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Research article

Image edge detection enhancement using coefficients of Sakaguchi type functions mapped onto petal shaped domain

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ARTICLE INFO

Dataset link: Source file of ''NATRAJAR''

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ABSTRACT

This research introduces a new approach to elevate the precision of image edge detection through a new algorithm rooted in the coefficients derived from the subclass $\mathcal{SC}^{l,\rho}$ (CSKP model). Our method employs convolution operations on input image pixels, utilizing the CSKP mask window in eight distinct directions, fostering a comprehensive and multi-directional analysis of edge features. To gauge the efficacy of our algorithm, image quality is assessed through perceptually significant metrics, including contrast, correlation, energy, homogeneity, and entropy. The study aims to contribute a valuable tool for diverse applications such as computer vision and medical imaging by presenting a robust and innovative solution to enhance image edge detection. The results demonstrate notable improvements, affirming the potential of the proposed algorithm to advance the current state-of-the-art in image processing.

1. Introduction and preliminaries

The first but essential stage in image analysis is the detection of edges in the image. Edge detection is crucial for image enhancement, analysis, compression, segmentation, object recognition, and other processes [2]. One of the best image enhancement methods for enhancing the quality of the image during the analysis process is edge detection. Accurate edge position is the goal of edge detection, which is essential for computer vision and image analysis. Edge detection is a step in the procedure known as image segmentation, which is primarily used to pinpoint specific areas in an image. Technically speaking, edge detection involves finding edge pixels, whereas edge enhancement involves boosting the contrast between the edge and background to make the edge more

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visible. There are various traditional algorithms for edge detection, such as Prewitts, Roberts, and Canny in ([1,3,6,7,11–13,18]). In our article, we have introduced a new concept from geometric function theory (GFT) for edge detection. GFT plays a central role in the development of complex analysis. It is the study of the geometrical properties of analytic functions. The investigation of univalent functions is one of the fundamental ideas of GFT. The theory of univalent functions is concerned primarily with relations between analytic structure and geometric behaviour. In GFT, so far, various researchers have studied the geometrical properties of various subclasses of analytic functions, whereas only a few have studied the application of GFT in image processing ([9,10,20]). Thus, the goal of this study is to create an innovative method for enhancing edge detection in images using the coefficients obtained for the subclass $\mathcal{SC}^{t,\rho}$ (CSKP model). Accordingly, the remaining portion of this research will be structured as follows:

- Detailed explanation of the model is given in section (2) and (3).
- Results of the experiment is discussed in section (4).
- Lastly, conclusion is presented in section (5).

In our method, five quality metrics have been used, which are mentioned below and using these quality metrics, we have showed the metric values for several images.

Contrast. Contrast is a measurement of the little differences that make up a picture.

$$C(k,n) = \sum_i \sum_j (i-j)^k P_{dis}[i,j]^n$$

If there is a lot of variety in an image, the $P[i,j]$'s will be concentrated away from the main diagonal and contrast will be strong.

Correlation. Image linearity

$$C_c = \frac{\sum_i \sum_j [ij P_{dis}[i,j]] - \mu_i \mu_j}{\sigma_i \sigma_j}$$

where $\mu_i = \sum i P_{dis}[i,j]$, $\sigma_i^2 = \sum i^2 P_{dis}[i,j] - \mu_i^2$.

Energy. Energy measures the sum of squared elements in the GLCM

$$C_{energy} = \sum_{i,j} P_{dis}[i,j]^2$$

Homogeneity. The co-occurrence matrix of a homogeneous image will contain both broad and narrow $P[i,j]$ values.

$$C_h = \sum_i \sum_j \frac{P_{dis}[i,j]}{1+|i-j|}$$

The $P[i,j]$ will typically cluster around the main diagonal when the range of gray levels is narrow. An uneven distribution of $P[i,j]$'s will be produced by a heterogeneous image.

Entropy. A measurement of the information content is entropy. It gauges the intensity distribution's unpredictability.

$$C_e = -\sum_i \sum_j P_{dis}[i,j] ln P_{dis}[i,j]$$

An image is represented by such a matrix, where the distance vector d has no favoured pairs of gray levels. Entropy is greatest when all $P[i,j]$ entries have identical magnitudes, and it is lowest when $P[i,j]$ entries have different magnitudes.

Edge detection is a technique that can be used to find the image edges required for computing the approximate absolute gradient magnitude at each point in a grayscale input image. The method being used raises the problem of calculating an appropriate absolute gradient magnitude for edges [4]. Images are subjected to a two-dimensional spatial gradient measured by the Sobel. To reduce the amount of data required to depict a digital image, a two-dimensional pixel array is transferred to a statistically uncorrelated dataset. A pair of 3×3 convolution masks are used in this edge detector, one to estimate gradients in the x-orientation and the other to estimate gradients in the y-orientation. The Sobel detector effectively exposes noise in images as edges because of its high sensitivity to it. Therefore, in conversations involving large amounts of data that are identified during data transmission, this operator is advised. Following are [4] provides a description of Sobel masks.

$$G_x = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \qquad G_y = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

The image is convolved with each of the masks. There are two numbers at each pixel location: S1 and S2, which represent the output from the mask of the row and column, respectively. Equations (1) and (2) are applied to those numbers to compute two matrices, the edge magnitude and orientation:

$$Edge\ magnitude = \sqrt{S_1^2 + S_2^2} \tag{1}$$

$$Edge\ direction = tan^{-1}\left(\frac{S_1}{S_2}\right) \tag{2}$$

2. A mathematical part

Let $\mathcal{A}$ be the class of all functions f which are holomorphic in the region $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ with the normalization $f(0) = f'(0) - 1 = 0$. Therefore, for $f \in \mathcal{A}$, one has

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \qquad (z \in \mathbb{D}). \tag{3}$$

We write $g_1 \prec g_2$, if there is an analytic function v in $\mathbb{D}$, with limitations $v(0) = 0$ and $|v(z)| < 1$, such that $g_1(z) = g_2(v(z))$, $(z \in \mathbb{D})$. In case of univalency of g_2 in $\mathbb{D}$, the following relation holds:

$$g_1(z) \prec g_2(z),\ (z \in \mathbb{D}) \iff g_1(0) = g_2(0) \quad \text{and} \quad g_1(\mathbb{D}) \subset g_2(\mathbb{D}).$$

In geometric function theory, the most basic and important subfamilies of the set $\mathcal{S}$ are the family $\mathcal{S}^*$ and $\mathcal{C}$ which are starlike and convex functions respectively.
By varying the subordination condition, we arrive at various geometrical sense. For example,

$$q(z) = 1 + sinh^{-1} z \tag{4}$$

then the class $\mathcal{S}^*_q := \mathcal{S}^*(1 + sinh^{-1} z)$ was provided by Kumar and Arora [14]. Clearly, the function $q(z)$ is a multivalued function and has the branch cuts about the line segments $(-i\infty, -i) \cup (i, i\infty)$, on imaginary axis and hence, it is holomorphic in $\mathbb{D}$. In a geometric point of view, the function $q(z)$ maps the unit disc $\mathbb{D}$ onto a petal-shaped region Ω_p,

$$\Omega_p = \{\omega \in C : |sinh(\omega - 1)| < 1\} \tag{5}$$

Some recent work on coefficient problems were discussed in [5,16,17].
Using this idea, E.K.Nithiyanandham and B.Sruthakeerthi [15] study a subfamily $\mathcal{SC}^{t,\rho}$ of analytic functions as

Definition. The function $f \in \mathcal{A}$ is in the class $\mathcal{SC}^{t,\rho}$ if

$$\frac{(1-t)[\rho z^2 f''(z) + z f'(z)]}{\rho z[f'(z) - t f'(tz)] + (1-\rho)[f(z) - f(tz)]} \prec \tilde{\Lambda}(z), \tag{6}$$

where $\tilde{\Lambda}(z)$ is given by (5), with $|t| \le 1$, $t \ne 1$, and $0 \le \rho \le 1$.

Theorem 1. *[15]If the function f of the form (3) belongs to $\mathcal{SC}^{t,\rho}$, then*

$$|a_2| \le \frac{H_1 U_2}{u_2},$$

$$|a_3| \le \frac{H_2 U_3}{H_1 U_2 u_3}\ max(1, U_2),$$

$$|a_4| \le \frac{H_3 U_4}{4 H_2 U_3 u_4}\left\{\left|\frac{5}{6} - U_2 - \frac{U_3}{U_2} + U_3\right| + \left|\frac{7}{3} - 2U_3\right| + \left|\frac{5}{6} + U_2 + \frac{U_3}{U_2} + U_3\right|\right\}$$

where

$$u_n = \frac{1-t^n}{1-t},\ U_n = \prod \frac{u_n}{n - u_n}\ ,n = 2,3,\cdots \ \text{and}\ H_n = \prod \frac{1}{1+n\rho}\ ,n = 1,2,\cdots$$

By fixing the parameter value, $\rho = 0$ in (6), we get the coefficient values as,
$\mathfrak{a}_1 = 1$,
$\mathfrak{a}_2 = \frac{1}{1-t}$,
$\mathfrak{a}_3 = \frac{1}{2-t-t^2}\ max\left(1, \frac{1+t}{1-t}\right)$.

3. Algorithm part

Edge detection is achieved using the coefficients $\mathfrak{a}_1, \mathfrak{a}_2, \mathfrak{a}_3$. The CSKP for edge detection is ED, which is computed from input image I:

$$ED(x, y) = Mask * I(x, y)$$

where $*$ is the convolution product.

The CSKP window's mask is represented by the coefficients a_1, a_2, a_3.
The below tables illustrates how eight masks are created in eight directions in this study.

$-a_1$	0	$-a_1$
a_1	a_2	a_3
$-a_1$	0	$-a_1$

(a)

0	$-a_1$	a_3
0	a_2	0
a_1	0	$-a_1$

(b)

0	a_3	$-a_1$
0	a_2	$-a_1$
$-a_1$	a_1	0

(c)

a_3	0	$-a_1$
0	a_2	0
$-a_1$	0	a_1

(d)

0	a_1	0
a_3	a_2	a_1
0	0	a_1

(e)

$-a_1$	0	a_1
0	a_2	0
a_3	0	$-a_1$

(f)

0	a_1	$-a_1$
0	a_2	$-a_1$
0	a_3	a_1

(g)

a_1	0	$-a_1$
0	a_2	0
$-a_1$	0	a_3

(h)

Dedicated CSKP directional mask.

Using the input image, the mask for the CSKP window is moved over it to calculate the pixel values.

Algorithm 1 Proposed CSKP model algorithm.

i. Load the original image into a variable A
ii. Convert the image to grayscale
iii. Perform edge detection using the Sobel operator
iv. Calculate GLCM (Gray-Level Co-occurrence Matrix) from the edge-detected image
v. Compute texture feature (Contrast, Correlation, Energy, Homogeneity and Entropy) from the GLCM
vi. Define convolution kernels for edge enhancement in different orientations: h1 to h8 for various orientations
vii. Convolve the edge-detected image with each kernel to enhance edges
viii. Calculate GLCM and texture feature for the enhanced image.
ix. Display the original edge-detected image, the enhanced image, and their values.

4. Application

Our goal in this section is to demonstrate how the CSKP enhances the edge detection images. Performance tests were implemented using MATLAB(Mathworks) in Windows 8.1 Pro, Processor: Intel(R) Pentium(R) CPU A1018@ 2.10GHz. A 3x3-pixel window is considered to be used in the proposed CSKP mask window. BSDS500: Berkeley Segmentation DataSet (BSDS), the first version has been published in 2001 [19] and consists of 300 images split up into 200 for training and 100 for validation, termed BSDS300; the last version [8], adds 200 new images for the testing and five images (Natarajar, Herbarium, Graph, Spine, Periya kovil) were taken into account.

The quantitative presentation of the suggested algorithm is calculated using contrast, correlation, energy, homogeneity, and entropy. Table (1) compares texture feature values for Sobel [4], Priya et al. [9] and our proposed model for five test images, and Table (2) compares average texture feature values for Sobel [4], Priya et al. [9] and our proposed model for the BSDS500 dataset. The results demonstrate in Table (1) and Table (2) notable improvements, affirming the potential of the proposed algorithm.
Each image was examined from each viewpoint in the research shown in Figure (1) and Figure (2). Better outcomes are obtained at $t = 0.4$ and at average rotation with regard to the constraints of contrast, correlation, energy, homogeneity, and entropy.

5. Conclusion

In this study, we have introduced a new edge detection enhancement algorithm by making use of coefficient bounds, kernels, and convolution concepts. The parameter values were adjusted for the betterment of the results. When compared with the other results, it is evident that the proposed approach produces good results. The drawback of the proposed method is that if we choose an image with high noise, our result fails to produce a better image. In the future, we aim to improve this shortcoming and produce an improvised edge detection algorithm.

CRediT authorship contribution statement

E.K. Nithiyanandham: Writing – review & editing, Writing – original draft, Software, Methodology, Conceptualization. **B. Srutha Keerthi:** Supervision, Investigation.

Table 1
Texture feature value comparison of test images.

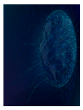
Images	Methods	Contrast	Correlation	Energy	Homogeneity	Entropy
	Sobel [4]	0.0653	0.2618	0.8505	0.9673	0.2706
	Priya et al. [9]	1.2535	0.8589	0.6331	0.9099	1.1248
	Proposed	2.2596	0.5851	0.6962	0.8949	1.1992
	Sobel [4]	0.0212	0.2830	0.9496	0.9894	0.1125
	Priya et al. [9]	0.5207	0.8481	0.8800	0.9740	0.4272
	Proposed	0.7478	0.5868	0.9039	0.9692	0.4306
	Sobel [4]	0.0888	0.2850	0.7949	0.9556	0.3526
	Priya et al. [9]	0.9522	0.9181	0.5913	0.9168	1.3218
	Proposed	2.6391	0.6671	0.6241	0.8738	1.4989
	Sobel [4]	0.0172	0.4522	0.9517	0.9914	0.1179
	Priya et al. [9]	0.4948	0.8718	0.8718	0.9730	0.4398
	Proposed	0.6181	0.6723	0.9019	0.9723	0.4555
	Sobel [4]	0.0599	0.5210	0.8185	0.9700	0.3541
	Priya et al. [9]	1.2125	0.9021	0.5448	0.9015	1.4040
	Proposed	2.2248	0.6916	0.6257	0.8914	1.5369

Table 2
Average texture feature value comparison of “BSDS500 dataset”.

Methods	Contrast	Correlation	Energy	Homogeneity	Entropy
Sobel [4]	0.04819	0.3252	0.8827	0.9758	0.2275
Priya et al. [9]	0.9862	0.8645	0.7039	0.9319	0.9356
Proposed	1.7274	0.6058	0.7594	0.9215	0.9776

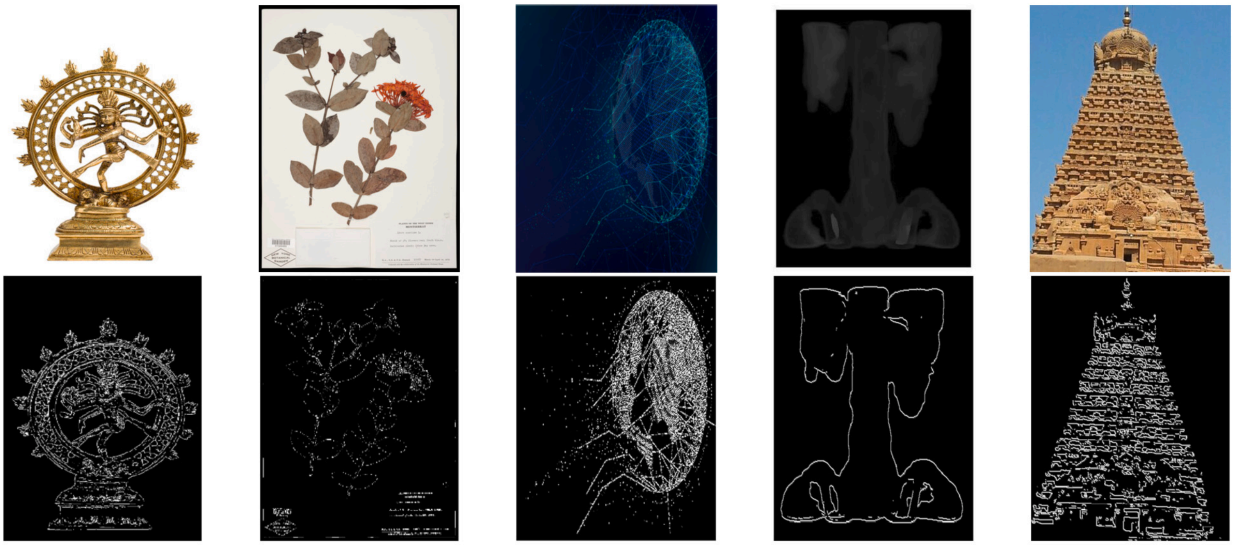


Fig. 1. Original images such as Natarajar, Herbarium, Graph, Spine, Periya kovil and their edge detected images.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Fig. 2. Edge detection of sample images from "BSDS500 dataset".

Data availability

Source files of the test images are given below:

Source file of ''NATRAJAR''
Source file of ''HERBARIUM''
Source file of ''GRAPH''
Source file of ''SPINE''
Source file of ''PERIYA KOVIL''
Source file of ''BSDS500 dataset''

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