



Quantum optics approach to radiation from atoms falling into a black hole

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We show that atoms falling into a black hole (BH) emit acceleration radiation which, under appropriate initial conditions, looks to a distant observer much like (but is different from) Hawking BH radiation. In particular, we find the entropy of the acceleration radiation via a simple laser-like analysis. We call this entropy horizon brightened acceleration radiation (HBAR) entropy to distinguish it from the BH entropy of Bekenstein and Hawking. This analysis also provides insight into the Einstein principle of equivalence between acceleration and gravity.

acceleration radiation | black hole entropy | equivalence principle | Hawking radiation | cavity QED

General relativity as originally developed by Einstein (1) is based on the union of geometry and gravity (2). Half a century later the union of general relativity and thermodynamics was found to yield surprising results such as Bekenstein–Hawking black hole entropy (3–6), particle emission from a black hole (5–9), and acceleration radiation (10–17). More recently the connection between black hole (BH) physics and optics, e.g., ultraslow light (18), fiber-optical analog of the event horizon (19), and quantum entanglement (20), has led to fascinating physics.

In their seminal works, Hawking, Unruh, and others (3–14) showed how quantum effects in curved space yield a blend of thermodynamics, quantum field theory, and gravity which continues to intrigue and stimulate. For problems as important and startling as Hawking and Unruh radiation, new and alternative approaches are of interest. In that regard it was shown (21, 22) that virtual processes in which atoms jump to an excited state while emitting a photon are an alternative way to view Unruh acceleration radiation. Namely, by breaking and interrupting the virtual processes which take place all around us, we can render the virtual photons real.

The present paper is an extension of that logic by considering what happens when atoms fall through a Boulware vacuum (23) into a BH as shown in Fig. 1. A mirror held at the event horizon shields infalling atoms from the Hawking radiation. The equivalence principle tells us that an atom falling in a gravitational field does not “feel” the effect of gravity; namely its 4 acceleration is equal to zero. However, as we discuss in *Appendix A*, there is relative acceleration between the atoms and the field modes. This leads to the generation of acceleration radiation. In *Appendix B* we provide a detailed calculation of the photon emission by atoms falling into a BH.

In the classic works (10–17) the atom (or other Unruh–DeWitt detector) was accelerated through flat space-time. The present work differs in that the atom is in free fall and the field is accelerated (or supported in a gravitational field) and contains a Boulware-like ground state of the quantized field. Qualitatively, the principle of equivalence suggests that the results should be analogous to those in refs. 10–17, but the notion that an atom in free fall should emit radiation is surprising to many

people (despite the results in refs. 24 and 25). For this and other reasons, the detailed calculation presented here, taking into account the quantitative differences between the two situations, has been necessary. (An example of another reason for including the detailed calculations of *Appendix B* is in the words of one of the reviewers: “How can the atom falling into a BH emit Unruh-like radiation which comes from a constant acceleration since the falling atom acceleration depends on the distance from the BH?” The answer to this and other such questions is given in *Appendix B*.)

Specifically we consider an atomic cloud consisting of two-level atoms emitting acceleration radiation (Fig. 1) (21, 22). We find that the quantum master equation technique, as developed in the quantum theory of the laser, provides a useful tool for the analysis of BH acceleration radiation and the associated entropy. (For the density matrix formulation of the quantum theory of the laser, see ref. 26. For pedagogical treatment and references, see refs. 27 and 28.) In particular, we derive a coarse-grained equation of motion for the density matrix of the emitted radiation of the form

$$\dot{\rho}_{nn} = (\mathcal{M}\rho)_{nn}, \quad [1]$$

where the time evolution of the diagonal elements of the density matrix ρ_{nn} is governed by the superoperator \mathcal{M} as given by Eq. 7.

Furthermore, we find that once we have cast the acceleration radiation problem in the language of quantum optics and cavity quantum electrodynamics (QED), the entropy follows directly. Specifically, once we calculate $\dot{\rho}$ for the field produced by accelerating atoms, we can use the von Neumann entropy relation to write

$$\dot{S}_p = -k_B \text{Tr}(\dot{\rho} \ln \rho) \quad [2]$$

Significance

Using a combination of quantum optics and general relativity, we show that the radiation emitted by atoms falling into a black hole looks like, but is different from, Hawking radiation. This analysis also provides insight into the Einstein principle of equivalence between acceleration and gravity.

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Reviewers: F.C., Harvard University; and M.D., Imperial College.

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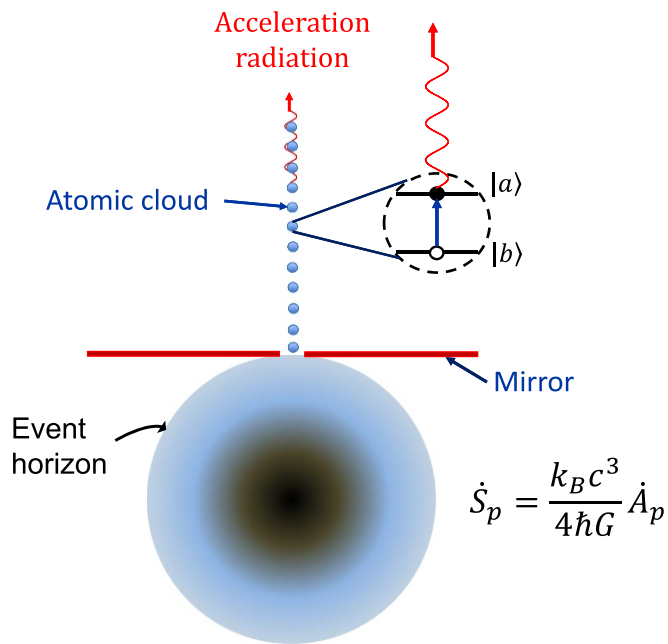


Fig. 1. A BH is bombarded by a pencil-like cloud of two-level atoms falling radially from infinity. As discussed in *Appendix B*, a “mirror” is held at the event horizon which shields infalling atoms from the Hawking radiation. One can imagine that there is a second mirror at large r so that atoms are falling through a cavity. The relative acceleration between the atoms and the field yields generation of acceleration radiation. The physics of the acceleration radiation process correspond to the excitation of the atom together with the emission of the photon (*Appendix B*).

to calculate the radiation entropy flux directly. From the present perspective the acceleration radiation–BH entropy problem is close in spirit to the quantum theory of the laser.

Hawking’s pioneering proof that BHs are not black (5, 6) is based on a quantum-field theoretic analysis showing that photon emission from a BH is characterized by a temperature T_{BH} and generalized BH entropy. York (29) gives an analogy between radiation from a BH and total internal reflection in classical optics. He argues that a light beam in a dense medium can undergo total internal reflection at a flat optical surface; but if we sprinkle dust particles on the surface, some light will be transmitted. Now the flat surface can be likened to the BH event horizon, the dust is replaced by vacuum fluctuations, and light is transmitted through the horizon.

Hawking showed that the radiation that comes out from the BH is described by the temperature

$$T_{\text{BH}} = \frac{\hbar c^3}{8\pi k_B G M}. \quad [3]$$

Hawking then associates the energy of the emitted radiation δE with the loss in energy of the BH $\delta(Mc^2)$ and writes the entropy loss as $\delta S = \delta(Mc^2)/T_{\text{BH}}$. Then using Eq. 3 he obtains

$$\delta S = k_B \frac{8\pi G}{\hbar c} M \delta M = \frac{k_B c^3}{4\hbar G} \delta A, \quad [4]$$

where the BH area in terms of the gravitational radius $r_g = 2GM/c^2$ is given by $A \equiv 4\pi r_g^2 = 16\pi G^2 M^2/c^4$.

In the present paper we analyze the problem of atoms outside the event horizon emitting acceleration radiation as they fall into the BH. The emitted radiation can be essentially, but not inevitably, thermal and has an entropy analogous to the BH result given by Eq. 4. However, the physics is very different. Here

we have radiation coming from the atoms, whereas Hawking radiation requires no extra matter (e.g., atoms).

Historically, Bekenstein (3, 4) introduced the BH entropy concept by information theory arguments. Hawking (5, 6) then introduced the BH temperature to calculate the entropy. In the present approach we calculate the radiation density matrix and then calculate the entropy directly. To distinguish this from the BH entropy we call it the horizon brightened acceleration radiation (HBAR) entropy.

The HBAR Entropy via Quantum Statistical Mechanics

As noted earlier, we here consider a BH bombarded by a beam of two-level atoms with transition frequency ω which fall into the event horizon at a rate κ (Fig. 1). The atoms emit and absorb the acceleration radiation.

We seek the density matrix of the field. As in the quantum theory of the laser (26), the (microscopic) change in the density matrix of the field due to any one atom, $\delta\rho^i$, is small. The (macroscopic) change due to ΔN atoms is then

$$\Delta\rho = \sum_i \delta\rho^i = \Delta N \delta\rho. \quad [5]$$

Writing $\Delta N = \kappa\Delta t$, where κ is the rate of atom injection at random times, we have the coarse-grained equation of motion

$$\frac{d\rho}{dt} = \kappa\delta\rho. \quad [6]$$

We thus obtain an evolution equation for the radiation following the approach used in the quantum theory of the laser (26). As is further discussed in *Appendices B* and *C*, the coarse-grained time rate of change of the radiation field density matrix for a particular field mode is found to be

$$\begin{aligned} \frac{1}{R} \frac{d\rho_{n,n}}{dt} = & -\frac{\kappa g^2}{\omega^2} e^{-\xi} [(n+1)\rho_{n,n} - n\rho_{n-1,n-1}] \\ & - \frac{\kappa g^2}{\omega^2} e^{\xi} [n\rho_{n,n} - (n+1)\rho_{n+1,n+1}], \end{aligned} \quad [7]$$

where g is the atom–field coupling constant, $\xi = 2\pi\nu r_g/c$,

$$R = \frac{\xi}{\sinh(\xi)}, \quad [8]$$

and ν is the photon frequency far from the BH. Using Eqs. 2 and 7, we find that the von Neumann entropy generation rate of the HBAR is (see *Appendix D* for details)

$$\dot{S}_p = \frac{4\pi k_B r_g}{c} \sum_{\nu} \dot{n}_{\nu} \nu, \quad [9]$$

where \dot{n}_{ν} is the flux of photons with frequency ν propagating away from the BH.

Taking into account that the BH mass change due to photon emission is $\dot{m}_p c^2 = \hbar \sum_{\nu} \dot{n}_{\nu} \nu$, we arrive at the HBAR entropy/area relation

$$\dot{S}_p = \frac{k_B c^3}{4\hbar G} \dot{A}_p. \quad [10]$$

Here $\dot{A}_p = (2\dot{m}_p/M)A$ is the rate of change of the BH area due to photon emission which we are interested in (*Appendix D*).

Discussion and Summary

Conversion of virtual photons into directly observable real photons is a subject not without precedent. Moore’s accelerating mirrors (30), the rapid change of refractive index considered by Yablonovitch (31), and the more recent observation of the

dynamical Casimir effect in a superconducting circuit (32) are a few examples.

The physics behind acceleration radiation are explained in ref. 21 (also ref. 33) where the following is stated:

In conclusion our simple model demonstrates that the ground-state atoms accelerated through a field vacuum-state radiate real photons. . . . The physical origin of the field energy in the cavity and of the internal energy in the atom is the work done by an external force driving the center-of-mass motion of the atom against the radiation reaction force. Both the present single mode and the many mode effect originate from the transition of the ground-state atom to the excited state with simultaneous emission of photon due to the counterrotating terms in the Hamiltonian.

In other words the virtual processes in which an atom jumps from the ground state to an excited state, together with the emission of a photon, followed by the reabsorption of the photon and return to the ground state, are altered by the acceleration. The atom is accelerated away from the original point of virtual emission, and there is a small probability that the virtual photon will “get away” before it is reabsorbed as is depicted in Fig. 1.

Acceleration radiation involves a combination of two effects: acceleration and nonadiabaticity that produce the emitted light. The energy is supplied by the external force field (e.g., the gravitational field of the star).

Gravitational acceleration of atoms is also a source of confusion. The equivalence principle tells us that the atom essentially falls “force-free” into the BH. How can it then be radiating? Indeed, the atomic evolution in the atom frame is described by the $e^{i\omega\tau}$ term in the Hamiltonian (Eq. 35). From the Hamiltonian we clearly see that it is the photon time (and space) evolution which contains effective acceleration. The radiation modes are fixed relative to the distant stars, and the photons (not the atoms) carry the seed of the acceleration effects in $\hat{V}(\tau)$.

In Fig. 2 we compare the probability of acceleration radiation P_{ex} for three configurations: (Fig. 2A) the atom is accelerated in Minkowski space-time relative to a fixed mirror; (Fig. 2B) the mirror is accelerated in Minkowski space-time relative to a fixed atom, with the field in a Rindler-like ground state (34); and (Fig. 2C) the atom freely falls in the gravitational field of a BH (assuming that BH Hawking radiation is shielded). In Fig. 2A P_{ex} is proportional to the Planck factor containing the atom frequency ω and the Unruh temperature $T_U = \hbar a / 2\pi k_B c$. In contrast, the Planck factor for Fig. 2B involves the photon frequency ν . For an atom freely falling in the gravitational field of a BH the Planck factor also contains the photon frequency (measured by an observer at infinity). This provides an insight into Einstein’s equivalence principle. Namely, a fixed atom near an accelerating mirror emits thermal radiation as in Fig. 2B; while an atom falling into a BH emits exactly the same thermal spectrum (Fig. 2C).

Please note that this is a very different perspective on the equivalence principle (35) than the usual elevator picture. There the elevator observer (the atom) feels the acceleration in his feet to be the same as a uniform gravitational field. Here the atom is stationary (Fig. 2B). It is the mirror which is accelerating and this changes the normal modes of the field. In Fig. 2C the atom is in free fall and thus it feels no gravity. However, the radiation normal modes are changed by the gravitational field of the BH. Moreover, the atom emits the same way in both cases, Fig. 2B and C. That is, the acceleration in Fig. 2B affects the atom in the same way as the gravitational field in Fig. 2C.

If atoms are ejected randomly, the photon statistics will be thermal (21, 22). For Fig. 2A the average photon occupation number in the mode with frequency ν reads (21, 22)

$$\bar{n}_\nu = \frac{1}{\exp\left(\frac{2\pi c\nu}{a}\right) - 1}. \quad [11]$$

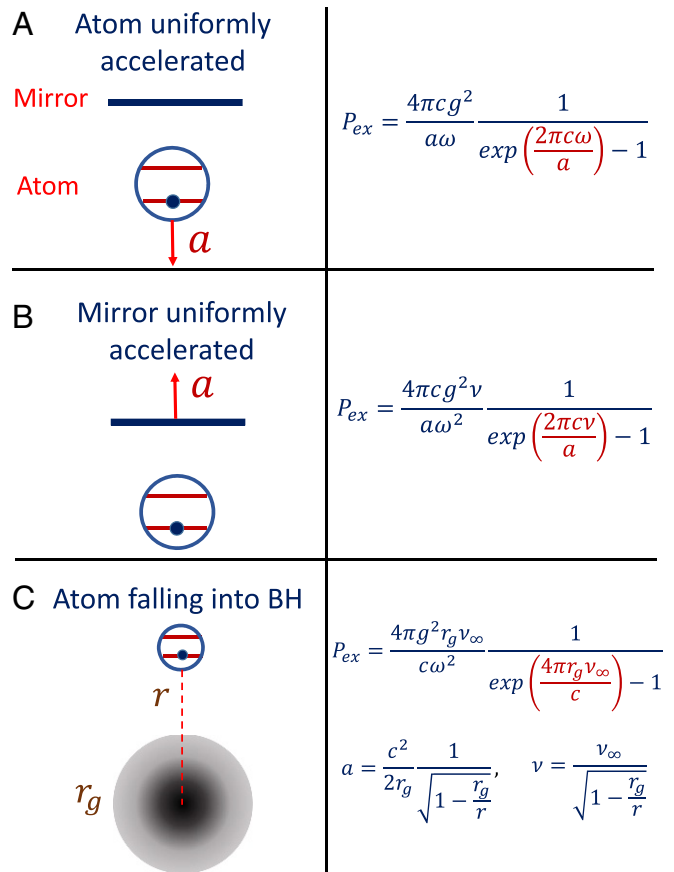


Fig. 2. Three configurations and the corresponding excitation probabilities wherein (A) the atom is accelerated in Minkowski space-time relative to a fixed mirror; (B) the mirror is accelerated in Minkowski space-time relative to a fixed atom, with the field in a Rindler-like ground state (34); and (C) the atom falls into a BH, and is shielded from Hawking radiation as discussed in Appendix B.

The photon spectrum is flat; that is, \bar{n}_ν is independent of the photon frequency ν . In contrast, for an accelerated mirror or atoms freely falling in the gravitational field \bar{n}_ν is given by the Planck distribution.

The present model is simple enough to allow a direct calculation of the HBAR entropy. It is a much more tractable problem than the daunting BH entropy issue. It is interesting that the answer for the HBAR entropy we found is essentially the same as the formula for the Bekenstein–Hawking BH entropy.

Appendix A. Motion of Particle in Rindler and Schwarzschild Space-Time

When atoms are in free fall, their operator time dependence in the interaction picture goes as $\hat{\sigma}^+(\tau) = \hat{\sigma}^+(0) e^{i\omega\tau}$, where τ is the proper time of the atom. The corresponding time evolution of the radiation-field operator is $\hat{a}_k^+(t) = \hat{a}_k^+(0) \psi[t(\tau), z(\tau)]$, where $\psi(t, z)$ is the mode function and the space and time parameterization of the field $t(\tau)$ and $z(\tau)$ are to be determined. In what follows we obtain the results in three steps: (i) special relativity, (ii) Rindler metric, and (iii) Schwarzschild metric.

Special Relativity. First of all we note that finding $t(\tau)$ and $z(\tau)$, i.e., the coordinate time and position of the atom in terms of the atom’s proper time, is really a problem in special relativity. Namely, from the 2D Minkowski line element

$$ds^2 = c^2 dt^2 - dz^2 \quad [12]$$

we can write

$$\tau = \int_0^\tau d\tau = \int_0^t \sqrt{1 - \frac{V^2}{c^2}} dt, \quad [13]$$

where $V = dz/dt$. One can show that for a particle moving with constant proper acceleration a

$$V = \frac{at}{\sqrt{1 + \frac{a^2 t^2}{c^2}}} \quad [14]$$

and, therefore,

$$\tau = \int_0^t \frac{dt}{\sqrt{1 + \frac{a^2 t^2}{c^2}}} = \frac{c}{a} \sinh^{-1} \left(\frac{at}{c} \right), \quad [15]$$

or

$$t(\tau) = \frac{c}{a} \sinh \left(\frac{a\tau}{c} \right). \quad [16]$$

Likewise, integration of $V(t)$ yields

$$z(t) - z(0) = \int_0^t V(t) dt = \frac{c^2}{a} \left(\sqrt{1 + \frac{a^2 t^2}{c^2}} - 1 \right). \quad [17]$$

Setting $z(0) = c^2/a$ and using Eq. 16 we obtain

$$z(\tau) = \frac{c^2}{a} \cosh \left(\frac{a\tau}{c} \right). \quad [18]$$

Rindler. The Rindler metric for a particle undergoing uniformly accelerated motion is obtained from the Minkowski line element 12 if we make a coordinate transformation

$$t = \frac{\bar{z}}{c} \sinh \left(\frac{\bar{a}\bar{t}}{c} \right), \quad [19]$$

$$z = \bar{z} \cosh \left(\frac{\bar{a}\bar{t}}{c} \right), \quad [20]$$

where \bar{a} is a constant. This leads to the line element

$$ds^2 = \left(\frac{\bar{a}\bar{z}}{c^2} \right)^2 c^2 d\bar{t}^2 - d\bar{z}^2, \quad [21]$$

which is the Rindler line element describing uniformly accelerated motion. Comparison of Eqs. 19 and 20 with Eqs. 16 and 18 shows that a particle moving along a trajectory with constant \bar{z} in Rindler space has $\tau = \bar{a}\bar{t}/a$ and is uniformly accelerating in Minkowski space with acceleration

$$a = \frac{c^2}{\bar{z}}. \quad [22]$$

Schwarzschild. Finally we make an observation that the $t - r$ part of the Schwarzschild metric,

$$ds^2 = \left(1 - \frac{r_g}{\bar{r}} \right) c^2 d\bar{t}^2 - \frac{1}{1 - \frac{r_g}{\bar{r}}} d\bar{r}^2, \quad [23]$$

where $r_g = 2GM/c^2$ is the gravitational radius, can be approximated around r_g by Rindler space by using the coordinate $0 < \bar{z} \ll r_g$ defined by

$$\bar{r} = r_g + \frac{\bar{z}^2}{4r_g}. \quad [24]$$

Expanding around r_g ,

$$1 - \frac{r_g}{\bar{r}} \approx \frac{\bar{z}^2}{4r_g^2}, \quad [25]$$

yields the Rindler metric (36)

$$ds^2 = \frac{\bar{z}^2}{4r_g^2} c^2 d\bar{t}^2 - d\bar{z}^2. \quad [26]$$

According to Eq. 22, curves of constant \bar{z} (or \bar{r}) correspond to uniformly accelerated motions with

$$a = \frac{c^2}{\bar{z}} = \frac{c^2}{2r_g} \frac{1}{\sqrt{1 - \frac{r_g}{\bar{r}}}}. \quad [27]$$

Appendix B. Acceleration Radiation from Atoms Falling into a Black Hole

Here we consider a two-level (a is the excited level and b is the ground state) atom with transition angular frequency ω freely falling into a nonrotating BH of mass M along a radial trajectory from infinity with zero initial velocity. We choose the gravitational radius $r_g = 2GM/c^2$ as a unit of distance and r_g/c as a unit of time and introduce the dimensionless distance, time, and frequency as

$$r \rightarrow r_g r, \quad t \rightarrow (r_g/c)t, \quad \omega \rightarrow (c/r_g)\omega.$$

In dimensionless Schwarzschild coordinates the atom trajectory is described by the equations

$$\frac{dr}{d\tau} = -\frac{1}{\sqrt{r}}, \quad \frac{dt}{d\tau} = \frac{r}{r-1}, \quad [28]$$

where t is the dimensionless time in Schwarzschild coordinates and τ is the dimensionless proper time for the atom. Integration of Eq. 28 yields

$$\tau = -\frac{2}{3} r^{3/2} + const, \quad [29]$$

$$t = -\frac{2}{3} r^{3/2} - 2\sqrt{r} - \ln \left(\frac{\sqrt{r}-1}{\sqrt{r}+1} \right) + const. \quad [30]$$

For a scalar photon in the Regge–Wheeler coordinate

$$r_* = r + \ln(r-1) \quad [31]$$

the field propagation equation reads

$$\left[\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r_*^2} + \left(1 - \frac{1}{r} \right) \left(\frac{1}{r^3} - \frac{\Delta}{r^2} \right) \right] \psi = 0, \quad [32]$$

where Δ is the angular part of the Laplacian. We are interested in solutions of this equation outside of the event horizon, that is, for $r > 1$. If the dimensionless photon angular frequency $\nu \gg 1$ and angular momentum is neglected, then the first two terms in Eq. 32 dominate and one can approximately write

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r_*^2} \right) \psi = 0. \quad [33]$$

We consider a solution of this equation describing an outgoing wave

$$\psi = e^{i\nu(t-r_*)} = e^{i\nu[t-r-\ln(r-1)]}, \quad [34]$$

where ν is the wave frequency measured by a distant observer. In general we will have many modes of the field (frequencies ν) which we will sum over as in Eq. 9. However, a proper cavity arrangement as alluded to in the caption of Fig. 1 could be envisioned as yielding effectively a single mode behavior. Furthermore, a properly configured dense atomic cloud could in itself be used to select the desired mode structure. Finally we note that the “mirror” of Fig. 1 could be thought of as completely

surrounding the BH. For the purpose of this appendix we assume that the Boulware vacuum has been arranged.

The interaction Hamiltonian between the atom and the field mode **34** is

$$\hat{V}(\tau) = \hbar g \left[\hat{a}_\nu e^{-i\nu t(\tau) + i\nu r_*(\tau)} + \text{H.c.} \right] \left(\hat{\sigma} e^{-i\omega\tau} + \text{H.c.} \right), \quad [35]$$

where the operator \hat{a}_ν is the photon annihilation operator, $\hat{\sigma}$ is the atomic lowering operator, and g is the atom–field coupling constant. We assume that $g \approx \text{const}$ which is the case for scalar (spin-0) “photons.” Initially the atom is in the ground state and there are no photons for the modes with frequency ν , so that the field is in the Boulware vacuum (23).

The probability of excitation of the atom (frequency ω) with simultaneous emission of a photon with frequency ν is due to a counterrotating term $\hat{a}_\nu^+ \hat{\sigma}^+$ in the interaction Hamiltonian. The probability of this event,

$$\begin{aligned} P_{exc} &= \frac{1}{\hbar^2} \left| \int d\tau \langle 1_\nu, a | \hat{V}(\tau) | 0, b \rangle \right|^2 \\ &= g^2 \left| \int d\tau e^{i\nu t(\tau) - i\nu r_*(\tau)} e^{i\omega\tau} \right|^2, \end{aligned}$$

can be written as an integral over the atomic trajectory from $r = \infty$ to the event horizon $r = 1$ as

$$P_{exc} = g^2 \left| \int_{\infty}^1 dr \left(\frac{d\tau}{dr} \right) e^{i\nu t(r) - i\nu r_*(r)} e^{i\omega\tau(r)} \right|^2. \quad [36]$$

Inserting here Eqs. 29–31 we obtain

$$P_{exc} = g^2 \left| \int_1^{\infty} dr \sqrt{r} e^{-i\nu \left[\frac{2}{3} r^{3/2} + r + 2\sqrt{r} + 2 \ln(\sqrt{r} - 1) \right]} e^{-\frac{2}{3} i\omega r^{3/2}} \right|^2.$$

Making a change of the integration variable into $y = r^{3/2}$ yields

$$P_{exc} = \frac{4g^2}{9} \left| \int_1^{\infty} dy e^{-i\nu \left[\frac{2}{3} y + y^{2/3} + 2y^{1/3} + 2 \ln(y^{1/3} - 1) \right]} e^{-\frac{2}{3} i\omega y} \right|^2. \quad [37]$$

Next we make another change of the integration variable $x = \frac{2\omega}{3}(y - 1)$ and find

$$P_{exc} = \frac{g^2}{\omega^2} \left| \int_0^{\infty} dx e^{-i\nu\phi(x)} e^{-ix} \right|^2, \quad [38]$$

where

$$\begin{aligned} \phi(x) &= \frac{x}{\omega} + \left(1 + \frac{3x}{2\omega} \right)^{2/3} + 2 \left(1 + \frac{3x}{2\omega} \right)^{1/3} \\ &\quad + 2 \ln \left[\left(1 + \frac{3x}{2\omega} \right)^{1/3} - 1 \right]. \end{aligned}$$

The asymptotic behavior of Eq. **38** at $\omega \gg 1$ can be obtained by expanding the function under the exponential in $1/\omega$. Keeping the leading terms we have

$$\phi(x) \approx 3 + 2 \ln \left(\frac{x}{2\omega} \right) + \frac{2x}{\omega}.$$

In the limit $\omega \gg 1$ Eq. **38** becomes

$$P_{exc} = \frac{g^2}{\omega^2} \left| \int_0^{\infty} dx e^{-2i\nu \ln x} e^{-ix(1 + \frac{2\nu}{\omega})} \right|^2$$

$$= \frac{g^2}{\omega^2 \left(1 + \frac{2\nu}{\omega} \right)^2} \left| \int_0^{\infty} dx x^{2i\nu} e^{ix} \right|^2. \quad [39]$$

Using

$$\int_0^{\infty} dx x^{2i\nu} e^{ix} = -\frac{\pi e^{-\pi\nu}}{\sinh(2\pi\nu)\Gamma(-2i\nu)},$$

where $\Gamma(z)$ is the gamma function, and the property $|\Gamma(-ix)|^2 = \pi/[x \sinh(\pi x)]$ we find

$$P_{exc} = \frac{4\pi g^2 \nu}{\omega^2 \left(1 + \frac{2\nu}{\omega} \right)^2} \frac{1}{e^{4\pi\nu} - 1}. \quad [40]$$

P_{exc} becomes exponentially small for $\nu \gg 1$. Thus, acceleration radiation will not be emitted with very large ν . On the other hand, typical atomic frequencies $\omega \gg 1$ and, therefore, in the following one can assume that $\omega \gg \nu$. Then, in the dimensional units Eq. **40** reads

$$P_{exc} = \frac{4\pi g^2 r_g \nu}{c\omega^2} \frac{1}{e^{\frac{4\pi r_g \nu}{c}} - 1}. \quad [41]$$

The probability of photon absorption is obtained by changing $\nu \rightarrow -\nu$, which for $\omega \gg \nu$ yields

$$P_{abs} = e^{\frac{4\pi r_g \nu}{c}} P_{exc}. \quad [42]$$

We note that the Planck factor in Eq. **40** comes from the detailed calculation, i.e., is not put in by hand. The result is equivalent to that for a constant acceleration because the main contribution comes from the event horizon. We note also that the mirror “edge effects” are not a problem.

Appendix C. Density Matrix for the Field Mode

The (microscopic) change in the density matrix of a field mode $\delta\rho^i$ due to an atom injected at time τ_i is

$$\delta\rho^i = -\frac{1}{\hbar^2} \int_{\tau_i}^{\tau_i + T_{\text{int}}} \int_{\tau_i}^{\tau_i + \tau'} d\tau' d\tau''$$

$$\text{Tr}_{atom} \left[\hat{V}(\tau'), \left[\hat{V}(\tau''), \rho^{\text{atom}}(\tau_i) \otimes \rho(t(\tau_i)) \right] \right], \quad [43]$$

where T_{int} is the proper atom–field interaction time, Tr_{atom} denotes the trace over atom states, and $\hat{V}(\tau)$ is the interaction Hamiltonian between the atom and the field mode given by Eq. **35**. The time τ is the atomic proper time, i.e., the time measured by an observer riding along with the atom.

In the case of random injection times, the equation of motion for the density matrix of the field is

$$\begin{aligned} \frac{d\rho_{n,n}}{dt} &= -\Gamma_e [(n+1)\rho_{n,n} - n\rho_{n-1,n-1}] - \\ &\quad - \Gamma_a [n\rho_{n,n} - (n+1)\rho_{n+1,n+1}], \end{aligned} \quad [44]$$

where Γ_e and Γ_a are emission and absorption rates due to coupling to a photon of frequency ν , $\Gamma_{e,a} = \kappa |gI_{e,a}|^2$, and $I_{e,a}$ are given by the integrals

$$g e^{-i\xi/\pi} I_{e,a} = -\frac{i}{\hbar} \int_{\tau_i}^{\tau_i + T_{\text{int}}} V_{e,a} d\tau,$$

where $\xi = 2\pi\nu r_g/c$ and ν is the mode frequency far from the BH. We note that the absorption and emission matrix elements of the interaction Hamiltonian are as in *Appendix B*,

$$V_a = \langle 0, a | \hat{V}(\tau) | 1, b \rangle, \quad V_e = \langle 1, a | \hat{V}(\tau) | 0, b \rangle,$$

and obtain Eq. **44**. Steady-state solution of Eq. **44** is given by the thermal distribution (26):

$$\rho_{n,n}^{S,S} = \exp(-2\xi n)[1 - \exp(-2\xi)]. \quad [45]$$

To approach this steady-state solution, we need a cavity to restrict the modes to a finite range of the Regge–Wheeler coordinate r_* , so the bottom mirror must be at $r_b > r_g$, and the top must be at $r_t < \infty$. This will modify the analysis of *Appendix B*, but we can then take the limit as $r_b \rightarrow r_g$ and $r_t \rightarrow \infty$.

Appendix D. Entropy Flux

The time rate of change of entropy due to photon generation,

$$\dot{S}_p = -k_B \sum_{n,\nu} \dot{\rho}_{n,n} \ln \rho_{n,n}, \quad [46]$$

to a good approximation can be written as

$$\dot{S}_p \approx -k_B \sum_{n,\nu} \dot{\rho}_{n,n} \ln \rho_{n,n}^{S,S}. \quad [47]$$

once one has approached the steady-state solution. The steady-state density matrix $\rho_{n,n}^{S,S}$ is given by Eq. 45. Inserting it into [47] gives

$$\dot{S}_p \approx \frac{4\pi k_B r_g}{c} \sum_{\nu} \dot{n}_{\nu} \nu, \quad [48]$$

where \dot{n}_{ν} is the photon flux from the infalling atoms.

Recalling the BH area $A \equiv 4\pi r_g^2$, where the gravitational radius $r_g = 2MG/c^2$ and $\dot{m}_p c^2 = \hbar \sum_{\nu} \dot{n}_{\nu} \nu$ is the power carried away by the emitted photons, we arrive at the HBAR entropy/area relation

$$\dot{S}_p = \frac{k_B c^3}{4\hbar G} \dot{A}_p. \quad [49]$$

Here $\dot{A}_p = 32\pi G^2 M \dot{m}_p / c^4$ is the rate of change of the BH area due to photon emission. The BH rest mass changes as $\dot{M} = \dot{m}_{\text{atom}} + \dot{m}_p$ due to the atomic cloud adding to and the emitted photons taking from the mass of the BH. The BH area A is proportional to M^2 and, hence, $\dot{A} = (2\dot{M}/M)A = \dot{A}_{\text{atom}} + \dot{A}_p$.

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