Article

Circular Modelling of Circumplex Measurements for Interpersonal Behavior

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Abstract

This article describes a new way to analyze data from the interpersonal circumplex (IPC) for interpersonal behavior. Instead of analyzing Agency and Communion separately or analyzing the IPC's octants, we propose using a circular regression model that allows us to investigate effects on a blend of Agency and Communion. The proposed circular model is called a projected normal (PN) model. We illustrate the use of a PN mixed-effects model on three repeated measures data sets with circumplex measurements from interpersonal and educational psychology. This model allows us to detect different types of patterns in the data and provides a more valid analysis of circumplex data. In addition to being able to investigate the effect on the location (mean) of scores on the IPC, we can also investigate effects on the spread (variance) of scores on the IPC. We also introduce new tools that help interpret the fixed and random effects of PN models.

Keywords

interpersonal circumplex, interpersonal circle, circular mixed-effects model, embedding approach, interpersonal theory

Good quality interpersonal relationships and behavior are regarded as very important for peoples' social functioning and mental health. In general, humans inherently feel a need for affiliation and power (aan het Rot, Hogenelst, & Moskowitz, 2013). Interpersonal behavior and relationships have been widely studied using interpersonal theory (e.g., Horowitz & Strack, 2011) and the two-dimensional interpersonal circle or circumplex (IPC).

The conceptual basis for the IPC was developed by Leary (1957), but the mathematical basis for the model was laid by Guttman (1954) who wanted to model an ordering "without a head and foot to it" (p. 325). Such an ordering can be called a circumplex or circular ordering and is described by Gurtman (2009, p. 602) as "a continuous order with no beginning or end." Until now, research on interpersonal behavior has heavily relied on linear statistical methods to analyze circumplex data (e.g., aan het Rot et al., 2017; Hopwood et al., 2020; Pennings et al., 2018). These linear methods, however, assume orderings that do have a beginning and end, which does not correspond to the circular nature of the IPC. Therefore, recently, several researchers are calling for new methods to analyze circumplex data, which allow for the analysis of IPC data in a circular fashion as a blend of its two dimensions, Agency and Communion (Gurtman, 2011; Pennings, 2017b; Wright, Pincus, Conroy, & Hilsenroth, 2009).

The goal of this article is twofold. First, to illustrate the advantages of using circular statistics on circumplex data, we will fit a mixed-effects model for circular data (Nuñez-Antonio & Gutiérrez-Peña, 2014) to three different repeated measures circumplex data sets. One data set is on blushing in social interactions (aan het Rot, Moskowitz, & de Jong, 2015) and two data sets on interpersonal teacher behavior (Brekelmans, Wubbels, & van Tartwijk, 2005; Pennings et al., 2018). We will refer to these three data sets as the interpersonal behavior data.

The second aim is to improve the interpretation of circular mixed-effects results. The circular mixed-effects model used in this article falls within the embedding approach to circular data. This approach is not only known for its flexible modelling of data but also has a disadvantage concerning the interpretation of results (Cremers, Mulder, & Klugkist, 2018; Maruotti, 2016). Therefore, we developed new interpretation measures for the mixed-effects model

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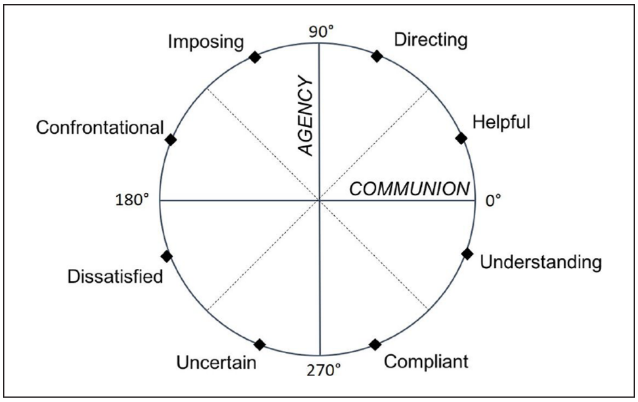

Figure 1. The interpersonal circle.
Note. The words presented in the circumference of the circle are anchor words to describe the type of behavior located in each part of the interpersonal circle.

for circular data by extending the methods for a (simple) circular regression model (Cremers et al., 2018) to the mixed-effects context.

Interpersonal Theory and the Interpersonal Circumplex

To study interpersonal behavior and relationships, it is very common to rely on insights from interpersonal theory (e.g., Horowitz & Strack, 2011). The most important premise of interpersonal theory is that all interpersonal behavior is represented as a blend of two orthogonal dimensions, Agency and Communion, which together form the basis for a circular, or circumplex, structure, which is called the IPC (see Figure 1; e.g., Horowitz & Strack, 2011; Leary, 1957). These two orthogonal dimensions are called Agency (i.e., vertical axis) and Communion (i.e., horizontal axis; Gurtman, 2009). Agency refers to the degree of power or control an individual exerts in interaction with others. Communion refers to the degree of friendliness or affiliation an individual conveys in interaction with others.

The IPC in Figure 1 is the IPC used in the educational context, the IPC for teachers. The IPC for teachers consists of octants (Directing, Helpful, etc.) that represent prototypical behaviors located in those eight parts of the IPC. Each octant is characterized by a corresponding degree of Agency and Communion (e.g., the upper right part of the IPC is characterized by high levels of both Agency and Communion).

The octants of the IPC are ordered in a circular fashion; interpersonal behavior, that is, represented by adjacent octants is more similar than behavior that is represented by opposite octants. Theoretically, however, the IPC is a continuous order meaning that an individuals' interpersonal behavior can be described using a continuous circular measurement (in degrees) along the edge of the circle drawn in Figure 1. Research nowadays focusses more on using Agency and Communion scores as continuous measures (e.g., Hopwood et al., 2020; Pennings et al., 2018) instead of using profiles based on octant scores (Wubbels, Brekelmans, den Brok, & van Tartwijk, 2006).

Analysis of Circumplex Data

In most circumplex research, the two axes Agency and Communion are analyzed separately (see, e.g., aan het Rot et al., 2013, aan het Rot et al., 2015, aan het Rot et al., 2017; Brekelmans et al., 2005; Hopwood et al., 2020; Mainhard, Brekelmans, den Brok, & Wubbels, 2011; Mainhard, Pennings, Wubbels, & Brekelmans, 2012; Pennings et al., 2014; Pennings et al., 2018; Sadler & Woody, 2003). Given that interpersonal theory states that interpersonal behavior should be described as a blend of Agency and Communion and cannot be interpreted correctly based on only one of these dimensions, this is not an ideal way of analyzing IPC data.

One way to analyze this blend is to use the octants as a categorical outcome in the analysis. For example, Wright et al. (2013) used a latent class analysis to distinguish between these different interpersonal subtypes in patients with borderline personality disorder. Using the octants as outcome variable has several drawbacks. By converting continuous measurements of the two dimensions to a single category, we lose important information about the data. This can be a disadvantage in research where Agency and Communion scores of participants are compared and related with other outcomes or in longitudinal research where the interest lies in change of interpersonal behavior over time. When data are categorized, small changes on the IPC are not automatically picked up as they do not necessarily imply a change from one octant to the other.

Circumplex data can also be studied using circular statistics, in which the Agency and Communion scores are converted to a continuous circular score, θ that lies between 0° and 360°, which depends on the ratio between Agency and Communion. For example, a positive score on Communion and a 0 score on Agency represent a circular value of 0°, which falls between the octants "understanding" and "helpful" of Figure 1. Wright et al. (2009) and Pennings et al. (2018) argue that circular statistics can help answer more precise questions, such as analyzing small changes over time.

Figure 2 illustrates the differences in the interpretation of circumplex data that are analyzed separately compared with data that are analyzed using circular statistics. Figure 2 shows how the scores of individuals change over two measurement occasions (left and right plot) both on the two axes, Agency and Communion, and on the circumplex/circle. We

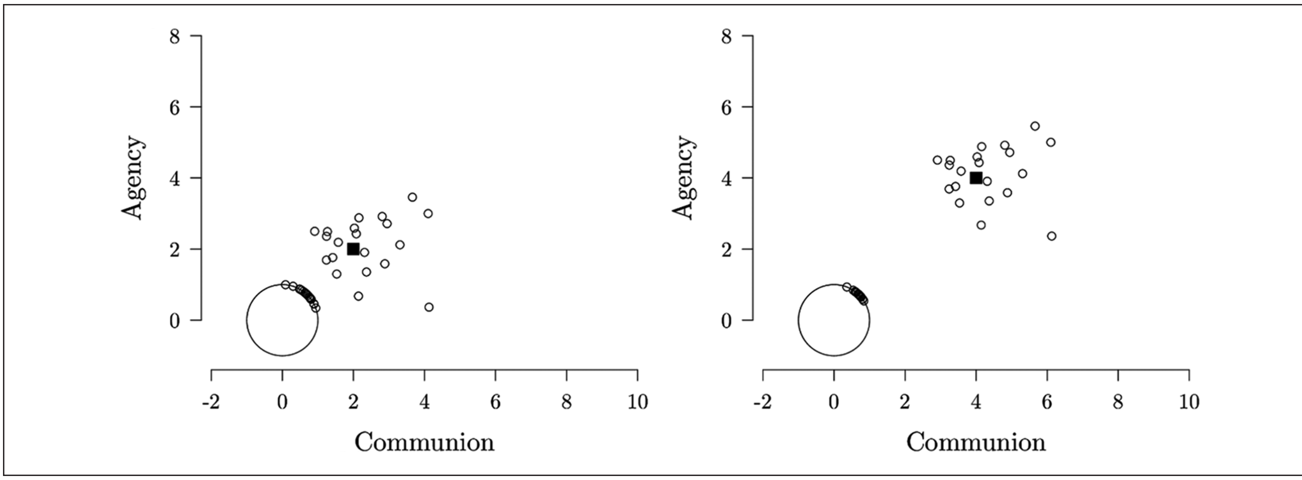

Figure 2. Communion (*x*-axis) and Agency scores (*y*-axis) and scores on the circumplex (circle) for two measurement occasions.
Note. The solid square indicates the average on Agency and Communion. Whereas on the Agency and Communion axes the mean changes, on the circumplex not the mean but the variance changes.

see that from the left to the right measurement occasion, the average score (solid square) on both the Agency and Communion axis increases, but their variance stays the same. On the circumplex itself, however, the average direction does not change, whereas the variance decreases. Patterns in scores on the separate components over time may thus reflect different patterns in scores on the circle and circular statistics can thus give us a different type of information about data on the IPC. Therefore, it is beneficial to consider a circular model for circumplex data. The new approach to circumplex data presented in this article is a method for circular statistics.

Methodological Background

The Embedding Approach to Circular Data

The circular mixed-effects model that we use to analyze data from the IPC falls within the embedding approach to circular data. In the embedding approach, we split the circular outcome in two parts and instead of using θ as outcome variable directly, we take its unit vector representation, $u = (\cos \theta, \sin \theta)$ and thus split it into a sine and cosine component (see Figure 3).

For data from the IPC, the two components of u, cos (θ) and sin (θ), can interpretation wise be referred to as a score on Communion and Agency. However, they do not represent exactly the same values since Agency and Communion scores are measured on a different scale. Communion and Agency range from $-\infty$ to ∞, while the sine and cosine range from −1 to 1. In the embedding approach it is assumed that u can be represented by an unobserved vector y in bivariate real space (R^2) as follows:

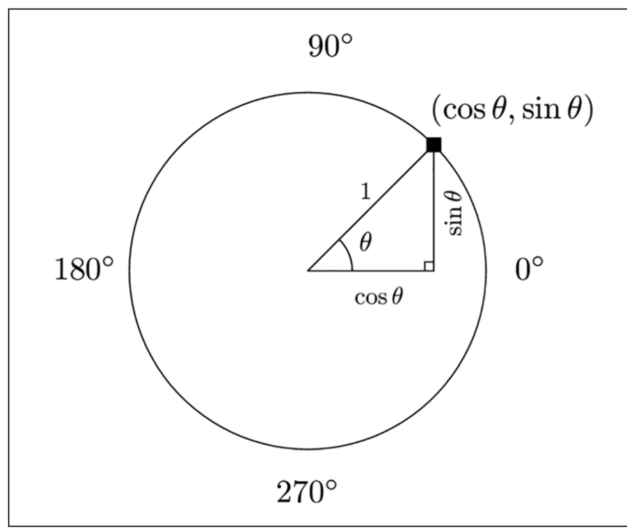

Figure 3. A decomposition of the angle θ into its sine (*y* coordinate) and cosine (*x* coordinate) component.
Note. The values on the sides of the right-angled triangle indicate their lengths.

$$\boldsymbol{u} = \frac{\boldsymbol{y}}{r},$$

where r represents the length of the vector y. The circular outcome, θ, thus originates from a projection onto the circle of a vector, y, in R^2. Note that when modelling circular data, we have only observed u, whereas y and r are considered latent variables.

In this article, we assume that the unobserved vector inR^2, y, is normally distributed with mean vector μ and

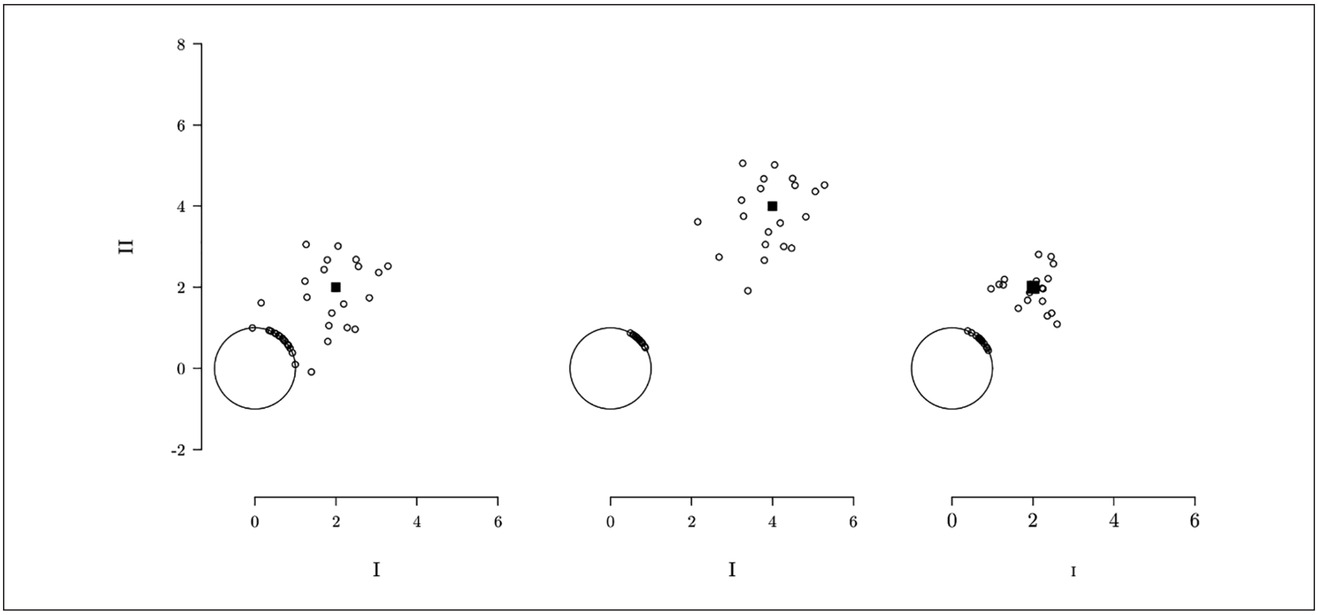

Figure 4. Three sets of bivariate normal data points with different mean vectors, from left to right (2, 2), (4, 4), (2, 2) and covariance matrices projected onto the circle, from left to right variances (0.9, 0.9, 0.5, covariances are 0).
Note. We see that the spread on the circle in the middle (higher mean) and right (smaller variance) plot is smaller than in the left plot. Thus, both mean and variance of the bivariate data influence the spread on the circle.

covariance matrix I ($y \sim N_2$ [μ, I]). It then follows that θ has a projected bivariate normal density PN (θ | μ, I; see Supplementary Material [available online] for full specification). Note that to be able to identify the model, the covariance matrix is fixed to be an identity matrix (I). We need to make a restriction because both the mean and the variance of a bivariate normal variable influence the spread of its projection onto the circle. This effect can be seen in Figure 4, where we have plotted three sets of data from different bivariate normal distributions (with components I and II) that are projected onto the circle. We see that the spread on the circle in the middle (higher mean) and right (smaller variance) plot is smaller than in the left plot. Thus, both the mean and the variance of bivariate normal data influence projected scores on the circle. Therefore, we need to fix either the mean or covariance matrix to identify and be able to estimate the model.[1] Instead of restricting the covariance matrix to be an identity matrix, as is done in this article, it is also possible to use the general projected normal distribution that imposes a different restriction on the covariance matrix (Wang & Gelfand, 2013).

A Mixed-Effects Model for Circular Data

A type of model that can be used to analyze longitudinal or repeated measures data are the so-called mixed-effects model. For circular data, a mixed-effects model based on the projected normal distribution is developed by Nuñez-Antonio and Gutiérrez-Peña (2014). It has independent observations of a design matrix for the fixed and random effect predictors, $\boldsymbol{X}$ and $\boldsymbol{Z}$ in addition to a circular outcome vector θ for each individual, $i = 1, \ldots, n$. The rows of the matrices $\boldsymbol{X}$ and $\boldsymbol{Z}$ and the indexes of the vector θ represent the measurement occasions $j = 1, \ldots, N$. The circular mixed-effects model can thus be regarded as a multilevel model with two levels. The model has the following mean structure:

$$\boldsymbol{\mu}_{ij} = \begin{pmatrix} \mu_{ij}^{I} \\ \mu_{ij}^{II} \end{pmatrix} = \begin{pmatrix} (\boldsymbol{\beta}^{I})^{t}\,\boldsymbol{x}_{ij}^{I} + (\boldsymbol{b}_i^{I})^{t}\,\boldsymbol{z}_{ij}^{I} \\ (\boldsymbol{\beta}^{II})^{t}\,\boldsymbol{x}_{ij}^{II} + (\boldsymbol{b}_i^{II})^{t}\,\boldsymbol{z}_{ij}^{II} \end{pmatrix}.$$

Here $\boldsymbol{\beta}^{I}$ and $\boldsymbol{\beta}^{II}$ are vectors with fixed effect coefficients and intercept and $\boldsymbol{b}^{I}$ and $\boldsymbol{b}^{II}$ are vectors with random effects for each individual. Note that each individual has two matrices of fixed and random effect predictors, one for the cosine and one for the sine component (denoted I and II, respectively) of the mean vector of the model. This structure is similar to that of a bivariate mixed-effects model.

For one individual i and components I and II, $\boldsymbol{X}$ and $\boldsymbol{Z}$ may look as follows:

$$\boldsymbol{X}_i^I = \begin{pmatrix} Intercept & x_1 & x_2 & Measurement \\ 1 & -28.31 & -10.40 & 0 \\ 1 & -13.94 & -10.40 & 2 \\ 1 & -6.79 & -10.40 & 3 \\ 1 & -1.29 & -10.40 & 5 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & -55.85 & -10.40 & 15 \end{pmatrix}, \boldsymbol{Z}_i^I = \begin{pmatrix} Intercept \\ 1 \\ 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}.$$

$$\boldsymbol{X}_i^{II} = \begin{pmatrix} Intercept & x_1 & x_2 & Measurement \\ 1 & -28.31 & -10.40 & 0 \\ 1 & -13.94 & -10.40 & 2 \\ 1 & -6.79 & -10.40 & 3 \\ 1 & -1.29 & -10.40 & 5 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & -55.85 & -10.40 & 15 \end{pmatrix}, \boldsymbol{Z}_i^{II} = \begin{pmatrix} Intercept \\ 1 \\ 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}.$$

In $\boldsymbol{X}_i^I$ and $\boldsymbol{X}_i^{II}$ we have included an intercept, a predictor x_1 that varies with each measurement, a predictor x_2 that is constant for each measurement and a predictor that indicates the measurement occasion, for example, week number. The design matrices $\boldsymbol{Z}_i^I$ and $\boldsymbol{Z}_i^{II}$ only contain an intercept, meaning that only the intercept of the model but none of the slopes are considered random.

Estimation Methods

In the embedding approach, we do not observe the bivariate normal variable, *y*, from which the circular outcome θ origins. This means that we need to use special techniques when fitting the model, for example, an expectation–maximization algorithm (Presnell, Morrison, & Littell, 1998) or auxiliary variables in a Bayesian setting (Nuñez-Antonio & Gutierrez-Pena, 2005). In this article, we use a Bayesian Markov Chain Monte Carlo (MCMC) as in Nuñez-Antonio and Gutiérrez-Peña (2014) and Hernandez-Stumpfhauser, Breidt, and van der Woerd (2017). The exact specifications of the priors and posteriors of the model and the MCMC sampler are given in the Supplementary Material available online. The MCMC sampler to estimate the circular mixed-effects model as well as a circular GLM (general linear model) have been implemented in the R-package bpnreg (Cremers, 2018). Example code and data for a circular GLM and mixed-effects model can be found in Cremers and Klugkist (2018).

Model Fit

To evaluate the fit and select a model, we use four different model selection criteria in this article: two versions of the deviance information criterion (DIC and DICalt) and two versions of the Watanabe–Akaike information criterion (WAIC1 and WAIC2; Gelman et al., 2014). We choose these four criteria because they are specifically useful in Bayesian models, where MCMC methods have been used to estimate the parameters. All four criteria have a fit part consisting of a measure based on the log likelihood and include a penalty in the form of an effective number of parameters. For all criteria lower values indicate better fit. Gelman et al. (2014) describe how to compute these criteria.

Assessing Circular Effects

In the embedding approach, model coefficients are estimated for the two bivariate components, I and II. For example, we have two sets of fixed-effects coefficients $\boldsymbol{\beta}^I$ and $\boldsymbol{\beta}^{II}$. For circumplex data this means that although Agency and Communion are analyzed together, the results are interpreted separately on the two components. However, we also want to be able to interpret the effect of predictors on a circular scale and interpret effects on the "blend" of Agency and Communion. In previous research it has been shown that we can transform the model coefficients of the two dimensional components in a circular regression model such that we get coefficients on the circle (Cremers et al., 2018). Three types of circular coefficients, the b_c, the *SAM* (slope at the mean) and the *AS* (average slope), have been developed. These three coefficients describe the slope at the inflection point of a circular regression line, at the data average and the average slope, respectively (see Figure 5). The *SAM* and the *AS* were introduced in Cremers et al. (2018) because b_c does

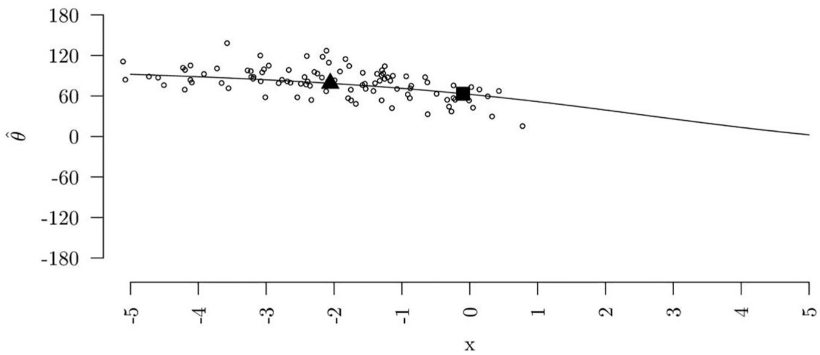

Figure 5. Predicted circular regression line for the relation between a linear predictor *x* and a predicted circular outcome θ (in degrees) together with the original data points.

Note. IPC = interpersonal circumplex. The square indicates the inflection point of the regression line and the triangle indicates the predicted value for *x*. Note that we have rescaled the *y*-axis to make sure the regression line is smooth. The IPC now ranges from −180° to 180° instead of 0° to 360°, where −60° = 300°, −120° = 240°, and −180° = 180° due to the periodicity of the circle.

not always lie close to the data. This may lead to unstable estimates with very large uncertainty intervals. Cremers et al. (2018) conclude that the *SAM* is the preferable estimate to use, both for its straightforward interpretation and absence of unstable estimates.

In this article, we have extended the method from Cremers et al. (2018) and developed new circular coefficients for the circular mixed-effects model. A description of this extension is given in the Supplementary Material available online. The circular coefficients for a circular GLM and mixed-effects model have also been implemented in the R-package bpnreg, which will be used to analyze the interpersonal behavior data.

IPC Measures, Data Sets, and Results

Data Set I concerns a study in personality psychology on blushing in social interactions (aan het Rot et al., 2015). In this study, the Interpersonal Grid (IG; Moskowitz & Zuroff, 2005) and the Social Behavior Inventory (SBI: Moskowitz, 1994) are used to obtain scores on the IPC. Both instruments consist of four interpersonal subscales representing different interpersonal behaviors: quarrelsomeness, agreeableness, dominance, and submissiveness (aan het Rot et al., 2013). The SBI contains 12 items that measure each of the four behaviors. In the IG, interpersonal behavior is indicated by points in an 11 by 11 grid where each of the sides represents a subscale. These points are then translated to a score from 0 to 5 on each of the four subscales. The four behavior subscales load on the two dimensions Communion (i.e., quarrelsomeness and agreeableness) and Agency (i.e., dominance and submissiveness; aan het Rot et al., 2013).

Data Set II and III concern two studies in educational psychology on teacher behavior (Brekelmans et al., 2005; Pennings et al., 2018). These studies both use the Questionnaire on Teacher Interaction (QTI; Wubbels et al., 2006) for obtaining scores on the IPC. The QTI consists of items that load on the eight octants of the IPC displayed in Figure 1 and that can be converted to scores on the two orthogonal dimensions: Agency and Communion. In the QTI, Agency and Communion are used as the metalabels for the degree of power or control a teacher exerts in interaction with his or her students and the degree of friendliness or affiliation a teacher conveys in interaction with his or her students.

Data Set I

Participants. This data set contains several repeated measures of social encounters for two samples of first-year University students gathered for the study of aan het Rot et al. (2015). In this article, we only use the data for their first sample of students ($N = 64$), where the number of encounters was 63.13 on average. Encounters that took place by phone and encounters that occurred within 3 hours of alcohol ingestion were removed. For more details on the sample, we refer the reader to the original study.

Measures. The measurements were collected using an event-contingent recording approach (Moskowitz, 1994), in which students were asked to report on social encounters soon after they occurred. Students reported their perception of both their own and their interaction partners' interpersonal behavior during the encounter. These perceptions are modeled separately as two outcome variables, Self and Interaction. Self is a measure that is constituted of the combined scores on the four interpersonal behaviors of the SBI. Interaction is a measurement on the IG that is translated to the circle. Participants were

Table 1. Descriptives for Data Set I.

Variable	$M/\hat{\theta}$	$SD/\hat{\rho}$	Range	Type
Self				
Blusher	16.80°	0.94	—	Circular
Nonblusher	28.50°	0.98	—	Circular
Interaction				
Blusher	11.26°	0.18	—	Circular
Nonblusher	349.90°	0.38	—	Circular

Note. $\hat{\theta}$ = circular mean; $\hat{\rho}$ = circular concentration; IPC = interpersonal circumplex. Self and Interaction represent the scores on the IPC of the students and their interaction partners, respectively.

categorized as blushers if they blushed during one or more social encounters and as nonblushers if they did not blush during any social encounter.

Research Question. The researchers of the original article hypothesized that blushing would be associated with high levels of submissive and agreeable behavior and that blushing would be relatively likely to occur during interactions with others who are perceived as powerful and low in affiliation.

Descriptives. Table 1 shows descriptives for the scores on the circumplex for the students and their interaction partners depending on whether they were blushers. Table 1 shows the circular means and concentration parameters of the Self and Interaction scores. The circular concentration parameter $\hat{\rho}$ is a value between 0 and 1, where 0 means that the scores are spread out across the entire circle and a score of 1 means that all scores are concentrated at the same spot. The circular variance equals $1-\hat{\rho}$ and an interpretation opposite to the concentration. We see from the circular means for the interpersonal behavior of the students that the nonblushers lean more toward dominant behavior (a circular score of 28.5° compared with 16.80°). Concerning the interpersonal behavior of the interaction partners, the partner of a blusher is perceived as more dominant. Last, we observe that the scores on the IPC of the students are much more concentrated on the circle compared with the scores of the interaction partners.

Model. We fit the same model to the scores on the IPC for the students and those of their interaction partners. The scores are predicted by whether the students are blushers. We use a model building procedure in which we first fit an intercept-only model and then include the blusher variable. We also include a random intercept. The full models that estimate the means of both components (I and II) for the scores of students and their interaction partners are as follows:

Table 2. Model Fit Statistics for the Models Fit to Data Set I.

Outcome	Self		Interaction	
Model	Intercept-only	Blusher	Intercept-only	Blusher
DIC	13,480	13,472	15,360	15,353
DIC ^{alt}	13,709	13,666	15,489	15,466
WAIC ^{1}	13,491	13,482	15,381	15,377
WAIC ^{2}	13,498	13,489	15,395	15,391

Note. DIC = deviance information criterion; WAIC = Watanake–Akaike information criterion. Each column contains the values of four model fit statistics for one of the models fit to Data Set I. Blusher represents the categorical variable measuring whether an individual is a blusher (1) or nonblusher (0).

$$\text{Self}\left(\mu_{ij}\right)=\begin{pmatrix}\mu_{ij}^{I}\\ \mu_{ij}^{II}\end{pmatrix}=\begin{pmatrix}\beta_0^{I}+\beta_1^{I}\text{blusher}_j+b_{0i}^{I}\\ \beta_0^{II}+\beta_1^{II}\text{blusher}_j+b_{0i}^{II}\end{pmatrix}.$$

$$\text{Interaction}\left(\mu_{ij}\right)=\begin{pmatrix}\mu_{ij}^{I}\\ \mu_{ij}^{II}\end{pmatrix}=\begin{pmatrix}\beta_0^{I}+\beta_1^{I}\text{blusher}_j+b_{0i}^{I}\\ \beta_0^{II}+\beta_1^{II}\text{blusher}_j+b_{0i}^{II}\end{pmatrix}.$$

The index j represents the individuals and i the social encounters. We fit the two models in R (R Core Team, 2017) using the package bpnreg (Cremers, 2018). The code can be found in the Supplementary Material available online. Note that for this data we are not interested in the random effects per se. We will therefore not include them in our discussion of the Results. However, because of the repeated measures structure of the data, we do fit a mixed-effects model.

Before evaluation of the results the convergence of the MCMC samplers was checked by means of traceplots. For all models, we reached convergence within 1,000 iterations (burn-in = 1,000, lag = 3).

Results

Model fit. Table 2 shows the fit for the two models predicting the scores on the circumplex for the students (Self) and their interaction partners (Interaction). All model fit statistics are closer to 0 for the models with predictor. We thus conclude that the variable blusher improves the fit of the model for both the students' and their interaction partners' scores on the IPC.

Fixed effects. Table 3 shows estimates of the posterior mean, mode, and standard deviation and the lower and upper bound of the 95% highest posterior density (HPD) interval for the predicted scores of the students' and their interaction partners' scores on the IPC for blushers and nonblushers. The standard deviation of a posterior distribution is an estimate for the standard error of the parameter. The HPD interval is the smallest interval in which 95% of the posterior mass is

Table 3. Descriptives of the Posterior Distributions for the Effect of Blushing on the Average Score on the IPC in Data Set I.

Variable	Mode	*M*	SD	LB HPD	UB HPD
Self					
Blusher	15.76°	15.98°	1.83°	12.20°	19.38°[a]
Nonblusher	28.16°	28.35°	2.61°	23.29°	33.74°[a]
Interaction					
Blusher	17.69°	24.50°	35.09°	−49.56°	88.14°
Nonblusher	−27.92°	−23.02°	16.44°	−55.47°	7.17°

Note. IPC = interpersonal circumplex; HPD = highest posterior density; LB = lower bound; UB = upper bound. Self and Interaction represent the scores on the IPC of the students and their interaction partners, respectively.
[a]Indicates that the HPD interval does not include 0.

Table 4. Descriptives of the Posterior Distributions for the Effect of Blushing on the Variance on the IPC in Data Set I.

	Mode	*M*	*SD*	LB HPD	UB HPD
Self					
Blusher	0.41	0.41	0.02	0.38	0.44[a]
Nonblusher	0.36	0.37	0.03	0.32	0.43[a]
Interaction					
Blusher	0.91	0.90	0.04	0.83	0.98[a]
Nonblusher	0.74	0.74	0.06	0.63	0.87[a]

Note. IPC = interpersonal circumplex; HPD = highest posterior density; LB = lower bound; UB = upper bound. Self and Interaction represent the scores on the IPC of the students and their interaction partners, respectively.
[a]Indicates that the HPD interval does not include 0.

located. In terms of interpretation, it is different from a frequentist confidence interval since HPD intervals allow for probability statements. For example, if the 95% HPD interval for a parameter μ runs from 2 to 4, we can say that the probability that μ lies between 2 and 4 is 0.95.

We see that the students' own scores differ depending on whether they are classified as blushers (the HPD intervals do not overlap). A blusher is predicted to have a score on the IPC of 15.76° and a nonblusher is predicted to have a score of the IPC of 28.16°. This is in line with the hypothesis of the original researchers that blushing is associated with higher levels of agreeable and submissive behaviors. The difference, however, is not very large. The interaction partners' scores on the IPC for blushers and nonblushers do not differ (the HPD intervals overlap).

Effects on spread. In addition to the average scores on the circumplex, we can also investigate the effect of being a blusher on the spread or variance of the scores on the circumplex. For example, the variance of the scores of students on the IPC may be different for blushers and nonblushers. From Table 4, we concluded that there is no association between blushing and the variance of scores on the IPC for the students and their interaction partners; the HPD intervals of the estimated circular variances overlap.

Table 5. Descriptives for Data Set II.

Variable	*M*/ $\hat{\theta}$	*SD*/ $\hat{\rho}$	Range	Type
IPC	25.21°	0.74	—	Circular
EX	5.61	4.68	0-29	Linear
Year	1992	6.31	1974-2006	Linear

Note. $\hat{\theta}$ = circular mean; $\hat{\rho}$ = circular concentration. IPC, EX, and Year represent the interpersonal cirucmplex, teacher experience, and the year of measurement, respectively.

Data Set II

Participants. The second data set contains data from Dutch secondary school teachers and was collected between 1982 and 2008. It contains circumplex data for over 7,199 teachers at different stages of their teaching career. A further description of the sample can be found in Brekelmans et al. (2005). For this article, we selected teachers whose data included at least three QTI measures during their career. This resulted in a data set with 126 teachers.

Measures. A circular score on the IPC was derived from the QTI measures of the teachers. Apart from the three circular scores on the IPC, we construct a variable EX that indicates the experience (in years) that a teacher had at a specific measurement occasion. Note that this means that instead of taking the number of the measurement occasion (first, second, third, etc.) or the year of the measurement, we take the experience of a teacher as the variable of interest. A teacher with 3 years of experience in 1990, thus has the same score on this time variable as a teacher with 3 years of experience in 2000. To control for possible biases due to the year in which the teacher started his or her career, we include a control variable that indicates the year in which a teacher had 0 years of experience.

Research Question. The research question of interest concerns how teachers' scores on the IPC change during their career.

Descriptives. Descriptives are shown in Table 5.

Model. To answer the research question, we predict the score on the circumplex (IPC), our circular outcome, using the variables EX and Year. We also include a random intercept. We will use a model building procedure for this data. This means that we will start by fitting a so-called intercept-only model, where we include only the intercept and no other predictors. In the subsequent models, predictors are

Table 6. Model Fit Statistics for the Models Fit to Data Set II.

Model	Intercept-only	EX	EX + Year
DIC	1,417	1,375	1,381
DIC ^{alt}	2,103	2,046	2,044
WAIC ^{1}	1,390	1,351	1,355
WAIC ^{2}	1,482	1,438	1,444

Note. DIC = deviance information criterion; WAIC = Watanake–Akaike information criterion. Each column contains the values of four model fit statistics for one of the models fit to Data Set II. EX and Year represent teacher experience and the year of measurement, respectively.

Table 7. Descriptives of the Posterior Distributions of Several Circular Regression Coefficients for the Effect of Experience in Data Set II.

Statistic	Mode	*M*	*SD*	LB HPD	UB HPD
b_c	5.49°	4.96°	2.55°	2.84°	8.37°[a]
SAM	2.01°	2.01°	0.81°	1.20°	2.59°[a]
AS	2.04°	2.08°	0.99°	1.20°	2.67°[a]

Note. b_c = slope at the inflection point; *SAM* = slope at the mean; *AS* = average slope; HPD = highest posterior density interval; LB = lower bound; UB = upper bound.
[a]Indicates that the HPD interval does not include 0.

added one by one. The full model that estimates the mean of both components (I and II) is as follows:

$$\mu_{ij} = \begin{pmatrix} \mu_{ij}^{I} \\ \mu_{ij}^{II} \end{pmatrix} = \begin{pmatrix} \beta_0^I + \beta_1^I \text{EX}_{ij} + \beta_2^I \text{Year}_i + b_{0i}^I \\ \beta_0^{II} + \beta_1^{II} \text{EX}_{ij} + \beta_2^{II} \text{Year}_i + b_{0i}^{II} \end{pmatrix}.$$

The fit of the models is assessed using the same criteria as in Data Set I. R-code for fitting the model can be found in the Supplementary Material available online. All continuous variables are centered on their grand mean before inclusion in the analysis. Before evaluation of the results, the convergence of the MCMC samplers was checked by means of traceplots. In all models, we reached convergence within 1,000 iterations (burn-in = 1,000, lag = 3).

Results

Model fit. Table 6 shows the model fit statistics. Both models with predictors fit better than the intercept-only model. All four model fit statistics are smaller, for example, the DIC decreases from 1,417 (intercept-only) to 1,375/1,381 (EX/EX + Year). The addition of the covariate Year does not improve the fit of the model. This indicates that the year of the first measurement does not have an effect on a teachers' average score on the IPC. We therefore decide to continue with the model that only contains years of experience (EX).

Fixed effects. In Table 7, descriptives of the posterior distribution for the three types of circular coefficient for the effect of experience (EX) are shown. All three coefficients are small but different from 0 as indicated by their HPD interval. That means that on average there is an effect of years of experience on a teachers' score on the circumplex. To be more precise, at the inflection point an increase of 1 unit in EX results in a counterclockwise move of b_c = 5.49° on the circumplex. At the grand mean of EX, an increase of 1 unit in EX results in a counterclockwise move of *SAM* = 2.01° on the circumplex. On average an increase of 1 unit in EX results in a counterclockwise move of *AS* = 2.04° on the circumplex. Note, however, that these effects need to be interpreted together with context, for example, at the grand mean of experience (5.61) the predicted score of a teacher is around 30° (see Figure 6, the "Helpful" octant), but for each unit increase in experience a teacher is expected to make a counterclockwise move of 2.01° and thus move toward the "Directing" octant of the IPC.

We plot the relation between years of experience and the score on the IPC in Figure 6. The line in this figure represents the effect of experience on the location of the score of a teacher on the IPC and the dots represent the data. The predicted value at 0 years of experience is 23° and at 29 years of experience it is 32°. Both these values fall in the "Helpful" octant of the IPC. The line is not steep, reflecting the small values of the estimates for the circular coefficients. However, the data does show that although the location of a teachers' score does not change much over time, the variance of the scores of the teachers on the circumplex does change. We see that the scores of teachers with a low amount of experience (EX = 0) are spread across the entire circumplex, while the scores of teachers with a high amount of experience (e.g., EX = 20) are much more concentrated at a specific region on the circumplex.

Effects on spread. We concluded from Figure 6 that there may be an effect of experience on the spread or variance of scores of teachers on the circumplex. In a model from the embedding approach, such as the circular mixed-effects model in this article, we automatically model an effect on the location and on the spread of the circular outcome. This is a remnant of the fact that the mean of the underlying bivariate distribution affects both the location and spread on the circle (see Figure 4). This means that each predictor in the model also has a potential effect on the spread of the circular outcome.

We compute the variance, $1-\hat{\rho}$, of the scores of teachers on the circumplex at different years of experience. Figure 7 shows these estimates. This plots reflects, but now also quantifies, the pattern that we already observed in the data. For less experienced teachers, for example, EX = 0, the variance of scores on the circle is larger at $1-\hat{\rho} \approx 0.2$ than for experienced teachers, $1-\hat{\rho} \to 0$.

Random effects. The variances of the linear random intercepts are estimated at 1.69 (HPD: 1.21, 2.32) for the

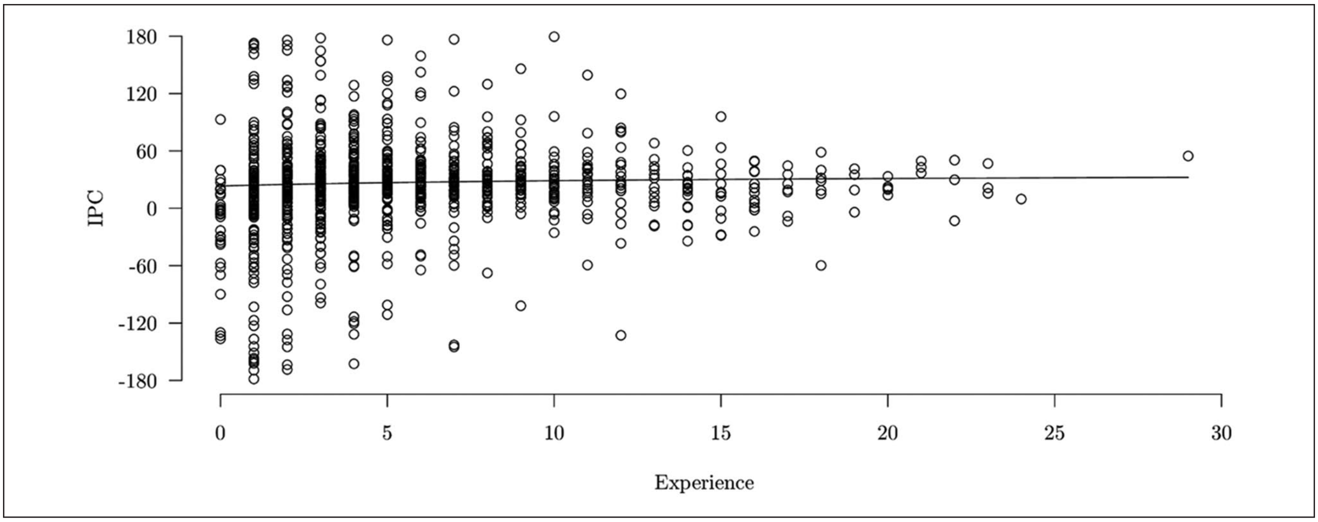

Figure 6. The effect of experience on the location of teachers' scores (in degrees) on the circumplex for Data Set II.
Note. The line represents the predicted circular regression line and the dots represent measurements for the teachers. Note that we have rescaled the *y*-axis to make sure the regression line is smooth. The interpersonal circumplex now ranges from −180° to 180° instead of 0° to 360°, where −60° = 300°, −120° = 240°, and −180° = 180° due to the periodicity of the circle.

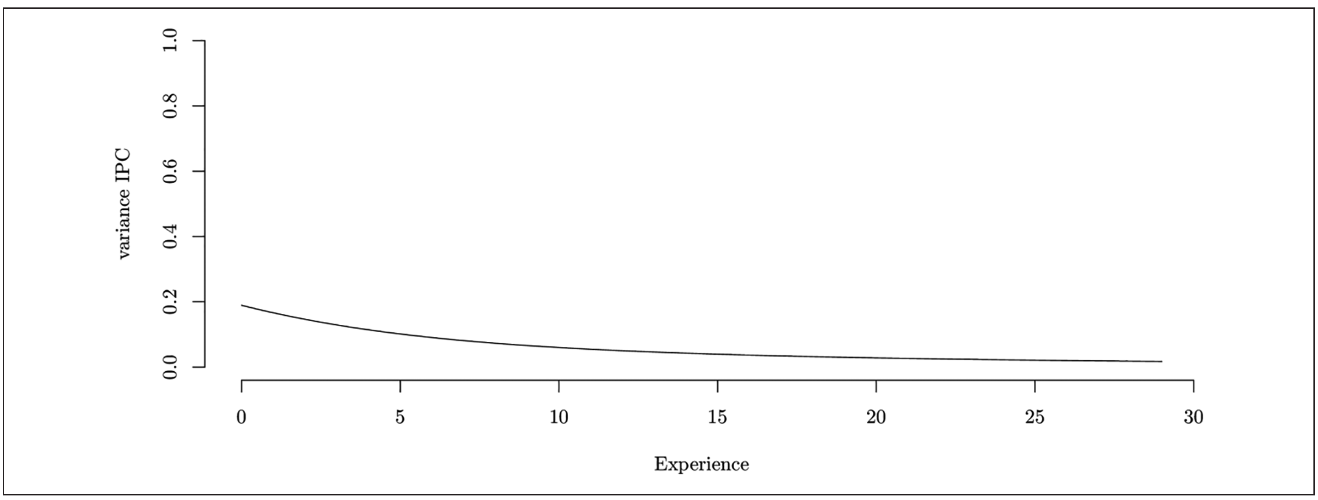

Figure 7. The effect of experience on the variance $1-\hat{\rho}$ of teachers' scores on the circumplex for Data Set II.

first or Communion component and 0.81 (HPD: 0.58, 1.09) for the second or Agency component. On the circle this translates to an estimated circular variance of 0.25 (HPD: 0.19, 0.30). This means that there is variance in the teachers' score at 0 years of experience, and thus the teachers' scores differ on the circumplex at that point. This was to be expected from the plotted data in Figure 6. It is clear from this figure that at 0 years of experience (EX = 0) there is a lot of variance between teachers in their score on the circumplex. The intercept variance could be explained by additional teacher characteristics. Data Set II, however, does not contain additional covariates to further explore this.

Data Set III

Participants. The third data set was collected between 2010 and 2015 and contains repeated measures on the QTI for 161 Dutch secondary school teachers. The data were gathered for the studies of Pennings (2017a), Claessens (2016), and van der Want (2015) and further details on the sample can be found in these studies.

Measures. The measurements on the QTI were taken in different years and in different classes. For this article, we only consider the year and take the measurements for the largest class if data for multiple classes were available in 1 year. We

Table 8. Descriptives for Data Set III.

Variable	$M/\hat{\theta}$	$SD/\hat{\rho}$	Range	Type
IPC				
Male	32.56°	0.80	—	Circular
Female	39.08°	0.80	—	Circular
Time	1.38	1.19	0-3	Linear
EX	5.27	1.15	1.83-7	Linear
SE	4.77	0.85	1.75-6.5	Linear

Note. $\hat{\theta}$ = circular mean; $\hat{\rho}$ = circular concentration. IPC, EX, and SE represent the interpersonal cirucmplex, teacher experience, and the self-efficacy, respectively.

ended up with a maximum of four measurements per teacher. A circular score on the IPC was constructed using the QTI measures of the teachers. In addition to a variable, Time, specifying the measurement occasion, the years of experience of the teacher at the first measurement (EX), a teachers' self-efficacy in dealing with student emotions (SE), and the gender of the teacher (Gender) will be used as covariates in the analysis. Note that both EX and Gender are constant, while SE is different for each measurement occasion.

Research Question. The research question of interest concerns how teachers' scores on the IPC change during their career and how this score is affected by the covariates experience, self-efficacy, and gender.

Descriptives. Table 8 shows descriptives for the circular outcome, IPC, and the predictors in the model.

Model. We model the score on the circumplex (IPC) using several predictor variables. We again use a model building procedure in which we first fit an intercept-only model and then the model including the Time predictor. We also include a random intercept. The full model includes the covariates SE, EX, and Gender:

$$\boldsymbol{\mu}_{ij} = \begin{pmatrix} \mu_{ij}^{I} \\ \mu_{ij}^{II} \end{pmatrix} = \begin{pmatrix} \beta_0^I + \beta_1^I \text{Time}_{ij} + \beta_2^I \text{SE}_{ij} + \beta_3^I \text{EX}_{ij} \\ + \beta_4^I \text{Gender}_{ij} + b_{0i}^I \\ \beta_0^{II} + \beta_1^{II} \text{Time}_{ij} + \beta_2^{II} \text{SE}_{ij} + \beta_3^{II} \text{EX}_{ij} + \\ \beta_4^{II} \text{Gender}_{ij} + b_{0i}^{II} \end{pmatrix}.$$

R-code for fitting the model can be found in the Supplementary Material available online. The fit of the models is assessed using the same criteria as for Data Set I and SE and EX are centered at their grand mean before inclusion in the analysis. Before evaluation of the results convergence was checked by means of traceplots. For all models convergence was reached within 1,000 iterations (burn-in = 1,000, lag = 3).

Table 9. Model Fit Statistics for the Models Fit to Data Set III.

Model	Intercept-only	Time	Time + SE + EX + Gender
DIC	462	469	460
DIC ^{alt}	1,476	1,262	1,354
WAIC ^{1}	443	441	438
WAIC ^{2}	513	511	510

Note. DIC = deviance information criterion; WAIC = Watanake–Akaike information criterion. Each columns contain the values of four model fit statistics for one of the models fit to Data Set III. Time, EX, and SE represent the measurement occasion, teacher experience, and self-efficacy, respectively.

Table 10. Descriptives of the Posterior Distributions of Several Circular Regression Coefficients for the Effect of the Continuous Predictors in Data Set III.

Variable	Statistic	Mode	*M*	*SD*	LB HPD	UB HPD
Gender (male)						
Time	b_c	1.52°	1.68°	2.42°	−2.90°	5.29°
	SAM	1.24°	0.49°	30.09°	−8.57°	12.25°
	AS	1.24°	1.98°	39.87°	−9.07°	11.04°
SE	b_c	15.99°	13.52°	11.00°	−18.38°	28.64°
	SAM	6.99°	10.17°	10.54°	2.79°	39.05°[a]
	AS	6.99°	8.46°	39.32°	2.25°	40.68°[a]
EX	b_c	1.29°	0.39°	28.83°	−0.59°	1.07°
	SAM	0.24°	0.27°	33.28°	−8.71°	8.63°
	AS	0.24°	–0.65°	25.67°	−5.75°	9.83°
Gender (female)						
Time	b_c	6.31°	11.78°	219.95°	−15.86°	26.63°
	SAM	2.19°	5.19°	215.79°	−6.99°	19.69°
	AS	3.46°	3.80°	81.22°	−5.99°	13.12°
SE	b_c	20.85°	20.12°	52.45°	5.00°	42.01°[a]
	SAM	8.06°	9.63°	50.08°	3.13°	19.81°[a]
	AS	9.72°	10.39°	7.86°	3.78°	19.24°[a]
EX	b_c	14.04°	10.36°	102.31°	−14.84°	27.71°
	SAM	10.23°	5.88°	100.86°	−6.87°	20.35°
	AS	3.86°	6.29°	2.92°	−4.14°	6.68°

Note. b_c = slope at the inflection point; *SAM* = slope at the mean; *AS* = average slope; HPD = highest posterior density; LB = lower bound; UB = upper bound. Time, EX, and SE represent the measurement occasion, teacher experience, and self-efficacy, respectively.
[a]Indicates that the HPD interval does not include 0.

Results

Model fit. Table 9 shows the model fit statistics. We conclude that both models with covariates fit better than the intercept-only model, the model fit statistics are smaller for 3 and 4 of the criteria. The full model shows improvement in 3 model fit criteria compared with the model with just Time. We therefore decide to continue with the full model.

Fixed effects. Table 10 shows the circular fixed effects estimates. Because the model contains a categorical covariate,

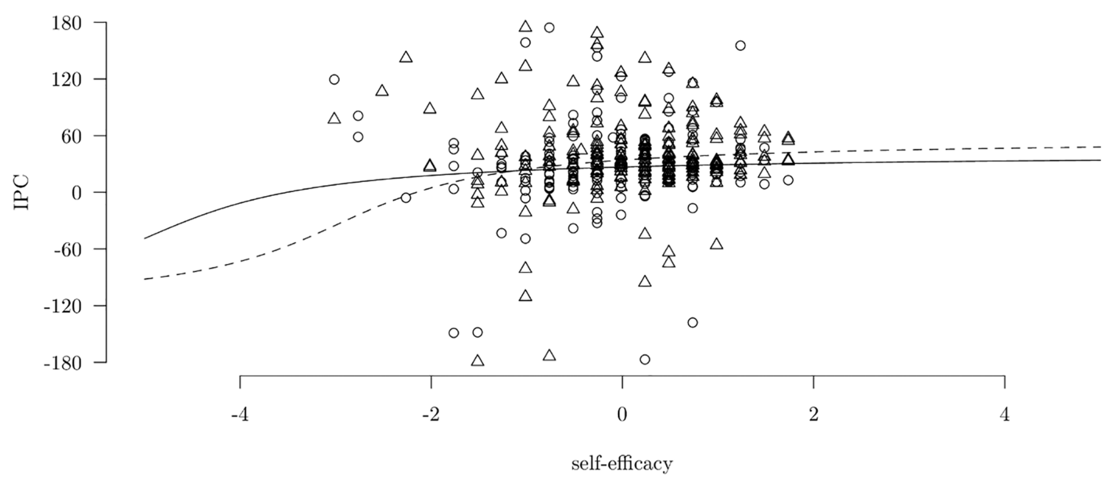

Figure 8. The effect of self-efficacy on the score on the IPC (in degrees) for Data Set II.
Note. IPC = interpersonal circumplex. We distinguish between the effect for male (solid regression line and circular data points) and female teachers (dashed regression line and triangular data points). Note that we have rescaled the *y*-axis to make sure the regression line is smooth. The IPC now ranges from −180° to 180° instead of 0° to 360° where −60° = 300°, −120° = 240°, and −180° = 180° due to the periodicity of the circle.

Table 11. Posterior Mode Estimates and their HPD Interval for the Linear and Circular Random Intercept Variances for Data Set III.

Model	Component I (HPD)	Component II (HPD)	Circular (HPD)
Intercept-only	2.51 (1.85, 4.50)	0.79 (0.36, 1.22)	0.11 (0.08, 0.14)
Time	2.73 (1.84, 4.32)	0.79 (0.43, 1.25)	0.12 (0.08, 0.16)
Time + SE + EX + Gender	2.72 (1.84, 4.17)	0.59 (0.29, 0.96)	0.11 (0.06, 0.17)

Note. HPD = Highest Posterior Density. Time, EX, and SE represent the measurement occasion, teacher experience, and self-efficacy, respectively.

Gender, we display a marginal effect for each variable at each of the levels of the covariate Gender. We do this because the marginal effect of a predictor in the circular mixed-effects model is different for different levels or values of the other predictors in the model. This is caused by the fact that we are fitting a model in which the relation between outcome and predictors is nonlinear.

Two coefficients, *SAM* and *AS*, for the effect of SE are small but different from 0 at both levels of Gender as indicated by their HPD intervals (see Table 10). That means that at Time = 0 and for a teacher with average experience, there is an effect of SE on a teachers' score on the circumplex at the grand mean and on average. To be more precise, at the grand mean of SE, an increase of 1 unit in SE results in a counterclockwise move on the IPC of *SAM* = 6.99° for Gender = male and *SAM* = 8.06° for Gender = female. On average, an increase of 1 unit in SE results in a counterclockwise move of *AS* = 6.99° for Gender = male and *AS* = 9.72°for Gender = female on the IPC. Figure 8 shows the effects of SE on the IPC for both male and female teachers. We see that the score on the circumplex slightly changes with increasing self-efficacy for both male and female teachers.

To investigate the effect of gender at Time = 0 for teachers with average scores on self-efficacy and experience, we look at the predicted scores on the IPC for males and females. The predicted score on the IPC for males is estimated at 17.50° (22.35°, 34.95°) and for females at 32.66° (28.65°, 40.11°). Their HPD intervals overlap which means that on average, male and female teachers do not differ in their scores on the IPC. For Time and EX none of the coefficients are different from 0 according to their HPD interval in Table 10.

Random effects. Table 11 shows the posterior modes of the linear and circular random intercept variances for the models that were fit to Data Set III. We see that from the intercept-only model to the model with Time the posterior mode of the circular variance increases slightly. This is a phenomenon that occurs more often in mixed-effects models that are fit to longitudinal data (Hox, 2002). The increase in variance is caused by the fact that the model is based on the assumption that the measurements within an individual are random samples from a population of possible measurements and assumes a certain variance for these measurements. However, in a longitudinal model, the repeated measures are usually fixed and therefore have a lot less variance than expected by the model. Therefore, it is possible that we see an increase in intercept variance from the model without the predictor Time, which indicates measurement occasion and the model with Time. However, from Table 11, we also see that the HPD intervals of the variances from these two models overlap. So, if there is any increase in variance that is different from 0 it is so small that it cannot be detected using the circular mixed-effects model on this data set.

In contrast to the best fitting model for Data Set II, the best fitting model for Data Set III contains additional covariates (apart from Time). Therefore, we may try to explain some of the intercept-variance from the model with Time using those covariates. From the model with Time to the model with Time, SE, EX, and Gender the posterior modes of all variances, linear, and circular decrease. However, their HPD intervals overlap meaning that the decrease in variance is not larger than 0. This means that the three additional covariates, Gender, SE, and EX do not explain a part of the circular or linear intercept variance.

Discussion

In this article, we have shown how a circular mixed-effects model can be used to model repeated measures from an IPC. Note that although we have focused on mixed-effects models and repeated measures data in this article, there are equivalent methods for the simpler circular GLM, regression and analysis of variance models (Cremers et al., 2018; Cremers & Klugkist, 2018). We have also developed new interpretation tools that have solved the interpretation problems associated with the circular (projected normal) mixed-effects model. This model together with the new interpretation tools has allowed us to interpret the effect of covariates on a circular score on the IPC itself instead of on its two separate components Agency and Communion.

In the previous analysis of the first data set, aan het Rot et al. (2015) conclude that participants that blush report fewer dominant behaviors and more submissive behaviors. Furthermore, they perceive their interaction partners as being less affiliative and more powerful. Their first finding is reflected in the results from the circular analysis in this article. We found that blushers on average report a score of 16.04°, whereas nonblushers report a score of 28.07° on the IPC. This means that compared with nonblushers, blushers not only show more submissive but also more affiliative behavior. Because we analyze the blend of Agency and Communion and restrict the score to the (edge of the) circle, a change in one of its components is forced to go hand in hand with a change in the other component. The advantage of this circular approach is that we can test hypotheses about the effect on the scores on the IPC directly instead of relying on two dimensions or four interpersonal behaviors. This is a more valid analysis because it is in line with the idea of the IPC being "a continuous order with no beginning or end" (Gurtman, 2009, p. 602).

For the second data set, we can also compare the results from the analysis with the circular mixed-effects model with previous analyses. In Brekelmans et al. (2005), the data set that we took a subset from to create Data Set II is analyzed using a similar model to the one in this article. In their analysis, students' perceptions of teachers, split in a Proximity (Communion) and Influence (Agency) score, are predicted by the teachers' experience in a multilevel growth model. Their findings are that students' perceptions of a teachers' Communion remain stable over the teacher career, while perceptions of a teachers' Agency grow in the first 6 years of a teacher career. In our analysis, we have, however, combined the effects on Agency and Communion which means that we can characterize a change in teachers' behavior on a blend of these dimensions. We have shown that there is a small effect of experience on a teachers' score on the IPC. They move from a score of 23° at 0 years of experience to a score of 32° at 29 years of experience. This means that over the course of the teaching career teachers move more toward the center (45°) of the "preferred" styles of teacher behavior which is composed of the "Directing" and "Helpful" types of Figure 1. Over time, teachers thus develop their style of teacher behavior more toward the preferred style of teacher behavior. Such a conclusion could not have been reached when we had used an analysis on the eight subtypes; 23° and 32° both fall within the "Helpful" subtype. The results for Agency and Communion from Brekelmans et al. (2005) could give us an indication of the effect we expect on the IPC itself, but this effect cannot be quantified such that we can test whether it is different from 0. In contrast, treating data on the IPC as circular does allow us to test for an effect.

In addition to being able to investigate the effect of a covariate on the location of a score on the IPC itself instead of on Agency and Communion separately, the circular mixed-effects model allows us to investigate effects on the spread of scores on the IPC. This gives us additional insights compared with the analysis from Brekelmans et al. (2005). From the analysis of Data Set II, we conclude that not only the location but also the spread of the score on the IPC changes with experience. The scores of more experienced teachers are more concentrated in the preferred type of teaching behavior than the scores of less experienced teachers. Note that although this is an indication that all teachers on average move toward the preferred type of behavior this does not mean that each individual teacher necessarily does so. To reach a more formal conclusion about this, we have to look at the effect of experience on the score on the IPC for each individual separately and include a random slope for experience in the model.

Since no covariates were available in the Data Set II, we used a third data set to show how covariates can be included to explain variance in the circular data. This is the first time that the longitudinal circular data from Data Set III have been analyzed. Although the models including effects over time and those of time combined with gender, experience, and self-efficacy improved model fit, the effects of the covariates were too small to explain any variance in the circular or linear intercepts. Still it enabled us to show how covariates can be included in the analysis and attends to the need for an analysis method to analyze Agency and Communion blended together. It thus provides researchers with the means to study associations between covariates and the circular IPC data without analyzing the two dimensions separately. The associations found in Data Set III may be quite interesting for practice. For example, we concluded from the analysis that gender does not matter for the quality of teacher–student relationships in Data Set III, and that although the self-efficacy of teachers may vary, it does not necessarily affect the quality of teacher–student relationships to a great extent. This is important because, especially, early career teachers may worry about their ability to teach. Fortunately, that does not necessarily seem to affect the quality of teacher–student relationships, which means that teachers can still develop toward establishing the preferred type of teacher behavior style (as found in Data Set II) even though they do not feel confident in their teaching yet.

In this article, we have used the embedding approach to circular data. Two other approaches to analyze circular data are the wrapping and intrinsic approach (Mardia & Jupp, 2000). In the intrinsic approach distributions that are directly defined on the circle, such as the von Mises distribution, are used to model the data. In the wrapping approach, the data are modeled by wrapping a univariate distribution defined on the real line, for example, the Normal distribution, onto the circle. Even though more complex models have been introduced for these approaches (Lagona, 2016; Wang & Gelfand, 2014), a mixed-effects model was not among them. An advantage of models from the intrinsic and wrapping approaches is that their results are easier to interpret compared with models from the embedding approach. Therefore, we have in this article introduced new interpretation tools for the circular mixed-effects model based on the projected normal distribution that solved the interpretation problems associated with the embedding approach.

A possible critique on the way we have analyzed circumplex data in this article is that we have only considered information on the angle resulting from the conversion of a score on Agency and Communion to the IPC. This includes directional information contained in the direction of the two-dimensional vector of an Agency and Communion score but excludes information on the "size" or "intensity" of this vector. In contrast, modelling Agency and Communion separately allows us to model the "intensity" but excludes directional information. Models that allow for simultaneous modelling of a circular and linear variable (direction and intensity), however, do exist. In the literature Mardia and Sutton (1978), Abe and Ley (2017), Mastrantonio, Maruotti, and Jona-Lasinio (2015), and Mastrantonio (2018) have introduced models for these so-called cylindrical data. However, thus far these models only model the relation between the linear and circular variable and their respective means but do not introduce a regression structure to predict their means using additional covariates. Additionally, the models do not allow for the modelling of multiple measurements over time. In future research, it would be useful to extend existing models for cylindrical data with a regression structure and a structure that allows for longitudinal data. We could then apply these extended cylindrical models to the data sets analyzed in this article.

In this article, we have included methods that allow for the analysis of a circumplex outcome variable, however, in certain cases these variables also serve as a predictor variable. At first glance modelling, circumplex predictors seems much easier than modelling circumplex outcomes. A circular predictor can namely be modelled in a standard model for "linear" outcomes by including the cosine and the sine of the predictor as two predictor variables into the model. However, if we do this for a circumplex predictor, this is equivalent to including the Agency and Communion component of the predictor into the model separately. If we are interested in the effect of a blend of Agency and Communion, we should come up with a different modelling strategy for circumplex predictors. The reparameterization used in this article for obtaining effects on the circle may provide a solution for this problem. Also different methods for including circular predictors have been developed in the literature (e.g., in Kim & SenGupta, 2015). In further research, it may be worthwhile to explore the case of circumplex predictors further.

In conclusion, we have shown that modelling longitudinal data from the IPC using a circular model is possible. It offers us a different perspective to the data and an analysis that is more in line with the original idea of the IPC. In addition, we have been able to solve interpretation issues of the specific model used in this article.

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Note

1. Restricting the covariance matrix also implies that we cannot simply take scores on Agency and Communion as y. Neither can we fit a bivariate linear mixed-effects model on Agency

and Communion scores directly and translate the results from this model to the circle easily. This is due to the fact the covariance matrix of such a model is unrestricted and thus both its mean and its variance influence the circular scores.

Supplemental Material

Supplemental material for this article is available online.

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