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Research article

Predicting construction cost under uncertainty using grey-fuzzy earned value analysis

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ABSTRACT

Insufficient emphasis on planning and control is one of the major causes of several delayed and cost-overrun construction projects. To improve such performances, many studies have been conducted on project control techniques such as Earned Value Analysis (EVA) and its modifications: fuzzy EVA and grey EVA. Since there is no analytical model integrating fuzzy theory and grey theory simultaneously with EVA, this research aimed at predicting construction cost under uncertainty using grey-fuzzy EVA. Consequently, simple and valid project cost control grey-fuzzy EVA algorithms were developed to ensure continuous project cost performance improvement in the presence of imprecise data. In addition, an analysis result interpretation scheme was presented. Grey-fuzzy EVA was compared with fuzzy EVA and grey EVA to check its validity. Then, a case study of a road project in Addis Ababa, Ethiopia, was presented to demonstrate the application of grey-fuzzy EVA. This research contributes determinations of the lower limit, median, and upper limit of predicted costs and degree of greyness using grey-fuzzy EVA, which simplifies cost analysis, requires only a small number of data points (BAC, PV, AC, and Progress), needs no experts to create a membership function, and is comprehensible for practitioners as compared to fuzzy EVA and grey EVA used separately.

1. Introduction

Project management, which entails project planning and control (PPC), is necessary to accomplish project success, which is generally measured by cost, time, and quality [1]. Because of insufficient planning and control, several projects have been delayed and incurred cost overruns [2]. Earned Value Analysis (EVA) is a commonly used cost-controlling method that compares actual value with planned value to provide an early warning signal [3]. Project cost performance must be analyzed regularly to discover variations and take corrective action to complete the project within budget [4]. EVA aids in the early detection of time and cost overruns, allowing managers to identify and control issues [5]. Scope, cost, and time control are all integrated into a single framework using EVA [6]. EVA can be applied to any project that has a carefully considered work plan, a cost accounting framework, and a timely data collection system with regular status cut-off points for tracking progress [7]. However, EVA ignores the cash flow of the contractor, the time value of money, and payment delays [8]. In addition, EVA ignores project risk assessments and variability [6]. Furthermore, EVA's premise that a project's previous performance can be used to forecast future performance is a flaw [9]. EVA is significantly impacted by macroeconomic variables and market instability in the construction sector [10].

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List of nomenclature

D_{max}	Maximum Value of grey number domain
D_{min}	Minimum Value of grey number domain
EV_l	Lower limit earned value
EV_m	Median earned value
EV_u	Upper limit earned value
PC_l	Lower limit percent complete
PC_m	Median percent complete
PC_{max}	Maximum domain of percent complete
PC_{min}	Minimum domain of percent complete
PC_u	Upper limit percent complete
$\otimes \widetilde{VAC}$	Grey-Fuzzy Variance at Completion
$\otimes \widetilde{CPI}$	Grey-Fuzzy Cost Performance Index
$\otimes \widetilde{CSI}$	Grey-Fuzzy Cost Schedule Performance Index
$\otimes \widetilde{EAC}$	Grey-Fuzzy Estimate at Completion
$\otimes \widetilde{SPI}$	Grey-Fuzzy Schedule Performance Index
$\otimes \widetilde{CV(\%)}$	Grey-Fuzzy Cost Variance Percent
$\otimes \widetilde{CV}$	Grey-Fuzzy Cost Variance
$\otimes \widetilde{ETC}$	Grey-Fuzzy Estimate to Complete
$\otimes \widetilde{EV}$	Grey-Fuzzy Earned Value
$\otimes \widetilde{PC}$	Grey Fuzzy Percent Complete
$\otimes \widetilde{TCPI}$	Grey-Fuzzy To-Complete Performance Index
AC	Actual Cost
BAC	Budget at Completion
ETB	Ethiopian Birr
EV	Earned Value
EVA	Earned Value Analysis
FST	Fuzzy set theory
Fuzzy EVA	Fuzzy earned value analysis
G°	Degree of greyness
GFN	Grey-Fuzzy Number
Grey EVA	Grey earned value analysis
Grey-Fuzzy EVA	Grey-Fuzzy earned value analysis
GST	Grey system theory
H	High
L	Lower Limit of grey number
L	Low
MH	Medium High
PC	Percent Complete
PPC	Project planning and control
PV	Planned value
TFN	Triangular fuzzy number
U	Upper Limit of grey number
VH	Very High
VL	Very Low

EVA is used to compute cost variance, performance index, and estimate at completion using three parameters: planned value, earned value, and actual cost [11]. The three key parameters of EVA are [12]: The expected cost of scheduled work is referred to as the planned value (PV). It serves as a benchmark against which project cost performance can be determined. It can be found in the cash flow diagram (S-curve). Actual cost (AC) indicates the money expended on the work completed. It is derived from accounting records that track actual financial expenses. On the other hand, earned value (EV) refers to the amount of executed work until the status date. To calculate EV, multiply the budget at completion (BAC) by the percent complete (PC) for the activity, milestone, or project. PC is a measure of how much work has been finished, represented as a percentage [13]. To reduce error, a PC should be assigned to each activity rather than the entire project [14]. Errors and uncertainty arise as a result of subjectivity in measuring PC, but they can be eliminated by employing linguistic variables [15]. PV is determined prior to the start of the project, whereas EV and AC are determined while the project is being executed and can only be determined until the status date [16]. Many applications in the real world are imprecise, vague, and ambiguous [17]. In projects, uncertainty is inevitable. Incomplete information and inaccurate data are the main sources of uncertainty [18]. Failing to take uncertainties into account in planning, performance evaluation, and forecasting results in erroneous results, which in turn results in poor decision-making [17]. Fuzzy set theory (FST) and grey system theory (GST) are most frequently used in the study of uncertain systems [18].

Zadeh [19] was the first to introduce fuzzy set theory (FST). FST is a simple and efficient method for illustrating the absence of accurate information or information that is difficult to get, interacts realistically with variables, and analyzes data in a way that is comparable to human reasoning [20]. The following are the advantages of using FST in project planning and control [21,22]: simplicity in computation and ease of usage; stochastic sampling and statistical computations are not required; activities' duration and cost are expressed as linguistic variables; these models directly include the mental reasoning, experience, and opinions of experts; and the results are precise. The membership function is a crucial component of FST, and its precise formulation is critical to the applicability of any fuzzy-based model [23]. If a precise description of an event's state is not available, a fuzzy number can be used to describe it [24]. Fuzzy numbers are created by converting linguistic variables using fuzzy principles [25]. Triangular and trapezoidal fuzzy numbers are commonly used in uncertainty [26]. Particularly triangular fuzzy numbers, which are represented by optimistic, most likely, and pessimistic values, are appropriate for long-term forecasting [27].

Grey System Theory (GST), which has both knowns and unknowns, was initially proposed by Deng [28]. GST is a viable technique for data analysis in uncertainty involving incomplete data [29] and small data [30]. GST is used when a probability distribution can not be determined or when the input data is insufficient or unclear [31]. GST is simpler to process data than FST since it does not need a membership function [32]. Furthermore, unlike FST, GST does not need the employment of experts to develop a suitable membership function [31]. FST has a clear intention but an unclear extension, whereas GST has an unclear intention but a clear extension [24]. In contrast to FST, which depends on subjective uncertainty, GST depends on objective uncertainty [25]. The fundamental benefit of a GST over FST is its minimal sample data demand and flexible pattern detection capacity [26]. The grey number is described as the unknown value that lies between two known bounds (the lower limit and the upper limit) [18,25]. The ratio of grey interval to grey number domain is known as the degree of greyness (G°) [26]. G° indicates the inadequacy or improbability of information [33]. Uncertainty decreases as G° gets closer to 0, whereas uncertainty increases as G° gets closer to 1 [34].

Despite its broad application, EVA has a significant flaw: it is founded on the erroneous premise that the parameters of EVA are entirely known, but in fact, these parameters are vague, imprecise, or unknown. Therefore, FST has been employed to address project uncertainty [35]. Naeni et al. [36] introduced fuzzy EVA as a way to improve the precision and dependability of project time and cost performance evaluation and forecasting in uncertain contexts. The primary benefit of fuzzy EVA is that it includes uncertainty when evaluating project performance, but the main drawback is that it is challenging to create a membership function when experts are not available [10]. To overcome this constraint, grey EVA was presented by Mahmoudi et al. [37] as a way to cope with uncertainty without the involvement of experts. A new grey-based earned value approach with key performance indicators (KPIs) was introduced by Eshghi et al. [38] for cost and time performance evaluation and forecasting in unpredictable situations. An EVA based on interval grey numbers was presented by Nadafi et al. [39] to predict project cost and time in unknown situations.

The main motivations for conducting this research are.

- Projects commonly overspend their budgets in Ethiopia since there is no proper method for cost control in uncertain circumstances. To help project managers easily evaluate and forecast project cost performance using an Excel spreadsheet in the presence of imprecise data and take corrective actions, the grey fuzzy EVA analytical model was developed.
- Grey-fuzzy EVA was proposed by integrating FST and GST with EVA because grey EVA and fuzzy EVA have drawbacks. Grey EVA only determines the lower and upper bounds of predicted costs, not the median and degree of greyness, while fuzzy EVA is difficult to utilize in practice as it requires subject-matter experts, and sometimes the experts may not be available.

The main contributions of this study are.

- This research primarily contributes estimates of the lower limit, median, and upper limit of predicted cost and degree of greyness using grey-fuzzy EVA based on deterministic progress data and a 5-point interval scale to consider uncertainty, unlike fuzzy EVA and grey EVA.
- When compared to fuzzy EVA and grey EVA used independently, grey-fuzzy EVA is suited for use by project managers to evaluate and predict project cost performance since it simplifies cost analysis (it uses GST to determine the triangular fuzzy number and can be done using an Excel spreadsheet), requires a minimal number of data (only four numbers of data), and does not require experts to create membership functions (it is based on deterministic progress and a 5-point interval scale).
- All project stakeholders, including contractors, consultants, clients, and regulatory bodies, can easily utilize grey-fuzzy EVA to control project cost performance at any stage and level of the project.

Section 1 introduces EVA, FST, GST, fuzzy EVA, and grey EVA and states the motivation and contribution of the study. Section 2 explains the available analytical models, the research gap, and the objective of the study. In Section 3, the methods for measuring uncertainty and grey-fuzzy number operations are presented. Section 4 deals with the research methodology, including research design, variables, and data processing. In Section 5, simple grey-fuzzy EVA algorithms are developed, and their interpretations are provided. The grey-fuzzy EVA is compared with fuzzy EVA and grey EVA to check its validity in Section 6. Section 7 demonstrates the application of grey-fuzzy EVA using a case study of a road project. In Section 8, the conclusions of the study and its limitations are provided. The paper concludes with a recommendation for future scope in Section 9.

2. Research rationale

Four types of analytical models are frequently employed: deterministic, probabilistic, fuzzy, and grey, as discussed briefly hereunder. Deterministic models are those in which all values are taken to be known or certain. Due to their ease of use and comprehension, deterministic models are the ones that project professionals employ the most frequently. Their accuracy is constrained because they do not account for uncertainty. A classic example of a deterministic model is earned value analysis (EVA), which excludes uncertainties from its evaluation and forecasting of project cost performance. The EVA method is especially useful for estimating a project's predicted cost and completion time based on actual performance until the project's status date [40]. EVA uses the core notion that patterns and trends in the past can be good predictors of the future [41]. Deterministic models must be complemented with probabilistic and/or fuzzy models to account for uncertainties [42].

To determine the probability distributions of outputs, probabilistic models take into account the uncertainties and risks related to the underlying assumptions and randomly sample those uncertain inputs. Variances, standard deviations, means, percentiles, and levels of confidence are important statistical expressions of the uncertainty related to the output distributions from probabilistic models [42]. The disadvantages of probabilistic or stochastic models include their complexity, difficulty in comprehension, necessity for software support, and time-consuming nature [43]. Acebes et al. [44] employed Monte Carlo simulation to determine the project's probability of success in terms of cost and time, and they used a regression technique to predict project cost and time. Soltan and Ashrafi [45] suggested a statistical method, in combination with EVA, for forecasting project cost and time, as well as determining the optimal action plan to improve project accuracy and performance.

Zadeh [19] was the first to create and apply a fuzzy model. According to Liu et al. [18], fuzzy models accommodate varying degrees of subjectivity, partial knowledge, and perception/vagueness. When a precise description of an event's state is not provided, fuzzy models can be used to represent linguistic events [24]. Fuzzy models enable an element to only partially belong to a set spanning from 0 to 1 using a membership function [23]. These models directly incorporate the mental reasoning, knowledge, and opinions of experts, and the results are extremely accurate [21]. One of the available fuzzy models is fuzzy EVA, which was proposed by Naeni et al. [36] to evaluate and forecast project cost and time performance.

Deng [28] was the pioneer to propose the grey model. Grey models are widely used because they can accurately predict system behavior with a limited number of data points [30] and incomplete data [29]. Grey models are practical tools for analyzing insufficient, incomplete, and imprecise input data [46]. The main advantages of a grey model over a probability model and a fuzzy model are that it requires fewer samples and is more adaptable in terms of model recognition [47]. Unlike the fuzzy model, the grey model does not need the employment of experts to develop a suitable membership function [32,37]. One of the available grey models is the grey EVA, which was proposed by Mahmoudi et al. [37] to take uncertainty into account without the requirement for experts to create membership functions.

2.1. The objective of the research

To the best of the authors' knowledge, there is no analytical model integrating FST and GST simultaneously with EVA. Consequently, the objective of this research is to predict construction cost under uncertainty using triangular grey-fuzzy EVA, taking into account the advantages of FST and GST. Accordingly, simple and valid project cost control grey-fuzzy EVA algorithms were developed to ensure continuous project cost performance improvement in the presence of imprecise data.

Comparisons of Grey-Fuzzy EVA with Fuzzy EVA and Grey EVA are as follows.

- The formation of a membership function for fuzzy EVA demands the expertise of subject-matter experts, which is difficult to get, while grey EVA does not show the median or the degree of greyness; it just shows the lower and upper bounds of predicted cost. Unlike fuzzy EVA and grey EVA, grey-fuzzy EVA determines the most important parameters, such as the lower limit, median, and upper limit of predicted costs, as well as the degree of greyness altogether, based on actual deterministic progress and a 5-point interval scale to consider uncertainties.
- Grey-fuzzy EVA yields results that are comparable to those of fuzzy EVA and grey EVA.
- Compared to fuzzy EVA and grey EVA, comprehensive grey-fuzzy EVA parameters are included.
- Compared to fuzzy EVA and grey EVA used separately, grey-fuzzy EVA simplifies cost analysis, requires only a small number of data points, does not require experts to establish a membership function, and is comprehensible for practitioners.
- Furthermore, grey-fuzzy EVA will be helpful for practitioners to control project cost easily and precisely in uncertain conditions, particularly when it is difficult to exactly determine the progress of activities, as compared to fuzzy EVA and grey EVA used separately.

3. Grey-fuzzy number (GFN) operations

Incompleteness and inadequacy of information are the main characteristics of uncertain systems. The incomplete information may include parameters, structure, boundaries, and behaviors of the system. Inaccuracies include conceptual, level, and prediction errors [18]. Depending on the causes of the uncertainty, the type and amount of data, the requirements, and other factors, several theories can be used to model the uncertainty [48]. The most popular techniques for quantifying uncertainty include fuzzy set theory (FST), grey system theory (GST), and probability theory (standard error approach, Bayesian method, Monte Carlo simulation method, etc.).

Probability theory is a non-deterministic mathematical technique that examines stochastic uncertainty or randomness using historical statistical laws, regression, and sampling [18,49]. To create predictive distributions, large samples must be available [18]. For many forms of studies involving uncertainty, including estimation and prediction, decision-making, planning, and control, probability theory is applied [49]. Probability theory enables precise cost predictions but may be difficult to understand and use [50]. Probabilistic theory has drawbacks [21]. First, because each project is unique, it is impossible to draw any conclusions using statistical inference. Instead, one must depend on the repeatability of activities to determine the probabilistic values, probabilistic distribution, mean, and variance of distributions, etc. Second, the inferential conclusion made by the expert is not adequately and explicitly taken into account in these models. Thirdly, considering multiple statistical distributions makes cost predictions for projects exceedingly difficult. Zadeh [51] argues that the idea of a fuzzy event, methods for dealing with fuzzy quantifiers, a system and methods for calculating fuzzy probabilities, and other challenges preclude probability theory from dealing with uncertainty and imprecision. He added that probability theory is less useful in fields where it is important to include human perception and reasoning. He also noted that it is less applicable in situations where variable dependencies are not precisely specified and probability information is vague or insufficient.

Fuzzy set theory (FST) offers a mathematical framework in which vague conceptual issues can be precisely and systematically examined and can also be taken as a modeling language that works well in fuzzy scenarios [48]. FST is a useful technique for modeling approximation reasoning and computation using linguistic concepts; it offers a way to derive conclusions from ambiguous information and the lack of complete and precise data [23]. FST examines issues with cognitive ambiguity or vagueness, where study objects have the trait of clear intension (concept) and unclear extension (domain), leveraging the expertise of experts to create the membership function [18]. FST uses a membership function to assign values between zero and one, in contrast to probability theory's assignment of zero or one [48]. The membership functions may be triangular, trapezoidal, pentagonal, hexagonal, and so on. However, a triangular fuzzy number has been chosen because of its simplicity in representing imprecise data and requires only three values, such as the lower limit, median, and upper limit. Secondly, a simple algorithm of arithmetic operations as well as easy computation and interpretation are applied. Thirdly, it is easily comprehensible for project managers to implement in real project cost evaluation and prediction.

Probability and FST, however, are unable to make precise predictions in the absence of sufficient data and experts. In these cases, grey system theory (GST) can be applied. GST addresses uncertainty problems caused by small data and inadequate information [18]. According to Liu et al. [34], the information in GST is divided into three categories: white for information that is entirely certain, grey for information that is only somewhat certain, and black for entirely ambiguous information. Not all data is entirely known (white information) or entirely unknown (black information). They combine to create the color grey. This is how the GST got its name to signify that it is used when there are gaps in the data [29]. Transforming grey (incomplete) information into significantly white (complete) information is the main concept of GST [28]. Grey and fuzzy numbers differ primarily in that a fuzzy number is defined as an interval, but the precise number of intervals is unknown and follows a membership function, whereas grey numbers have an unknown precise number but are known about the interval that contains the number [47]. While GST only considers the data's underlying meaning, FST relies on experts to capture intuitiveness [24]. GST can be enhanced with the expertise of experts, making it more flexible and responsive to any situation [52]. Compared to FST, GST has a smaller computational overhead [53]. Table 1 outlines the fundamental distinctions between probability theory, FST, and GST [18].

The main drawbacks of FST are its inability to learn from data and its reliance on expert knowledge for the creation of frequently context-dependent models. These restrictions can be removed by combining FST with complementary methods [23]. For measuring uncertainty, FST and GST are complementary methods [54]. Since establishing the membership function for a fuzzy number's left and right boundaries is complicated, utilizing grey numbers to calculate fuzzy numbers is simple [47]. GST has the benefit of being able to deal with fuzzy situations more flexibly than FST [55]. FST and GST are preferred in this study over probability theory primarily because they require fewer data points, whereas probability theory calls for a large amount of historical information or a database. FST and GST can be employed when it is challenging to exactly determine a project's progress since they deal with uncertainty by utilizing experts' views and the underlying meaning of data in the absence of significant prior data. Additionally, FST and GST are more practical and simpler to understand than probability theory. Grey-fuzzy number (GFN) is described as follows. Let $\widetilde{P}$ be a triangular fuzzy number (TFN) with a degree of greyness (G°), where G° is used to determine the level of uncertainty of data and $G^{\circ}\epsilon\,[0,1]$, $\widetilde{P} = (P_l,\ P_m, P_u)$, then $\otimes\widetilde{P} = (\widetilde{P}, G^{\circ})$ is GFN [33].

The mathematical operations of GFNs are defined as given below in Eqs. (1)–(3) [33].

Assume two grey fuzzy numbers $\otimes\widetilde{P} = (\widetilde{P}, G^{\circ}_{P}) \otimes \widetilde{Q} = (\widetilde{Q},\ G^{\circ}_{Q})$.

Table 1
Comparison among probability theory, FST, and GST [18].

Uncertainty research	Probability theory	FST	GST
Research objects	Stochastic	Cognitive	Poor information
Basic set	Cantor set	Fuzzy set	Grey number set
Describe method	Density function	Membership function	Possibility function
Procedure	Frequency	Cut set	Sequence operator
Data requirement	Known distribution	Known membership	Any distribution
Emphasis	Intension	Extension	Intension
Objective	Historical law	Cognitive expression	Law of reality
Characteristics	Large sample	Depend on experience	Small data

$$\otimes \widetilde{P} + \otimes \widetilde{Q} = \left(\widetilde{P} + \widetilde{Q}, G^{\circ}_{P} \vee G^{\circ}_{Q}\right) \tag{1}$$

$$\otimes \widetilde{P} * \otimes \widetilde{Q} = \left(\widetilde{P} * \widetilde{Q},\ G^{\circ}_{P} \wedge G^{\circ}_{Q}\right) \tag{2}$$

$$\frac{\otimes \widetilde{P}}{\otimes \widetilde{Q}} = \left(\frac{\widetilde{P}}{\widetilde{Q}},\ G^{\circ}{}_{P} \wedge G^{\circ}_{Q}\right) \tag{3}$$

The operations of two TFNs, $\widetilde{P} = (P_l,\ P_m,\ P_u), \widetilde{Q} = (Q_l,\ Q_m,\ Q_u)$, are presented as given in Eqs. (4)–(7) [56].

$$\widetilde{P} + \widetilde{Q} = (P_l + Q_l, P_m + Q_m,\ P_u + Q_u) \tag{4}$$

$$\widetilde{P} - \widetilde{Q} = (P_l - Q_u, P_m - Q_m, P_u - Q_l) \tag{5}$$

$$\widetilde{P} * \widetilde{Q} = (P_l * Q_l,\ \ P_m * Q_m, P_u * Q_u) \tag{6}$$

$$\frac{\widetilde{P}}{\widetilde{Q}} = \left(\frac{P_l}{Q_u}, \frac{P_m}{Q_m}, \frac{P_u}{Q_l}\right) \quad \text{Where } P_i > 0,\ Q_i > 0 \tag{7}$$

The degree of greyness (G°) is determined as shown in Eq. (8) [25,57].

$$G^{\circ} = \frac{|U - L|}{|D_{max} - D_{min}|} \tag{8}$$

Where: L: Lower Limit of grey number; U: Upper Limit of grey number; D_{min}: Minimum Value of the grey number domain; D_{max}: Maximum Value of the grey number domain; $D_{min} \le L$, $U \le D_{max}$.

4. Methodology

4.1. Research design

A quantitative technique was used to answer the research objective (a statistical approach using algorithms for the prediction of construction cost). Grey-fuzzy EVA algorithms for cost performance evaluation and prediction were developed by combining FST and GST with EVA. In addition, an analysis result interpretation scheme was presented. The grey-fuzzy EVA algorithms were validated by comparing them with fuzzy EVA and grey EVA using an example. Then, a case study was conducted to demonstrate the application of grey-fuzzy EVA in real-world road construction. For each project milestone, data on BAC, PV, AC, and progress were gathered using the case study. The data were analyzed using grey-fuzzy EVA algorithms. The data analysis results were interpreted. For data analysis, triangular fuzzy numbers, linguistic terms, and degree of greyness were used.

4.2. Variables

The independent variables for this research are variables collected through a case study, such as BAC, PV, AC, and $\otimes\ \widetilde{PC}$: Grey Fuzzy Percent Complete. The dependent variables for this research are Grey-Fuzzy EVA parameters for predicting construction cost ($\otimes$ $\widetilde{EV}$: Grey-Fuzzy Earned Value, $\otimes \widetilde{CV}$: Grey-Fuzzy Cost Variance, $\otimes \widetilde{CV(\%)}$: Grey-Fuzzy Cost Variance Percent, $\otimes\ \widetilde{CPI}$: Grey-Fuzzy Cost Performance Index, $\otimes\ \widetilde{EAC}$: Grey-Fuzzy Estimate at Completion, $\otimes\ \widetilde{ETC}$: Grey-Fuzzy Estimate to Complete, $\otimes\ \widetilde{VAC}$: Grey-Fuzzy Variance at Completion, and $\otimes\ \widetilde{TCPI}$: Grey-Fuzzy To-Complete Performance Index).

4.3. Data processing

Steps for Grey-Fuzzy EVA.

Step 1. BAC, PV, and AC for each milestone of a road project in Addis Ababa, Ethiopia, were collected.

Step 2. EV is calculated by multiplying the budget at completion (BAC) by the percent complete (PC) for each activity, milestone, and project. One of the most commonly used methods for calculating EV is percent complete, but it is prone to errors and uncertainties due to the following causes: As the person in charge estimates the percentage of the activity completed, it is subject to biased judgments [36]. Second, management pressure may affect the reported results as a result of company targets that must be achieved to qualify for project incentives or simply due to human nature, which involves avoiding "bad news" in the hopes that poor performance can be improved [12]. Thirdly, values can be distorted by an optimistic point of view at the commencement of an activity, when eager to demonstrate progress, or by a pessimistic point of view towards the finish as the intricacy of the activity is well grasped [58]. The solution to this problem is to use linguistic variables in estimating the percent complete of each activity [36]. Consequently, deterministic progress for each milestone of the road project was collected and converted into linguistic variables and then into grey-fuzzy numbers based on a 5-point interval scale, similar to the 5-point Likert scale, to consider 20% uncertainty. The most widely used psychometric scale is the Likert scale, which was first introduced by Rensis Likert in 1932. The Likert scale is the only tool available for the quantitative transformation of qualitative qualities like perceptions and opinions [59]. The Likert method has the following benefits [60]. A Likert scale may be quickly created and altered first. Second, the results of the numerical measurement can be applied directly to statistical inference. Third, studies using Likert scaling have shown strong measurement reliability. Fourth, Likert scaling makes it easier and quicker to gather and analyze a vast amount of data. Li [60] proposed the use of a fuzzy Likert scale to increase the precision of the conventional Likert scale and address information loss and distortion caused by closed-form scaling and the ordinal structure of this measurement method. However, employing a fuzzy Likert scale makes it more difficult for project professionals to use it in their work. In many studies, 5-point Likert scales were employed to gather respondents' opinions, whereas a 5-point interval scale was utilized in this study to convert deterministic percent complete into linguistic variables. The above-mentioned advantages of the Likert scale apply to the 5-point interval scale. It is simple for project managers to use in real-project cost evaluation and prediction. Additionally, it is simpler to comprehend, better suited for analysis, and results in better data distribution. The linguistic variables in Table 2 were categorized using a 5-point interval scale similar to a 5-point Likert scale.

Step 3. The dependent grey-fuzzy EVA Parameters were determined, such as $\otimes\widetilde{EV}, \otimes\widetilde{CV}, \otimes\widetilde{CV(\%)}, \otimes\widetilde{CPI}, \otimes\widetilde{EAC}, \otimes\widetilde{ETC}$, $\otimes\ \widetilde{VAC}$, and $\otimes\ \widetilde{TCPI}$.

Step 4. Interpretation of the analysis results was done.

5. Grey-fuzzy earned value analysis

Grey-fuzzy EVA algorithms were developed using basic parameters ($\otimes\ \widetilde{EV}$, BAC, PV, and AC) and linear formulas. In these algorithms, EV was considered grey-fuzzy, whereas BAC, PV, and AC were considered deterministic. These algorithms are used to control project costs. Project grey-fuzzy earned value, $\otimes\widetilde{EV}$, is computed as the sum of the grey-fuzzy earned value of each activity or milestone, $\otimes\widetilde{EV}_i$, as shown in Eq. (9).

$$\otimes\widetilde{EV} = \sum_{i=1}^{n} \otimes\widetilde{EV}_i \tag{9}$$

As shown in Eq. (10), the project budget at completion (BAC) is the sum of the budget of each activity or milestone BAC_i..

$$BAC = \sum_{i=1}^{n} BAC_i \tag{10}$$

The project's planned value until the status date, PV, is computed as the sum of the planned value of each activity or milestone until the status date , PV_i, as shown in Eq. (11).

$$PV = \sum_{i=1}^{n} PV_i \tag{11}$$

The total project actual cost until the status date, AC, is computed as the sum of the actual cost of each activity or milestone until the status date ,AC_i, as shown in Eq. (12).

$$AC = \sum_{i=1}^{n} AC_i \tag{12}$$

5.1. *Project cost performance evaluation*

The parameters $EV_l,\ EV_m, and\ \ EV_u$ are determined by multiplying the $\otimes\widetilde{PC}$ and the BAC of each activity or milestone of a project. Progresses of activity or milestone are converted into triangular grey-fuzzy percent completes. Algorithms for grey-fuzzy cost variance ($\otimes\ \widetilde{CV}$), grey-fuzzy cost variance percent ($\otimes\ CV\widetilde{(\%)}$), and grey-fuzzy cost performance index ($\otimes\ \widetilde{CPI}$) are hereunder developed to evaluate project cost performance.

Table 2
Conversion of deterministic percent complete into a 5-point interval scale, linguistic variables, and grey-fuzzy progress.

5-point interval scale	Linguistic Variables	Grey-Fuzzy Progress
0%–20%	Very Low (VL)	[(0.0, 0.1, 0.2), 0.2]
20%–40%	Low (L)	[(0.2, 0.3, 0.4), 0.2]
40%–60%	Medium (M)	[(0.4, 0.5, 0.6), 0.2]
60%–80%	High (H)	[(0.6, 0.7, 0.8), 0.2]
80%–100%	Very High (VH)	[(0.8, 0.9, 1.0), 0.2]

Grey-fuzzy earned value ($\otimes\widetilde{EV}$) is determined as follows in Eq. (13).

$$\otimes\widetilde{EV} = [(EV_l, EV_m, EV_u),\ g^\circ{}_{EV}] = \left[(PC_l * BAC,\ PC_m * BAC,\ PC_u * BAC), \frac{PC_u - PC_l}{PC_{max} - PC_{min}}\right] \tag{13}$$

Where: EV_l: Lower limit earned value, EV_m: Median earned value, EV_u: Upper limit earned value, PC_l: Lower limit percent complete, PC_m: Median percent complete, PC_u: Upper limit percent complete, BAC: Budget at Completion, PC_{max}: Maximum domain of percent complete, PC_{min}: Minimum domain of percent complete

Grey-fuzzy cost variance ($\otimes\widetilde{CV}$) is obtained by deducting deterministic actual cost (AC) from grey-fuzzy earned value ($\otimes\widetilde{EV}$) and is determined as follows in Eq. (14).

$$\otimes\widetilde{CV} = \otimes\widetilde{EV} - AC = \left[(EV_l - AC, EV_m - AC,\ EV_u - AC),\ g^\circ_{EV}\right] \tag{14}$$

The grey-fuzzy cost variance ($\otimes\widetilde{CV}$) results are interpreted according to Table 3.

Grey-fuzzy cost variance percent ($\otimes\widetilde{CV\%}$) is the ratio of grey fuzzy cost variance ($\otimes\widetilde{CV}$) to grey fuzzy earned value $\otimes\ \widetilde{EV}$. It shows the grey fuzzy percent of cost overrun or cost underrun. $\otimes\widetilde{CV}$ is determined as follows in Eq. (15).

$$\otimes\widetilde{CV\%} = \frac{\otimes\widetilde{CV}}{\otimes\widetilde{EV}} = \left[\left(\frac{CV_l}{EV_u}, \frac{CV_m}{EV_m}, \frac{CV_u}{EV_l}\right), g^\circ_{CV} \wedge g^\circ_{EV}\right] \tag{15}$$

Table 4 presents the interpretation of the grey-fuzzy cost variance percent ($\otimes\widetilde{CV\%}$) results.

The grey-fuzzy cost performance index ($\otimes\widetilde{CPI}$) is determined as the grey-fuzzy earned value ($\otimes\widetilde{EV}$) divided by actual cost (AC). It shows whether the project incurred a cost overrun or underrun. $\otimes\widetilde{CPI}$ is determined as follows in Eq. (16).

$$\otimes\widetilde{CPI} = \frac{\otimes\widetilde{EV}}{AC} = \left[\left(\frac{EV_l}{AC}, \frac{EV_m}{AC}, \frac{EV_u}{AC}\right),\ g^\circ_{EV}\right] \tag{16}$$

The grey-fuzzy cost performance index ($\otimes\widetilde{CPI}$) results are interpreted as shown in Table 5.

5.2. Project Cost Forecasting

The grey-fuzzy estimate at completion, $\otimes\ \widetilde{EAC}$, is determined based on the actual cost and grey-fuzzy estimate to complete, $\otimes\ \widetilde{ETC}$, using three alternative cases as follows:

Case I $\otimes\widetilde{\boldsymbol{ETC}}$ for the remaining work will be executed as planned.

This $\otimes\widetilde{EAC}$ approach takes the project's actual performance until the status date as the actual cost and forecasts that $\otimes\ \widetilde{ETC}$ for the remaining work will be accomplished as planned. As per Case I, $\otimes\widetilde{EAC}$ is determined as shown in Eq. (17).

$$\otimes\widetilde{EAC} = AC + (BAC - \otimes\widetilde{EV})$$
$$= [(AC + BAC - EV_u, AC + BAC - EV_m, AC + BAC - EV_l),\ g^\circ{}_{EV}] \tag{17}$$

Case II $\otimes\widetilde{\boldsymbol{ETC}}$ the for remaining work will be executed at $\otimes\ \widetilde{\boldsymbol{CPI}}$..

This $\otimes\widetilde{EAC}$ approach presumes that the project's previous performance will continue in the future. Based on Case II, $\otimes\ \widetilde{EAC}$ is computed as given in Eq. (18).

$$\otimes\widetilde{EAC} = \frac{BAC}{\otimes\widetilde{CPI}} = \left[\left(\frac{BAC}{CPI_u}, \frac{BAC}{CPI_m}, \frac{BAC}{CPI_l}\right), g^\circ_{CPI}\right] \tag{18}$$

Table 3
Project Cost Performance Evaluation based on Grey-Fuzzy Cost Variance ($\otimes\ \widetilde{CV}$).

Scenario	State of $\otimes\ \widetilde{\boldsymbol{CV}}$	Project Cost Performance Evaluation
1	$\otimes\ \widetilde{CV}_u\ < 0$	Cost overrun
2	$\otimes\ \widetilde{CV}_m\ < 0 < \otimes\ \widetilde{CV}_u$	Approximately cost overrun
3	$\otimes\ \widetilde{CV}_m\ = 0$	Breakeven
4	$\otimes\ \widetilde{CV}_l\ < 0 < \otimes\ \widetilde{CV}_m$	Approximately cost underrun
5	$\otimes\ \widetilde{CV}_l\ > 0$	Cost underrun

Table 4
Project Cost Performance Evaluation based on Grey-Fuzzy Cost Variance Percent ($\otimes \widetilde{CV}\%$).

Scenario	State of $\otimes \widetilde{CV}\%$	Project Cost Performance Evaluation
1	$\otimes \widetilde{CV}_u \% < 0\%$	Cost overrun
2	$\otimes \widetilde{CV}_m \% < 0\% < \otimes \widetilde{CV}_u \%$	Approximately cost overrun
3	$\otimes \widetilde{CV}_m = 0\%$	Breakeven
4	$\otimes \widetilde{CV}_l \% < 0\% < \otimes \widetilde{CV}_m \%$	Approximately cost underrun
5	$\otimes \widetilde{CV}_l \% > 0\%$	Cost underrun

Table 5
Project Cost Performance Evaluation based on the Grey-Fuzzy Cost Performance Index ($\otimes \widetilde{CPI}$).

Scenario	State of $\otimes \widetilde{CPI}$	Project Cost Performance Evaluation
1	$\otimes \widetilde{CPI}_u < 1$	Cost overrun
2	$\otimes \widetilde{CPI}_m < 1 < \otimes \widetilde{CPI}_u$	Approximately cost overrun
3	$\otimes \widetilde{CPI}_m = 1$	Breakeven
4	$\otimes \widetilde{CPI}_l < 1 < \otimes \widetilde{CPI}_m$	Approximately cost underrun
5	$\otimes \widetilde{CPI}_l > 1$	Cost underrun

Case III $\otimes \widetilde{ETC}$ for the remaining work will be executed at $\otimes \widetilde{CSI}$.

The $\otimes \widetilde{ETC}$ work will be completed at an efficiency that takes into consideration both grey-fuzzy cost and schedule performance indices. Based on Case III, $\otimes \widetilde{EAC}$ is calculated as shown in Eq. (19).

$$\otimes \widetilde{EAC} = AC + \left[\frac{BAC - \otimes \widetilde{EV}}{\otimes \widetilde{CSI}}\right] = \left[\left(AC + \frac{(BAC - EV_u)}{CSI_u}, AC + \frac{(BAC - EV_m)}{CSI_m}, AC + \frac{(BAC - EV_l)}{CSI_l}\right), \mathring{g}_{EV} \wedge \mathring{g}_{CSI}\right] \tag{19}$$

Grey-Fuzzy Schedule Performance Index ($\otimes \widetilde{SPI}$) is determined in Eq. (20).

$$\otimes \widetilde{SPI} = \frac{\otimes \widetilde{EV}}{PV} = \left[\left(\frac{EV_l}{PV}, \frac{EV_m}{PV}, \frac{EV_u}{PV}\right), \mathring{g}_{EV}\right] \tag{20}$$

Grey-Fuzzy Cost Schedule Performance Index ($\otimes \widetilde{CSI}$) is calculated in Eq. (21).

$$\otimes \widetilde{CSI} = \otimes \widetilde{CPI} * \otimes \widetilde{SPI} = \left[(CPI_l * SPI_l,\ CPI_m * SPI_m, CPI_u * SPI_u),\ \mathring{g}_{CPI} \wedge \mathring{g}_{SPI}\right] \tag{21}$$

Grey-Fuzzy Estimate To Complete ($\otimes \widetilde{ETC}$) is determined as follows in Eq. (22).

$$\otimes \widetilde{ETC} = AC - \otimes \widetilde{EAC} = \left[\left(AC - EAC_u, AC - EAC_m, AC - EAC_l\right),\ \mathring{g}_{EAC}\right] \tag{22}$$

Grey-fuzzy variance at completion ($\otimes \widetilde{VAC}$) is the discrepancy between the budget at completion and the grey-fuzzy estimate at completion. It indicates an expected cost overrun or underrun at the project's completion. $\otimes \widetilde{VAC}$ is determined as follows in Eq. (23).

$$\otimes \widetilde{VAC} = BAC - \otimes \widetilde{EAC} = \left[\left(BAC - EAC_u, BAC - EAC_m, BAC - EAC_l\right),\ \mathring{g}_{EAC}\right] \tag{23}$$

Table 6 illustrates how the grey-fuzzy variance at completion ($\otimes \widetilde{VAC}$) results are interpreted.

Grey-fuzzy to-complete performance index, $\otimes \widetilde{TCPI}$, which is the ratio of the remaining budget to the cost of remaining work, is a metric for the cost performance that needs to be maintained with the remaining budget. $\otimes \widetilde{TCPI}$ is used to evaluate whether it is easier

Table 6
Expected Project Cost Performance Evaluation based on Grey-Fuzzy Variance at Completion ($\otimes \widetilde{VAC}$).

Scenario	State of $\otimes \widetilde{VAC}$	Expected Project Cost Performance Evaluation
1	$\otimes \widetilde{VAC}_u < 0$	Expected to incur cost overrun
2	$\otimes \widetilde{VAC}_m < 0 < \otimes \widetilde{VAC}_u$	Expected to approximately incur cost overrun
3	$\otimes \widetilde{VAC}_m = 0$	Expected to be breakeven
4	$\otimes \widetilde{VAC}_l < 0 < \otimes \widetilde{VAC}_m$	Expected to be approximately cost underrun
5	$\otimes \widetilde{VAC}_l > 0$	Expected to be cost underrun

or difficult to complete the project on $BAC/\otimes\widetilde{EAC}$. $\otimes\widetilde{TCPI}$ is determined based on two alternatives as follows:

Alternative 1: Efficiency, which needs to be maintained to complete on BAC.

In alternative 1, $\otimes\widetilde{TCPI}$ is computed as shown in Eq. (24).

$$\otimes\widetilde{TCPI}=\frac{(BAC - \otimes\ \widetilde{EV})}{(BAC - AC)}=\left[\left(\frac{BAC\ - EV_u}{BAC - AC},\frac{BAC\ - EV_m}{BAC - AC},\frac{BAC - EV_l}{BAC -\ AC}\right),\ g^{\circ}_{EV}\right] \tag{24}$$

Alternative 2: Efficiency, which needs to be maintained to complete on $\otimes\ \widetilde{\boldsymbol{EAC}}$.

In alternative 2, $\otimes\widetilde{TCPI}$ is calculated as given in Eq. (25).

$$\otimes\ \widetilde{TCPI}=\frac{(BAC\ - \otimes\widetilde{EV})}{(\otimes\ \widetilde{EAC}\ - AC)}=\left[\left(\frac{BAC\ - EV_u}{EAC_u\ - AC},\frac{BAC\ - EV_m}{EAC_m\ - AC},\frac{BAC\ - EV_l}{EAC_l\ - AC}\right),g^{\circ}_{EV}\wedge g^{\circ}_{EAC}\right] \tag{25}$$

The results of the grey-fuzzy to-complete performance index ($\otimes\widetilde{TCPI}$) are interpreted as shown in Table 7.

6. Validation

6.1. Comparison of Grey-fuzzy EVA with fuzzy EVA and grey EVA

In this section, grey-fuzzy EVA is compared with fuzzy EVA [61] and grey EVA [37]. The BAC (Budget at Completion) and progress of each activity of the project are as seen in Table 8, and the PV (Planned Value) and the AC (Actual Cost) of the project on a weekly basis are also given below in Table 9. The BAC is $5000. AC and PV at week 6 are $2800 and $2300, respectively.

A. Comparison of $\otimes\widetilde{\boldsymbol{EV}}$ with $\widetilde{\boldsymbol{EV}}$ and $\otimes\ \boldsymbol{EV}$.

$\widetilde{EV}$ *and* $\otimes\ EV$ were computed based on the data presented in Tables 10 and 11, as shown in Eqs. (26) and (27), respectively.

$$\widetilde{EV}=\sum_{i=1}^{6}\widetilde{EV}_i=[1630,1980,2420,2780] \tag{26}$$

$$\otimes EV\ =\sum_{i=1}^{6}\otimes EV_i=[1830,2270] \tag{27}$$

$\otimes\widetilde{EV}$ is computed based on the data presented in Table 12, as shown in Eq. (28).

$$\otimes\widetilde{EV}\ =\sum_{i=1}^{6}\otimes\widetilde{EV}_i=[(1780,2220,2660),0.2] \tag{28}$$

The above results show similar values. The $\otimes\widetilde{EV}$ range is placed within the $\widetilde{EV}$ range.

If $\widetilde{A}\ =(a,b,c,d)$, defuzzified value of $\widetilde{A}$ is determined as shown below in Eq. (29).

$$\widetilde{A}=\frac{c^2+d^2+c\ *\ d\ -\ a^2\ -\ b^2\ -\ a\ *\ b}{3*\ (c+d\ -\ a\ -\ b)} \tag{29}$$

If $\widetilde{EV}$ is defuzzified based on Eq. (29), the $D\ (\widetilde{EV})=2203$ which is within the $\otimes\ \widetilde{EV}$.

If $\otimes\widetilde{EV}$ is defuzzified, then $D(\otimes\widetilde{EV})$ *is* determined as follows in Eq. (30).

$$D\ (\otimes\widetilde{EV})=\frac{EV_l+4\ *\ EV_m+EV_u}{6}=\frac{1780+4\ *\ 2220+2660}{6}=2220 \tag{30}$$

$D\ (\otimes\widetilde{EV})=2220$ is placed within the $\otimes EV$ range. Therefore, the grey-fuzzy EVA is valid.

Table 7
Project Cost Efficiency Evaluation based on the Grey-Fuzzy To-Complete Performance Index ($\otimes\ \widetilde{TCPI}$).

Scenario	State of $\otimes\ \widetilde{\boldsymbol{TCPI}}$	Project Cost Efficiency Evaluation
1	$\otimes\ \widetilde{TCPI}_u\ <\ 1$	Easier to complete on $BAC/\otimes\ \widetilde{EAC}$
2	$\otimes\ \widetilde{TCPI}_m\ <\ 1\ <\ \otimes\ \widetilde{TCPI}_u$	Approximately easier to complete on $BAC/\otimes\ \widetilde{EAC}$
3	$\otimes\ \widetilde{TCPI}_m\ =1$	Same to complete on $BAC/\otimes\ \widetilde{EAC}$
4	$\otimes\ \widetilde{TCPI}_l\ <1\ <\ \otimes\ \widetilde{TCPI}_m$	Approximately difficult to complete on $BAC/\otimes\ \widetilde{EAC}$
5	$\otimes\ \widetilde{TCPI}_l\ >\ 1$	Difficult to complete on $BAC/\otimes\ \widetilde{EAC}$

Table 8
BAC and progress of activities [61].

Activities	BAC($)	Progress
1. Filling Questionnaire	800	Very high (VH)
2. Checking up on people	1000	Medium High (MH)
3. Getting sample	500	Medium High (MH)
4. Testing sample	1200	Medium Low (ML)
5. Analyzing sample	900	Very low (VL)
6. Refining sample	600	Not Started

Table 9
Cumulative PV and AC for the project on a weekly basis [61].

Week	1	2	3	4	5	6	7	8	9	10	11	12
PV	100	400	750	1200	1700	2300	2950	3600	4150	4650	4850	5000
AC	200	500	1000	1500	2000	2800	–	–	–	–	–	–

Table 10
Calculation of $\widetilde{EV}_i$ based on BAC and $\widetilde{PC}_i$ [61].

Activities	*BAC* ($)	*Progress*	$\widetilde{PC}_i$	$\widetilde{EV}_i$
1. Filling questionnaire	800	Very High (VH)	[0.8, 0.9, 1, 1]	[640, 720, 800, 800]
2. Checking up on people	1000	Medium High (MH)	[0.5, 0.6, 0.7, 0.8]	[500, 600, 700, 800]
3. Getting Sample	500	Medium High (MH)	[0.5, 0.6, 0.7, 0.8]	[250, 300, 350, 400]
4. Testing Sample	1200	Medium Low (ML)	[0.2, 0.3, 0.4, 0.5]	[240, 360, 480, 600]
5. Analyzing Sample	900	Very Low (VL)	[0, 0, 0.1, 0.2]	[0, 0, 90, 180]
6. Refining Sample	600	Not started	[0, 0, 0, 0]	[0, 0, 0, 0]
Cumulative *BAC*	**5000**			**[1630, 1980, 2420, 2780]**

Table 11
Calculation of $\otimes EV_i$ based on BAC and $\otimes PC_i$ [37].

Activities	*BAC* ($)	Progress	$\otimes PC$	$\otimes EV_i$
1. Filling questionnaire	800	Very High (VH)	[0.9, 1]	[720, 800]
2. Checking up on people	1000	Medium High (MH)	[0.5, 0.6]	[500, 600]
3. Getting Sample	500	Medium High (MH)	[0.5, 0.6]	[250, 300]
4. Testing Sample	1200	Medium Low (ML)	[0.3, 0.4]	[360, 480]
5. Analyzing Sample	900	Very Low (VL)	[0, 0.1]	[0, 90]
6. Refining Sample	600	Not started	[0, 0]	[0, 0]
Cumulative *BAC*	**5000**			**[1830, 2270]**

Table 12
Calculation of $\otimes\widetilde{EV}_i$ based on BAC and $\otimes\widetilde{PC}_i$.

Activities	*BAC* ($)	Progress	$\otimes\widetilde{PC}_i$	$\otimes\widetilde{EV}_i$
1. Filling questionnaire	800	Very High (VH)	[(0.8, 0.9, 1), 0.2]	[(640, 720, 800), 0.2]
2. Checking up on people	1000	High (H)	[(0.6, 0.7, 0.8), 0.2]	[(600, 700, 800), 0.2]
3. Getting Sample	500	High (H)	[(0.6, 0.7, 0.8), 0.2]	[(300, 350, 400), 0.2]
4. Testing Sample	1200	Low (L)	[(0.2, 0.3, 0.4), 0.2]	[(240, 360, 480), 0.2]
5. Analyzing Sample	900	Very Low (VL)	[(0, 0.1, 0.2), 0.2]	[(0, 90, 180), 0.2]
6. Refining Sample	600	Not started	[(0, 0, 0), 0.2]	[(0, 0, 0), 0.2]
Cumulative *BAC*	**5000**			**[(1780, 2220, 2660), 0.2]**

Fig. 1 indicates the grey-fuzzy EV values are similar to fuzzy EV and grey EV values.

B. Comparison of $\otimes\widetilde{CPI}$ with $\widetilde{CPI}$ and $\otimes CPI$.

$\widetilde{CPI}$ *and* $\otimes CPI$ were calculated based on $\widetilde{EV}$ and AC, $\otimes EV$ and AC, as shown in Eqs. (31) and (32), respectively.

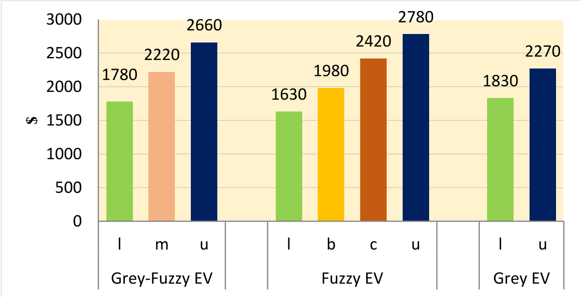


Fig. 1. Comparison of grey-fuzzy EV with fuzzy EV and grey EV

$$\widetilde{CPI} = \frac{\otimes\widetilde{EV}}{AC} = [0.58, 0.71, 0.86, 0.99] \tag{31}$$

$$\otimes CPI = \frac{\otimes EV}{AC} = [0.65, 0.81] \tag{32}$$

On the other hand, $\otimes\widetilde{CPI}$ is calculated as shown below in Eq. (33).

$$\otimes\widetilde{CPI} = \frac{\otimes\widetilde{EV}}{AC} = \left[\left(\frac{EV_l}{AC}, \frac{EV_m}{AC}, \frac{EV_u}{AC}\right), \ g^{\circ}_{EV}\right] \tag{33}$$

$= \left[\left(\frac{1780}{2800}, \frac{2200}{2800}, \frac{2660}{2800}\right), 0.2\right] = [(0.64, 0.79, 0.95), 0.2]$ Both $\widetilde{CPI}$ and $\otimes\widetilde{CPI}$ results show the project incurred cost overrun. The result is similar and $D\,(\widetilde{CPI}) = 0.79$ is within the $\otimes\widetilde{CPI}$ range. In addition, $\otimes\widetilde{CPI}$ is placed within the $\widetilde{CPI}$ range. Therefore, the grey-fuzzy EVA approach is valid for cost performance evaluation under uncertainty.

If $\otimes\widetilde{CPI}$ is defuzzified, then $D\,(\otimes\widetilde{CPI})$ is determined as follows in Eq. (34).

$$D\,(\otimes\widetilde{CPI}) = \frac{CPI_l + 4 \ * \ CPI_m + \ CPI_u}{6} = \frac{0.64 + 4 \ * \ 0.79 + 0.95}{6} = 0.79 \tag{34}$$

$D\,(\otimes\widetilde{CPI}) = 0.79$ is placed within the $\otimes EV$ range. Therefore, the grey-fuzzy EVA approach is valid for cost performance evaluation under uncertainty.

Fig. 2 indicates the grey-fuzzy CPI values are similar to fuzzy CPI and grey CPI values. The broken line indicates breakeven, above it indicates cost underrun, and below it indicates cost overrun.

C. Comparison of $\otimes\widetilde{EAC}$ with $\widetilde{EAC}$ and $\otimes$ ***EAC***.

The *BAC* for the project is \$5000. Then, $\widetilde{EAC}$ and $\otimes EAC$ were calculated as given below in Eqs. (35) and (36), assuming the current $\widetilde{CPI}$ and $\otimes CPI$ persist for the remaining works execution, respectively.

$$\widetilde{EAC} = \frac{BAC}{\widetilde{CPI}} = [5036, 5785, 7071, 8589] \tag{35}$$

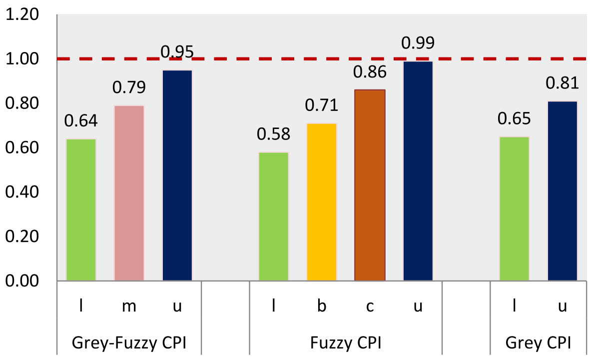


Fig. 2. Comparison of grey-fuzzy CPI with fuzzy CPI and grey CPI.

$$\otimes EAC = \frac{BAC}{\otimes EAC} = [6173, 7692] \tag{36}$$

On the other hand, $\otimes\widetilde{EAC}$ is computed as shown below in Eq. (37).

$$\otimes\widetilde{EAC} = \frac{BAC}{\otimes\widetilde{CPI}} = \left[\left(\frac{BAC}{CPI_u}, \frac{BAC}{CPI_m}, \frac{BAC}{CPI_l}\right), g^{\circ}_{CPI}\right] \tag{37}$$

$$= \left[\left(\frac{5000}{0.95}, \frac{5000}{0.79}, \frac{5000}{0.64}\right), 0.2\right] = [(5263, 6306, 7865), 0.2]$$

The results of $\widetilde{EAC}$ and $\otimes\widetilde{EAC}$ are similar and $D(\widetilde{EAC}) = 6650$ is within the $\otimes\widetilde{EAC}$ range. $\otimes\widetilde{EAC}$ is placed within the $\widetilde{EAC}$ range. Therefore, grey-fuzzy EVA is valid for cost forecasting under uncertainty.

If $\otimes\widetilde{EAC}$ is defuzzified, then $D(\otimes\widetilde{EAC})$ is determined as follows in Eq. (38).

$$D(\otimes\widetilde{EAC}) = \frac{EAC_l + 4 * EAC_m + EAC_u}{6} = \frac{5263 + 4 * 6306 + 7865}{6} = 6392 \tag{38}$$

$D(\otimes\widetilde{EAC}) = 6392$ is placed within $\otimes EAC$ range. Therefore, the grey-fuzzy EVA approach is valid for cost forecasting under uncertainty.

Fig. 3 indicates the grey-fuzzy EAC values are similar to fuzzy EAC and grey EAC values.

7. Case study

A case study on a road project in Addis Ababa, Ethiopia, shown in Fig. 4, whose road length is 5557 m and 10/15/20/30 m width, was conducted to demonstrate the application of grey-fuzzy EVA. The project was scheduled for one year, from July 2021 to June 2022. The status date of the project is June 30, 2022. The independent variables (BAC, PV, AC, and Progress) of the project were collected as given below in Tables 13–15. The project cost performance evaluation, forecasting, and efficiency measurement using grey-fuzzy EVA and interpretation of the results are presented hereunder.

$\otimes\widetilde{EV}$ is calculated based on the data presented in Table 16, as shown in Eq. (39).

$$\otimes\widetilde{EV} = \sum_{i=1}^{8} \otimes\widetilde{EV}_i = [(127, 911, 980.42, 159, 345, 889.00, 190, 779, 797.58), 0.2] \tag{39}$$

Taking 20% uncertainty in the deterministic progress data, the actual amount of work performed until the status date lies between ETB127, 911, 980.42 and ETB190,779,797.58. Most likely, ETB159, 345, 889.00 has been executed. The actual cost (ETB186,147,979) spent is higher than the lower limit and the median, but a little bit lower than the higher limit. In other words, the lower limit earned value is 69% of the actual cost, the median earned value is 86% of the actual cost, and the upper limit is 102% of the actual cost. These results indicate the project nearly incurred a cost overrun. So, the project manager should take remedial action to avert the situation.

BAC is calculated as shown in Eq. (40).

$$BAC = \sum_{i=1}^{8} BAC_i = 314, 339, 085.80 \tag{40}$$

PV is computed in Eq. (41).

$$PV = \sum_{i=1}^{8} PV_i = 314, 339, 085.80 \tag{41}$$

AC is calculated in Eq. (42).

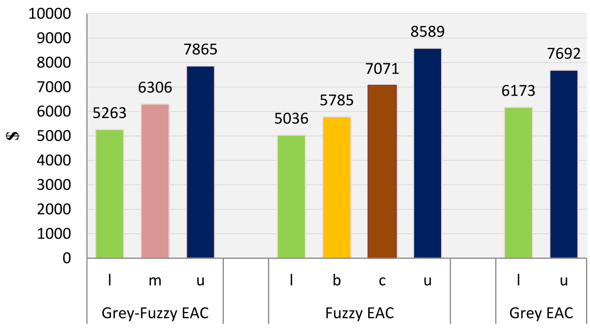


Fig. 3. Comparison of grey-fuzzy EAC with fuzzy EAC and grey EAC

Fig. 4. Case study road project for grey-fuzzy EVA

Table 13
BAC and Progress of the project until the status date.

No.	Milestone	*BAC* (ETB)	Progress
1.	General requirement and provision	2,572,332.00	Very High
2.	Earthworks	20,881,338.07	Low
3.	Sub-base and road bases	21,603,311.05	Low
4.	Bituminous pavements	61,573,708.07	Low
5.	Structures	171,000,868.85	High
6.	Incidental works	7,522,446.31	Very Low
7.	Electrical	16,975,471.08	Very Low
8.	Utilities	12,209,610.37	Low
	Total	**314,339,085.80**	

Table 14
Cumulative Planned Value (PV) for the project on a monthly basis.

Month	1	2	3	4	5	6
PV(ETB)	10,336,942.27	24,021,917.71	44,140,224.57	77,943,305.62	128,342,553.01	172,384,195.55
Month	**7**	**8**	**9**	**10**	**11**	**12**
PV(ETB)	194,021,356.91	204,904,734.56	214,379,730.48	233,581,054.71	283,428,637.42	314,339,085.80

Table 15
Actual Cost (AC) of the project in the fiscal year.

1. Direct Cost	Amount (ETB)	2. Indirect Cost	Amount (ETB)
1.1 Sub-Contractor	9,962,559	2.1 Project	13,216,065
1.2 Manpower	8,461,858	2.2 Head Office	12,177,905
1.3 Material	115,871,203	**Sub-Total (2)**	**25,393,970**
1.4 Equipment	20,926,235	**Actual Cost (AC) (1 + 2)**	**186,147,979**
1.5 Fuel	5,532,153		
Sub-Total (1)	**160,754,008**		

Table 16
Grey-Fuzzy Earned Value ($\otimes\widetilde{EV}_i$) for each Milestone of the Project.

	Milestones	*BAC*	Progress	$\otimes\widetilde{PC}_i$	$\otimes\widetilde{EV}_i$
1	General Reqt and Provision	2,572,332.00	Very High	[(0.8, 0.9, 1.0), 0.2]	[(2,057,865.60, 2,315,098.80, 2,572,332.00), 0.2]
2	Site clearance & earthwork	20,881,338.07	Low	[(0.2, 0.3, 0.4), 0.2]	[(4,176,267.61, 6,264,401.42, 8,352,535.23), 0.2]
3	Sub-base and road bases	21,603,311.05	Low	[(0.2, 0.3, 0.4), 0.2]	[(4,320,662.21, 6,480,993.32, 8,641,324.42), 0.2]
4	Bituminous pavement	61,573,708.07	Low	[(0.2, 0.3, 0.4), 0.2]	[(12,314,741.61, 18,472,112.42, 24,629,483.23), 0.2]
5	Structures	71,000,868.85	High	[(0.6, 0.7, 0.8), 0.2]	[(102,600,521.31, 119,700,608.19, 136,800,695.08), 0.2]
6	Incidental works	7,522,446.31	Very Low	[(0.0, 0.1, 0.2), 0.2]	[(0.00, 752,244.63, 1,504,489.26), 0.2]
7	Electrical works	16,975,471.08	Very Low	[(0.0, 0.1, 0.2), 0.2]	[(0.00, 1,697,547.11, 3,395,094.22), 0.2]
8	Utility line works	12,209,610.37	Low	[(0.2, 0.3, 0.4), 0.2]	[(2,441,922.07, 3,662,883.11, 4,883,844.15), 0.2]
Total Grey - Fuzzy EV					**[(127,911,980.42, 159,345,889.00, 190,779,797.58), 0.2]**

$$AC = \sum_{i=1}^{8} AC_i = 186,147,979.00 \tag{42}$$

A. Project Cost Performance Evaluation

$\otimes\widetilde{EV}$ for the general requirement and provision is determined in Eq. (43).

$$\otimes\widetilde{EV} = [(EV_l,\ EV_m, EV_u),\ g^{\circ}{}_{EV}]$$

$$= \left[(PC_l\ * BAC, PC_m * BAC, PC_u *\ BAC), \frac{PC_u - PC_l}{PC_{max}\ -\ PC_{min}}\right] \tag{43}$$

$$= \left[0.8\ *\ 2,572,332.00, 0.9\ * 2,572,332.00, 1.0\ *\ 2,572,332.00\ \right), \frac{1.0\ -\ 0.8}{1.0\ -\ 0.0}\right]$$

$$= [(2,057,865.60, 2,315,098.80, 2,572,332.00), 0.2]$$

A minimum of 2,057,865.60ETB and a maximum of 2,572,332.00ETB were earned in the fiscal year. On average, 2,315,098.80ETB was earned.

$\otimes\widetilde{CV}$ is determined as follows in Eq. (44).

$$\otimes\widetilde{CV} = \otimes\widetilde{EV} - \ AC = \left[(EV_l - AC, EV_m - AC, EV_u - AC), g^{\circ}_{EV}\right] \tag{44}$$

$$= [(127,911,980.42 - 186,147,979, 159,345,889.00 -\ 186,147,979, 190,779,797.58\ -\ 186,147,979), 0.2]$$

$$= [-58,235,998.58, -26,802,090.00, 4,631,818.58), 0.2]$$

This cost variance result indicates the lower limit earned value is lower than the actual cost by ETB58, 235, 998.58, whereas the median earned value is lower than the actual cost by ETB 26, 802, 090.00. On the other hand, the upper limit is higher than the actual cost by only ETB4, 631, 818.58. As per Table 3 $\otimes\ \widetilde{CV}_m\ =\ -\ 26,802,090.00 < 0 < \otimes\ \widetilde{CV}_u\ = 4,631,818.58$, the project has approximately incurred a cost overrun. So, corrective action should be taken by the project manager to avert the cost overrun. $\otimes\ \widetilde{CV\%}$ is determined as follows in Eq. (45).

$$\otimes\widetilde{CV\%} = \frac{\otimes\widetilde{CV}}{\otimes\widetilde{EV}} = \left[\left(\frac{CV_l}{EV_u}, \frac{CV_m}{EV_m}, \frac{CV_u}{EV_l}\right),\ g^{\circ}_{CV} \wedge g^{\circ}_{EV}\ \right] \tag{45}$$

$$= \left[\left(\frac{-58,235,998.58}{190,779,797.58}, \frac{-26,802,090.00}{159,345,889.00}, \frac{4,631,818.58}{127,911,980.42}\right), 0.2 \wedge 0.2\right]$$

$$= [-30.53\%, -16.82\%, 3.62\%), 0.2]$$

Considering 20% uncertainty in the deterministic data, the cost variance ranges between −30.53% and 3.62%. Most likely, the cost variance is −16.82%. These results imply, as per Table 4, $\otimes\ \widetilde{CV}_m\ \%\ =\ -\ 16.82\% < 0\ \% < \otimes\ \widetilde{CV}_u\ \%\ = 3.62\%$, the project approximately incurred a cost overrun. Hence, corrective action should be taken by the project manager to reduce the impact of cost overrun in future executions. To evaluate the project cost performance by comparing the grey-fuzzy earned value with the actual cost, $\otimes\ \widetilde{CPI}$, is determined as follows in Eq. (46).

$$\otimes\widetilde{CPI} = \frac{\otimes\widetilde{EV}}{AC} = \left[\left(\frac{EV_l}{AC}, \frac{EV_m}{AC}, \frac{EV_u}{AC}\right), g^{\circ}_{EV}\right] \tag{46}$$

$$= \left[\left(\frac{127,911,980.42}{186,147,979}, \frac{159,345,889.00}{186,147,979}, \frac{190,779,797.58}{186,147,979}\right), 0.2\right] = [(0.69, 0.86, 1.02), 0.2]$$

These results indicate the lower limit earned value (the executed amount of work) is 69% of the actual cost, while the median earned value is 86% of the actual cost. On the contrary, the upper limit of earned value is 102% of the actual cost. In other words, the earned value lacks 31% in the lower limit and 14% in the median, even to cover the money spent, and is higher by 2% (profit) in the upper limit. As per Table 5, $\otimes\ \widetilde{CPI}_m\ = 0.86 < 1 < \otimes\ \widetilde{CPI}_u\ = 1.02$, the project approximately incurred a cost overrun. So, appropriate actions should be taken by the project manager to avert the situation and ensure profitability. Otherwise, the cost overrun trend may continue in the execution of the remaining works, and as a consequence, the contractor may be bankrupt.

B. Project Cost Forecasting

$\otimes\widetilde{EAC}$ is determined based on AC and $\otimes\widetilde{ETC}$ using three alternative cases as follows:

Case I $\otimes\widetilde{ETC}$ **for the remaining work will be executed as planned.**

This case assumes the actual cost until the status date is taken, and the remaining works are assumed to be executed with the remaining budget. This requires a good strategy and commitment from the project team. As per case I, $\otimes\widetilde{EAC}$ is calculated in Eq. (47).

$$\otimes\widetilde{EAC} = AC + (BAC - \otimes\widetilde{EV}) = \left[(AC + BAC - EV_u, AC + BAC - EV_m, AC + BAC - EV_l),\ g^{\circ}_{EV}\right] \tag{47}$$

$$= [(186,147,979 + 314,339,085.80 \text{ - } 190,779,797.58, 186,147,979 + 314,339,085.80 \text{ - } 159,345,889.00,$$

$$186,147,979 + 314,339,085.80 \text{ - } 127,911,980.42), 0.2]$$

$$= [(309,707,267.22, 341,141,175.80, 372,575,084.38), 0.2]$$

The project may cost a minimum of 309,707,267.22ETB and a maximum of 372,575,084.38ETB to execute 314,339,085.80ETB worth of work. On average, it may cost 341,141,175.80ETB at the end of the work accomplished. Only the lower limit estimate at completion will be lower than the initial budget by ETB4,631,818.58 (1% of BAC), whereas the median and the upper limit will be higher than the approved budget by ETB26,802,090.00 (9% of BAC) and ETB58,235,998.58 (19% of BAC), respectively. In other words, the lower limit estimate at completion is 99% of the budget at completion, whereas the median and the upper limit are 109% and 119% of the budget at completion, respectively. Most likely, the cost of the project at completion may exceed the initial budget even if the remaining work is performed as initially planned. Hence, the project manager needs to take corrective action to minimize the possibility of a cost overrun in the remaining work. Assuming the remaining works will be performed as per the plan, the grey-fuzzy estimate to complete, $\otimes\widetilde{ETC}$ is determined as shown in Eq. (48).

$$\otimes\widetilde{ETC} = \otimes\widetilde{EAC} - AC = \left[(EAC_l - AC, EAC_m - AC, EAC_u - AC), g^{\circ}_{EAC}\right] \tag{48}$$

$$= [(309,707,267.22 \text{ - } 186,147,979, 341,141,175.80 \text{ - } 186,147,979, 372,575,084.38 \text{ - } 186,147,979), 0.2]$$

$$= [(123,559,288.22, 154,993,196.80, 186,427,105.38), 0.2]$$

If the remaining work is completed as planned, it may cost a minimum of 123,559,288.22ETB (39% of BAC), which is lower than the remaining budget of ETB128,191,106.80 (41% of BAC), and a maximum of 186,427,105.38ETB (59% of BAC), which is higher than the remaining budget, to execute the remaining works. On average, it may cost 154,993,196.80ETB (49% of BAC), which is higher than the remaining budget.

Assuming the remaining works will be executed as per the plan, the grey-fuzzy variance at completion, $\otimes\widetilde{VAC}$, is calculated as shown in Eq. (49).

$$\otimes\widetilde{VAC} = BAC - \otimes\widetilde{EAC} = \left[(BAC - EAC_u, BAC - EAC_m, BAC - EAC_l),\ g^{\circ}_{EAC}\right] \tag{49}$$

$$= [(314,339,085.80 \text{ - } 372,575,084.38, 314,339,085.80 \text{ - } 341,141,175.80, 314,339,085.80 \text{ - } 309,707,267.22), 0.2]$$

$$= [(\text{-}58,235,998.58, \text{-}26,802,090.00, 4,631,818.58), 0.2]$$

Considering the remaining works are executed as planned, the cost variance at completion results indicate the project may incur a loss of ETB58, 235, 998.58(19 % of BAC) pessimistically and get a profit of ETB4, 631, 818.58 (1 % of BAC) optimistically. Most likely, it may incur a cost overrun of ETB26, 802, 090.00(9 % of BAC) at the completion of the works. According to Table 6, $\otimes\ \widetilde{VAC}_m = -26,802,090.00 < 0 < \otimes\ \widetilde{VAC}_u = 4,631,818.58$, the project is expected to approximately incur a cost overrun.

Case II $\otimes\widetilde{ETC}$ **for the remaining work will be executed at** $\otimes\widetilde{CPI}$.

Assuming the remaining works will be performed as per the past cost performance, the grey-fuzzy estimate at completion $\otimes\widetilde{EAC}$ is determined in Eq. (50).

$$\otimes\widetilde{EAC} = \frac{BAC}{\otimes\widetilde{CPI}} = \left[\left(\frac{BAC}{CPI_u}, \frac{BAC}{CPI_m}, \frac{BAC}{CPI_l}\right),\ g^{\circ}_{CPI}\right] \tag{50}$$

$$= \left[\left(\frac{314,339,085.80}{1.02}, \frac{314,339,085.80}{0.86}, \frac{314,339,085.80}{0.69}\right), 0.2\right]$$

$$= [(306,707,451.65, 367,211,139.93, 457,451,955.25), 0.2]$$

According to the past performance ($\otimes\ \widetilde{CPI}$), the project may cost a minimum of 306,707,451.65ETB and a maximum of

457,451,955.25ETB to execute 314,339,085.80ETB worth of work. On average, it may cost 367,211,139.93ETB at the end of the work accomplished. The lower limit estimate at completion will be lower than the initial budget by ETB7,631,634.15, whereas the median and the upper limit will be higher than the approved budget by ETB52,872,054.13 and ETB143,112,869.45, respectively. In other words, the lower limit estimate at completion is 98% of the budget at completion, whereas the median and the upper limit are 117% and 146% of the budget at completion, respectively. There is a higher possibility in Case II than in Case I that the cost of the project at completion may exceed the initial budget if the remaining work is performed as per the past cost performance. Hence, remedial actions should be taken by the project manager to reduce cost overrun in the execution of the remaining works. Assuming the remaining works will be executed as per the trend of the past cost performance, the grey-fuzzy estimate to complete, $\otimes \widetilde{ETC}$, is determined as follows in Eq. (51).

$$\otimes\widetilde{ETC} = \otimes \widetilde{EAC} - AC = \left[(EAC_l - AC,\ EAC_m - AC, EAC_u - AC), \overset{\circ}{g}_{EAC}\right] \tag{51}$$

$$= [(306,707,451.65 - 186,147,979, 367,211,139.93 - 186,147,979.00, 457,451,955.25 - 186,147,979.00), 0.2]$$

$$= [(120,559,472.65, 181,063,160.93, 271,303,976.25), 0.2]$$

Taking the remaining work will be completed at grey-fuzzy CPI; it may cost a minimum of 120,559,472.65ETB (38% of BAC), which is lower than the remaining budget, ETB128,191,106.80 (41% of BAC), and a maximum of 271,303,976.25ETB (86% of BAC), more than double of the remaining budget, to execute the remaining works. On average, it may cost 181,063,160.93 TB (58% of BAC), which is higher than the remaining budget.

Assuming the remaining works are executed as per the past cost performance index, the grey-fuzzy cost variance at completion, $\otimes \widetilde{VAC}$ is computed as shown in Eq. (52).

$$\otimes\widetilde{VAC} = BAC - \otimes \widetilde{EAC} = \left[\left(BAC - EAC_u, BAC - EAC_m, BAC - EAC_l\right),\ \overset{\circ}{g}_{EAC}\right] \tag{52}$$

$$= [(314,339,085.80 - 457,451,955.25, 314,339,085.80 - 367,211,139.93, 314,339,085.80 - 306,707,451.65), 0.2]$$

$$= [(-143,112,869.45, -52,872,054.13, 7,631,634.15), 0.2]$$

Taking the remaining works will be performed as per past cost performance index, the cost variance at completion results indicate the project may incur a loss of ETB143,112,869.45 (46% of BAC) at worst case and get a profit of ETB7,631,634.15 (2% of BAC) at best. Most likely, it may incur a cost overrun of ETB52,872,054.13 (17% of BAC) at completion.

Case III $\otimes\widetilde{\boldsymbol{ETC}}$ for the remaining work will be executed at $\otimes \widetilde{\boldsymbol{CSI}}$.

Assuming the remaining work will be performed taking into account the combination effect of the cost performance index and the schedule performance index, the grey-fuzzy estimate at completion, $\otimes \widetilde{EAC}$, is determined in Eq. (53).

$$\otimes\widetilde{EAC} = AC + \left[\frac{BAC - \otimes\widetilde{EV}}{\otimes\widetilde{CSI}}\right]$$

$$= \left[\left(AC + \frac{(BAC - EV_u)}{CSI_u}, AC + \frac{(BAC - EV_m)}{CSI_m}, AC + \frac{(BAC - EV_l)}{CSI_l}\right), \overset{\circ}{g}_{EV} \wedge \overset{\circ}{g}_{CSI}\right] \tag{53}$$

The grey-fuzzy schedule performance index based on cost data, $\otimes \widetilde{SPI}$, is determined in Eq. (54).

$$\otimes \widetilde{SPI} = \frac{\otimes\widetilde{EV}}{PV} = \left[\left(\frac{EV_l}{PV}, \frac{EV_m}{PV}, \frac{EV_u}{PV}\right),\ \overset{\circ}{g}_{EV}\right] \tag{54}$$

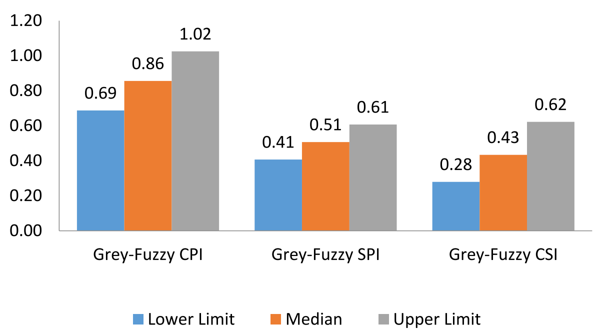


Fig. 5. Comparison of $\otimes \widetilde{CPI}$, $\otimes\widetilde{SPI}$, and $\otimes \widetilde{CSI}$.

$$= \left[\left(\frac{127,911,980.42}{314,339,085.80}, \frac{159,345,889.00}{314,339,085.80}, \frac{190,779,797.58}{314,339,085.80} \right), 0.2 \right] = [(0.41, 0.51, 0.61), 0.2]$$

The project achieved a minimum of 41% and a maximum of 61% of the plan. Most likely, it performed 51% of the plan, which is half of the plan. Hence, the project incurred delay. So, it is necessary to take appropriate action to accelerate the performance of the project. The grey-fuzzy cost schedule performance index, $\otimes \widetilde{CSI}$, is calculated in Eq. (55). It is a combination of the cost performance index and the schedule performance index.

$$\otimes \widetilde{CSI} = \otimes \widetilde{CPI} * \otimes \widetilde{SPI} = \left[(CPI_l * SPI_l,\ CPI_m * SPI_m, CPI_u * SPI_u),\ \overset{\circ}{g}_{CPI} \wedge \overset{\circ}{g}_{SPI} \right] \tag{55}$$

$$= [(0.69 * 0.41, 0.86 * 0.51, 1.02 * 0.61), 0.2 \wedge 0.2] = [(0.28, 0.43, 0.62), 0.2]$$

The cost schedule performance index ranges between 28% and 62%. Most likely, it performed 43% of the cost schedule plan. Thus, the project manager needs to take corrective action to change this worse scenario.

Fig. 5 indicates the cost performance is better than the schedule performance of the project, whereas the cost schedule performance is the lowest.

$$\otimes \widetilde{EAC} = \left[\left(186,147,979 + \frac{(314,339,085.80 - 190,779,797.58)}{0.62}, 186,147,979 + \frac{(314,339,085.80 - 159,345,889.00)}{0.43} \right. \right.$$

$$\left. \left. , 186,147,979 + \frac{(314,339,085.80 - 127,911,980.42)}{0.28} \right), 0.2 \wedge 0.2 \right]$$

$$= [(384,788,269.53, 543,328,379.68, 852,867,731.04), 0.2]$$

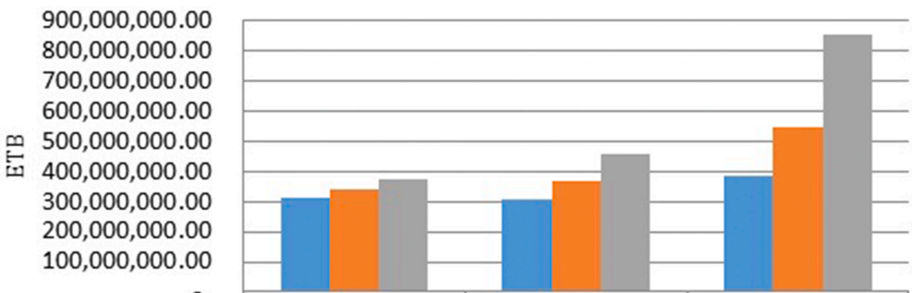
The project may cost a minimum of 384,788,269.53ETB and a maximum of 852,867,731.04ETB to execute 314,339,085.80ETB worth of work. On average, it may cost 543,328,379.68ETB at the end of the work accomplished. All the lower limit, median, and upper limit estimates at completion will be higher than the initial budget by ETB70,449,183.73 (22%), 228,989,293.87 (73%), and 538,528,645.24 (171%), respectively. In other words, the lower limit estimate at completion is 122% of the budget at completion, whereas the median and the upper limit are 173% and 271% of the budget at completion, respectively. This Case indicates the highest possibility, compared to Cases I and II, that the project will incur a cost overrun at completion if the remaining work is performed as per the past cost schedule performance. Hence, remedial actions should be taken by the project manager to reduce cost overrun in the execution of the remaining works. Assuming the remaining works will be executed as per the trend of the past cost schedule performance, the grey-fuzzy estimate to complete, $\otimes\widetilde{ETC}$ is determined in Eq. (56).

$$\otimes \widetilde{ETC} = \otimes \widetilde{EAC} - AC = \left[(EAC_l - AC,\ EAC_m - AC, EAC_u - AC),\ \overset{\circ}{g}_{EAC} \right] \tag{56}$$

$$= [(384,788,269.53 - 186,147,979, 543,328,379.68 - 186,147,979, 852,867,731.04 - 186,147,979), 0.2]$$

$$= [(198,640,290.53, 357,180,400.68, 666,719,752.04), 0.2]$$

If the remaining work is executed at grey-fuzzy CSI, it may cost a minimum of 198,640,290.53ETB (63% of BAC), which is higher than the remaining budget of ETB128,191,106.80 (41% of BAC) and a maximum of 666,719,752.04ETB (212% of BAC), more than five



	Case I: Grey-Fuzzy EAC	Case II: Grey-Fuzzy EAC	Case III: Grey-Fuzzy EAC
Lower Limit	309,707,267.22	306,707,451.65	384,788,269.53
Median	341,141,175.80	367,211,139.93	543,328,379.68
Upper Limit	372,575,084.38	457,451,955.25	852,867,731.04

Fig. 6. Comparison of grey-fuzzy EAC results.

times the remaining budget, to execute the remaining works. On average, it may cost 357,180,400.68ETB (114% of BAC), more than double the remaining budget.

Assuming the remaining works will be executed as per the cost schedule performance, the grey-fuzzy cost variance at completion, $\otimes \widetilde{VAC}$ is determined in Eq. (57).

$$\otimes\widetilde{VAC} = BAC - \otimes\widetilde{EAC} = \left[\left(BAC - EAC_u, BAC - EAC_{m,}\ BAC - EAC_l\right),\ g^{\circ}_{EAC}\right] \tag{57}$$

$$= [(314,339,085.80 - 852,867,731.04, 314,339,085.80 - 543,328,379.68, 314,339,085.80 - 384,788,269.53), 0.2]$$

$$= [(-538,528,645.24, -228,989,293.87, -70,449,183.73), 0.2]$$

If the remaining works are performed as per past cost schedule performance, the cost variance at completion results indicate the project may incur a loss of ETB538,528,645.24 (171% of BAC) at the worst scenario and ETB70,449,183.73 (22% of BAC) at the best scenario. Most likely, it may incur a cost overrun of ETB228,989,293.87 (73% of BAC) at completion.

Fig. 6 shows Case I: Grey-Fuzzy EAC provides the lowest estimates, while Case III: Grey-Fuzzy EAC provides the highest estimates. On the other hand, Case II: Grey-Fuzzy EAC provides values between the two cases.

Fig. 7 implies that Case I: Grey-Fuzzy ETC provides the lowest estimates, while Case III: Grey-Fuzzy ETC provides the highest estimates. Case II: Grey-Fuzzy ETC provides values between the two cases.

Fig. 8 shows Case I: Grey-Fuzzy VAC provides the lowest variances, while Case III: Grey-Fuzzy VAC provides the highest variances. Case II: Grey-Fuzzy VAC provides variances between the two cases.

C. Project Cost Efficiency Measurement

Alternative 1: Efficiency, which needs to be maintained to complete on BAC.

Based on alternative 1, $\otimes\widetilde{TCPI}$ is computed in Eq. (58).

$$\otimes\widetilde{TCPI} = \frac{(BAC - \otimes\widetilde{EV})}{(BAC - AC)} = \left[\left(\frac{BAC - EV_u}{BAC - AC}, \frac{BAC - EV_m}{BAC - AC}, \frac{BAC - EV_l}{BAC - AC}\right),\ g^{\circ}_{EV}\right] \tag{58}$$

$$= \left[\left(\frac{314,339,085.80 - 190,779,797.58}{314,339,085.80 - 186,147,979.00}, \frac{314,339,085.80 - 159,345,889.00}{314,339,085.80 - 186,147,979.00}, \frac{314,339,085.80 - 127,911,980.42}{314,339,085.80 - 186,147,979.00}\right), 0.2\right]$$

$$= [(0.96, 1.21, 1.45), 0.2]$$

The remaining work amount is a minimum of 96% and a maximum of 145% of the remaining budget. Most likely, it is 121% of the remaining budget. Based on Table 7, $\otimes\widetilde{TCPI}_l = 0.96 < 1 < \otimes\widetilde{TCPI}_m = 1.21$, it is approximately difficult to complete the project on *BAC*.

Alternative 2: Efficiency, which needs to be maintained to complete on $\otimes \widetilde{EAC}$.

Case I $\otimes\widetilde{\boldsymbol{TCPI}}$ Forecast for $\otimes\widetilde{\boldsymbol{ETC}}$ for the remaining work will be executed as planned.

Based on alternative 2: Case I, $\otimes\widetilde{TCPI}$ is calculated in Eq. (59).

$$\otimes\widetilde{TCPI} = \frac{(BAC - \otimes\widetilde{EV})}{(\otimes\widetilde{EAC} - AC)} = \left[\left(\frac{BAC - EV_u}{EAC_u - AC}, \frac{BAC - EV_m}{EAC_m - AC}, \frac{BAC - EV_l}{EAC_l - AC}\right), g^{\circ}_{EV} \wedge g^{\circ}_{EAC}\right] \tag{59}$$

$$= \left[\left(\frac{314,339,085.80\text{-}190,779,797.58}{372,575,084.38\text{-}186,147,979.00}, \frac{314\ 339\ 085.80\text{-}159,345,889.00}{341,141,175.80\text{-}186\ 147\ 979.00}, \frac{314,339,085.80\text{-}127,911,980.42}{309,707,267.22\text{-}186,147,979.00}\right), 0.2 \wedge 0.2\right]$$

$$= [(0.66, 1.00, 1.51), 0.2]$$

The remaining work amount is a minimum of 66% and a maximum of 151% of the estimate to complete. Most likely, it is 100% of the remaining budget. $\otimes\ \widetilde{TCPI}_m = 1.00$, which is the same as completing the project on $\otimes\ \widetilde{EAC}$.

Case II $\otimes\widetilde{\boldsymbol{TCPI}}$ Forecast for $\otimes\widetilde{\boldsymbol{ETC}}$ for the remaining work will be executed at $\otimes\ \widetilde{\boldsymbol{CPI}}$.

Based on alternative 2: Case II, $\otimes\widetilde{TCPI}$ is calculated in Eq. (60).

$$\otimes\widetilde{TCPI} = \frac{(BAC - \otimes\widetilde{EV})}{(\otimes\widetilde{EAC} - AC)} = \left[\left(\frac{BAC - EV_u}{EAC_u - AC}, \frac{BAC - EV_m}{EAC_m - AC}, \frac{BAC - EV_l}{EAC_l - AC}\right), g^{\circ}_{EV} \wedge g^{\circ}{}_{EAC}\right] \tag{60}$$

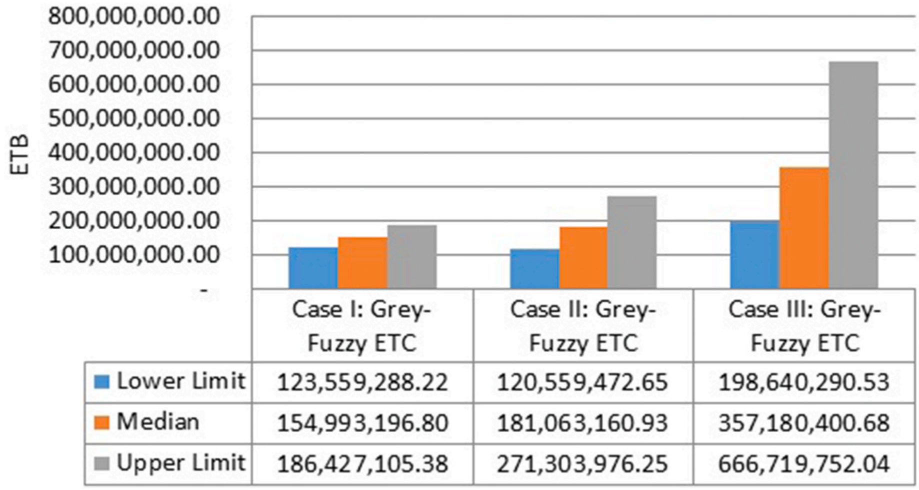


Fig. 7. Comparison of grey-fuzzy etc results.

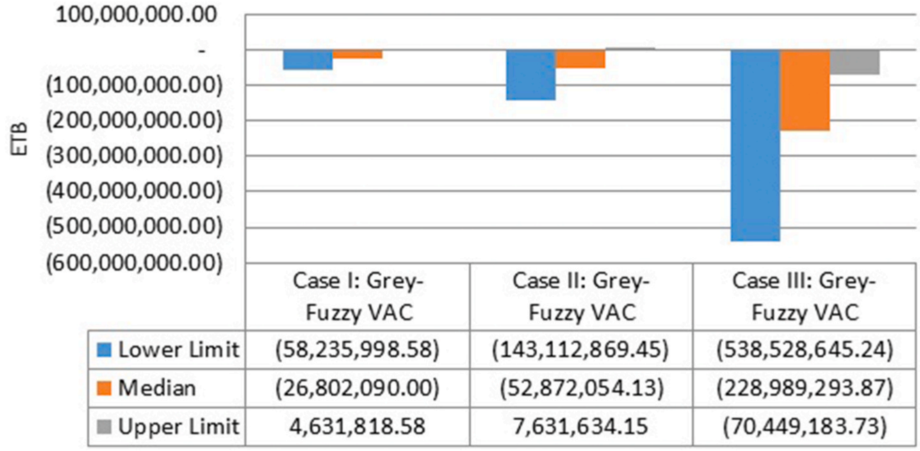


Fig. 8. Comparison of grey-fuzzy VAC results.

$$= \left[\left(\frac{314,339,085.80 \; - \; 190,779,797.58}{457,451,955.25 \; - \; 186,147,979.00}, \frac{314,339,085.80 \; - \; 159,345,889.00}{367,211,139.93 \; - \; 186,147,979.00}, \frac{314,339,085.80 \; - \; 127,911,980.42}{306,707,451.65 \; - \; 186,147,979.00}\right), 0.2 \wedge 0.2\right]$$

$$= [(0.46, 0.86, 1.55), 0.2]$$

The remaining work amount is a minimum of 46% and a maximum of 155% of the estimate to complete. Most likely, it is 86% of the remaining budget. Based on Table 7, $\otimes\widetilde{TCPI}_m = 0.86 < 1 < \otimes\widetilde{TCPI}_u = 1.55,$ it is approximately easier to complete the project on $\otimes \widetilde{EAC}$.

Case III $\otimes\boldsymbol{\widetilde{TCPI}}$ Forecast for $\otimes\boldsymbol{\widetilde{ETC}}$ for the remaining work will be executed at $\otimes \boldsymbol{\widetilde{CSI}}$.

Based on alternative 2: Case III, $\otimes\widetilde{TCPI}$ is calculated in Eq. (61).

$$\otimes\widetilde{TCPI} = \frac{(BAC \; - \otimes\widetilde{EV})}{(\otimes \; \widetilde{EAC} \; - \; AC)} = \left[\left(\frac{BAC \; - \; EV_u}{EAC_u \; - \; AC}, \frac{BAC \; - \; EV_m}{EAC_m - \; AC}, \frac{BAC \; - \; EV_l}{EAC_l - \; AC}\right), \; g^{\circ}_{EV} \wedge \; g^{\circ}{}_{EAC}\right] \tag{61}$$

$$= \left[\left(\frac{314,339,085.80 - \; 190,779,797.58}{852,867,731.04 \; - 186,147,979.00}, \frac{314,339,085.80 - \; 159,345,889.00}{543,328,379.68 \; - 186,147,979.00}, \frac{314,339,085.80 - \; 127,911,980.42}{384,788,269.53 \; - 186,147,979.00}\right), 0.2\right]$$

$$= [(0.19, 0.43, 0.94), 0.2]$$

The remaining work amount is a minimum of 19% and a maximum of 94% of the estimate to complete. Most likely, it is 43% of the remaining budget. Based on Table 7, $\otimes \widetilde{TCPI}_u = 0.94 < 1$, it is easier to complete the project on $\otimes \widetilde{EAC}$.

Fig. 9 indicates that the easiness of completing the project on $BAC/\otimes\widetilde{EAC}$ increases from left (Alt. 1: Grey-Fuzzy *TCPI*) to right (Alt. 2: Case III: Grey-Fuzzy *TCPI*).

8. Conclusion

Because of improper project control, projects incur cost overruns. EVA is a common deterministic project cost-control method and early warning signal for project cost monitoring and control under certain conditions. However, uncertainty needs to be taken into account in project cost evaluation and forecasting, as it is unavoidable in projects due to various factors. FST and GST are the two most common theories to take into account uncertainties in EVA. However, both theories have advantages and drawbacks. Taking into account the advantages of the two theories, this research integrated FST and GST simultaneously with EVA to predict construction costs using grey-fuzzy EVA in the presence of uncertain data.

- Simple grey-fuzzy EVA algorithms, which determine the lower limit, median, and upper limit of predicted costs and the degree of greyness, were developed for the evaluation and prediction of construction cost.
- Grey-fuzzy EVA was validated by comparing it with fuzzy EVA and grey EVA, as it provided comparable results.
- A case study on a road project was conducted to demonstrate how to evaluate project cost performance until the status date and predict project cost at completion using grey-fuzzy EVA in the uncertain progress of the project.
- According to the cost performance evaluation of the case study, since $\otimes \widetilde{CV}_m = -26{,}802{,}090.00 < 0 < \otimes \widetilde{CV}_u = 4{,}631{,}818.58$, $\otimes \widetilde{CV}_m \% = -16.82\% < 0\% < \otimes \widetilde{CV}_u \% = 3.62\%$, $\otimes \widetilde{CPI}_m = 0.86 < 1 < \otimes \widetilde{CPI}_u = 1.02$, the project approximately incurred a cost overrun.
- As per the project time performance forecasting based on planned performance, $\otimes \widetilde{VAC}_m = -26{,}802{,}090.00 < 0 < \otimes \widetilde{VAC}_u = 4,631,818.58$; based on current $\otimes\widetilde{CPI}$, $\otimes \widetilde{VAC}_m = -52,872,054.13 < 0 < \otimes \widetilde{VAC}_u = 7,631,634.15$; these results indicate the project is approximately expected to incur a cost overrun. On the other hand, based on $\otimes\widetilde{CSI}$, $\otimes \widetilde{VAC}_u = -70,449,183.73 < 0$, which indicates the project is expected to incur a cost overrun. Consequently, the project manager should take corrective action.
- The comparative results of the case study in percent are given below.

The percentage of CV based on Earned Value (EV)
Grey-fuzzy CV (%) = [(−31%, −17%, 4%), 20%]
The percentage of CPI based on Actual Cost (AC)
Grey-fuzzy CPI = [(69%, 86%, 102%), 20%]
The percentage of SPI based on Planned Value (PV)
Grey-fuzzy SPI = [(41%, 51%, 61%), 20%]
The percentage of CSI based on both AC and PV.
Grey-Fuzzy CSI = [(28%, 43%, 62%), 20%]
The percentages of comparative analysis results of EAC, ETC, and VAC based on BAC.
Alt. 1 Grey-fuzzy EAC = [(99%, 109%, 119%), 20%]
Alt. 2 Grey-fuzzy EAC = [(98%, 117%, 146%), 20%]
Alt. 3 Grey-fuzzy EAC = [(122%, 173%, 271%), 20%]
Alt. 1 Grey-fuzzy ETC = [(39%, 49%, 59%), 20%]
Alt. 2 Grey-fuzzy ETC = [(38%, 58%, 86%), 20%]
Alt. 3 Grey-fuzzy ETC = [(63%, 114%, 212%), 20%]
Alt. 1 Grey-fuzzy VAC = [(−19%, −9%, 1%), 20%]
Alt. 2 Grey-fuzzy VAC = [(−46%, −17%, 2%), 20%]
Alt. 3 Grey-fuzzy VAC = [(−171%, −73%, −22%), 20%]

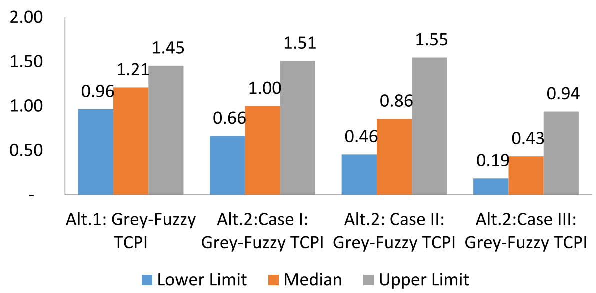


Fig. 9. Comparison of grey-fuzzy *TCPI* results.

Percentage of TCPI based on the remaining budget.
Alt. 1 Grey-fuzzy TCPI = [(96%, 121%, 145%), 20%]
Percentage of TCPI based on estimate to complete.
Alt. 2 Case I Grey-fuzzy TCPI = [(66%, 100%, 151%), 20%]
Alt. 2 Case II Grey-fuzzy TCPI = [(46%, 86%, 155%), 20%]
Alt. 2 Case III Grey-fuzzy TCPI = [(19%, 43%, 94%), 20%]
The limitations of this study include.

- This research is limited to using triangular grey-fuzzy EVA, not Gaussian, trapezoidal, hexagonal, pentagonal, and so on.
- This research integrated fuzzy theory and grey theory with EVA, excluding probability theory, which may increase the accuracy of the analysis results because there is no large amount of prior data or database available in our country.

9. Future scope

- Further study may be conducted on Gaussian, trapezoidal, hexagonal, and pentagonal grey fuzzy EVA and their comparison with triangular grey fuzzy EVA.
- Further study may also be conducted on the integration of probability theory, fuzzy theory, and grey theory all together with EVA to augment the precision of analysis results if a database is made available.

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Ethics statement

Review and/or approval by an ethics committee was not needed for this study because it does not involve animal experiments, and human and behavioral studies.

Informed consent was not required for this study because it does not involve human and behavioral studies.

Data availability statement

The data associated with this study has not been deposited into a publicly available repository.

Data will be made available on request.

CRediT authorship contribution statement

Endale Mamuye Desse: Writing – review & editing, Writing – original draft, Validation, Resources, Project administration, Methodology, Investigation, Formal analysis, Conceptualization. **Wubishet Jekale Mengesha:** Writing – review & editing, Validation, Supervision, Resources, Project administration, Methodology, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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