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Intuitionistic fuzzy set of Γ-submodules and its application in modeling spread of viral diseases, mutated COVID-*n*, via flights

Narjes Firouzkouhi[1] | **Abbas Amini**[2,3] | **Chun Cheng**[4] | **Ali Zarrabi**[5] | **Bijan Davvaz**[6]

[1]Department of Mathematics, Golestan University, Gorgan, Iran

[2]Department of Mechanical Engineering, Australian College of Kuwait, Mishref, Kuwait

[3]School of Engineering, Design and Built Environment, Centre for Infrastructure Engineering, Western Sydney University, Penrith, New South Wales, Australia

[4]Department of Materials Science and Engineering, Southern University of Science and Technology, Shenzhen, China

[5]Department of Biomedical Engineering, Faculty of Engineering and Natural Sciences, Istinye University, Sariyer, Istanbul, Turkey

[6]Department of Mathematics, Yazd University, Yazd, Iran

Correspondence
Abbas Amini, Department of Mechanical Engineering, Australian College of Kuwait, Mishref, Safat 13015, Kuwait.
Email: a.amini@ack.edu.kw and a.amini@westernsydney.edu.au

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Abstract

In this study, we generalize fuzzy Γ-module, as intuitionistic fuzzy Γ-submodule of Γ-module (IFΓM), and utilize it for modeling the spread of coronavirus in air travels. Certain fundamental features of intuitionistic fuzzy Γ-submodule are provided, and it is proved that IFΓM can be considered as a complete lattice. Some elucidatory examples are demonstrated to explain the properties of IFΓM. The relevance between the upper and lower α-level cut and intuitionistic fuzzy Γ-submodules are presented and the characteristics of upper and lower under image and inverse image of IFΓM are acquired. It is verified that the image and inverse image of intuitionistic fuzzy Γ-submodule are preserved under the module homomorphism. The obtained IFΓM is used to model the aerial transition of viral diseases, that is, COVID-*n*, via flights.

KEYWORDS

homomorphism, image and inverse image, intuitionistic fuzzy Γ-submodule, intuitionistic fuzzy set, level subsets

1 | INTRODUCTION

The theory of fuzzy set was established by Zadeh,[1] then Rosenfeld proposed a relation between fuzzy set and group theory and regulated the notion of fuzzy subgroups.[2] Atanassov established the *intuitionistic fuzzy set* (IFS) that involved basic and fundamental concepts as the extension of fuzzy sets.[3] In fact, the IFS has been beneficial to tackle incomplete and vague information. This theory is more effective as an IFS, related to the degree of nonmembership and membership in a unit interval, while a fuzzy set is associated to the degree of membership of an element in a specified set. Numerous ideas have been developed via IFS theory, for instance, Biwas[4] defined the intuitionistic fuzzy subgroups of a group, and Kim et al. surveyed the intuitionistic fuzzy ideals of semirings.[5] The authors presented the universal coefficient theorem in the category of intuitionistic fuzzy modules.[6] Sharma initiated the concept of t-intuitionistic fuzzy subgroup,[7] fuzzy quotient group,[8] (α, β)-cut of intuitionistic fuzzy group,[9] homomorphism of intuitionistic fuzzy group,[10] and direct product of intuitionistic fuzzy group.[11] Jun et al. investigated the quotient structures of intuitionistic fuzzy finite state machines,[12] they also studied the intuitionistic nil radicals of intuitionistic fuzzy ideals and Euclidean intuitionistic fuzzy ideals in rings.[13] Based on the intuitionistic fuzzy implications, Zhou et al. introduced the intuitionistic fuzzy rough sets.[14]

Studies on Γ-related were extended by Nobusawa[15] who characterized Γ-rings and afterwards Barnes[16] and Luh[17] improved the structure of Γ-rings. Sen et al. presented the idea of Γ-semigroup as a generalization of semigroup, after that, Rao defined the idea of Γ-semiring.[18] The authors introduced the theory of Γ-semihypergroup and expanded various classical concepts of semigroups.[19] Ameri et al. developed the concept of Γ-module over a Γ-ring and extended fuzzy Γ-hypermodules and fuzzy Γ-modules.[20,21] They also defined a connection between fuzzy Γ-hypermodules and Γ-modules through fundamental relations. Another study was done on fuzzy Γ-hypermodules and fuzzy Γ-hyperrings to obtain basic results.[22] Other researchers proposed the concept of IFSs in Γ-semigroups,[23] while Ersoy et al. studied the IFS in the Γ-semihyperring.[24] The authors extended the Atanassov intuitionistic fuzzy grade of hypergroups,[25] the Atanassov intuitionistic (S, T)-fuzzy n-ary subhypergroups and their traits,[26] and the Atanassov intuitionistic fuzzy interior ideals of Γ-semigroups.[27] Latif et al. explored basic theorems of t-intuitionistic fuzzy isomorphism of t-intuitionistic fuzzy subgroups.[28]

Gulzar et al. developed some classes of t-intuitionistic fuzzy subgroups,[29] and then determined the new applications of complex IFSs in group theory.[30] In fact, IFSs are helpful in advanced systems, systems theory, decision making, and so on. Recently, Ejegwa presented the correlation coefficient between IFSs and its applications in real-life decision-making problems.[31] Alcantud et al. studied the aggregation of infinite chains of IFSs and their applications with temporal IFSs.[32] Others extended the complex IFS by quaternion numbers along with utilizing them in decision making.[33] Wei et al. defined an information-based score function of interval-valued IFSs and its application in multiattribute decision-making.[34] Also, Tao et al. explored dynamic multicriteria decision making in real life.[35] There are many other potential applications of IFSs in chemistry, mathematics, programming, physics, medicine, and machine learning. Kumar De et al. used the IFSs for medical diagnosis,[36] while the authors proposed the applications of IFS in medicine.[37] Ejewa et al. utilized the IFSs in electoral systems.[38] Mahanta et al. surveyed a novel distance measure with various applications,[39] while others analyzed the measure of width-based distance on the interval-valued IFS.[40]

The coronavirus disease-2019 (COVID-19) pandemic is a serious global crisis that has quickly spread over the world, causing millions of mortalities till date. Although the first cases were reported in China, new cases were identified in all other nations in a short

period of time.[41] This viral disease has infected humanity worldwide with typical symptoms of fever, sore throat, cough, fatigue and dyspnea. Despite the capability of some countries on effective vaccination against coronavirus disease, the emergence of new infected cases is unpredictable and seriously worrying, as there is yet neither an adamant treatment against the mutated versions of COVID nor a prohibition methodology against the detrimental/deadly side effects of known vaccines.[42] As such, various countries implemented severe precautions to decelerate the diffusion of this disease after the World Health Organization (WHO) officially publicized the epidemic situation in mid-March 2019.[43]

Due to the COVID-19 outbreak, many countries have faced case threats through inbound international and national flights. After identifying the first cases of coronavirus in different countries, strict rules were imposed on the airlines that yielded the disruption of global transportation.[44] In fact, to lessen the chances of proliferation of COVID-19, very strict protocols were issued by governments on aerial sectors. These restrictions included installing high-efficiency air filters in aircrafts, imposing C-reactive protein (CRP) tests and vaccinations for travelers, wearing protective masks, and keeping social distances during the aerial trips.[45] While the air travels are considered as an essential transportation service worldwide, the surveillance/modeling of the corresponding global factors (studied here) is necessary to resume safe aerial trips with reduced/controlled COVID threats[46] (Figure 1).

The main contribution of this paper is the generalization of fuzzy Γ-module through the development of IFS, and the construction of new application for the spread of viral diseases, that is, coronavirus, among individuals in air travels. By using Γ-module, we expand the framework of IFS via the expression of some basic and significant characteristics with certain foundational traits. In Section 3, the intuitionistic fuzzy

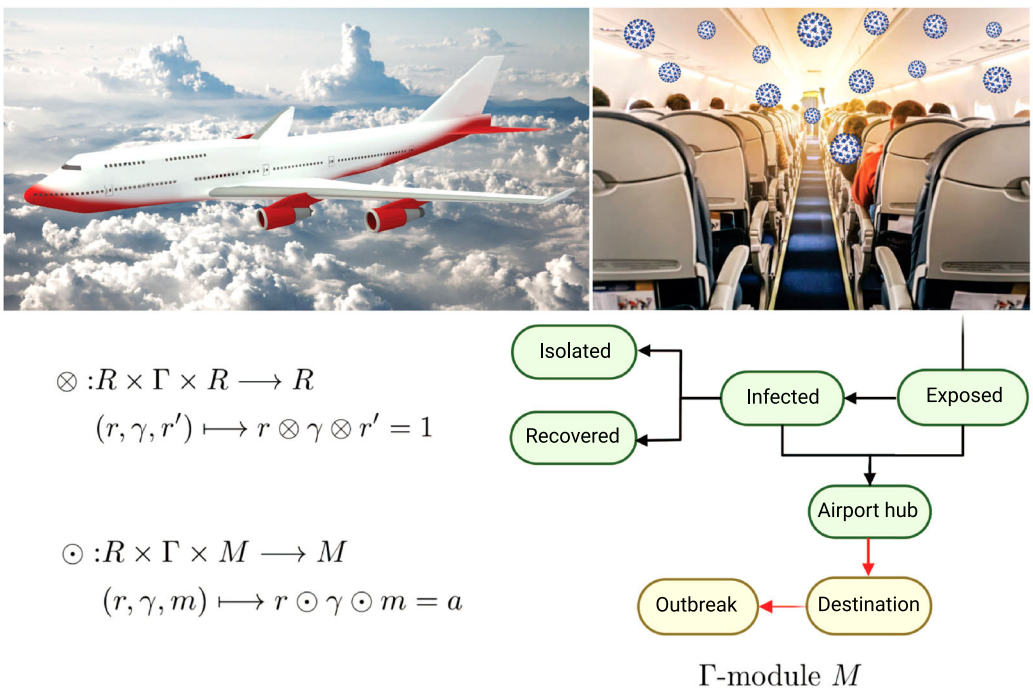

FIGURE 1 Impact of COVID-19 outbreak on flights [Color figure can be viewed at wileyonlinelibrary.com]

Γ-submodule (IFΓM) is established via the notion of Γ-modules to extend the fuzzy sets. Fundamental properties of intuitionistic fuzzy Γ-submodule are found, and it is verified that IFΓM can be regarded as a complete lattice. Furthermore, by considering the upper and lower α-level cut, we express the relationship between them and IFΓM, along with several traits of upper and lower via image and inverse image of IFΓM. It is shown that the image and inverse image of intuitionistic fuzzy Γ-submodule are preserved under the module homomorphism. In Section 4, the elucidatory examples address the application of IFΓM in the immunological transmission of COVID-n.

2 | PRELIMINARIES

The *IFSs* are the generalization of the fuzzy sets which were proposed by Atanassov.[3] An IFS A of a nonvoid set X is described by the formation $A = \{\langle t, \vartheta_A(t), \zeta_A(t)\rangle | t \in X\}$, where $\vartheta_A : X \longrightarrow [0,1]$ is the degree of membership and $\zeta_A : X \longrightarrow [0,1]$ is the degree of non-membership of the element $t \in X$, and we have $0 \leq \vartheta_A(t) + \zeta_A(t) \leq 1$. Note that we will write $A = (\vartheta_A, \zeta_A)$ instead of $A = \{\langle t, \vartheta_A(t), \zeta_A(t)\rangle\}$. Consider ϑ^c the complement of ϑ which is determined by $\vartheta^c_A(t) = 1 - \vartheta_A(t)$. Let $A = (\vartheta_A, \zeta_A)$ and $B = (\vartheta_B, \zeta_B)$ be two IFS of X. Thus, the next statements are introduced $\forall t \in X$, as follows:

(i) $A \subset B \Leftrightarrow \vartheta_A(t) \leq \vartheta_B(t), \zeta_A(t) \geq \zeta_B(t)$,
(ii) $A^c = \langle \zeta_A(t), \vartheta_A(t)\rangle$,
(iii) $A \cap B = \langle \vartheta_A(t) \wedge \vartheta_B(t), \zeta_A(t) \vee \zeta_B(t)\rangle$,
(iv) $A \cup B = \langle \vartheta_A(t) \vee \vartheta_B(t), \zeta_A(t) \wedge \zeta_B(t)\rangle$,
(v) $\Box A = \langle \vartheta_A(t), \vartheta^c_A(t)\rangle, \Diamond A = \langle \zeta^c_A(t), \zeta_A(t)\rangle$.

and if $\{A_i\}_{i \in I}$ be arbitary family of IFS in X, where $A_i = \langle \vartheta_{A_i}, \zeta_{A_i}\rangle$, thus

(i) $\bigcap_{i \in I} A_i = \langle \wedge_{i \in I} \vartheta_{A_i}(x), \vee_{i \in I} \zeta_{A_i}(x)\rangle$, that is, the intersection of A_i,
(ii) $\bigcup_{i \in I} A_i = \langle \vee_{i \in I} \vartheta_{A_i}(x), \wedge_{i \in I} \zeta_{A_i}(x)\rangle$, that is, the union of A_i.

Definition 2.1 (Barnes[16]). Suppose R and Γ be additive abelian groups. R is considered as a Γ-ring if a mapping exists:

$$\cdot : R \times \Gamma \times R \longrightarrow R$$

$$(r_1, \alpha_1, r_2) \longrightarrow r_1.\ \alpha_1.\ r_2 (= r_1 \alpha_1 r_2)$$

so that $\forall r_1, r_2, r_3 \in R, \alpha_1, \alpha_2 \in \Gamma$, the next circumstances hold:

(i) $(r_1 + r_2)\alpha_1 r_3 = r_1 \alpha_1 r_3 + r_2 \alpha_1 r_3$;
(ii) $r_1(\alpha_1 + \alpha_2) r_3 = r_1 \alpha_1 r_3 + r_1 \alpha_2 r_3$;
(iii) $r_1 \alpha_1 (r_2 + r_3) = r_1 \alpha_1 r_2 + r_1 \alpha_1 r_3$;
(iv) $(r_1 \alpha_1 r_2)\alpha_2 r_3 = r_1 \alpha_1 (r_2 \alpha_2 r_3)$.

Definition 2.2 (Ameri and Sadeghi).[20] Consider R as a Γ-ring. A *left* Γ*-module* under R is an additive abelian group M via a map $\cdot : R \times \Gamma \times M \longrightarrow M$ that $(r, \gamma, m) \longrightarrow r \cdot \gamma \cdot m$, so that for all $m, m_1, m_2 \in M$ and $\gamma, \gamma_1, \gamma_2 \in \Gamma$ and $r, r_1, r_2 \in R$ the next implications are satisfied:

(i) $r \cdot \gamma \cdot (m_1 + m_2) = r \cdot \gamma \cdot m_1 + r \cdot \gamma \cdot m_2$
(ii) $(r_1 + r_2) \cdot \gamma \cdot m = r_1 \cdot \gamma \cdot m + r_2 \cdot \gamma \cdot m$
(iii) $r \cdot (\gamma_1 + \gamma_2) \cdot m = r \cdot \gamma_1 \cdot m + r \cdot \gamma_2 \cdot m$
(iv) $r_1 \cdot \gamma_1 \cdot (r_2 \cdot \gamma_2 \cdot m) = (r_1 \cdot \gamma_1 \cdot r_2) \cdot \gamma_2 \cdot m$.

A nonvoid subset S *of* M *is considered as* left (right) Γ-submodule *of* M *provided for any* $S_1, S_2 \subseteq S$ *implies* $S_1 + S_2 \subseteq S$ *and also* $R\Gamma S \subseteq S (S\Gamma R \subseteq S)$.

3 | FUNDAMENTAL FEATURES OF IFS OF Γ-SUBMODULES

Definition 3.1. A *fuzzy left (right)* Γ*-module* over a Γ-ring R is introduced to be a couple (M, ϑ), where, M is a left Γ-module and function $\vartheta : M \longrightarrow [0, 1]$ that holds the following circumstances:

(i) $\vartheta(0) = 1$,
(ii) $\vartheta(x + y) \geq min\{\vartheta(x), \vartheta(y)\}$,
(iii) $\vartheta(r\gamma x) \geq \vartheta(x) (\vartheta(x\gamma r) \geq \vartheta(x))$. ϑ *is considered as a fuzzy* Γ*-module of* M *supposing* ϑ *is a fuzzy left* Γ*-module and also fuzzy right* Γ*-module of* M.

Example 3.2. Assume $M = \mathbb{Z}_n$ for prime integer n, and $R = \mathbb{Z}$ and $\Gamma = \mathbb{Z}$. Define

$$\cdot : \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}_n \longrightarrow \mathbb{Z}_n$$

with $(r, \gamma, \overline{x}) \longmapsto r \cdot \gamma \cdot \overline{x} := \overline{r\gamma x}$, for every $r \in R, \gamma \in \Gamma, \overline{x} \in M$, thus M is a Γ-module under a Γ-ring R (Figure 2).

Moreover, introduce the fuzzy set ϑ of M as follows:

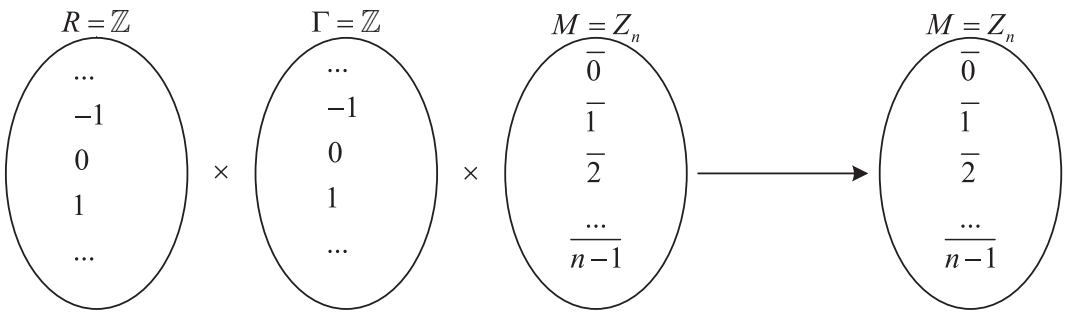

FIGURE 2 Γ-module M

$$\vartheta(\bar{x}) = \begin{cases} 1, & \text{if } \bar{x} = \bar{0}, \\ 1/3, & \text{otherwise}, \end{cases}$$

Thus, ϑ is a fuzzy Γ-module of M.

Example 3.3. Suppose $M = \mathbb{Z}$ and $R = \mathbb{Z}$ and Γ be a subring of $(\mathbb{Z}, +, \cdot)$. Hence, R is a Γ-ring and $(M, +)$ is an abelian group. Define

$$\cdot : \mathbb{Z} \times \Gamma \times \mathbb{Z} \longrightarrow \mathbb{Z}$$

with $(r, \gamma, m) \longmapsto r \cdot \gamma \cdot m := r\gamma m$ for every $r \in R, \gamma \in \Gamma, m \in M$. Therefore, M is a Γ-module. Now, describe ϑ in the following way:

$$\vartheta(m) = \begin{cases} 1, & \text{if } m = 0, \\ 3/5, & \text{otherwise}. \end{cases}$$

Hence, ϑ is a fuzzy Γ-module of M.

Definition 3.4. Assume M be a left Γ-module under a Γ-ring. An IFS $A = \langle \vartheta_A, \zeta_A \rangle$ of M is described as *left intuitionistic fuzzy* Γ*-submodule* if for all $x, y \in M, r \in R, \gamma \in \Gamma$ the next statements is satisfied:

(i) $\vartheta_A(0) = 1$ and $\zeta_A(0) = 0$,
(ii) $\vartheta_A(x + y) \geq min\{\vartheta_A(x), \vartheta_A(y)\}$ and $\zeta_A(x + y) \leq max\{\zeta_A(x), \zeta_A(y)\}$,
(iii) $\vartheta_A(x) \leq \vartheta_A(r\gamma x)$ and $\zeta_A(x) \geq \zeta_A(r\gamma x)$.

Denote that IFΓM *is intuitionistic fuzzy* Γ*-submodule. Also, it is defined for right* Γ*-submodule, the IFS of* $A = \langle \vartheta_A, \zeta_A \rangle$ *of* M *is considered an IF*ΓM *provided it is left and right IF*ΓM.

Example 3.5. Assume $M = \mathbb{Z}$ and $R = \mathbb{Z}$ and $\Gamma = \mathbb{Z}$. Then, $(M, +)$ is an abelian group and R is a Γ-ring. Define

$$\cdot : \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z} \longrightarrow \mathbb{Z}$$

written by $(r, \gamma, x) \longmapsto r\gamma x$, for every $r \in R, \gamma \in \Gamma, x \in M$. Thus, M is a Γ-module. Describe two fuzzy sets ϑ and ζ of M, in the following way:

$$\vartheta_A(x) = \begin{cases} 1, & \text{if } x = 0, \\ 1/3, & \text{otherwise}, \end{cases}$$

and

$$\zeta_A(x) = \begin{cases} 0, & \text{if } x = 0, \\ 2/3, & \text{if } x = 1, \\ 2/5, & \text{otherwise}. \end{cases}$$

Hence, $A = \langle \vartheta_A, \zeta_A \rangle$ is an IFΓM of M.

Proposition 3.6. *Suppose $\{A_i\}_{i\in I}$ be a family of IFΓM. Hence, $\bigcap_{i\in I} A_i$ and $\bigcup_{i\in I} A_i$ are IFΓM.*

Proof. We will verify $\bigcap_{i\in I} A_i \in IF\Gamma M$, and the rest is similar. Let $\{A_i\}$ be IFΓM for every $i \in I$. So, we prove the statements:

(i) $\vartheta_{(\cap_{i\in I} A_i)}(0) = \vartheta_{A_1}(0) \wedge ... \wedge \vartheta_{A_n}(0) = 1 \wedge ... \wedge 1 = 1$ and $\zeta_{(\cap_{i\in I} A_i)}(1) = \zeta_{A_1}(1) \wedge ... \wedge \zeta_{A_n}(1) = 0 \wedge ... \wedge 0 = 0$.

(ii)
$$\begin{aligned}\vartheta_{(\cap_{i\in I} A_i)}(x+y) &= \vartheta_{A_1}(x+y) \wedge ... \wedge \vartheta_{A_n}(x+y)\\ &\geq (\vartheta_{A_1}(x) \wedge \vartheta_{A_1}(y)) \wedge ... \wedge (\vartheta_{A_n}(x) \wedge \vartheta_{A_n}(y))\\ &= (\vartheta_{A_1}(x) \wedge ... \wedge \vartheta_{A_1}(x)) \wedge (\vartheta_{A_1}(y)) \wedge ... \wedge \vartheta_{A_n}(y)\Big)\\ &= \vartheta_{(\cap_{i\in I} A_i)}(x) \wedge \vartheta_{(\cap_{i\in I} A_i)}(y)\end{aligned}$$

(iii) $\vartheta_{(\cap_{i\in I} A_i)}(x) = \vartheta_{A_1}(x) \wedge ... \wedge \vartheta_{A_n}(x) \leq \vartheta_{A_1}(r\gamma x) \wedge ... \wedge \vartheta_{A_n}(r\gamma x) = \vartheta_{(\cap_{i\in I} A_i)}(r\gamma x)$, and $\zeta_{(\cap_{i\in I} A_i)}(x) = \zeta_{A_1}(x) \wedge ... \wedge \zeta_{A_n}(x) \geq \zeta_{A_1}(r\gamma x) \wedge ... \wedge \zeta_{A_n}(r\gamma x) = \zeta_{(\cap_{i\in I} A_i)}(r\gamma x)$.

This completes the proof. □

Proposition 3.7. *Assume M be a Γ-module under Γ-ring R. Thus, IF ΓM is a complete lattice under the inclusion $\subset$.*

Proof. Assume $\{A_i\}_{i\in I}$ be any subset of IFΓM, hence $\bigcap_{i\in I} A_i \in IF\Gamma M$. Evidently, $\bigcap_{i\in I} A_i$ is the largest intuitionistic fuzzy Γ-submodule contained in A_i. Therefore, $\bigcap_{i\in I} A_i = \wedge_{i\in I} A_i$. Also, $\bigcup_{i\in I} A_i \in IF\Gamma M$, and it is the least intuitionistic fuzzy Γ-submodule containing A_i. So, $\bigcup_{i\in I} A_i = \vee_{i\in I} A_i$. It yields that IFΓ$M$ is a complete lattice. □

Theorem 3.8. *If S_1 is a Γ-submodule of M, hence $\tilde{S}_1 = \langle \chi_{S_1}, \chi^c_{S_1} \rangle$ is an IF Γ M of M.*

Proof. Assume $x, y \in S_1, r \in R, \gamma \in \Gamma$. Since S_1 is Γ-submodule, so $x + y \in S_1$ and $r\gamma x \in S_1$. We verify the next statements.

(i) $\chi_{S_1}(x) = 1$ and $\chi^c_{S_1}(x) = 0$,

(ii) $\chi_{S_1}(x+y) = 1 \geq min\{\chi_{S_1}(x), \chi_{S_1}(y)\} = 1 \wedge 1 = 1$, and

$$\begin{aligned}\chi^c_{S_1}(x+y) = 1 - \chi_{S_1}(x+y) &\leq 1 - min\{\chi_{S_1}(x), \chi_{S_1}(y)\}\\ &= max\{1 - \chi_{S_1}(x), 1 - \chi_{S_1}(y)\}\\ &= max\Big\{\chi^c_{S_1}(x), \chi^c_{S_1}(y)\Big\},\end{aligned}$$

(iii) $\chi_{S_1}(r\gamma x) = 1 \geq \chi_{S_1}(x)$ and $\chi^c_{S_1}(r\gamma x) = 1 - \chi_{S_1}(r\gamma x) \leq 1 - \chi_{S_1}(x) = \chi^c_{S_1}(x)$.

Supposing $x \notin S_1$ or $y \notin S_1$, thus $\chi_{S_1}(x) = 0$ or $\chi_{S_1}(y) = 0$. Therefore,

$$\chi_{S_1}(x + y) \geq 0 = min\{\chi_{S_1}(x), \chi_{S_1}(y)\},$$

and

$$\begin{aligned} max\left\{\chi^c_{S_1}(x), \chi^c_{S_1}(y)\right\} &= max\{1 - \chi_{S_1}(x), 1 - \chi_{S_1}(y)\} \\ &= 1 - min\left\{\chi_{S_1}(x), \chi_{S_1}\right. \\ &= 1 \geq \chi^c_{S_1}(x + y) \end{aligned}$$

□

Theorem 3.9. *Consider S_1 be a nonvoid subset of M. If $\tilde{S}_1 = \langle \chi_{S_1}, \chi^c_{S_1} \rangle$ is an IF ΓM of M, then S_1 is a Γ-submodule of M.*

Proof. Assume that $\tilde{S}_1 = \langle \chi_{S_1}, \chi^c_{S_1} \rangle$ is an IFΓM of M. We should verify for $x, y \in S_1, r \in R, \gamma \in \Gamma$ that $x + y \in S_1$ and $r\gamma x \in S_1$. It yields that

$$\chi_{S_1}(x + y) \geq min\{\chi_{S_1}(x), \chi_{S_1}(y)\} = 1 \wedge 1 = 1$$

and

$$\chi^c_{S_1}(x + y) \leq max\left\{\chi^c_{S_1}(x), \chi^c_{S_1}(y)\right\} = 0 \wedge 1 = 1$$

So, $\chi_{S_1}(x + y) = 1$ then, $x + y \in S_1$. Also, we have

$$\chi_{S_1}(r\gamma x) \geq \chi_{S_1}(x) = 1$$

and

$$\chi^c_{S_1}(r\gamma x) \leq \chi_{S_1}(x) = 0$$

It means that $r\gamma x \in S_1$. □

Proposition 3.10. *Assume that $A = \langle \vartheta_A, \zeta_A \rangle$ be an IFΓM of M, and $0 \leq \alpha \leq 1$. Introduce an IFS $B = \langle \vartheta_B, \zeta_B \rangle$ on M by $\vartheta_B(x) = \alpha\vartheta_A(x)$ and $\zeta_B(x) = (1 - \alpha)\zeta_A(x)$, for all $x \in M$. Hence, $B = \langle \vartheta_B, \zeta_B \rangle$ is an IFΓM of M.*

Proof. We have

$$0 \leq \vartheta_B(x) + \zeta_B(x) = \alpha\vartheta_A(x) + (1 - \alpha)\zeta_A(x) \leq 1.$$

□

Proposition 3.11. *Suppose that $A = \langle \vartheta_A, \zeta_A \rangle$ be an IFΓM of M. Describe an IFS $B = \langle \vartheta_B, \zeta_B \rangle$ on M, by $\vartheta_B(x) = (\vartheta_A(x))^2$ and $\zeta_B(x) = 1 - (1 - \zeta_A(x))^2$, for all $x \in M$. Thus, $B = \langle \vartheta_B, \zeta_B \rangle$ is an IFΓS of M.*

Proof. Consider $A = \langle \vartheta_A, \zeta_A \rangle$ be an IFΓM of M. So, we have $\vartheta_A(x + y) \geq min\{\vartheta_A(x), \vartheta_A(y)\}$. Then,

$$(\vartheta_A(x+y))^2 \geq (min\{\vartheta_A(x), \vartheta_A(y)\})^2 = min\{(\vartheta_A(x))^2, (\vartheta_A(y))^2\} = min\{\vartheta_B(x), \vartheta_B(y)\}.$$

Since $\vartheta_A(x) \leq \vartheta_A(r\gamma x)$, therefore $(\vartheta_A(x))^2 \leq (\vartheta_A(r\gamma x))^2$, that implies $\vartheta_B(x) \leq \vartheta_B(r\gamma x)$. Also, we have

$$\begin{aligned}
&\zeta_A(x+y) \leq max\{\zeta_A(x), \zeta_A(y)\}\\
&\Rightarrow (-\zeta_A(x+y)) \geq min\{(-\zeta_A(x)), (-\zeta_A(y))\}\\
&\Rightarrow (1-\zeta_A(x+y))^2 \geq (min\{1-\zeta_A(x), 1-\zeta_A(y)\})^2\\
&\Rightarrow (1-\zeta_A(x+y))^2 \geq min\{(1-\zeta_A(x))^2, (1-\zeta_A(y))^2\}\\
&\Rightarrow -(1-\zeta_A(x+y))^2 \leq max\{-(1-\zeta_A(x))^2, -(1-\zeta_A(y))^2\}\\
&\Rightarrow 1-(1-\zeta_A(x+y))^2 \leq max\{1-(1-\zeta_A(x))^2, 1-(1-\zeta_A(y))^2\}\\
&\Rightarrow \zeta_B(x+y) \leq max\{\zeta_B(x), \zeta_B(y)\}.
\end{aligned}$$

In addition, we have

$$\begin{aligned}
&\zeta_A(x) \geq \zeta_A(r\gamma x)\\
&\Rightarrow -\zeta_A(x) \leq -\zeta_A(r\gamma x)\\
&\Rightarrow (1-\zeta_A(x))^2 \leq (1-\zeta_A(r\gamma x))^2\\
&\Rightarrow -(1-\zeta_A(x))^2 \geq -(1-\zeta_A(r\gamma x))^2\\
&\Rightarrow 1-(1-\zeta_A(x))^2 \geq 1-(1-\zeta_A(r\gamma x))^2\\
&\Rightarrow \zeta_B(x) \geq \zeta_B(r\gamma x).
\end{aligned}$$

The proof is completed. □

Theorem 3.12. *An IFS $A = \langle \vartheta_A, \zeta_A \rangle$ of left (right) Γ-module M is an IF ΓM if and only if the fuzzy sets ϑ_A and ζ_A^c are fuzzy left (right) Γ-module.*

Proof. Let $A = \langle \vartheta_A, \zeta_A \rangle$ be IFΓM of M. By definition, ϑ_A is left fuzzy Γ-module. Moreover, for $x, y \in M, \gamma \in \Gamma$, we attain

(i) $\zeta_A^c(0) = 1 - \zeta_A(0) = 1 - 0 = 1.$

(ii)
$$\begin{aligned}
\zeta_A^c(x+y) &= 1 - \zeta_A(x+y)\\
&\geq 1 - max\{\zeta_A(x), \zeta_A(y)\}\\
&= min\{1-\zeta_A(x), 1-\zeta_A(y)\}\\
&= min\left\{\zeta_A^c(x), \zeta_A^c(y)\right\},
\end{aligned}$$

(iii)
$$\begin{aligned}
\zeta_A^c(r\gamma x) &= 1 - \zeta_A(r\gamma x)\\
&\geq 1 - \zeta_A(x)\\
&= \zeta_A^c(x),
\end{aligned}$$

Hence, ζ_A^c is fuzzy left Γ-module.

On the contrary, assume that the fuzzy sets ϑ_A and ζ_A^c are fuzzy left (right) Γ-module. So, $\vartheta(0) = 1$, $\vartheta(x+y) \geq min\{\vartheta(x), \vartheta(y)\}$, and $\vartheta(r\gamma x) \geq \vartheta(x)$, for all $x, y \in M, r \in R, \gamma \in \Gamma$. Also,

(i) $\zeta_A(0) = 1 - \zeta_A^c(0) = 1 - 1 = 0$,

(ii)
$$\begin{aligned}\zeta_A(x+y) &= 1 - \zeta_A^c(x+y)\\ &\leq 1 - min\left\{\zeta_A^c(x), \zeta_A^c(y)\right\}\\ &= max\left\{1 - \zeta_A^c(x), 1 - \zeta_A^c(y)\right\}\\ &= max\{\zeta_A(x), \zeta_A(y)\},\end{aligned}$$

(iii) $\zeta_A(r\gamma x) = 1 - \zeta_A^c(r\gamma x) \leq 1 - \zeta_A^c(x) = \zeta_A(x)$.

Thus, $A = \langle \vartheta_A, \zeta_A \rangle$ is an IFΓM of M. □

Theorem 3.13. *Assume $A = \langle \vartheta_A, \zeta_A \rangle$ be IFΓM of M. Hence, $\Box A$ and $\Diamond A$ are also IFΓM of M.*

Proof. Suppose $A = \langle \vartheta_A, \zeta_A \rangle$ be IFΓM of M. For all $x, y \in M, r \in R, \gamma \in \Gamma$, we attain

(i) $\vartheta(0) = 1$,
(ii) $\vartheta(x+y) \geq min\{\vartheta(x), \vartheta(y)\}$,
(iii) $\vartheta(r\gamma x) \geq \vartheta(x) (\vartheta(x\gamma r) \geq \vartheta(x))$.

Therefore, we have

$$\vartheta_A^c(0) = 1 - \vartheta_A(0) = 1 - 1 = 0,$$

and

$$\begin{aligned}\vartheta_A^c(x+y) &= 1 - \vartheta_A(x+y)\\ &\leq 1 - min\{\vartheta_A(x), \vartheta_A(y)\}\\ &= max\{1 - \vartheta_A(x), 1 - \vartheta_A(y)\}\\ &= max\left\{\vartheta_A^c(x), \vartheta_A^c(y)\right\},\end{aligned}$$

and

$$\begin{aligned}\vartheta_A^c(r\gamma x) &= 1 - \vartheta_A(r\gamma x)\\ &\leq 1 - \vartheta_A(x)\\ &= \vartheta_A^c(x).\end{aligned}$$

It implies that $\Box A$ is an IFΓM of M. Similarly, we can verify for $\Diamond A$. □

Remark 3.14. For a proper IFS of A, we have $\Box A \subset A \subset \Diamond A$ and $\Box A \neq A \neq \Diamond A$, but if A is a fuzzy Γ-module, then we have $\Box A = A = \Diamond A$.

$U(\vartheta_A;\alpha)$ is described as an *upper bound α-level cut* of ϑ, and written by $U(\vartheta_A;\alpha) = \{x \in M | \vartheta_A(x) \geq \alpha\}$, and also $L(\vartheta_A;\alpha)$ is considered as *lower bound α-level cut* of ϑ, and written by $L(\vartheta_A;\alpha) = \{x \in M | \vartheta_A(x) \leq \alpha\}$, for any fuzzy set ϑ of M and $\alpha \in [0,1]$.

Theorem 3.15. *An IFS A of a Γ-module M is a left (right) IFΓM if and only if for every* $\alpha, \beta \in [0,1]$, *the subsets* $U(\vartheta_A;\alpha)$ *and* $L(\zeta_A;\beta)$ *of M are left (right) Γ-submodule.*

Proof. Suppose that $A = \langle \vartheta_A, \zeta_A \rangle$ be IFΓM of M. Let $x, y \in U(\vartheta_A;\alpha)$. Since $\vartheta_A(x+y) \geq min\{\vartheta_A(x), \vartheta_A(y)\}$, and $\vartheta_A(x) \geq \alpha, \vartheta_A(y) \geq \alpha$ so we have $\vartheta_A(x+y) \geq \alpha \wedge \alpha = \alpha$, it means that $x + y \subseteq U(\vartheta_A;\alpha)$. Also, since $\vartheta(r\gamma x) \geq \vartheta(x)$, and $\vartheta_A(x) \geq \alpha$, so we have $\vartheta(r\gamma x) \geq \vartheta(x) \geq \alpha$, it yields that $r\gamma x \subseteq U(\vartheta_A;\alpha)$.

Now, assume that $x, y \in L(\zeta_A;\beta)$. Since $\zeta_A(x+y) \leq max\{\zeta_A(x), \zeta_A(y)\}$, and $\zeta_A(x) \leq \beta, \zeta_A(y) \leq \beta$, so we have $\zeta_A(x+y) \leq \beta \vee \beta = \beta$, it follows $x + y \subseteq L(\zeta_A;\beta)$. Moreover, since $\zeta(r\gamma x) \leq \zeta(x)$, and $\zeta_A(x) \leq \beta$, so we have $\zeta(r\gamma x) \leq \zeta(x) \leq \beta$, we conclude that $r\gamma x \subseteq L(\zeta_A;\beta)$.

On the contrary, assume that the subsets $U(\vartheta_A;\alpha)$ and $L(\zeta_A;\beta)$ of M are left Γ-submodule. Let $x, y \in M, \gamma \in \Gamma$, and $\vartheta_A(x) = \alpha_0, \vartheta_A(y) = \alpha_1, \zeta_A(x) = \beta_0$, and $\zeta_A(y) = \beta_1$, that $\alpha_0 \leq \alpha_1$ and $\beta_0 \leq \beta_1$. If $x, y \in U(\vartheta_A;\alpha_0)$ and $x, y \in L(\zeta_A;\beta_1)$, by hypothesis we attain $x + y \subseteq U(\vartheta_A;\alpha_0)$, and $x + y \subseteq L(\zeta_A;\beta_1)$. Therefore,

$$\alpha_0 = min\{\vartheta_A(x), \vartheta_A(y)\} \leq \vartheta_A(x+y),$$

$$\beta_1 = max\{\zeta_A(x), \zeta_A(y)\} \geq \zeta_A(x+y).$$

Also, $r\gamma x \subseteq U(\vartheta_A;\alpha_0)$, and $r\gamma x \subseteq L(\zeta_A;\beta_1)$, so we have $\vartheta_A(r\gamma x) \geq \alpha_0$, and $\zeta_A(r\gamma x) \leq \beta_1$. Thus, $\vartheta_A(r\gamma x) \geq \vartheta_A(x)$, and $\zeta_A(r\gamma x) \leq \zeta_A(x)$. Hence, $A = \langle \vartheta_A, \zeta_A \rangle$ is an IFΓM of M. □

Definition 3.16. Assume that $A = \langle \vartheta_A, \zeta_A \rangle$ and $B = \langle \vartheta_B, \zeta_B \rangle$ be two IFS of M and $\overline{M}$. Consider $\pi : M \longrightarrow \overline{M}$ be a map. Hence, we have

(i) The image of A under the map π is signified by $\pi(A)$, that is written $\pi(A) = (\vartheta_{\pi(A)}, \zeta_{\pi(A)})$, which $\forall\ \overline{m} \in \overline{M}$, we note

$$\vartheta_{\pi(A)}(\bar{m}) = \begin{cases} \bigvee\limits_{m \in \pi^{-1}(\bar{m})} \vartheta_A(m), & \text{if } \pi^{-1}(\bar{m}) \neq \varnothing, \\ 0, & \text{otherwise,} \end{cases}$$

and

$$\zeta_{\pi(A)}(\bar{m}) = \begin{cases} \bigwedge\limits_{m \in \pi^{-1}(\bar{m})} \zeta_A(m), & \text{if } \pi^{-1}(\bar{m}) \neq \varnothing, \\ 1, & \text{otherwise.} \end{cases}$$

(ii) The inverse image of B is signified by $\pi^{-1}(B)$, that is written $\pi^{-1}(B) = (\vartheta_{\pi^{-1}(B)}, \zeta_{\pi^{-1}(B)})$, which for $m \in M$, we note

$$\vartheta_{\pi^{-1}(B)}(m) = \vartheta_B(\pi(m)), \zeta_{\pi^{-1}(B)}(m) = \zeta_B(\pi(m)).$$

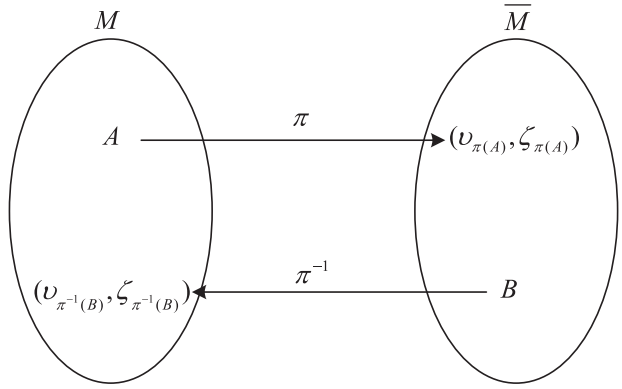

FIGURE 3 Image and inverse image of IFS

The image and inverse image are depicted in Figure 3.

Proposition 3.17. *Assume M_1 and M_2 be two Γ-modules over Γ-ring R and $\pi : M_1 \longrightarrow M_2$ be a surjective homomorphism. Suppose $A = \langle \vartheta_A, \zeta_A \rangle$ is an IFTM of M_1, thus for every $\alpha, \beta \in [0, 1]$, we have*

(i) $\pi(U(\vartheta_A; \alpha)) = U(\vartheta_{\pi(A)}; \alpha)$,
(ii) $\pi(L(\zeta_A; \beta)) = L(\zeta_{\pi(A)}; \beta)$.

Proof. We prove (i),

$$\begin{aligned} y \in \pi(U(\vartheta_A; \alpha)) &\Leftrightarrow \exists\, x_0 \in U(\vartheta_A; \alpha);\, \pi(x_0) = y \\ &\Leftrightarrow \exists\, x_0 \in U(\vartheta_A; \alpha);\, \exists\, x_0 \in \pi^{-1}(y) \\ &\Leftrightarrow \vartheta_A(x_0) \geq \alpha;\, \exists\, x_0 \in \pi^{-1}(y) \\ &\Leftrightarrow \left(\vee_{x_0 \in \pi^{-1}(y)} \vartheta_A(x_0)\right) \geq \alpha \\ &\Leftrightarrow \vartheta_{\pi(A)}(y) \geq \alpha \\ &\Leftrightarrow y \in U(\vartheta_{\pi(A)}; \alpha) \end{aligned}$$

also, we prove (ii) in the following:

$$\begin{aligned} y \in \pi(L(\zeta_A; \beta)) &\Leftrightarrow \exists\, x_0 \in L(\zeta_A; \beta);\, \pi(x_0) = y \\ &\Leftrightarrow \exists\, x_0 \in L(\zeta_A; \beta);\, \exists\, x_0 \in \pi^{-1}(y) \\ &\Leftrightarrow \zeta_A(x_0) \leq \beta;\, \exists\, x_0 \in \pi^{-1}(y) \\ &\Leftrightarrow \left(\wedge_{x_0 \in \pi^{-1}(y)} \zeta_A(x_0)\right) \leq \beta \\ &\Leftrightarrow \zeta_{\pi(A)}(y) \leq \beta \\ &\Leftrightarrow y \in L(\zeta_{\pi(A)}; \beta). \end{aligned}$$

□

Proposition 3.18. *Suppose M_1 and M_2 be two Γ-modules over Γ-ring R and $\pi : M_1 \longrightarrow M_2$ be a surjective homomorphism. Assume $B = \langle \vartheta_B, \zeta_B \rangle$ be an IFΓM of M_2, hence for every $\alpha, \beta \in [0, 1]$, we have*

(i) $\pi^{-1}(U(\vartheta_B; \alpha)) = U\left(\vartheta_{\pi^{-1}(B)}; \alpha\right)$,
(ii) $\pi^{-1}(L(\zeta_B; \beta)) = L\left(\zeta_{\pi^{-1}(B)}; \beta\right)$.

Proof. We verify (i) in the following:

$$\begin{aligned} x \in \pi^{-1}(U(\vartheta_B; \alpha)) &\Leftrightarrow \pi(x) \in U(\vartheta_B; \alpha) \\ &\Leftrightarrow \vartheta_B(\pi(x)) \geq \alpha \\ &\Leftrightarrow \vartheta_{\pi^{-1}(B)}(x) \geq \alpha \\ &\Leftrightarrow x \in U\left(\vartheta_{\pi^{-1}(B)}; \alpha\right). \end{aligned}$$

Moreover, we prove (ii) as follows:

$$\begin{aligned} x \in \pi^{-1}(L(\zeta_B; \beta)) &\Leftrightarrow \pi(x) \in L(\zeta_B; \beta) \\ &\Leftrightarrow \zeta_B(\pi(x)) \leq \beta \\ &\Leftrightarrow \zeta_{\pi^{-1}(B)}(x) \leq \beta \\ &\Leftrightarrow x \in L\left(\zeta_{\pi^{-1}(B)}; \beta\right). \end{aligned}$$

□

Definition 3.19. Assume M be Γ-module over R, and $\overline{M}$ be $\overline{\Gamma}$-module over $\overline{R}$. If the map $\pi : M \longrightarrow \overline{M}$ and bijection $\varphi : \Gamma \longrightarrow \overline{\Gamma}$ and $\psi : R \longrightarrow \overline{R}$ exist. (π, φ, ψ) is called a *homomorphism* of M to $\overline{M}$, provided for all $x, y \in M, \gamma \in \Gamma$, we attain

$$\pi(x + y) = \pi(x) + \pi(y),$$

$$\pi(r\gamma x) = \psi(r)\varphi(\gamma)\pi(x).$$

Moreover, if π be a bijection, then we call (π, φ, ψ) is an *isomorphism.*

Theorem 3.20. *Assume M be Γ-module, and $\overline{M}$ be $\overline{\Gamma}$-module. Let (π, φ, ψ) be homomorphism from M to $\overline{M}$. Hence,*

(i) *if $A = \langle \vartheta_A, \zeta_A \rangle$ is an IFΓM of M, thus $\pi(A)$ is an IF$\overline{\Gamma}$M of $\overline{M}$.*
(ii) *if $B = \langle \vartheta_B, \zeta_B \rangle$ is an IF$\overline{\Gamma}$M of $\overline{M}$, thus $\pi^{-1}(B)$ is an IFΓM of M.*

Proof. (i): Since $\pi(A) = (\vartheta_{\pi(A)}, \zeta_{\pi(A)})$, hence for all $x \in M, \gamma \in \Gamma, r \in R, x', y' \in \bar{M}, \gamma' \in \bar{\Gamma}, r' \in \bar{R}$, we have

$$\begin{aligned}\vartheta_{\pi(A)}(x'+y') &= \bigvee_{t\in\pi^{-1}(x'+y')} \vartheta_A(t)\\ &\geq \bigvee_{\pi(z)=x'+y'} \vartheta_A(z)\\ &= \bigvee_{\pi(z)=\pi(x)+\pi(y)} \vartheta_A(z) = \bigvee_{\pi(z)=\pi(x+y)} \vartheta_A(z) = \bigvee_{z=x+y} \vartheta_A(z)\\ &\geq \bigvee_{\pi(x)=x',\pi(y)=y'} min\{\vartheta_A(x), \vartheta_A(y)\}\\ &= min\{\bigvee_{\pi(x)=x'} \vartheta_A(x), \bigvee_{\pi(y)=y'} \vartheta_A(y)\}\\ &= min\{\vartheta_{\pi(A)}(x'), \vartheta_{\pi(A)}(y')\},\end{aligned}$$

Moreover, $\vartheta_{\pi(A)}(r'\gamma'x') = \vee_{t\in\pi^{-1}(r'\gamma'x')}\ \vartheta_A(t) \geq \vee_{\pi(z)=r'\gamma'x'}\ \vartheta_A(z) = \vee_{\pi(z)=\psi(r)\varphi(\gamma)\pi(x)}\ \vartheta_A(z) = \vee_{\pi(z)=\pi(r\gamma x)}\vartheta_A(z) = \vee_{z=r\gamma x}\ \vartheta_A(z) \geq \vee_{\pi(x)=x'}\ \vartheta_A(x) = \vartheta_{\pi(A)}(x')$. It is straightforward to prove for $\zeta_{\pi(A)}$. Thus, $\pi(A)$ is an IF$\overline{\Gamma}$M of $\overline{M}$.

The proof of (ii) is analogous to (i). □

4 | APPLICATION OF IFΓM FOR THE SPREAD TREND OF COVID-*n* VIA AIR TRAVELS

The application of an IFS on-submodules is expressed for the diffusion of coronavirus disease 2019 (COVID-19) via flights. COVID-19 is the most recent epidemic disease which has affected all over the world yielding nearly 4 million deaths till July 2021. This viral disease was first emerged in Wuhan, China, and quickly spread across the world in a short period of time, entangling all the countries and devastating numerous infrastructures.[41] Air travels have negatively assisted the global epidemic of viral diseases, specifically those highly infectious diseases, that is, COVID-*n*.[46] It was reported that after a major flight, there have been some new patients infected with coronavirus.[47] Here, we utilize the developed IFΓM to model the dispersion of coronavirus disease between individuals who traveled to different countries via different airlines. In this transition, we appoint Γ as the set of airlines, *R* as the set of countries, and *M* as the set of family members (Figure 4).

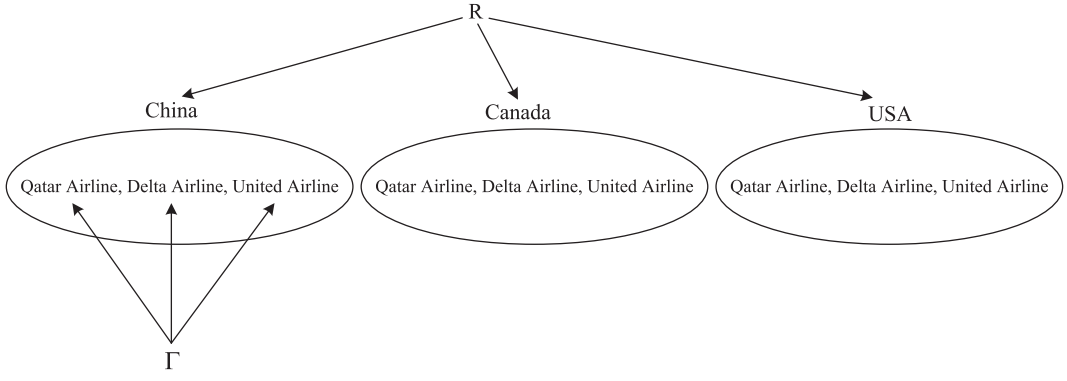

FIGURE 4 The set *R* and Γ

Assume Γ be important airlines which operate in different countries. Consider $\Gamma = \{Qatar\ Airline, Delta\ Airline, United\ Airline\}$ with the operation "+" that is defined as follows:

$$x + y = \text{The airline which plays a role in disease transmission to } x \text{ and } y$$

The set Γ with the operation + is shown in Table 1.

Thus, $(\Gamma, +)$ is an abelian group.

Suppose R be the countries that participated in our model. Let $R = \{China, Canada, USA\}$ and the operation $\uplus$ determined in the following manner:

$$a \uplus b = \text{The country which contaminates a and b.}$$

The set R via the operation $\uplus$ is given in Table 2.

Therefore, $(R, \uplus)$ is an abelian group. Now, we introduce the operation "$\otimes$" in the next way:

$$\begin{aligned} \otimes: \quad & R \times \Gamma \times R \longrightarrow R \\ & (r, \gamma, r') \longmapsto r \otimes \gamma \otimes r' = 1 \end{aligned}$$

which $r \otimes \gamma \otimes r'$ means the country infected by COVID-19 in relation with the airlines. Hence, $(R, \uplus, \otimes)$ is Γ-ring.

Consider the set M as the family members who travel to countries R with airlines Γ. Let $M = \{Bob, Jack, Sara, Nancy\}$. Describe the operation "$\oplus$" as follows:

$$t \oplus s = \text{The person who transmits the disease to t and s}$$

In Table 3. $(M, \oplus)$ is defined.

Then, $(M, \oplus)$ is the abelian group. Introduce the operation "$\odot$" for all $r \in R, \gamma \in \Gamma, m \in M$, in the following manner:

$$\begin{aligned} & \odot : R \times \Gamma \times M \longrightarrow M \\ & (r, \gamma, m) \longmapsto r \odot \gamma \odot m = a \end{aligned}$$

TABLE 1 Group $(\Gamma, +)$

+	Qatar Airline = A	Delta Airline = B	United Airline = C
Qatar Airline = A	A	B	C
Delta Airline = B	B	C	A
United Airline = C	C	A	B

TABLE 2 Ring $(R, \uplus)$

$\uplus$	China = 1	Canada = 2	USA = 3
China = 1	1	2	3
Canada = 2	2	1	3
USA = 3	3	2	1

TABLE 3 Module $(M, \oplus)$

$\oplus$	**Bob = *a***	**Jack = *b***	**Sara = *c***	**Nancy = *d***
Bob = a	a	b	c	d
Jack = b	b	a	d	c
Sara = c	c	d	a	b
Nancy = d	d	c	b	a

TABLE 4 Intuitionistic fuzzy set A

A	**Degree of membership and nonmembership of COVID-19**
Bob = a	(1,0)
Jack = b	(0.6, 0.4)
Sara = c	(0.5,0.3)
Nancy = d	(0.5,0.4)

Therefore, $(M, \oplus, \odot)$ is Γ-module over Γ-ring R.

The IFS A of M is determined as follows.

The degree of membership can be interpreted as a percentage of dependence. Table 4 depicts that the disease transmission power of Bob is more than the others, Jack is in the second rank and so on. To verify that A is IFΓM of M, we pursue the following procedure for all elements of A. For example, $\vartheta_A(b \oplus d) = \vartheta_A(c) = 0.5 \geq \vartheta_A(b) \wedge \vartheta_A(d) = 0.6 \wedge 0.5$, and $\zeta_A(b \oplus d) = \zeta_A(c) = 0.3 \leq \zeta_A(b) \wedge \zeta_A(d) = 0.4 \wedge 0.4$. Also, $\vartheta_A(r \odot \gamma \odot b) = \vartheta_A(a) = 1 \geq \vartheta_A(b) = 0.6$, and $\zeta_A(r \odot \gamma \odot b) = \zeta_A(a) = 0 \leq \zeta_A(b) = 0.4$. Therefore, an IFS $A = \langle \vartheta_A, \zeta_A \rangle$ is IFΓM of M.

5 | CONCLUSION

In this paper, a framework for the IFS associated to Γ-submodule was constructed to generalize the fuzzy set. Certain features of IFS of Γ-modules were expressed along with illustrative examples, and a link between upper and lower α-level cut and intuitionistic fuzzy Γ-submodules was also presented. By applying the module homomorphism, the image and inverse image of intuitionistic fuzzy Γ-submodule were preserved under the homomorphism. In addition, the convenient circumstance was carried out to create the t-IFS of Γ-modules, (α, β)-IFS of Γ-modules, homomorphism and direct product of IFS of Γ-modules which were the main characteristics of the intuitionistic fuzzy Γ-submodules. The effective application of this survey was demonstrated in modeling the spread of COVID-19 via air travels. The results rationalized the immunological case by using the developed intuitionistic fuzzy Γ-submodules. There is a potential to exploit the capability of IFS of Γ-subrings and IFS of Γ-subgroups in other fields.

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CONFLICT OF INTERESTS

The authors declare that there are no conflict of interests.

ORCID

Narjes Firouzkouhi https://orcid.org/0000-0003-2716-549X
Abbas Amini https://orcid.org/0000-0002-7987-9243
Chun Cheng https://orcid.org/0000-0001-7145-8893
Ali Zarrabi https://orcid.org/0000-0003-0391-1769
Bijan Davvaz https://orcid.org/0000-0003-1941-5372

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