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Models of Epidemiological Security Management in the Spread of the SARS-CoV-2 Virus

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Abstract: The task of managing epidemic security during COVID-19 is considered. The spread of the SARS-CoV-2 virus without and with vaccination is described by mathematical and computer models built on the basis of the epidemic control protocol adopted by the Georgian authorities. The mathematical model of the spread of the SARS-CoV-2 virus is described using the Cauchy problem for a system of ordinary differential equations. For the management of epidemiological safety, a objective function has been built, which takes into account: the financial consequences of introducing a lockdown in the country and the cost of treating the infected. Among the parameters of the model, the governing ones are highlighted. The control parameters are used to minimize the objective function. In the work, mainly theoretical research is given. However, computer simulation and a computational experiment on the proposed computer model with constant parameters allows us to answer the question: what is the number of infected citizens in the country, in which the economy does not need a lockdown, and the recovery prognosis of those infected with the SARS-CoV-2 virus is favorable.

Keywords: Mathematical, computer model, SARS-CoV-2, management, epidemic, vaccination.

1. INTRODUCTION

The COVID19 pandemic caused by the SARS-CoV-2 virus did not pass by Georgia. The reaction of the Georgian authorities to the SARS-Cov-2 epidemic was different in the spring and autumn of 2020. For example, in the spring of 2020, the authorities listened to the advice of experts from the health care system and actually started a lockdown in the country. The result - minimum number of cases and deaths of the new corona virus. In the fall of 2020, the authorities changed their tactics and did not introduce a lockdown, preferring to fight pointwise with the hearths of infection. The result: thousands of infected and hundreds of deaths. WHO in the fall of 2020 transferred Georgia from the green zone, prosperous countries, to the red – disadvantaged ones, according to the number of infections. The reason for the change in the tactics of combating the epidemic is the economy, the implementation of the tactics of complete quarantine in the country, i.e. lockdown turned out to be too expensive and the country's budget could not afford it. To improve the economy quarantine was lifted, but as a result, the number of infected people increased, who are being treated from budget funds and in this case, it is about economic costs.

In this work, in my opinion, there is a clear commitment to the ideas and goals of TC 9.5. Technology, culture and international stability (TECIS) given in Kopacek P. (2016). As noted in Kopacek P., Stapleton L., Dimirovski M. (2017), unstable, unstable, unviable and unsustainable systems pose serious problems and threaten even humanity itself, therefore it is vitally important that scientists and technologists, especially researchers and control and automation system practitioners are able to contribute to making the world more stable for all. This assessment probably applies to the global fight against the COVID19 pandemic. The World Health Organization has repeatedly called on the world community to show more solidarity in the fight against the COVID19 pandemic. So on August 4, 2021, WHO Director General Tedros Adanom Ghebreyesus appealed to countries to donate their vaccine doses to those who have not yet been able to vaccinate priority populations. It was noted that so far 80% of the doses of the vaccine available worldwide have been used in high-income countries, while many other countries are still waiting to vaccinate their priority populations. Therefore, WHO considers it necessary to introduce a moratorium on booster (third dose of vaccine) vaccinations against COVID-19 in developed countries. Therefore, for low-income countries, it is relevant to fight the epidemic by observing regulations including: wearing medical masks, keeping a distance, etc., and in extreme cases, a complete lockdown.

Thus, the task is, to what extent and how quarantine measures should be applied so that the population and economy of the country do not suffer greatly. In fact, the matter concerns the management of population life safety and state economy. This control problem can be partly solved with the help of a computer experiment, when implementing the optimal control problem for a dynamic system - mathematical models for the spread of SARS-CoV-2.

As a prototype of the model for the spread of the virus in Georgia similar existing models for countries with low or middle income could be used, for example, Mezei Alex, Cohen Jamie, Renwick J Matthew, Atwell Jessica, Portnoy Allison. (2021)., Zirhumanana Balike Dieudonne. (2021). But, as a basis for constructing a mathematical model for the spread of SARS-Cov-2, a protocol developed by the

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Georgian health care system was adopted, which is binding on all authorities of the country. When constructing the model, the ideas outlined in Kermack W.O., McKendrick A.G. (1927), Kereselidze N. (2018), Kereselidze N. (2019), Kereselidze N. (2020) were used.

2. BUSINESS LOGIC OF THE EPIDEMIC CONTROL PROCESS

Let us say that at time moment t the number of citizens in the country is $N(t)$. At the same time, $N_e(t)$ number of citizens are entering the country. According to the protocol, all of them should be sent to the places designated for quarantine - hotels, sanatoriums, rest houses, etc. However, let's say that not all arriving citizens are transferred to quarantine, some managed to somehow avoid this. So, From $N_e(t)$ citizens, $\alpha_{e1}(t)N_e(t)$ were quarantined and $\alpha_{e2}(t)N_e(t)$ escaped the quarantine. We have $N_e(t)=\alpha_{e1}(t)N_e(t)+\alpha_{e2}(t)N_e(t)$. Or $1=\alpha_{e1}(t)+\alpha_{e2}(t)$. Note that the country has a group of people entering - E , a group in quarantine - Q . Let us say that at time moment t the number $N_q(t)$ of citizens are in quarantine. After some time, a certain number of people, who test positive for SARS-Cov-2 virus is transferred to a hospital for treatment - they are infected and there is documentary evidence of this, we will denote this group of people by I . Let us say that at time moment t the number of citizens sent from quarantine for treatment is $N_{qi}(t)$, and $N_{qh}(t)$ number of citizens are released from quarantine and they replenish the group of citizens - HS, specifically groups of healthy people without immunity - H , their number is $N_{hs}(t)$. After treatment from the I group, the recovered patients join the group of healthy people with immunity HI , Their number is $N_{hi}(t)$, unfortunately, a certain number of patients $N_{di}(t)$ cannot be saved. The group of those who died from the virus will be denoted by D . Note that in addition to knowingly infected people, there is a group S of sick people in society who carry the virus, but there is no documentary evidence of this, the number of these people is $N_s(t)$. It is the members of the group S who are the main distributors of the virus, they freely contact with members of the healthy people group without immunity - H , in which $N_h(t)$ are citizens infecting them. The complexity of the situation is that the relevant authorities do not know the exact number of these people, but also the distributors of the infection themselves. Note that group HS is a union of groups H and S , $N_{hs}(t)=N_h(t)+N_s(t)$.

Epidemiological services identify infected people $N_{si}(t)$ from the group S and transfer them to hospitals for treatment, thereby replenishing the group I . At the same time, the circle of their contacts is determined, let in the amount of $N_c(t)$ citizens from group HS and transfer them into quarantine, replenishing group Q , respectively $N_{ch}(t)$ from group H and $N_{cs}(t)$ from group S , $N_c(t)=N_{ch}(t)+N_{cs}(t)$. Unfortunately, the group S also has cases of death from the virus, let us designate their number through $N_{ds}(t)$, which replenishes the group D . We will assume that the people who have recovered from the new corona virus acquire immunity at the time of observation and do not become infected again. Note that the number of group E,Q,I,HI,D,HS members is known at every moment in time t . However, the exact number of members of groups H and S, respectively, is not known. Meanwhile, contacts of H and S group members can worsen the epidemiological situation, as patients from S can infect healthy people from H. Let's build a scheme for combating the epidemic and its business logic in the form of a directed graph (Fig. 1):

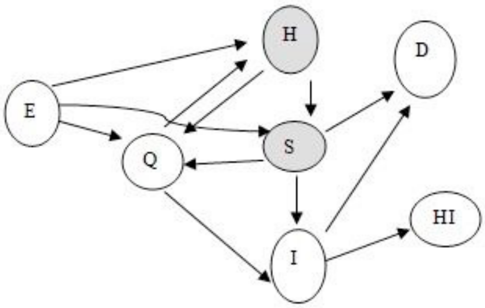

Figure 1. Directed graph battle with the epidemic

Comment. Analysis of the graph shown in Fig. 1, which is an illustration of the epidemic control flow, shows that from quarantine, in which the citizen was at the expense of the state, is sent to the group H , which is freely in contact with group G, infected, but not yet officially confirmed. Therefore, some of them (people) return to Q again, as contacts in group HS with an identified infected person from this group. Thus, the same person can go through quarantine several times at the expense of government funds.

3. BUILDING MODELS

In the directed graph in Fig. 1, the arc EH has weight $\alpha_{e21}(t)N_e(t)$ and arc ES - $\alpha_{e22}(t)N_e(t)$. The fact is that some citizens who entered the country and did not get into quarantine can be both healthy and infected – sick. Their exact number is not known, but it is known that $\alpha_{e21}(t)+\alpha_{e22}(t)=\alpha_{e2}(t)$, but together they make up the HS = HUS group, a union of the H and S groups.

The rate of change in the number of healthy people without immunity - $N_h(t)$ from group H depends on the intensity: contacts between members of groups H and S; replenishment of group Q from group H with $N_{ch}(t)$ people; replenishment of group H from group Q with $N_{qh}(t)$

people, replenishment of group H from group E with $N_e(t)$ people. Therefore, we have

$$\begin{aligned}\frac{dN_h(t)}{dt} &= \alpha_{qh}(t)N_q(t)+\alpha_{eh}(t)\alpha_{e21}(t)N_e(t)- \\ &-\alpha_{hc}(t)N_h(t)-\alpha_{hs}(t)N_h(t)N_s(t)\end{aligned} \tag{1}$$

Where $\alpha_{qh}(t)$, $\alpha_{eh}(t)$, $\alpha_{e21}(t)$, $\alpha_{hc}(t)$, $\alpha_{hs}(t)$ are corresponding coefficients.

In a similar way, we can write out relations similar to (1) and for the rate of change in the number of groups Q, I, HI, S. As a result, we get a system of ordinary differential equations :

$$\begin{cases}\dfrac{dN_h(t)}{dt} = \alpha_{qh}(t)N_q(t)+\alpha_{eh}(t)\alpha_{e21}(t)N_e(t)- \\ \qquad -\alpha_{hc}(t)N_h(t)-\alpha_{hs}(t)N_h(t)N_s(t), \\ \dfrac{dN_q(t)}{dt} = \alpha_{hc}(t)N_h(t)+\alpha_{eq}(t)\alpha_{e1}(t)N_e(t)+ \\ \qquad +\alpha_{sc}(t)N_s(t)-\alpha_{qh}(t)N_q(t)- \\ \qquad -\alpha_{qi}(t)N_q(t), \\ \dfrac{dN_s(t)}{dt} = \alpha_{hs}(t)N_h(t)N_s(t)+ \\ \qquad +\alpha_{es}(t)\alpha_{e22}(t)N_e(t)- \\ \qquad -\alpha_{si}(t)N_s(t)-\alpha_{sc}(t)N_s(t)- \\ \qquad -\alpha_{sd}(t)N_s(t), \\ \dfrac{dN_i(t)}{dt} = \alpha_{si}(t)N_s(t)+\alpha_{qi}(t)N_q(t)- \\ \qquad -\alpha_{ihi}(t)N_i(t)-\alpha_{id}(t)N_i(t), \\ \dfrac{dN_d(t)}{dt} = \alpha_{id}(t)N_i(t)+\alpha_{sd}(t)N_s(t).\end{cases} \tag{2}$$

Where, the coefficients in the system are non-negative, the epidemic is observed over a period of time $[t_0;T]$. In (2), the function $N_e(t)$ is known in principle - the number of arriving citizens. At the initial moment of time t_0, the following are known:

$$\begin{cases}N_q(t_0)=N_{q0}, N_i(t_0)=N_{i0}, \\ N_d(t_0)=N_{d0}, N_h(t_0)+N_s(t_0)=N_{00}\end{cases} \tag{3}$$

System (2) with initial conditions (3) constitutes a mathematical model of the SARS-Cov-2 virus epidemic.

4. THE TASK OF MANAGING THE EPIDEMIC

Analyzing the protocol for fighting the epidemic and its mathematical model, it should be noted that the control of the spread of infections has special control levers. For example, by improving the control over the arriving citizens, it is possible to practically exclude the penetration of sick citizens into society, bypassing quarantine. In the model, for example, the values of the coefficients $\alpha_{e21}(t),\alpha_{e22}(t)$ should be reduced. Also, by choosing the values of the coefficient $\alpha_{hs}(t)$, in fact, it is possible to achieve a hard lockdown or a liberal policy of containing the epidemic. Let us say when selecting $\alpha_{e21}(t)$- the impact on budget expenditures is $B:\alpha_{e21}(t)\to R$, and the costs of treating infected people can be expressed in terms of $W:\int_{t_0}^{T} N_i(\tau)d\tau X\alpha_{hs}(t)\to R$, then the total costs of identifying the infected and their treatment, taking into account their minimization, can be expressed as follows:

$$\begin{aligned}&J(\alpha_{e21}(t),\alpha_{hs}(t)) = B(\alpha_{e21}(t))+ \\ &+W\left(\int_{t_0}^{T} N_i(\tau)d\tau,\alpha_{hs}(t)\right)\to \inf .\end{aligned} \tag{4}$$

Minimization of Functional (4) must be achieved under conditions (2), (3) and the constraint:

$$B(\alpha_{e21}(t))\geq L>0. \tag{5}$$

Constraint (5) means that budgetary expenditures for these activities cannot be less than a certain amount. It is clear that we have restrictions from above - budgetary funds are limited!

Note that functional $W(*)$ also takes into account the fact that by choosing the values of the coefficient $\alpha_{hs}(t)$ (thereby determining the level of lockdown), we actually change financial receipts - specifically, we reduce them. Therefore, we need to minimize this value as well.

To manage the safety of the population life and the economy of the state, an extreme problem of the type (4), (5), (2), (3) is considered.

Computer implementation of the model (2), (3) and the extremal problem (4), (5) was carried out in the MatLab environment for various values of constant coefficients of system (2), initial conditions (3) and specific functionals B,W.

5. EPIDEMIC MODELS WITH VACCINATION

Until now, we have considered models of the spread of the SARS-Cov-2 virus without taking into account the vaccination process in the community. There is an explanation for this. At the first stage of the fight against COVID19, there was simply no appropriate vaccine. In Georgia, which belongs to the developing countries, a small amount of the vaccine appeared only in the spring of 2021. At the same time, with the help of vaccination, the number of healthy people with immunity can be dramatically increased. According to experts from the World Health Organization, COVID19 can be considered defeated if 70% of the population is vaccinated against the virus, that is, the population acquires collective immunity.

When vaccinated in the oriented graph in Figure 1, an arc from group H to HI will appear. This arc shows the process of vaccinating people from a group of healthy people without immunity, as a result of which vaccinated people from H go to a group of healthy people with HI immunity. Let us designate the number of these people at the time moment t

who received immunity through vaccination through $N_{hiv}(t)$. Since there are people in the HI group who acquired immunity after recovery, we will designate their number at a time point t through $N_{hit}(t)$. Therefore, we will get a new directed graph in Figure 2, reflecting the process of fighting the epidemic during vaccination.

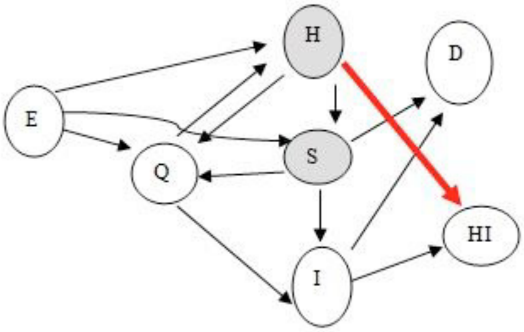

Figure 2. Directed graph of the fight against the epidemic, taking into account vaccination

Notice, that $N_{hi}(t)=N_{hit}(t)+N_{hiv}(t)$. The growth rate of the number of healthy people with immunity depends on the intensity of the recovery of infected people and the vaccination of healthy people without immunity -

$$\frac{dN_{hi}(t)}{dt}=\alpha_{ihi}(t)N_i(t)+\alpha_{vac}(t)N_h(t). \qquad (6)$$

By adding differential equation (6) to system (2), we obtain a pandemic control model taking into account vaccination. As a result, we get a system of ordinary differential equations of the fight against the epidemic using vaccination:

$$\begin{cases}
\frac{dN_h(t)}{dt}=\alpha_{qh}(t)N_q(t)+\alpha_{eh}(t)\alpha_{e21}(t)N_e(t)-\\
\quad -\alpha_{hc}(t)N_h(t)-\alpha_{hs}(t)N_h(t)N_s(t)-\\
\quad -\alpha_{vac}(t)N_h(t),\\
\frac{dN_q(t)}{dt}=\alpha_{hc}(t)N_h(t)+\alpha_{eq}(t)\alpha_{e1}(t)N_e(t)+\\
\quad +\alpha_{sc}(t)N_s(t)-\alpha_{qh}(t)N_q(t)-\\
\quad -\alpha_{qi}(t)N_q(t),\\
\frac{dN_s(t)}{dt}=\alpha_{hs}(t)N_h(t)N_s(t)+\\
\quad +\alpha_{es}(t)\alpha_{e22}(t)N_e(t)-\\
\quad -\alpha_{si}(t)N_s(t)-\alpha_{sc}(t)N_s(t)-\\
\quad -\alpha_{sd}(t)N_s(t),\\
\frac{dN_i(t)}{dt}=\alpha_{si}(t)N_s(t)+\alpha_{qi}(t)N_q(t)-\\
\quad -\alpha_{ihi}(t)N_i(t)-\alpha_{id}(t)N_i(t),\\
\frac{dN_d(t)}{dt}=\alpha_{id}(t)N_i(t)+\alpha_{sd}(t)N_s(t),\\
\frac{dN_{hi}(t)}{dt}=\alpha_{ihi}(t)N_i(t)+\alpha_{vac}(t)N_h(t).
\end{cases} \qquad (7)$$

To control the safety of life of the population and the economy of the state during vaccination of the population, we obtain an extreme problem of the type (4), (5), (7), (3). Where vaccination costs are included: the cost of the vaccine and the organization of the vaccination. At the same time, another limitation appears, the purpose of which is to achieve herd immunity:

$$N_{hi}(T)=N_{hit}(T)+N_{hiv}(T)\geq 0{,}7N(T).$$

6. CONCLUSIONS

Thus, with the spread of the SARS-CoV-2 virus in the absence of vaccination, an extreme problem (4), (5), (2), (3) was formulated to manage the safety of life of the population and the economy of the state. If the state has the opportunity to vaccinate its population, then an extreme problem arises (7), (5), (2), (3). By solving these extreme problems, the safety of life of the population and the economy is achieved with minimal budgetary expenditures. For low-income countries, this is critical. However, these extreme targets can also be beneficial for high-income countries, in addition to saving money, they can help other countries to fight the epidemic with these savings. And this act has, in addition to a humanitarian character, a pragmatic calculation, since the Pandemic around the world will not end if an epidemic is raging in at least one country.

A computational experiment carried out on a computer model built on the basis of mathematical models (2), (3) and (7), (3) with constant coefficients allows us to conclude that by choosing the values of parameters α_{e21} and α_{si} it is possible to select such a number of infected citizens $N_i(t)$, during which the economy does not need a lockdown, and the prognosis for the recovery of those infected with the SARS-CoV-2 virus is favorable.

Further research and improvement of Control Tasks (4), (5), (2), (3) and (7), (5), (2), (3) requires synergistic efforts of epidemiologists, economists; management specialists and politicians. Depending on the priorities of society, it will be necessary to modify the target functions (4) and (5) accordingly. In the future, when changing the virus strain, it is naturally necessary to make changes in the parameters of the model (5), (2), (3).

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