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Cubic-quartic bright optical solitons with improved Adomian decomposition method

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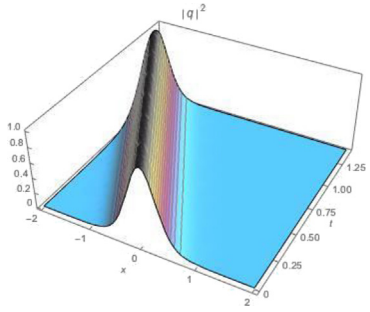
HIGHLIGHTS

- Optical soliton solutions for the perturbed nonlinear Schrodinger's equation are revealed.
- Quadratic-cubic nonlinearity is considered.
- Bright optical solitons are retrieved by the help of the IADM.
- The numerical results together with high level accuracy plots are exhibited.
- The method proposed herein works with high degree of accuracy.

GRAPHICAL ABSTRACT

Cubic-quartic soliton transmission having power law of nonlinearity refractive index.



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ABSTRACT

This paper numerically retrieves cubic-quartic solitons having power law of nonlinearity refractive index. An improvement of the Adomian decomposition scheme is the adopted algorithm of this work. The results are displayed along with the established error analysis.

Introduction

One of the emerging concepts from mathematical photonics is "cubic-quartic (CQ) solitons" [1–7,9,10]. This appears when group velocity dispersion (GVD) runs low and hence discarded. This was first introduced a couple of years ago as a follow-up to the concept of "quartic solitons" which was a prequel paper to the first paper on CQ solitons [8]. It is noted that quartic solitons cannot be analytically studied and therefore one must remain contended with numerical solutions only. In order to understand the behavior of solitons in absence of GVD the concept of CQ solitons was subsequently introduced. Later spectrums of results have started pouring in with CQ solitons.

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While all of the works thus far on CQ solitons are analytical in character, it now time to take a fresher look at such solitons from a numerical perspective. The current paper thus addresses CQ solitons from a numerical standpoint. The algorithm that displays the results is an improved version of Adomian decomposition method (IADM) scheme. The focus is on power law nonlinearity refractive index. The details of the scheme are inked and the results are displayed in the upcoming sections.

The model

The cubic-quartic (CQ) NLSE including third and fourth order dispersion but without GVD is given by [1]:

$$iq_t + iaq_{xxx} + bq_{xxxx} + c \mid q|^{2m}q = 0, \quad m \in \mathbb{N}. \tag{1}$$

Here, in Eq. (1), $q(x,t)$ the complex-valued wave amplitude that governs the evolution of a nonlinear wave, x is a longitudinal variable and t is a co-moving time and $i = \sqrt{-1}$. Besides a and b respectively represent coefficients of third and fourth order dispersions. Finally c is the coefficient of power law of refractive index where m stands for the power law factor.

Exact analytical bright and singular soliton solutions for the CQ model (1) were recently obtained in [1] with the help the of the undetermined coefficients method and before in [8] pure-quartic solitons propagation was studied.

Cubic-quartic bright optical solitons

The bright CQ 1-soliton solution to (1) was found by the authors in [1] and is given by

$$q(x,t) = A\text{sech}^{\frac{2}{m}}[B(x-vt)]e^{i[-\kappa x+\omega t+\theta_0]}. \tag{2}$$

In Eq. (2), v is the soliton velocity, ω is the angular velocity, κ is the soliton frequency, and θ_0 is the phase center.

The amplitude A and the inverse width B of the CQ 1-soliton are given by

$$A = \left[-\frac{(m+2)(3m+2)P_2^2}{2bc(m^2+2m+2)^2}\right]^{\frac{1}{2m}}, \qquad B = \frac{m}{2}\left[-\frac{P_2}{b(m^2+2m+2)}\right]^{\frac{1}{2}}, \tag{3}$$

where $P_2 = \frac{3a^2}{8b}$.

The velocity v and the angular velocity ω of the CQ 1-soliton are given by

$$v = -3a\kappa^2 + 4b\kappa^3, \qquad \omega = \frac{b\kappa^3(b\kappa-a)(m^2+2m+2)^2 - 9\kappa^2(m+1)^2(a-2b\kappa)^2}{b(m^2+2m+2)^2}, \tag{4}$$

where the soliton frequency κ, is related to the coefficients of the model by $\kappa = \frac{a}{4b}$.

Material and methods

A modification of the standard Adomian decomposition method (ADM) was proposed by A. M. Wazwaz first in [11] and shortly after the modification was improved in [12] by A. M. Wazwaz and S. M. El-Sayed. This improvement to the Adomian decomposition method was established based on the assumption that the initial condition can be decomposed into a series of functions in the spatial variable. We will use the IADM to solve the Eq. (1) in the case of bright solitons through several examples.

Supposing that $q(x,t) = u_1(x,t) + iu_2(x,t)$, Eq. (1) can be split into real and imaginary parts

$$-L_t u_2 - aR_3 u_2 + bR_4 u_1 + cN_1(u_1,u_2) = 0$$

$$L_t u_1 + aR_3 u_1 + bR_4 u_2 + cN_2(u_1,u_2) = 0, \tag{6}$$

where $L_t = \frac{\partial}{\partial t}$ and $L_t^{-1} = \int_0^t (\cdot)ds$. Each R_j is a linear differential operator, that is, $R_j = \frac{\partial^j}{\partial x^j}$, $j = 3,4$ and the nonlinear terms N_1 and N_2 are given for the cases $m = 1$ and $m = 2$ respectively, by

$$\begin{cases} N_1(u_1,u_2) = u_1^2 + u_1u_2^2 \\ N_2(u_1,u_2) = u_2^3 + 2u_1^2u_2 - u_1u_2, \end{cases} \tag{7}$$

$$\begin{cases} N_1(u_1,u_2) = u_1^5 + 2u_1^3u_2^2 + u_1u_2^4 \\ N_2(u_1,u_2) = u_2^5 + 2u_1^2u_2^3 + u_1^4u_2. \end{cases} \tag{8}$$

As the operator L_t is invertible, applying the operator L_t^{-1} to both sides of Eqs. (5) and (6), we get

$$u_1(x,t) = \Re e(q(x,0)) - L_t^{-1}(aR_3u_1 + bR_4u_2 + cN_2(u_1,u_2)) \tag{9}$$

$$u_2(x,t) = \Im m(q(x,0)) + L_t^{-1}(-aR_3u_2 + bR_4u_1 + cN_1(u_1,u_2)), \tag{10}$$

where $u_{1,0}(x,0) = \Re e(q(x,0))$ and $u_{2,0}(x,0) = \Im m(q(x,0))$.

Assume that Eq. (1) has the following series solution [13]:

$$q(x,t) = u_1(x,t) + iu_2(x,t) = \sum_{n=0}^{\infty} u_{1,n}(x,t) + i\sum_{n=0}^{\infty} u_{2,n}(x,t)., \tag{11}$$

The components $u_{j,n}$ for $j = 1,2$ will be determined recurrently. Also the nonlinear operators N_1 and N_2 are decomposed as follows:

$$N_j(u_1,u_2) = \sum_{n=0}^{\infty} A_{j,n}(u_{j,0}, u_{j,1}, \ldots, u_{j,n}), \quad j = 1,2 \tag{12}$$

where $\{A_{j,n}\}_{n=0}^{\infty}$ is the so-called Adomian polynomials sequence. A novel method to calculate the Adomian polynomials was recently proposed in [14], namely

$$A_{j,0}(u_{j,0}) = N_j(u_{j,0}) \tag{13}$$

$$A_{j,n}(u_{j,0}, u_{j,1}, \ldots, u_{j,n}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} N_j\left(\sum_{k=0}^{n} u_{j,k}e^{ik\omega}\right)e^{-in\omega}d\omega, \quad n \geqslant 1. \tag{14}$$

As we can see, in this algorithm tedious calculations of high derivatives are not required.

Hence from Eqs. (9), (10) (11) and (12), we have the following iterative algorithm to compute the solution components:

$$u_1(x,t) = \sum_{n=0}^{\infty} f_n(x) - L_t^{-1}\left(aR_3\left(\sum_{n=0}^{\infty} u_{1,n}(x,t)\right) + bR_4\left(\sum_{n=0}^{\infty} u_{2,n}(x,t)\right) + c\sum_{n=0}^{\infty} A_{2,n}\right), \tag{15}$$

$$u_2(x,t) = \sum_{n=0}^{\infty} g_n(x) + L_t^{-1}\left(-aR_3\left(\sum_{n=0}^{\infty} u_{2,n}(x,t)\right) + bR_4\left(\sum_{n=0}^{\infty} u_{1,n}(x,t)\right) + c\sum_{n=0}^{\infty} A_{1,n}\right), \tag{16}$$

According to IADM, we are assuming that the initial conditions will be decomposed in series, namely:

Table 1
The absolute error when t = 0.1, t = 0.2, t = 0.3 and t = 0.5 for case m = 1 and subcase (i).

x	Error when t = 0.1	Error when t = 0.2	Error when t = 0.3	Error when t = 0.5
−1.00	1.7×10^{-9}	3.7×10^{-9}	3.1×10^{-8}	6.4×10^{-8}
−0.50	3.1×10^{-8}	6.8×10^{-8}	4.5×10^{-7}	5.1×10^{-7}
0.00	2.0×10^{-8}	5.5×10^{-7}	2.1×10^{-7}	6.6×10^{-8}
0.50	2.6×10^{-9}	1.4×10^{-8}	6.1×10^{-8}	8.1×10^{-8}
1.00	4.4×10^{-10}	7.1×10^{-9}	2.3×10^{-8}	5.5×10^{-8}

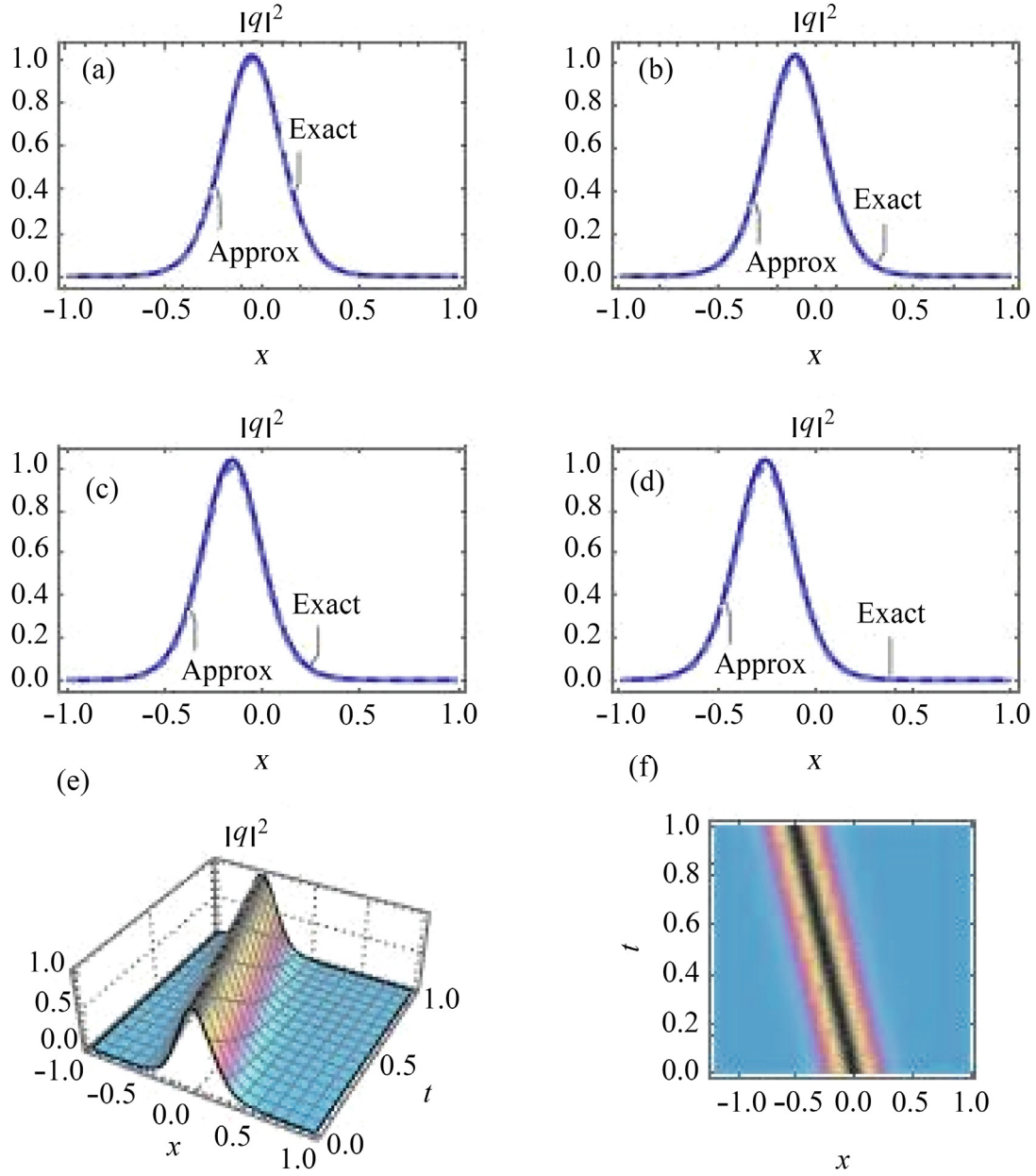

Fig. 1. Comparison of proposed method solution by IADM and exact solution for −1 < x < 1 and (a) t = 0.1, (b) t = 0.2, (c) t = 0.3, (d) t = 0.5. (e) Profile of the solution q(x,t) and (f) density plot of the solution. Case m = 1 and subcase (i) with N = 15.

$$\Re e(q(x,0))) = \sum_{n=0}^{\infty} f_n(x), \quad \Im m(q(x,0)) = \sum_{n=0}^{\infty} g_n(x). \tag{17}$$

Now we proceed to approximate solution components $u_{1,n}(x,t)$ and $u_{2,n}(x,t)$ for $n \geqslant 0$ using IADM by the following recursive relationships:

$$\begin{cases} u_{1,0}(x,t) = f_0(x), \\ u_{1,k+1}(x,t) = f_{k+1}(x) - \int_0^t (aR_3(u_{1,k}(x,\zeta) \\ +bR_4(u_{2,k}(x,\zeta)) + cA_{2,k})d\zeta, \ k \geqslant 0. \end{cases} \tag{18}$$

Table 2
The absolute error when t = 0.1, t = 0.2, t = 0.3 and t = 0.5 for case m = 1 and subcase (ii).

x	Error when t = 0.1	Error when t = 0.2	Error when t = 0.3	Error when t = 0.5
−1.50	1.9×10^{-10}	3.5×10^{-10}	4.0×10^{-9}	5.2×10^{-8}
−1.00	2.1×10^{-10}	3.7×10^{-10}	4.8×10^{-9}	7.4×10^{-7}
−0.50	2.6×10^{-9}	1.8×10^{-8}	3.3×10^{-8}	2.5×10^{-6}
0.00	2.8×10^{-8}	2.0×10^{-9}	5.6×10^{-8}	5.0×10^{-7}
0.50	1.8×10^{-9}	2.4×10^{-9}	4.1×10^{-9}	2.3×10^{-7}
1.00	2.2×10^{-9}	3.0×10^{-10}	3.7×10^{-9}	2.0×10^{-8}
1.50	1.4×10^{-10}	2.2×10^{-10}	1.1×10^{-10}	4.1×10^{-9}

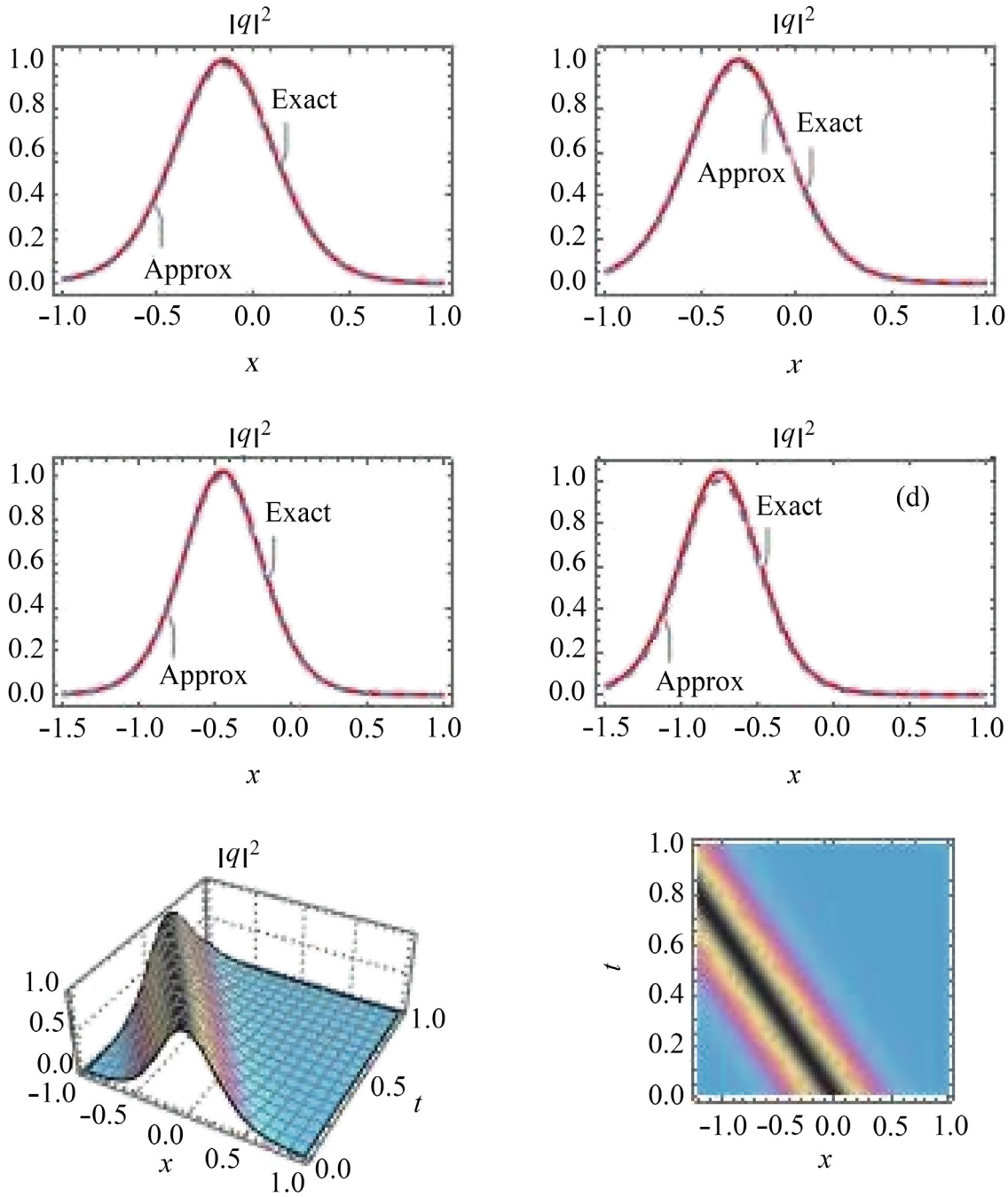

Fig. 2. Comparison of proposed method solution by IADM and exact solution for −1.5 < x < 1.5 and (a) t = 0.1, (b) t = 0.2, (c) t = 0.3, (d) t = 0.5. (e) Profile of the solution q(x,t) and (f) density plot of the solution. Case m = 1 and subcase (ii) with N = 15.

Table 3
The absolute error when t = 0.1, t = 0.2, t = 0.3 and t = 0.5 for case m = 2 and subcase (iii).

x	Error when t = 0.1	Error when t = 0.2	Error when t = 0.3	Error when t = 0.5
−1.00	2.0×10^{-10}	1.7×10^{-9}	6.2×10^{-9}	7.2×10^{-8}
−0.50	3.3×10^{-10}	5.3×10^{-8}	8.8×10^{-8}	6.1×10^{-7}
0.00	5.0×10^{-9}	6.9×10^{-7}	7.1×10^{-7}	7.8×10^{-6}
0.50	4.6×10^{-10}	6.0×10^{-8}	5.3×10^{-8}	6.8×10^{-7}
1.00	6.4×10^{-10}	3.8×10^{-9}	6.3×10^{-9}	2.9×10^{-8}

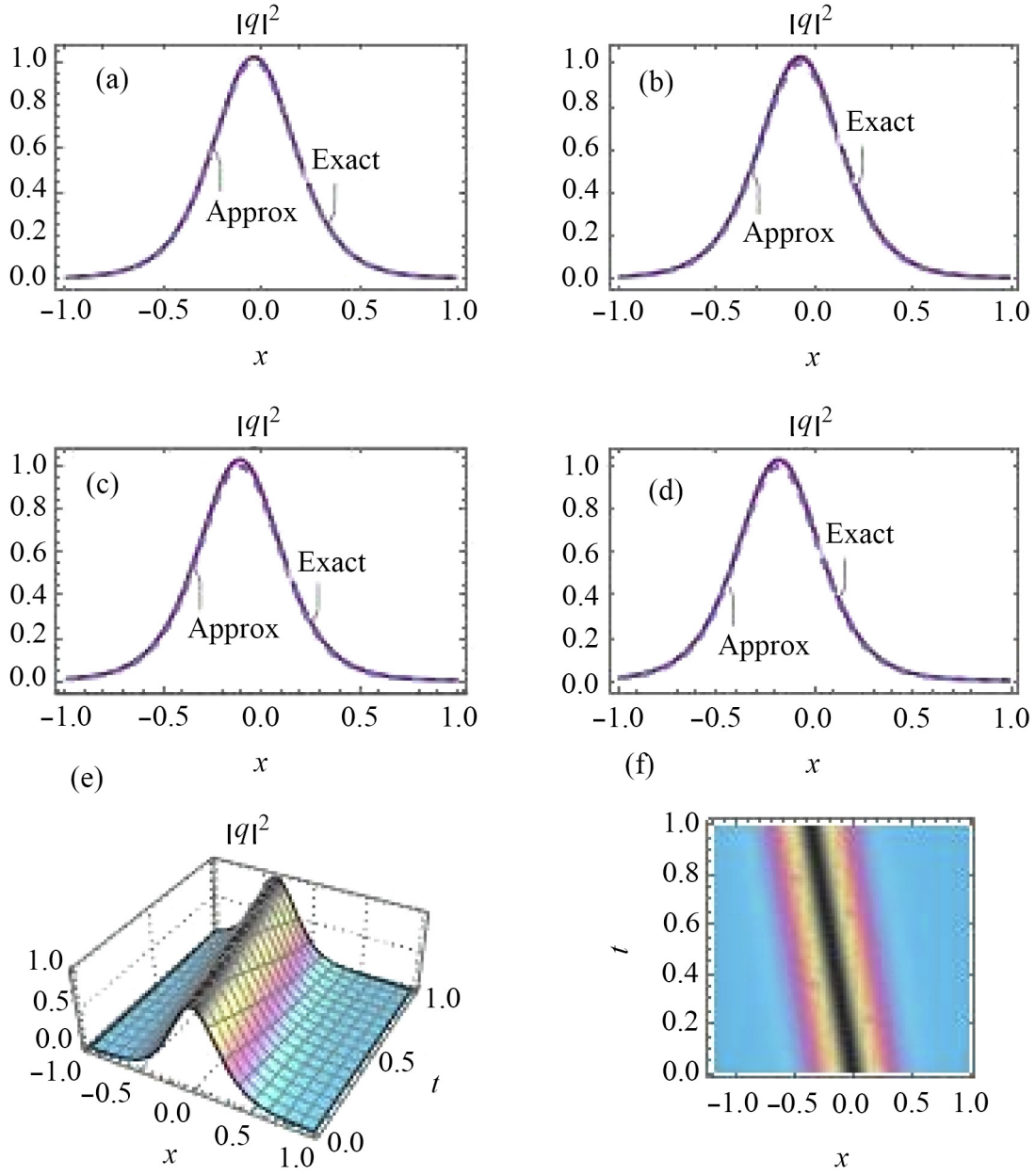

Fig. 3. Comparison of proposed method solution by IADM and exact solution for −1 < x < 1 and (a) t = 0.1, (b) t = 0.2, (c) t = 0.3, (d) t = 0.5. (e) Profile of the solution q(x,t) and (f) density plot of the solution. Case m = 2 and subcase (iii) with N = 15.

Table 4
The absolute error when t = 0.1, t = 0.2, t = 0.3 and t = 0.5 for case m = 2 and subcase (iv).

x	Error when t = 0.1	Error when t = 0.2	Error when t = 0.3	Error when t = 0.5
−1.50	8.9×10^{-10}	7.5×10^{-9}	5.1×10^{-9}	7.3×10^{-8}
−1.00	7.6×10^{-9}	7.4×10^{-9}	4.9×10^{-8}	6.7×10^{-7}
−0.50	1.2×10^{-9}	6.8×10^{-8}	8.3×10^{-7}	6.7×10^{-6}
0.00	2.9×10^{-8}	6.0×10^{-9}	5.6×10^{-7}	4.3×10^{-6}
0.50	1.0×10^{-9}	1.6×10^{-9}	7.2×10^{-8}	6.4×10^{-7}
1.00	2.9×10^{-9}	5.5×10^{-9}	8.7×10^{-9}	3.1×10^{-8}
1.50	4.0×10^{-10}	3.9×10^{-10}	6.4×10^{-9}	8.1×10^{-8}

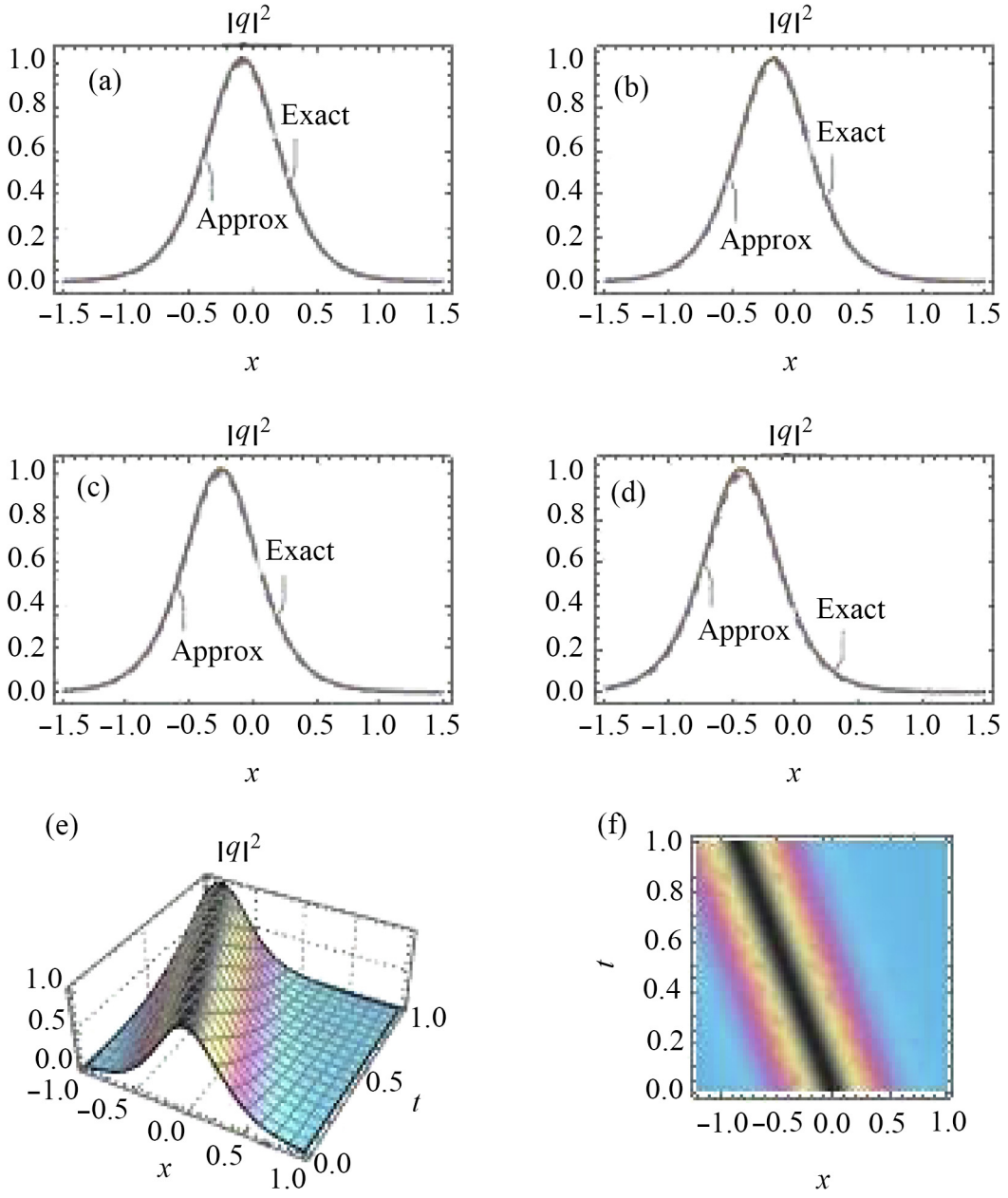

Fig. 4. Comparison of proposed method solution by IADM and exact solution for −1.5 < x < 1.5 and (a) t = 0.1, (b) t = 0.2, (c) t = 0.3, (d) t = 0.5. (e) Profile of the solution q(x,t) and (f) density plot of the solution. Case m = 2 and subcase (iv) with N = 15.

$$\begin{cases} u_{2,0}(x,t) = g_0(x), \\ u_{2,k+1}(x,t) = g_{k+1}(x) + \int_0^t (-aR_3(u_{2,k}(x,\zeta)) \\ +bR_4(u_{1,k}(x,\zeta)) + cA_{1,k})d\zeta,\ k \geqslant 0. \end{cases} \tag{19}$$

From the above consideration, the solution will be approximated by two truncated series:

$$q(x,t) = u_1(x,t) + iu_2(x,t) \approx \sum_{n=0}^{N} u_{1,n}(x,t) + i\sum_{n=0}^{N} u_{2,n}(x,t). \quad (20)$$

Results and discussion

In this section we give several examples to illustrate the efficiency and validity of the IADM and its application for the solution of the Eq. (1) in the case of bright solitons.

Case $m = 1$

Consider the CQ-NLSE model in Eq. (1) with the dispersion parameters and the power law of refractive index given in the following subcases:

(i) $a = \frac{1}{2}, b = -1$ and $c = 1$.
(ii) $a = \frac{1}{2}, b = -2$ and $c = -1$.

Case $m = 2$

Consider the CQ-NLSE model in Eq. (1) with the dispersion parameters and the power law of refractive index given in the following subcases:

(iii) $a = 1, b = -2$ and $c = 2$.
(iv) $a = 2, b = -1$ and $c = -3$.

To perform the simulations, we also consider the initial condition at $t = 0$ from Eq. (2)

$$q(x,0) = A\text{sech}^2[B(x)]e^{i[-\kappa x+\theta_0]}. \quad (21)$$

Next we will present the simulation of the two cases (and the two subcases of each) above:

- In Table 1, we examine some values of t and compare with the results obtained from the exact solution for case $m = 1$ and subcase (i). For the same case and with the same parameters, the 2D simulations for values of from $t = 0.1,\ 0.2,\ 0.3,\ 0.5$ and 3D profile of the approximate solution and its respective density plot are shown in Fig. 1(a), (b), (c), (d), (e) and (f), respectively.
- In Table 2, we examine some values of t and compare with the results obtained from the exact solution for case $m = 1$ and subcase (ii). For the same case and with the same parameters, the 2D simulations for values of from $t = 0.1,\ 0.2,\ 0.3,\ 0.5$ and 3D profile of the approximate solution and its respective density plot are shown in Fig. 2(a), (b), (c), (d), (e) and (f), respectively.
- In Table 3, we examine some values of t and compare with the results obtained from the exact solution for case $m = 2$ and subcase (iii). For the same case and with the same parameters, the 2D simulations for values of from $t = 0.1,\ 0.2,\ 0.3,\ 0.5$ and 3D profile of the approximate solution and its respective density plot are shown in Fig. 3 (a), (b), (c), (d), (e) and (f), respectively.
- Finally, in Table 4, we examine some values of t and compare with the results obtained from the exact solution for case $m = 2$ and subcase (d). For the same case and with the same parameters, the 2D simulations for values of from $t = 0.1,\ 0.2,\ 0.3,\ 0.5$ and 3D profile of the approximate solution and its respective density plot are shown in Fig. 4(a), (b), (c), (d), (e) and (f), respectively.

Conclusions

This paper discussed CQ solitons by the aid of IADM. The numerical results speak for itself with the display of impressive profiles for bright solitons. The results of the scheme thus pave way for future results. They will stem from CQ solitons from birefringent fibers having various nonlinear structures. Further along, the results will be extended to the model with DWDM networks. Those results will be available shortly down the road. Therefore those knowledge-hungry folks are suggested to hold it with patience!.

Declaration of Competing Interest

The authors declare they have no conflict of interest.

Compliance with Ethics Requirements

This article does not contain any studies with human or animal subjects.

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