.5}^{1} z_h \nu_u^{-1} dz_h - \int_{0.5}^{1} z_h \nu_l^{-1} dz_h}{\int_{0.5}^{1} \nu_u^{-1} dz_h - \int_{0.5}^{1} \nu_l^{-1} dz_h},$$

$$Z_{\nu}(\tilde{c}_h^{IF}) = \frac{\int_{0.5}^{1} (a_2 z_h + (a_3 - a_2) z_h^2) dz_h - \int_{0.5}^{1} (a_2 z v + (a_1 - a_2) z_h^2) dz_h}{\int_{0.5}^{1} (a_2 + (a_3 - a_2) z_h) dz_h - \int_{0.5}^{1} (a_2 + (a_1 - a_2) z_h) dz_h},$$

$$Z_{\nu}(\tilde{c}_h^{IF}) = \frac{7}{9}. \tag{10}$$

The rank function $d(\tilde{c}_h^{IF})$ for crisp value of triangular intuitionistic fuzzy per-unit measurement cost $\tilde{c}_h^{IF}$ is defined as:

$$d(\tilde{c}_h^{IF}) = \sqrt{\frac{1}{2}\left[\left(A_{\mu}(\tilde{c}_h^{IF}) - Z_{\mu}(\tilde{c}_h^{IF})\right)^2 + \left(A_{\nu}(\tilde{c}_h^{IF}) - Z_{\nu}(\tilde{c}_h^{IF})\right)^2\right]}.$$

Using the results given in Eqs. (7)-(10), we get

$$d(\tilde{c}_h^{IF}) = \sqrt{\frac{1}{2}\left[\left(\frac{a_1 + a_2 + a_3}{3} - \frac{2}{3}\right)^2 + \left(\frac{2a_1 - a_2 + 2a_3}{3} - \frac{7}{9}\right)^2\right]}. \tag{11}$$

Using the ranking function given in Eq. (11), the intuitionistic fuzzy cost constrain multi-objective optimization problem defined in Eq. (6) can be transformed into a classical multi-objective optimization problem as follows:

$$\left.\begin{aligned} \text{Minimize}\left(\left[V_{1,L}\left(\bar{y}_{st}\right), V_{1,U}\left(\bar{y}_{st}\right)\right], \ldots, \left[V_{k,L}\left(\bar{y}_{st}\right), V_{k,U}\left(\bar{y}_{st}\right)\right], \ldots, \left[V_{K,L}\left(\bar{y}_{st}\right), V_{K,U}\left(\bar{y}_{st}\right)\right]\right), \\ \text{subject to} \\ c_o + \sum_{h=1}^{H} d(\tilde{c}_h^{IF}) n_h \le B, \\ 2 \le n_h \le M_h, \\ n_h (h = 1, 2, ..., H) \text{ are an integers.} \end{aligned}\right\} \tag{12}$$

3. Solution methodology

The fuzzy approach-based methods are proposed to solve the compromise optimum allocation problem formulated in Eq. (12). The classical methods are also presented to solve the formulated compromise allocation problem for the efficiency comparison of fuzzy approaches.

3.1. Neutrosophic fuzzy programming method

Multi-objective decision-making models are the most commonly applicable mathematical models to achieve specified goals that have a conflicting nature. The compromise solution that optimizes all conflicting objective functions simultaneously in the model is to achieve this in such situations. The selection of multi-objective optimization methods for a compromise solution depends on the availability of information about preferences and the nature of the targeted goals. The neutrosophic optimization techniques are applied to find a compromise solution for many real-life multi-objective decision-making problems when the targeted goals are

Table 3
Payoff matrix.

Objectives	$V_{1,L}\left(\bar{y}_{st}\right)$	$V_{1,U}\left(\bar{y}_{st}\right)$	…	$V_{k,L}\left(\bar{y}_{st}\right)$	$V_{k,U}\left(\bar{y}_{st}\right)$	…	$V_{K,L}\left(\bar{y}_{st}\right)$	$V_{K,U}\left(\bar{y}_{st}\right)$
$n_{h(1,L)}$	$V_{1,L}^{1,L}\left(\bar{y}_{st}\right)$	$V_{1,U}^{1,L}\left(\bar{y}_{st}\right)$	…	$V_{k,L}^{1,L}\left(\bar{y}_{st}\right)$	$V_{k,U}^{1,L}\left(\bar{y}_{st}\right)$	…	$V_{K,L}^{1,L}\left(\bar{y}_{st}\right)$	$V_{K,U}^{1,L}\left(\bar{y}_{st}\right)$
$n_{h(1,U)}$	$V_{1,L}^{1,U}\left(\bar{y}_{st}\right)$	$V_{1,U}^{1,U}\left(\bar{y}_{st}\right)$	…	$V_{k,L}^{1,U}\left(\bar{y}_{st}\right)$	$V_{k,U}^{1,U}\left(\bar{y}_{st}\right)$	…	$V_{K,L}^{1,U}\left(\bar{y}_{st}\right)$	$V_{K,U}^{1,U}\left(\bar{y}_{st}\right)$
⋮	⋮	⋮		⋮	⋮		⋮	⋮
$n_{h(k,L)}$	$V_{1,L}^{q,L}\left(\bar{y}_{st}\right)$	$V_{1,U}^{q,L}\left(\bar{y}_{st}\right)$	…	$V_{k,L}^{q,L}\left(\bar{y}_{st}\right)$	$V_{k,U}^{q,L}\left(\bar{y}_{st}\right)$	…	$V_{K,L}^{q,L}\left(\bar{y}_{st}\right)$	$V_{K,U}^{q,L}\left(\bar{y}_{st}\right)$
$n_{h(k,U)}$	$V_{1,L}^{q,U}\left(\bar{y}_{st}\right)$	$V_{1,U}^{q,U}\left(\bar{y}_{st}\right)$	…	$V_{k,L}^{q,U}\left(\bar{y}_{st}\right)$	$V_{k,U}^{q,U}\left(\bar{y}_{st}\right)$	…	$V_{K,L}^{q,U}\left(\bar{y}_{st}\right)$	$V_{K,U}^{1,U}\left(\bar{y}_{st}\right)$
⋮	⋮	⋮		⋮	⋮		⋮	⋮
$n_{h(K,L)}$	$V_{1,L}^{K,L}\left(\bar{y}_{st}\right)$	$V_{1,U}^{K,L}\left(\bar{y}_{st}\right)$	…	$V_{k,L}^{K,L}\left(\bar{y}_{st}\right)$	$V_{k,U}^{K,L}\left(\bar{y}_{st}\right)$	…	$V_{K,L}^{K,L}\left(\bar{y}_{st}\right)$	$V_{K,U}^{K,L}\left(\bar{y}_{st}\right)$
$n_{h(K,U)}$	$V_{1,L}^{K,U}\left(\bar{y}_{st}\right)$	$V_{1,U}^{K,U}\left(\bar{y}_{st}\right)$	…	$V_{k,L}^{K,U}\left(\bar{y}_{st}\right)$	$V_{k,U}^{K,U}\left(\bar{y}_{st}\right)$	…	$V_{K,L}^{K,U}\left(\bar{y}_{st}\right)$	$V_{K,U}^{1,U}\left(\bar{y}_{st}\right)$

specified as neutrosophic numbers. The existence of an indeterminacy degree encounters some practical aspects of decision-making problems in applied survey sampling and other areas of research. Rani and Mishra [26] proposed a neutrosophic programming model for the compromise selection of a recycling partner for electronics equipment sustainable waste. Khan et al. [17] solved the industrial production problem to maximize the net profit under resource and system constraints using a neutrosophic programming approach. Ahmad [1] suggested a neutrosophic optimization method for a compromise solution to intuitionistic multi-objective pricing and managerial problems.

In multi-objective compromise sample allocation problem formulated in Eq. (12), we have to achieve a minimum level of $V_{k,m}\left(\bar{y}_{st}\right),\ k=1,2,\ldots,K$ and $m=L,U$ under the set of constraints. The optimum level of each targeted gaol is impossible to obtain in a set of multiple conflicting objectives satisfying the recourse constraint in sample surveys. The objective function $V_{k,m}\left(\bar{y}_{st}\right)$ can assume value under compromise allocation within an interval obtained by optimum allocation. For determining the lower and upper bounds of each objective, we solved each objective function individually to find the optimum allocation, ignoring the other objectives. Let $n_{h(k,m)}$ be the optimum allocation that minimizes the objective $V_{k,m}\left(\bar{y}_{st}\right)$. The trade-off among *2K* objectives is given in Table 3.

The domains of the objective functions are determined by individual optimum allocation. Using Table 3, the lower bound $(L_{k,m})$ and upper bound $(U_{k,m})$ for the k^{th} objective are defined as:

$$L_{k,m} = \text{Minimize } V_{k,m}^{q,r}\left(\bar{y}_{st}\right), k=1,2,3,\ldots,K,\ m=L,U,\ q=1,2,3,\ldots,K \text{ and } r=L,U.$$

$$U_{k,m} = \text{Max } V_{k,m}^{q,r}\left(\bar{y}_{st}\right), k=1,2,3,\ldots,K,\ m=L,U,\ q=1,2,3,\ldots,K \text{ and } r=L,U.$$

Let A be a feasible solution set, the neutrosophic decision set $\tilde{A}_{k,m}\in A$ is characterized by truth membership function $\alpha_{\tilde{A}_{k,m}}:\mathbb{R}^+\to[0,1]$, falsity membership function $\beta_{\tilde{A}_{k,m}}:\mathbb{R}^+\to[0,1]$ and indeterminacy membership function $\theta_{\tilde{A}_{k,m}}:\mathbb{R}^+\to[0,1]$, written as:

$$\tilde{A}_{k,m}(x)=\left(x,\alpha_{\tilde{A}_{k,m}}(x),\theta_{\tilde{A}_{k,m}}(x),\beta_{\tilde{A}_{k,m}}(x)\right).$$

The domains of these three membership functions of $V_{k,m}\left(\bar{y}_{st}\right)$ using $L_{k,m}$ and $U_{k,m}$ are defined as:

- For truth membership function: $L_{k,m}^{T}=L_{k,m},\ U_{k,m}^{T}=U_{k,m}$.
- For indeterminate membership function: $L_{k,m}^{I}=L_{k,m},\ U_{k,m}^{I}=L_{k,m}+s_{k,m}(U_{k,m}-L_{k,m})$.
- For false membership function: $L_{k,m}^{F}=L_{k,m}+t_{k,m}(U_{k,m}-L_{k,m}),\ U_{k,m}^{F}=U_{k,m}$,

where $s_{k,m}\in[0,1]$ and $t_{k,m}\in[0,1]$ are tolerance variables whose values are chosen by decision makers according to their preferences. The membership functions used in the neutrosophic compromise allocation model are defined as follows:

$$\alpha_{\tilde{A}_{k,m}}=\begin{cases}1, & V_{k,m}\left(\bar{y}_{stk}\right)<L_{k,m}^{T}\\ \frac{U_{k,m}^{T}-V_{k,m}\left(\bar{y}_{st}\right)}{U_{k,m}^{T}-L_{k,m}^{T}}, & V_{k,m}\left(\bar{y}_{st}\right)\in\left[L_{k,m}^{T},U_{k,m}^{T}\right]\\ 0, & V_{k,m}\left(\bar{y}_{st}\right)>U_{k,m}^{T}.\end{cases}$$

$$\theta_{\tilde{A}_{k,m}}=\begin{cases}0, & V_{k,m}\left(\bar{y}_{st}\right)<L_{k,m}^{I}\\ \frac{U_{k,m}^{I}-V_{k,m}\left(\bar{y}_{st}\right)}{U_{k,m}^{I}-L_{k,m}^{I}}, & V_{k,m}\left(\bar{y}_{st}\right)\in\left[L_{k,m}^{I},U_{k,m}^{I}\right]\\ 0, & V_{k,m}\left(\bar{y}_{st}\right)>U_{k,m}^{I}.\end{cases}$$

$$\beta_{\tilde{A}_{k,m}} = \begin{cases} 0, & V_{k,m}\left(\bar{y}_{st}\right) < L^{F}_{k,m} \\ \frac{V_{k,m}\left(\bar{y}_{st}\right) - L^{F}_{k,m}}{U^{F}_{k,m} - L^{F}_{k,m}}, & V_{k,m}\left[\bar{y}_{st}\right] \in \left[L^{F}_{k,m}, U^{F}_{k,m}\right] \\ 1, & V_{k,m}\left(\bar{y}_{st}\right) > U^{F}_{k,m}. \end{cases}$$

For the compromise solution of the problem formulated in Eq. (12), the neutrosophic multi-objective mathematical programming model is formulated as:

$$\left.\begin{aligned} &\text{Maximize } \alpha_{\tilde{A}_{k,m}}, \text{ Maximize } \theta_{\tilde{A}_{k,m}}, \text{ Minimize } \beta_{\tilde{A}_{k,m}}, \\ &\text{subject to} \\ &c_o + \sum_{h=1}^{H} d(\tilde{c}_h^{IF}) n_h \le B, \\ &2 \le n_h \le M_h, \\ &\alpha_{\tilde{A}_{k,m}} \ge \theta_{\tilde{A}_{k,m}} \text{ and } \alpha_{\tilde{A}_{k,m}} \ge \beta_{\tilde{A}_{k,m}}, \\ &\alpha_{\tilde{A}_{k,m}} + \theta_{\tilde{A}_{k,m}} + \beta_{\tilde{A}_{k,m}} \le 3, \\ &\alpha_{\tilde{A}_{k,m}}, \theta_{\tilde{A}_{k,m}} \text{ and } \beta_{\tilde{A}_{k,m}} \in [0,1], \forall m = L, U \text{ and } k = 1,2,\ldots K, \\ &n_h (h = 1,2,\ldots,H) \text{ are an integers.} \end{aligned}\right\} \tag{13}$$

Using the auxiliary variates $\alpha_{k,m}$, $\theta_{k,m}$, and $\beta_{k,m}$, the single objective neutrosophic fuzzy optimization model equivalent to model formulated in Eq. (13) is defined as:

$$\left.\begin{aligned} &\text{Maximize} \sum_{k=1}^{K} \sum_{m=L}^{U} \left(\alpha_{k,m} + \theta_{k,m} - \beta_{k,m}\right), \\ &\text{subject to} \\ &U^{T}_{k,m} - \left(U^{T}_{k,m} - L^{T}_{k,m}\right) \alpha_{k,m} \ge V_{k,m}\left(\bar{y}_{st}\right), \\ &U^{I}_{k,m} - \left(U^{I}_{k,m} - L^{I}_{k,m}\right) \theta_{k,m} \ge V_{k,m}\left(\bar{y}_{st}\right), \\ &L^{F}_{k,m} + \left(U^{F}_{k,m} - L^{F}_{k,m}\right) \beta_{k,m} \ge V_{k,m}\left(\bar{y}_{st}\right), \\ &c_o + \sum_{h=1}^{H} d(\tilde{c}_h^{IF}) n_h \le B, \\ &2 \le n_h \le M_h, \\ &\alpha_{k,m} \le \alpha_{\tilde{A}_{k,m}}, \theta_{k,m} \le \theta_{\tilde{A}_{k,m}} \text{ and } \beta_{k,m} \ge \beta_{\tilde{A}_{k,m}}, \\ &\alpha_{k,m} \ge \theta_{k,m}, \alpha_{k,m} \ge \beta_{k,m} \text{ and } \alpha_{k,m} + \theta_{k,m} + \beta_{k,m} \le 3, \\ &\alpha_{k,m}, \theta_{k,m} \text{ and } \beta_{k,m} \in [0,1], \forall m = L, U \text{ and } k = 1,2,\ldots K, \\ &n_h (h = 1,2,\ldots,H) \text{ are an integers.} \end{aligned}\right\} \tag{14}$$

3.2. Intuitionistic fuzzy programming method

Angelov [4] extended the traditional fuzzy optimization method to an intuitionistic fuzzy (IF) decision-making approach by incorporating the degree of dissatisfaction (non-membership) of the objectives and the constraints together with the degree of stratification (membership). Wan and Li [36] applied the *IF* programming method to solve the selection problem of green suppliers under a set of constraints. Mahajan and Gupta [21] proposed *IF* optimistic, pessimistic, and mixed approaches for finding compromise solutions and applied them to transportation and production problems. Gupta et al. [14] and Raghav et al. [24] suggested compromise sample allocation procedures using the *IF* programming approach for estimating multivariate population means in stratified random sampling under a deterministic cost function.

The problem formulated in Eq. (12) is transformed as a *IF* multi-objective optimization problem for compromise allocation in multivariate neutrosophic stratified random sampling, given as follows:

$$\left.\begin{aligned}\text{Maximize } \alpha_{\tilde{A}_{k,m}}, \text{Minimize } \beta_{\tilde{A}_{k,m}}, \\ \text{subject to} \\ c_o + \sum_{h=1}^{H} d(\tilde{c}_h^{IF}) n_h \leq B, \\ 2 \leq n_h \leq M_h, \\ \alpha_{\tilde{A}_{k,m}} \geq \beta_{\tilde{A}_{k,m}}, \\ \alpha_{\tilde{A}_{k,m}} + \beta_{\tilde{A}_{k,m}} \leq 1, \\ \alpha_{\tilde{A}_{k,m}} \text{ and } \beta_{\tilde{A}_{k,m}} \in [0,1], m = L, U \text{ and } k = 1,2,\ldots K, \\ n_h (h = 1,2,\ldots,H) \text{ are an integers.}\end{aligned}\right\} \tag{15}$$

Using the auxiliary variates $\alpha_{k,m}$ and $\beta_{k,m}$, the single objective intuitionistic fuzzy programming model, equivalent to the model formulated in Eq. (15), is defined as:

$$\left.\begin{aligned}\text{Maximize} \sum_{k=1}^{K} \sum_{m=L}^{U} \left(\alpha_{k,m} - \beta_{k,m}\right), \\ \text{subject to} \\ U_{k,m}^{T} - \left(U_{k,m}^{T} - L_{k,m}^{T}\right) \alpha_{k,m} \geq V_{k,m}\left(\bar{y}_{st}\right), \\ L_{k,m}^{F} + \left(U_{k,m}^{F} - L_{k,m}^{F}\right) \beta_{k,m} \geq V_{k,m}\left(\bar{y}_{st}\right), \\ c_o + \sum_{h=1}^{H} d(\tilde{c}_h^{IF}) n_h \leq B, \\ 2 \leq n_h \leq M_h, \\ \alpha_{k,m} \leq \alpha_{\tilde{A}_{k,m}} \text{ and } \beta_{k,m} \geq \beta_{\tilde{A}_{k,m}}, \\ \alpha_{k,m} \geq \beta_{k,m} \text{ and } \alpha_{k,m} + \beta_{k,m} \leq 1, \\ \alpha_{k,m} \text{ and } \beta_{k,m} \in [0,1], m = L, U \text{ and } k = 1,2,\ldots K, \\ n_h (h = 1,2,\ldots,H) \text{ are an integers.}\end{aligned}\right\} \tag{16}$$

3.3. Classical methods

The compromise solution to the problem (12) can be obtained using classical procedures suggested by Cochran [9], Sukhatme [29], and Khan et al. [18].

1. Sukhatme [29] proposed the compromise allocation by minimizing the trace of the mean square errors of an estimator in multivariate stratified random sampling. The compromise allocation using the Sukhtame's Method is defined as:

$$\left.\begin{aligned}\text{Minimize} \sum_{k=1}^{K} \sum_{m=L}^{U} V_{k,m}\left(\bar{y}_{st}\right), \\ \text{subject to} \\ c_o + \sum_{h=1}^{H} d(\tilde{c}_h^{IF}) n_h \leq B, \\ 2 \leq n_h \leq M_h, \\ n_h (h = 1,2,\ldots,H) \text{ are an integers.}\end{aligned}\right\} \tag{17}$$

2. The mathematical model for compromise allocation using Khan et al. [18] weighted method is formulated as:

$$\left.\begin{array}{r}\text{Minimize } \sum_{k=1}^{K}\sum_{m=L}^{U} W_{k,m}V_{k,m}\left(\bar{y}_{st}\right), \\ \text{subject to} \\ c_o+\sum_{h=1}^{H} d(\tilde{c}_h^{IF})n_h \leq B, \\ 2 \leq n_h \leq M_h, \\ \sum_{k=1}^{K}\sum_{m=L}^{U} W_{k,m}=1, \\ n_h(h=1,2,...,H) \text{ are an integers.}\end{array}\right\} \tag{18}$$

Where $W_{k,m}:(k=1,2,3,..,K;m=L,U)$ are relative weights defined as:

$$W_{k,m}=\frac{S^2_{Y_{k,m}}}{\sum_{k=1}^{K}\sum_{m=L}^{U} S^2_{Y_{k,m}}}.$$

3. Cochran [9] suggested the compromise allocation as an average of individual optimum allocations in multi-character surveys. Using the individual optimum allocation $n_{h_{k,m}}(k=1,2,3,...,K)$, the compromise allocation under Cochran's approach is defined as:

$$n_h=\frac{1}{2K}\sum_{k=1}^{K}\sum_{m=L}^{U} n_{h_{(k,m)}}.$$

4. Numerical study

Researchers have been studying changes in atmospheric climate variables over the last few decades. The survey sampling techniques are used to collect the data for estimating the population mean and other parameters of these variables, presented in interval form. For example, daily temperature, humidity, visibility, and wind speed fluctuate within a minimum and maximum value and are presented in interval form like [min, max]. This interval-valued data is linked with linguistic variables such as low, normal, or high, which have fuzzy meanings with some degree of indeterminacy. Therefore, neutrosophic statistical techniques are best suited to analyze the fuzzy data of these atmospheric climate variables, which have some degree of indeterminacy.

We performed the numerical study based on the daily temperature, daily humidity, daily visibility, and daily wind speed data of Qassim city, Saudi Arabia, for the year 2018. We stratified the date into four strata according to seasonal variation. The description of the study variables and their mathematical presentation are given in Table 4. The statistical summary is given in Table 5. We assumed that $\tilde{c}_1^{IF}=(5,6,8)$, $\tilde{c}_2^{IF}=(4,8,12)$, and $\tilde{c}_3^{IF}=(3,5,8)$, $\tilde{c}_h^{IF}=(4,6,10)$, $c_o=30$, and $B=250$. Using the ranking function defined in Eq. (11), the intuitionistic fuzzy measurement costs $\tilde{c}_h^{IF}(h=1,2,3,4)$ are transformed into classical numbers $d(\tilde{c}_h^{IF})(h=1,2,3,4)=(5.78,7.28,4.78,6.48)$, respectively.

Table 4
Variables description and their symbolic presentation.

Variables description	Notations				
	Population	Winter: h = 1	Spring: h = 2	Summer: h = 3	Autumn: h = 4
Daily humidity percentage	Y_{1_N}	$Y_{1_{1N}}$	$Y_{1_{2N}}$	$Y_{1_{3N}}$	$Y_{1_{4N}}$
Daily lower humidity percentage	Y_{1_L}	$Y_{1_{1L}}$	$Y_{1_{2L}}$	$Y_{1_{3L}}$	$Y_{1_{4L}}$
Daily upper humidity percentage	Y_{1_U}	$Y_{1_{1U}}$	$Y_{1_{2U}}$	$Y_{1_{3U}}$	$Y_{1_{4U}}$
Daily temperature	Y_{2_N}	$Y_{2_{1N}}$	$Y_{2_{2N}}$	$Y_{2_{3N}}$	$Y_{2_{4N}}$
Daily lower temperature	Y_{2_L}	$Y_{2_{1L}}$	$Y_{2_{2L}}$	$Y_{2_{3L}}$	$Y_{2_{4L}}$
Daily upper temperature	Y_{2_U}	$Y_{2_{1U}}$	$Y_{2_{2U}}$	$Y_{2_{3U}}$	$Y_{2_{4U}}$
Daily visibility	Y_{3_N}	$Y_{3_{1N}}$	$Y_{3_{2N}}$	$Y_{3_{3N}}$	$Y_{3_{4N}}$
Daily lower visibility	Y_{3_L}	$Y_{3_{1L}}$	$Y_{3_{2L}}$	$Y_{3_{3L}}$	$Y_{3_{4L}}$
Daily upper visibility	Y_{3_U}	$Y_{3_{1U}}$	$Y_{3_{2U}}$	$Y_{3_{3U}}$	$Y_{3_{4U}}$
Daily wind speed	Y_{4_N}	$Y_{4_{1N}}$	$Y_{4_{2N}}$	$Y_{4_{3N}}$	$Y_{4_{4N}}$
Daily lower wind speed	Y_{4_L}	$Y_{4_{1L}}$	$Y_{4_{2L}}$	$Y_{4_{3L}}$	$Y_{4_{4L}}$
Daily upper wind speed	Y_{4_U}	$Y_{4_{1U}}$	$Y_{4_{2U}}$	$Y_{4_{3U}}$	$Y_{4_{4U}}$

The objective is to obtain the most precise estimate of the mean of these atmospheric study variables. How many sampling units (days) be selected from each stratum (season) to make the sample representative and achieve this objective for each study variable at optimum utilization resources. This problem is formulated as a multi-objective minimization problem as follows:

Table 5
Statistical summary of climates variables data.

h	$\bar{Y}_{1_{hN}}$	$\bar{Y}_{2_{hN}}$	$\bar{Y}_{3_{hN}}$	$\bar{Y}_{4_{hN}}$	$\sigma^2_{Y_{1_{hN}}}$	$\sigma^2_{Y_{2_{hN}}}$
1	[41.50 , 78.63]	[10.31 , 22.69]	[3.92 , 15.06]	[3.31 , 20.80]	[296.77 , 309.25]	[15.21 , 16.82]
2	[26.01 , 60.80]	[20.79 , 33.96]	[2.60 , 15.91]	[4.62 , 28.55]	[174.69 , 479.61]	[15.79 , 21.32]
3	[14.65 , 34.72]	[28.52 , 43.47]	[11.83 , 16.00]	[3.64 , 27.32]	[3.53 , 67.13]	[5.57 , 4.34]
4	[39.08 , 63.87]	[20.34 , 32.84]	[4.46 , 14.31]	[2.97 , 22.16]	[751.49 , 845.76]	[29.20 , 79.16
h	$\sigma^2_{Y_{3_{hN}}}$	$\sigma^2_{Y_{4_{hN}}}$	M_h	j	$\bar{Y}_{j_N}$	$\sigma^2_{Y_{j_N}}$
1	[53.31 , 10.52]	[10.98 , 51.82]	90	1	[30.22 , 59.39]	[418.58 , 670.51]
2	[26.75 , 244.70]	[15.43 , 103.13]	92	2	[20.04 , 33.30]	[57.87 , 84.00]
3	[45.86 , 0.00]	[11.84 , 96.09]	92	3	[5.72 , 15.32]	[59.31 , 67.76]
4	[61.47 , 15.06]	[12.52 , 44.41]	92	4	[3.64 , 24.74]	[12.94 , 84.07]

Table 6
Optimum allocation for individual objectives.

n_h	$V_{1,L}(\bar{y}_{st})$	$V_{1,U}(\bar{y}_{st})$	$V_{2,L}(\bar{y}_{st})$	$V_{2,U}(\bar{y}_{st})$	$V_{3,L}(\bar{y}_{st})$	$V_{3,U}(\bar{y}_{st})$	$V_{4,L}(\bar{y}_{st})$	$V_{4,U}(\bar{y}_{st})$
n_1	11	10	10	9	11	6	9	9
n_2	8	10	9	9	7	23	11	11
n_3	3	5	7	4	12	2	11	14
n_4	18	15	14	18	12	6	10	8

$$\left.\begin{aligned}
&\text{Minimize}\\
V_{1,L}(\bar{y}_{st}) &= \frac{18.0434}{n_1}+\frac{11.0985}{n_2}+\frac{0.2240}{n_3}+\frac{46.7114}{n_4}\\
V_{1,U}(\bar{y}_{st}) &= \frac{18.8020}{n_1}+\frac{30.4704}{n_2}+\frac{4.2648}{n_3}+\frac{52.5708}{n_4}\\
V_{2,L}(\bar{y}_{st}) &= \frac{0.9245}{n_1}+\frac{1.0033}{n_2}+\frac{0.3539}{n_3}+\frac{1.8153}{n_4}\\
V_{2,U}(\bar{y}_{st}) &= \frac{1.0229}{n_1}+\frac{1.3543}{n_2}+\frac{0.2757}{n_3}+\frac{4.9205}{n_4}\\
V_{3,L}(\bar{y}_{st}) &= \frac{3.2411}{n_1}+\frac{1.6994}{n_2}+\frac{2.9135}{n_3}+\frac{3.8211}{n_4}\\
V_{3,U}(\bar{y}_{st}) &= \frac{0.6399}{n_1}+\frac{15.5459}{n_2}+\frac{0.0000}{n_3}+\frac{0.9361}{n_4}\\
V_{4,L}(\bar{y}_{st}) &= \frac{0.6676}{n_1}+\frac{0.9800}{n_2}+\frac{0.7520}{n_3}+\frac{0.7783}{n_4}\\
V_{4,U}(\bar{y}_{st}) &= \frac{3.1509}{n_1}+\frac{6.5519}{n_2}+\frac{6.1045}{n_3}+\frac{2.7602}{n_4}\\
&\text{subject to}\\
&30+5.78n_1+7.27n_2+4.78n_3+6.48n_4 \le 250\\
&2\le n_1\le 90, 2\le n_2\le 92, 2\le n_3\le 92, 2\le n_4\le 91\\
&n_1\ n_2,\ n_3 \text{ and } n_4 \text{ are an integers.}
\end{aligned}\right\} \tag{19}$$

The compromise optimum value of the bounded decision variables $n_h(h=1,2,3,4)$ will produce the required objectives. For this, we found the optimum allocation of each objective individually, which is presented in Table 6. The trade-off among these objectives under the optimum allocation is summarized in Table 7. For using the neutrosophic fuzzy (NF) programming and intuitionistic fuzzy (IF) programming approaches, we specified the domain of each objective function for the compromise optimum allocation, given in Table 8.

The true membership functions $\alpha_{\tilde{A}_{k,m}} : k=1,2,3,4\ \&\ m=L,U$, false membership functions $\beta_{\tilde{A}_{k,m}} : k=1,2,3,4\ \&\ m=L,U$ and indeterminate membership functions $\theta_{\tilde{A}_{k,m}} : k=1,2,3,4\ \&\ m=L,U$ are designed graphically which are presented in Fig. 1.

4.1. Neutrosophic fuzzy programming method

The compromise optimum solution for the problem formulated in Eq. (19) can be obtained by solving the following neutrosophic fuzzy optimization model using mixed integer non-linear programming technique.

$$\left.\begin{array}{r}
\text{Maximize } \displaystyle\sum_{k=1}^{4}\sum_{m=L}^{U}(\alpha_{k,m}+\theta_{k,m}-\beta_{k,m}) \\
\text{subject to} \\
\frac{18.0434}{n_1}+\frac{11.0985}{n_2}+\frac{0.2240}{n_3}+\frac{46.7114}{n_4}\le 11.3870-5.6896\alpha_{1,L} \\
\frac{18.8020}{n_1}+\frac{30.4704}{n_2}+\frac{4.2648}{n_3}+\frac{52.5708}{n_4}\le 15.3527-6.0677\alpha_{1,U} \\
\frac{0.9245}{n_1}+\frac{1.0033}{n_2}+\frac{0.3539}{n_3}+\frac{1.8153}{n_4}\le 0.6772-0.2931\alpha_{2,L} \\
\frac{1.0229}{n_1}+\frac{1.3543}{n_2}+\frac{0.2757}{n_3}+\frac{4.9205}{n_4}\le 1.1873-0.5809\alpha_{2,U} \\
\frac{3.2411}{n_1}+\frac{1.6994}{n_2}+\frac{2.9135}{n_3}+\frac{3.8211}{n_4}\le 2.7077-1.6090\alpha_{3,L} \\
\frac{0.6399}{n_1}+\frac{15.5459}{n_2}+\frac{0.0000}{n_3}+\frac{0.9361}{n_4}\le 2.3570-1.4184\alpha_{3,U} \\
\frac{0.6676}{n_1}+\frac{0.9800}{n_2}+\frac{0.7520}{n_3}+\frac{0.7783}{n_4}\le 0.6596-0.3501\alpha_{4,L} \\
\frac{3.1509}{n_1}+\frac{6.5519}{n_2}+\frac{6.1045}{n_3}+\frac{2.7602}{n_4}\le 4.3223-2.5955\alpha_{4,U} \\
\frac{18.0434}{n_1}+\frac{11.0985}{n_2}+\frac{0.2240}{n_3}+\frac{46.7114}{n_4}\le 10.24908-4.55168\theta_{1,L} \\
\frac{18.8020}{n_1}+\frac{30.4704}{n_2}+\frac{4.2648}{n_3}+\frac{52.5708}{n_4}\le 14.13914-4.854240\theta_{1,U} \\
\frac{0.9245}{n_1}+\frac{1.0033}{n_2}+\frac{0.3539}{n_3}+\frac{1.8153}{n_4}\le 0.61858-0.234480\theta_{2,L} \\
\frac{1.0229}{n_1}+\frac{1.3543}{n_2}+\frac{0.2757}{n_3}+\frac{4.9205}{n_4}\le 1.07112-0.464720\theta_{2,U} \\
\frac{3.2411}{n_1}+\frac{1.6994}{n_2}+\frac{2.9135}{n_3}+\frac{3.8211}{n_4}\le 2.38588-1.287280\theta_{3,L} \\
\frac{0.6399}{n_1}+\frac{15.5459}{n_2}+\frac{0.0000}{n_3}+\frac{0.9361}{n_4}\le 2.07332-1.134720\theta_{3,U} \\
\frac{0.6676}{n_1}+\frac{0.9800}{n_2}+\frac{0.7520}{n_3}+\frac{0.7783}{n_4}\le 0.58958-0.280080\theta_{4,L} \\
\frac{3.1509}{n_1}+\frac{6.5519}{n_2}+\frac{6.1045}{n_3}+\frac{2.7602}{n_4}\le 3.8032-2.0764\theta_{4,U} \\
\frac{18.0434}{n_1}+\frac{11.0985}{n_2}+\frac{0.2240}{n_3}+\frac{46.7114}{n_4}\le 8.5422+2.8448\beta_{1,L} \\
\frac{18.8020}{n_1}+\frac{30.4704}{n_2}+\frac{4.2648}{n_3}+\frac{52.5708}{n_4}\le 12.3188+3.0339\beta_{1,U} \\
\frac{0.9245}{n_1}+\frac{1.0033}{n_2}+\frac{0.3539}{n_3}+\frac{1.8153}{n_4}\le 0.53065+0.1466\beta_{2,L} \\
\frac{1.0229}{n_1}+\frac{1.3543}{n_2}+\frac{0.2757}{n_3}+\frac{4.9205}{n_4}\le 0.89685+0.2905\beta_{2,U} \\
\frac{3.2411}{n_1}+\frac{1.6994}{n_2}+\frac{2.9135}{n_3}+\frac{3.8211}{n_4}\le .1.90315+0.8046\beta_{3,L} \\
\frac{0.6399}{n_1}+\frac{15.5459}{n_2}+\frac{0.0000}{n_3}+\frac{0.9361}{n_4}\le 1.6478+0.7092\beta_{3,U} \\
\frac{0.6676}{n_1}+\frac{0.9800}{n_2}+\frac{0.7520}{n_3}+\frac{0.7783}{n_4}\le 0.48455+0.1751\beta_{4,L} \\
\frac{3.1509}{n_1}+\frac{6.5519}{n_2}+\frac{6.1045}{n_3}+\frac{2.7602}{n_4}\le 3.02455+1.2978\beta_{4,U} \\
\text{and constraints given in Eq. (21)}
\end{array}\right\} \tag{20}$$

Table 7
Payoff matrix.

Optimum allocation	Objectives function values							
	$V_{1,L}(\bar{y}_{st})$	$V_{1,U}(\bar{y}_{st})$	$V_{2,L}(\bar{y}_{st})$	$V_{2,U}(\bar{y}_{st})$	$V_{3,L}(\bar{y}_{st})$	$V_{3,U}(\bar{y}_{st})$	$V_{4,L}(\bar{y}_{st})$	$V_{4,U}(\bar{y}_{st})$
$n_{h_{1,L}}$	5.6974	9.8603	0.4283	0.6275	1.6905	2.0534	0.4771	3.2936
$n_{h_{1,U}}$	6.0731	9.2849	0.3846	0.6209	1.3315	1.6810	0.3670	2.3752
$n_{h_{2,L}}$	6.4060	9.6301	0.3841	0.6436	1.2021	1.8582	0.3387	2.1123
$n_{h_{2,U}}$	5.8891	9.4615	0.4035	0.6064	1.4896	1.8504	0.4143	2.7576
$n_{h_{3,L}}$	7.1371	10.7985	0.4081	0.7195	1.0986	2.3570	0.3282	1.9612
$n_{h_{3,U}}$	11.3870	15.3527	0.6772	1.1873	2.7077	0.9386	0.6596	4.3223
$n_{h_{4,L}}$	7.7053	10.5039	0.4076	0.7539	1.1616	1.5780	0.3095	1.7767
$n_{h_{4,U}}$	8.8687	11.7351	0.4461	0.8715	1.2004	1.6014	0.3143	1.7268

Table 8
Domain of membership functions of $V_{k,m}(\bar{y}_{st})$.

Limits	$V_{1,L}(\bar{y}_{st})$	$V_{1,U}(\bar{y}_{st})$	$V_{2,L}(\bar{y}_{st})$	$V_{2,U}(\bar{y}_{st})$	$V_{3,L}(\bar{y}_{st})$	$V_{3,U}(\bar{y}_{st})$	$V_{4,L}(\bar{y}_{st})$	$V_{4,U}(\bar{y}_{st})$
L^T	5.6974	9.2849	0.3841	0.6064	1.0986	0.9386	0.3095	1.7268
U^T	11.3870	15.3527	0.6772	1.1873	2.7077	2.3570	0.6596	4.3223
L^F	8.5422	12.3188	0.5307	0.8969	1.9032	1.6478	0.4846	3.0246
U^F	11.3870	15.3527	0.6772	1.1873	2.7077	2.3570	0.6596	4.3223
L^I	5.6974	9.2849	0.3841	0.6064	1.0986	0.9386	0.3095	1.7268
U^I	10.2491	14.1391	0.6186	1.0711	2.3859	2.0733	0.5896	3.8032

$$\left.\begin{aligned} 30+5.78n_1+7.27n_2+4.78N_3+6.48N_4 \le 250 \\ 2\le n_1\le 90, 2\le n_2\le 92, 2\le n_3\le 92, 2\le n_4\le 91 \\ \alpha_{k,m}\le \alpha_{\tilde{A}_{k,m}}, \theta_{k,m}\le \theta_{\tilde{A}_{k,m}}, \beta_{k,m}\ge \beta_{\tilde{A}_{k,m}}, \alpha_{k,m}, \theta_{k,m}, \beta_{k,m}\in[0,1] \\ \alpha_{k,m}\ge\theta_{k,m}, \alpha_{k,m}\ge\beta_{k,m}, \alpha_{k,m}+\theta_{k,m}+\beta_{k,m}=1; k=1,2,3,4; m=L,U \\ n_1\ n_2,\ n_3 \text{ and } n_4 \text{ are an integers.} \end{aligned}\right\} \tag{21}$$

The compromise optimum solution is $n_1=9,\ n_2=11,\ n_3=7,\ n_4=13$ and cost is 249.69.

4.2. Intuitionistic fuzzy programming method

The compromise optimum solution for the problem formulated in Eq. (19) can be obtained by solving the following intuitionistic fuzzy optimization model using mixed integer non-linear programming technique.

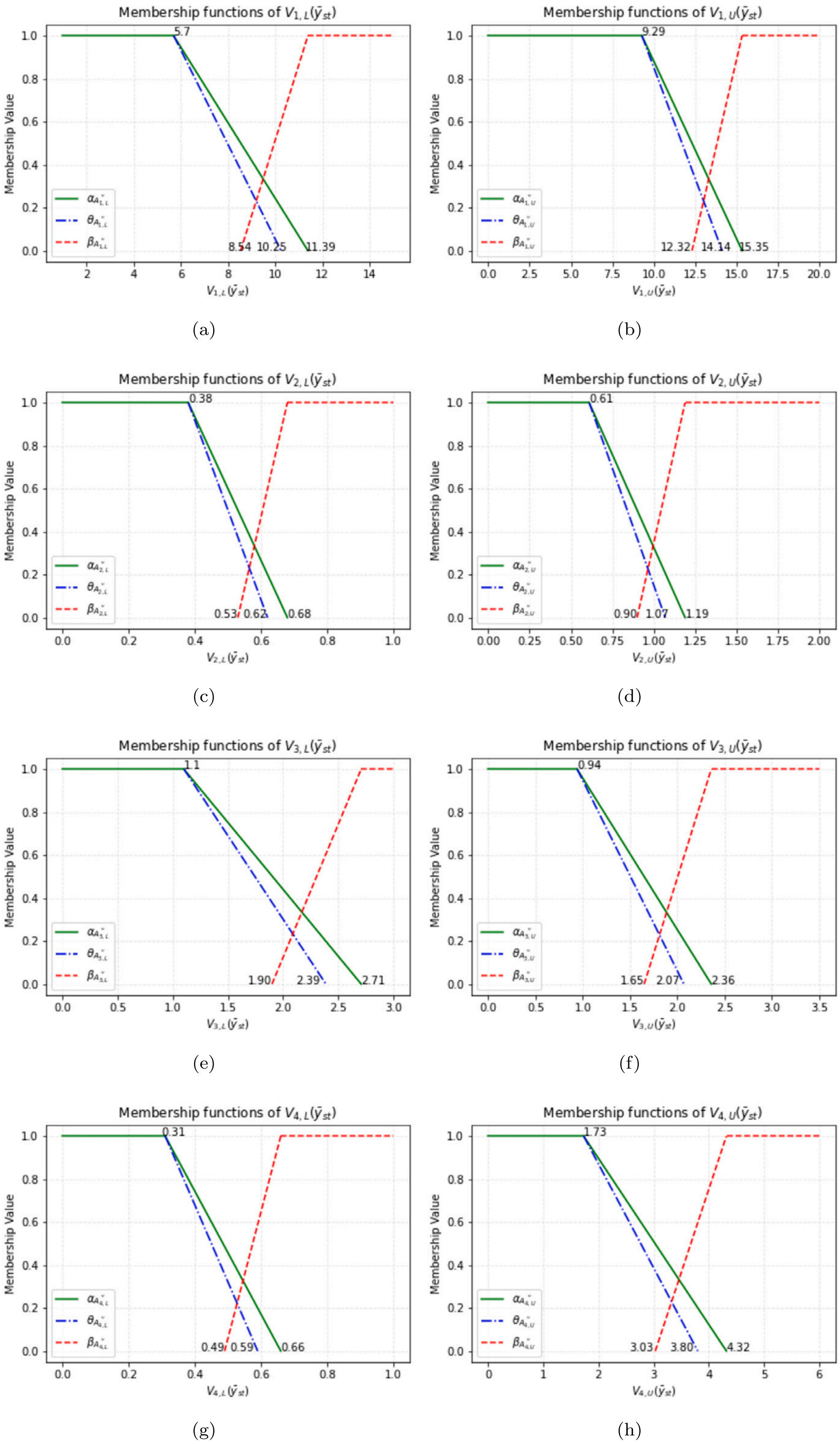


Fig. 1. Membership functions.

$$
\left.\begin{array}{r}
\text{Maximize } \sum_{k=1}^{4}\sum_{m=L}^{U}\left(\alpha_{k,m}-\beta_{k,m}\right) \\
\text{subject to} \\
\frac{18.0434}{n_1}+\frac{11.0985}{n_2}+\frac{0.2240}{n_3}+\frac{46.7114}{n_4}\le 11.3870-5.6896\alpha_{1,L} \\
\frac{18.8020}{n_1}+\frac{30.4704}{n_2}+\frac{4.2648}{n_3}+\frac{52.5708}{n_4}\le 15.3527-6.0677\alpha_{1,U} \\
\frac{0.9245}{n_1}+\frac{1.0033}{n_2}+\frac{0.3539}{n_3}+\frac{1.8153}{n_4}\le 0.6772-0.2931\alpha_{2,L} \\
\frac{1.0229}{n_1}+\frac{1.3543}{n_2}+\frac{0.2757}{n_3}+\frac{4.9205}{n_4}\le 1.1873-0.5809\alpha_{2,U} \\
\frac{3.2411}{n_1}+\frac{1.6994}{n_2}+\frac{2.9135}{n_3}+\frac{3.8211}{n_4}\le 2.7077-1.6090\alpha_{3,L} \\
\frac{0.6399}{n_1}+\frac{15.5459}{n_2}+\frac{0.0000}{n_3}+\frac{0.9361}{n_4}\le 2.3570-1.4184\alpha_{3,U} \\
\frac{0.6676}{n_1}+\frac{0.9800}{n_2}+\frac{0.7520}{n_3}+\frac{0.7783}{n_4}\le 0.6596-0.3501\alpha_{4,L} \\
\frac{3.1509}{n_1}+\frac{6.5519}{n_2}+\frac{6.1045}{n_3}+\frac{2.7602}{n_4}\le 4.3223-2.5955\alpha_{4,U} \\
\frac{18.0434}{n_1}+\frac{11.0985}{n_2}+\frac{0.2240}{n_3}+\frac{46.7114}{n_4}\le 8.5422+2.8448\beta_{1,L} \\
\frac{18.8020}{n_1}+\frac{30.4704}{n_2}+\frac{4.2648}{n_3}+\frac{52.5708}{n_4}\le 12.3188+3.0339\beta_{1,U} \\
\frac{0.9245}{n_1}+\frac{1.0033}{n_2}+\frac{0.3539}{n_3}+\frac{1.8153}{n_4}\le 0.53065+0.1466\beta_{2,L} \\
\frac{1.0229}{n_1}+\frac{1.3543}{n_2}+\frac{0.2757}{n_3}+\frac{4.9205}{n_4}\le 0.89685+0.2905\beta_{2,U} \\
\frac{3.2411}{n_1}+\frac{1.6994}{n_2}+\frac{2.9135}{n_3}+\frac{3.8211}{n_4}\le .1.90315+0.8046\beta_{3,L} \\
\frac{0.6399}{n_1}+\frac{15.5459}{n_2}+\frac{0.0000}{n_3}+\frac{0.9361}{n_4}\le 1.6478+0.7092\beta_{3,U} \\
\frac{0.6676}{n_1}+\frac{0.9800}{n_2}+\frac{0.7520}{n_3}+\frac{0.7783}{n_4}\le 0.48455+0.1751\beta_{4,L} \\
\frac{3.1509}{n_1}+\frac{6.5519}{n_2}+\frac{6.1045}{n_3}+\frac{2.7602}{n_4}\le 3.02455+1.2978\beta_{4,U} \\
30+5.78n_1+7.27n_2+4.78N_3+6.48N_4\le 250 \\
2\le n_1\le 90, 2\le n_2\le 92, 2\le n_3\le 92, 2\le n_4\le 91 \\
\alpha_{k,m}\le \alpha_{\tilde{A}_{k,m}}, \beta_{k,m}\ge \beta_{\tilde{A}_{k,m}}, \alpha_{k,m}\ge \beta_{k,m} \\
\alpha_{k,m}+\beta_{k,m}=1, \alpha_{\tilde{A}_{k,m}}, \beta_{\tilde{A}_{k,m}}\in[0,1]; k=1,2,3,4; m=L,U \\
n_1\ n_2,\ n_3 \text{ and } n_4 \text{ are an integers.}
\end{array}\right\} \tag{22}
$$

The compromise optimum solution is $n_1 = 11,\ n_2 = 11,\ n_3 = 6,\ n_4 = 12$ and cost is 249.99.

4.3. Classical methods

1. The compromise optimum allocation using Sukhatme's method is obtained by solving the model which is given as follows:

$$\left.\begin{array}{r}\text{Minimize}\\ \left(\frac{18.0434}{n_1}+\frac{11.0985}{n_2}+\frac{0.2240}{n_3}+\frac{46.7114}{n_4}\right)\\ +\left(\frac{18.8020}{n_1}+\frac{30.4704}{n_2}+\frac{4.2648}{n_3}+\frac{52.5708}{n_4}\right)\\ +\left(\frac{0.9245}{n_1}+\frac{1.0033}{n_2}+\frac{0.3539}{n_3}+\frac{1.8153}{n_4}\right)\\ +\left(\frac{1.0229}{n_1}+\frac{1.3543}{n_2}+\frac{0.2757}{n_3}+\frac{4.9205}{n_4}\right)\\ +\left(\frac{3.2411}{n_1}+\frac{1.6994}{n_2}+\frac{2.9135}{n_3}+\frac{3.8211}{n_4}\right)\\ +\left(\frac{0.6399}{n_1}+\frac{15.5459}{n_2}+\frac{0.0000}{n_3}+\frac{0.9361}{n_4}\right)\\ +\left(\frac{0.6676}{n_1}+\frac{0.9800}{n_2}+\frac{0.7520}{n_3}+\frac{0.7783}{n_4}\right)\\ +\left(\frac{3.1509}{n_1}+\frac{6.5519}{n_2}+\frac{6.1045}{n_3}+\frac{2.7602}{n_4}\right)\\ \text{subject to}\\ 30+5.78n_1+7.27n_2+4.78n_3+6.48n_4\le 250\\ 2\le n_1\le 90, 2\le n_2\le 92, 2\le n_3\le 92, 2\le n_4\le 91\\ n_1\ n_2,\ n_3 \text{ and } n_4 \text{ are an integers.}\end{array}\right\} \tag{23}$$

We get $n_1 = 6,\ n_2 = 3,\ n_3 = 5,\ n_4 = 27$ and cost is 249.98.

2. The mathematical model for Khan weighted method is given as follows:

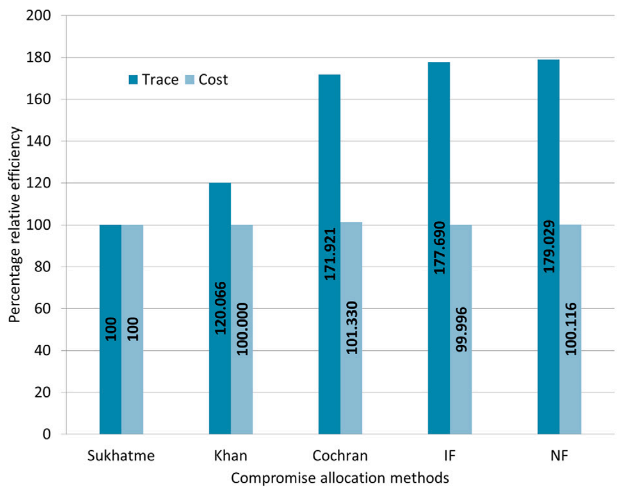
$$\left.\begin{array}{r}\text{Minimize}\\ 0.2877\left(\frac{18.0434}{n_1}+\frac{11.0985}{n_2}+\frac{0.2240}{n_3}+\frac{46.7114}{n_4}\right)\\ +0.4608\left(\frac{18.8020}{n_1}+\frac{30.4704}{n_2}+\frac{4.2648}{n_3}+\frac{52.5708}{n_4}\right)\\ +0.0398\left(\frac{0.9245}{n_1}+\frac{1.0033}{n_2}+\frac{0.3539}{n_3}+\frac{1.8153}{n_4}\right)\\ +0.0577\left(\frac{1.0229}{n_1}+\frac{1.3543}{n_2}+\frac{0.2757}{n_3}+\frac{4.9205}{n_4}\right)\\ +0.0408\left(\frac{3.2411}{n_1}+\frac{1.6994}{n_2}+\frac{2.9135}{n_3}+\frac{3.8211}{n_4}\right)\\ +0.0466\left(\frac{0.6399}{n_1}+\frac{15.5459}{n_2}+\frac{0.0000}{n_3}+\frac{0.9361}{n_4}\right)\\ +0.0089\left(\frac{0.6676}{n_1}+\frac{0.9800}{n_2}+\frac{0.7520}{n_3}+\frac{0.7783}{n_4}\right)\\ +0.0578\left(\frac{3.1509}{n_1}+\frac{6.5519}{n_2}+\frac{6.1045}{n_3}+\frac{2.7602}{n_4}\right)\\ \text{subject to}\\ 30+5.78n_1+7.27n_2+4.78n_3+6.48n_4\le 250\\ 2\le n_1\le 90, 2\le n_2\le 92, 2\le n_3\le 92, 2\le n_4\le 91\\ n_1\ n_2,\ n_3 \text{ and } n_4 \text{ are an integers.}\end{array}\right\} \tag{24}$$

Khan's approach based compromise optimum solution is $n_1 = 23,\ n_2 = 7,\ n_3 = 2,\ n_4 = 9$ and cost is 249.98.

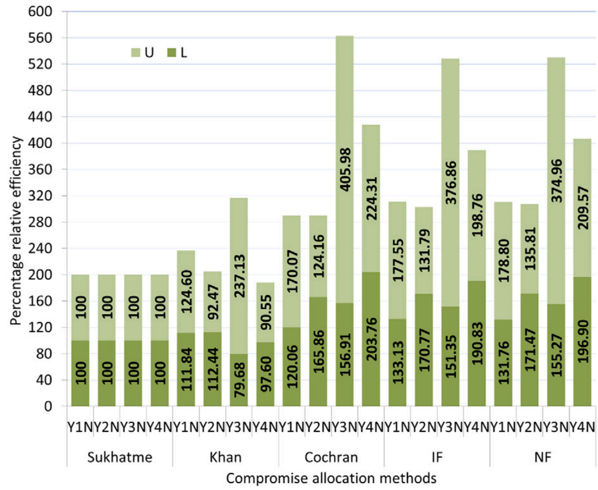
3. The integer compromise allocation under Cochran approach is obtained by rounding the average of individual optimum allocation (Table 6) to the integer values. We get $n_1 = 9,\ n_2 = 12,\ n_3 = 8,\ n_4 = 11$ and cost is 246.70.

Table 9
Variance of neutrosophic estimators and utilized cost under compromise allocation methods.

Methods	$V_1(\bar{y}_{st_N})$	$V_2(\bar{y}_{st_N})$	$V_3(\bar{y}_{st_N})$	$V_4(\bar{y}_{st_N})$	Trace	Cost
Sukhatme	[7.6447,14.9263]	[0.5816,0.7761]	[1.7027,5.1369]	[0.5824,3.8293]	35.1801	249.98
Khan	[6.8353,11.980]	[0.5172,0.8394]	[2.1369,2.1663]	[0.5967,4.2290]	29.3006	249.98
Cochran	[6.3673,8.7764]	[0.3506,0.6251]	[1.0852,1.2653]	[0.2858,1.7072]	20.4629	246.70
IF	[5.7423,8.4068]	[0.3405,0.5889]	[1.1250,1.3631]	[0.3052,1.9266]	19.7985	249.99
NF	[5.8021,8.3481]	[0.3392,0.5715]	[1.0966,1.3700]	[0.2958,1.8272]	19.6505	249.69



(a) Over-all efficiency performence.



(b) Trade-off among the characteristics.

Fig. 2. Efficiency comparison of compromise allocation methods.

5. Comparative analysis and discussion

In this section, we performed the comparative analysis among compromise optimum allocation methods based on numerical study. The results of this analysis are given in Table 9 and Fig. 2.

In this section, we performed a comparative study of the classical methods, intuitionistic fuzzy (IF) programming approach, and neutrosophic fuzzy (NF) programming method for the compromise optimum allocation (COA) in multi-variate stratified random sampling (MVSS). The variance of the neutrosophic estimates of the means of atmospheric variables, daily humidity (Y_{1N}), daily temperature (Y_{2N}), daily visibility level (Y_{3N}), and daily wind speed (Y_{4N}), under these compromise optimum allocation methods are given in Table 9 along with utilized resources and trace value. The greater trace value produces the overall lower precision of these estimates. The Sukhatme's method produced a greater trace value as compared to other methods. The *NF* programming method produced a minimum trace value at minimum utilization of resources as compared to the rest of these methods. The comparative analysis in terms of percentage relative efficiency is carried out and graphically visualized in Fig. 2. Fig. 2 shows the effect of compromise optimum allocation methods on the precision of means estimates of atmospheric variables ($Y_{jN}(j = 1, 2, 3, 4)$). The Khan method produced 20% more efficient results overall as compared to the Sukhatme's method at the same utilization level of the sample survey budget (Fig. 2(a)). But the Khan method compromised the efficiency of mean estimates of Y_{4N} (Fig. 2(b)). Fig. 2(a) indicates that the Cochran method produced 71.92%, the IF programming approach produced 77.69%, and the NF programming method produced 79.029% more efficient results overall as compared to the Sukhatme's method. But the Cochran method utilized 1.33% less resources, the *NF* method utilized 0.116% less resources, and the *IF* method utilized 0.014% more resources as compared to the Sukhatme's method. From Fig. 2(b), we observed that all compromise optimum allocation methods produced a more efficient estimate of the mean Y_{3N} as compared to other variables $Y_{jN}(j = 1, 2, 4)$. From Table 9 and Fig. 2(a), it is clear that the IF method and NF method performed better than the Sukhatme's method, Khan and Cochran's classical methods. Using the results of Table 9, we found that the *NF* method produced 0.753% more efficient results overall as compared to the *IF* method while saving 0.12% resources available for the sample survey. This superiority of the *NF* programming method is due to two reasons. First, the better trade-off among the multiple conflicting objectives at optimal utilization of sample survey resources Second, the advantage of taking into account the indeterminacy degree in compromise optimum allocation decision-making in an applied multi-character sample survey from a heterogeneous population.

6. Conclusion

In this study, we investigated the compromise optimum allocation problem for estimating the means of heterogeneous neutrosophic multi-characteristics under the study in stratified random sampling design, dealing with fuzzy uncertainty in per-unit measurement cost. We proposed the neutrosophic fuzzy programming (NF) approach and the intuitionistic fuzzy programming (IF) method for deciding the compromise optimum value of stratum sample size under the intuitionistic fuzzy cost function. The comparative analysis based on a numerical study showed that the proposed *NF* and *IF* methods produced more precise estimates of population means of atmospheric variables; daily air temperature, daily humidity, daily visibility, and daily wind speed as compared to existing classical methods. All the proposed and existing compromise optimal allocation techniques are compared with the Sukhatme's method. According to the results, the *IF* approach gave 77.69% more precise estimates of means overall, which is high compared to the existing methods. The *NF* method produced 79.03% more efficient results and saved 0.12% survey resources as compared to the Sukhatme's method. The greater efficiency of proposed methods over classical methods is due to incorporating expert knowledge and indeterminacy degree in the compromise optimum allocation decision-making process. Moreover, the *NF* method produced more efficient results with less utilization of survey resources relative to the *IF* method. The neutrosophic fuzzy programming methods for solving the compromise allocation problem in neutrosophic multi-variate stratified random sampling are more efficient and economical in terms of precision of estimates of means and optimal utilization of survey sampling resources. This study can be extended to solve optimum allocation problem in other sampling design for estimating the means of heterogeneous neutrosophic multi-characteristics.

CRediT authorship contribution statement

Atta Ullah: Writing – original draft, Visualization, Software, Methodology, Investigation, Formal analysis, Conceptualization. **Javid Shabbir:** Writing – review & editing, Validation, Supervision, Methodology, Conceptualization. **Abdullah Mohammed Alomair:** Writing – review & editing, Visualization, Resources, Project administration, Funding acquisition, Formal analysis, Data curation. **Fawaz Khaled Alarfaj:** Writing – review & editing, Validation, Software, Resources, Project administration, Investigation, Funding acquisition, Data curation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The statistical data summary of data is given within article. The detail data set is online available at https://www.kaggle.com/ and https://climateknowledgeportal.worldbank.org/.

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