# The Odd Weibull Inverse Topp–Leone Distribution with Applications to COVID-19 Data

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## Abstract

This paper aims at defining an optimal statistical model for the COVID-19 distribution in the United Kingdom, and Canada. A combining the inverted Topp–Leone distribution and the odd Weibull family introduces a new lifetime distribution with a three-parameter to formulate the odd Weibull inverted Topp–Leone (OWITL) distribution. As a simple linear representation, hazard rate function, and moment function, this new distribution has several nice properties. To estimate the unknown parameters of OWITL distribution, maximum likelihood, least-square, weighted least-squares, maximum product spacing, Cramér–von Mises estimators, and Anderson–Darling estimation methods are used. To evaluate the use of estimation techniques, a numerical outcome of the Monte Carlo simulation is obtained.

**Keywords** Odd Weibull family · Inverted Topp–Leone distribution · Maximum likelihood estimation · Maximum product spacing · COVID-19

## 1 Introduction

Over the years, statistical lifetime distributions have gained a lot of coverage. Its interest has therefore evolved over time. Distribution theory researchers do this either by adding a new parameter to make the distribution of interest more versatile or even creating a new distribution family or modeling data in a variety of fields, including economics, engineering, reliability, and medical sciences Anake et al. [1]. Because of their applicability in many fields such as biological sciences, life test issues, medical, etc., inverted (or inverse) distributions are of great importance. The density and hazard ratio of the inverted distributions illustrate a distinct structure from the non-inverted distributions of conformation. The applications of inverted distributions have been discussed with many researchers, and the reader can refer

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to Abd AL-Fattah et al. [2], Barco et al. [3], Hassan and Abd-Allah [4], Hassan and Mohamed [5], Muhammed [6], Chesneau et al. [7], Usman and ul Haq [8], Efer-honore et al. [9] among others. The modeling of COVID-19 data have been dis-cussed with many researchers, and the reader can refer to Kumar [10], Khakharia et al. [11], Li et al. [12], Liu et al. [13], Wang [14], Lalmuanawma et al. [15] and Bullock et al. [16].

Hassan et al. [17] suggested the cumulative distribution function (CDF) and prob-ability density function (PDF) of the inverted Topp–Leone distribution (ITL) distri-bution with form parameter $\delta > 0$ as follows:

$$G(x;\delta) = 1 - \frac{(1+2x)^\delta}{(1+x)^{2\delta}}; \quad x > 0, \delta > 0 \tag{1}$$

and,

$$g(x;\delta) = 2\delta x\,(1+x)^{-2\delta-1}(1+2x)^{\delta-1}; \quad x, \delta > 0 \tag{2}$$

Kumar and Dharmaja [18] presented the exponentiated Kies distribution and some of its properties for this distribution. Dey et al. [19]. derived the product moments of the modified Kies distribution under Type II progressive censored sample, as well as an approximation of the distribution parameters. Bourguignon et al. [20] introduced the unusual Weibull-G family. Al-Babtain et al. [21] submitted a new distribution family based on the modified Kies (MK) distribution and the T-X family. A spe-cial case of the odd Weibull-G (OW) family with one parameter is the MK family. Almetwally et al. [22] introduced modified Kies inverted Topp–Leone distribution. If $G(x;\delta)$ is the baseline CDF depending on a parameter vector $\delta$, then the CDF of the OW family is defined by

$$F(x;\Theta) = 1 - e^{-\lambda\left[\frac{G(x;\delta)}{1-G(x;\delta)}\right]^\alpha}, x > 0, \lambda, \alpha > 0, \tag{3}$$

where $\Theta$ is parameter vector $(\alpha, \lambda, \delta)$. The corresponding PDF of (3) is given by

$$f(x;\Theta) = \alpha\lambda \frac{g(x;\delta)G(x;\delta)^{\alpha-1}}{[1-G(x;\delta)]^{\alpha+1}} e^{-\lambda\left[\frac{G(x;\delta)}{1-G(x;\delta)}\right]^\alpha}, x > 0, \lambda, \alpha > 0, \tag{4}$$

The motivation of the new distribution is modeling the COVID-19. We used the COVID-19 of the United Kingdom and Canada as real data to evaluate the use of the model techniques. The new cases or new deaths of COVID-19 data are discrete data (count). We used the daily mortality rate of COVID-19 for the United Kingdom and Canada. The daily mortality rate is continuous data.

The modulated three-parameter odd Weibull inverted Topp–Leone (OWITL) dis-tribution, which has many desirable properties, which is obtained in this paper. The OWITL distribution has a very flexible PDF, can be positively skewed, symmetrical, and negatively skewed, and can allow tails to be more flexible. It is capable of mode-ling down, up, bathtub, upside-down bathtub, and reverse-J hazard rates monotonically. Also, it has a closed-form CDF and is very simple to manage, making the distribution a candidate for use in various fields, such as life testing, reliability, biomedical research,

and study of survival. Three real data applications show that certain conventional distributions with scale and shape parameters such as ITL, the Marshall-Olkin exponential, modified Kies exponential, inverse Weibull, inverse exponential, and inverse Rayleigh distributions are very competitive with the suggested distribution.

In alternative estimation methods, the maximum product spacing approach is used to estimate the continuous univariate model parameters as an alternative to the Maximum Likelihood method developed for complete sample by Cheng and Amin [23] and this developed to use under censored sample by Singh et al. [24], Basu et al. [25], Almetwally et al. [26], El-Sherpieny et al. [27], Alshenawy et al. [28, 29]. The least-square and weighted least-square methods are used to estimate the parameters of the beta distribution by Swain et al. [30]. Based on the discrepancy between CDF estimates and the empirical distribution function, the Cramér–von–Mises has been introduced by Cramér [31] and von Mises [32]. Luceño [33], used the Cramér–von–Mises estimators to Fit the generalized Pareto distribution.

We plan to make a new extension bivariate OWITL based on copula in future studies, such as done in Almetwally et al. [34], Muhammed and Almetwally [35] and Kim et al. [36]. We plan to discuss a new application for the OWITL distribution quest based on a censored sample such as done in Almetwally et al. [37] and Aslam et al. [38].

The remainder of this paper is structured as follows: We get the OWITL distribution in Sect. 2. We address some of the mathematical properties of the OWITL distribution in Sect. 3. In Sect. 4, we get the OWITL distribution by an estimation process. In Sect. 5, OWITL distribution simulation results are obtained. Three implementations of real data analytics were obtained in Sect. 6. In Sect. 7, the paper is summarized and concluded.

## 2 OWITL Distribution

Consider the ITL distribution of the positive scale factor $\delta$ and the CDF of Eqs. (1, 2) given (for $x > 0$). We define the OWITL distribution's CDF by inserting the ITL distribution's CDF into (3), such as:

$$F(x;\Theta) = 1 - e^{-\lambda\left[(1+x)^{2\delta}(1+2x)^{-\delta}-1\right]^{\alpha}}, x > 0, \alpha, \lambda, \delta > 0, \tag{5}$$

$$f(x;\Theta) = 2\alpha\lambda\delta x \frac{\left[(1+x)^{2\delta}-(1+2x)^{\delta}\right]^{\alpha-1}}{(1+2x)^{\alpha\delta+1}(1+x)^{1-2\delta}} e^{-\lambda\left[(1+x)^{2\delta}(1+2x)^{-\delta}-1\right]^{\alpha}}, x > 0, \alpha, \lambda, \delta > 0. \tag{6}$$

where $\Theta$ is parameter vector $(\alpha, \lambda, \delta)$. The OWITL distribution's hazard rate (HR) feature is shown as

$$h(x;\Theta) = 2\alpha\lambda\delta \, x \frac{\left[(1+x)^{2\delta}-(1+2x)^{\delta}\right]^{\alpha-1}}{(1+2x)^{\alpha\delta+1}(1+x)^{1-2\delta}}. \tag{7}$$

Figures 1 and 2 displays some PDF and HR plots of the OWITL distribution for the values specified for the $\alpha$, $\lambda$, and $\delta$ functions. The diagrams shown in Fig. 2 indicate
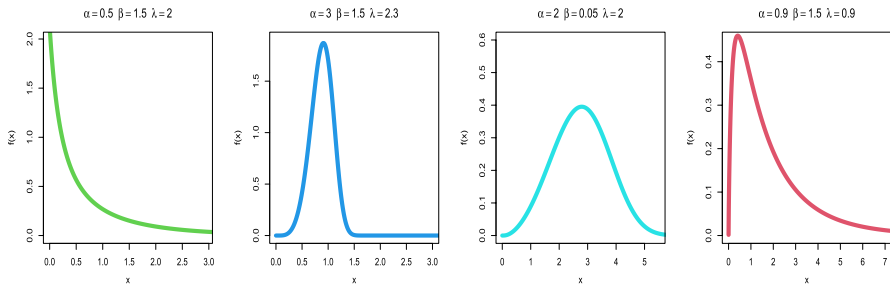
**Fig. 1** PDF of the OWITL distribution for certain parameter values
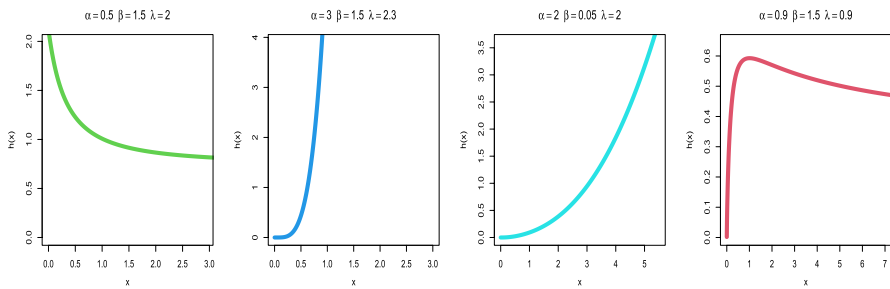


**Fig. 2** HF of the OWITL distribution for certain parameter values

that it is possible to increase, decrease and shape a bathtub for the HR feature in the OWITL distribution. One benefit of the distribution of OWITL over an ITL distribution is that the last of them can not model a phenomenon that shows increasing, decreasing shapes, failure rates of the bathtub, and therefore becomes more flexible to analyze data about lifetime.

## 3 Some Mathematical Properties of the OWITL Distribution

### 3.1 Linear Representation for the OWITL Distribution

For the OW family, we have a helpful linear representation and use it to provide a useful linear representation for the distribution of OWITL. A combination depiction of the OW family can be given as follows,

$$f(x;\Theta) = \alpha \sum_{j,k=0}^{\infty} \frac{(-1)^{k+j}}{j!} \binom{\alpha(j+1)+k}{k} \lambda^{j+1} g(x;\delta) [G(x;\delta)]^{\alpha(j+1)+k-1}. \tag{8}$$

Using the ITL distribution's PDF and CDF, the last OWITL distribution equation can be rewritten as

$$f(x;\Theta) = 2\delta(h+1)x \sum_{j,k,h=0}^{\infty} \zeta_{j,k,h}(1+x)^{-2\delta(h+1)-1}(1+2x)^{\delta(h+1)-1}, \tag{9}$$

where $\quad \zeta_{j,k,h} = \frac{\alpha\lambda^{j-1}}{(h+1)} \frac{(-1)^{j+k+h}}{j!} \begin{pmatrix} \alpha(j+1)+k \\ k \end{pmatrix} \begin{pmatrix} \alpha(j+1)+k-1 \\ h \end{pmatrix}.$ Equation (9) denotes the ITL density with parameter $\delta(h+1)$.

A combination depiction of the CDF of OW family can be given as follows,

$$F(x;\Theta) = 1 - \sum_{j,k=0}^{\infty} \begin{pmatrix} \alpha(j+1)+k \\ k \end{pmatrix} \frac{(-1)^{k+j}}{j!} \lambda^{j+1} [G(x;\delta)]^{\alpha(j+1)+k-1}. \tag{10}$$

By using Eq. 10, the CDF of OWITL distribution equation can be rewritten as

$$F(x;\Theta) = 1 - \sum_{j,k,h=0}^{\infty} \zeta_{j,k,h}(1+x)^{-2\delta(h+1)}(1+2x)^{\delta(h+1)}, \tag{11}$$

## 3.2 Quantile for the OWITL Distribution

The OWITL distribution's quantile function, i.e. $x = Q(x) = F(x,\Theta)^{-1}(Q)$, is derived as follows by inverting (5):

$$x_Q = \left( \left\{ 1 - \left( 1 + \left[ \frac{-1}{\lambda} ln(1-Q) \right]^{\frac{1}{\alpha}} \right)^{\frac{-1}{\delta}} \right\}^{-0.5} - 1 \right)^{-1} ; 0 < Q < 1 \tag{12}$$

In particular, the first quartile Q1, the second quartile Q2, and the third quartile Q3 are obtained by setting $Q = 0.25, 0.5, 0.75$, respectively, in Eq. (12).

## 3.3 Moments for the OWITL Distribution

According Hassan et al. [17], the $r_{th}$ moment of $X$ follows simply from Eq. (9) as

$$\begin{aligned} \acute{\mu}_r &= E(X^r) \\ &= 2\delta \sum_{j,k,h=0}^{\infty} \zeta_{j,k,h}(h+1)\, \beta(j+r+2,\ \delta(h+1)-r) \end{aligned} \tag{13}$$

The $r_{th}$ incomplete moment of $X$ can be obtained from (9) as

$$\omega_r(t) = \int_0^t x^r f(x, \Theta) dx$$

$$= \sum_{j,k,h=0}^{\infty} \zeta_{j,k,h} 2\delta(h+1) \sum_{b=0}^{\infty} \binom{\delta(h+1)-1}{b} \tag{14}$$

$$B(b+r+2, \ \delta(h+1)-r, \ \frac{t}{1+t})$$

where $B(b+r+2, \ \delta(h+1)-r, \ \frac{t}{1+t})$ is the incomplete beta function.

## 4 Estimation Methods

This section uses six different estimation methods called: maximum likelihood, least-square, the maximum product of spacing, weighted least-square, Cramér-von Mises, and Anderson–Darling, to analyze the estimation problem of the OWITL distribution parameters.

### 4.1 Maximum Likelihood Estimators

Let $x_1, \ldots, x_n$ be a random sample with the parameters $\alpha$, $\lambda$ and $\delta$ from an OWITL distribution. The log-likelihood function for the distribution of OWITL is given by

$$l(\Theta) = n[\ln(2) + \ln(\alpha) + \ln(\lambda) + \ln(\delta)] + \sum_{i=1}^{n} \ln(x_i) - (1 - 2\delta) \sum_{i=1}^{n} \ln(1 + x_i)$$

$$+ (\alpha - 1) \sum_{i=1}^{n} \ln \left[ (1 + x_i)^{2\delta} - (1 + 2x_i)^{\delta} \right] - (\alpha\delta + 1) \sum_{i=1}^{n} \ln(1 + 2x_i)$$

$$- \lambda \sum_{i=1}^{n} \left[ (1 + x_i)^{2\delta} (1 + 2x_i)^{-\delta} - 1 \right]^{\alpha} \tag{15}$$

The partial derivatives of $l(\Theta)$ with respect to the model parameters $\alpha$, $\lambda$ and $\delta$ are

$$\frac{\partial l(\Theta)}{\partial \alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n} \ln \left[ (1 + x_i)^{2\delta} - (1 + 2x_i)^{\delta} \right] - \delta \sum_{i=1}^{n} \ln(1 + 2x_i)$$

$$- \lambda \sum_{i=1}^{n} \left[ (1 + x_i)^{2\delta} (1 + 2x_i)^{-\delta} - 1 \right]^{\alpha} \ln \left[ (1 + x_i)^{2\delta} (1 + 2x_i)^{-\delta} - 1 \right] \tag{16}$$

$$\frac{\partial l(\Theta)}{\partial \lambda} = \frac{n}{\lambda} + - \sum_{i=1}^{n} \left[ (1 + x_i)^{2\delta} (1 + 2x_i)^{-\delta} - 1 \right]^{\alpha} \tag{17}$$

and

$$\frac{\partial l(\Theta)}{\partial \delta} = \frac{n}{\delta} + 2 \sum_{i=1}^{n} \ln(1 + x_i) - \alpha \sum_{i=1}^{n} \ln(1 + 2x_i)$$

$$+ (\alpha - 1) \sum_{i=1}^{n} \frac{2(1 + x_i)^{2\delta} \ln(1 + x_i) - (1 + 2x_i)^{\delta} \ln(1 + 2x_i)}{(1 + x_i)^{2\delta} - (1 + 2x_i)^{\delta}} \tag{18}$$

$$- \alpha \sum_{i=1}^{n} \left[ \frac{(1 + x_i)^{2\delta}}{(1 + 2x_i)^{\delta}} - 1 \right]^{\alpha-1} \left[ \frac{(1 + x_i)^2}{1 + 2x_i} \right]^{\delta} \ln \left[ \frac{(1 + x_i)^2}{1 + 2x_i} \right]$$

It is possible to obtain the maximum likelihood estimation (MLE) of $\alpha$, $\lambda$, and $\delta$ by maximizing the last equation for $\alpha$, $\lambda$, and $\delta$. By using the Newton-Rapshon method, the R packages can be used to optimize the log-likelihood function for obtaining the MLE.

## 4.2 Least-Squares and Weighted Least-Squares Methods

To estimate the parameters of various distributions, the least-squares (LS) and weighted least-square (WLS) methods are used. Let $x_{(1)} < \cdots < x_{(n)}$ be a random sample with the $\alpha$, $\lambda$ and $\delta$ parameters from the OWITL distribution. LS estimators (LSE) and WLS estimators (WLSE) of the $\alpha$, $\lambda$ and $\delta$ distribution parameters of OWITL can be obtained by minimizing the following:

$$V(\Theta) = \sum_{i=1}^{n} v_i \left[ 1 - e^{-\lambda \left[ (1 + x_{(i)})^{2\delta} (1 + 2x_{(i)})^{-\delta} - 1 \right]^{\alpha}} - \frac{i}{n + 1} \right]^2 \tag{19}$$

$v_i = 1$ for LSE and $v_i = \frac{(n+1)^2(n+2)}{[i(n-I+1)]}$ for WLSE with respect to $\alpha$, $\lambda$ and $\delta$. Furthermore, by resolving the nonlinear equations, the LSE and WLSE follow:

$$\frac{\partial V(\Theta)}{\partial \alpha} = 2\lambda \sum_{i=1}^{n} v_{(i)} \left[ 1 - e^{-\lambda \left[ (1 + x_{(i)})^{2\delta} (1 + 2x_{(i)})^{-\delta} - 1 \right]^{\alpha}} \right.$$

$$\left. - \frac{i}{n + 1} \right] e^{-\lambda \left[ (1 + x_{(i)})^{2\delta} (1 + 2x_{(i)})^{-\delta} - 1 \right]^{\alpha}} \tag{20}$$

$$\left[ (1 + x_{(i)})^{2\delta} (1 + 2x_{(i)})^{-\delta} - 1 \right]^{\alpha} \ln \left[ (1 + x_{(i)})^{2\delta} (1 + 2x_{(i)})^{-\delta} - 1 \right],$$

$$\frac{\partial V(\Theta)}{\partial \lambda} = 2 \sum_{i=1}^{n} v_i \left[ 1 - e^{-\lambda \left[ (1 + x_{(i)})^{2\delta} (1 + 2x_{(i)})^{-\delta} - 1 \right]^{\alpha}} \right.$$

$$\left. - \frac{i}{n + 1} \right] e^{-\lambda \left[ (1 + x_{(i)})^{2\delta} (1 + 2x_{(i)})^{-\delta} - 1 \right]^{\alpha}} \tag{21}$$

$$\left[ (1 + x_{(i)})^{2\delta} (1 + 2x_{(i)})^{-\delta} - 1 \right]^{\alpha},$$

and

$$
\frac{\partial V(\Theta)}{\partial \delta} = 2\alpha\lambda \sum_{i=1}^{n} v_i \left[ 1 - e^{-\lambda\left[(1+x_{(i)})^{2\delta}(1+2x_{(i)})^{-\delta}-1\right]^{\alpha}} \right.
$$

$$
\left. - \frac{i}{n+1} \right] e^{-\lambda\left[(1+x_{(i)})^{2\delta}(1+2x_{(i)})^{-\delta}-1\right]^{\alpha}}
$$

$$
\left[ \frac{(1+x_{(i)})^{2\delta}}{(1+2x_{(i)})^{\delta}} - 1 \right]^{\alpha-1} \left[ \frac{(1+x_{(i)})^2}{1+2x_{(i)}} \right]^{\delta} \ln \left[ \frac{(1+x_{(i)})^2}{1+2x_{(i)}} \right].
$$

(22)

### 4.3 Maximum Product of Spacings Method

If $x_{(1)} < \cdots < x_{(n)}$ is a random sample of the size $n$, you can describe the uniform spacing of the OWITL distribution as:

$$
D_i(\Theta) = F(x_{(i)}, \Theta) - F(x_{(i-1)}, \Theta); \ i = 1, \ldots, n+1
$$

(23)

where $D_i(\Theta)$ denotes to the uniform spacings, $F(x_{(0)}, \Theta) = 0$, $F(x_{(n+1)}, \Theta) = 1$ and $\sum_{i=1}^{n+1} D_i(\Theta) = 1$. The maximum product of spacing (MPS) estimators (MPSE) of the OWITL parameters can be obtained by maximizing

$$
G(\Theta) = \frac{1}{n+1} \sum_{i=1}^{n+1} \ln \left[ e^{-\lambda\left(\left[\frac{(1+x_{(i-1)})^2}{1+2x_{(i-1)}}\right]^{\delta}-1\right)^{\alpha}} - e^{-\lambda\left(\left[\frac{(1+x_{(i)})^2}{1+2x_{(i)}}\right]^{\delta}-1\right)^{\alpha}} \right]
$$

(24)

with respect to $\alpha$, $\lambda$ and $\delta$. Further, the MPSE of the OWITL parameters can also be obtained by solving

$$
\frac{\partial G(\Theta)}{\partial \alpha} = \frac{\lambda}{n+1} \sum_{i=1}^{n+1} \frac{e^{-\lambda\left(\left[\tau_i\right]^{\delta}-1\right)^{\alpha}} \left(\left[\tau_i\right]^{\delta}-1\right)^{\alpha} \ln\left(\left[\tau_i\right]^{\delta}-1\right)}{e^{-\lambda\left(\left[\tau_{i-1}\right]^{\delta}-1\right)^{\alpha}} - e^{-\lambda\left(\left[\tau_i\right]^{\delta}-1\right)^{\alpha}}}
$$

$$
- \frac{e^{-\lambda\left(\left[\tau_{i-1}\right]^{\delta}-1\right)^{\alpha}} \left(\left[\tau_{i-1}\right]^{\delta}-1\right)^{\alpha} \ln\left(\left[\tau_{i-1}\right]^{\delta}-1\right)}{e^{-\lambda\left(\left[\tau_{i-1}\right]^{\delta}-1\right)^{\alpha}} - e^{-\lambda\left(\left[\tau_i\right]^{\delta}-1\right)^{\alpha}}},
$$

(25)

$$
\frac{\partial G(\Theta)}{\partial \lambda} = \frac{1}{n+1} \sum_{i=1}^{n+1} \frac{e^{-\lambda\left(\left[\tau_i\right]^{\delta}-1\right)^{\alpha}} \left(\left[\tau_i\right]^{\delta}-1\right)^{\alpha}}{e^{-\lambda\left(\left[\tau_{i-1}\right]^{\delta}-1\right)^{\alpha}} - e^{-\lambda\left(\left[\tau_i\right]^{\delta}-1\right)^{\alpha}}}
$$

$$
- \frac{e^{-\lambda\left(\left[\tau_{i-1}\right]^{\delta}-1\right)^{\alpha}} \left(\left[\tau_{i-1}\right]^{\delta}-1\right)^{\alpha}}{e^{-\lambda\left(\left[\tau_{i-1}\right]^{\delta}-1\right)^{\alpha}} - e^{-\lambda\left(\left[\tau_i\right]^{\delta}-1\right)^{\alpha}}},
$$

(26)

and

$$\frac{\partial G(\Theta)}{\partial \delta} = \frac{\alpha \lambda}{n+1} \sum_{i=1}^{n+1} \frac{e^{-\lambda \left( [\tau_i]^\delta - 1 \right)^\alpha} \left[ (\tau_i)^\delta - 1 \right]^{\alpha-1} [\tau_i]^\delta \ln [\tau_i]}{e^{-\lambda \left( [\tau_{i-1}]^\delta - 1 \right)^\alpha} - e^{-\lambda \left( [\tau_i]^\delta - 1 \right)^\alpha}}$$
$$- \frac{e^{-\lambda \left( [\tau_{i-1}]^\delta - 1 \right)^\alpha} \left[ (\tau_{i-1})^\delta - 1 \right]^{\alpha-1} [\tau_{i-1}]^\delta \ln [\tau_{i-1}]}{e^{-\lambda \left( [\tau_{i-1}]^\delta - 1 \right)^\alpha} - e^{-\lambda \left( [\tau_i]^\delta - 1 \right)^\alpha}}. \tag{27}$$

where $\tau_i = \frac{(1+x_{(i)})^2}{1+2x_{(i)}}$.

### 4.4 Cramér–von–Mises Method

The Cramér–von–Mises (CVM) can be obtained for OWITL by minimizing the following function with respect to $\alpha$, $\lambda$, and $\delta$, the CVM estimators (CVME) of the OWITL parameters $\alpha$, $\lambda$, and $\delta$ are obtained.

$$C(\Theta) = \frac{1}{12} + \sum_{i=1}^{n} \left( 1 - e^{-\lambda \left( \left[ \left[ \frac{(1+x_{(i)})^2}{1+2x_{(i)}} \right]^\delta \right] - 1 \right)^\alpha} - \frac{2i-1}{2n} \right)^2 \tag{28}$$

In addition, by resolving the nonlinear equations, the CVME as follows

$$\frac{\partial C(\Theta)}{\partial \alpha} = 2\lambda \sum_{i=1}^{n} \left( 1 - e^{-\lambda \left( [\tau_i]^\delta - 1 \right)^\alpha} - \frac{2i-1}{2n} \right) e^{-\lambda \left( [\tau_i]^\delta - 1 \right)^\alpha}$$
$$\ln \left( [\tau_i]^\delta - 1 \right), \tag{29}$$

$$\frac{\partial C(\Theta)}{\partial \lambda} = 2 \sum_{i=1}^{n} \left( 1 - e^{-\lambda \left( [\tau_i]^\delta - 1 \right)^\alpha} - \frac{2i-1}{2n} \right) e^{-\lambda \left( [\tau_i]^\delta - 1 \right)^\alpha}$$
$$\left( [\tau_i]^\delta - 1 \right)^\alpha, \tag{30}$$

and

$$\frac{\partial C(\Theta)}{\partial \delta} = \alpha 2 \sum_{i=1}^{n} \left( 1 - e^{-\lambda \left( [\tau_i]^\delta - 1 \right)^\alpha} - \frac{2i-1}{2n} \right) e^{-\lambda \left( [\tau_i]^\delta - 1 \right)^\alpha}$$
$$\left( [\tau_i]^\delta - 1 \right)^{\alpha-1} [\tau_i]^\delta \ln [\tau_i]. \tag{31}$$

### 4.5 Anderson–Darling Method

In Anderson–Darling (AD), other forms of minimum distance estimators are the AD estimators (ADE). The ADE of the parameters of the OWITL is acquired by minimizing

$$
A(\Theta) = -n - \frac{1}{n} \sum_{i=1}^{n} (2i-1) \left( \ln \left( 1 - e^{-\lambda \left( \left[ \frac{(1+x_{(i)})^2}{1+2x_{(i)}} \right]^{\delta} - 1 \right)^{\alpha}} \right) \right.
$$
$$
\left. - \left( \left[ \frac{(1+x_{(n+1-i)})^2}{1+2x_{(n+1-i)}} \right]^{\delta} - 1 \right)^{\alpha} \right)^2 \right)
\tag{32}
$$

Regarding $\alpha$, $\lambda$ and $\delta$, respectively. It is also possible to obtain the ADE by resolving the nonlinear equations.

## 5 Simulation Results

In this portion, a simulation analysis evaluates the output of six different estimators of the OWITL parameters. For the different values of parameter $\alpha = (0.5, 3)$, $\lambda = (0.5, 3)$, and $\delta = (0.5, 3)$, we consider the different sample sizes $n = 50, 100, 150$. We create 10000 iteration of random samples for OWLTL distribution. We get the average values of relative bias (RB) and their corresponding mean square error (MSE) for each calculation.

The output of the different estimators is evaluated in terms of RB and MSE, i.e. those whose MSE values are closer to zero would be the most effective method of estimating. Simulation results are obtained via the R program. Tables 1 and 2 show the RB and MSE values for MLE, LS, WLS, MPS, CVM, and AD. Also, as the sample size increases in all situations, the mean depending on all estimation methods tends towards the true parameter values, suggesting that all estimators are asymptotically unbiased. If $\delta < 1$, and $\alpha > 1$, the MPS is the best estimation methods in most times. If $\delta > 1$, and $\alpha > 1$, the AD method is the best estimation methods in most times. If $\delta < 1$, $\lambda < 1$ and $\alpha < 1$, the LS is the best estimation methods in most times. If $\delta < 1$, $\lambda < 1$ and $\alpha < 1$, the LS is the best estimation methods in most times. If $\delta < 1$, $\lambda > 1$ and $\alpha < 1$, the LS is the best estimation methods in most times. If $\delta$, and $\lambda$ increases and $\alpha < 1$, the LS is the best estimation methods in most times (Figs. 3, 4).

## 6 Application of Real Data Analysis

This section is dedicated to demonstrate the potential of two real data sets for the OWITL distribution. Compared with other competitive models, OWITL delivery, namely: extended odd Weibull inverse Rayleigh (EOWIR) which is introduced by Almetwally [39], generalized inverse Weibull (GIW) distribution which is introduced
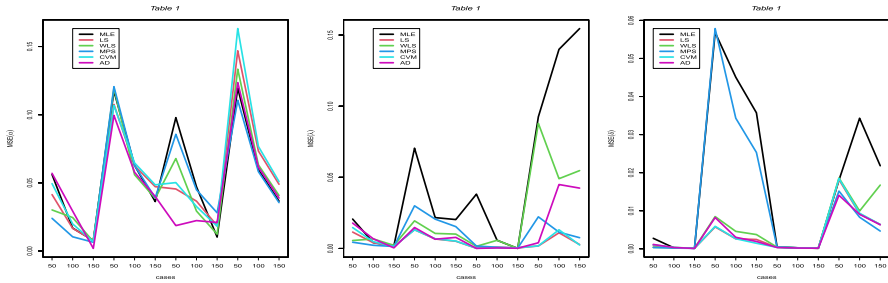
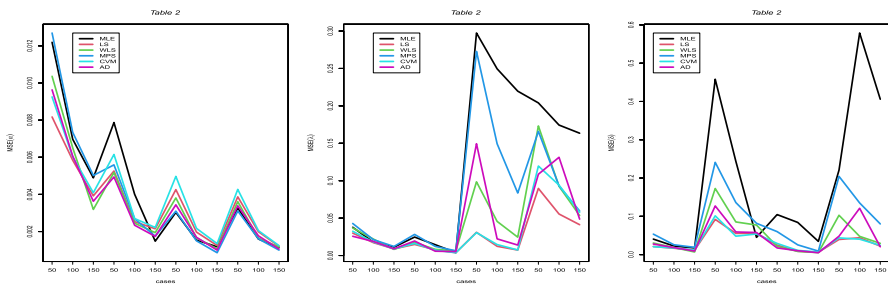**Fig. 3** MSE for different cases in Table 1



**Fig. 4** MSE for different cases in Table 2

by De Gusmao et al. [40], exponential Lomax (ELo) distribution which is introduced by El-Bassiouny et al. [41], modified Kies exponential (MKEx) which is introduced by Al-Babtain et al. [21], and power Lomax (PL) distribution by Rady et al. [42].

For both models fitted on the basis of two real data sets, Tables 3 and 4 include Cramér-von Mises (W*), Anderson–Darling (A*) and Kolmogorov–Smirnov (KS) statistic values along with its *P* value. Furthermore, these tables include the parameters MLE and Standard Errors (SE) for the models considered (Fig. 5).

The OWITL distribution has the highest *P* value and the lowest distance of the Kolmogorov–Smirnov(KS), W* and A* values in Tables 3 and 4 when compared to all other models used here to suit the COVID-19 results. Figures 6 and 7 show the empirical, histogram, QQ-plot, and PP-plot fit for the OWITL distribution of Canadian and United Kingdom COVID-19 results. These applications show that the OWITL model can yield better fit than some other distribution.

In order to classify the possible shapes behind these data of the unknown hrf, we plot the total time on test (TTT) plot in Fig. 5 (see Aarset [43] for further details on the use of TTT plots in data analysis). In Fig. 5, since the blue line is convex, then concave, the unknown hrf probably presents a bathtub shape. Therefore, the OWITL distribution is appropriate to fit the data, where probTV is the accumulated probability distribution for time value and sorted_time_value is the vector time Value sorted of data. If data include outliers, we can use the Robust methods as least trimmed square, least median square, M, S, and MM, see Almongy and Almetwally [44, 45]. Artificial intelligence techniques can also be used see Olson et al. [46], Shi et al. [47] and Tien [48].

**Table 1** Mean and MSE of the MLE, LS, MPS, WLS, CVM and AD estimates for OWITL distribution when $\alpha = 3$

| $\beta$ | $\lambda$ | $n$ | | MLE | | LS | | WLS | | MPS | | CVM | | AD | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | RB | MSE | RB | MSE | RB | MSE | RB | MSE | RB | MSE | RB | MSE |
| 0.5 | 0.5 | 50 | $\alpha$ | − 0.02041 | 0.05612 | 0.01571 | 0.04136 | − 0.01411 | 0.03010 | − 0.02038 | 0.02395 | 0.02738 | 0.04941 | 0.00774 | 0.05697 |
| | | | $\beta$ | 0.03888 | 0.02066 | − 0.06825 | 0.01152 | 0.03552 | 0.00558 | 0.03882 | 0.00438 | − 0.10389 | 0.01468 | − 0.04320 | 0.01784 |
| | | | $\lambda$ | − 0.00829 | 0.00273 | 0.01926 | 0.00077 | − 0.00357 | 0.00035 | − 0.00835 | 0.00031 | 0.02911 | 0.00103 | 0.01841 | 0.00111 |
| | | 100 | $\alpha$ | − 0.01339 | 0.01675 | 0.00437 | 0.01660 | 0.00382 | 0.02450 | − 0.01338 | 0.01039 | 0.01156 | 0.01981 | − 0.00590 | 0.02924 |
| | | | $\beta$ | 0.03074 | 0.00412 | − 0.02187 | 0.00403 | − 0.01950 | 0.00669 | 0.03073 | 0.00223 | − 0.04199 | 0.00480 | 0.00923 | 0.00681 |
| | | | $\lambda$ | − 0.00718 | 0.00022 | 0.00658 | 0.00027 | 0.00657 | 0.00029 | − 0.00721 | 0.00016 | 0.01113 | 0.00031 | 0.00165 | 0.00030 |
| | | 150 | $\alpha$ | − 0.00892 | 0.00773 | − 0.00975 | 0.00769 | 0.00267 | 0.00685 | − 0.00890 | 0.00640 | − 0.00742 | 0.00673 | − 0.00100 | 0.00187 |
| | | | $\beta$ | 0.01931 | 0.00194 | 0.02515 | 0.00150 | − 0.01238 | 0.00220 | 0.01926 | 0.00139 | 0.01921 | 0.00135 | 0.00212 | 0.00041 |
| | | | $\lambda$ | − 0.00433 | 0.00011 | − 0.00389 | 0.00010 | 0.00370 | 0.00014 | − 0.00433 | 0.00009 | − 0.00248 | 0.00010 | 0.00026 | 0.00007 |
| | 3 | 50 | $\alpha$ | − 0.04226 | 0.11728 | − 0.01304 | 0.10754 | 0.00370 | 0.11935 | − 0.04216 | 0.12054 | 0.01241 | 0.10678 | 0.00143 | 0.09947 |
| | | | $\beta$ | 0.03994 | 0.07046 | 0.02221 | 0.01317 | 0.00131 | 0.01940 | 0.03996 | 0.02997 | − 0.00648 | 0.01310 | 0.00569 | 0.01472 |
| | | | $\lambda$ | 0.01190 | 0.05683 | − 0.00029 | 0.00572 | 0.00435 | 0.00844 | 0.01180 | 0.05786 | 0.00432 | 0.00588 | 0.00362 | 0.00818 |
| | | 100 | $\alpha$ | − 0.02897 | 0.06284 | − 0.00519 | 0.06256 | − 0.00011 | 0.05594 | − 0.02895 | 0.06268 | 0.00783 | 0.06459 | 0.00093 | 0.05772 |
| | | | $\beta$ | 0.05447 | 0.02168 | 0.00892 | 0.00665 | 0.01908 | 0.01056 | 0.05449 | 0.02061 | − 0.00625 | 0.00683 | 0.00346 | 0.00648 |
| | | | $\lambda$ | 0.00131 | 0.04507 | 0.00057 | 0.00259 | − 0.00109 | 0.00455 | 0.00126 | 0.03430 | 0.00303 | 0.00261 | 0.00144 | 0.00299 |
| | | 150 | $\alpha$ | − 0.01986 | 0.03623 | − 0.00209 | 0.04718 | 0.00444 | 0.03856 | − 0.01981 | 0.03901 | 0.00716 | 0.04849 | 0.00322 | 0.03994 |
| | | | $\beta$ | 0.03670 | 0.02031 | − 0.00082 | 0.00507 | 0.02338 | 0.01010 | 0.03662 | 0.01534 | − 0.01130 | 0.00520 | 0.01585 | 0.00777 |
| | | | $\lambda$ | 0.00207 | 0.03572 | 0.00024 | 0.00242 | − 0.00236 | 0.00371 | 0.00207 | 0.02530 | 0.00409 | 0.00143 | − 0.00027 | 0.00190 |

**Table 1** (continued)

| β | λ | n | | MLE RB | MLE MSE | LS RB | LS MSE | WLS RB | WLS MSE | MPS RB | MPS MSE | CVM RB | CVM MSE | AD RB | AD MSE |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 0.5 | 50 | α | −0.04415 | 0.09784 | −0.00583 | 0.04548 | −0.00094 | 0.06783 | −0.04409 | 0.08560 | 0.00386 | 0.05016 | 0.00045 | 0.01860 |
| | | | β | 0.00088 | 0.03817 | 0.00025 | 0.00014 | 0.00006 | 0.00149 | 0.00088 | 0.00170 | −0.00024 | 0.00016 | −0.00009 | 0.00007 |
| | | | λ | −0.01541 | 0.00046 | −0.00310 | 0.00035 | −0.00169 | 0.00039 | −0.01546 | 0.00043 | 0.00130 | 0.00035 | 0.00053 | 0.00032 |
| | | 100 | α | −0.03090 | 0.04711 | −0.01938 | 0.03654 | −0.01164 | 0.02927 | −0.03088 | 0.04488 | −0.01209 | 0.03260 | −0.00171 | 0.02228 |
| | | | β | 0.00130 | 0.00575 | 0.00121 | 0.00087 | 0.00298 | 0.00573 | 0.00130 | 0.00086 | 0.00073 | 0.00047 | 0.00034 | 0.00038 |
| | | | λ | −0.01061 | 0.00022 | −0.00709 | 0.00022 | −0.00407 | 0.00022 | −0.01065 | 0.00023 | −0.00381 | 0.00021 | −0.00022 | 0.00017 |
| | | 150 | α | −0.02027 | 0.01019 | −0.00612 | 0.01858 | −0.01044 | 0.01231 | −0.02022 | 0.02799 | 0.00061 | 0.01816 | −0.00658 | 0.02089 |
| | | | β | 0.00090 | 0.00003 | 0.00060 | 0.00041 | 0.00067 | 0.00006 | 0.00089 | 0.00028 | 0.00001 | 0.00046 | 0.00027 | 0.00016 |
| | | | λ | −0.00682 | 0.00010 | −0.00223 | 0.00017 | −0.00305 | 0.00011 | −0.00681 | 0.00013 | 0.00053 | 0.00017 | −0.00195 | 0.00013 |
| | 3 | 50 | α | −0.03719 | 0.11941 | −0.00116 | 0.14687 | 0.00741 | 0.13336 | −0.03711 | 0.11054 | 0.02698 | 0.16317 | 0.00937 | 0.12355 |
| | | | β | −0.00908 | 0.09253 | −0.00087 | 0.00175 | 0.00866 | 0.08792 | −0.00902 | 0.02222 | 0.00032 | 0.00182 | −0.00024 | 0.00400 |
| | | | λ | −0.01163 | 0.01801 | −0.00417 | 0.01817 | −0.00129 | 0.01859 | −0.01170 | 0.01514 | 0.00570 | 0.01844 | 0.00061 | 0.01404 |
| | | 100 | α | −0.02732 | 0.05961 | −0.00364 | 0.07334 | 0.00088 | 0.06315 | −0.02730 | 0.05821 | 0.01042 | 0.07671 | 0.00154 | 0.06154 |
| | | | β | −0.00228 | 0.13994 | −0.00166 | 0.01102 | −0.00709 | 0.04903 | −0.00225 | 0.01145 | 0.00026 | 0.01313 | −0.00072 | 0.04493 |
| | | | λ | −0.00913 | 0.03429 | −0.00201 | 0.00916 | 0.00212 | 0.00991 | −0.00917 | 0.00827 | 0.00267 | 0.00931 | 0.00072 | 0.00903 |
| | | 150 | α | −0.01857 | 0.03656 | −0.00020 | 0.04892 | 0.00386 | 0.04068 | −0.01851 | 0.03546 | 0.00912 | 0.05086 | 0.00307 | 0.03881 |
| | | | β | −0.00280 | 0.15454 | −0.00124 | 0.00272 | 0.00461 | 0.05472 | −0.00279 | 0.00756 | −0.00086 | 0.00243 | 0.00065 | 0.04241 |
| | | | λ | −0.00559 | 0.02180 | −0.00076 | 0.00634 | 0.00109 | 0.01672 | −0.00559 | 0.00463 | 0.00254 | 0.00635 | 0.00102 | 0.00629 |

**Table 2** Mean and MSE of the MLE, LS, MPS, WLS, CVM and AD estimates for OWITL distribution when $\alpha = 0.5$

| $\beta$ | $\lambda$ | $n$ | | MLE RB | MLE MSE | LS RB | LS MSE | WLS RB | WLS MSE | MPS RB | MPS MSE | CVM RB | CVM MSE | AD RB | AD MSE |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.5 | 0.5 | 50 | $\alpha$ | − 0.04867 | 0.01219 | 0.00189 | 0.00817 | 0.00471 | 0.01036 | − 0.04826 | 0.01269 | 0.03848 | 0.00925 | 0.00822 | 0.00963 |
| | | | $\beta$ | 0.08763 | 0.03776 | 0.05293 | 0.02998 | 0.06975 | 0.03687 | 0.08956 | 0.04283 | 0.05268 | 0.03244 | 0.03595 | 0.02575 |
| | | | $\lambda$ | 0.11416 | 0.04050 | 0.03572 | 0.02112 | 0.05382 | 0.03020 | 0.11242 | 0.05351 | 0.02407 | 0.02073 | 0.06526 | 0.02760 |
| | | 100 | $\alpha$ | − 0.02960 | 0.00697 | − 0.00568 | 0.00584 | − 0.00104 | 0.00651 | − 0.02889 | 0.00734 | 0.01149 | 0.00602 | 0.00282 | 0.00606 |
| | | | $\beta$ | 0.04788 | 0.01791 | 0.01561 | 0.01723 | 0.03086 | 0.02202 | 0.05036 | 0.02121 | 0.01428 | 0.01808 | 0.02295 | 0.01880 |
| | | | $\lambda$ | 0.06096 | 0.02235 | 0.05465 | 0.01665 | 0.04867 | 0.01846 | 0.05879 | 0.02565 | 0.05080 | 0.01729 | 0.04960 | 0.01838 |
| | | 150 | $\alpha$ | − 0.02542 | 0.00489 | − 0.00633 | 0.00392 | 0.00890 | 0.00319 | − 0.02487 | 0.00502 | 0.00543 | 0.00409 | 0.00719 | 0.00363 |
| | | | $\beta$ | 0.03298 | 0.01117 | − 0.00381 | 0.00888 | 0.01781 | 0.00822 | 0.03175 | 0.01209 | − 0.00496 | 0.00896 | 0.01502 | 0.00897 |
| | | | $\lambda$ | 0.04789 | 0.01750 | 0.04803 | 0.01145 | 0.00904 | 0.00686 | 0.04738 | 0.01893 | 0.04507 | 0.01183 | 0.02104 | 0.00937 |
| | 3 | 50 | $\alpha$ | − 0.01846 | 0.00787 | 0.01257 | 0.00526 | 0.01144 | 0.00518 | − 0.01826 | 0.00559 | 0.04447 | 0.00615 | 0.02625 | 0.00494 |
| | | | $\beta$ | 0.09194 | 0.02467 | 0.03070 | 0.01485 | 0.02619 | 0.01752 | 0.09198 | 0.02820 | 0.02243 | 0.01617 | 0.03971 | 0.01941 |
| | | | $\lambda$ | − 0.02033 | 0.45807 | − 0.01133 | 0.09195 | 0.00421 | 0.17277 | − 0.02055 | 0.24099 | − 0.01263 | 0.10126 | − 0.01495 | 0.12711 |
| | | 100 | $\alpha$ | − 0.00678 | 0.00401 | 0.00459 | 0.00253 | 0.00912 | 0.00245 | − 0.00655 | 0.00269 | 0.02005 | 0.00270 | 0.00731 | 0.00234 |
| | | | $\beta$ | 0.06590 | 0.01448 | 0.02348 | 0.00768 | 0.02529 | 0.00825 | 0.06689 | 0.01219 | 0.01729 | 0.00703 | 0.01145 | 0.00579 |
| | | | $\lambda$ | − 0.02393 | 0.24358 | − 0.00367 | 0.05540 | − 0.00304 | 0.08529 | − 0.02439 | 0.13591 | − 0.00420 | 0.04809 | 0.00641 | 0.05908 |
| | | 150 | $\alpha$ | − 0.00685 | 0.00148 | 0.00129 | 0.00215 | 0.01140 | 0.00194 | − 0.00677 | 0.00175 | 0.01198 | 0.00222 | 0.01386 | 0.00174 |
| | | | $\beta$ | 0.03757 | 0.00385 | − 0.00002 | 0.00379 | 0.01704 | 0.00625 | 0.03768 | 0.00629 | − 0.00307 | 0.00375 | 0.01599 | 0.00527 |
| | | | $\lambda$ | − 0.01991 | 0.04503 | 0.00568 | 0.05564 | − 0.01122 | 0.07723 | − 0.01996 | 0.08180 | 0.00430 | 0.05388 | − 0.01407 | 0.05773 |

**Table 2** (continued)

| β | λ | n | | MLE | | LS | | WLS | | MPS | | CVM | | AD | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | RB | MSE | RB | MSE | RB | MSE | RB | MSE | RB | MSE | RB | MSE |
| 3 | 0.5 | 50 | α | − 0.02950 | 0.00303 | 0.01069 | 0.00426 | 0.01427 | 0.00381 | − 0.02937 | 0.00310 | 0.03928 | 0.00497 | 0.01753 | 0.00344 |
| | | | β | 0.00422 | 0.29736 | 0.00406 | 0.03102 | − 0.00436 | 0.09851 | − 0.01702 | 0.27278 | 0.00536 | 0.03061 | 0.01234 | 0.14940 |
| | | | λ | 0.04629 | 0.10453 | 0.01975 | 0.02492 | 0.04551 | 0.02152 | 0.04527 | 0.06005 | 0.08555 | 0.02908 | 0.02836 | 0.01743 |
| | | 100 | α | − 0.01355 | 0.00154 | 0.01109 | 0.00197 | 0.01452 | 0.00166 | − 0.01328 | 0.00150 | 0.02497 | 0.00217 | 0.01389 | 0.00169 |
| | | | β | − 0.01736 | 0.24951 | − 0.00221 | 0.01235 | 0.01687 | 0.04560 | 0.00496 | 0.14935 | 0.00256 | 0.01465 | 0.00160 | 0.02226 |
| | | | λ | − 0.01096 | 0.08405 | 0.00167 | 0.00875 | − 0.00942 | 0.00835 | − 0.01235 | 0.02543 | 0.03781 | 0.01095 | 0.01408 | 0.01015 |
| | | 150 | α | − 0.00616 | 0.00118 | 0.01142 | 0.00125 | 0.01308 | 0.00107 | − 0.00594 | 0.00086 | 0.02071 | 0.00135 | 0.01311 | 0.00100 |
| | | | β | 0.01367 | 0.21963 | 0.00208 | 0.00735 | 0.00502 | 0.02452 | 0.01459 | 0.08353 | 0.00372 | 0.00738 | 0.00528 | 0.01399 |
| | | | λ | − 0.03757 | 0.03456 | 0.00283 | 0.00632 | 0.00490 | 0.00526 | − 0.03942 | 0.00915 | 0.02396 | 0.00679 | 0.00281 | 0.00492 |
| | 3 | 50 | α | − 0.03478 | 0.00327 | − 0.01459 | 0.00387 | 0.00337 | 0.00361 | − 0.03468 | 0.00311 | 0.00935 | 0.00427 | 0.00046 | 0.00339 |
| | | | β | − 0.00249 | 0.20393 | 0.00183 | 0.08968 | 0.01124 | 0.17315 | − 0.01247 | 0.16593 | 0.00207 | 0.11958 | − 0.00247 | 0.10862 |
| | | | λ | − 0.03852 | 0.22066 | − 0.00733 | 0.03950 | − 0.00248 | 0.10276 | − 0.03876 | 0.20435 | 0.00770 | 0.04365 | 0.00078 | 0.04806 |
| | | 100 | α | − 0.02500 | 0.00160 | − 0.00363 | 0.00199 | 0.00132 | 0.00165 | − 0.02498 | 0.00161 | 0.00988 | 0.00205 | 0.00190 | 0.00177 |
| | | | β | − 0.00035 | 0.17435 | − 0.00680 | 0.05548 | 0.00651 | 0.09257 | − 0.00255 | 0.09308 | 0.01554 | 0.09410 | 0.01220 | 0.13127 |
| | | | λ | − 0.04053 | 0.57885 | − 0.00553 | 0.04404 | − 0.00931 | 0.04760 | − 0.04065 | 0.13495 | 0.00392 | 0.04027 | − 0.00016 | 0.12101 |
| | | 150 | α | − 0.01705 | 0.00114 | − 0.00455 | 0.00121 | 0.00301 | 0.00116 | − 0.01705 | 0.00100 | 0.00366 | 0.00124 | 0.00119 | 0.00106 |
| | | | β | − 0.01242 | 0.16346 | − 0.02066 | 0.04118 | 0.00596 | 0.05344 | − 0.00035 | 0.05952 | 0.00177 | 0.05779 | 0.00150 | 0.04860 |
| | | | λ | − 0.02932 | 0.40590 | 0.00404 | 0.02209 | − 0.00063 | 0.02942 | − 0.02932 | 0.08014 | 0.00912 | 0.02416 | 0.00360 | 0.02043 |
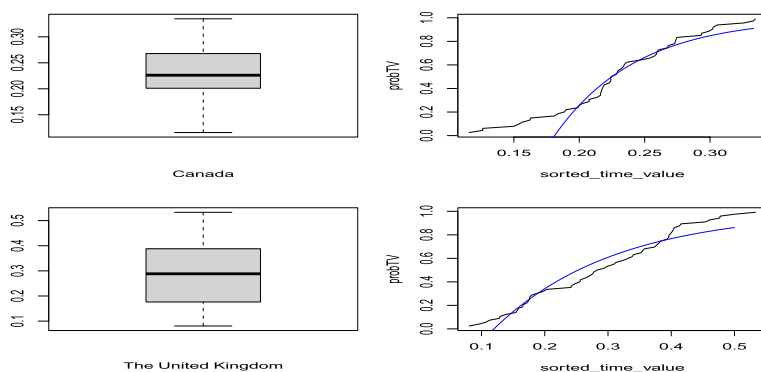
**Fig. 5** Boxplot and TTT plot of these data

**Table 3** MLE, SE, KS test and *P* values for COVID-19 data of Canada

|  | $\alpha$ | $\lambda$ | $\delta$ | KS | *P* Value | W* | A* |
|---|---|---|---|---|---|---|---|
| **OWITL** | | | | | | | |
| Estimate | 2.7178 | 29.9649 | 6.1262 | 0.1030 | 0.5576 | 0.0699 | 0.3791 |
| SE | 0.4372 | 116.9170 | 6.8681 | | | | |
| **EOWIR** | | | | | | | |
| Estimate | 1.8700 | 0.0337 | 0.0420 | 0.1176 | 0.5475 | 0.0707 | 0.3919 |
| SE | 0.4922 | 0.3806 | 0.0052 | | | | |
| **GIW** | | | | | | | |
| Estimate | 0.3850 | 0.0850 | 3.6875 | 0.1703 | 0.0688 | 0.3963 | 2.3662 |
| SE | 0.8440 | 0.6864 | 0.3405 | | | | |
| **Elo** | | | | | | | |
| Estimate | 55.9357 | 111.6019 | 5.5394 | 0.1346 | 0.2396 | 0.1790 | 1.1295 |
| SE | 22.7480 | 172.2316 | 8.8683 | | | | |
| **MKEx** | | | | | | | |
| Estimate | 3.6641 | | 2.7451 | 0.1139 | 0.4300 | 0.0853 | 0.4615 |
| SE | 0.3845 | | 0.0759 | | | | |
| **PL** | | | | | | | |
| Estimate | 15.0575 | 4.1503 | 0.0433 | 0.1293 | 0.2811 | 0.1867 | 1.1323 |
| SE | 12.6272 | 0.4413 | 0.0477 | | | | |

Firstly: The data represent a COVID-19 data belong to Canada of 56 days, from 1 November to 26 December 2020 [https://covid19.who.int/]. these data formed of drought mortality rate. The data are as follows: 0.1622 0.1159 0.1897 0.1260 0.3025 0.2190 0.2075 0.2241 0.2163 0.1262 0.1627 0.2591 0.1989 0.3053 0.2170 0.2241 0.2174 0.2541 0.1997 0.3333 0.2594 0.2230 0.2290 0.1536 0.2024 0.2931 0.2739 0.2607 0.2736 0.2323 0.1563 0.2677 0.2181 0.3019 0.2136 0.2281 0.2346 0.1888 0.2729 0.2162 0.2746 0.2936 0.3259 0.2242 0.1810 0.2679 0.2296 0.2992 0.2464 0.2576 0.2338 0.1499 0.2075 0.1834 0.3347 0.2362.
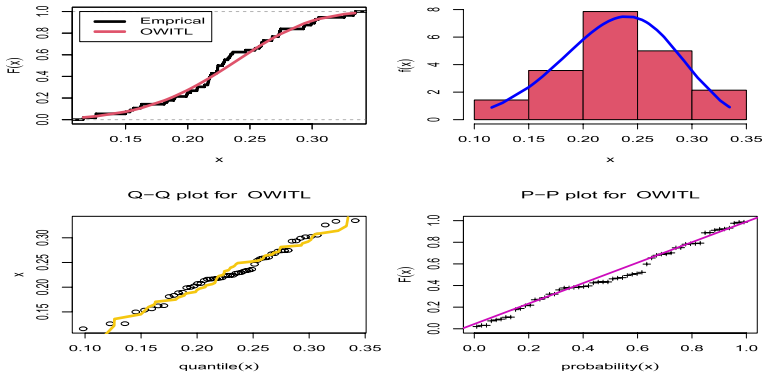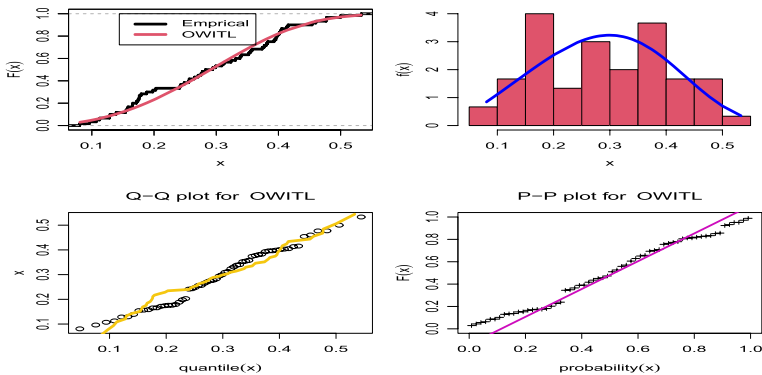
**Fig. 6** Cumulative function and empirical cdf, histogram, Q–Q plot and P–P plot for the OWITL distribution for COVID-19 data of Canada

**Table 4** MLE, SE, KS test and *P* values for COVID-19 data of The United Kingdom

|  | $\alpha$ | $\lambda$ | $\delta$ | KS | *P* Value | W* | A* |
|---|---|---|---|---|---|---|---|
| **OWITL** | | | | | | | |
| Estimate | 1.2330 | 0.7076 | 13.1360 | 0.1021 | 0.5262 | 0.0817 | 0.5149 |
| SE | 0.3265 | 0.9580 | 9.8608 | | | | |
| **EOWIR** | | | | | | | |
| Estimate | 0.3763 | − 0.7678 | 0.1165 | 0.1323 | 0.2240 | 0.1181 | 0.6706 |
| SE | 0.0541 | 0.0436 | 0.0052 | | | | |
| **GIW** | | | | | | | |
| Estimate | 0.4682 | 0.1871 | 2.0434 | 0.1509 | 0.1171 | 0.3966 | 2.2908 |
| SE | 4.1902 | 3.4222 | 0.1887 | | | | |
| **Elo** | | | | | | | |
| Estimate | 7.3456 | 64.4861 | 6.9706 | 0.1029 | 0.5128 | 0.2072 | 1.1858 |
| SE | 1.9085 | 95.1558 | 10.6752 | | | | |
| **MKEx** | | | | | | | |
| Estimate | 2.0555 | | 2.0902 | 0.1042 | 0.4994 | 0.0866 | 0.5468 |
| SE | 0.2206 | | 0.0982 | | | | |
| **PL** | | | | | | | |
| Estimate | 278.7158 | 2.7716 | 12.3695 | 0.1089 | 0.4441 | 0.1050 | 0.6327 |
| SE | 1479.239 | 0.2891 | 65.4890 | | | | |

Secondly: The data represent a COVID-19 data belong to The United Kingdom of 60 days, from 1 December 2020 to 29 January 2021 [https://covid19.who.int/]. These data formed of drought mortality rate. The data are as follows: 0.1292 0.3805 0.4049 0.2564 0.3091 0.2413 0.1390 0.1127 0.3547 0.3126 0.2991 0.2428 0.2942 0.0807 0.1285 0.2775 0.3311 0.2825 0.2559 0.2756 0.1652 0.1072 0.3383 0.3575 0.2708 0.2649 0.0961 0.1565 0.1580 0.1981 0.4154 0.3990 0.2483 0.1762 0.1760 0.1543 0.3238 0.3771 0.4132 0.4602 0.3523 0.1882 0.1742 0.4033 0.4999 0.3930

**Fig. 7** Cumulative function and empirical cdf, histogram, Q–Q plot and P–P plot for the OWITL distribution for COVID-19 data of The United Kingdom

0.3963 0.3960 0.2029 0.1791 0.4768 0.5331 0.3739 0.4015 0.3828 0.1718 0.1657 0.4542 0.4772 0.3402.

## 7 Summary

We suggest a new three-parameter model in this paper, called the odd Weibull inverted Topp–Leone distribution, which can be denoted as an OWITL distribution. The distribution of OWITL is motivated by the wide use in life testing of the ITL model and provides more flexibility for evaluate lifetime data. Survival function, hazard function, linear, quantile representation, and OWITL distribution moments are given. We compare the methods of MLE, LSE, MPSE, WLSE, CVME, and ADE and conclude that the alternative methods of MLE are better than the MLE method. In the sense of statistics, we have two implementations of the OWITL distribution for COVID-19 data. The OWITL distribution estimation parameters are derived from MLE, LSE, MPSE, WLSE, CVME, and ADE. Estimation methods are used to estimate the parameters of the model and results of the simulation are given to test the performance of the model. The proposed model of two real-life data offers a consistently better fit than the distributions EOWIR, GIW, WLo, MKEx, and PL.

**Authors' contributions** Modeling for the COVID-19 distribution in the United Kingdom and Canada is studied. A new lifetime distribution with a three-parameter of odd Weibull inverted Topp–Leone is introduced. Important properties are studied. Parameter estimation was obtained by using different estimation methods.

**Ethical statements** All of the followed procedures were in accordance with the ethical and scientific standards. This article does not contain any studies with human participants performed by the author.

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