Applied Soft Computing 124 (2022) 109055

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

# A novel behavioral three-way decision model with application to the treatment of mild symptoms of COVID-19

Shi-Fan He [a,b], Ying-Ming Wang [a,*], Xiaohong Pan [a,b], Kwai-Sang Chin [b]

[a] Decision Sciences Institute, Fuzhou University, Fujian, 350108, China

[b] Department of Systems Engineering and Engineering Management, City University of Hong Kong, Hong Kong

ARTICLE INFO

*Article history:*
Received 8 October 2021
Received in revised form 19 April 2022
Accepted 16 May 2022
Available online 25 May 2022

*Keywords:*
Three-way decisions
Evidential reasoning
Psychological behavior
Coronavirus disease 2019
Multi-attribute decision making

ABSTRACT

The Coronavirus Disease 2019 (COVID-19) has popularized since late December 2019. In present, it is still highly transmissible and has severe impact on the public health and global economy. Due to the lack of specific drug and the appearance of different variants, the selection of the antiviral therapy to treat the patients with mild symptom is of vital importance. Hence, in this paper, we propose a novel behavioral Three-Way Decision (3WD) model and apply it to the medicine selection decision. First, a new relative utility function is constructed by considering the risk-aversion behavior and regret-aversion behavior of human beings. Second, based on the relative utility function, some new rules are defined to calculate the thresholds and conditional probabilities in 3WD and some corresponding theorems are explored and proved. Next, a new information fusion mechanism in the framework of evidential reasoning algorithm is developed. Then, the decision results are obtained based on the Bayesian decision procedure and the principle of maximum utility. Finally, an example with large-scale data set and an example about medicine selection for COVID-19 are provided to show the implementation process and effectiveness of the proposed method. Comparative analysis and sensitivity analysis are also performed to illustrate the superiority and the robustness of the current proposal.



## 1. Introduction

The Coronavirus Disease 2019 (COVID-19) caused by SARA-CoV-2 was initially found in late December 2019 [1]. It is highly transmissible and has severe impact on the public health and global economy. On March 2020, the World Health Organization (WHO) declared the COVID-19 as a global pandemic. This disease has influenced billions of humans and swept across almost all countries in the world. Fig. 1 shows the COVID-19 situation by WHO region until April 2022. From Fig. 1, we can notice that the development of this disease is fluctuating and has not shown an ending tendency. To make matters worse, the SARA-CoV-2 has different variants. Variant is the virus with one or several new mutations (mentioned as the changes in the process of virus replication). Some mutations will change the characteristics of virus, such as transmission, severity, therapeutic medicines and diagnostic tools etc. These mutations will make the virus more adaptable to the environment. Although there is evidence that the change of the SARA-CoV-2 is more slowly than other viruses such as HIV, more than hundreds of variations of this virus have been reported worldwide. Some SARS-CoV-2 variants such as Alpha variant founded in United Kingdom, Beta variant founded in South Africa, Gamma variant founded in Brazil and Delta variant founded in India, have shown greater perniciousness than the original version. Specifically, those variants (1) have high transmissibility, or (2) increase harmful change in clinical disease presentation, or (3) decrease the effectiveness of the available measures, diagnostics, vaccines and therapeutics. Unfortunately, there is no specific drug for the COVID-19 for now. Hence, there still is a long way to end this pandemic.

Recently, numerous studies on COVID-19 have been published. To predict the COVID-19 confirmed, death and cured cases of India, Gupta et al. [2] texted five machine learning methods and found that the random forest model has the best performance in this prediction. Wang et al. [3] employed machine learning method to predict the infection severity of people based on their genetic data, which helps to identify who is more vulnerable to the COVID-19. Ghorui et al. [4] evaluated the risk factors contributed to the spread of COVID-19 by multi-attribute decision making (MADM) tools and concluded that “long duration of contact with the infected person”, “hospitals and clinic” and “verbal spread” are the top three significant risk factors. Recognizing and separating the infected people is an important step in controlling infection. Hence, Wu et al. [5] developed a novel joint classification and segmentation system to conduct COVID-19 diagnosis

* Corresponding author.
*E-mail address:* ymwang@fzu.edu.cn (Y.-M. Wang).

https://doi.org/10.1016/j.asoc.2022.109055
1568-4946/

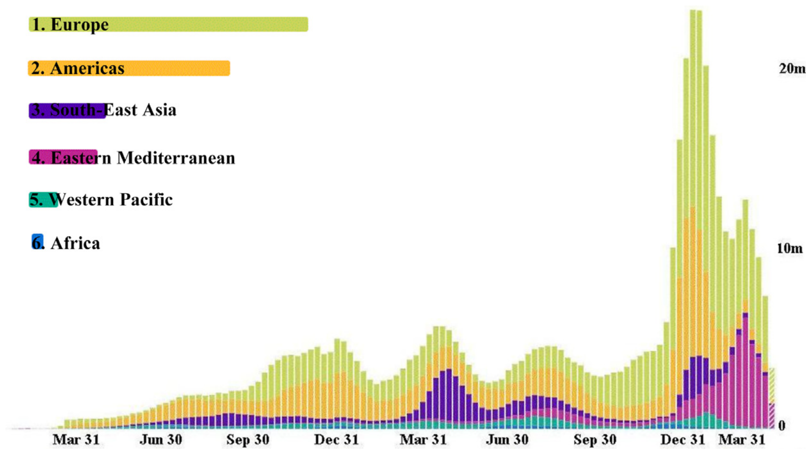


**Fig. 1.** The COVID-19 situation by WHO region (data source: https://www.who.int).

and testified its effectiveness by extensive experiments. Ahmad et al. [6] proposed a fuzzy cloud-based COVID-19 diagnosis assistant to distinguish confirmed, suspects or suspicious patients, which classifies the patients into mild, moderate, severe or critical patients.

The knowledge about COVID-19 is limited and there is no proven specific treatment for patients expect for supportive care. Hence, the researches on the supportive care may be the only way to find the effective and safe treatment for the COVID-19 and other future possible outbreaks [7]. To select the best therapy for the mild symptoms of COVID-19, Mishra et al. [8] employed the hesitant fuzzy decision-making method to rank five medicines with respect to seven attributes. Yildirim et al. [7] employed fuzzy PROMETHEE and VIKOR methods to evaluate the available COVID-19 treatment options. Chai et al. [9] proposed MADM method under Z-uncertain probabilistic linguistic variables to make emergency decision for treatment of COVID-19 patients. However, these methods provide decision references only by ranking alternatives. Because of the limitation of knowledge, it is of highly possibility that the alternatives provided are all not good. In this case, although one alternative is the first option, it still not good enough. Simply adopting it may not cure the patients and even cause dangerous situation. To conquer this drawback, 3WD method provides an effective tool to deal with decision regarding COVID-19. 3WD was first proposed by Yao [10] based on the Decision-Theoretic Rough Sets (DTRSs). It classifies alternatives into three pair-wise disjoint regions, i.e., positive, negative and boundary regions, which means acceptance, rejection, and non-commitment, respectively [11]. Different from other selection methods, the 3WD-based method can not only provide ranking order of alternatives, but also classify them into different regions. For the alternatives in the positive regions, they can be further measured by their ranking order; for the alternative in the boundary regions, the decision should be suspended until obtaining more information; for the alternatives in the negative regions, they should not be chosen. This contributes to reduce the decision risks, especially in medical decision. Hence, in this paper, a 3WD-based MADM method is proposed to assist in the medicine selection for the treatment of mild symptoms of COVID-19.

The main contributions of this paper are briefly summarized as follows: (1) Considering the regret-aversion behavior and risk-aversion behavior of human beings, this paper proposes a novel behavioral 3WD model involving the definition of relative utility function, the determination of thresholds, the calculation of conditional probabilities and the ranking regulations. (2) To correctly fuse the decision information in 3WD, the Evidential Reasoning (ER) algorithm is introduced into 3WD for the first time, which can preserve as much original decision information as possible. (3) Apply the proposed 3WD-based method to solve medicine decision problems. Different from other medicine decision methods, the proposed 3WD-based method can not only produce the ranking order and classification of alternatives, but also reduce the risks involved in the decision-making process.

The reminder of this paper is organized as follows: Section 2 reviews literature related to medical decision based on MADM, 3WD and ER algorithm. Section 3 introduces some relative theories, including the 3WD, the prospect theory, the shadowed sets and the ER algorithm, which will be used in the following sections. Section 4 defines the improved behavioral 3WD, including the relative utility functions derived from evaluation values, the determination of thresholds, the calculation of conditional probabilities and the ranking regulations of alternatives. The complete decision process based on the proposed 3WD model is proposed in Section 5. Section 6 provides two numerical examples to demonstrate the implementation and effectiveness of the proposed method. Comparative analysis and sensitivity analysis are also conducted in this section to show its superiority and robustness. Conclusions are drawn in Section 7.

## 2. Literature review

In this section, we will briefly review some studies related to medical decision based on MADM, 3WD and ER algorithm.

### 2.1. Medical decision based on MADM

Healthcare and medical industry play an important role in the living standard and well-being of people [12]. Making an accurate medical decision is a difficult task and normally needs to consider several attributes from different aspects [13,14]. For example, when making a medical diagnosis, the doctor need to evaluate the disease from multiple aspects because diseases are normally accompanied by multiple symptoms [15]. Hence, the medical decision can be considered as a kind of MADM problems [13,16]. For that, MADM methods have been applied in various fields of medical decision and healthcare [16]. Based on fuzzy integral and fuzzy measure, Dursun et al. [17] proposed a multi-attribute group decision making method to deal with healthcare waste in Lstanbul, Turkey. Li and Wei [18] developed a large-scale group decision making method to conduct healthcare management, which considers the complexity of the management and the opinions of stakeholders. Considering experts may lack knowledge to handle critical diseases, Das and Kar [16] proposed an algorithm based on intuitionistic fuzzy soft set to explore a method that can reflect the opinions of all experts. To solve medical decision about acute inflammatory demyelinating disease, Chen et al. [19] defined an extended QUALIFLEX method to conduct MADM analysis. Tolga et al. [13] defined finite interval Type-2 Gaussian fuzzy numbers and extended TODIM method with FIT2 Gaussian fuzzy numbers to select healthcare device.

### 2.2. Three-way decisions

3WD initially proposed by Yao [10,20,21] divides a whole into three regions, i.e., positive region (POS), negative region (NEG) and boundary region (BND), which can be interpreted as three decision actions, i.e., acceptance, rejection and non-commitment. The idea of 3WD is in line with people's cognition because they innovatively provide a deferment strategy [11]. When people have a full acknowledge about an event, they can make a quick rejection or acceptance judgments; but if they cannot make an immediate decision, they are usually willing to postpone the decision, that is, deferment [15]. The extensive studies of 3WD have led to the extension from narrow 3WD to wide 3WD. In narrow sense, 3WD was firstly introduced to interpret three types of classification rules in rough set theory [22]. Until now, narrow 3WD has developed many generalized models, such as three-way approximation models [23], three-way analysis models [24], three-way concept lattice models [25] and so on. These models are 3WD in various context, which have specific mathematical expressions [26]. In recent years, the wide 3WD has been studied in depth based on the common existed "three" phenomena in the fields of computer sciences, management, cognitive science and so on [27]. For wide 3WD, decision is viewed as computing, processing, analysis etc. [28]. It is thinking, problem solving and information processing in threes [22]. The wide 3WD changes the two-way consideration such as true/false, white/black, into three-way consideration like true/unsure/false, white/gray/black [28], which is flexible and simple enough [22]. Hence, the wide 3WD has been a method that can be used in various research topics.

3WD offers new opportunities for studying MADM problems [29]. It provides not only a reasonable semantic interpretation for decision results, but also a powerful and scientific tool to address MADM problems [30]. Hence, the fusion of 3WD with MADM has become a hot research topic. Jia and Liu [31] applied 3WD to MADM by using attribute values to express loss functions and preliminarily manifested the correlation between them. Huang et al. [32] explored a new 3WD method to MADM, in which they provided a new calculation method of loss function and conditional probabilities. Zhu et al. [33] defined a new 3WD method based on the regret theory, which includes optimistic, neutral and pessimistic strategies. Liang et al. [34] proposed a behavioral 3WD model based on the prospect theory under interval type-2 fuzzy environment and applied it to solve MADM problems. Considering several departments or agents may be involved in the decision-making process, Sun et al. [35] proposed a 3WD method to handle multiple attribute group decision making problems with linguistic information. Wang et al. [36] took the hesitancy of decision makers into consideration and developed a three-way MADM method under hesitant fuzzy environments. In real world, there exist many MADM problems with incomplete information. To deal with these problems, Zhan et al. [37] proposed a novel 3WD-based MADM model based on utility theory.

**Table 1**
The loss functions.

| | $X$ | $X^c$ |
|---|---|---|
| $a_P$ | $\lambda_{PP}$ | $\lambda_{PN}$ |
| $a_B$ | $\lambda_{BP}$ | $\lambda_{BN}$ |
| $a_N$ | $\lambda_{NP}$ | $\lambda_{NN}$ |

### 2.3. Evidential reasoning algorithm

Dempster–Shafer (D–S) theory is easy to understand and can comprehensively process the uncertain and inaccurate information [38]. However, D–S theory has some drawbacks in dealing with conflicting evidences [39]. To tackle this issue, Yang and Xu [39] developed the ER algorithm based on the combination rule of D–S theory. Till now, the ER algorithm has been testified as a good tool to aggregate the information and has many successful applications in aggregating the uncertain and imprecise information in MADM problems. Xue et al. [40] introduced the ER algorithm to multi-scale hesitant fuzzy linguistic environment to combine the attributes in the hazard assessment of landslide dames. To reasonably assess the renewable energy projects, Liang et al. [41] and Pan et al. [42] respectively proposed a multi-granular linguistic ER algorithm and an interval type-2 fuzzy ER algorithm to aggregate the information under multiple attributes. Yuan and Luo [43] replaced aggregation operators with ER algorithm to aggregate the initial decision information under intuitionistic fuzzy environment. Zhang et al. [44] deduced a general analytical ER algorithm, which can explicitly aggregate attributes with interval belief structures. Loughney et al. [45] utilized the ER method to determine a suitable wireless sensor network orientation for monitoring asset integrity of an offshore gas turbine driven generator.

## 3. Preliminary

In this section, some basic concepts about 3WD, prospect theory, shadowed sets and ER algorithm are briefly reviewed, which will be used in the following sections.

### 3.1. 3WD based on DTRSs

The idea of the 3WD was initially proposed by Yao [10,20,21] based on DTRSs. It employs loss functions to explain the risks of losses, calculates the expected loss of three actions based on conditional probabilities and classifies objects into three regions based on Bayesian minimum risk decision rules.

Suppose there are 2 states, denoted as $\Omega = \{X, X^c\}$, which respectively represent an element is in $X$ and not in $X$, and 3 actions for each state, denoted as $\Theta = \{a_P, a_B, a_N\}$ where $a_P$, $a_B$ and $a_N$ indicate $x \in POS(X)$, $x \in BND(X)$ and $x \in NEG(X)$, respectively. The loss function in different states is given by a $3 \times 2$ matrix, as shown in Table 1.

In Table 1, $\lambda_{PP}$, $\lambda_{BP}$ and $\lambda_{NP}$ represent the losses caused by taking actions $a_P$, $a_B$ and $a_N$ when an element belongs to $X$; $\lambda_{PN}$, $\lambda_{BN}$ and $\lambda_{NN}$ denote the losses of taking actions $a_P$, $a_B$ and $a_N$ when an element belongs to $X^c$. The expected loss $R(a_i|[x])$ when taking different actions ($a_P$, $a_B$ or $a_N$) for objects in $[x]$ can be expressed as:

$$R(a_P|[x]) = \lambda_{PP}\Pr(X|[x]) + \lambda_{PN}\Pr\left(X^c|[x]\right) \tag{1}$$

$$R(a_B|[x]) = \lambda_{BP}\Pr(X|[x]) + \lambda_{BN}\Pr\left(X^c|[x]\right) \tag{2}$$

$$R(a_N|[x]) = \lambda_{NP}\Pr(X|[x]) + \lambda_{NN}\Pr\left(X^c|[x]\right) \tag{3}$$

where $\Pr(X|[x])$ is the conditional probability of an object belonging to $X$.

According to the minimum-risk decision rules derived from the Bayesian decision procedure, the decision rules can be determined as:

(P) If $R(a_P\,|[x])\le R(a_B\,|[x])$ and $R(a_P\,|[x])\le R(a_N\,|[x])$, then $x\in POS(X)$;

(B) If $R(a_B\,|[x])\le R(a_P\,|[x])$ and $R(a_B\,|[x])\le R(a_N\,|[x])$, then $x\in BND(X)$;

(N) If $R(a_N\,|[x])\le R(a_P\,|[x])$ and $R(a_N\,|[x])\le R(a_B\,|[x])$, then $x\in NEG(X)$.

This decision rules can be further simplified based on the fact that $\Pr(X\,|[x])+\Pr(X^c\,|[x])=1$ and the assumption that $\lambda_{PP}\le\lambda_{BP}<\lambda_{NP}$ and $\lambda_{NN}\le\lambda_{BN}<\lambda_{PN}$. The simplified rules are provided as follows:

(P1) If $\Pr(X\,|[x])\ge\alpha$ and $\Pr(X\,|[x])\ge\gamma$, then $x\in POS(X)$;

(B1) If $\Pr(X\,|[x])\le\alpha$ and $\Pr(X\,|[x])\ge\beta$, then $x\in BND(X)$;

(N1) If $\Pr(X\,|[x])\le\beta$ and $\Pr(X\,|[x])\le\gamma$, then $x\in NEG(X)$.

where $\alpha$, $\beta$ and $\gamma$ are the decision thresholds, which are determined by:

$$\alpha=\frac{(\lambda_{PN}-\lambda_{BN})}{(\lambda_{PN}-\lambda_{BN})+(\lambda_{BP}-\lambda_{PP})} \tag{4}$$

$$\beta=\frac{(\lambda_{BN}-\lambda_{NN})}{(\lambda_{BN}-\lambda_{NN})+(\lambda_{NP}-\lambda_{BP})} \tag{5}$$

$$\gamma=\frac{(\lambda_{PN}-\lambda_{NN})}{(\lambda_{PN}-\lambda_{NN})+(\lambda_{NP}-\lambda_{PP})} \tag{6}$$

If there is an assumption that $(\lambda_{PN}-\lambda_{BN})(\lambda_{NP}-\lambda_{BP})>(\lambda_{BP}-\lambda_{PP})(\lambda_{BN}-\lambda_{NN})$, then the thresholds meet $0<\beta<\gamma<\alpha<1$. The rules (P1)–(N1) can be simplified as follows:

(P2) If $\Pr(X\,|[x])\ge\alpha$, then $x\in POS(X)$;

(B2) If $\beta<\Pr(X\,|[x])<\alpha$, then $x\in BND(X)$;

(N2) If $\Pr(X\,|[x])\le\beta$, then $x\in NEG(X)$.

### 3.2. Prospect theory

Prospect theory was developed by Kahneman and Tversky [46] on the basis of utility theory. It can forecast the actual decision behavior of decision maker under risk [47]. Prospect theory consists of two phases: the editing phase and the evaluation phase [46]. In the editing phase, the outcomes of alternatives are coded as gains or losses relative to the reference point. In the evaluation phase, the prospect values are calculated by a value function and the alternative with the highest prospect value is chosen. The prospect value function is defined as follows:

$$v(\Delta x)=\begin{cases}(\Delta x)^p & \Delta x\ge 0\\ -\theta(-\Delta x)^q & \Delta x<0\end{cases} \tag{7}$$

where $p$ and $q$ are coefficients of risk attitude, $0\le p,q\le 1$; $\Delta x$ represents the deviation between the existing value and reference point, which denotes gain ($\Delta x\ge 0$) or loss ($\Delta x<0$); and $\theta$ is the risk aversion coefficient, $\theta>1$. In [48], Kahneman and Tversky found that setting $p=q=0.88$ and $\theta=2.25$ will make the results keep consistent with empirical data.

### 3.3. The shadowed sets

Shadowed set coined by Pedrycz [49,50] shows its superiority in charactering fuzzy information. The construction of shadowed set is based on balancing the uncertainty, which is also called as uncertainty relocation. It maps the membership grade of object in universe to a set {0, 1, [0, 1]}, which is defined as:

**Definition 1** (*[49,50]*). Let $U$ be a given universe of discourse, a shadowed set $S$ can be represented as follows:

$$S:U\to\{0,1,[0,1]\} \tag{8}$$

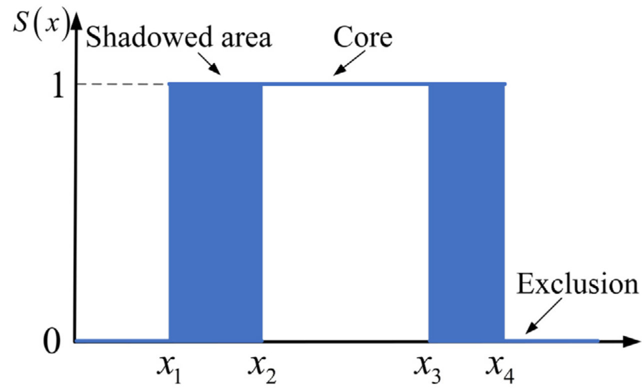


**Fig. 2.** An illustrate example of a shadowed number $S=(x_1,x_2,x_3,x_4)$.

where 0, 1 and [0, 1] respectively mean full exclusion, full belongingness and uncertainty, which respectively correspond to the exclusion, the core and the shadowed area in the shadowed set. The exclusion area of shadowed set consists of $\{x\,|S(x)=0,x\in U\}$; $\{x\,|S(x)=1,x\in U\}$ composes the core of shadowed set; and the shadowed area is the regions of $U$ where $S(x)=[0,1]$.

According to the definition of shadowed set, Landowski [51] defined the shadowed number $S=(x_1,x_2,x_3,x_4)$ where $x_1\le x_2\le x_3\le x_4$. The membership degree of the shadowed set is 1 when values between $x_2$ and $x_3$. It is 0 for values less than or equal to $x_1$ and for values larger than or equal to $x_4$. And for the values between $x_1$ and $x_2$ as well as between $x_3$ and $x_4$, it is an interval [0, 1]. An illustration of shadowed number $S=(x_1,x_2,x_3,x_4)$ is shown in Fig. 2.

Li et al. [52] proposed a data-driven method to construct shadowed sets used to model linguistic terms. First, interval data pre-processing is conducted on the collected interval data. After bad data processing, outlier processing, tolerance limit processing and reasonable interval processing, ineffective data will be deleted, and the data remained will be used in phase 2, i.e., construction of shadowed sets based on interval data. As word means different things for different people, this difference can be considered by shadowed sets. Inspired by their innovative work, we also tried to construct the shadowed sets corresponding to seven-level linguistic terms in our previous research [53] as shown in Fig. 3. For more details, please kindly refer to [52,53].

### 3.4. ER algorithm

The ER algorithm is developed based on the D–S theory proposed by Dempster and Shafer, which is well suited to address the imprecise and uncertain information. To better understand ER algorithm, we first introduce some primary concepts about traditional D–S theory and ER algorithm.

**Definition 2** (*[54,55]*). Let $H=\{H_p\,|p\in[1,N]\}$ be a collectively exhaustive and mutually exclusive set, which is called the frame of discernment. A basic probability assignment (also called a belief structure or a basic belief assignment) is a function $m:2^H\to[0,1]$, satisfying:

$$m(\Phi)=0 \tag{9}$$

$$\sum_{A\subseteq H}m(A)=1 \tag{10}$$

where $\Phi$ is an empty set, $A$ is any subset of $H$, $2^H$ is the power set of $H$ and consists of all the subsets of $H$ including empty set and universal set.

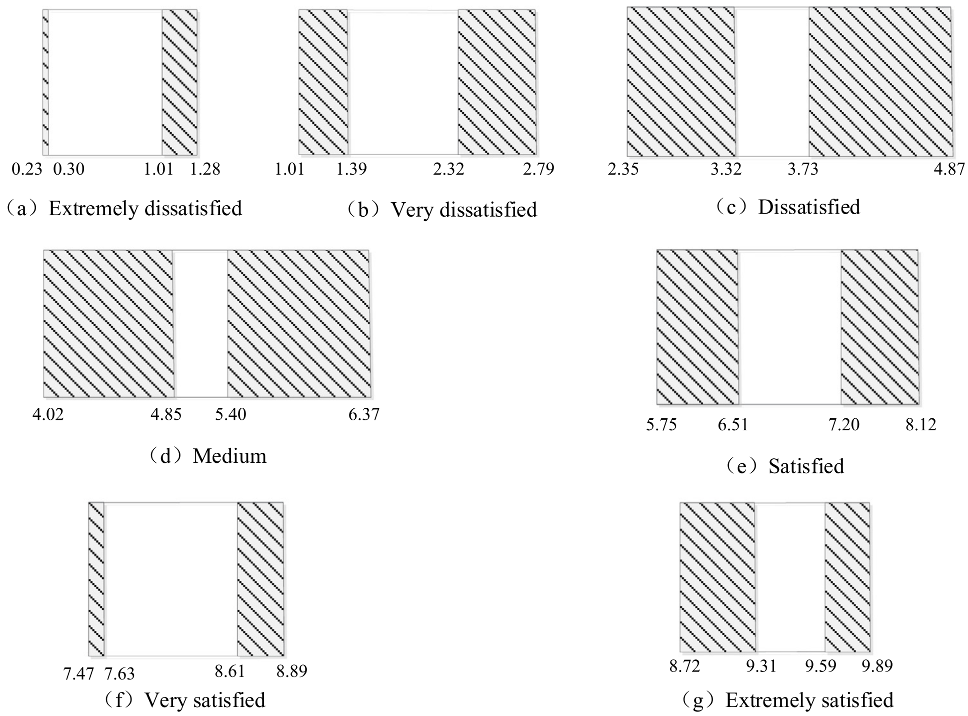


**Fig. 3.** The shadowed sets for seven-level linguistic terms.

The probability mass $m(A)$ measures the belief exactly assigned to $A$ and depicts the degree of the evidence supporting $A$. The assigned probability to $H$ is called the degree of ignorance, which is denoted as $m(H)$. For each subset $A \subseteq H$, if $m(A) > 0$, $A$ is called a focal element of $m$.

A belief measure $Bel$ and a plausibility measure $Pl$ are defined as following:

$$Bel(A) = \sum_{B \subseteq A} m(B) \tag{11}$$

$$Pl(A) = \sum_{A \cap B \neq \Phi} m(B) \tag{12}$$

where $A$ and $B$ are subsets of $H$. $Bel(A)$ represents the exact support for $A$ and $Pl(A)$ denotes the possible support for $A$. Their relationship is $Pl(A) = 1 - Bel(\overline{A})$, where $\overline{A}$ is the complement of $A$. $[Bel(A), Pl(A)]$ constitutes the interval that supports $A$. The difference between the belief and the plausibility of set $A$ describes the ignorance of the assessment for the set $A$.

**Definition 3** ([54,55]). The core of the D–S theory of evidence is the Dempster's combination rule. It provides a way to fuse evidence with different sources, which is defined as follows:

$$[m_1 \oplus m_2](C) = \begin{cases} 0, & C = \Phi \\ \dfrac{\sum_{A \cap B = C} m_1(A) m_2(B)}{1 - \sum_{A \cap B = \Phi} m_1(A) m_2(B)}, & C \neq \Phi \end{cases} \tag{13}$$

where $A$ and $B$ are both focal elements, $1 - \sum_{A \cap B = \Phi} m_1(A) m_2(B)$ is named as the normalization factor.

## 4. The improved behavioral 3WD

In classical 3WD, the loss function is fixed for alternatives belonging to the same state and taking the same action. That cannot effectively distinguish each alternative. To conquer this drawback, Jia and Liu [31] defined relative loss functions with respect to multiple attributes. Based on their work, many scholars have studied the relative utility functions considering the risk attitude of decision maker based on the prospect theory, such as [34,56]. This makes a meaningful extension for 3WD theory. But some researchers think that feeling such as regret and rejoice is also a fact of life and it is irrational to ignore them [57]. Hence, in this paper, we are going to simultaneously take the risk-aversion and the regret-aversion behavior into account by combining the prospect theory and the regret–rejoice function.

### 4.1. The utility functions derived from evaluation values

The definition of the prospect theory is introduced in Section 3.2 and the regret–rejoice function is defined as follows:

**Definition 4** ([58,59]). The regret–rejoice function $reg - rej(\Delta\phi)$ is defined as follows:

$$reg - rej(\Delta\phi) = 1 - \exp(-\delta(\Delta\phi)) \tag{14}$$

where $\delta(\delta > 0)$ is the regret aversion coefficient of the decision maker and $\Delta\phi$ is the difference between two alternatives. If $reg - rej(\Delta\phi) > 0$, the decision maker will feel rejoice; otherwise, the decision maker will feel regret.

Assume a MDAM problem includes $M$ alternatives $a_l$ $(l = 1, \ldots, M)$ to be evaluated regarding $L$ attributes $e_i$ $(i = 1, \ldots, L)$ by linguistic term $s_t$, $t \in [0, N]$. The initial decision-making matrix is expressed as $D = (x_{li})_{M \times L}$. The weight of attribute $e_i$ is denoted by $w_i$, where $0 \leq w_i \leq 1$ and $\sum_{i=1}^{L} w_i = 1$. For states $X_i$ and $X_i{}^c$, $a_l \in X_i$ means that alternative $a_l$ meets the requirement of attribute $e_i$; $a_l \in X_i{}^c$ denotes that alternative $a_l$ does not meet the requirement of attribute $e_i$. Besides, $x^i_{\min} = s_0$ and $x^i_{\max} = s_N$. Then, the relative regret function and the relative rejoice function can be respectively determined by the formulas in Tables 2 and 3.

After obtaining the relative regret function and the relative rejoice function, the regret or rejoice perceived by the decision maker is calculated by: $\Delta(x_{li}) = b(x_{li}) - \lambda(x_{li})$. If $\Delta(x_{li}) < 0$, the decision maker feels regret; if $\Delta(x_{li}) \geq 0$, the decision maker

**Table 2**
The relative regret function $\lambda(x_{li})$ of alternative $a_l$ on attribute $e_i$.

| $x_i$ | $X_i$ | ${X_i}^c$ |
|---|---|---|
| $a_P$ | $\lambda_{PP}^{li}=0$ | $\lambda_{PN}^{li}=1-\exp\left(-\delta\cdot d\left(x_{li},x_{\max}^{i}\right)\right)$ |
| $a_B$ | $\lambda_{BP}^{li}=\eta_i\cdot\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\min}^{i}\right)\right)\right]$ | $\lambda_{BN}^{li}=\eta_i\cdot\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\max}^{i}\right)\right)\right]$ |
| $a_N$ | $\lambda_{NP}^{li}=1-\exp\left(-\delta\cdot d\left(x_{li},x_{\min}^{i}\right)\right)$ | $\lambda_{NN}^{li}=0$ |

**Table 3**
The relative rejoice function $b(x_{li})$ of alternative $a_l$ on attribute $e_i$.

| $x_i$ | $X_i$ | ${X_i}^c$ |
|---|---|---|
| $a_P$ | $b_{PP}^{li}=1-\exp\left(-\delta\cdot d\left(x_{li},x_{\min}^{i}\right)\right)$ | $b_{PN}^{ij}=0$ |
| $a_B$ | $b_{BP}^{li}=(1-\eta_i)\cdot\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\min}^{i}\right)\right)\right]$ | $b_{BN}^{li}=(1-\eta_i)\cdot\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\max}^{i}\right)\right)\right]$ |
| $a_N$ | $b_{NP}^{li}=0$ | $b_{NN}^{li}=1-\exp\left(-\delta\cdot d\left(x_{li},x_{\max}^{i}\right)\right)$ |

**Table 4**
The utility function $\upsilon(x_{li})$ of alternative $a_l$ on attribute $e_i$.

| $x_{ij}$ | $X_i$ | ${X_i}^c$ |
|---|---|---|
| $a_P$ | $\upsilon_{PP}^{li}=\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\min}^{i}\right)\right)\right]^p$ | $\upsilon_{PN}^{li}=-\theta\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\max}^{i}\right)\right)\right]^q$ |
| $a_B$ | $\upsilon_{BP}^{li}=\left\{(1-2\eta_i)\cdot\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\min}^{i}\right)\right)\right]\right\}^p$ | $\upsilon_{BN}^{li}=\left\{(1-2\eta_i)\cdot\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\max}^{i}\right)\right)\right]\right\}^p$ |
| $a_N$ | $\upsilon_{NP}^{li}=-\theta\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\min}^{i}\right)\right)\right]^q$ | $\upsilon_{NN}^{li}=\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\max}^{i}\right)\right)\right]^p$ |

Where $\eta_i$ is a parameter used to calculate the value of adopting non-commitment, $0<\eta_i\le 0.5$; $\delta$ is the regret aversion coefficient, $\delta>0$; $p$ and $q$ are coefficients of risk attitude, $0\le p,q\le 1$ and $\theta$ is the risk aversion coefficient, $\theta>1$.

perceives rejoice. According to the description in Section 3.2, we can observe that the regret and rejoice the decision maker perceived are consistent with the outcomes of prospect theory, thus the utility function $\upsilon(x_{li})$ of alternative $a_l$ on attribute $e_i$ can be obtained as shown in Table 4.

**Remark 1.** The regret–rejoice function is derived from regret theory, which can effectively describe the psychological behavior of decision maker. That is, the decision maker will rejoice when the gain of selected alternative is more than others and will regret if the loss of the selected alternative is more than others [60]. However, as stated in prospect theory, the decision maker is more sensitive to losses than to equal gains [47]. In other word, the decision maker is loss aversion. This characteristic is largely ignored by the regret–rejoice function including in regret theory. For that, this paper combines the regret–rejoice function with prospect theory to describe the psychological behavior of decision maker more comprehensively. In this way, both the regret aversion and loss aversion of decision maker can be well reflected.

After obtaining the utility function, we can determine the expected utilities of taking different actions, which are calculated as follows:

$$R(a_P|x_{li})=\upsilon_{PP}^{li}(x)\cdot\Pr(X|a_l)+\upsilon_{PN}^{li}(x)\cdot\Pr\left(X^c|a_l\right) \tag{15}$$

$$R(a_B|x_{li})=\upsilon_{BP}^{li}(x)\cdot\Pr(X|a_l)+\upsilon_{BN}^{li}(x)\cdot\Pr\left(X^c|a_l\right) \tag{16}$$

$$R(a_N|x_{li})=\upsilon_{NP}^{li}(x)\cdot\Pr(X|a_l)+\upsilon_{NN}^{li}(x)\cdot\Pr\left(X^c|a_l\right) \tag{17}$$

Inspired by the Bayesian minimum-risk decision rules and the idea of maximizing expected utility, Lei et al. [56] proposed decision rules that maximize the expected utility. In this paper, we also adopt these decision rules, therefore, the three decision rules are derived as follows:

(P3) If $R(a_P|x_{li})\ge R(a_B|x_{li})$ and $R(a_P|x_{li})\ge R(a_N|x_{li})$, then $x_{li}\in POS(X)$;

(B3) If $R(a_B|x_{li})\ge R(a_P|x_{li})$ and $R(a_B|x_{li})\ge R(a_N|x_{li})$, then $x_{li}\in BND(X)$;

(N3) If $R(a_N|x_{li})\ge R(a_P|x_{li})$ and $R(a_N|x_{li})\ge R(a_B|x_{li})$, then $x_{li}\in NEG(X)$.

**Theorem 1.** *For $\forall x\in(x_{\min},x_{\max})$, when $0<\eta_i\le 0.5$, the utility functions meet the conditions $\upsilon_{PP}^{li}\ge\upsilon_{BP}^{li}\ge\upsilon_{NP}^{li}$ and $\upsilon_{NN}^{li}\ge\upsilon_{BN}^{li}\ge\upsilon_{PN}^{li}$.*

**Proof.** First, we prove $\upsilon_{PP}^{li}\ge\upsilon_{BP}^{li}$ and $\upsilon_{BP}^{li}\ge\upsilon_{NP}^{li}$ as follows:

$$\begin{aligned}\upsilon_{PP}^{li}-\upsilon_{BP}^{li}&=\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\min}^{i}\right)\right)\right]^p\\&\quad-\left\{(1-2\eta_i)\cdot\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\min}^{i}\right)\right)\right]\right\}^p\\&=\left[1-(1-2\eta_i)^p\right]\cdot\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\min}^{i}\right)\right)\right]^p\end{aligned}$$

Since $0<\eta_i\le 0.5$, therefore $0\le 1-2\eta_i\le 1$. Then, we can prove $\upsilon_{PP}^{li}-\upsilon_{BP}^{li}\ge 0$.

Similarly, $\upsilon_{BP}^{li}-\upsilon_{NP}^{li}=\left\{(1-2\eta_i)\cdot\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\min}^{i}\right)\right)\right]\right\}^p+\theta\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\min}^{i}\right)\right)\right]^p$.

Obviously, $\upsilon_{BP}^{li}-\upsilon_{NP}^{li}\ge 0$. Hence, we can obtain $\upsilon_{PP}^{li}\ge\upsilon_{BP}^{li}\ge\upsilon_{NP}^{li}$.

Then, we prove $\upsilon_{NN}^{li}\ge\upsilon_{BN}^{li}$ and $\upsilon_{BN}^{li}\ge\upsilon_{PN}^{li}$ as follows:

$$\begin{aligned}\upsilon_{NN}^{li}-\upsilon_{BN}^{li}&=\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\max}^{i}\right)\right)\right]^p\\&\quad-\left\{(1-2\eta_i)\cdot\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\max}^{i}\right)\right)\right]\right\}^p\\&=\left[1-(1-2\eta_i)^p\right]\cdot\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\max}^{i}\right)\right)\right]^p\ge 0\\\upsilon_{BN}^{li}-\upsilon_{PN}^{li}&=\left\{(1-2\eta_i)\cdot\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\max}^{i}\right)\right)\right]\right\}^p\\&\quad+\theta\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\max}^{i}\right)\right)\right]^q\ge 0\end{aligned}$$

Thus, $\upsilon_{NN}^{li}\ge\upsilon_{BN}^{li}\ge\upsilon_{PN}^{li}$ holds. □

**Remark 2.** For the three actions, namely acceptance $a_p$, delay $a_B$ and rejection $a_N$, the utility of accepting the right alternative exceeds those of delaying and rejecting the right alternative. The utility of accepting the right alternative is the highest and the utility of rejecting the right alternative is the least. In similar ways, the utility of rejecting the wrong alternative exceeds those of delaying and accepting the wrong alternative. The utility of rejecting the wrong alternative is the highest and the utility of accepting the wrong alternative is the least. Thus, $\upsilon_{PP}^{li}\ge\upsilon_{BP}^{li}\ge\upsilon_{NP}^{li}$ and $\upsilon_{NN}^{li}\ge\upsilon_{BN}^{li}\ge\upsilon_{PN}^{li}$ are reasonable in semantics.

### 4.2. The determination of thresholds

In this section, we will discuss the determination of thresholds. Based on the fact that $\Pr(X|a_l)+\Pr(X^c|a_l)=1$ and the

$$\alpha_{li}-\beta_{li}=\frac{v_{BN}^{li}-v_{PN}^{li}}{\left(v_{BN}^{li}-v_{PN}^{li}\right)+\left(v_{PP}^{li}-v_{BP}^{li}\right)}-\frac{v_{NN}^{li}-v_{BN}^{li}}{\left(v_{NN}^{li}-v_{BN}^{li}\right)+\left(v_{BP}^{li}-v_{NP}^{li}\right)}$$
$$=\frac{\left(v_{BP}^{li}-v_{NP}^{li}\right)\left(v_{NN}^{li}-v_{PN}^{li}\right)-\left(v_{PP}^{li}-v_{NP}^{li}\right)\left(v_{NN}^{li}-v_{BN}^{li}\right)}{\left(v_{BN}^{li}-v_{PN}^{li}+v_{PP}^{li}-v_{BP}^{li}\right)\left(v_{NN}^{li}-v_{BN}^{li}+v_{BP}^{li}-v_{NP}^{li}\right)}$$
$$=\frac{\left(\left\{(1-2\eta_i)\cdot\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\min}^i\right)\right)\right]\right\}^p+\theta\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\min}^i\right)\right)\right]^p\right)\cdot\left\{\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\max}^i\right)\right)\right]^p +\theta\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\max}^i\right)\right)\right]^q\right\}-\left(\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\max}^i\right)\right)\right]^p-\left\{(1-2\eta_i)\cdot\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\max}^i\right)\right)\right]\right\}^p\right)\cdot\left\{\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\min}^i\right)\right)\right]^p+\theta\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\min}^i\right)\right)\right]^p\right\}}{\left(v_{BN}^{li}-v_{PN}^{li}+v_{PP}^{li}-v_{BP}^{li}\right)\left(v_{NN}^{li}-v_{BN}^{li}+v_{BP}^{li}-v_{NP}^{li}\right)}$$

**Box I.**

$$\alpha_{li}-\beta_{li}\geq \frac{\theta^2\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\min}^i\right)\right)\right]^p\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\max}^i\right)\right)\right]^q-\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\min}^i\right)\right)\right]^p\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\max}^i\right)\right)\right]^p}{\left(v_{BN}^{li}-v_{PN}^{li}+v_{PP}^{li}-v_{BP}^{li}\right)\left(v_{NN}^{li}-v_{BN}^{li}+v_{BP}^{li}-v_{NP}^{li}\right)}$$

**Box II.**

conditions $v_{PP}^{li}\geq v_{BP}^{li}\geq v_{NP}^{li}$ and $v_{NN}^{li}\geq v_{BN}^{li}\geq v_{PN}^{li}$, we can rewrite the constraints (P3)–(N3) as follows:

For (P3), we can obtain $\Pr(X|a_l)\geq\frac{v_{BN}^{li}-v_{PN}^{li}}{\left(v_{BN}^{li}-v_{PN}^{li}\right)+\left(v_{PP}^{li}-v_{BP}^{li}\right)}$ according to $R(a_P|x_{li})\geq R(a_B|x_{li})$ and $\Pr(X|a_l)\geq\frac{v_{NN}^{li}-v_{PN}^{li}}{\left(v_{NN}^{li}-v_{PN}^{li}\right)+\left(v_{PP}^{li}-v_{NP}^{li}\right)}$ according to $R(a_P|x_{li})\geq R(a_N|x_{li})$.

For (B3), we can get $\Pr(X|a_l)\leq\frac{v_{BN}^{li}-v_{PN}^{li}}{\left(v_{BN}^{li}-v_{PN}^{li}\right)+\left(v_{PP}^{li}-v_{BP}^{li}\right)}$ according to $R(a_B|x_{li})\geq R(a_P|x_{li})$ and $\Pr(X|a_l)\geq\frac{v_{NN}^{li}-v_{BN}^{li}}{\left(v_{NN}^{li}-v_{BN}^{li}\right)+\left(v_{BP}^{li}-v_{NP}^{li}\right)}$ according to $R(a_B|x_{li})\geq R(a_N|x_{li})$.

For (N3), we can get $\Pr(X|a_l)\leq\frac{v_{NN}^{li}-v_{PN}^{li}}{\left(v_{NN}^{li}-v_{PN}^{li}\right)+\left(v_{PP}^{li}-v_{NP}^{li}\right)}$ according to $R(a_N|x_{li})\geq R(a_P|x_{li})$ and $\Pr(X|a_l)\leq\frac{v_{NN}^{li}-v_{BN}^{li}}{\left(v_{NN}^{li}-v_{BN}^{li}\right)+\left(v_{BP}^{li}-v_{NP}^{li}\right)}$ according to $R(a_N|x_{li})\geq R(a_B|x_{li})$.

Then,

$$\alpha_{li}=\frac{v_{BN}^{li}-v_{PN}^{li}}{\left(v_{BN}^{li}-v_{PN}^{li}\right)+\left(v_{PP}^{li}-v_{BP}^{li}\right)} \tag{18}$$

$$\beta_{li}=\frac{v_{NN}^{li}-v_{BN}^{li}}{\left(v_{NN}^{li}-v_{BN}^{li}\right)+\left(v_{BP}^{li}-v_{NP}^{li}\right)} \tag{19}$$

$$\gamma_{li}=\frac{v_{NN}^{li}-v_{PN}^{li}}{\left(v_{NN}^{li}-v_{PN}^{li}\right)+\left(v_{PP}^{li}-v_{NP}^{li}\right)} \tag{20}$$

Therefore, the thresholds are determined.

**Theorem 2.** *For $x\in(x_{\min},x_{\max})$, if $0<\eta_i\leq 0.5$, then the decision problem can be regarded as a three-way decision problem.*

**Proof.** The 3WD requires $\alpha_{li}>\beta_{li}$ [31], that is to say, $\alpha_{li}-\beta_{li}>0$. (See the equation in Box I.)

Because of $0<\eta_i\leq 0.5$, it means $0\leq 1-2\eta_i\leq 1$, Therefore, we have the equation in Box II.

It is known that $d(x_{li},x_{\max})\geq 0$ and $d(x_{li},x_{\min})\geq 0$, then we have $\exp(-\delta\cdot d(x_{li},x_{\min}))\leq 1$ and $\exp(-\delta\cdot d(x_{li},x_{\max}))\leq 1$. Thus, $1-\exp(-\delta\cdot d(x_{li},x_{\min}))\geq 0$ and $1-\exp(-\delta\cdot d(x_{li},x_{\max}))\geq 0$.

Hence, see the equation in Box III.

According to prospect theory, the decision maker is more sensitive to the losses than to the equal gains [47,61]. Hence, we can prove $\alpha_{li}-\beta_{li}>0$. That is to say, if $0<\eta_i\leq 0.5$, then the decision problem can be regarded as a 3WD problem. □

**Table 5**
Relationship between $\eta_i$ and $(\alpha_{li},\beta_{li},\gamma_{li})$.

| $\eta_i$ | $\alpha_i,\beta_i,\gamma_i$ |
|---|---|
| $\eta_i=0$ | $\alpha_{li}=1,\beta_{li}=0$ |
| $0<\eta_i\leq 0.5$ | $0<\beta_{li}<\gamma_{li}<\alpha_{li}<1$ |
| $0.5<\eta_i<1$ | $0<\alpha_{li}<\gamma_{li}<\beta_{li}<1$ |
| $\eta_i=1$ | $\alpha_{li}=0,\beta_{li}=1$ |

The relationships between 3WD models and different $\eta_i$ are shown in Fig. 4; and the relationships between $\eta_i$ and thresholds $(\alpha_{li},\beta_{li},\gamma_{li})$ are summarized in Table 5.

From Fig. 4 and Table 5, we can know that when $\eta_i=0$, the 3WD degrades into Pawlak's rough set; when $0<\eta_i\leq 0.5$, the thresholds meet the requirement of the 3WD; and when $0.5<\eta_i\leq 1$, the 3WD degrades into the two-way decision.

### 4.3. The calculation of conditional probability

In 3WD, the estimation and evaluation of the conditional probability is a crucial problem. Liang et al. [62] demonstrated that the positive ideal solution (PIS) and negative ideal solution (NIS) in TOPSIS method correspond to the two decision states in 3WD, and the conditional probability can be calculated by means of the relative closeness degree. Inspired by [62], this paper implies the TOPSIS method to elaborate the determination of conditional probability.

Normally speaking, the maximum of the attribute will be selected as PIS and the minimum of the attribute will be as NIS.

$$\alpha_{li}-\beta_{li}\geq\frac{\theta^2\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\max}^i\right)\right)\right]^q-\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\max}^i\right)\right)\right]^p}{\left(\upsilon_{BN}^{li}-\upsilon_{PN}^{li}+\upsilon_{PP}^{li}-\upsilon_{BP}^{li}\right)\left(\upsilon_{NN}^{li}-\upsilon_{BN}^{li}+\upsilon_{BP}^{li}-\upsilon_{NP}^{li}\right)}$$
$$\geq\frac{\theta\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\max}^i\right)\right)\right]^q-\left[1-\exp\left(-\delta\cdot d\left(x_{li},x_{\max}^i\right)\right)\right]^p}{\left(\upsilon_{BN}^{li}-\upsilon_{PN}^{li}+\upsilon_{PP}^{li}-\upsilon_{BP}^{li}\right)\left(\upsilon_{NN}^{li}-\upsilon_{BN}^{li}+\upsilon_{BP}^{li}-\upsilon_{NP}^{li}\right)}.$$

**Box III.**

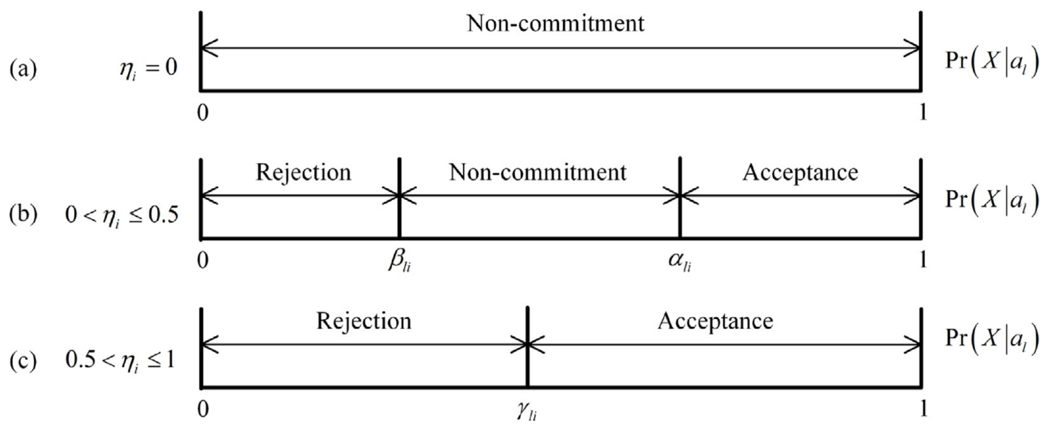


**Fig. 4.** 3WD model with different values of $\eta_i$.

Hence, the PIS and NIS of alternatives $a_l$ are first confirmed by:

$$a_l^+=\left\{a_1^+,a_2^+,\ldots,a_M^+\right\}=\left\{\max_i x_{1i},\max_i x_{2i},\ldots,\max_i x_{Mi}\right\}=\max_i x_{li} \tag{21}$$

$$a_l^-=\left\{a_1^-,a_2^-,\ldots,a_M^-\right\}=\left\{\min_i x_{1i},\min_i x_{2i},\ldots,\min_i x_{Mi}\right\}=\min_i x_{li} \tag{22}$$

Then, the relative closeness of alternative $a_l$ is calculated as follows.

$$RC\left(a_l\right)=\frac{AD\left(a_l,a_l^-\right)}{AD\left(a_l,a_l^-\right)+AD\left(a_l,a_l^+\right)} \tag{23}$$

$$AD\left(a_l,a_l^-\right)=\sum_{i=1}^{L}w_iD\left(x_{li},a_l^-\right) \tag{24}$$

$$AD\left(a_l,a_l^+\right)=\sum_{i=1}^{L}w_iD\left(x_{li},a_l^+\right) \tag{25}$$

In which $AD\left(a_l,a_l^-\right)$ is the distance between alternative $a_l$ and alternative $a_l^-$, and $AD\left(a_l,a_l^+\right)$ is the distance between alternative $a_l$ and alternative $a_l^+$, $RC\left(a_l\right)$ represents the probability of alternative $a_l$ belonging to the state $X$.

Hence, the value of the conditional probability of alternative $a_l$ is determined by $\Pr\left(X\left|a_l\right.\right)=RC\left(a_l\right)$.

### 4.4. The ranking regulations for alternatives

3WD can not only classify all alternatives into *POS*, *BND* or *NEG* regions, but also can provide a complete ranking order. This is important because in some case, it may be hard to make final decision only based on the classification [30]. Besides, the ranking of alternatives can assist decision maker in selecting an optimal alternative and allocating limited resources [63]. Hence, the ranking regulation is necessary.

In this paper, the ranking regulations consist of two phases. First, the priority principle is determined. To be specific, the alternatives in $POS\left(X\right)$ are superior to the alternatives in $BND\left(X\right)$ and alternatives in $BND\left(X\right)$ are better than those in $NEG\left(X\right)$. Hence, the regions of alternatives are firstly determined by the following rules:

Based on Theorem 1 in Section 3.3, the decision rules are provided as follows:

(P4) If $\Pr\left(X|a_l\right)\geq\alpha$ and $\Pr\left(X|a_l\right)\geq\gamma$, then $a_l\in POS\left(X\right)$;
(B4) If $\Pr\left(X|a_l\right)\leq\alpha$ and $\Pr\left(X|a_l\right)\geq\beta$, then $a_l\in BND\left(X\right)$;
(N4) If $\Pr\left(X|a_l\right)\leq\beta$ and $\Pr\left(X|a_l\right)\leq\gamma$, then $a_l\in NEG\left(X\right)$.

If there is an assumption that $\left(\upsilon_{BP}-\upsilon_{PN}\right)\left(\upsilon_{BP}-\upsilon_{NP}\right)\leq\left(\upsilon_{PP}-\upsilon_{BP}\right)\left(\upsilon_{NN}-\upsilon_{BN}\right)$, then the thresholds meet $0<\beta<\gamma<\alpha<1$. The rules (P4)–(N4) can be simplified as follows:

(P5) If $\Pr\left(X|a_l\right)\geq\alpha$, then $a_l\in POS\left(X\right)$;
(B5) If $\beta<\Pr\left(X|a_l\right)<\alpha$, then $a_l\in BND\left(X\right)$;
(N5) If $\Pr\left(X|a_l\right)\leq\beta$, then $a_l\in NEG\left(X\right)$.

Second, the ranking of alternatives in the same regions follows the principle of maximizing utility. Utility is the main factor deciding whether an alternative should be chosen or not, which can also be used to explain the semantics of three rules in 3WD [63]. The overall utility value can be calculated by the following function.

$$\text{Utility}=\begin{cases}\sum_{i=1}^{L}R\left(a_P\left|x_{li}\right.\right), & if\ a_l\in POS\left(X\right)\\ \sum_{i=1}^{L}R\left(a_B\left|x_{li}\right.\right), & if\ a_l\in BND\left(X\right)\\ \sum_{i=1}^{L}R\left(a_N\left|x_{li}\right.\right), & if\ a_l\in NEG\left(X\right)\end{cases} \tag{26}$$

The higher the utility value is, the better the alternative will be.

**Table 6**
The decision matrix in the form of linguistic terms.

| | $e_1$ | $e_2$ | ... | $e_L$ |
|---|---|---|---|---|
| $a_1$ | $x_{11}$ | $x_{12}$ | ... | $x_{1L}$ |
| $a_2$ | $x_{21}$ | $x_{22}$ | ... | $x_{2L}$ |
| ... | ... | ... | ... | ... |
| $a_M$ | $x_{M1}$ | $x_{M2}$ | ... | $x_{ML}$ |

## 5. The decision process based on the proposed 3WD method

In this section, we present a complete decision process based on the proposed 3WD method. In this decision process, we first construct the utility function based on the prospect theory and regret–rejoice function. Then, we design a mechanism to employ the ER algorithm to combine multiple attributes information. Finally, the 3WD rules are developed to obtain the decision results.

For a MADM problem, it involves $M$ alternatives $a_l(l = 1, \ldots, M)$ to be evaluated regarding $L$ attributes $e_i$ $(i = 1, \ldots, L)$. The weight of attribute $e_i$ is denoted by $w_i$, where $0 \leq w_i \leq 1$ and $\sum_{i=1}^{L} w_i = 1$. The decision maker gives opinion about this MADM problem by linguistic terms. The initial preference information can be collected and expressed by a matrix $D = (x_{li})_{M\times L}$ as shown in Table 6.

**Step 1**. Determine the utility function $v(x_{li})$ according to Table 4. In the initial decision matrix $D = \left(x_{ij}\right)_{M\times L}$, the information is characterized by linguistic terms, which cannot be computed directly. In our previous study [53], we proposed a method that employs the shadowed sets to model linguistic terms and proposed a distance measure model based on shadowed sets to measure the relationships between linguistic terms, which is defined as follows:

**Definition 5** (*[53]*). Let $A = (A_1, A_2, \ldots, A_n)$ and $B = (B_1, B_2, \ldots, B_n)$ be any two shadowed sets. Then, the centroid-based generalized Minkowski distance model can be calculated by

$$D(A, B) = \left[\sum_{k=1}^{n} |C(A_k) - C(B_k)|^{\tau}\right]^{\frac{1}{\tau}} \tag{27}$$

$$C(A_k) = \frac{\int_{x_{\min}}^{x_{\max}} \mu_{A_k}(x)\, xdx}{\int_{x_{\min}}^{x_{\max}} \mu_{A_k}(x)\, dx};\ C(B_k) = \frac{\int_{x_{\min}}^{x_{\max}} \mu_{B_k}(x)\, xdx}{\int_{x_{\min}}^{x_{\max}} \mu_{B_k}(x)\, dx}$$

where $\mu(x)$ is the membership function of shadowed set.

If $\tau = 1$, then the centroid-based generalized Minkowski distance model will be reduced to the centroid-based generalized Hamming distance model.

If $\tau = 2$, then the centroid-based generalized Minkowski distance model will be reduced to the centroid-based generalized Euclidean distance model.

If $\tau \to \infty$, then the centroid-based generalized Minkowski distance model will be reduced to the centroid-based generalized Chebyshev distance model.

For more details, please refer to [53].

**Step 2**. Calculate the thresholds $\alpha_{li}$ and $\beta_{li}$ by Eqs. (18) and (20).

**Step 3**. Construct decision matrix with distributed assessment $S(e_i(a_l)) = \left\{\left(H_n, \zeta_{n,i}(a_l)\right) n = 1, \ldots, N\right\} = \left\{(N, \beta_{li}), (B, \alpha_{li} - \beta_{li}), (P, 1-\alpha_{li})\right\}$, $l = 1, \ldots, M$, $i = 1, \ldots, L$. In ER algorithm, when the alternative $a_l$ is evaluated as grade $H_n$ regarding attribute $e_i$ with a degree of belief $\zeta_{n,i}(a_l)$, we could denote this evaluation by a distributed assessment or belief structure $S(e_i(a_l)) = \left\{\left(H_n, \zeta_{n,i}(a_l)\right) n = 1, \ldots, N\right\}$, where $\zeta_{n,i}(a_l) \geq 0$ and $\sum_{n=1}^{N} \zeta_{n,i}(a_l) \leq 1$. If $\sum_{n=1}^{N} \zeta_{n,i}(a_l) = 1$, the assessment is complete; if not, it is incomplete.

According to the decision rules of 3WD, we can conclude that for $\Pr(X_i\,|[x_{li}]) \geq \alpha_{li}$, $x_{li} \in POS(X)$; for $\beta_{li} < \Pr(X_i\,|[x_{li}]) < \alpha_{li}$, $x_{li} \in BND(X)$ and for $\Pr(X_i\,|[x_{li}]) \leq \beta_{li}$, $x_{li} \in NEG(X)$. That means for alternative $x_{li}$, it has three possible outcomes, i.e., $x_{li} \in POS(X)$, $x_{li} \in BND(X)$ and $x_{li} \in NEG(X)$ with the possibilities of $1 - \alpha_{li}$, $\alpha_{li} - \beta_{li}$ and $\beta_{li}$, which can be directly observed from Fig. 4(b). In other word, the *POS*, *BND* and *NEG* compose a collectively exhaustive and mutually exclusive set, and the possibilities $1-\alpha_{li}$, $\alpha_{li} - \beta_{li}$ and $\beta_{li}$ are referred to the degree of belief. The evaluation set and its corresponding belief degree compose a belief structure, which is in coordinate with the belief structure in ER algorithm. Then, it can be written as $S(e_i(a_l)) = \{(N, \beta_{li}), (B, \alpha_{li} - \beta_{li}), (P, 1-\alpha_{li})\}$, where $1-\alpha_{li}$, $\alpha_{li} - \beta_{li}$ and $\beta_{li}$ are not less than 0 and $(1-\alpha_{li}) + (\alpha_{li} - \beta_{li}) + \beta_{li} = 1$, which means the assessment is complete.

**Step 4**. Obtain the basic probability masses $\zeta_n$ by the ER algorithm. Then, we can obtain the combined decision information $S(e(a_l)) = \{(H_n, \zeta_n(a_l)), n = 1, \ldots, N\}$, $l = 1, \ldots, M$.

The combination way of evidence in the recursive ER algorithm is clear in concept, but it may be undesirable in the situations requiring an explicit ER aggregation function, such as in optimization. Therefore, Wang et al. [64] proposed an analytical ER algorithm, which is equivalence to the recursive ER algorithm and makes the ER algorithm more flexible in aggregating attributes.

First, the basic probability masses are obtained by combining the relative weights and the degree of belies by the following equations:

$$m_{n,i} = m_i(H_n) = w_i \zeta_{n,i}(a_l),\ \ n = 1, \ldots, N;\ i = 1, \ldots, L \tag{28}$$

$$m_{H,i} = m_i(H) = 1 - \sum_{n=1}^{N} m_{n,i} = 1 - w_i \sum_{n=1}^{N} \zeta_{n,i}(a_l),\ \ i = 1, \ldots, L \tag{29}$$

$$\overline{m}_{H,i} = \overline{m}_i(H) = 1 - w_i,\ \ i = 1, \ldots, L \tag{30}$$

$$\widetilde{m}_{H,i} = \widetilde{m}_i(H) = w_i\left(1 - \sum_{n=1}^{N} \zeta_{n,i}(a_l)\right),\ \ i = 1, \ldots, L \tag{31}$$

$$m_{H,i} = \overline{m}_{H,i} + \widetilde{m}_{H,i} \tag{32}$$

$$\sum_{i=1}^{L} w_i = 1 \tag{33}$$

Then, the basic probability masses are combined by the analytical ER algorithm:

$$\{H_n\}: m_n = k\left[\prod_{i=1}^{L}\left(m_{n,i} + \overline{m}_{H,i} + \widetilde{m}_{H,i}\right) - \prod_{i=1}^{L}\left(\overline{m}_{H,i} + \widetilde{m}_{H,i}\right)\right],\quad n = 1, \ldots, N, \tag{34}$$

$$\{H\}: \widetilde{m} = k\left[\prod_{i=1}^{L}\left(\overline{m}_{H,i} + \widetilde{m}_{H,i}\right) - \prod_{i=1}^{L} \overline{m}_{H,i}\right], \tag{35}$$

$$\{H\}: \overline{m} = k\left[\prod_{i=1}^{L} \overline{m}_{H,i}\right], \tag{36}$$

$$k = \left[\sum_{n=1}^{N}\prod_{i=1}^{L}\left(m_{n,i} + \overline{m}_{H,i} + \widetilde{m}_{H,i}\right) - (N-1)\prod_{i=1}^{L}\left(\overline{m}_{H,i} + \widetilde{m}_{H,i}\right)\right]^{-1}, \tag{37}$$

**Table 7**
The linguistic terms and their corresponding shadowed numbers.

| Linguistic terms | Shadowed numbers |
|---|---|
| Extremely dissatisfied (*ED*) | (0.23, 0.30, 1.01, 1.28) |
| Very dissatisfied (*VD*) | (1.01, 1.39, 2.32, 2.79) |
| Dissatisfied (*D*) | (2.35, 3.32, 3.73, 4.87) |
| Medium (*M*) | (4.02, 4.85, 5.40, 6.37) |
| Satisfied (*S*) | (5.75, 6.51, 7.20, 8.12) |
| Very satisfied (*VS*) | (7.47, 7.63, 8.61, 8.89) |
| Extremely satisfied (*ES*) | (8.72, 9.31, 9.59, 9.89) |

$$\{H_n\} : \zeta_n = \frac{m_n}{1-\overline{m}_H}, \ n = 1, \ldots, N, \tag{38}$$

$$\{H\} : \zeta_H = \frac{\widetilde{m}_H}{1-\overline{m}_H}. \tag{39}$$

**Step 5**. Calculate the thresholds $\alpha_l$ and $\beta_l$ according to the combined information $S(e(a_l)) = \{(H_n, \zeta_n(a_l)), n = 1, \ldots, N\} = \{(N, \zeta_1(a_l)), (B, \zeta_2(a_l)), (P, \zeta_3(a_l))\}$.

$$\alpha_l = 1 - \zeta_3(a_l) \tag{40}$$

$$\beta_l = \zeta_1(a_l) \tag{41}$$

**Step 6**. Determine the conditional probability $\Pr(X|a_l)$ by Eqs. (21)–(25).

**Step 7**. Classify all alternatives based on the decision rules (P5) -(N5) and rank all alternatives in the same region according to the expected utilities.

## 6. Numerical analysis

In this section, two numerical examples are presented. The first one is emulation study with a large-scale data set, which aims at manifesting the feasibility and applicability of the proposed method. The other one is from the case study about COVID-19 drug selection in Mishra et al.'s research [8], which is an application of the proposed method and contributes to illustrate the implementation process of the proposed method. Then, a comparative analysis is conducted to show its superiority and a sensitivity analysis is designed to test the robustness of the proposed method.

### 6.1. Numerical example

#### 6.1.1. Numerical example 1—illustrative example with large-scale data

In this section, a numerical example with large-scale data set is provided to illustrate the feasibility and applicability of the proposed method. The numerical example used in this section is randomly generated, which consists of 300 alternatives denotes as $\{a_1, a_2, \ldots, a_{300}\}$ and 10 attributes represented by $\{e_1, e_2, \ldots, e_{10}\}$. For the sake of space, the preference information has been included as a supplementary material document, which is available online.[1]

In this example, the weights of attributes are set to be the same and the values of parameters are: $p = q = 0.88$, $\theta = 2.25$, $\delta = 0.3$ and $\eta_i = 0.45$. As stated in Section 3.3, a data-driven method is employed to obtain the shadowed sets corresponding to linguistic terms in our previous study. According to this method, the seven-level linguistic terms and their corresponding shadowed numbers are shown in Table 7. For more details, please refer to [53].

[1] [Online] Available: http://sinbad2.ujaen.es/sites/default/files/2022-04/Numerical%20example%201_0.pdf.

**Table 8**
The evaluation matrix in the form of linguistic terms.

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ | $e_6$ | $e_7$ |
|---|---|---|---|---|---|---|---|
| $a_1$ | *S* | *M* | *D* | *S* | *M* | *D* | *D* |
| $a_2$ | *M* | *S* | *M* | *M* | *M* | *D* | *M* |
| $a_3$ | *M* | *S* | *M* | *M* | *D* | *VD* | *VD* |
| $a_4$ | *VS* | *S* | *M* | *M* | *M* | *S* | *S* |
| $a_5$ | *M* | *S* | *D* | *M* | *VD* | *D* | *M* |

Due to the restriction of space, we only present the decision results as shown in Fig. 5. The detail implementation process of the proposed method will be given in numerical example 2. From Fig. 5, we can not only know the ranking order, that is, for all 300 alternatives, the 200rd alternative ranks the first and the 176th alternative ranks the last, but also know the classification results. With the help of 3WD rules, all alternatives are divided into three parts, i.e., POS, BND and NEG. For alternatives in NEG regions, they should not be chosen in any case; for alternatives in BND regions, selecting it or not needs more information and consideration; for alternatives in POS regions, they can be chosen and the order of selecting them is further determined by expected utilities.

#### 6.1.2. Numerical example 2-application to COVID-19 drug selection

To show the effectiveness and the superiority of proposed method in tackling drug selection problem, we further applied the proposed method to solve the COVID-19 drug selection problem. This numerical example is from the case study in Mishra et al.'s research [8].

**(1) Background description**

To select the suitable drug for the treatment of the patients with mild symptoms of COVID-19, Mishra et al. developed a questionnaire to collect information from doctors and patients. The doctors investigated are qualified virologists with rich experience in medical field and have been trained about drug usage on patients who have appeared COVID-19 symptoms. After the processes of questionnaire collection, Delphi surveys and language normalization, the decision matrix was obtained.

To be specific, five medicines are selected to be alternatives to treat the patients who are suffering COVID-19, which are LPV/RTV-IFNb ($a_1$), Favipiravir ($a_2$), LPV/PTV ($a_3$), Remdesivir ($a_4$) and Hydroxychloroquine ($a_5$), and seven attributes are determined to measure the impact, the performance and the possible side effects of these medicines, namely, Anorexia ($e_1$), Cough ($e_2$), Fatigue ($e_3$), Fever ($e_4$), Myalgia ($e_5$), Shortness of breath ($e_6$) and Sputum production ($e_7$). The corresponding weights of these attributes are $w_i = \{0.150, 0.163, 0.145, 0.176, 0.117, 0.127, 0.123\}$. Mishra et al. [8] had collected the performances of five medicines with regard to the seven attributes of three experts, which are in the form of seven-level linguistic terms {Extremely dissatisfied (ED), Very dissatisfied (VD), Dissatisfied (D), Medium (M), Satisfied (S), Very satisfied (VS) and Extremely satisfied (ES)}. In this paper, we employed the decision matrix of expert 1 to show the implement process of the proposed method. The initial decision matrix is shown in Table 8.

**(2) Implementation of the proposed method**

Employing the proposed method to solve this problem mainly includes the following steps:

**Step 1**. Determine the utility function $v(x_{li})$ according to Table 4. The results are shown in Table 9.

**Step 2**. Calculate the thresholds $\alpha_{li}$ and $\beta_{li}$ by Eqs. (18) and (20). The results are shown in Table 10.

**Step 3**. Construct decision matrix with distributed assessment $S(e_i(a_l)) = \{(H_n, \zeta_{n,i}(a_l)) \ n = 1, \ldots, N\} = \{(N, \beta_{li}), (B, \alpha_{li} - \beta_{li}), (P, 1 - \alpha_{li})\}$, $l = 1, \ldots, M$, $i = 1, \ldots, L$ as shown in Table 11.

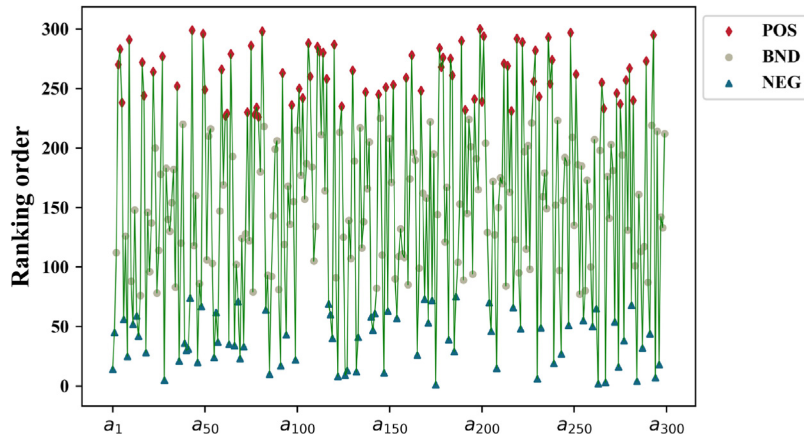


**Fig. 5.** Results of all alternatives in numerical example 1.

**Table 9**
The utility function $v(x_{li})$.

| | | $e_1$ | | $e_2$ | | $e_3$ | | $e_4$ | | $e_5$ | | $e_6$ | | $e_7$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $X$ | $X^c$ | $X$ | $X^c$ | $X$ | $X^c$ | $X$ | $X^c$ | $X$ | $X^c$ | $X$ | $X^c$ | $X$ | $X^c$ |
| $a_1$ | P | 0.862 | −1.256 | 0.766 | −1.669 | 0.618 | −1.892 | 0.862 | −1.256 | 0.766 | −1.669 | 0.618 | −1.892 | 0.618 | −1.892 |
| | B | 0.114 | 0.074 | 0.101 | 0.098 | 0.081 | 0.111 | 0.114 | 0.074 | 0.101 | 0.098 | 0.081 | 0.111 | 0.081 | 0.111 |
| | N | −1.939 | 0.558 | −1.723 | 0.742 | −1.391 | 0.841 | −1.939 | 0.558 | −1.723 | 0.742 | −1.391 | 0.841 | −1.391 | 0.841 |
| $a_2$ | P | 0.766 | −1.669 | 0.862 | −1.256 | 0.766 | −1.669 | 0.766 | −1.669 | 0.766 | −1.669 | 0.618 | −1.892 | 0.766 | −1.669 |
| | B | 0.101 | 0.098 | 0.114 | 0.074 | 0.101 | 0.098 | 0.101 | 0.098 | 0.101 | 0.098 | 0.081 | 0.111 | 0.101 | 0.098 |
| | N | −1.723 | 0.742 | −1.939 | 0.558 | −1.723 | 0.742 | −1.723 | 0.742 | −1.723 | 0.742 | −1.391 | 0.841 | −1.723 | 0.742 |
| $a_3$ | P | 0.766 | −1.669 | 0.862 | −1.256 | 0.766 | −1.669 | 0.766 | −1.669 | 0.618 | −1.892 | 0.343 | −2.037 | 0.343 | −2.037 |
| | B | 0.101 | 0.098 | 0.114 | 0.074 | 0.101 | 0.098 | 0.101 | 0.098 | 0.081 | 0.111 | 0.045 | 0.119 | 0.045 | 0.119 |
| | N | −1.723 | 0.742 | −1.939 | 0.558 | −1.723 | 0.742 | −1.723 | 0.742 | −1.391 | 0.841 | −0.771 | 0.905 | −0.771 | 0.905 |
| $a_4$ | P | 0.905 | −0.774 | 0.862 | −1.256 | 0.766 | −1.669 | 0.766 | −1.669 | 0.766 | −1.669 | 0.862 | −1.256 | 0.862 | −1.256 |
| | B | 0.119 | 0.045 | 0.114 | 0.074 | 0.101 | 0.098 | 0.101 | 0.098 | 0.101 | 0.098 | 0.114 | 0.074 | 0.114 | 0.074 |
| | N | −2.036 | 0.344 | −1.939 | 0.558 | −1.723 | 0.742 | −1.723 | 0.742 | −1.723 | 0.742 | −1.939 | 0.558 | −1.939 | 0.558 |
| $a_5$ | P | 0.766 | −1.669 | 0.862 | −1.256 | 0.618 | −1.892 | 0.766 | −1.669 | 0.343 | −2.037 | 0.618 | −1.892 | 0.766 | −1.669 |
| | B | 0.101 | 0.098 | 0.114 | 0.074 | 0.081 | 0.111 | 0.101 | 0.098 | 0.045 | 0.119 | 0.081 | 0.111 | 0.101 | 0.098 |
| | N | −1.723 | 0.742 | −1.939 | 0.558 | −1.391 | 0.841 | −1.723 | 0.742 | −0.771 | 0.905 | −1.391 | 0.841 | −1.723 | 0.742 |

**Table 10**
The thresholds $\alpha_{li}$ and $\beta_{li}$.

| | $e_1$ | | $e_2$ | | $e_3$ | | $e_4$ | | $e_5$ | | $e_6$ | | $e_7$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\alpha_{l1}$ | $\beta_{l1}$ | $\alpha_{l2}$ | $\beta_{l2}$ | $\alpha_{l3}$ | $\beta_{l3}$ | $\alpha_{l4}$ | $\beta_{l4}$ | $\alpha_{l5}$ | $\beta_{l5}$ | $\alpha_{l6}$ | $\beta_{l6}$ | $\alpha_{l7}$ | $\beta_{l7}$ |
| $a_1$ | 0.640 | 0.191 | 0.727 | 0.261 | 0.789 | 0.331 | 0.640 | 0.191 | 0.727 | 0.261 | 0.789 | 0.331 | 0.789 | 0.331 |
| $a_2$ | 0.727 | 0.261 | 0.640 | 0.191 | 0.727 | 0.261 | 0.727 | 0.261 | 0.727 | 0.261 | 0.789 | 0.331 | 0.727 | 0.261 |
| $a_3$ | 0.727 | 0.261 | 0.640 | 0.191 | 0.727 | 0.261 | 0.727 | 0.261 | 0.789 | 0.331 | 0.879 | 0.491 | 0.879 | 0.491 |
| $a_4$ | 0.511 | 0.122 | 0.640 | 0.191 | 0.727 | 0.261 | 0.727 | 0.261 | 0.727 | 0.261 | 0.640 | 0.191 | 0.640 | 0.191 |
| $a_5$ | 0.727 | 0.261 | 0.640 | 0.191 | 0.789 | 0.331 | 0.727 | 0.261 | 0.879 | 0.491 | 0.789 | 0.331 | 0.727 | 0.261 |

**Table 11**
Decision matrix with distributed assessment.

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ | $e_6$ | $e_7$ |
|---|---|---|---|---|---|---|---|
| $a_1$ | $\{(N,0.191),(B,0.449),(P,0.360)\}$ | $\{(N,0.261),(B,0.466),(P,0.273)\}$ | $\{(N,0.331),(B,0.457),(P,0.211)\}$ | $\{(N,0.191),(B,0.449),(P,0.360)\}$ | $\{(N,0.261),(B,0.466),(P,0.273)\}$ | $\{(N,0.331),(B,0.457),(P,0.211)\}$ | $\{(N,0.331),(B,0.457),(P,0.211)\}$ |
| $a_2$ | $\{(N,0.261),(B,0.466),(P,0.273)\}$ | $\{(N,0.191),(B,0.449),(P,0.360)\}$ | $\{(N,0.261),(B,0.466),(P,0.273)\}$ | $\{(N,0.261),(B,0.466),(P,0.273)\}$ | $\{(N,0.261),(B,0.466),(P,0.273)\}$ | $\{(N,0.331),(B,0.457),(P,0.211)\}$ | $\{(N,0.261),(B,0.466),(P,0.273)\}$ |
| $a_3$ | $\{(N,0.261),(B,0.466),(P,0.273)\}$ | $\{(N,0.191),(B,0.449),(P,0.360)\}$ | $\{(N,0.261),(B,0.466),(P,0.273)\}$ | $\{(N,0.261),(B,0.466),(P,0.273)\}$ | $\{(N,0.331),(B,0.457),(P,0.211)\}$ | $\{(N,0.491),(B,0.388),(P,0.121)\}$ | $\{(N,0.491),(B,0.388),(P,0.121)\}$ |
| $a_4$ | $\{(N,0.122),(B,0.389),(P,0.489)\}$ | $\{(N,0.191),(B,0.449),(P,0.360)\}$ | $\{(N,0.261),(B,0.466),(P,0.273)\}$ | $\{(N,0.261),(B,0.466),(P,0.273)\}$ | $\{(N,0.261),(B,0.466),(P,0.273)\}$ | $\{(N,0.191),(B,0.449),(P,0.360)\}$ | $\{(N,0.191),(B,0.449),(P,0.360)\}$ |
| $a_5$ | $\{(N,0.261),(B,0.466),(P,0.273)\}$ | $\{(N,0.191),(B,0.449),(P,0.360)\}$ | $\{(N,0.331),(B,0.457),(P,0.211)\}$ | $\{(N,0.261),(B,0.466),(P,0.273)\}$ | $\{(N,0.491),(B,0.388),(P,0.121)\}$ | $\{(N,0.331),(B,0.457),(P,0.211)\}$ | $\{(N,0.261),(B,0.466),(P,0.273)\}$ |

**Table 12**
The combined information.

| | $e$ |
|---|---|
| $a_1$ | $\{(N, 0.253), (B, 0.481), (P, 0.266)\}$ |
| $a_2$ | $\{(N, 0.245), (B, 0.486), (P, 0.269)\}$ |
| $a_3$ | $\{(N, 0.307), (B, 0.464), (P, 0.230)\}$ |
| $a_4$ | $\{(N, 0.196), (B, 0.467), (P, 0.337)\}$ |
| $a_5$ | $\{(N, 0.285), (B, 0.474), (P, 0.240)\}$ |

**Table 13**
The thresholds $\alpha_l$ and $\beta_l$.

| | $\alpha_l$ | $\beta_l$ |
|---|---|---|
| $a_1$ | 0.734 | 0.253 |
| $a_2$ | 0.731 | 0.245 |
| $a_3$ | 0.770 | 0.307 |
| $a_4$ | 0.663 | 0.196 |
| $a_5$ | 0.760 | 0.285 |

**Step 4**. Obtain the basic probability masses $\zeta_n$ according to Eqs. (28)–(41). The results are shown in Table 12.

**Step 5**. Calculate the thresholds $\alpha_l$ and $\beta_l$. The results are shown in Table 13.

**Step 6**. Determine the conditional probability $\Pr(X|a_l)$ by Eqs. (21)–(25). Then, we can obtain $\Pr(X|a_1) = 0.475$, $\Pr(X|a_2) = 0.523$, $\Pr(X|a_3) = 0.245$, $\Pr(X|a_4) = 0.895$ and $\Pr(X|a_5) = 0.312$.

**Step 7**. Classify all alternatives based on the decision rules (P5)–(N5) and rank all alternatives in the same region according to the expected utilities. Then, we can obtain $POS(X) = \{a_4\}$, $BND(X) = \{a_1, a_2, a_5\}$ and $NEG(X) = \{a_3\}$. And based on the expected utility, the ranking order is $a_4 \succ a_2 \succ a_5 \succ a_1 \succ a_3$.

The decision result reveals: (1) Based on the opinion of expert 1, Remdesivir ($a_4$) is the most desirable medicine, the effectiveness of LPV/RTV-IFNb ($a_1$), Favipiravir ($a_2$) and Hydroxychloroquine ($a_5$) should be further tested and LPV/PTV ($a_3$) should not be chosen. (2) In the same way, we also can obtain the decision results of expert 2 and expert 3. For expert 2, the decision result is $POS(X) = \{a_3\}$, $BND(X) = \{a_1, a_4, a_5\}$ and $NEG(X) = \{a_2\}$. The ranking order are $a_3 \succ a_5 \succ a_4 \succ a_1 \succ a_2$. For expert 3, the decision result is $POS(X) = \varnothing$, $BND(X) = \{a_1, a_2, a_3, a_4, a_5\}$ and $NEG(X) = \varnothing$. The ranking order is $a_5 \succ a_4 \succ a_2 \succ a_1 \succ a_3$. The decision results are different even paradox. This is common when dealing with a new disease. In this case, the involvement of new experts and the discussion between group is advised to reach consensus. (3) When making the final group decision, the classification should be priority to the ranking. To be specific, the alternative medicines in the positive region will be first considered. If all experts agree with the medicines in the positive region, then the decision results will be generated based on the ranking order.

### 6.2. Comparative analysis

In order to show the effectiveness and superiority of the proposed method, we respectively compare it with the hesitant fuzzy group decision-making method proposed by Mishra et al. [8], 3WD under MADM proposed by Jia and Liu [31], 3WD based on regret theory proposed by Huang and Zhan [65] and 3WD based on prospect theory proposed by Liang et al. [34]. Except for Mishra et al.'s method, other methods did not involve group decision. To make a fair comparison, we employ the aggregated matrix in [8] (as shown in Table 14) to conduct the comparative analysis and the results are summarized in Table 15.

**Table 14**
The aggregated matrix in [8].

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ | $e_6$ | $e_7$ |
|---|---|---|---|---|---|---|---|
| $a_1$ | 0.684 | 0.608 | 0.512 | 0.633 | 0.450 | 0.423 | 0.370 |
| $a_2$ | 0.666 | 0.554 | 0.572 | 0.608 | 0.476 | 0.454 | 0.373 |
| $a_3$ | 0.696 | 0.633 | 0.549 | 0.633 | 0.454 | 0.469 | 0.401 |
| $a_4$ | 0.804 | 0.649 | 0.680 | 0.624 | 0.729 | 0.600 | 0.604 |
| $a_5$ | 0.604 | 0.690 | 0.615 | 0.644 | 0.548 | 0.547 | 0.381 |

(1) **The superiority of 3WD in medical decision**. Mishra et al.'s method [8] was also developed for the drug selection to treat the mild symptoms of COVID-19. Compared with other 3WD-based methods, their method directly output the ranking order of alternatives. That means that the decision maker only relies on the ranking order to make the decision regardless of whether the alternative is good or not. This will increase the risk in medical decision and even lead to a dangerous situation. For example, when evaluating the performance of medicines, five alternatives may all belong to negative region, namely, $NEG = \{a_1, a_2, a_3, a_4, a_5\}$. But when ranking them by two-way decision methods, a ranking order will be obtained such as $a_4 \succ a_5 \succ a_3 \succ a_1 \succ a_2$. According to the result of two-way decision, $a_4$ will be chosen. However, even though $a_4$ is the first option, it is still of bad quality because it is in the negative region. Choosing it will inevitably increase the decision risk. In medical decision, this might be a threat to the patients' life. In this case, 3WD is a sensible tool. It can not only provide a complete ranking order of alternatives, but also provide their classification. This can largely decrease the risk in medical decision.

(2) **The consideration of psychological behavior**. Mishra et al.'s method [8] as well as Jia and Liu's method [31] did not consider the psychological behavior of decision makers. However, human is bounded rational, which was initially proposed by Simon [66] and has been approved by many scholars. That means in the decision-making process, the decision results are inevitably affected by the psychological behavior of decision maker. Huang and Zhan [65] proposed a 3WD method based on regret theory to reflect the regret-aversion behavior of decision maker. From Table 15, we can notice that the results obtained by Huang and Zhan's method is different from our method. The reason is that their calculation of conditional probabilities is based on the cardinality of class, which ignore the degree of difference. Liang et al. [34] proposed a 3WD method based on the prospect theory to reflect the risk-aversion behavior of decision maker. The different outcomes between Liang et al.'s method and Jia and Liu's method can also reflect the great influence of psychological behavior on decision results. Besides, from Table 15, we can observe that the result obtained by Liang et al.'s method is different from that produced by our method. The reason is that Liang et al.'s method does not take into account the regret-aversion of decision maker. However, the regret feeling is also a fact of life and it is irrational to ignore it [57]. Hence, the proposed method considers the risk-aversion behavior and regret-aversion behavior at the same time and produces a more reliable result.

### 6.3. Sensitivity analysis

In this section, sensitivity analysis is conducted to explore the robustness of the proposed method. The parameter involved in this method includes: the parameter used to calculate the value of adopting non-commitment $\eta$ $(0 < \eta \leq 0.5)$, the regret aversion coefficient $\delta$ $(\delta > 0)$, the coefficients of risk attitude $p$ and $q$ $(0 \leq p, q \leq 1)$ and the risk aversion coefficient $\theta$ $(\theta > 1)$. In this

**Table 15**
The results of comparative analysis.

| Methods | Classification | Ranking order |
|---|---|---|
| Mishra et al's method [8] | | $a_4 \succ a_5 \succ a_3 \succ a_1 \succ a_2$ |
| Jia and Liu's method [31] | $Pos = \{a_4, a_5\}$, $BND = \{a_1, a_2, a_3\}$ | $a_4 \succ a_5 \succ a_3 \succ a_1 \succ a_2$ |
| Huang and Zhan's method [65] | $Pos = \{a_3, a_4, a_5\}$, $NEG = \{a_1, a_2\}$ | $a_5 \succ a_4 \succ a_3 \succ a_2 \succ a_1$ |
| Liang et al.'s method [34] | $Pos = \{a_4\}$ $BND = \{a_1, a_2, a_3, a_5\}$ | $a_4 \succ a_3 \succ a_2 \succ a_5 \succ a_1$ |
| The proposed method | $Pos = \{a_4\}$ $BND = \{a_1, a_2, a_3, a_5\}$ | $a_4 \succ a_2 \succ a_3 \succ a_5 \succ a_1$ |

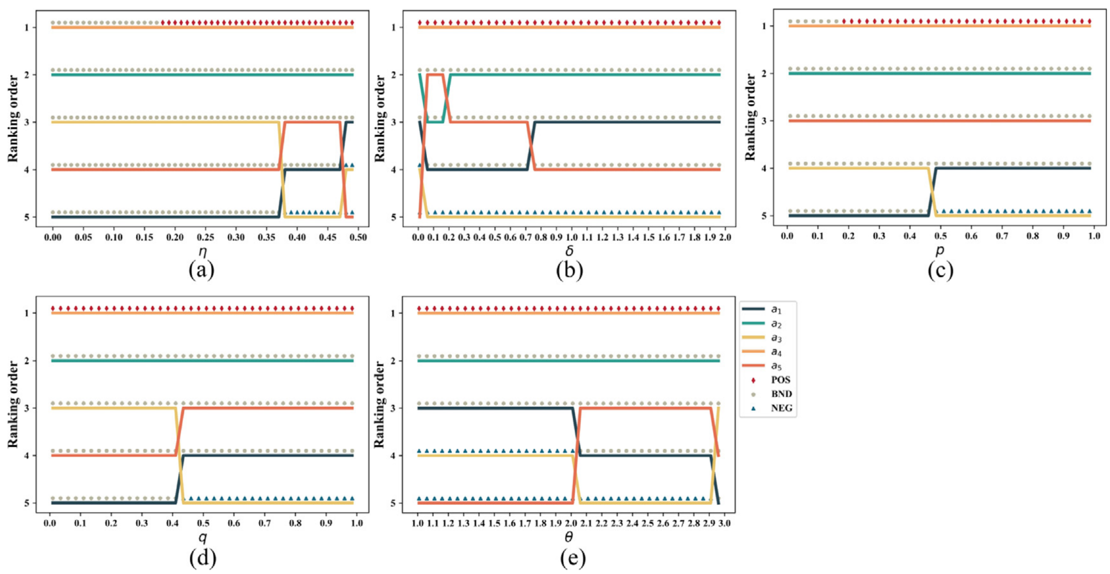


**Fig. 6.** The influence of parameters $\eta$, $\delta$, $p$, $q$ and $\theta$.

analysis, we respectively change the values of parameters $\eta$, $\delta$, $p$, $q$ and $\theta$ to analyze their influence on classification and ranking order. The results are summarized in Fig. 6.

From Fig. 6, it can be noticed that: (1) No matter how the parameters values change, it always obeys the priority principle. That is, the alternatives in *POS* are superior to the alternatives in *BND* and alternatives in *BND* are better than those in *NEG*. (2) The classification and ranking order vary with different value of $\eta$, $\delta$, $p$, $q$ and $\theta$. From the definition of prospect theory and regret–rejoice function, we already know that different risk and regret attitudes lead to different results. Hence, in a specific decision-making problem, their values should be determined according to the specific decision-making situation and decision maker. (3) Although the classification and ranking order may have some fluctuation, the variation of the decision results is almost stable. That is, alternative $a_4$ will rank first in all case even though $a_4$ is not always in the positive region. For example, for $\eta \leq 0.18$ and $p \leq 0.185$, all alternatives will be in the boundary region. The ranking results will be further based on their expected utilities and alternative $a_4$ will still be the first option. In other words, the proposed method maintains its validity when selecting the best alternative.

## 7. Conclusions

The epidemic of COVID-19 has led to unprecedented societal influence, especially for the public health and the global economy. The scientists from all over the world are trying their best to control this epidemic. In this paper, a new method with respect to the treatment of mild symptoms of COVID-19 is proposed, which helps to select the most desirable therapy. The method developed in this paper is based on behavioral 3WD model, which can help the managers and the doctors in making sensible judgments, reducing the decision risks and working on practical applications. From the results of numerical example and comparative analysis, we can conclude that different from other therapy selection methods, the proposed method not only can provide the ranking order of alternatives, but also classifies them into the positive region, boundary region and negative region, which can decrease the decision risks involved in the medical decision. Medical decision is related with the life of patient. When facing some unknown or unfamiliar diseases, the ranking of alternative medicines may not be as the same importance as their classification. For medicines in negative region, it should not be chosen even though it ranks the first. Besides, the sensitivity analysis also shows the robustness of the proposed method. The medicine selection method provided in this paper provides a new perspective for the therapy selection of COVID-19, which can be used in other or future possible medical problems.

## CRediT authorship contribution statement

**Shi-Fan He:** Conceptualization, Methodology, Software, Investigation, Writing, Revision. **Ying-Ming Wang:** Funding acquisition, Conceptualization, Supervision, Methodology. **Xiaohong Pan:** Software, Investigation, Revision. **Kwai-Sang Chin:** Validation, Writing – review & editing, Visualization.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Acknowledgments

This research was supported by the National Natural Science Foundation of China (Grant no. 61773123).

## References


[1] L. Ocampo, K. Yamagishi, Modeling the lockdown relaxation protocols of the philippine government in response to the COVID-19 pandemic: An intuitionistic fuzzy DEMATEL analysis, Socio-Econ. Plan. Sci. 72 (2020) 100911.

[2] V.K. Gupta, A. Gupta, D. Kumar, A. Sardana, Prediction of COVID-19 confirmed, death, and cured cases in India using random forest model, ig Data Min. Anal. 4 (2021) 116–123.

[3] Y. Wang, Q. Guo, G. Li, Y.W.Y. Jiao, Predictions of COVID-19 infection severity based on Co-associations between the SNPs of Co-morbid diseases and COVID-19 through machine learning of genetic data, in: 2020 IEEE 8th International Conference on Computer Science and Network Technology, ICCSNT, IEEE, 2020, pp. 92–96.

[4] N. Ghorui, A. Ghosh, S.P. Mondal, M.Y. Bajuri, A. Ahmadian, S. Salahshour, M. Ferrara, Identification of dominant risk factor involved in spread of COVID-19 using hesitant fuzzy MCDM methodology, Results Phys. 21 (2021) 103811.

[5] Y. Wu, S. Gao, J. Mei, J. Xu, D. Fan, R. Zhang, M. Cheng, JCS: An explainable COVID-19 diagnosis system by joint classification and segmentation, IEEE Trans. Image Process. 30 (2021) 3113–3126.

[6] S. Ahmad, S. Mehfuz, J. Beg, N. Ahmad Khan, A. Husain Khan, Fuzzy cloud based COVID-19 diagnosis assistant for identifying affected cases globally using MCDM, Mater. Today: Proc. (2021) http://dx.doi.org/10.1016/j.matpr.2021.1001.1240.

[7] F.S. Yildirim, M. Sayan, T. Sanlidag, B. Uzun, D.U. Ozsahin, I. Ozsahin, Comparative evaluation of the treatment of COVID-19 with multicriteria decision-making techniques, J. Healthc. Eng. 2021 (2021) http://dx.doi.org/10.1155/2021/8864522.

[8] A.R. Mishra, P. Rani, R. Krishankumar, K.S. Ravichandran, S. Kar, An extended fuzzy decision-making framework using hesitant fuzzy sets for the drug selection to treat the mild symptoms of coronavirus disease 2019 (COVID-19), Appl. Soft Comput. 103 (2021) 107155.

[9] J. Chai, S. Xian, S. Lu, Z-uncertain probabilistic linguistic variables and its application in emergency decision making for treatment of COVID-19 patients, Int. J. Intell. Syst. 36 (2020) 362–402.

[10] Y. Yao, Three-way decisions with probabilistic rough sets, Inform. Sci. 180 (2010) 341–353.

[11] J. Zhu, Z. Chen, B. Shuai, W. Pedrycz, K. Chin, L. Martínez, Failure mode and effect analysis: A three-way decision approach, Eng. Appl. Artif. Intell. 106 (2021) 104505.

[12] H. Omrani, K. Shafaat, A. Emrouznejad, An integrated fuzzy clustering cooperative game data envelopment analysis model with application in hospital efficiency, Expert Syst. Appl. 114 (2018) 615–628.

[13] A.C. Tolga, I.B. Parlak, O. Castillo, Finite-interval-valued type-2 Gaussian fuzzy numbers applied to fuzzy TODIM in a healthcare problem, Eng. Appl. Artif. Intell. 87 (2020) 103352.

[14] A. Mardani, R.E. Hooker, S. Ozkul, S. Yifan, M. Nilashi, H.Z. Sabzi, G.C. Fei, Application of decision making and fuzzy sets theory to evaluate the healthcare and medical problems: A review of three decades of research with recent developments, Expert Syst. Appl. 137 (2019) 202–231.

[15] J. Wang, X. Ma, J. Dai, J. Zhan, A novel three-way decision approach under hesitant fuzzy information, Inform. Sci. 578 (2021) 482–506.

[16] S. Das, S. Kar, Group decision making in medical system: An intuitionistic fuzzy soft set approach, Appl. Soft Comput. 24 (2014) 196–211.

[17] M. Dursun, E.E. Karsak, M.A. Karadayi, A fuzzy multi-criteria group decision making framework for evaluating health-care waste disposal alternatives, Expert Syst. Appl. 38 (2011) 11453–11462.

[18] S. Li, C. Wei, A large scale group decision making approach in healthcare service based on sub-group weighting model and hesitant fuzzy linguistic information, Comput. Ind. Eng. 144 (2020) 106444.

[19] T. Chen, C. Chang, J. Rachel Lu, The extended QUALIFLEX method for multiple criteria decision analysis based on interval type-2 fuzzy sets and applications to medical decision making, European J. Oper. Res. 226 (2013) 615–625.

[20] Y. Yao, S. Wang, X. Deng, Constructing shadowed sets and three-way approximations of fuzzy sets, Inform. Sci. 412 (2017) 132–153.

[21] Y. Yao, The superiority of three-way decisions in probabilistic rough set models, Inform. Sci. 181 (2011) 1080–1096.

[22] Y. Chen, P. Zhu, Extending characteristic relations on an incomplete data set by the three-way decision theory, Internat. J. Approx. Reason. 119 (2020) 108–121.

[23] Y. Yao, S. Wang, X. Deng, Constructing shadowed sets and three-way approximations of fuzzy sets, Inform. Sci. 412–413 (2017) 132–153.

[24] H. Fujita, A. Gaeta, V. Loia, F. Orciuoli, Resilience analysis of critical infrastructures: A cognitive approach based on granular computing, IEEE Trans. Cybern. 49 (2019) 1835–1848.

[25] P.K. Singh, Three-way fuzzy concept lattice representation using neutrosophic set, Int. J. Mach. Learn. Cybern. 8 (2016) 69–79.

[26] J. Wang, X. Ma, Z. Xu, J. Zhan, A three-way decision approach with risk strategies in hesitant fuzzy decision information systems, Inform. Sci. 588 (2022) 293–314.

[27] C. Jiang, D. Guo, Y. Duan, Y. Liu, Strategy selection under entropy measures in movement-based three-way decision, Internat. J. Approx. Reason. 119 (2020) 280–291.

[28] Y. Yao, Three-way decision and granular computing, Internat. J. Approx. Reason. 103 (2018) 107–123.

[29] B. Yang, J. Li, Complex network analysis of three-way decision researches, Int. J. Mach. Learn. Cybern. 11 (2020) 973–987.

[30] J. Ye, J. Zhan, B. Sun, A three-way decision method based on fuzzy rough set models under incomplete environments, Inform. Sci. 577 (2021) 22–48.

[31] F. Jia, P. Liu, A novel three-way decision model under multiple-criteria environment, Inform. Sci. 471 (2019) 29–51.

[32] X. Huang, J. Zhan, B. Sun, A three-way decision method with pre-order relations, Inform. Sci. 595 (2022) 231–256.

[33] J. Zhu, X. Ma, J. Zhan, A regret theory-based three-way decision approach with three strategies, Inform. Sci. 595 (2022) 89–118.

[34] D. Liang, M. Wang, Z. Xu, Heterogeneous multi-attribute nonadditivity fusion for behavioral three-way decisions in interval type-2 fuzzy environment, Inform. Sci. 496 (2019) 242–263.

[35] B. Sun, W. Ma, B. Li, X. Li, Three-way decisions approach to multiple attribute group decision making with linguistic information-based decision-theoretic rough fuzzy set, Internat. J. Approx. Reason. 93 (2018) 424–442.

[36] J. Wang, X. Ma, Z. Xu, J. Zhan, Three-way multi-attribute decision making under hesitant fuzzy environments, Inform. Sci. 552 (2021) 328–351.

[37] J. Zhan, J. Ye, W. Ding, P. Liu, A novel three-way decision model based on utility theory in incomplete fuzzy decision systems, IEEE Trans. Fuzzy Syst. (2021) http://dx.doi.org/10.1109/TFUZZ.2021.3078012.

[38] Z. Zhou, L. Chen, X. Han, S. Tang, G. Li, An interval evidential reasoning-based dynamic performance evaluation method for complex systems, Comput. Ind. Eng. 162 (2021) 107735.

[39] J. Yang, D. Xu, On the evidential reasoning algorithm for multiple attribute decision analysis under uncertainty, IEEE Trans. Syst. Man Cybern. A 32 (2002) 289–304.

[40] W. Xue, Z. Xu, H. Wang, Z. Ren, Hazard assessment of landslide dams using the evidential reasoning algorithm with multi-scale hesitant fuzzy linguistic information, Appl. Soft Comput. 79 (2019) 74–86.

[41] Y. Liang, Y. Ju, J. Qin, W. Pedrycz, Multi-granular linguistic distribution evidential reasoning method for renewable energy project risk assessment, Inf. Fusion 65 (2021) 147–164.

[42] X. Pan, Y. Wang, S. He, The evidential reasoning approach for renewable energy resources evaluation under interval type-2 fuzzy uncertainty, Inform. Sci. 576 (2021) 432–453.

[43] J. Yuan, X. Luo, Approach for multi-attribute decision making based on novel intuitionistic fuzzy entropy and evidential reasoning, Comput. Ind. Eng. 135 (2019) 643–654.

[44] M. Zhang, Y. Wang, L. Li, S. Chen, A general evidential reasoning algorithm for multi-attribute decision analysis under interval uncertainty, European J. Oper. Res. 257 (2017) 1005–1015.

[45] S. Loughney, J. Wang, D.B. Matellini, T.T. Nguyen, Utilizing the evidential reasoning approach to determine a suitable wireless sensor network orientation for asset integrity monitoring of an offshore gas turbine driven generator, Expert Syst. Appl. (2021) 115583.

[46] D. Kahneman, A. Tversky, Prospect theory: An analysis of decision under risk, Econometrica 47 (1979) 263–292.

[47] Y. Dong, N. Luo, H. Liang, Consensus building in multiperson decision making with heterogeneous preference representation structures: A perspective based on prospect theory, Appl. Soft Comput. 35 (2015) 898–910.

[48] A. Tversky, D. Kahneman, Advances in prospect theory: Cumulative representation of uncertainty, J. Risk Uncertain. 5 (1992) 297–323.

[49] W. Pedrycz, From fuzzy sets to shadowed sets: Interpretation and computing, Int. J. Intell. Syst. 24 (2009) 48–61.

[50] W. Pedrycz, Interpretation of clusters in the framework of shadowed sets, Pattern Recognit. Lett. 26 (2005) 2439–2449.

[51] M. Landowski, Shadowed numbers and their standard and multidimensional arithmetic, Inform. Sci. 507 (2020) 485–502.

[52] C. Li, J. Yi, H. Wang, G. Zhang, J. Li, Interval data driven construction of shadowed sets with application to linguistic word modelling, Inform. Sci. 507 (2020) 503–521.

[53] S. He, X. Pan, Y. Wang, A shadowed set-based TODIM method and its application to large-scale group decision making, Inform. Sci. 544 (2021) 135–154.
[54] J. Yang, Y. Wang, D. Xu, K. Chin, The evidential reasoning approach for MADA under both probabilistic and fuzzy uncertainties, European J. Oper. Res. 171 (2006) 309–343.
[55] Y. Wang, J. Yang, D. Xu, K. Chin, The evidential reasoning approach for multiple attribute decision analysis using interval belief degrees, European J. Oper. Res. 175 (2006) 35–66.
[56] W. Lei, W. Ma, B. Sun, Multigranulation behavioral three-way group decisions under hesitant fuzzy linguistic environment, Inform. Sci. 537 (2020) 91–115.
[57] H. Levy, Regret theory: State dominance and expected utility, J. Math. Psych. 79 (2017) 1–12.
[58] X. Peng, Y. Yang, Algorithms for interval-valued fuzzy soft sets in stochastic multi-criteria decision making based on regret theory and prospect theory with combined weight, Appl. Soft Comput. 54 (2017) 415–430.
[59] D.E. Bell, Regret in decision making under uncertainty, Oper. Res. 30 (1982) 961–981.
[60] H. Zhou, J. Wang, H. Zhang, Grey stochastic multi-criteria decision-making based on regret theory and TOPSIS, Int. J. Mach. Learn. Cybern. 8 (2015) 651–664.
[61] M. Abdellaoui, H. Bleichrodt, C. Paraschiv, Loss aversion under prospect theory: A parameter-free measurement, Manage. Sci. 53 (2007) 1659–1674.
[62] D. Liang, Z. Xu, D. Liu, Y. Wu, Method for three-way decisions using ideal TOPSIS solutions at Pythagorean fuzzy information, Inform. Sci. 435 (2018) 282–295.
[63] H. Jiang, B.Q. Hu, A novel three-way group investment decision model under intuitionistic fuzzy multi-attribute group decision-making environment, Inform. Sci. 569 (2021) 557–581.
[64] Y. Wang, J. Yang, D. Xu, Environmental impact assessment using the evidential reasoning approach, European J. Oper. Res. 174 (2006) 1885–1913.
[65] X. Huang, J. Zhan, TWD-R: A three-way decision approach based on regret theory in multi-scale decision information systems, Inform. Sci. 581 (2021) 711–739.
[66] H.A. Simon, Effects of increased productivity upon the ratio of urban to rural population, Econometrica 15 (1947) 31–42.