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Research article

Effective multi-attribute group decision-making approach to study astronomy in the probabilistic linguistic q-rung orthopair fuzzy VIKOR framework

Sumera Naz [a,*], Areej Fatima [a], Shariq Aziz But [b], Dragan Pamucar [c,d,e], Ronald Zamora-Musa [f], Melisa Acosta-Coll [g]

[a] *Department of Mathematics, Division of Science and Technology, University of Education, Lahore, Pakistan*
[b] *School of Systems and Technology, Department of Computer Science, University of Management and Technology, Lahore, Pakistan*
[c] *University of Belgrade, Faculty of Organizational Sciences, Department of Operations Research and Statistics, Belgrade, Serbia*
[d] *Yuan Ze University, College of Engineering, Taoyuan, Taiwan*
[e] *Western Caspian University, Department of Mechanics and Mathematics, Baku, Azerbaijan*
[f] *Department of Industrial Engineering, Universidad Cooperativa de Colombia UCC, Barrancabermeja 687031, Colombia*
[g] *Department of Computer Science and Electronics, Universidad de la Costa, Barranquilla 080002, Colombia*

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ABSTRACT

This study employs a novel fuzzy logic-based framework to address multi-attribute group decision-making problems commonly encountered in modern astronomy. Our approach utilizes the probabilistic linguistic q-rung orthopair fuzzy set (PLq-ROFS) to handle the inherent uncertainties associated with astronomical data. The PLq-ROFS offers significant advantages over existing fuzzy sets like probabilistic hesitant, linguistic intuitionistic, and linguistic Pythagorean fuzzy sets, which comprise both stochastic and non-stochastic uncertainties simultaneously. To aggregate the probabilistic linguistic decision information effectively, we propose two novel operators: the PLq-ROF weighted power average (PLq-ROFWPA) and the PLq-ROF weighted power geometric (PLq-ROFWPG). These operators form the foundation of a novel method within the PLq-ROF environment. Furthermore, this study integrates the PLq-ROF framework with the VIseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) model, a widely used decision-making (DM) tool known for its ability to balance group utility maximization with individual regret minimization. This integration leads to the PLq-ROF-VIKOR model, a novel approach for ranking alternative solutions based on the subjective preferences of decision-makers. The effectiveness of the proposed method is demonstrated through a real-world case study in astronomy, accompanied by both parameter and comparative analyses. These analyses highlight the efficiency and accuracy of the PLq-ROF-VIKOR model, ultimately leading to the conclusion that cosmology is the most optimal key finding in this case study.

* Corresponding author.
E-mail addresses: sumera.naz@ue.edu.pk (S. Naz), shariq2315@gmail.com (S.A. But), dpamucar@gmail.com (D. Pamucar), ronald.zamora@campusucc.edu.co (R. Zamora-Musa), macosta10@cuc.edu.co (M. Acosta-Coll).

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1. Introduction

Group DM is a complex process that involves multiple individuals with different perspectives, preferences, and priorities. The fuzzy set (FS) theory can be a valuable tool in facilitating group DM by considering and accommodating the uncertainties and vagueness associated with human judgments. The FS theory allows for the representation and manipulation of imprecise or ambiguous information, which is often present in DM situations. The general framework for utilizing FS theory in group DM is as follows: (1) Clearly define the decision problem and establish the attributes and alternatives involved. This step is crucial to ensure that everyone in the group has a common understanding of the decision context. (2) Each group member expresses their preferences or evaluations of the alternatives using linguistic terms (LTs) or fuzzy numbers. For example, instead of assigning a crisp numerical value to a criterion, they may use terms like low, medium, or high to indicate their assessment. (3) Combine the individual evaluations into a group evaluation by aggregating the FSs provided by the group members. There are various aggregation methods available, such as the arithmetic mean, geometric mean, or fuzzy integral. (4) Convert the aggregated FS into a crisp value to facilitate further analysis and comparison. Defuzzification methods, such as centroid or height methods, can be used to obtain a representative value from the FS. (5) Assess the robustness of the decision by examining how sensitive the results are to changes in individual evaluations. This step helps to identify influential group members or attributes and highlights potential areas of disagreement or inconsistency. (6) Engage the group members in discussions and negotiations to address any conflicts or differences in opinions. FS theory provides a flexible framework to incorporate multiple perspectives and reconcile conflicting viewpoints. (7) Utilize the defuzzified values and sensitivity analysis results to evaluate and compare the alternatives based on the defined criteria. Various DM techniques, such as multi-criteria decision analysis, can be applied to support the final DM process. Implementing FS theory in group DM requires effective communication and collaboration among the group members. Establishing clear guidelines, facilitating discussions, and promoting mutual understanding can enhance the effectiveness of the process. Furthermore, several software tools and decision support systems incorporate FS theory to facilitate group DM. These tools often provide a user-friendly interface for inputting fuzzy evaluations, performing aggregation, conducting sensitivity analysis, and visualizing the results, which can streamline the DM process and improve its transparency. Overall, FS theory can be a powerful approach for handling uncertainties and vagueness in group DM, promoting inclusivity, and accommodating diverse perspectives within a DM context. The process of DM holds significant importance and has become a central focus for researchers.

Decision-makers often encounter multi-attribute DM (MADM) issues when evaluating data with vague information. Researchers have made substantial contributions to the field of FS by introducing intuitionistic FS (IFS) and Pythagorean FS (PFS). PFS relaxed the constraint of IFS $0 \leq \alpha + \beta \leq 1$ to $0 \leq \alpha^2 + \beta^2 \leq 1$. Consequently, PFS offers a more generalized approach than IFS. However, FS, IFS and PFS provide incomplete information about the elements of the data set. Yager [50] came up with the idea of q-rung orthopair FS (q-ROFS) with the condition $0 \leq \alpha^q + \beta^q \leq 1$, which is a more general form of PFS, to solve this problem. The q-ROFS has been extensively applied in MAGDM and has introduced numerous fresh methods to decision-makers. This extension of traditional FS theory provides a flexible framework for representing uncertainty and vagueness in DM problems. It introduced the notion of rung levels to capture different degrees of uncertainty and enables a more detailed modeling of imprecise information. The applicability of our method, particularly the incorporation of the PLq-ROF set (PLq-ROFS) in our study, is fundamental in addressing the complex challenges of MAGDM prevalent in modern astronomy. Our innovative fuzzy logic-based framework is designed to navigate the complexities inherent in astronomical data analysis and DM processes. The necessity of employing PLq-ROFS in our study lies in its adept handling of uncertainties inherent to astronomical data. Astronomical data often exhibit inherent uncertainties due to factors such as measurement errors, observational limitations, and the innate variability of celestial phenomena. PLq-ROFS offers a robust framework for representing and managing uncertainties within linguistic data, thereby enabling a more nuanced and precise approach to DM within the astronomical context. Through the integration of PLq-ROFS, our study adeptly captures the vagueness and imprecision inherent in subjective assessments of astronomical attributes, thereby ensuring the reliability and accuracy of decision outcomes. By incorporating PLq-ROFS, our framework empowers astronomers to navigate the complexities of DM with greater confidence and clarity. Furthermore, the fusion of PLq-ROFS with the VIKOR model marks a significant advancement in DM methodologies customized specifically for astronomy. By effectively managing uncertainties and harmonizing competing objectives, our approach contributes to the generation of informed and dependable decision outcomes within the field of astronomy, and represents a pioneering leap forward in addressing the unique challenges of MAGDM in modern astronomy, paving the way for more robust and insightful DM processes within the discipline. To understand the origin, evolution, and interactions of these astronomical entities, astronomy relies on mathematics, physics, and chemistry. The study acknowledges the historical significance of astronomy, noting that it was one of the earliest scientific disciplines. Ancient civilizations played a vital role in its development by observing and studying the night sky and tracking the motions of celestial bodies. This historical foundation contributes to the understanding of astronomy today. Group DM in astronomy typically involves collaboration among scientists, researchers, and experts in the field. These decisions can pertain to various aspects, including observation planning, data analysis, project proposals, mission design, and policy-making. Group DM is conducted in astronomy by the following ways:

1. Astronomers often form research collaborations to tackle complex scientific questions. These collaborations consist of groups of researchers from different institutions working together on a common project. DM within these collaborations is typically conducted through discussions, brainstorming sessions, and consensus-building. Scientists exchange ideas, analyze data, propose hypotheses, and collectively decide on the best course of action for their research.
2. Observatories, both ground-based and space-based, often have limited observing time and resources. Astronomers must collectively decide which celestial objects to observe, how long to observe them, and what instruments to use. These decisions

are made based on scientific priorities, feasibility, and the interests of the participating researchers. Observational planning committees or working groups are commonly formed to evaluate proposals and allocate observing time fairly.

3. After the data is collected, decisions must be made regarding how to analyze and interpret it. Astronomical data sets can be vast and complex, requiring collaborative efforts. Scientists may form analysis teams to collectively develop data reduction techniques, algorithms, and statistical methods. Group discussions and peer reviews are conducted to ensure the reliability and accuracy of the results.
4. When it comes to designing and planning space missions, group DM plays a crucial role. Mission teams include scientists, engineers, and managers who work together to define mission objectives, instrument specifications, launch timelines, and mission trajectories. These decisions involve considerations such as scientific goals, budget constraints, technical feasibility, and risk management. Regular meetings, reviews, and consultations with stakeholders are conducted to make informed decisions throughout the mission's lifecycle.
5. Public authorities, academic institutions, or private organizations frequently fund astronomical research. Committees or panels made up of subject-matter experts frequently decide on funding distribution, policy creation, and strategic planning. These committees review proposals, evaluate scientific merit, and distribute resources according to predetermined criteria. Public input and community engagement may also be sought to ensure transparency in DM.
6. Astronomy is an international field, and many significant discoveries and projects involve collaboration among astronomers from different countries. International collaborations require coordination, resource-sharing, and DM at various levels. Organizations such as the International Astronomical Union facilitate global collaborations, establish standards, and provide a platform for discussions and DM on matters such as naming celestial objects and promoting international cooperation.

1.1. Motivation

The motivation behind the need for robust MAGDM tool known as the PLq-ROF-VIKOR model is illustrated below, which is capable of handling uncertainty in astronomy.

- Despite the advancements in DM methodologies, the application of fuzzy logic-based frameworks to addressing MAGDM issues specific to astronomy remains relatively limited [1,12,46]. There is a lack of research focusing on developing adaptable fuzzy logic-based models to handle the complexities of astronomical data and DM scenarios.
- A novel framework is required to effectively address MAGDM problems in astronomy by incorporating both stochastic and non-stochastic uncertainties.
- The utilization of PLq-ROFS, a promising FS model capable of addressing uncertainties in DM, is not extensively explored in the field of astronomy. Existing studies in astronomy often rely on conventional DM techniques, overlooking the benefits that PLq-ROFS could offer in handling uncertainties inherent in astronomical data.
- In order to effectively combine individual preferences into a single decision, the incorporation of PLq-ROFS with AOs can be an effective assessment technique to handle ambiguities in the aggregation phase.
- There is a deficiency of research exploring the integration of advanced DM models, such as the VIKOR model, with fuzzy logic-based frameworks in the context of astronomy. The development of integrated models modified to the unique requirements of astronomical DM scenarios is yet to be adequately addressed.

1.2. Novelty

The novelty of this research article behind the motivational factors is as follows:

- Astronomical data is inherently uncertain due to limitations in observation and instrumentation. Our proposed PLq-ROF-VIKOR model can handle this inherent uncertainty efficiently.
- Existing fuzzy logic approaches have limitations in capturing both stochastic and non-stochastic uncertainties present in astronomy. The presented approach can handle both stochastic and non-stochastic uncertainties related to astronomy in a better way.
- PLq-ROFS offers significant advantages over existing FSs by providing a more comprehensive representation of uncertainty in astronomical data.
- To effectively combine decision information within the PLq-ROF framework, the utilization of PLq-ROFWPA and PLq-ROFWPG operators offers a novel aggregation techniques.
- The PLq-ROF-VIKOR model allows for incorporating the subjective preferences of decision-makers while ranking alternatives in astronomy.

1.3. Objectives

The primary objectives of this research are:

1. To contribute into the field of decision science by presenting a comprehensive and innovative approach that combines fuzzy logic, MAGDM, and the VIKOR framework. This contribution seeks to offer decision-makers a more robust and accurate tool for addressing complex decision scenarios and fostering well-informed choices.
2. To enhance the VIKOR framework by incorporating the PLq-ROFS. This integration aims to provide a more comprehensive and effective approach for MADM scenarios.
3. To leverage the PLq-ROFWPA and PLq-ROFWPG operators as aggregation methods within the enhanced VIKOR framework. These operators will facilitate the effective combination of linguistic decision information from various decision-makers, accommodating both stochastic and non-stochastic uncertainties.
4. To establish a novel approach for ranking alternatives within the MAGDM context. The enhanced VIKOR framework, enriched with PLq-ROFS and related operators, will allow decision-makers to assign subjective preferences to alternatives, leading to more accurate and balanced rankings.
5. To validate the effectiveness of the proposed approach by applying it to a real-world numerical example. Through this validation process, the study aims to demonstrate the practical utility and improved performance of the enhanced VIKOR framework with PLq-ROFS, PLq-ROFWPA operator, and PLq-ROFWPG operator.
6. To perform a comparative analysis of the proposed approach with existing approaches in the field. By comparing the results and efficiency of the enhanced VIKOR framework with alternative methods, the study aims to showcase the advantages and strengths of the proposed approach.

1.4. Outline

The structure of this work is divided into several sections to comprehensively address the research objectives. Section 2 provides an interpretation of the existing research. Section 3 serves as an introductory part, providing background information on the topic and defining important terms essential for the study. Section 4 delves into a detailed exploration of the PLq-ROFWPA and PLq-ROFWPG operators. It thoroughly explains the formulation, principles, properties of these operators, and their applicability in various DM contexts. In Section 5, the research methodology is presented, outlining the approach, techniques, and procedures adopted in developing the PLq-ROFWPA-VIKOR model. Next, Section 6 showcases a compelling case study, demonstrating the practical application of the PLq-ROFWPA-VIKOR model. This section discusses the DM results obtained from the model, conducts a parameter analysis to show the impact of the parameter, and conducts a comparative analysis with existing approaches. Finally, Section 7 concludes the study by summarizing the key findings, emphasizing the limitations of the research, and proposing future avenues for further investigation and enhancement in the field.

2. Literature review

DM is a fundamental aspect of various aspects of life, including daily routines, management, social interactions, and economics. Many real-world DM challenges fall within the realm of MAGDM, where a group of experts evaluates alternatives based on multiple criteria, and the preferences of decision-makers determine the final ranking of these alternatives [39]. Akram et al. introduced extended MABAC [2] and CODAS [3] methods to rank the alternatives with 2-tuple linguistic T-spherical fuzzy set. Similarly, to solve various MAGDM problems, Naz et al. put forward the ideas of combining the Heronian mean operators with 2-tuple linguistic bipolar fuzzy set [27] and hybrid DEMATEL-TOPSIS approach with 2-tuple linguistic q-rung orthopair fuzzy information [28]. The MAGDM process consists of four primary components: defining the decision problem, articulating the viewpoints of decision-makers, establishing the ranking order of alternatives, and executing the DM recommendations. These responsibilities are distributed among the involved parties. Defining the decision problem involves identifying potential alternatives and the attributes necessary for selecting the optimal choice. Decision-makers utilize various methods to express their opinions on how each alternative performs across each criterion. The ranking order of alternatives is determined by decision-makers using specific methodologies and assessment matrices. Finally, the DM advice is put into action by selecting the best alternative based on the established ranking order. DM with LTs [54] is an approach that aims to bridge the gap between human subjective judgments and the formal representation of decision problems. It recognizes that DM involves complex and often vague concepts that cannot always be expressed in precise numerical terms. By incorporating LTs such as high, medium, and low to describe variables and preferences, decision-makers can better articulate their thoughts and preferences. This framework enables a more intuitive and human-centric approach to DM, allowing individuals to express their preferences in a manner that aligns with their natural language. By utilizing LTs, decision-makers can capture the subtleties and nuances that are often lost in traditional numerical models, leading to more meaningful and informed decisions. Additionally, this approach facilitates communication and collaboration among stakeholders, as it allows for a common language that is easily understood and interpreted by all parties involved. Ultimately, DM with LTs empowers decision-makers to incorporate their subjective judgments and personal experiences into the DM process, resulting in more holistic and context-aware outcomes. Group DM with probabilistic LTs is a dynamic and versatile approach that allows a collective evaluation of alternatives while considering uncertainty and imprecision. It harnesses the power of LTs, which provide a more expressive and intuitive means of expressing opinions and preferences. In this context, each decision-maker assigns LTs to express their subjective judgments regarding the relative likelihood of outcomes. These terms are then converted into probabilistic values, allowing for the quantification of uncertainty. By aggregating the individual probabilistic assessments, a group consensus can be reached, providing a comprehensive view of the collective preferences. This methodology empowers groups to make informed decisions in complex and uncertain scenarios [55], fosters transparency [10], and ensures a more robust and inclusive [6] DM process.

The probabilistic linguistic term set (PLTS) [30] serves as a versatile means of expressing linguistic evaluations with varying weights, making them a promising tool in DM. PLTS is particularly adept at handling the complexity and ambiguity inherent in real-world scenarios and expert thinking. Consequently, researchers have integrated PLTS with DM techniques to enhance their effectiveness. Various models have been proposed to leverage PLTS in DM processes. For instance, Nie and Wang [29] explored prospect theory-based consistency recovery strategies and these strategies are used to build the group DM support model, considering the different risk attitudes of decision-makers. Li et al. [20] developed a model to convert score values into PLTS, reducing computational complexity in DM. Chen et al. [8] proposed a probabilistic linguistic and dual trust network-based collaborative filtering model to capture user preferences and improve recommendation accuracy. Limboo and Dutta [21] introduced a novel concept of a q-rung orthopair basic probability assignment, presenting its application in the domain of medical diagnosis. The research explored the integration of this theoretical framework into the context of healthcare DM, suggesting potential advancements in medical diagnostic processes through the utilization of q-rung orthopair structures. Mao et al. [25] introduced PLTS to capture the fuzziness and unpredictability of choice information, along with a model for determining subjective and objective weights of criteria. Ma et al. [24] addressed app evaluation in a PL setting using an integrated MADM method. Kong and Wu [16] combined PLTSs with the PAMSSEM II and MAUT methods to develop the PL-MAUT-PAMSSEM II approach for destination selection in inbound tourism. The membership function (MF) serves as a means to depict data within a FS [53]. FS theory proves highly effective in managing uncertainties encountered in real-life situations. Atanassov [5] introduced the concept of an IFS as an extension of FS theory. In IFS, information is conveyed through the utilization of both MF and non-membership function (NMF). These functions assign values within the unit interval $[0,1]$, with the requirement that their summation does not exceed one. In other words, if we denote the MF and NMF as μ and ν, respectively, then $0 \leq \mu + \nu \leq 1$. The PFS, as defined by Yager [49], represents a broader concept than the IFS, allowing for a wider range of expression in terms of MF and NMF. In PFS, decision-makers have greater flexibility in conveying their judgments compared to IFS. Both the MF and NMF must still adhere to certain conditions: $0 \leq \mu^2 + \nu^2 \leq 1$. Yager [50] introduced the notion of q-rung orthopair FS (q-ROFS), which serves as a generalization encompassing both IFS and PFS. The stipulated condition for the MF μ and NMF ν is that they must satisfy $0 \leq \mu^q + \nu^q \leq 1$, where the parameter $q \geq 1$.

Aggregation operators (AOs) have proven to be valuable in solving MADM problems. The objective of Seikh and Mandal [34] was to expand the possibility of Archimedean t-norm and Archimedean t-conorm in the context of q-ROFS. They presented new operators for q-ROFS created from Archimedean t-norm and Archimedean t-conorm, examining their desirable characteristics and subsequently utilizing them to construct q-ROF Archimedean weighted averaging (geometric), Archimedean order weighted averaging (geometric), and Archimedean hybrid averaging (geometric) operators. Seikh and Mandal [35] discussed many AOs for combining q-ROF information based on Frank t-norm and t-conorm. Certain AOs, such as Yager's power average (PA) operator [48] and power geometric (PG) operator [45], have been developed by researchers worldwide to address diverse scenarios. These operators are particularly effective as they can mitigate the adverse effects of negative data on the final ranking outcomes. For instance, Jana et al. [15] introduced the Pythagorean fuzzy power Dombi weighted averaging mean operators and applied them to MAGDM. This application aims to alleviate the impact of inaccurate data. Their research showcased the usefulness of these operators in dealing with various circumstances. Wang et al. [42] introduced and explored the application of complex intuitionistic fuzzy Dombi prioritized AOs, specifically addressing their relevance in the context of resilient green supplier selection. Kumar and Chen presented a sophisticated linguistic intuitionistic fuzzy weighted AO [17] and developed an improved linguistic interval-valued Atanassov intuitionistic fuzzy weighted averaging AO [18]. Naseem et al. [26] introduced Aczel-Alsina AOs rooted in complex single-valued neutrosophic information, demonstrating their application in addressing DM problems. Sunthrayuth et al. [33] formulated the Pythagorean fuzzy hypersoft Einstein weighted average operator to address the DM problems in real-life agricultural farming. Ali et al. [4] extended the interaction geometric AO to facilitate material selection through the utilization of interval-valued intuitionistic fuzzy hypersoft sets. Du et al. [11] established a model to solve the positional weight using cross-entropy and Orness measures and designed a group decision information fusion process based on interval intuitionistic fuzzy combinatorically weighted average operators. Liu et al. [22] proposed the fundamental Archimedean operational laws, and these laws served as the basis for developing various complex Pythagorean fuzzy Archimedean-weighted averaging operators. Verma and Mittal [41] formulated a new MAGDM approach based on the generalized Pythagorean fuzzy probabilistic ordered weighted cosine similarity operator and illustrated it with a numerical example regarding the selection of robots in the Aeronautics Company.

Yang and Chen [51] developed a technique for evaluating the water quality of a tributary of the Songhua River by combining the Monte Carlo, CRITIC, and VIKOR approaches. Zhang et al. [56] presented a VIKOR approach based on regret theory to address the MADM problem with fully unknowable weight information and a Pythagorean hesitate fuzzy assessment value. Yadav et al. [47] proposed the hybrid entropy-VIKOR approach, which has importance in the biomedical field because of its capacity to successfully handle complex DM circumstances. Riaz et al. [32] established a new hybrid methodology for MAGDM using Einstein averaging aggregation operators and the cubic bipolar fuzzy-VIKOR method. By simultaneously taking into account environmental, economic, safety, and technological aspects, Tuskan and Basari [40] looked into how the AHP and VIKOR approaches may be used to create a sustainable design for anti-slide piles. Dagistanli [9] presented a novel interval-valued intuitionistic fuzzy VIKOR approach for the selection of research and development projects within the context of defense industry investment decisions. The study explored the integration of interval-valued IFSs into the VIKOR DM model, providing a specialized methodology to address the complex and multifaceted nature of project selection in the defense industry. To solve the MAGDM in probabilistic uncertain linguistic conditions, Lei et al. [19] developed the TODIM-VIKOR model. Taherdoost and Madanchian [38] looked at the VIKOR approach, a multi-criteria DM method, to handle complicated DM problems in diverse domains like engineering, management, and finance. Yildirim and Kuzu Yildirim [52] delved into evaluating the satisfaction level of citizens with municipality services using the picture fuzzy VIKOR method. Their analysis covers the period from 2014 to 2019. The authors employ innovative methods to assess citizen

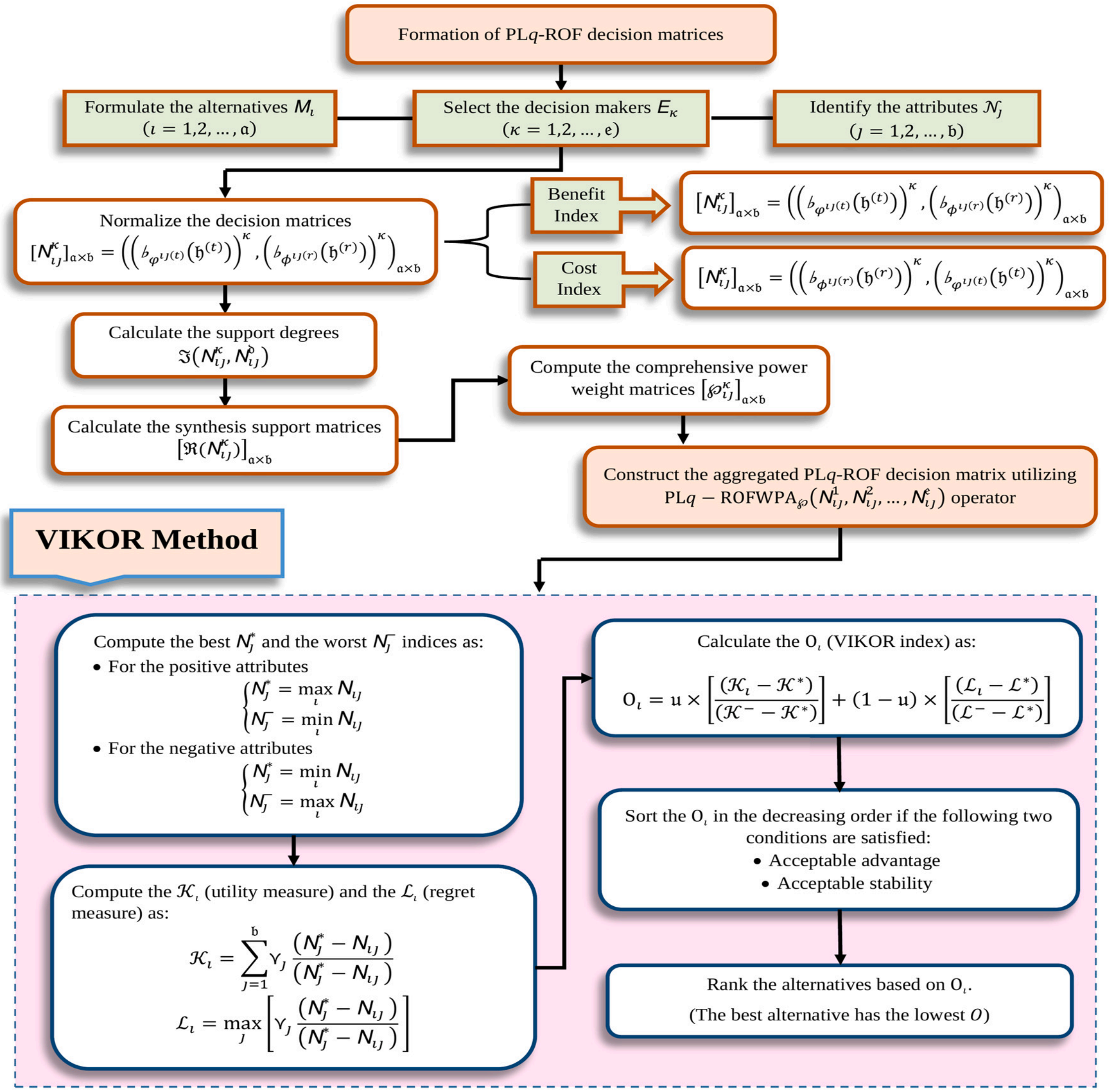

Fig. 1. Organizational layout of the research study.

satisfaction, contributing valuable insights to DM processes in public service management. In order to aggregate the linguistic decision information within the MAGDM framework, this research study employs two operators: the PLq-ROFWPA and the PLq-ROFWPG operators. It utilizes the VIKOR model and develops the PLq-ROF-VIKOR model to rank alternatives based on decision-makers' subjective preferences. The study aims to contribute in the field of astronomy by offering a novel methodology for addressing MAGDM challenges. Fig. 1 visually illustrates the organizational layout of the research study.

3. Preliminaries

To make the discussion in the next sections easier to understand, some fundamental ideas, including the PLTS and the normalization of the PLTS, are recapped in this section.

3.1. The concept of probabilistic linguistic term set

In DM scenarios, decision-makers often face challenges when it comes to selecting appropriate LTs to express their preferences, primarily due to hesitancy or uncertainty. The decision-makers may hesitate or struggle to precisely define their preferences using LTs, which can make the DM process more complex. Furthermore, accurately providing a complete probabilistic distribution for these LTs can be difficult in practice. It may be challenging to gather comprehensive and precise probabilistic information for all possible LTs that can adequately represent the decision-makers' preferences. In such situations, when there is hesitancy in selecting LTs and difficulties in providing complete probabilistic distributions, it becomes necessary to rely solely on the available probability information. This means that DM processes need to be based on the limited probability information that is available, even if it does not represent the entire range of possibilities. The decision-makers must work with the provided probabilities and make decisions or draw conclusions based on the available information. The definition of PLTS can be outlined as follows:

Definition 1. [30] Let $\mathcal{L}$ be a linguistic term set (LTS) defined as $\{\flat_\omega|\omega=-\zeta,\dots,-2,-1,0,1,2,\dots,\zeta\}$. A PLTS can be described as follows:

$$\mathsf{N}_\flat(\mathfrak{h})=\{\flat_{\omega^{(k)}}(\mathfrak{h}^{(k)})|\flat_{\omega^{(k)}}\in\mathcal{L},\mathfrak{h}^{(k)}\geq 0,k=1,2,\dots,\#\mathsf{N}_\flat(\mathfrak{h}),\sum_{k=1}^{\#\mathsf{N}_\flat(\mathfrak{h})}\mathfrak{h}^{(k)}\leq 1\}\tag{1}$$

where $\flat_{\omega^{(k)}}(\mathfrak{h}^{(k)})$ represents the linguistic term $\flat_{\omega^{(k)}}$ with the associated probability $\mathfrak{h}^{(k)}$. The term $\#\mathsf{N}_\flat(\mathfrak{h})$ denotes the number of distinct LTs in $\mathsf{N}_\flat(\mathfrak{h})$. Note that when $\sum_{k=1}^{\#\mathsf{N}_\flat(\mathfrak{h})}\mathfrak{h}^{(k)}=1$, it indicates complete information on the probabilistic distribution of all possible LTs. If $\sum_{k=1}^{\#\mathsf{N}_\flat(\mathfrak{h})}\mathfrak{h}^{(k)}<1$, it implies partial ignorance due to insufficient knowledge for a comprehensive assessment, which is common in practical group DM problems. Particularly, $\sum_{k=1}^{\#\mathsf{N}_\flat(\mathfrak{h})}\mathfrak{h}^{(k)}=0$ represents complete ignorance. Effectively managing the ignorance of $\mathsf{N}_\flat(\mathfrak{h})$ is crucial in the application of PLTS. To ensure straightforward determination of operational results among PLTS, the positions of elements in a set can be arbitrarily swapped, hence the use of ordered PLTS.

Initially, Xu [44] introduced the concept of an additive linguistic evaluation scale, while Gou et al. [14] defined a transition function that relates LTs to the range $[0,1]$.

Definition 2. [14,44] Let the linguistic term $\flat_\omega$ represents equivalent information as β, which can be obtained using the transition function $\mathfrak{g}$:

$$\mathfrak{g}:[\flat_{-\zeta},\flat_\zeta]\to[0,1],\ \mathfrak{g}(\flat_\omega)=\frac{\omega+\zeta}{2\zeta}=\beta.\tag{2}$$

At the same time, β can express information equivalent to the linguistic term $\flat_\omega$, which can be derived using the inverse transition function $\mathfrak{g}^{-1}$:

$$\mathfrak{g}^{-1}:[0,1]\to[\flat_{-\zeta},\flat_\zeta],\ \mathfrak{g}^{-1}=\flat_{(2\beta-1)\zeta}=\flat_\omega.\tag{3}$$

3.2. The basic concepts of PLq-ROFS

Definition 3. [23] Let $\mathcal{X}=(\mathfrak{x}_1,\mathfrak{x}_2,\dots,\mathfrak{x}_\mathfrak{b})$ be a fixed set and $\mathcal{L}=\{\flat_\omega|\omega=-\zeta,\dots,-2,-1,0,1,2,\dots,\zeta\}$ be a LTS, then the PLq-ROFS $\mathsf{N}_\flat(\mathfrak{h})$ on $\mathcal{L}$ can be established as:

$$\mathsf{N}_\flat(\mathfrak{h})=\{(\mathfrak{x},H_\flat(\mathfrak{h})(\mathfrak{x}),G_\flat(\mathfrak{h})(\mathfrak{x}))|\mathfrak{x}\in\mathcal{X}\}$$

where $H_\flat(\mathfrak{h})(\mathfrak{x})=\{\flat_{\varphi^{(t)}}(\mathfrak{h}^{(t)})|\flat_{\varphi^{(t)}}\in\mathcal{L}_{[-\zeta,\zeta]},\mathfrak{h}^{(t)}\geq 0,\sum_{t=1}^{T}\mathfrak{h}^{(t)}\leq 1\}$ is the MF and $G_\flat(\mathfrak{h})(\mathfrak{x})=\{\flat_{\phi^{(r)}}(\mathfrak{h}^{(r)})|\flat_{\phi^{(r)}}\in\mathcal{L}_{[-\zeta,\zeta]},\mathfrak{h}^{(r)}\geq 0,\sum_{r=1}^{R}\mathfrak{h}^{(r)}\leq 1\}$ is the NMF, respectively. For any $\mathfrak{x}\in\mathcal{X}$, they satisfy the condition: $0\leq(\max_{t=1}^{T}\{\varphi^{(t)}\})^q+(\max_{r=1}^{R}\{\phi^{(r)}\})^q\leq\zeta^q(q\geq 1)$ where $\varphi^{(t)}$ and $\phi^{(r)}$ are the subscripts of $\flat_{\varphi^{(t)}}$ and $\flat_{\phi^{(r)}}$, respectively.

If $\mathcal{X}=\{\mathfrak{x}\}$, the PL$q$-ROFS $\mathsf{N}_\flat(\mathfrak{h})$ is degenerated to a probabilistic linguistic q-rung orthopair fuzzy number (PLq-ROFN) $\mathsf{N}_\flat(\mathfrak{h})\triangleq(\{\flat_{\varphi^{(t)}}(\mathfrak{h}^{(t)})\},\{\flat_{\phi^{(r)}}(\mathfrak{h}^{(r)})\})$ where $\flat_{\varphi^{(t)}},\flat_{\phi^{(r)}}\in\mathcal{L}_{[-\zeta,\zeta]}$, $\sum_{t=1}^{T}\mathfrak{h}^{(t)}\leq 1$ and $\sum_{r=1}^{R}\mathfrak{h}^{(r)}\leq 1$.

Definition 4. [23] Let $\mathsf{N}^1_\flat(\mathfrak{h})=(\{\flat_{\varphi^{1(t)}}(\mathfrak{h}^{(t)})\},\{\flat_{\phi^{1(r)}}(\mathfrak{h}^{(r)})\})$ and $\mathsf{N}^2_\flat(\mathfrak{h})=(\{\flat_{\varphi^{2(t)}}(\mathfrak{h}^{(t)})\},\{\flat_{\phi^{2(r)}}(\mathfrak{h}^{(r)})\})$ $(t=1,2,\dots,T;\ r=1,2,\dots,R)$ are two PLq-ROFNs and $\eta>0$, then the operational laws of the PLq-ROFNs can be expressed as follows:

(a) $\mathrm{neg}(\mathsf{N}^1_\flat(\mathfrak{h}))=(\{\mathfrak{g}^{-1}(\mathfrak{g}(\flat_{\varphi^{1(r)}}))(\mathfrak{h}^{(r)})\},\{\mathfrak{g}^{-1}(\mathfrak{g}(\flat_{\phi^{1(t)}}))(\mathfrak{h}^{(t)})\})$;

(b) $\mathsf{N}^1_\flat(\mathfrak{h})\oplus\mathsf{N}^2_\flat(\mathfrak{h})=(\{\mathfrak{g}^{-1}(\sqrt[q]{(\mathfrak{g}(\flat_{\varphi^{1(t)}}))^q+(\mathfrak{g}(\flat_{\varphi^{2(t)}}))^q-((\mathfrak{g}(\flat_{\varphi^{1(t)}}))(\mathfrak{g}(\flat_{\varphi^{2(t)}})))^q})(\mathfrak{h}^{(t)})\},\ \{\mathfrak{g}^{-1}((\mathfrak{g}(\flat_{\phi^{1(r)}}))(\mathfrak{g}(\flat_{\phi^{2(r)}})))(\mathfrak{h}^{(r)})\})$;

(c) $\mathsf{N}^1_\flat(\mathfrak{h})\otimes\mathsf{N}^2_\flat(\mathfrak{h})=(\{\mathfrak{g}^{-1}((\mathfrak{g}(\flat_{\varphi^{1(t)}}))(\mathfrak{g}(\flat_{\varphi^{2(t)}})))(\mathfrak{h}^{(t)})\},\ \{\mathfrak{g}^{-1}(\sqrt[q]{(\mathfrak{g}(\flat_{\phi^{1(r)}}))^q+(\mathfrak{g}(\flat_{\phi^{2(r)}}))^q-((\mathfrak{g}(\flat_{\phi^{1(r)}}))(\mathfrak{g}(\flat_{\phi^{2(r)}})))^q})(\mathfrak{h}^{(r)})\})$;

(d) $\eta\mathsf{N}^1_\flat(\mathfrak{h})=(\{(\mathfrak{g}^{-1}(\sqrt[q]{1-(1-(\mathfrak{g}(\flat_{\varphi^{1(t)}}))^q)^\eta})(\mathfrak{h}^{(t)})\},\ \{\mathfrak{g}^{-1}((\mathfrak{g}(\flat_{\phi^{1(r)}}))^\eta)(\mathfrak{h}^{(r)})\})$;

(e) $(\mathsf{N}^1_\flat(\mathfrak{h}))^\eta=(\{\mathfrak{g}^{-1}((\mathfrak{g}(\flat_{\varphi^{1(t)}}))^\eta)(\mathfrak{h}^{(t)})\},\ \{\mathfrak{g}^{-1}(\sqrt[q]{1-(1-(\mathfrak{g}(\flat_{\phi^{1(r)}}))^q)^\eta})(\mathfrak{h}^{(r)})\})$.

Theorem 1. *[23] Let any two PLq-ROFNs $N^1_\flat(\mathfrak{h}) = (\{\flat_{\varphi^{1(t)}}(\mathfrak{h}^{(t)})\}, \{\flat_{\phi^{1(r)}}(\mathfrak{h}^{(r)})\})$ and $N^2_\flat(\mathfrak{h}) = (\{\flat_{\varphi^{2(t)}}(\mathfrak{h}^{(t)})\}, \{\flat_{\phi^{2(r)}}(\mathfrak{h}^{(r)})\})$ $(t = 1, 2, \ldots, T;$ $r = 1, 2, \ldots, R)$, and η, η_1, $\eta_2 > 0$, then*

(1) $N^1_\flat(\mathfrak{h}) \oplus N^2_\flat(\mathfrak{h}) = N^2_\flat(\mathfrak{h}) \oplus N^1_\flat(\mathfrak{h})$;
(2) $N^1_\flat(\mathfrak{h}) \otimes N^2_\flat(\mathfrak{h}) = N^2_\flat(\mathfrak{h}) \otimes N^1_\flat(\mathfrak{h})$;
(3) $\eta(N^1_\flat(\mathfrak{h}) \oplus N^2_\flat(\mathfrak{h})) = \eta N^1_\flat(\mathfrak{h}) \oplus \eta N^2_\flat(\mathfrak{h})$;
(4) $\eta_1 N^1_\flat(\mathfrak{h}) \oplus \eta_2 N^1_\flat(\mathfrak{h}) = (\eta_1 + \eta_2) N^1_\flat(\mathfrak{h})$;
(5) $(N^1_\flat(\mathfrak{h}))^{\eta_1} \otimes (N^1_\flat(\mathfrak{h}))^{\eta_2} = (N^1_\flat(\mathfrak{h}))^{\eta_1+\eta_2}$;
(6) $(N^1_\flat(\mathfrak{h}))^{\eta} \otimes (N^2_\flat(\mathfrak{h}))^{\eta} = (N^1_\flat(\mathfrak{h}) \otimes N^2_\flat(\mathfrak{h}))^{\eta}$.

The score and accuracy functions of a PLq-ROFN $\mathsf{N}_\flat(\mathfrak{h})$ can be defined as:

Definition 5. [23] For any PLq-ROFN $\mathsf{N}_\flat(\mathfrak{h}) = (\{\flat_{\varphi^{(t)}}(\mathfrak{h}^{(t)})\}, \{\flat_{\phi^{(r)}}(\mathfrak{h}^{(r)})\})$ where $\flat_{\varphi^{(t)}}$ and $\flat_{\phi^{(r)}} \in \mathcal{L}_{[-\zeta,\zeta]}$ $(t = 1, 2, \ldots, T;\ r = 1, 2, \ldots, R)$, then the score function F of $\mathsf{N}_\flat(\mathfrak{h})$ is

$$F(\mathsf{N}_\flat(\mathfrak{h})) = \sum_{t=1}^{\#T_\varphi} (\mathfrak{g}(\flat_{\varphi^{(t)}}) \cdot \mathfrak{h}^{(t)})^q - \sum_{r=1}^{\#R_\phi} (\mathfrak{g}(\flat_{\phi^{(r)}}) \cdot \mathfrak{h}^{(r)})^q, \tag{4}$$

and the accuracy function $\beth$ of $\mathsf{N}_\flat(\mathfrak{h})$ is

$$\beth(\mathsf{N}_\flat(\mathfrak{h})) = \sum_{t=1}^{\#T_\varphi} (\mathfrak{g}(\flat_{\varphi^{(t)}}) \cdot \mathfrak{h}^{(t)})^q + \sum_{r=1}^{\#R_\phi} (\mathfrak{g}(\flat_{\phi^{(r)}}) \cdot \mathfrak{h}^{(r)})^q \tag{5}$$

where $\#T_\varphi$ and $\#R_\phi$ represent the number of elements in the corresponding set.

In order to compare the order relation of PLq-ROFNs, the comparison rules can be presented as follows:

Theorem 2. *[23] Let $F(N^1_\flat(\mathfrak{h}))$ and $F(N^2_\flat(\mathfrak{h}))$ are the score functions of any two PLq-ROFNs $N^1_\flat(\mathfrak{h})$ and $N^2_\flat(\mathfrak{h})$, the accuracy functions of both numbers are $\beth(N^1_\flat(\mathfrak{h}))$ and $\beth(N^1_\flat(\mathfrak{h}))$, respectively, then the order relation of $N^1_\flat(\mathfrak{h})$ and $N^2_\flat(\mathfrak{h})$ are as follows:*

(1) *If $F(N^1_\flat(\mathfrak{h})) > F(N^2_\flat(\mathfrak{h}))$, then $N^1_\flat(\mathfrak{h}) \succ N^2_\flat(\mathfrak{h})$;*
(2) *If $F(N^1_\flat(\mathfrak{h}^1)) < F(N^2_\flat(\mathfrak{h}))$, then $N^1_\flat(\mathfrak{h}) \prec N^2_\flat(\mathfrak{h})$;*
(3) *If $F(N^1_\flat(\mathfrak{h})) = F(N^2_\flat(\mathfrak{h}))$, then*
   (a) *If $\beth(N^1_\flat(\mathfrak{h})) < \beth(N^2_\flat(\mathfrak{h}))$, then $N^1_\flat(\mathfrak{h}) \prec N^2_\flat(\mathfrak{h})$;*
   (b) *If $\beth(N^1_\flat(\mathfrak{h})) > \beth(N^2_\flat(\mathfrak{h}))$, then $N^1_\flat(\mathfrak{h}) \succ N^2_\flat(\mathfrak{h})$;*
   (c) *If $\beth(N^1_\flat(\mathfrak{h})) = \beth(N^2_\flat(\mathfrak{h}))$, then $N^1_\flat(\mathfrak{h}) \cong N^2_\flat(\mathfrak{h})$.*

Definition 6. [23] Let $\mathsf{N}^1_\flat(\mathfrak{h}) = (\{\flat_{\varphi^{1(t)}}(\mathfrak{h}^{(t)})\}, \{\flat_{\phi^{1(r)}}(\mathfrak{h}^{(r)})\})$ and $\mathsf{N}^2_\flat(\mathfrak{h}) = (\{\flat_{\varphi^{2(t)}}(\mathfrak{h}^{(t)})\}, \{\flat_{\phi^{2(r)}}(\mathfrak{h}^{(r)})\})(t = 1, 2, \ldots, T;\ r = 1, 2, \ldots, R)$ are the two PLq-ROFNs where $\flat_{\varphi^{j(t)}}$ and $\flat_{\phi^{j(r)}} \in \mathcal{L}_{[-\zeta,\zeta]}(j = 1, 2)$, the normalized Hamming distance measure d between $\mathsf{N}^1_\flat(\mathfrak{h})$ and $\mathsf{N}^2_\flat(\mathfrak{h})$ can be established as:

$$\begin{aligned} d(\mathsf{N}_{\flat_1}(\mathfrak{h}^1), \mathsf{N}_{\flat_2}(\mathfrak{h}^2)) = &\sum_{t=1}^{\#T_\varphi} ((\mathfrak{h}^{(1t)}.\mathfrak{h}^{(2t)}).|(\mathfrak{g}(\flat_{\varphi_{1(t)}})^q - \mathfrak{g}(\flat_{\varphi_{2(t)}})^q)|) + \\ &\sum_{r=1}^{\#R_\phi} ((\mathfrak{h}^{(1r)}.\mathfrak{h}^{(2r)}).|(\mathfrak{g}(\flat_{\phi_{1(r)}})^q - \mathfrak{g}(\flat_{\phi_{2(r)}}))^q|) \end{aligned} \tag{6}$$

where $q \geq 1$, $\#T_\varphi$ and $\#R_\phi$ represent the number of elements in the corresponding set.

Definition 7. [48] The PA operator is a nonlinear weighted average AO defined as follows:

$$PA(\breve{a}_1, \breve{a}_2, \ldots, \breve{a}_{\mathfrak{b}}) = \frac{\sum_{\iota=1}^{\mathfrak{b}} (1 + \mathfrak{R}(\breve{a}_\iota))\breve{a}_\iota}{\sum_{\iota=1}^{\mathfrak{b}} (1 + \mathfrak{R}(\breve{a}_\iota))} \tag{7}$$

where $\mathfrak{R}(\breve{a}_\iota) = \sum_{\iota,j=1,j\neq\iota}^{\mathfrak{b}} \mathfrak{I}(\breve{a}_\iota, \breve{a}_j)$ and $\mathfrak{I}(\breve{a}, \breve{b})$ is the support for $\breve{a}$ and $\breve{b}$ which satisfies the following three properties:

– $\mathfrak{I}(\breve{a},\breve{b}) \in [0,1]$;
– $\mathfrak{I}(\breve{a},\breve{b}) = \mathfrak{I}(\breve{b},\breve{a})$;
– $\mathfrak{I}(\breve{a},\breve{b}) \geq \mathfrak{I}(\breve{x},\breve{y})$, if $|\breve{a}-\breve{b}| < |\breve{x}-\breve{y}|$.

Definition 8. [45] Based on the PA operator and geometric mean, the PG operator is defined as;

$$PG(\breve{a}_1,\breve{a}_2,\ldots,\breve{a}_{\mathfrak{b}}) = \prod_{\imath=1}^{\mathfrak{b}} \breve{a}_\imath^{\frac{1+\mathfrak{R}(\breve{a}_\imath)}{\sum_{\imath=1}^{\mathfrak{b}}(1+\mathfrak{R}(\breve{a}_\imath))}}. \tag{8}$$

The PA and PG operators are widely used nonlinear weighted aggregation techniques that incorporate weighting vectors to combine input values in a manner that enhances mutual support and reinforcement. These operators are based on the concept that when two values, $\breve{a}_\imath$ and $\breve{b}_\imath$, are closer to each other, they are deemed more similar and provide stronger mutual support.

4. The PLq-ROF power aggregation operators

Group DM utilizing AOs is a powerful approach that harnesses the collective wisdom of a team or community. AOs provide a systematic and structured framework for combining individual opinions, preferences, or judgments into a consolidated group decision. These operators take into account various factors, such as importance weights, consensus measures, and the degree of agreement among group members. By leveraging AOs, the DM process becomes more objective and transparent, ensuring that diverse viewpoints are considered and evaluated. This approach enables teams to make informed and robust decisions, promoting fairness, inclusivity, and ultimately enhancing the quality of outcomes. Whether it is selecting the best course of action, prioritizing alternatives, or evaluating complex criteria, group DM utilizing AOs offers a reliable and effective strategy for achieving consensus and optimizing collective intelligence. This section introduces two distinct types of weighted information AOs, namely PLq-ROFWPA and PLq-ROFWPG operators. The properties of these proposed operators, such as idempotency, monotonicity, and boundedness, are also discussed.

Definition 9. Assume $\mathcal{L} = \{\flat_\omega | \omega = -\zeta,\ldots,-2,-1,0,1,2,\cdots,\zeta\}$ is a LTS, $\mathsf{N}^\jmath_\flat(\mathfrak{h}) = ((\flat_{\varphi^{\jmath(t)}}(\mathfrak{h}^{(t)})),(\flat_{\phi^{\jmath(r)}}(\mathfrak{h}^{(r)})))(\jmath = 1,2,\ldots,\mathfrak{b};\ t = 1,2,\ldots,T;\ r = 1,2,\ldots,R)$ be the adjusted PLq-ROFNs with weight vector $\curlyvee = (\curlyvee_1,\curlyvee_2,\ldots,\curlyvee_{\mathfrak{b}})^T$, such that $\curlyvee_\jmath \in [0,1]$ and $\sum_{\jmath=1}^{\mathfrak{b}} \curlyvee_\jmath = 1$, then the PL$q$-ROFWPA operator is defined as follows:

$$\text{PL}q\text{-ROFWPA}_\curlyvee(\mathsf{N}^1_\flat(\mathfrak{h}),\mathsf{N}^2_\flat(\mathfrak{h}),\ldots,\mathsf{N}^{\mathfrak{b}}_\flat(\mathfrak{h})) = \bigoplus_{\jmath=1}^{\mathfrak{b}} \wp_\jmath(\mathsf{N}^\jmath_\flat(\mathfrak{h})) \tag{9}$$

where the comprehensive power weights $\wp_\jmath$ can be calculated as:

Step 1. Calculate the support degrees between $\mathsf{N}^\jmath_\flat(\mathfrak{h})$ and $\mathsf{N}^\imath_\flat(\mathfrak{h})$ where $\jmath,\ \imath = 1,2,\ldots,\mathfrak{b}$ and $\jmath \neq \imath$.

$$\mathfrak{I}(\mathsf{N}^\jmath_\flat(\mathfrak{h}),\mathsf{N}^\imath_\flat(\mathfrak{h})) = 1 - d(\mathsf{N}^\jmath_\flat(\mathfrak{h}),\mathsf{N}^\imath_\flat(\mathfrak{h})), \tag{10}$$

here, $d(\mathsf{N}^\jmath_\flat(\mathfrak{h}),\mathsf{N}^\imath_\flat(\mathfrak{h}))$ represents the distance between $\mathsf{N}^\jmath_\flat(\mathfrak{h})$ and $\mathsf{N}^\imath_\flat(\mathfrak{h})$.

Step 2. Calculate the synthesis support degrees $\mathfrak{R}(\mathsf{N}^\jmath_\flat(\mathfrak{h}))$ of $\mathsf{N}^\jmath_\flat(\mathfrak{h})$.

$$\mathfrak{R}(\mathsf{N}^\jmath_\flat(\mathfrak{h})) = \sum_{\jmath,\imath=1,\jmath\neq\imath}^{\mathfrak{b}} \mathfrak{I}(\mathsf{N}^\jmath_\flat(\mathfrak{h}),\mathsf{N}^\imath_\flat(\mathfrak{h})). \tag{11}$$

Step 3. Compute the comprehensive power weights as:

$$\wp_\jmath = \frac{\curlyvee_\jmath\left(1+\mathfrak{R}(\mathsf{N}^\jmath_\flat(\mathfrak{h}))\right)}{\sum_{\jmath=1}^{\mathfrak{b}} \curlyvee_\jmath\left(1+\mathfrak{R}(\mathsf{N}^\jmath_\flat(\mathfrak{h}))\right)}. \tag{12}$$

By employing the innovative operational laws of PLq-ROFS, Theorem 3 can be derived.

Theorem 3. *Let $N^\jmath_\flat(\mathfrak{h}) = ((\flat_{\varphi^{\jmath(t)}}(\mathfrak{h}^{(t)})),(\flat_{\phi^{\jmath(r)}}(\mathfrak{h}^{(r)})))(\jmath = 1,2,\ldots,\mathfrak{b};\ t = 1,2,\ldots,T;\ r = 1,2,\ldots,R)$ be the collection of PLq-ROFNs with the weight vector $\curlyvee = (\curlyvee_1,\curlyvee_2,\ldots,\curlyvee_{\mathfrak{b}})^T$, such that $\curlyvee_\jmath \in [0,1]$ and $\sum_{\jmath=1}^{\mathfrak{b}} \curlyvee_\jmath = 1$, then*

$$PLq\text{-}ROFWPA_\curlyvee(N^1_\flat(\mathfrak{h}),N^2_\flat(\mathfrak{h}),\ldots,N^{\mathfrak{b}}_\flat(\mathfrak{h}))$$

$$=\left(\left(\mathfrak{g}^{-1}\left(\sqrt[q]{1-\prod_{\jmath=1}^{\mathfrak{b}}\left(1-\left(\mathfrak{g}(\flat_{\varphi^{\jmath(t)}})\right)^{q}\right)^{\wp_\jmath}}\right)(\prod_{\jmath=1}^{\mathfrak{b}}\mathfrak{h}^{(t)})\right),\left(\mathfrak{g}^{-1}\left(\prod_{\jmath=1}^{\mathfrak{b}}\left(\mathfrak{g}(\flat_{\phi^{\jmath(r)}})\right)^{\wp_\jmath}\right)(\prod_{\jmath=1}^{\mathfrak{b}}\mathfrak{h}^{(r)})\right)\right) \tag{13}$$

where $\flat_{\varphi^{\jmath(t)}}$ and $\flat_{\phi^{\jmath(r)}} \in \mathcal{L}_{[-\zeta,\zeta]}$, and $\wp_\jmath$ are the comprehensive power weights.

Proof. We prove that Eq. (13) holds by using the mathematical induction method for positive integer $\mathfrak{b}$.

(a) When $\mathfrak{b}=1$, then $\wp_1 \mathsf{N}_\flat^1(\mathfrak{h})=\left(\left(\mathfrak{g}^{-1}\left(\sqrt[q]{1-\left(1-\left(\mathfrak{g}(\flat_{\varphi^{1(t)}})\right)^{q}\right)^{\wp_1}}\right)(\mathfrak{h}^{(t)})\right),\left(\mathfrak{g}^{-1}\left(\left(\mathfrak{g}(\flat_{\phi^{1(r)}})\right)^{\wp_1}\right)(\mathfrak{h}^{(r)})\right)\right)$.
Thus, Eq. (13) holds for $\mathfrak{b}=1$.

(b) Suppose that Eq. (13) holds for $\mathfrak{b}=\mathfrak{a}$,

$$\begin{aligned}&\mathrm{PL}q\text{-ROFWPA}_{\wp}(\mathsf{N}_\flat^1(\mathfrak{h}),\mathsf{N}_\flat^2(\mathfrak{h}),\ldots,\mathsf{N}_\flat^{\mathfrak{a}}(\mathfrak{h}))\\&=\left(\left(\mathfrak{g}^{-1}\left(\sqrt[q]{1-\prod_{\jmath=1}^{\mathfrak{a}}\left(1-\left(\mathfrak{g}(\flat_{\varphi^{\jmath(t)}})\right)^{q}\right)^{\wp_\jmath}}\right)(\prod_{\jmath=1}^{\mathfrak{a}}\mathfrak{h}^{(t)})\right),\left(\mathfrak{g}^{-1}\left(\prod_{\jmath=1}^{\mathfrak{a}}\left(\mathfrak{g}(\flat_{\phi^{\jmath(r)}})\right)^{\wp_\jmath}\right)(\prod_{\jmath=1}^{\mathfrak{a}}\mathfrak{h}^{(r)})\right)\right).\end{aligned}$$

Then, when $\mathfrak{b}=\mathfrak{a}+1$, by inductive assumption, then

$$\begin{aligned}&\mathrm{PL}q\text{-ROFWPA}_{\wp}(\mathsf{N}_\flat^1(\mathfrak{h}),\mathsf{N}_\flat^2(\mathfrak{h}),\ldots,\mathsf{N}_\flat^{\mathfrak{a}}(\mathfrak{h}),\mathsf{N}_\flat^{\mathfrak{a}+1}(\mathfrak{h}))\\&=\mathrm{PL}q\text{-ROFWPA}_{\wp}(\mathsf{N}_\flat^1(\mathfrak{h}),\mathsf{N}_\flat^2(\mathfrak{h}),\ldots,\mathsf{N}_\flat^{\mathfrak{a}}(\mathfrak{h}))\oplus\wp_{\mathfrak{a}+1}\mathsf{N}_\flat^{\mathfrak{a}+1}(\mathfrak{h})\\&=\left(\left(\mathfrak{g}^{-1}\left(\sqrt[q]{1-\prod_{\jmath=1}^{\mathfrak{a}}\left(1-\left(\mathfrak{g}(\flat_{\varphi^{\jmath(t)}})\right)^{q}\right)^{\wp_\jmath}}\right)(\prod_{\jmath=1}^{\mathfrak{a}}\mathfrak{h}^{(t)})\right),\left(\mathfrak{g}^{-1}\left(\prod_{\jmath=1}^{\mathfrak{a}}\left(\mathfrak{g}(\flat_{\phi^{\jmath(r)}})\right)^{\wp_\jmath}\right)(\prod_{\jmath=1}^{\mathfrak{a}}\mathfrak{h}^{(r)})\right)\right)\\&\oplus\left(\left(\mathfrak{g}^{-1}\left(\sqrt[q]{1-\left(1-\left(\mathfrak{g}(\flat_{\varphi^{\mathfrak{a}+1(t)}})\right)^{q}\right)^{\wp_{\mathfrak{a}+1}}}\right)(\mathfrak{h}^{(t)})\right),\left(\mathfrak{g}^{-1}\left(\left(\mathfrak{g}(\flat_{\phi^{\mathfrak{a}+1(r)}})\right)^{\wp_{\mathfrak{a}+1}}\right)(\mathfrak{h}^{(r)})\right)\right),\\&=\left(\left(\mathfrak{g}^{-1}\left(\sqrt[q]{1-\prod_{\jmath=1}^{\mathfrak{a}+1}\left(1-\left(\mathfrak{g}(\flat_{\varphi^{\jmath(t)}})\right)^{q}\right)^{\wp_\jmath}}\right)(\prod_{\jmath=1}^{\mathfrak{a}+1}\mathfrak{h}^{(t)})\right),\left(\mathfrak{g}^{-1}\left(\prod_{\jmath=1}^{\mathfrak{a}+1}\left(\mathfrak{g}(\flat_{\phi^{\jmath(r)}})\right)^{\wp_\jmath}\right)(\prod_{\jmath=1}^{\mathfrak{a}+1}\mathfrak{h}^{(r)})\right)\right).\end{aligned}$$

Consequently, it can be deduced that Eq. (13) holds for a positive integer $\mathfrak{b}=\mathfrak{a}+1$. Therefore, utilizing the mathematical induction method, conclude that Eq. (13) holds for any $\mathfrak{b}\geq 1$. □

Theorem 4. *Let $\mathsf{N}_\flat^\jmath(\mathfrak{h})=((\flat_{\varphi^{\jmath(t)}}(\mathfrak{h}^{(t)})),(\flat_{\phi^{\jmath(r)}}(\mathfrak{h}^{(r)})))(\jmath=1,2,\ldots,\mathfrak{b};\ t=1,2,\ldots,T;\ r=1,2,\ldots,R)$ be the collection of PLq-ROFNs with weight vector $\Upsilon=(\Upsilon_1,\Upsilon_2,\ldots,\Upsilon_{\mathfrak{b}})^T$, such that $\Upsilon_\jmath\in[0,1]$ and $\sum_{\jmath=1}^{\mathfrak{b}}\Upsilon_\jmath=1$, then the PL$q$-ROFWPA operator has the following properties:*

1. *(Idempotency) If all $\mathsf{N}_\flat^\jmath(\mathfrak{h})=((\flat_{\varphi^{\jmath(t)}}(\mathfrak{h}^{(t)})),(\flat_{\phi^{\jmath(r)}}(\mathfrak{h}^{(r)})))(\jmath=1,2,\ldots,\mathfrak{b})$ are equal, for all $\jmath$, then*

$$\mathrm{PL}q\text{-ROFWPA}_\Upsilon(\mathsf{N}_\flat^1(\mathfrak{h}),\mathsf{N}_\flat^2(\mathfrak{h}),\ldots,\mathsf{N}_\flat^{\mathfrak{b}}(\mathfrak{h}))=\mathsf{N}_\flat(\mathfrak{h}).$$

2. *(Monotonicity) Let $\mathsf{N}_\flat^\jmath(\mathfrak{h}')=(\mathsf{N}_\flat^1(\mathfrak{h}),\mathsf{N}_\flat^2(\mathfrak{h}),\ldots,\mathsf{N}_\flat^{\mathfrak{b}}(\mathfrak{h}))$ and $\mathsf{N}_\flat'^{\jmath}(\mathfrak{h}')=(\mathsf{N}_\flat'^{1}(\mathfrak{h}'),\mathsf{N}_\flat'^{2}(\mathfrak{h}'),\ldots,\mathsf{N}_\flat'^{\mathfrak{b}}(\mathfrak{h}'))$ be two collections of adjusted PLq-ROFNs, for all $\jmath$, $\flat_{\varphi^{\jmath(t)}}<\flat_{\varphi'^{\jmath(t)}}$ and $\flat_{\phi^{\jmath(r)}}>\flat_{\phi'^{\jmath(r)}}$ then*

$$\mathrm{PL}q\text{-ROFWPA}_\Upsilon(\mathsf{N}_\flat^1(\mathfrak{h}),\mathsf{N}_\flat^2(\mathfrak{h}),\ldots,\mathsf{N}_\flat^{\mathfrak{b}}(\mathfrak{h}))<\mathrm{PL}q\text{-ROFWPA}_\Upsilon(\mathsf{N}_\flat'^{1}(\mathfrak{h}'),\mathsf{N}_\flat'^{2}(\mathfrak{h}'),\ldots,\mathsf{N}_\flat'^{\mathfrak{b}}(\mathfrak{h}')).$$

3. *(Boundedness) Let $\flat_{\varphi^{\jmath(+)}}=\max_{t=1}^{T}\flat_{\varphi^{\jmath(t)}}$, $\flat_{\varphi^{\jmath(-)}}=\min_{t=1}^{T}\flat_{\varphi^{\jmath(t)}}$, $\flat_{\phi^{\jmath(+)}}=\max_{r=1}^{R}\flat_{\phi^{\jmath(r)}}$ and $\flat_{\phi^{\jmath(-)}}=\min_{r=1}^{R}\flat_{\phi^{\jmath(r)}}$ then*

$$((\flat_{\varphi^{\jmath(-)}}(\mathfrak{h}^{(t)})),(\flat_{\phi^{\jmath(+)}}(\mathfrak{h}^{(r)})))\leq\mathrm{PL}q\text{-ROFWPA}_\Upsilon(\mathsf{N}_\flat^1(\mathfrak{h}),\mathsf{N}_\flat^2(\mathfrak{h}),\ldots,\mathsf{N}_\flat^{\mathfrak{b}}(\mathfrak{h}))\leq((\flat_{\varphi^{\jmath(+)}}(\mathfrak{h}^{(t)})),(\flat_{\phi^{\jmath(-)}}(\mathfrak{h}^{(r)}))).$$

Definition 10. Assume $\mathsf{N}_\flat^\jmath(\mathfrak{h}^\jmath)=((\flat_{\varphi^{\jmath(t)}}(\mathfrak{h}^{(t)})),(\flat_{\phi^{\jmath(r)}}(\mathfrak{h}^{(r)})))(\jmath=1,2,\ldots,\mathfrak{b};\ t=1,2,\ldots,T;\ r=1,2,\ldots,R)$ be the adjusted PLq-ROFNs with the weight vector $\Upsilon=(\Upsilon_1,\Upsilon_2,\ldots,\Upsilon_{\mathfrak{b}})^T$ such that $\Upsilon_\jmath\in[0,1]$ and $\sum_{\jmath=1}^{\mathfrak{b}}\Upsilon_\jmath=1$. The PL$q$-ROFWPG operator is defined as follows:

$$\mathrm{PL}q\text{-ROFWPG}_\Upsilon(\mathsf{N}_\flat^1(\mathfrak{h}^1),\mathsf{N}_\flat^2(\mathfrak{h}),\ldots,\mathsf{N}_\flat^{\mathfrak{b}}(\mathfrak{h}))=\bigotimes_{\jmath=1}^{\mathfrak{b}}(\mathsf{N}_\flat^\jmath(\mathfrak{h}))^{\wp_\jmath} \tag{14}$$

where the comprehensive power weights $\wp_\jmath$ can be calculated as:

Step 1. Calculate the support degrees between $\mathsf{N}_{\flat}^{\jmath}(\mathfrak{h})$ and $\mathsf{N}_{\flat}^{\imath}(\mathfrak{h})(\jmath,\ \imath=1,2,\ldots,\mathfrak{b},;\jmath\neq\imath)$.

$$\mathfrak{T}(\mathsf{N}_{\flat}^{\jmath}(\mathfrak{h}),\mathsf{N}_{\flat}^{\imath}(\mathfrak{h}))=1-d(\mathsf{N}_{\flat}^{\jmath}(\mathfrak{h}),\mathsf{N}_{\flat}^{\imath}(\mathfrak{h})) \tag{15}$$

where the $d(\mathsf{N}_{\flat}^{\jmath}(\mathfrak{h}),\mathsf{N}_{\flat}^{\imath}(\mathfrak{h}))$ is the distance between $\mathsf{N}_{\flat}^{\jmath}(\mathfrak{h})$ and $\mathsf{N}_{\flat}^{\imath}(\mathfrak{h})$.

Step 2. Calculate the synthesis support degrees $\mathfrak{R}(\mathsf{N}_{\flat}^{\jmath}(\mathfrak{h}))$ of $(\mathsf{N}_{\flat}^{\jmath}(\mathfrak{h}))$.

$$\mathfrak{R}(\mathsf{N}_{\flat}^{\jmath}(\mathfrak{h}))=\sum_{\jmath,\imath=1,\jmath\neq\imath}^{\mathfrak{b}}\mathfrak{T}(\mathsf{N}_{\flat}^{\jmath}(\mathfrak{h}),\mathsf{N}_{\flat}^{\imath}(\mathfrak{h})). \tag{16}$$

Step 3. Compute the comprehensive power weights as:

$$\wp_{\jmath}=\frac{\curlyvee_{\jmath}\left(1+\mathfrak{R}(\mathsf{N}_{\flat}^{\jmath}(\mathfrak{h}))\right)}{\sum_{\jmath=1}^{\mathfrak{b}}\curlyvee_{\jmath}\left(1+\mathfrak{R}(\mathsf{N}_{\flat}^{\jmath}(\mathfrak{h}))\right)}. \tag{17}$$

By utilizing the novel operational laws of PLq-ROFS, Theorem 5 can be obtained.

Theorem 5. *Let $N_{\flat}^{\jmath}(\mathfrak{h})=((\flat_{\varphi^{\jmath(t)}}(\mathfrak{h}^{(t)}),(\flat_{\phi^{\jmath(r)}}(\mathfrak{h}^{(r)})))(\jmath=1,2,\ldots,\mathfrak{b};\ t=1,2,\ldots,T;\ r=1,2,\ldots,R)$ be the collection of PLq-ROFNs with the weight vector $\curlyvee=(\curlyvee_1,\curlyvee_2,\ldots,\curlyvee_{\mathfrak{b}})^T$, such that $\curlyvee_{\jmath}\in[0,1]$ and $\sum_{\jmath=1}^{\mathfrak{b}}\curlyvee_{\jmath}=1$. The aggregation value obtained by the PLq-ROFWPG operator remains a PLq-ROFN, and*

$$\begin{aligned}&PLq\text{-}ROFWPG_{\curlyvee}(N_{\flat}^{1}(\mathfrak{h}),N_{\flat}^{2}(\mathfrak{h}),\ldots,N_{\flat}^{\mathfrak{b}}(\mathfrak{h}))\\&=\left(\left(\mathfrak{g}^{-1}\left(\prod_{\jmath=1}^{\mathfrak{b}}\left(\mathfrak{g}(\flat_{\varphi^{\jmath(t)}})\right)^{\wp_{\jmath}}\right)(\prod_{\jmath=1}^{\mathfrak{b}}\mathfrak{h}^{(t)})\right),\left(\mathfrak{g}^{-1}\left(\sqrt[q]{1-\prod_{\jmath=1}^{\mathfrak{b}}\left(1-\left(\mathfrak{g}(\flat_{\phi^{\jmath(r)}})\right)^{q}\right)^{\wp_{\jmath}}}\right)(\prod_{\jmath=1}^{\mathfrak{b}}\mathfrak{h}^{\jmath(r)})\right)\right)\end{aligned} \tag{18}$$

where $\flat_{\varphi^{\jmath(t)}}$ and $\flat_{\phi^{\jmath(r)}}\in\mathcal{L}_{[-\zeta,\zeta]}$, and $\wp_{\jmath}$ are the comprehensive power weights.

Proof. The proof of the PLq-ROFWPG operator is analogous to that of PLq-ROFWPA operator in Theorem 3. □

The PLq-ROFWPG operator possesses the same properties idempotency, monotonicity and boundedness as of the PLq-ROFWPA operator.

5. VIKOR model to MAGDM approach based on PLq-ROFWPA operator

Many models have been proposed in recent years to address MAGDM issues. However, each of these strategies has advantages and disadvantages. To address MAGDM issues, Opricovic presented the VIKOR method, a compromise index-based MAGDM solution. The VIKOR technique ranks alternatives using reference points, similar to the TOPSIS model. The VIKOR technique has been used in a variety of contexts. The multi-criterion complex framework was intended to be optimized by the VIKOR method. In the procedure, a compromised ranking might be established by comparing the measure of proximity to the ideal alternative. This approach describes an index that is closer to the ideal solution's positive value. VIKOR measures an alternative's overall performance under all criteria as group value and the alternative's worst performance under all criteria as individual regret. However, VIKOR is unable to handle MAGDM issues where the criteria of various alternatives are inconsistent (different) and the decision information is uncertain. This section presents a novel MAGDM method that utilizes the PLq-ROFWPA operator within the framework of PLq-ROFNs. The proposed method aims to address MAGDM problems effectively. The following provides a detailed explanation of the proposed approach:

Step 1. Construct the PLq-ROF decision matrices.
Let us consider a scenario where we have a set of '$\mathfrak{a}$' alternatives denoted as $\mathsf{M}=\{\mathsf{M}_1,\mathsf{M}_2,\ldots,\mathsf{M}_{\mathfrak{a}}\}$ and a set of '$\mathfrak{b}$' attributes denoted as $\mathcal{N}=\{\mathcal{N}_1,\mathcal{N}_2,\ldots,\mathcal{N}_{\mathfrak{b}}\}$. To capture the preferences of a group of $\mathfrak{e}$ decision-makers, create a set $\mathsf{E}=\{\mathsf{E}_1,\mathsf{E}_2,\ldots,\mathsf{E}_{\mathfrak{e}}\}$ where each decision-maker $\mathsf{E}_{\kappa}(\kappa=1,2,\ldots,\mathfrak{e})$ is associated with a weight value $\curlyvee'_{\kappa}$ in the weight vector $\curlyvee'=(\curlyvee'_1,\curlyvee'_2,\ldots,\curlyvee'_{\mathfrak{e}})^T$. The weights $\curlyvee'_{\kappa}$ satisfy the conditions $\curlyvee'\in[0,1]$ and $\sum_{\kappa=1}^{\mathfrak{e}}\curlyvee'_{\kappa}=1$, representing the relative importance of each decision-maker's opinion. Each decision-maker provides their assessments for the alternatives $\mathsf{M}_{\imath}(\imath=1,2,\ldots,\mathfrak{a})$ regarding the attributes $\mathcal{N}_{\jmath}(\jmath=1,2,\ldots,\mathfrak{b})$ using PLq-ROFNs. These assessments are captured in a PLq-ROF decision matrix $[\mathsf{N}_{\imath\jmath}^{\kappa}]_{\mathfrak{a}\times\mathfrak{b}}=((\flat_{\varphi^{\imath\jmath(t)}}(\mathfrak{h}^{(t)}))^{\kappa},(\flat_{\phi^{\imath\jmath(r)}}(\mathfrak{h}^{(r)}))^{\kappa})_{\mathfrak{a}\times\mathfrak{b}}$.

Step 2. Normalize the decision matrices:

$$\mathsf{N}_{\imath\jmath}^{\kappa}=\begin{cases}((\flat_{\varphi^{\imath\jmath(t)}}(\mathfrak{h}^{(t)}))^{\kappa},(\flat_{\phi^{\imath\jmath(r)}}(\mathfrak{h}^{(r)}))^{\kappa})\in BI,\\((\flat_{\phi^{\imath\jmath(r)}}(\mathfrak{h}^{(r)}))^{\kappa},(\flat_{\varphi^{\imath\jmath(t)}}(\mathfrak{h}^{(t)}))^{\kappa})\in CI\end{cases} \tag{19}$$

where BI is the benefit index and CI is the cost index.

Step 3. Calculate the support degrees $\mathfrak{I}(\mathsf{N}_{ij}^{\kappa},\mathsf{N}_{ij}^{\mathfrak{d}})$:

$$\mathfrak{I}(\mathsf{N}_{ij}^{\kappa},\mathsf{N}_{ij}^{\mathfrak{d}})=1-d(\mathsf{N}_{ij}^{\kappa},\mathsf{N}_{ij}^{\mathfrak{d}}),\quad \kappa,\mathfrak{d}=1,2,\dots,\mathfrak{e};\kappa\neq\mathfrak{d} \tag{20}$$

where $d(\mathsf{N}_{ij}^{\kappa},\mathsf{N}_{ij}^{\mathfrak{d}})$ represents the normalized Hamming distance between N_{ij}^{κ} and $\mathsf{N}_{ij}^{\mathfrak{d}}$ that is calculated by Eq. (6).

Step 4. Calculate the synthesis support matrices $[\mathfrak{R}(\mathsf{N}_{ij}^{\kappa})]_{\mathfrak{a}\times\mathfrak{b}}$:

$$\mathfrak{R}(\mathsf{N}_{ij}^{\kappa})=\sum_{\kappa,\mathfrak{d}=1;\kappa\neq\mathfrak{d}}^{\mathfrak{e}}\mathfrak{I}(\mathsf{N}_{ij}^{\kappa},\mathsf{N}_{ij}^{\mathfrak{d}}). \tag{21}$$

Step 5. Compute the comprehensive power weight matrices $[\wp_{ij}^{\kappa}]_{\mathfrak{a}\times\mathfrak{b}}$:

$$\wp_{ij}^{\kappa}=\frac{\Upsilon'_{\kappa}\left(1+\mathfrak{R}(\mathsf{N}_{ij}^{\kappa})\right)}{\sum_{\kappa=1}^{\mathfrak{e}}\Upsilon'_{\kappa}\left(1+\mathfrak{R}(\mathsf{N}_{ij}^{\kappa})\right)}. \tag{22}$$

Step 6. In order to construct the aggregated PLq-ROF decision matrix, the individual decisions made by decision-makers need to be merged into a collective decision using the PLq-ROFWPA operator. The PLq-ROFWPA operator combines the individual assessments provided by the decision-makers and generates a comprehensive decision where

$$\begin{aligned}&\text{PL}q\text{-ROFWPA}_{\wp}(\mathsf{N}_{ij}^{1},\mathsf{N}_{ij}^{2},\dots,\mathsf{N}_{ij}^{\mathfrak{e}})\\&=\left(\left(\mathfrak{g}^{-1}\left(\sqrt[q]{1-\prod_{\kappa=1}^{\mathfrak{e}}\left(1-\left(\mathfrak{g}(\flat_{\varphi^{\kappa(t)}})\right)^{q}\right)^{\wp_{\kappa}}}\right)(\mathfrak{h}^{(t)})\right),\left(\mathfrak{g}^{-1}\left(\prod_{\kappa=1}^{\mathfrak{e}}\left(\mathfrak{g}(\flat_{\phi^{\kappa(r)}})\right)^{\wp_{\kappa}}\right)(\mathfrak{h}^{(r)})\right)\right).\end{aligned} \tag{23}$$

The VIKOR approach is an exceptional tool in MADM. It is additionally recognized as a compromise ranking technique. The best compromise solution can be developed employing a set of specific rules based on the three rank values $\mathcal{K}_{\imath}$, $\mathcal{L}_{\imath}$, and $O_{\imath}$. The following are the precise steps for the conventional VIKOR method:

Step 7. $\mathsf{N}_{\jmath}^{*}$ and $\mathsf{N}_{\jmath}^{-}$ have to be calculated in the first step of the procedure. The benefit and cost criterion must be considered separately.

The best N_{ij} is defined as $\mathsf{N}_{\jmath}^{*}$ and the worst N_{ij} as $\mathsf{N}_{\jmath}^{-}$ for each attribute $\jmath$. Utilize Eqs. (22) and (25) to compute the $\mathsf{N}_{\jmath}^{*}$ and $\mathsf{N}_{\jmath}^{-}$ indices for the positive and the negative attributes, respectively.

$$\begin{cases}\mathsf{N}_{\jmath}^{*}=\max_{\imath}\mathsf{N}_{ij};\\ \mathsf{N}_{\jmath}^{-}=\min_{\imath}\mathsf{N}_{ij},\end{cases}\text{for positive attributes} \tag{24}$$

$$\begin{cases}\mathsf{N}_{\jmath}^{*}=\min_{\imath}\mathsf{N}_{ij};\\ \mathsf{N}_{\jmath}^{-}=\max_{\imath}\mathsf{N}_{ij}\end{cases}\text{for negative attributes} \tag{25}$$

where $\imath=1,2,\dots,\mathfrak{a}$, $\jmath=1,2,\dots,\mathfrak{b}$.

Step 8. Estimate the $\mathcal{K}_{\imath}$ and $\mathcal{L}_{\imath}$ through the Eqs. (20) and (21), respectively:

$$\mathcal{K}_{\imath}=\sum_{\jmath=1}^{\mathfrak{b}}\Upsilon_{\jmath}\frac{\left(\mathsf{N}_{\jmath}^{*}-\mathsf{N}_{ij}\right)}{\left(\mathsf{N}_{\jmath}^{*}-\mathsf{N}_{\jmath}^{-}\right)}, \tag{26}$$

$$\mathcal{L}_{\imath}=\max_{\jmath}\left[\Upsilon_{\jmath}\frac{\left(\mathsf{N}_{\jmath}^{*}-\mathsf{N}_{ij}\right)}{\left(\mathsf{N}_{\jmath}^{*}-\mathsf{N}_{\jmath}^{-}\right)}\right] \tag{27}$$

where $\mathcal{K}_{\imath}$ and $\mathcal{L}_{\imath}$ represent the utility measure and regret measure for each alternative $\imath$; respectively, among which, the smaller the value $\mathcal{K}_{\imath}$; the greater the utility of the alternative. And on the contrary, the smaller the value $\mathcal{L}_{\imath}$; the lower the regret. The value $\Upsilon_{\jmath}$ expresses the weight of the $\jmath$th criteria. Also, rank the alternatives based on $\mathcal{K}_{\imath}$ and $\mathcal{L}_{\imath}$ in the ascending order.

Step 9. The VIKOR index $O_{\imath}$ is also determined for each alternative as shown in Eq. (28)

$$O_{\imath}=\mathfrak{u}\times\left[\frac{(\mathcal{K}_{\imath}-\mathcal{K}^{*})}{(\mathcal{K}^{-}-\mathcal{K}^{*})}\right]+(1-\mathfrak{u})\times\left[\frac{(\mathcal{L}_{\imath}-\mathcal{L}^{*})}{(\mathcal{L}^{-}-\mathcal{L}^{*})}\right]. \tag{28}$$

$$\mathcal{K}^{*}=\min_{\imath}\mathcal{K}_{\imath},\quad \mathcal{K}^{-}=\max_{\imath}\mathcal{K}_{\imath},\quad \mathcal{L}^{*}=\min_{\imath}\mathcal{L}_{\imath},\quad \mathcal{L}^{-}=\max_{\imath}\mathcal{L}_{\imath}$$

where $\mathfrak{u}$ denotes the strategic weight, which is commonly assumed to be 0.5.

Step 10. A compromise solution alternative M_1 with the lowest value in O can potentially be identified if it fulfills two conditions:

Condition 1: Acceptable advantage $O(\mathsf{M}_2) - O(\mathsf{M}_1) \geq DO$ where M_2 is the second lowest value and $O(.)$ represents the actual value of the alternative in O where $DO = 1/(\mathfrak{a} - 1)$.

Condition 2: Acceptable stability in DM Based on $\mathcal{K}$ and $\mathcal{L}$, M_1 can also be considered the most appropriate alternative. Once the value $\mathfrak{u}$ is determined, the DM process will maintain stability. This stability could involve majority rule voting when $\mathfrak{u} > 0.5$, consensus when $\mathfrak{u} = 0.5$, or veto when $\mathfrak{u} < 0.5$.

Although if either of the two conditions is not fulfilled, compromise solutions should be indicated as follows:

- If the second condition is not fulfilled, the compromise solutions become M_1 and M_2.
- If the first condition is not fulfilled, the compromise solutions are $\mathsf{M}_1, \mathsf{M}_2, \ldots, \mathsf{M}_{\mathfrak{a}}$. $\mathsf{M}_{\mathfrak{a}}$ fulfills $O(\mathsf{M}_{\mathfrak{a}}) - O(\mathsf{M}_1) < DO$. Indistinguishable alternatives are represented as M_1, $\mathsf{M}_2, \ldots$, $\mathsf{M}_{\mathfrak{a}}$.

Step 11. Rank the alternatives based on O_ι in the ascending order. A compromise solution of alternative with the lowest value in O can be identified as the best alternative.

6. Application and analysis

Astronomy has emerged as a prominent field for leveraging machine learning and probabilistic modeling techniques. This is primarily due to the availability of extensive public data sets, predominantly comprising digital imaging data. These datasets enable astronomers to apply these powerful tools for data analysis and exploration. Simple yet effective models have been developed to understand and study key astronomical phenomena, including stars, quasars, and galaxies. Additionally, accurate models of telescopes, cameras, and detectors facilitate precise observations and measurements. Astronomy, the scientific study of celestial phenomena beyond Earth's atmosphere, dates back to ancient cultures that meticulously observed and documented the motions of celestial bodies. It is among the oldest branches of natural science. Astronomy employs principles from mathematics, physics, and chemistry to elucidate the origin, evolution, physical properties, and chemical composition of celestial objects. Moreover, astronomers investigate the structure, history, and future of the universe as a whole, delving into profound questions about its nature. The two primary fields of astronomy are observational and theoretical. Observational astronomy entails gathering and evaluating information from telescopes and other devices that find various kinds of space-derived electromagnetic radiation or particles. To describe and forecast the behavior of celestial objects and phenomena based on the laws of physics and chemistry, theoretical astronomy develops models and simulations. Both branches support one another and increase our knowledge of the universe. The science of astronomy is one that continuously challenges and advances our understanding of the cosmos. It is interesting and exciting. It also arouses interest and awe regarding where humans fit into the big cosmic scheme. In addition to being a science, astronomy is also a cultural and aesthetic pursuit that embodies the human desire for understanding and adventure. Astronomy is the scientific study of the universe and its celestial bodies, such as planets, stars, galaxies, and black holes. Astronomy is a fascinating and rewarding field of research, but it is not the only one that deals with the cosmos.

A detailed description of eight branches of astronomy is given below:

Astrophotography (M_1): Astrophotography focuses on capturing celestial objects and phenomena in the night sky. This fascinating discipline allows photographers to unveil the breathtaking beauty of the cosmos, from capturing the glow of distant galaxies to the intricate details of our moon's surface or the ethereal streaks of meteor showers. Astrophotographers employ advanced camera equipment and long exposure techniques and often venture to dark and remote locations to minimize light pollution, enabling them to reveal the hidden wonders of the universe. It requires a deep understanding of both photography and astronomy, making it a unique and rewarding hobby or profession for those who seek to explore the cosmos through the lens of a camera. Astrophotography can seem daunting to a beginner, but the good news is that it is now easier than ever.

Astrobiology (M_2): Astrobiology is a multidisciplinary field of scientific inquiry that seeks to understand the potential for life beyond Earth. It explores the origins, evolution, and distribution of life in the universe, aiming to answer fundamental questions such as whether life exists elsewhere in the cosmos and what conditions might support it. Astrobiologists study extreme environments on Earth, like deep-sea hydrothermal vents and acidic hot springs, to learn about life's adaptability and the potential habitability of other celestial bodies, including Mars, Europa, and exoplanets. By investigating the chemistry of life, the environments that could harbor it, and the cosmic processes that shape planetary systems, astrobiology sheds light on the profound question of our place in the universe and the possibility of life elsewhere.

Cosmology (M_3): Cosmology is the scientific study of the origin, evolution, and structure of the universe as a whole. It seeks to answer fundamental questions about the nature of the cosmos, such as the Big Bang theory, the expansion of the universe, the distribution of galaxies, and the formation of cosmic structures. Cosmologists use a combination of theoretical models, observational data, and advanced technologies to explore the vast expanse of space and unravel the mysteries of the universe's past, present, and future. Cosmology not only deepens our understanding of the cosmos but also challenges our perceptions of the universe's scale, composition, and ultimate fate, making it a captivating and continuously evolving field of scientific inquiry.

Astrometry (M_4): Astrometry focuses on the precise measurement and study of the positions and motions of celestial objects in the sky. It plays a fundamental role in our understanding of the universe by providing crucial data for various astronomical studies. Astrometrists use sophisticated instruments like telescopes and cameras to record the apparent positions of stars, planets, and other celestial bodies. By tracking their movements over time, astrometry helps astronomers determine the orbits of planets, discover new celestial objects, and study the dynamics of galaxies and star systems. Astrometric data also contributes to the

Table 1
Overview of evaluation attributes.

Attributes	Overview
Observational ($\mathcal{N}_1$)	The only thing astronomers can do with faraway stars or galaxies is study them using a variety of tools and methods; they are too far away to conduct controlled experiments on them. In order to analyze their observations and evaluate their theories, they make use of the laws of physics and mathematics.
Interdisciplinary ($\mathcal{N}_2$)	A wide range of topics is covered by astronomy, including the origin and future of the universe as well as the development and evolution of stars, planets, and galaxies. Chemistry, biology, geology, computer science, and engineering are just a few of the other disciplines that it draws from for its knowledge and techniques.
Exploratory ($\mathcal{N}_3$)	As new technology makes it possible for astronomers to study farther away and fainter objects and to investigate various electromagnetic radiation wavelengths, astronomy continuously pushes the boundaries of human knowledge and discovery. In addition, astronomers look for signs of extraterrestrial life and try to comprehend how the universe works.
Inspirational ($\mathcal{N}_4$)	Astronomy causes interest and curiosity in the natural world and our relationship to it. It also forces us to critically and creatively consider difficult and fundamental issues like how the cosmos came into being, how it functions, and what our place in it is.

accurate navigation of spacecraft and the search for exoplanets. In essence, astrometry serves as the celestial GPS, enabling us to map and comprehend the vast cosmic landscape.

Astrochemistry (M_5): Astrochemistry explores the chemistry of the universe beyond Earth. It delves into the composition, reactions, and processes that occur in the vast expanse of space, including within stars, interstellar clouds, and planetary atmospheres. Astrochemists use spectroscopy and laboratory experiments to identify and understand the complex molecules and chemical reactions occurring in these cosmic environments. This field not only sheds light on the origins of life and the formation of celestial bodies but also helps us better comprehend the fundamental building blocks of the universe itself. It is an interdisciplinary science that bridges the gap between astronomy and chemistry, offering profound insights into the cosmic chemistry that shapes our universe.

Astronautics (M_6): Astronautics focuses on the design, development, and operation of spacecraft and the science of space travel. It encompasses a wide range of activities, including launching, controlling, and navigating spacecraft, conducting experiments in outer space, and studying celestial bodies and phenomena. Astronautics has played a pivotal role in expanding our understanding of the universe, enabling human exploration of space, and advancing technology for both civilian and military applications. It continues to be a dynamic and exciting field, pushing the boundaries of human knowledge and capabilities as we explore the cosmos beyond Earth's atmosphere.

Astrodynamics (M_7): Astrodynamics focuses on the study of the motion and behavior of objects in space, including spacecraft, satellites, and celestial bodies. It involves the application of principles from physics and mathematics to understand the orbital mechanics, trajectory planning, and the control of objects in space. Astrodynamics plays a crucial role in space exploration, satellite deployment, and mission planning, enabling us to calculate precise trajectories, perform orbital maneuvers, and ensure the successful navigation of spacecraft in the complex and dynamic environment of space. It is a fundamental field in the realm of space science and technology, contributing to the advancement of our understanding of the universe and our ability to explore and utilize space for various purposes.

Astrogeology (M_8): Astrogeology focuses on the study of the geological processes and features of celestial bodies beyond Earth. It combines principles of geology with planetary science to analyze and interpret the surfaces, interiors, and histories of planets, moons, asteroids, and comets. Astrogeologists use remote sensing data, spacecraft missions, and geological techniques to unravel the mysteries of these extraterrestrial bodies. By understanding the geological evolution of these objects, astrogeologists contribute to our broader understanding of the solar system's history and the potential for past or present extraterrestrial life. Astrogeology plays a vital role in planning and executing space missions, including those to Mars, the Moon, and other celestial destinations, and it continues to expand our knowledge of the universe beyond our home planet.

As a result, astronomy, which stands for the study of celestial objects and phenomena, could indeed be categorized as a classical MAGDM problem. We intend to employ the PLq-ROF-VIKOR method that is suggested in this paper to evaluate astronomy. In this case, eight alternatives, $\mathsf{M} = \{\mathsf{M}_1, \mathsf{M}_2, \mathsf{M}_3, \dots, \mathsf{M}_8\}$ are evaluated by four decision-makers, $\mathsf{E} = \{\mathsf{E}_1, \mathsf{E}_2, \mathsf{E}_3, \mathsf{E}_4\}$, with weights $\curlyvee' = (0.1085, 0.1319, 0.6330, 0.1266)^T$, to address the given problem. The four decision-makers select the best alternative using the four attributes, $\mathcal{N} = \{\mathcal{N}_1, \mathcal{N}_2, \mathcal{N}_3, \mathcal{N}_4\}$ (shown in Table 1), with the associated weight vectors being $\curlyvee = (0.2668, 0.2493, 0.2427, 0.2412)^T$. In order to quantify each LTS $\mathcal{S} = \{\flat_{-4}$: irrelevant, $\flat_{-3}$: low relevance, $\flat_{-2}$: moderate relevance, $\flat_{-1}$: high relevance, $\flat_0$: not effective, $\flat_1$: low effectiveness, $\flat_2$: moderate effectiveness, $\flat_3$: high effectiveness, $\flat_4$: reliable$\}$, four decision-makers provide their opinions. The evaluation values provided by the four decision-makers for each alternative are condensed into decision matrices displayed in Tables 2-5.

6.1. DM based on the PLq-ROFWPA operator under VIKOR method

In this subsection, the detailed numerical explanation of the proposed method is described.

Table 2
PLq-ROF decision matrix provided by E_1.

Alternatives	Attributes	
	$\mathcal{N}_1$	$\mathcal{N}_2$
M_1	$(\{ƀ_{1}(0.1), ƀ_{2}(0.4), ƀ_{3}(0.5)\}, \{ƀ_{-4}(0.4), ƀ_{-3}(0.2), ƀ_{-1}(0.4)\})$	$(\{ƀ_{-2}(0.5), ƀ_{1}(0.3), ƀ_{2}(0.2)\}, \{ƀ_{-3}(0.1), ƀ_{-2}(0.2), ƀ_{1}(0.7)\})$
M_2	$(\{ƀ_{-2}(0.3), ƀ_{-1}(0.3), ƀ_{1}(0.4)\}, \{ƀ_{-1}(0.1), ƀ_{0}(0.1), ƀ_{1}(0.8)\})$	$(\{ƀ_{-3}(0.3), ƀ_{-2}(0.5), ƀ_{-1}(0.2)\}, \{ƀ_{-4}(0.4), ƀ_{-3}(0.4), ƀ_{-2}(0.2)\})$
M_3	$(\{ƀ_{-4}(0.3), ƀ_{-2}(0.6), ƀ_{1}(0.1)\}, \{ƀ_{-3}(0.2), ƀ_{-2}(0.4), ƀ_{-1}(0.4)\})$	$(\{ƀ_{-1}(0.1), ƀ_{2}(0.2), ƀ_{3}(0.7)\}, \{ƀ_{-3}(0.1), ƀ_{-1}(0.1), ƀ_{2}(0.8)\})$
M_4	$(\{ƀ_{-2}(0.3), ƀ_{-1}(0.2), ƀ_{2}(0.5)\}, \{ƀ_{-4}(0.4), ƀ_{-3}(0.4), ƀ_{-2}(0.2)\})$	$(\{ƀ_{-2}(0.3), ƀ_{1}(0.3), ƀ_{2}(0.4)\}, \{ƀ_{-2}(0.3), ƀ_{0}(0.4), ƀ_{1}(0.3)\})$
M_5	$(\{ƀ_{1}(0.6), ƀ_{2}(0.1), ƀ_{3}(0.3)\}, \{ƀ_{-1}(0.5), ƀ_{1}(0.4), ƀ_{2}(0.1)\})$	$(\{ƀ_{1}(0.5), ƀ_{2}(0.2), ƀ_{3}(0.3)\}, \{ƀ_{-3}(0.1), ƀ_{-2}(0.2), ƀ_{-1}(0.7)\})$
M_6	$(\{ƀ_{-3}(0.2), ƀ_{-2}(0.4), ƀ_{1}(0.4)\}, \{ƀ_{-4}(0.2), ƀ_{-3}(0.4), ƀ_{-2}(0.4)\})$	$(\{ƀ_{-3}(0.3), ƀ_{-2}(0.3), ƀ_{1}(0.4)\}, \{ƀ_{-4}(0.2), ƀ_{-3}(0.3), ƀ_{-2}(0.5)\})$
M_7	$(\{ƀ_{-2}(0.3), ƀ_{-1}(0.3), ƀ_{0}(0.4)\}, \{ƀ_{-2}(0.6), ƀ_{-1}(0.2), ƀ_{1}(0.2)\})$	$(\{ƀ_{-4}(0.1), ƀ_{-3}(0.4), ƀ_{-2}(0.5)\}, \{ƀ_{-2}(0.3), ƀ_{-1}(0.1), ƀ_{1}(0.6)\})$
M_8	$(\{ƀ_{-1}(0.1), ƀ_{2}(0.2), ƀ_{4}(0.7)\}, \{ƀ_{-4}(0.1), ƀ_{-3}(0.1), ƀ_{-2}(0.8)\})$	$(\{ƀ_{-2}(0.3), ƀ_{-1}(0.3), ƀ_{1}(0.4)\}, \{ƀ_{-3}(0.2), ƀ_{-2}(0.6), ƀ_{1}(0.2)\})$
	$\mathcal{N}_3$	$\mathcal{N}_4$
M_1	$(\{ƀ_{-4}(0.1), ƀ_{-3}(0.4), ƀ_{1}(0.5)\}, \{ƀ_{-3}(0.1), ƀ_{-1}(0.2), ƀ_{2}(0.7)\})$	$(\{ƀ_{1}(0.2), ƀ_{2}(0.2), ƀ_{3}(0.6)\}, \{ƀ_{-4}(0.4), ƀ_{-3}(0.5), ƀ_{-2}(0.1)\})$
M_2	$(\{ƀ_{-2}(0.4), ƀ_{-1}(0.3), ƀ_{3}(0.3)\}, \{ƀ_{-4}(0.3), ƀ_{-3}(0.3), ƀ_{-2}(0.4)\})$	$(\{ƀ_{-4}(0.5), ƀ_{1}(0.3), ƀ_{2}(0.2)\}, \{ƀ_{-3}(0.3), ƀ_{-2}(0.2), ƀ_{-1}(0.5)\})$
M_3	$(\{ƀ_{-3}(0.1), ƀ_{1}(0.1), ƀ_{2}(0.8)\}, \{ƀ_{-2}(0.4), ƀ_{-1}(0.4), ƀ_{1}(0.2)\})$	$(\{ƀ_{1}(0.3), ƀ_{2}(0.6), ƀ_{3}(0.1)\}, \{ƀ_{-1}(0.2), ƀ_{1}(0.4), ƀ_{2}(0.4)\})$
M_4	$(\{ƀ_{-4}(0.7), ƀ_{-3}(0.2), ƀ_{-2}(0.1)\}, \{ƀ_{-3}(0.8), ƀ_{-2}(0.1), ƀ_{-1}(0.1)\})$	$(\{ƀ_{-4}(0.5), ƀ_{-3}(0.2), ƀ_{-2}(0.3)\}, \{ƀ_{-3}(0.4), ƀ_{-2}(0.1), ƀ_{-1}(0.5)\})$
M_5	$(\{ƀ_{-2}(0.4), ƀ_{-1}(0.4), ƀ_{1}(0.2)\}, \{ƀ_{-2}(0.2), ƀ_{-1}(0.4), ƀ_{1}(0.4)\})$	$(\{ƀ_{1}(0.4), ƀ_{2}(0.4), ƀ_{3}(0.2)\}, \{ƀ_{-1}(0.3), ƀ_{1}(0.3), ƀ_{2}(0.4)\})$
M_6	$(\{ƀ_{-4}(0.1), ƀ_{-3}(0.1), ƀ_{-2}(0.8)\}, \{ƀ_{-3}(0.5), ƀ_{-2}(0.3), ƀ_{-1}(0.2)\})$	$(\{ƀ_{-2}(0.1), ƀ_{-1}(0.1), ƀ_{1}(0.8)\}, \{ƀ_{-2}(0.5), ƀ_{-1}(0.2), ƀ_{1}(0.3)\})$
M_7	$(\{ƀ_{1}(0.2), ƀ_{2}(0.4), ƀ_{3}(0.4)\}, \{ƀ_{-3}(0.4), ƀ_{-1}(0.4), ƀ_{2}(0.2)\})$	$(\{ƀ_{-2}(0.2), ƀ_{1}(0.4), ƀ_{2}(0.4)\}, \{ƀ_{-3}(0.1), ƀ_{-2}(0.1), ƀ_{-1}(0.8)\})$
M_8	$(\{ƀ_{-3}(0.4), ƀ_{-2}(0.3), ƀ_{1}(0.3)\}, \{ƀ_{-1}(0.1), ƀ_{0}(0.6), ƀ_{1}(0.3)\})$	$(\{ƀ_{-1}(0.3), ƀ_{2}(0.2), ƀ_{3}(0.5)\}, \{ƀ_{-4}(0.2), ƀ_{-3}(0.6), ƀ_{-2}(0.2)\})$

Table 3
PLq-ROF decision matrix provided by E_2.

Alternatives	Attributes	
	$\mathcal{N}_1$	$\mathcal{N}_2$
M_1	$(\{ƀ_{1}(0.1), ƀ_{2}(0.4), ƀ_{3}(0.5)\}, \{ƀ_{-4}(0.4), ƀ_{-3}(0.2), ƀ_{-2}(0.4)\})$	$(\{ƀ_{-3}(0.5), ƀ_{-2}(0.3), ƀ_{-1}(0.2)\}, \{ƀ_{-4}(0.1), ƀ_{-3}(0.2), ƀ_{-2}(0.7)\})$
M_2	$(\{ƀ_{-2}(0.3), ƀ_{-1}(0.3), ƀ_{2}(0.4)\}, \{ƀ_{-1}(0.1), ƀ_{1}(0.1), ƀ_{2}(0.8)\})$	$(\{ƀ_{-2}(0.3), ƀ_{1}(0.5), ƀ_{2}(0.2)\}, \{ƀ_{-3}(0.4), ƀ_{-2}(0.4), ƀ_{-1}(0.2)\})$
M_3	$(\{ƀ_{-4}(0.3), ƀ_{-2}(0.6), ƀ_{1}(0.1)\}, \{ƀ_{-3}(0.2), ƀ_{-2}(0.4), ƀ_{-1}(0.4)\})$	$(\{ƀ_{-3}(0.1), ƀ_{2}(0.2), ƀ_{3}(0.7)\}, \{ƀ_{-4}(0.1), ƀ_{-1}(0.1), ƀ_{2}(0.8)\})$
M_4	$(\{ƀ_{-2}(0.3), ƀ_{1}(0.2), ƀ_{2}(0.5)\}, \{ƀ_{-4}(0.4), ƀ_{-3}(0.4), ƀ_{-2}(0.2)\})$	$(\{ƀ_{-4}(0.3), ƀ_{-3}(0.3), ƀ_{2}(0.4)\}, \{ƀ_{-1}(0.3), ƀ_{2}(0.4), ƀ_{3}(0.3)\})$
M_5	$(\{ƀ_{1}(0.6), ƀ_{2}(0.1), ƀ_{3}(0.3)\}, \{ƀ_{-1}(0.5), ƀ_{1}(0.4), ƀ_{2}(0.1)\})$	$(\{ƀ_{1}(0.5), ƀ_{2}(0.2), ƀ_{3}(0.3)\}, \{ƀ_{-3}(0.1), ƀ_{-2}(0.2), ƀ_{-1}(0.7)\})$
M_6	$(\{ƀ_{-3}(0.2), ƀ_{-2}(0.4), ƀ_{1}(0.4)\}, \{ƀ_{-4}(0.2), ƀ_{-3}(0.4), ƀ_{-2}(0.4)\})$	$(\{ƀ_{-3}(0.3), ƀ_{-1}(0.3), ƀ_{1}(0.4)\}, \{ƀ_{-4}(0.2), ƀ_{-3}(0.3), ƀ_{-2}(0.5)\})$
M_7	$(\{ƀ_{-2}(0.3), ƀ_{-1}(0.3), ƀ_{2}(0.4)\}, \{ƀ_{-3}(0.6), ƀ_{-2}(0.2), ƀ_{-1}(0.2)\})$	$(\{ƀ_{-3}(0.1), ƀ_{-2}(0.4), ƀ_{-1}(0.5)\}, \{ƀ_{-2}(0.3), ƀ_{-1}(0.1), ƀ_{2}(0.6)\})$
M_8	$(\{ƀ_{-1}(0.1), ƀ_{1}(0.2), ƀ_{4}(0.7)\}, \{ƀ_{-4}(0.1), ƀ_{-3}(0.1), ƀ_{-2}(0.8)\})$	$(\{ƀ_{-1}(0.3), ƀ_{1}(0.3), ƀ_{2}(0.4)\}, \{ƀ_{-1}(0.2), ƀ_{0}(0.6), ƀ_{1}(0.2)\})$
	$\mathcal{N}_3$	$\mathcal{N}_4$
M_1	$(\{ƀ_{-4}(0.1), ƀ_{-3}(0.4), ƀ_{-2}(0.5)\}, \{ƀ_{-4}(0.1), ƀ_{-3}(0.2), ƀ_{-2}(0.7)\})$	$(\{ƀ_{-1}(0.3), ƀ_{2}(0.2), ƀ_{3}(0.5)\}, \{ƀ_{-1}(0.2), ƀ_{1}(0.6), ƀ_{2}(0.2)\})$
M_2	$(\{ƀ_{1}(0.4), ƀ_{2}(0.3), ƀ_{3}(0.3)\}, \{ƀ_{-3}(0.3), ƀ_{-2}(0.3), ƀ_{-1}(0.4)\})$	$(\{ƀ_{-3}(0.5), ƀ_{1}(0.3), ƀ_{2}(0.2)\}, \{ƀ_{-3}(0.3), ƀ_{-2}(0.2), ƀ_{-1}(0.5)\})$
M_3	$(\{ƀ_{-2}(0.1), ƀ_{-1}(0.1), ƀ_{1}(0.8)\}, \{ƀ_{-2}(0.4), ƀ_{1}(0.4), ƀ_{2}(0.2)\})$	$(\{ƀ_{-4}(0.5), ƀ_{-3}(0.2), ƀ_{-2}(0.3)\}, \{ƀ_{-4}(0.4), ƀ_{-3}(0.1), ƀ_{-2}(0.5)\})$
M_4	$(\{ƀ_{-3}(0.7), ƀ_{-2}(0.2), ƀ_{-1}(0.1)\}, \{ƀ_{-3}(0.8), ƀ_{-2}(0.1), ƀ_{-1}(0.1)\})$	$(\{ƀ_{1}(0.3), ƀ_{2}(0.6), ƀ_{3}(0.1)\}, \{ƀ_{-1}(0.2), ƀ_{1}(0.4), ƀ_{2}(0.4)\})$
M_5	$(\{ƀ_{-2}(0.4), ƀ_{-1}(0.4), ƀ_{1}(0.2)\}, \{ƀ_{-1}(0.2), ƀ_{1}(0.4), ƀ_{2}(0.4)\})$	$(\{ƀ_{-2}(0.1), ƀ_{-1}(0.1), ƀ_{1}(0.8)\}, \{ƀ_{-3}(0.5), ƀ_{-1}(0.2), ƀ_{1}(0.3)\})$
M_6	$(\{ƀ_{-4}(0.1), ƀ_{-3}(0.1), ƀ_{-2}(0.8)\}, \{ƀ_{-3}(0.5), ƀ_{-2}(0.3), ƀ_{-1}(0.2)\})$	$(\{ƀ_{1}(0.4), ƀ_{2}(0.4), ƀ_{3}(0.2)\}, \{ƀ_{-2}(0.3), ƀ_{1}(0.3), ƀ_{2}(0.4)\})$
M_7	$(\{ƀ_{-1}(0.4), ƀ_{1}(0.3), ƀ_{2}(0.3)\}, \{ƀ_{-1}(0.1), ƀ_{0}(0.6), ƀ_{1}(0.3)\})$	$(\{ƀ_{-2}(0.2), ƀ_{1}(0.4), ƀ_{2}(0.4)\}, \{ƀ_{-3}(0.1), ƀ_{-2}(0.1), ƀ_{1}(0.8)\})$
M_8	$(\{ƀ_{1}(0.2), ƀ_{2}(0.4), ƀ_{3}(0.4)\}, \{ƀ_{-3}(0.4), ƀ_{-2}(0.4), ƀ_{-1}(0.2)\})$	$(\{ƀ_{-3}(0.1), ƀ_{-2}(0.4), ƀ_{-1}(0.5)\}, \{ƀ_{-4}(0.4), ƀ_{-3}(0.5), ƀ_{-2}(0.1)\})$

Table 4
PLq-ROF decision matrix provided by E_3.

Alternatives	Attributes	
	$\mathcal{N}_1$	$\mathcal{N}_2$
M_1	$(\{ƀ_{-1}(0.1), ƀ_{1}(0.2), ƀ_{4}(0.7)\}, \{ƀ_{-4}(0.1), ƀ_{-3}(0.1), ƀ_{-2}(0.8)\})$	$(\{ƀ_{-1}(0.3), ƀ_{1}(0.3), ƀ_{2}(0.4)\}, \{ƀ_{-3}(0.2), ƀ_{-2}(0.6), ƀ_{-1}(0.2)\})$
M_2	$(\{ƀ_{-2}(0.3), ƀ_{-1}(0.3), ƀ_{1}(0.4)\}, \{ƀ_{-3}(0.1), ƀ_{-2}(0.1), ƀ_{-1}(0.8)\})$	$(\{ƀ_{-3}(0.3), ƀ_{-2}(0.5), ƀ_{-1}(0.2)\}, \{ƀ_{-4}(0.4), ƀ_{-3}(0.4), ƀ_{-2}(0.2)\})$
M_3	$(\{ƀ_{1}(0.3), ƀ_{2}(0.2), ƀ_{3}(0.5)\}, \{ƀ_{-4}(0.4), ƀ_{-3}(0.4), ƀ_{-2}(0.2)\})$	$(\{ƀ_{-2}(0.3), ƀ_{1}(0.3), ƀ_{2}(0.4)\}, \{ƀ_{-2}(0.3), ƀ_{-1}(0.4), ƀ_{1}(0.3)\})$
M_4	$(\{ƀ_{-4}(0.3), ƀ_{-2}(0.6), ƀ_{1}(0.1)\}, \{ƀ_{-3}(0.2), ƀ_{-2}(0.4), ƀ_{-1}(0.4)\})$	$(\{ƀ_{-4}(0.5), ƀ_{-3}(0.2), ƀ_{-2}(0.3)\}, \{ƀ_{-3}(0.1), ƀ_{-2}(0.2), ƀ_{-1}(0.7)\})$
M_5	$(\{ƀ_{-3}(0.2), ƀ_{1}(0.4), ƀ_{2}(0.4)\}, \{ƀ_{-4}(0.2), ƀ_{-3}(0.4), ƀ_{-2}(0.4)\})$	$(\{ƀ_{-1}(0.1), ƀ_{1}(0.2), ƀ_{2}(0.7)\}, \{ƀ_{-3}(0.1), ƀ_{-1}(0.1), ƀ_{2}(0.8)\})$
M_6	$(\{ƀ_{-2}(0.3), ƀ_{-1}(0.3), ƀ_{1}(0.4)\}, \{ƀ_{-3}(0.6), ƀ_{-2}(0.2), ƀ_{-1}(0.2)\})$	$(\{ƀ_{-4}(0.1), ƀ_{-3}(0.4), ƀ_{-2}(0.5)\}, \{ƀ_{-2}(0.3), ƀ_{-1}(0.1), ƀ_{1}(0.6)\})$
M_7	$(\{ƀ_{-4}(0.6), ƀ_{-3}(0.1), ƀ_{-2}(0.3)\}, \{ƀ_{1}(0.5), ƀ_{2}(0.4), ƀ_{4}(0.1)\})$	$(\{ƀ_{-3}(0.3), ƀ_{-2}(0.3), ƀ_{1}(0.4)\}, \{ƀ_{-3}(0.2), ƀ_{-2}(0.3), ƀ_{-1}(0.5)\})$
M_8	$(\{ƀ_{1}(0.1), ƀ_{2}(0.4), ƀ_{3}(0.5)\}, \{ƀ_{-4}(0.4), ƀ_{-3}(0.2), ƀ_{-1}(0.4)\})$	$(\{ƀ_{-2}(0.3), ƀ_{1}(0.5), ƀ_{2}(0.2)\}, \{ƀ_{-4}(0.1), ƀ_{-3}(0.2), ƀ_{1}(0.7)\})$
	$\mathcal{N}_3$	$\mathcal{N}_4$
M_1	$(\{ƀ_{-4}(0.4), ƀ_{-3}(0.3), ƀ_{1}(0.3)\}, \{ƀ_{-2}(0.1), ƀ_{-1}(0.6), ƀ_{1}(0.3)\})$	$(\{ƀ_{1}(0.3), ƀ_{2}(0.6), ƀ_{3}(0.1)\}, \{ƀ_{-1}(0.2), ƀ_{1}(0.4), ƀ_{2}(0.4)\})$
M_2	$(\{ƀ_{-2}(0.4), ƀ_{-1}(0.3), ƀ_{3}(0.3)\}, \{ƀ_{-4}(0.3), ƀ_{-3}(0.3), ƀ_{-2}(0.4)\})$	$(\{ƀ_{-4}(0.5), ƀ_{1}(0.3), ƀ_{2}(0.2)\}, \{ƀ_{-3}(0.3), ƀ_{-2}(0.2), ƀ_{-1}(0.5)\})$
M_3	$(\{ƀ_{-3}(0.1), ƀ_{1}(0.1), ƀ_{2}(0.8)\}, \{ƀ_{-2}(0.4), ƀ_{-1}(0.4), ƀ_{1}(0.2)\})$	$(\{ƀ_{-2}(0.1), ƀ_{-1}(0.1), ƀ_{1}(0.8)\}, \{ƀ_{-2}(0.5), ƀ_{-1}(0.2), ƀ_{1}(0.3)\})$
M_4	$(\{ƀ_{-4}(0.1), ƀ_{-3}(0.4), ƀ_{1}(0.5)\}, \{ƀ_{-3}(0.1), ƀ_{-2}(0.2), ƀ_{2}(0.7)\})$	$(\{ƀ_{1}(0.4), ƀ_{2}(0.4), ƀ_{3}(0.2)\}, \{ƀ_{-1}(0.3), ƀ_{1}(0.3), ƀ_{2}(0.4)\})$
M_5	$(\{ƀ_{-2}(0.4), ƀ_{-1}(0.4), ƀ_{2}(0.2)\}, \{ƀ_{-4}(0.2), ƀ_{-3}(0.4), ƀ_{-2}(0.4)\})$	$(\{ƀ_{-2}(0.2), ƀ_{1}(0.4), ƀ_{2}(0.4)\}, \{ƀ_{-3}(0.1), ƀ_{-2}(0.1), ƀ_{1}(0.8)\})$
M_6	$(\{ƀ_{1}(0.2), ƀ_{2}(0.4), ƀ_{3}(0.4)\}, \{ƀ_{-2}(0.4), ƀ_{-1}(0.4), ƀ_{2}(0.2)\})$	$(\{ƀ_{-3}(0.3), ƀ_{-2}(0.2), ƀ_{-1}(0.5)\}, \{ƀ_{-1}(0.2), ƀ_{1}(0.6), ƀ_{2}(0.2)\})$
M_7	$(\{ƀ_{-4}(0.7), ƀ_{-3}(0.2), ƀ_{-2}(0.1)\}, \{ƀ_{-3}(0.8), ƀ_{-2}(0.1), ƀ_{4}(0.1)\})$	$(\{ƀ_{1}(0.1), ƀ_{2}(0.4), ƀ_{3}(0.5)\}, \{ƀ_{-4}(0.4), ƀ_{-3}(0.5), ƀ_{-2}(0.1)\})$
M_8	$(\{ƀ_{-2}(0.1), ƀ_{1}(0.1), ƀ_{4}(0.8)\}, \{ƀ_{-3}(0.5), ƀ_{-2}(0.3), ƀ_{-1}(0.2)\})$	$(\{ƀ_{-4}(0.5), ƀ_{-3}(0.2), ƀ_{-2}(0.3)\}, \{ƀ_{-3}(0.4), ƀ_{-2}(0.1), ƀ_{-1}(0.5)\})$

Table 5
PLq-ROF decision matrix provided by E_4.

Alternatives	Attributes	
	$\mathcal{N}_1$	$\mathcal{N}_2$
M_1	$(\{\flat_{-2}(0.3),\flat_{-1}(0.2),\flat_{2}(0.5)\},\{\flat_{-4}(0.4),\flat_{-3}(0.4),\flat_{-2}(0.2)\})$	$(\{\flat_{-2}(0.3),\flat_{1}(0.3),\flat_{2}(0.4)\},\{\flat_{-2}(0.3),\flat_{-1}(0.4),\flat_{1}(0.3)\})$
M_2	$(\{\flat_{-4}(0.3),\flat_{-3}(0.6),\flat_{1}(0.1)\},\{\flat_{-3}(0.2),\flat_{-2}(0.4),\flat_{-1}(0.4)\})$	$(\{\flat_{-3}(0.3),\flat_{-2}(0.3),\flat_{1}(0.4)\},\{\flat_{-1}(0.2),\flat_{1}(0.6),\flat_{2}(0.2)\})$
M_3	$(\{\flat_{-3}(0.1),\flat_{-2}(0.4),\flat_{-1}(0.5)\},\{\flat_{-4}(0.4),\flat_{-3}(0.2),\flat_{2}(0.4)\})$	$(\{\flat_{-2}(0.5),\flat_{1}(0.3),\flat_{2}(0.2)\},\{\flat_{-4}(0.1),\flat_{-3}(0.2),\flat_{-2}(0.7)\})$
M_4	$(\{\flat_{-4}(0.2),\flat_{-3}(0.4),\flat_{-2}(0.4)\},\{\flat_{-3}(0.2),\flat_{-2}(0.4),\flat_{-1}(0.4)\})$	$(\{\flat_{-4}(0.1),\flat_{-3}(0.4),\flat_{-2}(0.5)\},\{\flat_{-2}(0.3),\flat_{-1}(0.1),\flat_{1}(0.6)\})$
M_5	$(\{\flat_{1}(0.6),\flat_{2}(0.1),\flat_{3}(0.3)\},\{\flat_{-1}(0.5),\flat_{1}(0.4),\flat_{2}(0.1)\})$	$(\{\flat_{-3}(0.3),\flat_{-2}(0.5),\flat_{-1}(0.2)\},\{\flat_{-4}(0.4),\flat_{-3}(0.4),\flat_{-2}(0.2)\})$
M_6	$(\{\flat_{-2}(0.3),\flat_{-1}(0.3),\flat_{1}(0.4)\},\{\flat_{-3}(0.6),\flat_{-2}(0.2),\flat_{-1}(0.2)\})$	$(\{\flat_{-2}(0.5),\flat_{1}(0.2),\flat_{2}(0.3)\},\{\flat_{-3}(0.1),\flat_{-2}(0.2),\flat_{-1}(0.7)\})$
M_7	$(\{\flat_{-3}(0.1),\flat_{-2}(0.6),\flat_{2}(0.3)\},\{\flat_{-4}(0.1),\flat_{-3}(0.1),\flat_{1}(0.8)\})$	$(\{\flat_{-1}(0.1),\flat_{2}(0.2),\flat_{3}(0.7)\},\{\flat_{-2}(0.1),\flat_{-1}(0.1),\flat_{2}(0.8)\})$
M_8	$(\{\flat_{-4}(0.5),\flat_{-3}(0.4),\flat_{-1}(0.1)\},\{\flat_{1}(0.1),\flat_{2}(0.1),\flat_{3}(0.8)\})$	$(\{\flat_{-3}(0.3),\flat_{-2}(0.3),\flat_{1}(0.4)\},\{\flat_{-3}(0.2),\flat_{-2}(0.3),\flat_{-1}(0.5)\})$
	$\mathcal{N}_3$	$\mathcal{N}_4$
M_1	$(\{\flat_{-3}(0.1),\flat_{1}(0.1),\flat_{2}(0.8)\},\{\flat_{-2}(0.4),\flat_{-1}(0.4),\flat_{1}(0.2)\})$	$(\{\flat_{-2}(0.1),\flat_{-1}(0.1),\flat_{1}(0.8)\},\{\flat_{-2}(0.5),\flat_{-1}(0.2),\flat_{1}(0.3)\})$
M_2	$(\{\flat_{1}(0.4),\flat_{2}(0.3),\flat_{3}(0.3)\},\{\flat_{-3}(0.1),\flat_{-2}(0.6),\flat_{-1}(0.3)\})$	$(\{\flat_{-3}(0.4),\flat_{-2}(0.5),\flat_{-1}(0.1)\},\{\flat_{-4}(0.4),\flat_{-3}(0.5),\flat_{-2}(0.1)\})$
M_3	$(\{\flat_{-2}(0.4),\flat_{-1}(0.4),\flat_{1}(0.2)\},\{\flat_{-2}(0.2),\flat_{-1}(0.4),\flat_{1}(0.4)\})$	$(\{\flat_{-2}(0.2),\flat_{1}(0.4),\flat_{2}(0.4)\},\{\flat_{-3}(0.1),\flat_{-2}(0.1),\flat_{-1}(0.8)\})$
M_4	$(\{\flat_{-4}(0.7),\flat_{-3}(0.2),\flat_{-2}(0.1)\},\{\flat_{-3}(0.8),\flat_{-2}(0.1),\flat_{-1}(0.1)\})$	$(\{\flat_{-1}(0.3),\flat_{3}(0.2),\flat_{4}(0.5)\},\{\flat_{-4}(0.2),\flat_{-3}(0.6),\flat_{-2}(0.2)\})$
M_5	$(\{\flat_{-2}(0.4),\flat_{-1}(0.3),\flat_{3}(0.3)\},\{\flat_{-4}(0.3),\flat_{-3}(0.3),\flat_{-2}(0.4)\})$	$(\{\flat_{-3}(0.5),\flat_{-2}(0.2),\flat_{-1}(0.3)\},\{\flat_{-2}(0.4),\flat_{1}(0.1),\flat_{2}(0.5)\})$
M_6	$(\{\flat_{-3}(0.1),\flat_{1}(0.1),\flat_{2}(0.8)\},\{\flat_{-3}(0.5),\flat_{-2}(0.3),\flat_{-1}(0.2)\})$	$(\{\flat_{-2}(0.1),\flat_{-1}(0.1),\flat_{1}(0.8)\},\{\flat_{-3}(0.3),\flat_{-2}(0.2),\flat_{-1}(0.5)\})$
M_7	$(\{\flat_{-4}(0.1),\flat_{-3}(0.4),\flat_{1}(0.5)\},\{\flat_{-4}(0.1),\flat_{-3}(0.2),\flat_{2}(0.7)\})$	$(\{\flat_{1}(0.4),\flat_{-3}(0.6),\flat_{-2}(0.2)\},\{\flat_{-4}(0.3),\flat_{-3}(0.3),\flat_{3}(0.4)\})$
M_8	$(\{\flat_{-3}(0.1),\flat_{-2}(0.1),\flat_{-1}(0.8)\},\{\flat_{-3}(0.4),\flat_{-2}(0.4),\flat_{1}(0.2)\})$	$(\{\flat_{-1}(0.3),\flat_{1}(0.6),\flat_{2}(0.1)\},\{\flat_{-3}(0.5),\flat_{-2}(0.4),\flat_{-1}(0.1)\})$

Step 1. Construct the PLq-ROF decision matrices.

$$[N^{\kappa}_{\imath\jmath}]_{8\times 4} = ((\flat_{\varphi^{\imath\jmath(t)}}(\mathfrak{h}^{\imath\jmath(t)}))^{\kappa}, (\flat_{\phi^{\imath\jmath(r)}}(\mathfrak{h}^{(r)}))^{\kappa})_{8\times 4}$$

where $\imath = 1,2,\ldots,8, \jmath = 1,2,3,4,$ and $\kappa = 1,2,3,4$ as shown in Tables 2-5.

Step 2. As all the attributes are of benefit type so there is no need to normalize the data.

Step 3. Calculate the support degrees $\mathfrak{I}(N^{\kappa}_{\imath\jmath}, N^{d}_{\imath\jmath})$ according to Eq. (26):

$$\mathfrak{I}^{12}=\mathfrak{I}^{21}=\begin{array}{c c}& \begin{array}{cccc}\mathcal{N}_1 & \mathcal{N}_2 & \mathcal{N}_3 & \mathcal{N}_4\end{array}\\ \begin{array}{c}M_1\\M_2\\M_3\\M_4\\M_5\\M_6\\M_7\\M_8\end{array} & \begin{bmatrix}0.9937 & 0.7894 & 0.7030 & 1.0888\\ 1.2056 & 1.1395 & 1.1560 & 1.0001\\ 1.0000 & 0.9980 & 0.9419 & 0.5164\\ 1.0266 & 1.1895 & 1.0025 & 1.4852\\ 1.0000 & 1.0000 & 1.1218 & 0.6239\\ 1.0000 & 1.0048 & 1.0000 & 1.4671\\ 1.0696 & 1.1077 & 0.7921 & 1.1063\\ 0.9672 & 1.1492 & 1.2078 & 0.6484\end{bmatrix}\end{array}\quad \mathfrak{I}^{13}=\mathfrak{I}^{31}=\begin{array}{c c}& \begin{array}{cccc}\mathcal{N}_1 & \mathcal{N}_2 & \mathcal{N}_3 & \mathcal{N}_4\end{array}\\ \begin{array}{c}M_1\\M_2\\M_3\\M_4\\M_5\\M_6\\M_7\\M_8\end{array} & \begin{bmatrix}1.1218 & 0.9150 & 0.8857 & 1.1153\\ 0.8859 & 1.0000 & 1.0000 & 1.0000\\ 1.2688 & 0.6461 & 1.0000 & 0.6122\\ 0.9185 & 0.7637 & 1.0445 & 1.4852\\ 0.7092 & 1.0307 & 0.9647 & 0.7051\\ 1.0149 & 1.0203 & 1.5786 & 0.9754\\ 1.2875 & 0.9935 & 0.7405 & 1.1901\\ 0.7363 & 1.1031 & 1.4035 & 0.6451\end{bmatrix}\end{array}$$

$$\mathfrak{I}^{14}=\mathfrak{I}^{41}=\begin{array}{c c}& \begin{array}{cccc}\mathcal{N}_1 & \mathcal{N}_2 & \mathcal{N}_3 & \mathcal{N}_4\end{array}\\ \begin{array}{c}M_1\\M_2\\M_3\\M_4\\M_5\\M_6\\M_7\\M_8\end{array} & \begin{bmatrix}0.7252 & 1.0035 & 1.0286 & 0.6770\\ 0.8789 & 1.1579 & 1.1560 & 0.8875\\ 1.1039 & 0.5248 & 0.8560 & 0.6481\\ 0.8434 & 0.8110 & 1.0000 & 1.4175\\ 1.0000 & 0.6795 & 1.0187 & 0.6960\\ 1.0149 & 1.1203 & 1.2653 & 0.9552\\ 1.0895 & 1.5179 & 0.6617 & 1.2965\\ 0.7613 & 0.9676 & 0.9230 & 0.8378\end{bmatrix}\end{array}\quad \mathfrak{I}^{23}=\mathfrak{I}^{32}=\begin{array}{c c}& \begin{array}{cccc}\mathcal{N}_1 & \mathcal{N}_2 & \mathcal{N}_3 & \mathcal{N}_4\end{array}\\ \begin{array}{c}M_1\\M_2\\M_3\\M_4\\M_5\\M_6\\M_7\\M_8\end{array} & \begin{bmatrix}1.1281 & 1.1256 & 1.1827 & 1.0398\\ 0.6803 & 0.8605 & 0.8440 & 0.9999\\ 1.2688 & 0.6481 & 1.0581 & 1.1283\\ 0.8919 & 0.5742 & 1.0420 & 1.0000\\ 0.7092 & 1.0307 & 0.8429 & 1.1412\\ 1.0149 & 1.0156 & 1.5786 & 0.7055\\ 1.2179 & 0.8857 & 1.0698 & 1.0838\\ 0.7691 & 0.9539 & 1.1969 & 0.9941\end{bmatrix}\end{array}$$

$$\mathfrak{I}^{24}=\mathfrak{I}^{42}=\begin{array}{c c}& \begin{array}{cccc}\mathcal{N}_1 & \mathcal{N}_2 & \mathcal{N}_3 & \mathcal{N}_4\end{array}\\ \begin{array}{c}M_1\\M_2\\M_3\\M_4\\M_5\\M_6\\M_7\\M_8\end{array} & \begin{bmatrix}0.7316 & 1.2141 & 1.3256 & 0.6035\\ 0.6733 & 1.0184 & 1.0000 & 0.8873\\ 1.1039 & 0.5268 & 0.9141 & 1.1346\\ 0.8168 & 0.6215 & 0.9975 & 0.9735\\ 1.0000 & 0.6795 & 0.8968 & 0.9693\\ 1.0149 & 1.1156 & 1.2653 & 0.5708\\ 1.0198 & 1.4102 & 0.9071 & 1.1903\\ 0.7941 & 0.8184 & 0.6445 & 1.2133\end{bmatrix}\end{array}\quad \mathfrak{I}^{34}=\mathfrak{I}^{43}=\begin{array}{c c}& \begin{array}{cccc}\mathcal{N}_1 & \mathcal{N}_2 & \mathcal{N}_3 & \mathcal{N}_4\end{array}\\ \begin{array}{c}M_1\\M_2\\M_3\\M_4\\M_5\\M_6\\M_7\\M_8\end{array} & \begin{bmatrix}0.4933 & 1.0321 & 1.0949 & 0.6122\\ 0.9930 & 1.1579 & 1.1560 & 0.8875\\ 0.6711 & 0.9464 & 0.8560 & 1.0995\\ 0.9829 & 1.0965 & 0.7180 & 0.9609\\ 1.3938 & 0.5087 & 1.0540 & 0.9674\\ 1.0000 & 1.1352 & 0.7290 & 0.9182\\ 0.8068 & 1.4269 & 0.9459 & 0.6406\\ 0.9259 & 0.7996 & 0.1135 & 1.1341\end{bmatrix}\end{array}$$

Step 4. Calculate the synthesis support matrices $[\mathfrak{R}(\mathrm{N}_{ij}^{\kappa})]_{8\times 4}$ according to Eq. (27):

$$\mathfrak{R}^1 = \begin{array}{c} \\ \mathrm{M}_1 \\ \mathrm{M}_2 \\ \mathrm{M}_3 \\ \mathrm{M}_4 \\ \mathrm{M}_5 \\ \mathrm{M}_6 \\ \mathrm{M}_7 \\ \mathrm{M}_8 \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{N}_1 & \mathcal{N}_2 & \mathcal{N}_3 & \mathcal{N}_4 \end{array} \\ \begin{bmatrix} 2.8406 & 2.7079 & 2.6172 & 2.8811 \\ 2.9704 & 3.2974 & 3.3120 & 2.8876 \\ 3.3727 & 2.1690 & 2.7979 & 1.7767 \\ 2.7885 & 2.7643 & 3.0470 & 4.3879 \\ 2.7092 & 2.7103 & 3.1052 & 2.0250 \\ 3.0299 & 3.1454 & 3.8439 & 3.3977 \\ 3.4466 & 3.6190 & 2.1943 & 3.5929 \\ 2.4649 & 3.2199 & 3.5344 & 2.1313 \end{bmatrix} \end{array} \quad \mathfrak{R}^2 = \begin{array}{c} \\ \mathrm{M}_1 \\ \mathrm{M}_2 \\ \mathrm{M}_3 \\ \mathrm{M}_4 \\ \mathrm{M}_5 \\ \mathrm{M}_6 \\ \mathrm{M}_7 \\ \mathrm{M}_8 \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{N}_1 & \mathcal{N}_2 & \mathcal{N}_3 & \mathcal{N}_4 \end{array} \\ \begin{bmatrix} 2.8533 & 3.1291 & 3.2113 & 2.7321 \\ 2.5593 & 3.0184 & 3.0000 & 2.8873 \\ 3.3727 & 2.1729 & 2.9141 & 2.7793 \\ 2.7353 & 2.3853 & 3.0420 & 3.4587 \\ 2.7092 & 2.7103 & 2.8615 & 2.7344 \\ 3.0299 & 3.1359 & 3.8439 & 2.7435 \\ 3.3074 & 3.4036 & 2.7689 & 3.3804 \\ 2.5304 & 2.9215 & 3.0492 & 2.8558 \end{bmatrix} \end{array}$$

$$\mathfrak{R}^3 = \begin{array}{c} \\ \mathrm{M}_1 \\ \mathrm{M}_2 \\ \mathrm{M}_3 \\ \mathrm{M}_4 \\ \mathrm{M}_5 \\ \mathrm{M}_6 \\ \mathrm{M}_7 \\ \mathrm{M}_8 \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{N}_1 & \mathcal{N}_2 & \mathcal{N}_3 & \mathcal{N}_4 \end{array} \\ \begin{bmatrix} 2.7432 & 3.0726 & 3.1634 & 2.7674 \\ 2.5593 & 3.0184 & 3.0000 & 2.8873 \\ 3.2086 & 2.2406 & 2.9141 & 2.8400 \\ 2.7933 & 2.4345 & 2.8046 & 3.4461 \\ 2.8122 & 2.5701 & 2.8615 & 2.8136 \\ 3.0299 & 3.1711 & 3.8862 & 2.5992 \\ 3.3122 & 3.3061 & 2.7562 & 2.9146 \\ 2.4313 & 2.8566 & 2.7138 & 2.7733 \end{bmatrix} \end{array} \quad \mathfrak{R}^4 = \begin{array}{c} \\ \mathrm{M}_1 \\ \mathrm{M}_2 \\ \mathrm{M}_3 \\ \mathrm{M}_4 \\ \mathrm{M}_5 \\ \mathrm{M}_6 \\ \mathrm{M}_7 \\ \mathrm{M}_8 \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{N}_1 & \mathcal{N}_2 & \mathcal{N}_3 & \mathcal{N}_4 \end{array} \\ \begin{bmatrix} 1.9502 & 3.2497 & 3.4491 & 1.8927 \\ 2.5452 & 3.3342 & 3.3120 & 2.6622 \\ 2.8790 & 1.9980 & 2.6262 & 2.8822 \\ 2.6432 & 2.5291 & 2.7155 & 3.3519 \\ 3.3938 & 1.8678 & 2.9694 & 2.6328 \\ 3.0299 & 3.3711 & 3.2595 & 2.4442 \\ 2.9160 & 4.3549 & 2.5147 & 3.1274 \\ 2.4813 & 2.5855 & 1.6810 & 3.1851 \end{bmatrix} \end{array}$$

Step 5. Compute the comprehensive power weight matrices $[\wp_{ij}^{\kappa}]_{8\times 4}(\kappa = 1,2,3,4)$ according to Eq. (24):

$$\wp^1 = \begin{array}{c} \\ \mathrm{M}_1 \\ \mathrm{M}_2 \\ \mathrm{M}_3 \\ \mathrm{M}_4 \\ \mathrm{M}_5 \\ \mathrm{M}_6 \\ \mathrm{M}_7 \\ \mathrm{M}_8 \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{N}_1 & \mathcal{N}_2 & \mathcal{N}_3 & \mathcal{N}_4 \end{array} \\ \begin{bmatrix} 0.1136 & 0.0990 & 0.0946 & 0.1149 \\ 0.1196 & 0.1140 & 0.1149 & 0.1093 \\ 0.1128 & 0.1077 & 0.1066 & 0.0809 \\ 0.1091 & 0.1175 & 0.1140 & 0.1288 \\ 0.1042 & 0.1145 & 0.1142 & 0.0888 \\ 0.1085 & 0.1074 & 0.1096 & 0.1295 \\ 0.1128 & 0.1117 & 0.0945 & 0.1222 \\ 0.1088 & 0.1183 & 0.1324 & 0.0902 \end{bmatrix} \end{array} \quad \wp^2 = \begin{array}{c} \\ \mathrm{M}_1 \\ \mathrm{M}_2 \\ \mathrm{M}_3 \\ \mathrm{M}_4 \\ \mathrm{M}_5 \\ \mathrm{M}_6 \\ \mathrm{M}_7 \\ \mathrm{M}_8 \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{N}_1 & \mathcal{N}_2 & \mathcal{N}_3 & \mathcal{N}_4 \end{array} \\ \begin{bmatrix} 0.1386 & 0.1340 & 0.1340 & 0.1343 \\ 0.1303 & 0.1296 & 0.1295 & 0.1329 \\ 0.1371 & 0.1311 & 0.1336 & 0.1339 \\ 0.1308 & 0.1285 & 0.1384 & 0.1296 \\ 0.1267 & 0.1392 & 0.1305 & 0.1333 \\ 0.1319 & 0.1302 & 0.1332 & 0.1340 \\ 0.1329 & 0.1295 & 0.1356 & 0.1417 \\ 0.1348 & 0.1336 & 0.1437 & 0.1350 \end{bmatrix} \end{array}$$

$$\wp^3 = \begin{array}{c} \\ \mathrm{M}_1 \\ \mathrm{M}_2 \\ \mathrm{M}_3 \\ \mathrm{M}_4 \\ \mathrm{M}_5 \\ \mathrm{M}_6 \\ \mathrm{M}_7 \\ \mathrm{M}_8 \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{N}_1 & \mathcal{N}_2 & \mathcal{N}_3 & \mathcal{N}_4 \end{array} \\ \begin{bmatrix} 0.6460 & 0.6345 & 0.6356 & 0.6508 \\ 0.6255 & 0.6221 & 0.6216 & 0.6377 \\ 0.6333 & 0.6424 & 0.6410 & 0.6531 \\ 0.6376 & 0.6255 & 0.6254 & 0.6202 \\ 0.6250 & 0.6429 & 0.6265 & 0.6534 \\ 0.6330 & 0.6303 & 0.6448 & 0.6182 \\ 0.6384 & 0.6077 & 0.6485 & 0.6078 \\ 0.6288 & 0.6308 & 0.6326 & 0.6341 \end{bmatrix} \end{array} \quad \wp^4 = \begin{array}{c} \\ \mathrm{M}_1 \\ \mathrm{M}_2 \\ \mathrm{M}_3 \\ \mathrm{M}_4 \\ \mathrm{M}_5 \\ \mathrm{M}_6 \\ \mathrm{M}_7 \\ \mathrm{M}_8 \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{N}_1 & \mathcal{N}_2 & \mathcal{N}_3 & \mathcal{N}_4 \end{array} \\ \begin{bmatrix} 0.1018 & 0.1324 & 0.1358 & 0.0999 \\ 0.1246 & 0.1342 & 0.1340 & 0.1201 \\ 0.1167 & 0.1189 & 0.1188 & 0.1320 \\ 0.1225 & 0.1285 & 0.1221 & 0.1214 \\ 0.1441 & 0.1033 & 0.1288 & 0.1245 \\ 0.1266 & 0.1321 & 0.1124 & 0.1183 \\ 0.1159 & 0.1511 & 0.1214 & 0.1282 \\ 0.1276 & 0.1173 & 0.0913 & 0.1407 \end{bmatrix} \end{array}$$

Step 6. Construct the comprehensive PLq-ROF decision matrix of the individual decision matrices with the help of PLq-ROFWPA operator (Table 6).

Step 7. Calculate the best and the worst amounts of each attribute according to the Eq. (24) as all of the attributes are of benefit type (Table 7).

Step 8. Estimate the $\mathcal{K}_\iota$ and $\mathcal{L}_\iota$ through Eq. (26) (Table 8):

Step 9. Determine the VIKOR index for each alternative according to Eq. (28):

$$O_1 = 0.2492, \quad O_2 = 0.8275, \quad O_3 = 0.0000, \quad O_4 = 0.9475,$$
$$O_5 = 0.1530, \quad O_6 = 0.8475, \quad O_7 = 0.9970, \quad O_8 = 0.1593.$$

Step 10. Condition 1: Acceptable advantage

$$O(\mathrm{M}_8) - O(\mathrm{M}_3) = 0.1593 - 0.0000 \geq 1/(8-1) = 0.1428$$

Table 6
Comprehensive PLq-ROF decision matrix by the PLq-ROFWPA operator.

Alternatives	Attributes
	$\mathcal{N}_1$
M_1	$(\{ƀ_{-0.1513}(0.1000), ƀ_{1.2487}(0.4000), ƀ_{4.0000}(0.5000)\}, \{ƀ_{-4.0000}(0.4000), ƀ_{-3.0000}(0.2000), ƀ_{-1.9057}(0.4000)\})$
M_2	$(\{ƀ_{-2.0653}(0.3000), ƀ_{-1.0960}(0.3000), ƀ_{1.1815}(0.4000)\}, \{ƀ_{-2.6840}(0.1000), ƀ_{-1.5515}(0.1000), ƀ_{-0.5095}(0.8000)\})$
M_3	$(\{ƀ_{0.4940}(0.3000), ƀ_{1.4489}(0.6000), ƀ_{2.5616}(0.1000)\}, \{ƀ_{-4.0000}(0.2000), ƀ_{-2.8109}(0.4000), ƀ_{-1.4838}(0.4000)\})$
M_4	$(\{ƀ_{-2.5997}(0.3000), ƀ_{-0.7869}(0.2000), ƀ_{1.2041}(0.5000)\}, \{ƀ_{-4.0000}(0.4000), ƀ_{-2.3065}(0.4000), ƀ_{-1.2781}(0.2000)\})$
M_5	$(\{ƀ_{-0.0349}(0.6000), ƀ_{1.4676}(0.1000), ƀ_{2.4921}(0.3000)\}, \{ƀ_{-4.0000}(0.5000), ƀ_{-2.1714}(0.4000), ƀ_{-0.9804}(0.1000)\})$
M_6	$(\{ƀ_{-2.1235}(0.2000), ƀ_{-1.1555}(0.4000), ƀ_{1.0000}(0.4000)\}, \{ƀ_{-4.0000}(0.2000), ƀ_{-2.3070}(0.4000), ƀ_{-1.2786}(0.4000)\})$
M_7	$(\{ƀ_{-2.5811}(0.3000), ƀ_{-1.8212}(0.3000), ƀ_{0.4892}(0.4000)\}, \{ƀ_{-4.0000}(0.6000), ƀ_{-0.1044}(0.2000), ƀ_{2.3066}(0.2000)\})$
M_8	$(\{ƀ_{0.5348}(0.1000), ƀ_{1.7200}(0.2000), ƀ_{4.0000}(0.7000)\}, \{ƀ_{4.0000}(0.1000), ƀ_{2.7431}(0.1000), ƀ_{0.9719}(0.8000)\})$
	$\mathcal{N}_2$
M_1	$(\{ƀ_{-1.2721}(0.5000), ƀ_{0.8405}(0.3000), ƀ_{1.8327}(0.2000)\}, \{ƀ_{-4.0000}(0.1000), ƀ_{-2.0769}(0.2000), ƀ_{-0.8021}(0.7000)\})$
M_2	$(\{ƀ_{-2.6897}(0.3000), ƀ_{-0.8329}(0.5000), ƀ_{0.2891}(0.2000)\}, \{ƀ_{-4.0000}(0.4000), ƀ_{-2.6422}(0.4000), ƀ_{-1.5572}(0.2000)\})$
M_3	$(\{ƀ_{-1.8570}(0.1000), ƀ_{1.3158}(0.2000), ƀ_{2.3366}(0.7000)\}, \{ƀ_{-4.0000}(0.1000), ƀ_{-1.3673}(0.1000), ƀ_{0.6835}(0.8000)\})$
M_4	$(\{ƀ_{-2.8285}(0.3000), ƀ_{-1.0113}(0.3000), ƀ_{0.4060}(0.4000)\}, \{ƀ_{-2.6343}(0.3000), ƀ_{-1.3677}(0.4000), ƀ_{-0.2070}(0.3000)\})$
M_5	$(\{ƀ_{-0.1542}(0.5000), ƀ_{1.2398}(0.2000), ƀ_{2.2617}(0.3000)\}, \{ƀ_{-4.0000}(0.1000), ƀ_{-1.5837}(0.2000), ƀ_{0.4924}(0.7000)\})$
M_6	$(\{ƀ_{-2.7612}(0.3000), ƀ_{-0.8273}(0.3000), ƀ_{0.3566}(0.4000)\}, \{ƀ_{-4.0000}(0.2000), ƀ_{-1.8097}(0.3000), ƀ_{-0.2406}(0.5000)\})$
M_7	$(\{ƀ_{-2.0982}(0.1000), ƀ_{-0.0637}(0.4000), ƀ_{1.4220}(0.5000)\}, \{ƀ_{-2.6875}(0.3000), ƀ_{-1.6551}(0.1000), ƀ_{-0.1418}(0.6000)\})$
M_8	$(\{ƀ_{-1.8102}(0.3000), ƀ_{0.7219}(0.3000), ƀ_{1.8242}(0.4000)\}, \{ƀ_{-4.0000}(0.2000), ƀ_{-2.5830}(0.6000), ƀ_{0.7092}(0.2000)\})$
	$\mathcal{N}_3$
M_1	$(\{ƀ_{-3.3929}(0.1000), ƀ_{-0.9035}(0.4000), ƀ_{1.0513}(0.5000)\}, \{ƀ_{-4.0000}(0.1000), ƀ_{-1.4106}(0.2000), ƀ_{0.4994}(0.7000)\})$
M_2	$(\{ƀ_{-0.3062}(0.4000), ƀ_{0.5924}(0.3000), ƀ_{3.0000}(0.3000)\}, \{ƀ_{-4.0000}(0.3000), ƀ_{-2.7996}(0.3000), ƀ_{-1.7745}(0.4000)\})$
M_3	$(\{ƀ_{-2.5206}(0.1000), ƀ_{0.7171}(0.1000), ƀ_{1.8107}(0.8000)\}, \{ƀ_{-2.0000}(0.4000), ƀ_{-0.7882}(0.4000), ƀ_{1.1233}(0.2000)\})$
M_4	$(\{ƀ_{-3.3900}(0.7000), ƀ_{-2.6752}(0.2000), ƀ_{0.5175}(0.1000)\}, \{ƀ_{-3.0000}(0.8000), ƀ_{-2.0000}(0.1000), ƀ_{0.6279}(0.1000)\})$
M_5	$(\{ƀ_{-2.0000}(0.4000), ƀ_{-1.0000}(0.4000), ƀ_{2.0385}(0.2000)\}, \{ƀ_{-4.0000}(0.2000), ƀ_{-2.6013}(0.4000), ƀ_{-1.4370}(0.4000)\})$
M_6	$(\{ƀ_{0.5131}(0.1000), ƀ_{1.5521}(0.1000), ƀ_{2.5815}(0.8000)\}, \{ƀ_{-2.4365}(0.5000), ƀ_{-1.4024}(0.3000), ƀ_{0.6906}(0.2000)\})$
M_7	$(\{ƀ_{-1.0619}(0.2000), ƀ_{-0.0926}(0.4000), ƀ_{0.9436}(0.4000)\}, \{ƀ_{-4.0000}(0.4000), ƀ_{-1.9013}(0.4000), ƀ_{3.0541}(0.2000)\})$
M_8	$(\{ƀ_{-0.7881}(0.4000), ƀ_{0.9628}(0.3000), ƀ_{4.0000}(0.3000)\}, \{ƀ_{-1.6827}(0.1000), ƀ_{-0.0861}(0.6000), ƀ_{1.2140}(0.3000)\})$
	$\mathcal{N}_4$
M_1	$(\{ƀ_{0.7259}(0.2000), ƀ_{1.8774}(0.2000), ƀ_{2.9064}(0.6000)\}, \{ƀ_{-4.0000}(0.4000), ƀ_{-0.0511}(0.5000), ƀ_{1.1929}(0.1000)\})$
M_2	$(\{ƀ_{-3.2907}(0.5000), ƀ_{0.8579}(0.3000), ƀ_{1.8511}(0.2000)\}, \{ƀ_{-4.0000}(0.3000), ƀ_{-2.1598}(0.2000), ƀ_{-1.1426}(0.5000)\})$
M_3	$(\{ƀ_{-1.1398}(0.3000), ƀ_{0.0178}(0.6000), ƀ_{1.3877}(0.1000)\}, \{ƀ_{-4.0000}(0.2000), ƀ_{-1.4419}(0.4000), ƀ_{0.1955}(0.4000)\})$
M_4	$(\{ƀ_{0.6975}(0.5000), ƀ_{2.0382}(0.2000), ƀ_{4.0000}(0.3000)\}, \{ƀ_{-4.0000}(0.4000), ƀ_{-0.3453}(0.1000), ƀ_{0.8023}(0.5000)\})$
M_5	$(\{ƀ_{-1.0837}(0.4000), ƀ_{0.8500}(0.4000), ƀ_{1.9026}(0.2000)\}, \{ƀ_{-2.7981}(0.3000), ƀ_{-1.4332}(0.3000), ƀ_{1.1983}(0.4000)\})$
M_6	$(\{ƀ_{-0.8834}(0.1000), ƀ_{-0.0968}(0.1000), ƀ_{1.0412}(0.8000)\}, \{ƀ_{-1.6326}(0.5000), ƀ_{0.1992}(0.2000), ƀ_{1.3986}(0.3000)\})$
M_7	$(\{ƀ_{0.6652}(0.2000), ƀ_{1.6017}(0.4000), ƀ_{2.6283}(0.4000)\}, \{ƀ_{-4.0000}(0.1000), ƀ_{-2.7992}(0.1000), ƀ_{-1.1901}(0.8000)\})$
M_8	$(\{ƀ_{-1.9128}(0.3000), ƀ_{-0.0976}(0.2000), ƀ_{0.7957}(0.5000)\}, \{ƀ_{-4.0000}(0.2000), ƀ_{-2.2891}(0.6000), ƀ_{-1.2618}(0.2000)\})$

Table 7
The best amount and the worst amount of each attribute.

Attributes	Best amount
$\mathcal{N}_1$	$(\{ƀ_{0.5348}(0.1000), ƀ_{1.7200}(0.2000), ƀ_{4.0000}(0.7000)\}, \{ƀ_{-4.0000}(0.1000), ƀ_{-2.7431}(0.1000), ƀ_{-0.9719}(0.8000)\})$
$\mathcal{N}_2$	$(\{ƀ_{-1.8570}(0.1000), ƀ_{1.3158}(0.2000), ƀ_{2.3366}(0.7000)\}, \{ƀ_{-4.0000}(0.1000), ƀ_{-1.3673}(0.1000), ƀ_{0.6835}(0.8000)\})$
$\mathcal{N}_3$	$(\{ƀ_{0.5131}(0.1000), ƀ_{1.5521}(0.1000), ƀ_{2.5815}(0.8000)\}, \{ƀ_{-2.4365}(0.5000), ƀ_{-1.4024}(0.3000), ƀ_{0.6906}(0.2000)\})$
$\mathcal{N}_4$	$(\{ƀ_{0.7259}(0.2000), ƀ_{1.8774}(0.2000), ƀ_{2.9064}(0.6000)\}, \{ƀ_{-4.0000}(0.4000), ƀ_{-0.0511}(0.5000), ƀ_{1.1929}(0.1000)\})$
	Worst amount
$\mathcal{N}_1$	$(\{ƀ_{-2.0653}(0.3000), ƀ_{-1.0960}(0.3000), ƀ_{1.1815}(0.4000)\}, \{ƀ_{-2.6840}(0.1000), ƀ_{-1.5515}(0.1000), ƀ_{-0.5095}(0.8000)\})$
$\mathcal{N}_2$	$(\{ƀ_{-0.1542}(0.5000), ƀ_{1.2398}(0.2000), ƀ_{2.2617}(0.3000)\}, \{ƀ_{-4.0000}(0.1000), ƀ_{-1.5837}(0.2000), ƀ_{0.4924}(0.7000)\})$
$\mathcal{N}_3$	$(\{ƀ_{-3.3929}(0.1000), ƀ_{-0.9035}(0.4000), ƀ_{1.0513}(0.5000)\}, \{ƀ_{-4.0000}(0.1000), ƀ_{-1.4106}(0.2000), ƀ_{0.4994}(0.7000)\})$
$\mathcal{N}_4$	$(\{ƀ_{-1.0837}(0.4000), ƀ_{0.8500}(0.4000), ƀ_{1.9026}(0.2000)\}, \{ƀ_{-2.7981}(0.3000), ƀ_{-1.4332}(0.3000), ƀ_{1.1983}(0.4000)\})$

Table 8
The values of $\mathcal{K}_\iota$ and $\mathcal{L}_\iota$ of alternatives.

	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8
$\mathcal{K}_\iota$	−2.2226	2.2872	−4.6524	2.8507	−3.5196	2.4646	3.7316	−2.8504
Ranking of $\mathcal{K}_\iota$	$M_3 \succ M_5 \succ M_8 \succ M_1 \succ M_2 \succ M_6 \succ M_4 \succ M_7$							
$\mathcal{L}_\iota$	0.4222	2.5298	−0.2884	3.1183	0.2940	2.5939	3.0978	−0.2079
Ranking of $\mathcal{L}_\iota$	$M_3 \succ M_8 \succ M_5 \succ M_1 \succ M_2 \succ M_6 \succ M_7 \succ M_4$							

Table 9
Parameter analysis with the parameter q by the PLq-ROFWPA-VIKOR method.

Parameters	Scores	Ranking
$q = 2$	0.2426 0.8246 0.0000 0.9432 0.1231 0.8521 0.9970 0.1067	$M_3 \succ M_8 \succ M_5 \succ M_1 \succ M_2 \succ M_6 \succ M_4 \succ M_7$
$q = 4$	0.2492 0.8275 0.0000 0.9475 0.1530 0.8475 0.9970 0.1193	$M_3 \succ M_8 \succ M_5 \succ M_1 \succ M_2 \succ M_6 \succ M_4 \succ M_7$
$q = 6$	0.2783 0.8338 0.0057 0.9581 0.1894 0.8477 0.9970 0.1597	$M_3 \succ M_8 \succ M_5 \succ M_1 \succ M_2 \succ M_6 \succ M_4 \succ M_7$
$q = 8$	0.3113 0.8323 0.0064 0.9653 0.2392 0.8363 1.0000 0.2337	$M_3 \succ M_8 \succ M_5 \succ M_1 \succ M_2 \succ M_6 \succ M_4 \succ M_7$
$q = 10$	0.3397 0.7971 0.0057 0.9369 0.3006 0.7861 0.9957 0.3215	$M_3 \succ M_8 \succ M_5 \succ M_1 \succ M_2 \succ M_6 \succ M_4 \succ M_7$
$q = 11$	0.3482 0.7708 0.0054 0.9138 0.3310 0.7516 0.9808 0.3587	$M_3 \succ M_8 \succ M_5 \succ M_1 \succ M_2 \succ M_6 \succ M_4 \succ M_7$
$q = 13$	0.3611 0.7398 0.0048 0.8712 0.3918 0.7113 0.9523 0.4241	$M_3 \succ M_8 \succ M_5 \succ M_1 \succ M_2 \succ M_6 \succ M_4 \succ M_7$
$q = 17$	0.3632 0.7099 0.0039 0.7927 0.4855 0.6553 0.9058 0.5000	$M_3 \succ M_8 \succ M_5 \succ M_1 \succ M_2 \succ M_6 \succ M_4 \succ M_7$
$q = 19$	0.3473 0.6878 0.0035 0.7435 0.5014 0.6263 0.8793 0.5000	$M_3 \succ M_8 \succ M_5 \succ M_1 \succ M_2 \succ M_6 \succ M_4 \succ M_7$
$q = 20$	0.3405 0.6806 0.0033 0.7239 0.5089 0.6161 0.8709 0.5000	$M_3 \succ M_8 \succ M_5 \succ M_1 \succ M_2 \succ M_6 \succ M_4 \succ M_7$
$q = 22$	0.3289 0.6716 0.0029 0.7044 0.5226 0.6015 0.8604 0.5000	$M_3 \succ M_8 \succ M_5 \succ M_1 \succ M_2 \succ M_6 \succ M_4 \succ M_7$
$q = 24$	0.3191 0.6669 0.0026 0.7008 0.5348 0.5921 0.8555 0.5000	$M_3 \succ M_8 \succ M_5 \succ M_1 \succ M_2 \succ M_6 \succ M_4 \succ M_7$
$q = 26$	0.3037 0.6558 0.0023 0.6876 0.5325 0.5793 0.8446 0.4871	$M_3 \succ M_8 \succ M_5 \succ M_1 \succ M_2 \succ M_6 \succ M_4 \succ M_7$

Condition 2: Acceptable stability in DM According to $\mathcal{K}$ and $\mathcal{L}$, M_3 is likewise the most suitable alternative.
Step 11. Hence the ranking of alternatives is:

$$M_3 \succ M_8 \succ M_5 \succ M_1 \succ M_2 \succ M_6 \succ M_4 \succ M_7$$

6.2. Parameter analysis

The PLq-ROFWPA operator is a mathematical tool used in DM and aggregation processes within a probabilistic linguistic framework. It expands upon the classical ordered weighted average operator to handle uncertainty and linguistic preferences. The parameter q plays a crucial role in shaping the behavior and characteristics of the aggregation process. Different values of q enable decision-makers to adjust the process based on their preferences and the relative importance they assign to extreme values and dominant LTs. When we assign different values to parameter q based on the PLq-ROFWPA operator, then the ranking order remains the same for all values of q, that is, $M_3 \succ M_8 \succ M_5 \succ M_1 \succ M_2 \succ M_6 \succ M_4 \succ M_7$. Thus, M_3 consistently identifies as the best alternative, which indicates that M_3 is the preferred choice regardless of the specific q value. This consistency suggests that M_3 possesses certain attributes or characteristics that consistently make it the most desirable alternative, according to the decision-maker's preferences and the linguistic information provided. The PLq-ROFWPA-VIKOR method allows decision-makers to incorporate linguistic information, probabilities, and weights into their DM process. Despite variations in the q parameter, the operator consistently recognizes M_3 as the optimal choice. This implies that M_3 consistently demonstrates strong performance or possesses attributes that align closely with the decision-maker's preferences, as expressed through the linguistic information and weights. The stability of M_3 as the best alternative across different values of q indicates its robust selection. It suggests that M_3 consistently stands out among the alternatives, exhibiting favorable characteristics that make it the most preferable option. Therefore, if M_3 is consistently chosen as the best alternative by the PLq-ROFWPA-VIKOR method for all the given q values, it signifies a strong preference for M_3 across various parameter settings and reinforces its status as the preferred choice in the DM process. Table 9 illustrates the ordering of alternatives according to the parameter q by the PLq-ROFWPA-VIKOR method, providing a clear representation of how different q values impact the selection of alternatives. Fig. 2 illustrates the parameter analysis of the proposed approach. At the end, we come to this point that the application of the PLq-ROFWPA operator and the PLq-ROFWPA-VIKOR method in DM processes within a probabilistic linguistic framework demonstrates a remarkable consistency in the selection of M_3 as the preferred alternative, regardless of the chosen parameter values. This unwavering preference underscores the method's ability to accommodate diverse linguistic information, probabilities, and weights, while maintaining the robustness and stability of the DM process. M_3 consistently exhibits attributes that strongly align with the decision-maker's preferences, reinforcing its status as the optimal choice. Overall, this approach proves highly effective in managing uncertainty and accommodating linguistic preferences, making it a valuable tool for complex decision scenarios where maintaining a consistent and preferred choice is outstanding.

6.3. Comparative analysis

A comparative analysis was conducted. To evaluate the VIKOR framework for model selection, we compared it to established methods such as probabilistic linguistic MABAC (PL-MABAC) [43] for preference aggregation, probabilistic linguistic CODAS (PL-CODAS) [7] for consensus building, PLq-ROF-TOPSIS [23], PLq-ROF power Archimedean weighted average (PLq-ROFAWA) AO [31], and Fermatean fuzzy-VIKOR (FF-VIKOR) method [13] for health care waste treatment technologies. This thorough comparison serves two purposes: first, it evaluates the usefulness of the VIKOR method in model selection, and second, it compares its performance against established methods. By evaluating numerous approaches, we gain a more comprehensive knowledge of each method's strengths and limitations, ultimately demonstrating the clear advantages of our proposed approach. This not only enhances knowledge in model selection but also enables better DM in real-world applications.

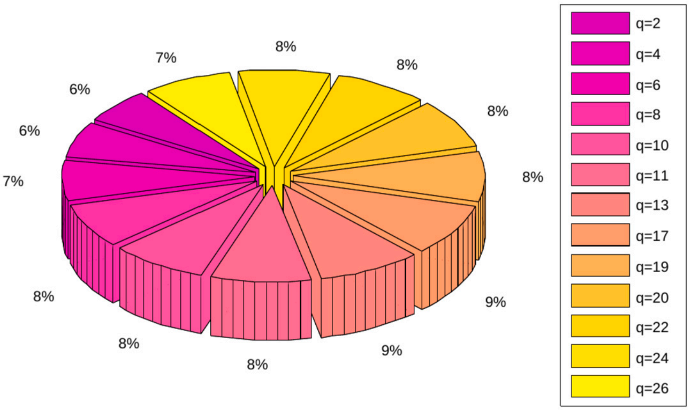

Fig. 2. Parameter analysis of the PLqROFWPA-VIKOR methodology.

Table 10
Comparison analysis of the proposed PLq-ROF-VIKOR method with the PL-MABAC method.

PLq-ROF-VIKOR method				
VIKOR index	0.2676	0.8502	0.6173	0.9159
	0.7501	0.8263	0.6049	0.0000
Ranking	$M_8 \succ M_1 \succ M_7 \succ M_3 \succ M_5 \succ M_6 \succ M_2 \succ M_4$			
PL-MABAC method [43]				
Sum	0.6360	−0.4774	1.1661	−0.3512
	0.2887	1.0411	−0.0224	0.4179
Ranking	$M_3 \succ M_6 \succ M_1 \succ M_8 \succ M_5 \succ M_7 \succ M_4 \succ M_2$			

6.3.1. Comparison analysis of the PLq-ROF-VIKOR method with the PL-MABAC method

MABAC and VIKOR are both methods used for making decisions when there are multiple criteria to consider, but they have different strengths. MABAC is particularly good at dealing with situations where there is uncertainty. It also has the advantage of being transparent, meaning it is clear how the decision was made without needing to assign specific weights to the criteria. On the other hand, VIKOR is known for its ability to find a single solution that balances all the different criteria, even when they conflict with each other. It is really good at handling situations where we need to make trade-offs between different criteria to reach a compromise. To choose the best branch of astronomy based on observational, interdisciplinary, exploratory, and inspirational criteria, the VIKOR method appears to be a suitable choice. VIKOR is renowned for its efficiency in finding compromise solutions and achieving a balance among multiple criteria. In this context, where we want to take various factors into account, VIKOR can assist us in pinpointing the branch of astronomy that performs well across these criteria in a balanced manner. It is particularly useful in MADM situations, especially when we need to reconcile conflicting objectives or criteria, which appears to align with our DM challenge. When we compare the proposed PLq-ROF-VIKOR method with the PL-MABAC method, the resulting rankings are as follows: $M_8 \succ M_1 \succ M_7 \succ M_3 \succ M_5 \succ M_6 \succ M_2 \succ M_4$ and $M_3 \succ M_6 \succ M_1 \succ M_8 \succ M_5 \succ M_7 \succ M_4 \succ M_2$. Hence, the alternatives M_8 and M_3 are the most favorable choices among the available options. To further illustrate this comparison, a detailed comparative analysis can be found in Table 10. Additionally, this information is visually presented in Fig. 3, offering a graphical representation of how these two DM methods compare with each other.

6.3.2. Comparison analysis of the PLq-ROF-VIKOR method with the PL-CODAS method

CODAS and VIKOR are both techniques for making decisions when we have multiple factors to consider. They each have their own unique advantages. CODAS is great when we need a thorough evaluation of different options, especially when these options are related in complex ways. It uses distance measurements to help us compare alternatives, which can be really helpful in tricky decision scenarios. VIKOR, on the other hand, is excellent at finding compromise solutions quickly. This makes it a good choice when we want to reach a balanced decision, especially when there are multiple conflicting criteria. However, it is crucial to understand that VIKOR and CODAS are not directly comparable because they have different strengths and applications. To say that VIKOR is better than CODAS, we need to assess their performance in a specific situation, taking into account factors like the quality of the data, the criteria, and the preferences of the decision-makers involved. Each method has its own strengths and should be chosen based on the specific needs of the decision we are trying to make. VIKOR's ability to strike a balance between conflicting criteria and offer a single ranking for alternatives makes it a preferred choice in many applications, demonstrating that it is often a better option than CODAS for decision-makers seeking a practical and well-rounded DM approach. When we compare the proposed PLq-ROF-VIKOR method with the PL-CODAS method, the resulting rankings are: $M_8 \succ M_1 \succ M_7 \succ M_3 \succ M_5 \succ M_6 \succ M_2 \succ M_4$ and

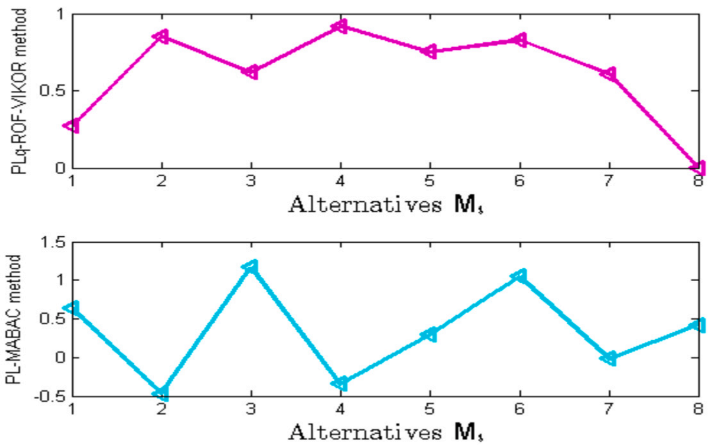

Fig. 3. Graphical representation of Table 10.

Table 11
Comparison analysis of the proposed PLq-ROF-VIKOR method with the PL-CODAS method.

PLq-ROF-VIKOR method				
VIKOR index	0.2676	0.8502	0.6173	0.9159
	0.7501	0.8263	0.6049	0.0000
Ranking	$M_8 \succ M_1 \succ M_7 \succ M_3 \succ M_5 \succ M_6 \succ M_2 \succ M_4$			
PL-CODAS method [7]				
Average solution	−5.8359	18.6321	−16.6665	12.8497
	2.0653	−10.3488	3.1834	−3.8794
Ranking	$M_2 \succ M_4 \succ M_7 \succ M_5 \succ M_8 \succ M_1 \succ M_6 \succ M_3$			

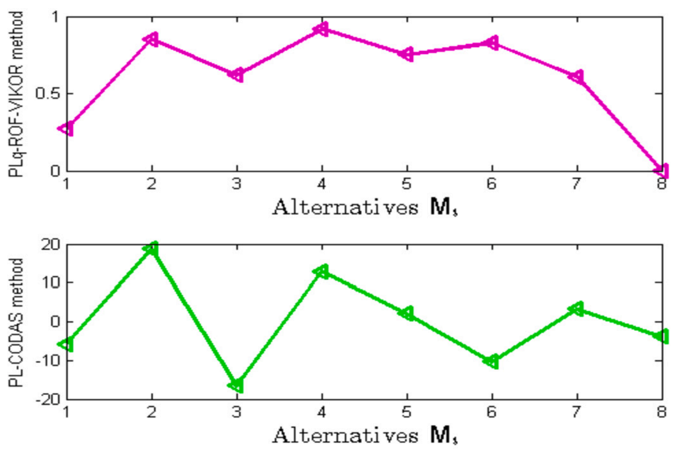

Fig. 4. Graphical representation of Table 11.

$M_2 \succ M_4 \succ M_7 \succ M_5 \succ M_8 \succ M_1 \succ M_6 \succ M_3$, and alternatives M_2 and M_8 are the most favorable choice among the available options, respectively. To further illustrate this comparison, a detailed comparative analysis can be found in Table 11. Additionally, this information is visually presented in Fig. 4, offering a graphical representation of how these two DM methods compare with each other.

6.3.3. Comparison analysis of the PLq-ROF-VIKOR method with the PLq-ROF-TOPSIS method

TOPSIS and VIKOR are both methods for making decisions when we have multiple factors to consider. Each has its own strengths. TOPSIS is known for being straightforward and easy to use. It ranks options by measuring how close they are to the best and worst possible outcomes. VIKOR, on the other hand, is often seen as more versatile. It not only ranks choices but also finds a compromise solution that balances the best and worst outcomes. This is handy when you need to make a decision that involves some level of

Table 12
Comparison analysis of the proposed PLq-ROF-VIKOR method with the PLq-ROF-TOPSIS method.

PLq-ROF-VIKOR method				
VIKOR index	0.3663	0.9183	0.0247	0.3992
	0.8215	0.1884	0.5784	0.0617
Ranking	$M_3 \succ M_8 \succ M_6 \succ M_1 \succ M_4 \succ M_7 \succ M_5 \succ M_2$			
PLq-ROF-TOPSIS method [23]				
Closeness index	−0.2570	−0.1089	−0.1486	−0.1420
	−0.1579	−0.1580	−0.1748	−0.0471
Ranking	$M_8 \succ M_2 \succ M_4 \succ M_3 \succ M_5 \succ M_6 \succ M_7 \succ M_1$			

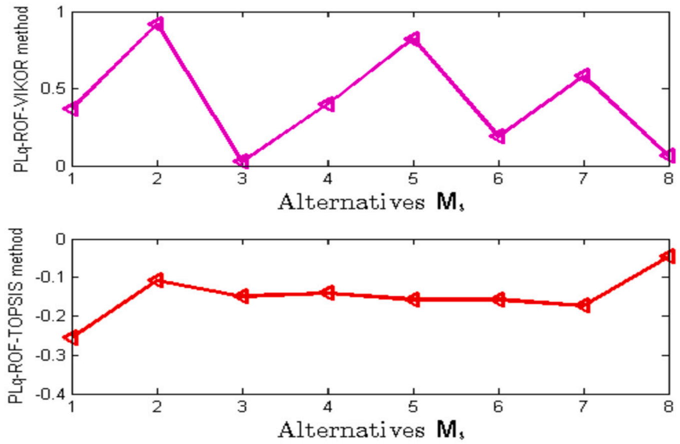

Fig. 5. Graphical representation of Table 12.

compromise. VIKOR has a few extra benefits, as it can deal with uncertainty and sensitivity analysis, which makes it more suitable for complex or uncertain decisions, and it provides a more comprehensive DM framework, making it a better choice in many cases. Hence, VIKOR is preferred over TOPSIS because it can handle situations where we need a compromise and offers a more robust approach, especially in complex and uncertain DM scenarios. When we compare the proposed PLq-ROF-VIKOR method with the PLq-ROF-TOPSIS method, the resulting rankings are: $M_3 \succ M_6 \succ M_1 \succ M_8 \succ M_5 \succ M_7 \succ M_4 \succ M_2$ and $M_8 \succ M_1 \succ M_4 \succ M_5 \succ M_3 \succ M_2 \succ M_7 \succ M_6$ and alternatives M_3 and M_8 are the most favorable choice among the available options, respectively. To further illustrate this comparison, a detailed comparative analysis can be found in Table 12. Additionally, this information is visually presented in Fig. 5, offering a graphical representation of how these two DM methods compare with each other.

6.3.4. *Comparison analysis of the PLq-ROF-VIKOR method with PLq-ROFAWA operator*

Both Archimedean AO and VIKOR have their strengths, but VIKOR is often seen as the better choice in many DM situations. While Archimedean operators help combine preferences and manage compromises, VIKOR offers several key advantages. VIKOR excels at finding balanced solutions when criteria conflict and provides a clear ranking of options. It is especially useful when there is uncertainty or a need to juggle multiple goals. Moreover, VIKOR's simplicity and transparency make it easier for decision-makers to grasp and use. In general, VIKOR is preferred over Archimedean operators because it is more robust, flexible, and effective at handling complex multi-criteria decisions. When we compare the proposed PLq-ROF-VIKOR method with the PLq-ROFAWA AO, the resulting rankings are: $M_3 \succ M_6 \succ M_1 \succ M_8 \succ M_5 \succ M_7 \succ M_4 \succ M_2$ and $M_7 \succ M_2 \succ M_3 \succ M_1 \succ M_8 \succ M_5 \succ M_6 \succ M_4$ and alternatives M_3 and M_7 are the most favorable choice among the available options, respectively. To further illustrate this comparison, a detailed comparative analysis can be found in Table 13. Additionally, this information is visually presented in Fig. 6, offering a graphical representation of how these two DM methods compare with each other.

6.3.5. *Comparison analysis of the PLq-ROF-VIKOR method with the FF-VIKOR method*

In the realm of FSs, they take distinct approaches. The PLq-ROFS enhances the concept of orthopair FSs by incorporating a parameter q that controls the degree of orthopairness, providing a flexible framework for capturing nuanced uncertainties. On the other hand, the Fermatean FS leverages the Fermat average and optimization principles to determine membership degrees, aiming to model gradual transitions more effectively than traditional FSs. While both approaches extend FS theory, they do so in distinct ways, offering researchers versatile tools for addressing complex uncertainties in various applications. Choosing the right one depends on our needs: PLq-ROFS shines when uncertainty fluctuates and needs quantification, while Fermatean FS excels when independent truth and falsity representations are most important. Ultimately, understanding their strengths and limitations guides us towards the most

Table 13
Comparison analysis of the proposed PLq-ROF-VIKOR method with the PLq-ROF Archimedean AO.

PLq-ROF-VIKOR method				
VIKOR index	0.3663	0.9183	0.0247	0.3992
	0.8215	0.1884	0.5784	0.0617
Ranking	$M_3 \succ M_8 \succ M_6 \succ M_1 \succ M_4 \succ M_7 \succ M_5 \succ M_2$			
PLq-ROFAWA AO [31]				
Scores	−5.3203	−5.7456	−5.5014	−2.1546
	−4.4205	−2.3478	−6.3722	−5.3149
Ranking	$M_7 \succ M_2 \succ M_3 \succ M_1 \succ M_8 \succ M_5 \succ M_6 \succ M_4$			

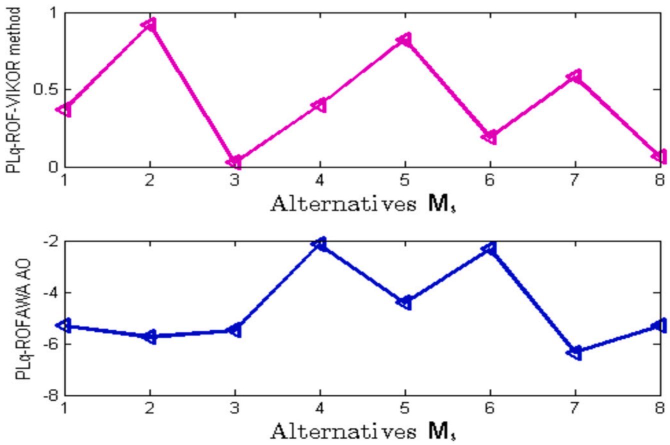

Fig. 6. Graphical representation of Table 13.

Table 14
Comparison analysis of the proposed PLq-ROF-VIKOR method with the FF-VIKOR method.

PLq-ROF-VIKOR method				
VIKOR index	0.4202	0.9288	0.0426	0.4631
	0.8416	0.2247	0.5964	0.1446
Ranking	$M_3 \succ M_8 \succ M_6 \succ M_1 \succ M_4 \succ M_7 \succ M_5 \succ M_2$			
FF-VIKOR method [13]				
VIKOR index	0.8354	0.4854	0.6446	0.5777
	0.0000	0.9316	1.0000	0.2862
Ranking	$M_5 \succ M_8 \succ M_2 \succ M_4 \succ M_3 \succ M_1 \succ M_6 \succ M_7$			

effective tool for navigating the world of fuzzy uncertainties. When we compare the proposed PLq-ROF-VIKOR method with the FF-VIKOR method the resulting rankings are: $M_3 \succ M_8 \succ M_6 \succ M_1 \succ M_4 \succ M_7 \succ M_5 \succ M_2$ and $M_5 \succ M_8 \succ M_2 \succ M_4 \succ M_3 \succ M_1 \succ M_6 \succ M_7$ and alternatives M_3 and M_5 are the most favorable choice among the available options, respectively. To further illustrate this comparison, a detailed comparative analysis can be found in Table 14. Additionally, this information is visually presented in Fig. 7, offering a graphical representation of how these two DM methods compare with each other.

7. Conclusions

The scientific study of the stars, planets, and other celestial bodies that make up the sky is known as astronomy. One of the first fields of science, it has roots in prehistoric times, when people first began to study the movements of the sun, moon, and stars. In order to study the secrets of the cosmos, astronomy has developed over time, embracing new techniques and technology. Astronomers can now see and measure the characteristics of far-off objects, such as their size, shape, color, temperature, composition, and motion, using telescopes, satellites, probes, and spacecraft. Theoretical elements of comprehending the operation of the universe, including the formation and development of stars, galaxies, black holes, and the cosmos as a whole, are also included in astronomy. There are numerous chances for study and exploration in the interesting and exciting discipline of astronomy. The fascinating and

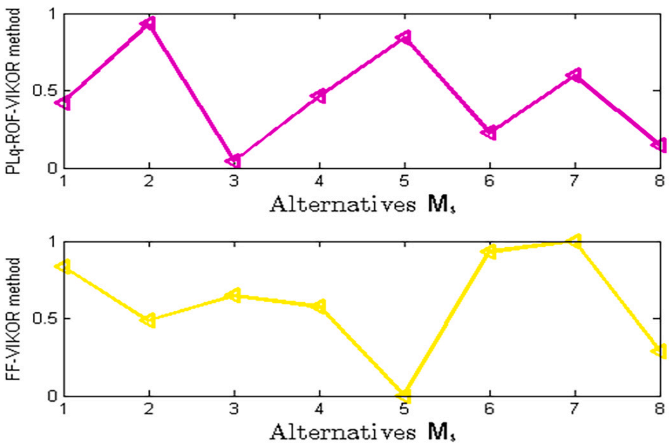

Fig. 7. Graphical representation of Table 14.

gratifying field of astronomy offers several employment options. When decision-makers consider astronomical objects, they aim to understand how different components of astronomical objects interact with each other and how changes in one component can effect other components. They also consider the values, preferences, and behaviors of individuals and communities that are effected by the decision. The criteria for contemporary astronomical objects are examined in this article within the context of fuzzy logic-based group DM. In this paper, we used the PLq-ROFS, which is regarded as the generalized form of PLTS and q-ROFS to address the uncertainty and imprecision associated with group DM problems. In order to aggregate PLq-ROF information, we presented the concepts of the PLq-ROFWA and the PLq-ROFWG operators, and some of their important properties such as idempotency, monotonicity, and boundedness. In particular, the PLq-ROFWA and the PLq-ROFWG operators can objectively deal with the MF and NMF of PLq-ROF information. Furthermore, we established the PLq-ROF-VIKOR model and provided a full explanation of the calculation steps for solving the real-world case study related to astronomy. The suggested model takes into consideration a compromise between individual regret reduction and collective utility maximization, which has been shown to be more accurate and useful. The most preferred alternative is then determined using a group DM process utilizing the PLq-ROF-VIKOR model. In the end, the results of the comparative analysis presented that the established approach can be successfully applied to address MADM problems in the PLq-ROF environment.

While our proposed model represents a significant advancement in both theoretical formulation and practical implementation, it has several inherent limitations. Firstly, the complexity of the proposed model entails the integration of multiple parameters, each contributing to the overall DM process. Variations in these parameters could potentially yield divergent results, thereby highlighting the sensitivity of the model to parameter adjustments. This aspect necessitates careful consideration and sensitivity analysis to assess the robustness and reliability of the model across different parameter configurations. Moreover, although our current methodology utilizes a general approach for determining attribute weights, there exists the opportunity to incorporate more sophisticated weighting methods. Implementing specific weighting techniques adapted to the unique characteristics of the decision context could enhance the precision and accuracy of the weight determination process. Exploring alternative weighting methodologies may further refine the model's performance and optimize decision outcomes. Furthermore, additional limitations may include the scalability of the proposed model to accommodate larger datasets and more complex decision scenarios. The computational demands associated with processing extensive datasets and executing intricate algorithms may pose practical constraints, potentially impacting the model's efficiency and scalability. Addressing these limitations requires a concerted effort to enhance the model's robustness, flexibility, and applicability in real-world DM contexts. Future research will focus on refining parametrization strategies, exploring advanced weighting methodologies, and optimizing computational algorithms to overcome existing constraints and propel the model towards greater efficacy. Moreover, future research will be extended further by using the presented AOs with (p, q) quasirung orthopair FS [36] and 3,4-quasirung FS [37].

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CRediT authorship contribution statement

Sumera Naz: Validation, Investigation, Formal analysis, Conceptualization. **Areej Fatima:** Writing – original draft. **Shariq Aziz But:** Writing – review & editing, Methodology, Conceptualization. **Dragan Pamucar:** Writing – review & editing, Funding acquisition.

Ronald Zamora-Musa: Writing – review & editing, Funding acquisition. **Melisa Acosta-Coll:** Writing – review & editing, Funding acquisition.

Declaration of competing interest

The authors declare no conflict of interest.

Data availability

Enquiries about data availability should be directed to the authors.

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