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Method Article

A mathematical programming formulation for long-term infrastructure investment planning in Small Island Developing States

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A B S T R A C T

Mixed-integer programming is a common method used in electricity generation and transmission optimization models. However, the size of the problem can result in extraordinarily long run times. Solve time also increases exponentially with the number of variables to optimize. There is therefore a constant trade-off between a realistic representation of the network and computational tractability. Additionally, actual data and publicly available, real-world application are scare. This is particularly true for Small Island Developing States. This paper bridges these gaps by describing a customized mathematical formulation for co-optimizing generation and transmission infrastructure investments. Data from the island of Jamaica and program scripts are available for reproduction. Key customizations to a mixed-integer programming model for long-term generation and transmission infrastructure investment planning include:

- Hours are treated as representative hour categories and multiplied by the number of hour types within a given period.
- Simulated construction is limited to every other year.
- While fossil fuel plants are treated as discrete variables, renewable energy plants are treated as continuous variables.

A R T I C L E I N F O

Method name: Mixed-integer programming for electricity generation and transmission capacity investments
Keywords: Small island developing states, Energy modeling, Generation capacity, Transmission capacity, Co-optimization
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Specifications table

Subject Area	Energy
More specific subject area	*Energy Modelling*
Method name	*Mixed-integer programming for electricity generation and transmission capacity investments*
Name and reference of original method	*Our approach to this problem is based on a combination of mathematical formulations for co-optimizing generation and transmission resources based on the following two papers:* 1 *Krishnan V, Ho J, Hobbs BF, Liu AL, McCalley JD, Shahidehpour M, et al. Co-optimization of electricity transmission and generation resources for planning and policy analysis: review of concepts and modeling approaches. Energy Syst 2016;7:297–332.* (a) *This paper provides the basic framework for the mathematical formulation used. However, the structure is general and lacked specificity of the actual use case.* 2 *Sparrow FT, Yu Z, Gotham DJ, Bowen BH, Nderitu G, Wang J, et al. A Multi-Regional Electricity Trade Study for Southern African Power Pool. vol. Vol.2, Proceedings of the American Power Conference: 1998, p. 715–20.* (a) *This paper was developed as a power pool model for several countries on the African continent and was not designed for use in a single country. However, it provided useful insights on how the general formulation above could be customized for use in the case of Jamaica.*
Resource availability	*Data and math programming script are available at:* https://github.com/AtkinsTPhD/GTEP_SIDS.git *The GAMS software can be downloaded at:* https://www.gams.com/

Method details

The general outline of mixed integer programming for generation and transmission investment planning can be found in [1,2]. In this paper, the emphasis is on the specific customizations used to operationalize these methods.

Program set-up

The program is implemented using the General Algebraic Modeling System (GAMS). While a "light" version of GAMS can be downloaded from the official GAMS website, the size of this problem will likely require the purchase of a GAMS license in order to utilize more advanced programming features and solve large, complex problems. Additionally, because large programs can take weeks to run, setting tolerance levels for the optimization problem can help to reduce run-time but still obtain useful results. Since the objective function used in this paper is in millions of dollars, an absolute gap of one thousand dollars and a relative gap of 0.01% are used.

Equations

The general idea is to satisfy future electricity demand by building new infrastructure at least cost in net present value (NPV) terms. Consequently, the objective function minimizes the operating and investment costs over the planning time horizon.

$$\min A \times \sum_{t}^{T} \left\{ \left[TOC_t + \sum_{c} \left(I_{c,\,t} \times \sum_{\tau \le t} x_{c,\tau} \right) + \sum_{k} \left(I_{k,t} \times \sum_{\tau \le t} w_{k,\tau} \right) \right] \right\} \div (1+r)^t \tag{1}$$

A represents the number years per period. Not only does pre-defining the number of years per period reflect cases where construction takes place in more than a year, but it also reduces the size of the problem. If seasonal adjustments are required, the objective function can also be preceded by

Notation

sets

l	transmission lines (directed by definition)
g	generation plants
n, z	nodes in network
h	hour types
$t,\ \tau$	time-period index

Subsets

e	existing generator (subset of g)
c	candidate generator (subset of g)
f	fossil fuel generator (subset of g)
i	renewable energy generators (subset of g)
j	existing transmission line (subset of l)
k	candidate transmission line (subset of l)

Parameters

B_l	susceptance of transmission line l (siemens)
r	discount rate $\in (0, 1)$ (fraction)
$y_{e,t}$	0 if existing generator is retired in period t, 1 otherwise (indicator)
$I_{c,\ t}$	annualized investment cost of candidate generation plant ($ millions)
$I_{k,t}$	annualized investment cost of candidate transmission lines ($ millions)
F_g	fixed operating and maintenance (O&M) cost of generator g ($ per year)
V_g	variable operating and maintenance (O&M) cost of generator g ($ per MWh)
ϕ_h	number of hours of hour type h (hours)
P_g^{MAX}	maximum generation capacity of generator g (MW)
λ_g	forced outage rate of generator g $\in [0, 1]$ (fraction)
$\psi_{g,h}$	unforced outage rate of generator g for hour type h $\in (0, 1)$ (fraction)
S_l^{MAX}	maximum power flow across line l (MW)
$D_{n,h,t}$	demand at node n for hour type h in year t (MW
K_t	infrastructure investment budget in USD millions in year t ($ millions)
$q_{g,h}$	availability factor of generator g in hour type $h \in (0, 1]$ (fraction)
q_g^{peak}	availability factor of generator during peak hours $\in (0, 1]$ (fraction)
α_t	peak demand in year t (MW)
R	reserve margin $\in (0, 1)$, i.e. share of installed capacity that must be available above peak demand (fraction)
Z	renewable energy (RE) target (fraction)
T	time period in which RE target becomes active (scalar)
A	number of years per period (scalar)
Q_i	maximum cumulative capacity expansion for renewable energy generators (scalar)

Infrastructure build variables

$x_{c,t}$	1 if candidate generator is built – these are binary variables for fossil fuel generators and continuous variables for renewable energy generators
$w_{k,t}$	1 if candidate transmission line is built – these are binary variables

Variables

$P_{g,h,t}$	Real power produced by generator g for hour type h in year t (MW)
$S_{l,n,z,h,t}$	Power flow across line l from node n to node z, for hour type h in year t. (This will be negative if flows is from z to n). (MW)
$\theta_{l,\ n,h,t}$	Bus voltage angle for line l at node n for hour type h in year t (Radians)
U_{t,l^c}	Slack variable for use with big “M” method (MW)

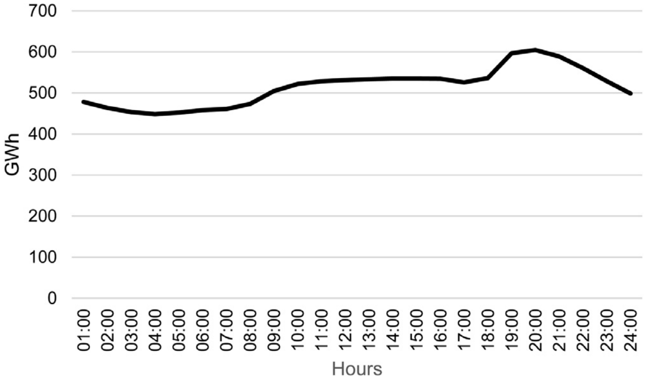

Fig. 1. Typical hourly load in Jamaica.

a multiple representing the number of seasons per period. TOC_t represents the total operating cost of a generator in period t as specified by

$$TOC_t = \left[\sum_{e} (F_e \times y_{e,t}) + \sum_{c} \left(F_c \times \sum_{\tau \le t} x_{c,\tau} \right) + \sum_{g,n,h} \left(V_g \times P_{g,h,t} \times \phi_h \right) \right] \div 1,000,000 \tag{2}$$

In (2), F_e and F_c represent fixed operating and maintenance costs for existing generators (e) and candidate generators (c). These are measured in dollars per year. The parameter $y_{e,t}$ takes a value of 1 if an existing unit is available in period t and a value of 0 if the unit has been retired by period t. In real-world applications, this is a useful way to account for plans that already exist to retire generation units. Generators are often described as "lumpy", meaning that they can typically only be built in fixed/discrete sizes. To capture this "lumpiness" in a model, candidate generators are treated as binary variables. The variable $x_{c,\tau}$ is a build variable for candidate generators. When treated as binary, $x_{c,\tau}$ takes a value of 1 when a generator is built in period t and 0 otherwise. However, in practice, it was found that it could take over a month to solve a single run of this model given the size of the problem. Consequently, only fossil fuel generators are treated as binary; renewable energy (RE) generators are treated as continuous variables. This was deemed reasonable because it not only reduced run-time, but RE generators are typically modular, allowing for marginal increases in capacity sizes. For instance, it would not be impractical to marginally increase the capacity of a solar plant by installing an additional solar panel. On the other hand, since natural gas plants are built in discrete sizes, one would not expect only an additional MW of capacity to be added at a given time. Hence, while the build variable for candidate generators is written as a single variable in (2) ($x_{c,\tau}$), in practice, this is coded as two separate variables; one binary and the other, continuous. Ultimately in the case of fossil fuel generators, $\sum_{\tau \le t} x_{c,\tau}$ represents whether or that generator has been built by period t. For RE units, $\sum_{\tau \le t} x_{c,\tau}$ represents how much capacity has been built by period t.

Continuing with (2), V_g denotes the variable operating and maintenance cost of generator g. (Recall that e and c are subsets of g.). V_g is measured in \$/MWh. Here, V_g represents all variable costs. That is, operation and maintenance costs as well as fuel costs. Fuel costs for fossil fuel generators are calculated as the product of the heat rate of the generator (mmBtu/MWh) and the forecasted fuel prices (\$/mmBtu). The parameter $P_{g,h,t}$ represents real power generation (MW) by generator g in hour type h in period t. Importantly, hours are not simulated individually. For a 24-hour period, hours are grouped into hour-categories based on similar load levels. Fig. 1 illustrates the typical hourly load for Jamaica. Based on the load curve, hours were grouped into five hour-categories: "off-peak night" (hr. 1–6), "off-peak day" (hr. 7–8), "average day" (hr. 9–18), "peak hour" (hr. 19–21) and "average night" (hr. 22–24). To obtain generation in MWh, $P_{g,h,t}$ is multiplied by the number of hours in each hour type ϕ_h. Categorization is necessary to make the program tractable and help balance the trade-off between accurate results and computation time such that meaningful results can be obtained while running the program on a reasonably equipped, standard computer. This is a fairly standard approach that has been used in a variety of other models, notably the MARKAL [3] and TIMES [4,5] models. To preserve diversity among load levels, the script accompanying this paper distinguishes hour categories/hour types by season, week day types (e.g. weekday or weekend) and hour grouping for the day (e.g. "peak hour", "off-peak day", etc.) The number of these instances of grouped hours is accounted for by the term A in Eq. (1). However, this significantly increases the notation and number of equations that would be required for use in this paper. Therefore, assume that each parameter and variable indexed by h is also indexed by seasons and day types. In the specific case used in the accompanying program file, "seasons" correspond with quarters based on the island's historical load. The resulting values are divided by 1 million to convert TOC_t to millions of dollars, consistent with the unit of measurement for the objective function (1).

Returning to (1), $I_{c,\ t}$ denotes the annualized investment cost of candidate generator c in period t and $I_{k,t}$ represents the annualized investment cost of candidate transmission line k in period t. Annualized costs are calculated as annual payments based on the overnight capital cost of the infrastructure. The variable $w_{k,t}$ is a binary variable representing the construction of candidate transmission line k in period t. Accordingly, $\sum_{\tau\le t} w_{k,\tau}$ takes on a value of 1 if a candidate transmission line has been built by period t.

As described previously, fossil fuel plants are modeled as binary variables (3) and can only be built once (4). RE generators however, are modeled to be limited by some finite capacity (5). (Note that all generators are already mapped to specific zones by definition.) The idea behind these constraints is to simulate real physical restrictions on where new infrastructure can be built. For example, for hydro resources used in this test case, previous studies had already determined the capacity of each river to generate electricity. These are used to define the maximum allowable capacity at the location for each potential hydro generator.

$$x_{f,t} \in \{0,1\}\ \forall\ c,t \tag{3}$$

$$\sum_t x_{f,t} \le 1\ \forall\ c \tag{4}$$

$$0 \le \sum_t x_{i,t} \le Q_i\ \forall\ c \tag{5}$$

Likewise, candidate transmission lines are binary (6) and can only be built once (7). These too, are already mapped to specific geographic zones/nodes in the network by definition.

$$w_{k,t} \in \{0,1\}\ \forall\ k,\ \ t \tag{6}$$

$$\sum_t w_{k,t} \le 1\ \forall\ k \tag{7}$$

In [1], a general constraint on power generation is presented. The main customization in this paper are adjustments to generator capacity P_g^{MAX} based on forced λ_g and unforced $\psi_{g,h}$ outage rates and the existence/retirement of plants $Operational_{g,t}$ (8). Here, $Operational_{g,t} = \ y_{e,\ t}$ for existing units and $Operational_{g,t} = \ \sum_{\tau\le t} x_{c,\ \tau}$ for candidate units. Note that $\psi_{g,h}$ is indexed on h. This is because, for peak hours, $\psi_{g,h} = 0$. This accounts for the fact that planned, precautionary maintenance is usually conducted outside of peak hours.

$$0 \le P_{g,\ h,t} \le P_g^{MAX} \times \left(1-\lambda_g\right) \times \left(1-\psi_{g,h}\right) \times q_{g,h}\ \times Operational_{g,t}\ \forall\ g,h,t \tag{8}$$

Another customization is the reserve margin constraint to ensure resource adequacy. In (9), q_g^{peak} represents the availability factor for a generating unit during peak hours. This is a value between 0 and 1.

$$\sum_g P_g^{MAX} \times q_g^{peak}\ \times Operational_{g,t} \ge\ \alpha_t \times\ \ (1+R)\ \forall\ t \tag{9}$$

In terms of transmission constraints, power flow across existing lines $S_{j,n,z,h,t}$ are limited by transmission capacity S_j^{MAX}. Depending on the direction of power flow between node n and node z, flow will be denoted as negative or positive. Additionally, line losses are already explicitly accounted for in electricity demand. Furthermore, power flow is modeled as direct current, not alternating current.

$$-S_j^{MAX} \le S_{j,n,z,h,t} \le S_j^{MAX} \ \forall \ j, \ n, z, h, t \tag{10}$$

Using notation already defined, (11) defines the power flow constraint for candidate transmission lines.

$$-\sum_{\tau \le t} w_{k,\tau} \times S_k^{MAX} \le S_{k,n,z,h,t} \le \sum_{\tau \le t} w_{k,\tau} \times S_k^{MAX} \ \forall \ k, n, z, h, t \tag{11}$$

Bus voltage angles $\theta_{l,n,h,t}$, measured in radians, do not exceed pi (set to 10 decimal places).

$$-\pi \le \ \theta_{l,n,h,t} \le \pi \ \forall \ l, n, h, t \tag{12}$$

Power flow is also defined as the product of suseptance, B_l, and difference between bus voltage angles at each node connecting an existing transmission line.

$$S_{j,n,z,h,t} = B_j \times \left(\theta_{j,n,h,t} - \theta_{j,z,h,t}\right) \ \forall \ j, n, z, h, t \tag{13}$$

For candidate transmission lines, the "Big-M" method is used to avoid creating a non-linear problem. Here, M is a large constant and $U_{t,k}$ is a slack variable defined by (15) and (16).

$$S_{k,n,z,h,t} = B_k \times \left(\theta_{k,n,h,t} - \theta_{k,z,h,t}\right) + \left(\sum_{\tau \le t} w_{k,\tau} - 1\right) M + U_{t,k} \ldots \forall \ k, t \ n, \ z \tag{14}$$

$$U_{t,k} \ge 0 \ \forall \ t, k \tag{15}$$

$$U_{t,k} \le 2 \ \times \left(1 - \sum_{\tau \le t} w_{k, \ \tau}\right) \times M \ \forall \ t, \ k \tag{16}$$

Consequently, if $\sum_{\tau \le t} w_{k,\tau} = 0$, Eq. (14) is non-binding. However, if $\sum_{\tau \le t} w_{k,\tau} = 1$, then (14) is binding, $U_{t,k} = 0$.

In this paper, demand must be always be satisfied. (Minor modifications to the model can account for load shedding). In (17), total generation $\phi_h \times \sum_g P_{g,h,t}$ and net inflows $\phi_h \times \sum_n (S_{l,n,z,h,t} - \ S_{l,z,n,h,t})$ sum to demand $D_{n,h,t}\phi_h$ at every point in time.

$$\phi_h \times \left[\sum_g P_{g,h, \ t} + \sum_n \left(S_{l,n,z,h,t} - \ S_{l,z,n,h,t}\right)\right] = D_{n,h,t} \times \ \phi_h \ \forall \ n, h, t \tag{17}$$

Finally, renewable energy targets are common energy policies in Small Island Developing States. To represent this, total renewable energy generation $\phi_h \times \sum_{i,n,h} P_{i,n,h,t}$ is constrained to surpass some fraction Z of total demand $\phi_h \times \sum_{n,h} D_{n,h,t}$. This constraint holds for each period t starting from a pre-defined date T.

$$\phi_h \times \sum_{i,n,h} P_{i,n,h,t} \ge Z \times \phi_h \times \sum_{n,h} D_{n,h,t} \ \forall \ t \ge T \tag{18}$$

These equations and attending discussion of the procedure describe the customized application of mixed-integer programming to long-term generation and transmission expansion planning in the context of a Small Island Developing State. Key differences between this model and the models by [1,2] are summarized below:

Krishnan et al. [1] outline a generalized model sufficient for understanding broad GTEP optimization but lacks the specificity of a more real-world application. First, Krishnan et al. [1] model new generation as capacity. Furthermore, no distinction is made between renewable energy (RE) investments and fossil fuel investments. In our model, only RE generation investments are treated as continuous variables while fossil fuel generation investments are treated as binary. The purpose of doing this is to more accurately reflect the investment decision faced by the decision maker and the modularity of wind and solar installations [6]. Second, our model begins with an existing set of generators and transmission corridors that determine the initial network topology. Optimizing over an entirely new set of generation and transmission infrastructure as illustrated by Krishnan et al. [1] would yield a lower cost solution. This is because, instead of relying partially on existing infrastructure, which may have been sub-optimal to begin with, the model would be free to find an optimal solution unconstrained by decisions made prior to the start of the planning horizon. Third, our model introduces new constraints. These are the reserve margin constraint (Eq. (9)) and the renewable and a renewable portfolio standard (Eq. (18)). Of note, the renewable portfolio standard is a binding constraint, suggesting that the optimal solution would have a lower total cost if this constraint was excluded. However, this constraint is important in the SIDS context given the ambitious renewable energy goals SIDS tend to have.

The model presented in this paper also differs from the model by Sparrow et al. [2] in key ways. In particular, a major gap in the Sparrow et al. model is that it ignores power flow constraints; it is formulated as a transportation model. Our model takes the approach of Krishnan et al. [1] and includes specific power flow constraints that reflect Kirchhoff's laws Eqs. (12)–((16)) and therefore offer a more realistic representation of power flow across the network. This is of importance for situations where the transmission structure is complex, including many loops, tighter transmission limits, etc. Sparrow et al. [2] also optimize generation and transmission expansion across several nations in a power pool. Our model focuses on a closed network (i.e. an island). This is more suited to the focus of the paper on power planning for Small Island Developing States.

This paper not only delineates the general method, but it provides specific details on (1) setting up the GAMS environment for successfully reproducing these results, (2) particular customizations too lengthy to describe in a general research publication and (3) issues encountered in previous versions of the model before arriving at a final version that can be shared publicly. Except for indivisibilities associated with binary investment variables, this is a convex program. It is solved using a branch and bound method with a relative tolerance between the upper and lower bound on the objective value of 0.01%. The solution can therefore only be guaranteed within this tolerance level. It is possible to set the tolerance level to zero to achieve global optimality. However, even in that circumstance, it is not possible to guarantee uniqueness of the optimal solution beyond the objective value; there may be alternative configurations of generation and transmission investments, as well as generation and power flows, that results in equally good solutions.

The result of the modified method illustrated in this paper is a model that balances a real-world representation of an island network with manageable run times while generating meaningful results. The modifications to this model reduced run times from over two months to a few weeks using a computer with a 9th generation Intel® i7 processor at 2.6Ghz with 16 GB of RAM on a 64-bit Windows operating system. Once data is available, the model can be scaled and applied to a different country or context. These alternative contexts can be larger, more topologically complex, and policy alternatives can be extended to include emissions targets, taxes, etc. Hour categories is another dimension that can be extended. When extending hour categories, the important feature to maintain is the diversity in load being grouped across time, space or another dimension. The model is therefore scalable, flexible, and useful for long-term planning while minimizes computation time.

Supplementary material and/or additional information

Program complications and failed attempts

Initially, the model formulation was indexed on years, allowing for new infrastructure to be built in any year during the planning horizon. However, the addition of a renewable portfolio constraint increased the run time from just over a week to longer than a month. This influenced the decision to allow new infrastructure to be built only every other year and demonstrates how each successive constraint substantially increases model complexity.

There were also failed attempts to incorporate energy storage in the model procedure to more effectively account for a future with potentially higher renewable energy resources. These failed attempts were due, in part, to the treatment of hour types in the model. Explicitly modeling hours vs hour types may help to resolve this, but risks increasing computation time exponentially. This is an area of possible future research.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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