Heliyon 10 (2024) e27237

Contents lists available at ScienceDirect

Heliyon

journal homepage: www.cell.com/heliyon

Research article

Assessment and solutions for vulnerability of urban rail transit network based on complex network theory: A case study of Chongqing

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ARTICLE INFO

Keywords:
Complex network theory
Rail transit network
network structure characteristic
vulnerability analysis
key node identification

ABSTRACT

As a typical complex network system, the operating environment of rail transit network (RTN) is complex and demanding. This study aims to accurate assess the weaknesses and vulnerability of RTN, which is crucial for ensuring its smooth operation. Taking Chongqing Rail Transit (CRT) as an example, this study developed a network topology model using the spatial L method and analyzed the network structure characteristics, along with the importance of key nodes under different indicators, based on complex network theory. Additionally, this study analyzed the geographical spatial distribution characteristics of nodes based on the topography and urban spatial structure of Chongqing. Then, this study classified the nodes in the RTN according to basic topological indicators, namely degree, betweenness centrality, network efficiency, and passenger flow volume (PFV). The results indicated six cluster of nodes, reflecting the variability in node vulnerability concerning overall influence (providing alternative paths, reducing path length), regional aggregation capacity, and transportation capacity. Finally, this study proposed targeted management strategies for different clusters of nodes and their respective geographical locations, providing necessary references for rational planning, safety protection, and sustainable construction of RTN.

1. Introduction

Rail transit has become an essential tool for alleviating urban traffic demand due to its large capacity, fast speed, safety, and reliability. As new lines are continuously planned, constructed, and operated, China's metropolitan areas have seen the development of rail transit network (RTN). Moreover, the growing passenger flow volume (PFV) and interconnectedness of stations in the expanding RTN have brought attention to the ripple effect and local problem linkage on the entire network. From a network structure perspective, any failure of a node within the network can reduce network connectivity and efficiency, resulting in weakened RTN service, passenger congestion, and potential safety issues [1,2]. Detecting the most critical station that impact the RTN during disturbances is crucial. By efficiently maintaining and recovering these critical station, the anti-disturbance capability, stability, and resilience of RTN can be enhanced, thereby mitigating adverse impacts of disturbance events on the RTN.

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https://doi.org/10.1016/j.heliyon.2024.e27237
Received 12 October 2023; Received in revised form 8 January 2024; Accepted 26 February 2024
Available online 27 February 2024
2405-8440/

Therefore, research on key station identification in RTN has become a hot topic, and related scientific-technical advancements are constantly emerging. The first category includes intelligent monitoring systems such as image detection and sensors. In recent years, computer vision technology has made significant progress. Powerful algorithms like convolutional neural networks (CNN) were used to analyze images or videos from surveillance cameras or other sources to monitor PFV and human behavior [3]. By mining long-term PFV data for inertia features closely related to people's travel habits [4], key stations and key lines can be identified. The second category included Geographic Information System (GIS) and Building Information Modelling (BIM). These technologies enabled intelligent functions such as precise mapping, visualization analysis, intelligent equipment inspection, equipment failure warning, intelligent maintenance, and emergency response [5,6]. However, the application of GIS and BIM technology in rail transit was mainly focused on the design and construction stages, with limited applications during the operation and maintenance stages. The third category was big data analytics. The rapid development of big data technology provided support for studying passenger travel in rail transit, such as Smart Card Data [7] and Automatic Fare Collection System (AFCS) [8]. These technologies could mine spatiotemporal characteristics of passenger travel based on historical and real-time data, predict passenger demand, identify key stations, and optimize station operations.

These technological advancements are continuously evolving, improving the accuracy, efficiency, and intelligence of key station identification in RTN. However, existing scientific and technical approaches to key station identification in RTN often relied on single PFV data, real-time monitoring, and historical pattern analysis of factors such as travel time and PFV to define key stations that have the most significant impact on the performance of the transportation network. They overlooked the topological characteristics of the overall RTN and the impact of stations on the structural performance of the transportation network, making it challenging to predict unforeseen circumstances.

The development of graph theory has made network analysis the most effective method for transportation research [9,10]. Numerous studies have explored the vulnerability of RTN using complex network theory [11–14]. In this theory, stations are treated as nodes, and the connections between stations are called edges. The structure of the stations is established and analyzed based on the relationship between nodes and edges. Some nodes have been found to play a critical role in network performance [15,16]. Previous literature has used various indicators, including degree and betweenness centrality, to select critical nodes, revealing that intentional attacks based on these indicators were more likely to cause network collapse than random attacks [17–19]. Vulnerability assessments of RTN attacked nodes have also been conducted based on overall network connectivity or network efficiency loss [20,21]. Results have shown that the vulnerability and resilience of the RTN depended not only on the degree of disrupted nodes but also on their contribution to overall network connectivity [22]. In general, different importance assessment indicators had diverse impacts on RTN performance. Many previous studies on key node selection were relatively simple and lacked a unified indicator system, resulting in limited analysis of network structure characteristics.

Currently, some scholars have recognized the variations in node importance across different environments and attempted to enhance or even create new assessment methods to identify key nodes with the greatest impact on the structure of RTN [23,24]. For instance, some scholars have combined network efficiency with global indicators like the maximum connected subgraph [25,26] and the largest connected component (LCC) ratio [27]. Others have integrated degree and betweenness centrality with accessibility [28], connectivity [29], and network efficiency [30,31] using the entropy method and weighting method. Compared to using a single indicator, these new assessment methods offered a more accurate evaluation of the key nodes in RTN [28,32]. Moreover, some scholars have considered PFV (passenger flow volume) while exploring network features, as it reflected the transportation capacity of RTN. The importance ranking of RTN nodes has been determined based on the variance contribution of each indicator using techniques like Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) and factor analysis [33,34]. It has been pointed out that attacks based on the highest load node could result in more damages compared with the attacks based on the largest degree node [35]. Networks that take PFV into account differ to some extent from those that don't, with PFV-inclusive networks tending to be more vulnerable [36]. Despite these developments, previous research often focused on individual indicators or limited combinations of indicators. The theoretical foundations and factors considered varied among different methods, resulting in different identified key nodes. There is a lack of comprehensive approaches that consider global, regional, and transportation indicators together when evaluating key nodes and intervals in the RTN.

Table 1 provided an overview of the research objects, vulnerability assessment indicators, and strategy-related suggestions derived from pertinent literature within the preceding five-year period. Notably, several novel assessment methodologies have been devised to ascertain the importance of nodes in RTN. Nevertheless, these approaches have seldom integrated global, regional, and transportation indicators comprehensively. When appraising key nodes and intervals within the RTN, it is imperative to consider both their localized

Table 1
Literature in related fields in the last five years.

Research object	Vulnerability assessment indicator	Suggestion or not	Publication year
Suzhou	Degree, betweenness centrality, and operation accessibility	No	2023 [28]
Shanghai	Network efficiency, and Largest connected component ratio	Yes	2022 [27]
Shenzhen	Several different centrality measures	No	2020 [37]
Nanjing	Degree distribution, and passenger flow volume	No	2019 [35]
Shenzhen	Network efficiency	No	2019 [20]
Shanghai	Degree, connectedness, and network efficiency	No	2019 [30]
Shenzhen	Largest connected component ratio, and network efficiency	No	2018 [25]
Beijing	Largest connected component ratio, and network efficiency	No	2018 [26]

significance and overall network influence, while also the influence of PFV on network operation. Additionally, previous literature has mainly focused on large plain cities such as Beijing, Shanghai, and Shenzhen, with limited studies on mountainous cities with complex terrain. Furthermore, few studies have provided specific policy or strategy-related impacts and suggestions, based on the importance of key nodes, indicating a weak link between theory and actual policy-making.

Taking Chongqing Rail Transit (CRT) as an example, this study addresses the gaps in previous research by developing a network topology model using the spatial L method and applying complex network theory to analyze the characteristics of the network structure. The nodes in the RTN are categorized based on three fundamental topological indicators: degree, betweenness centrality, network efficiency, and PFV. These indicators help assess the vulnerability of nodes concerning global, local, and transportation performance, enabling us to propose tailored management recommendations for nodes in different clusters. Noteworthy innovations in this study include the assessment of key nodes, considering global, local, and transportation indicators; the selection of CRT located in mountainous regions with complex terrain as the research subject, taking into account practical factors such as the geographic location of each node in the network; and the clustering of importance assessment results, leading to targeted management strategies for nodes in different clusters. As a result, this research offers valuable insights for identifying and maintaining critical nodes in RTN.

The rest of the paper was organized as follows. Section 2 introduces the case study and methodology of the study; proposes four key

Fig. 1. Group Analysis of rail nodes based on Chongqing's topography and urban spatial structure.

node identification indicators; gives the vulnerability assessment method and data sources. Section 3 shows the results of vulnerability of CRT based on the proposed method. Section 4 analyzes the nodes within different clustering levels based on the results presented in Section 3, and proposes corresponding targeted management strategies. Section 5 provides the conclusions.

2. Methodology

2.1. Case study

RTN plays an important role in Chongqing's urban transportation system and urban economic development. The CRT network has been steadily expanding, and there are plans to establish a "22-line-1-ring" layout covering 1252 km by 2035, making it one of the longest RTNs in Chinese metropolitan cities. However, the CRT faces unique challenges compared to other RTNs due to its extensive mountainous routes. Fig. 1 illustrates the terrain and urban spatial structure of Chongqing's main urban area (within the ring expressway). Shaped by natural mountains and water bodies, the city has evolved into a diverse urban landscape with multiple groups. Adhering to the principles of balanced local communities, Chongqing, in contrast to similarly sized flat cities, demonstrates a higher prevalence of short-distance travel within groups. Residents' travel patterns reveal a distinctive distribution characterized by "closer proximity and shorter distances." The CRT plays a pivotal role in connecting the central group with others. The spatial configuration of the RTN intricately aligns with the mountainous city's organization, featuring a "radial and circular" layout pattern that harmonizes with natural contours. Given the exceptional characteristics and significance for Chongqing of CRT, this study has chosen it as the primary research subject, warranting further investigation and exploration.

Fig. 2 illustrates the CRT system plan, comprising 9 rail lines and a loop rail, encompassing a total of 233 nodes. This "nine lines and one loop" RTN structure forms a non-oriented network denoted as "G," composed of rail lines and nodes. To construct the network topology model for CRT, the Space L method was employed, and the resulting model is depicted in Fig. 3.

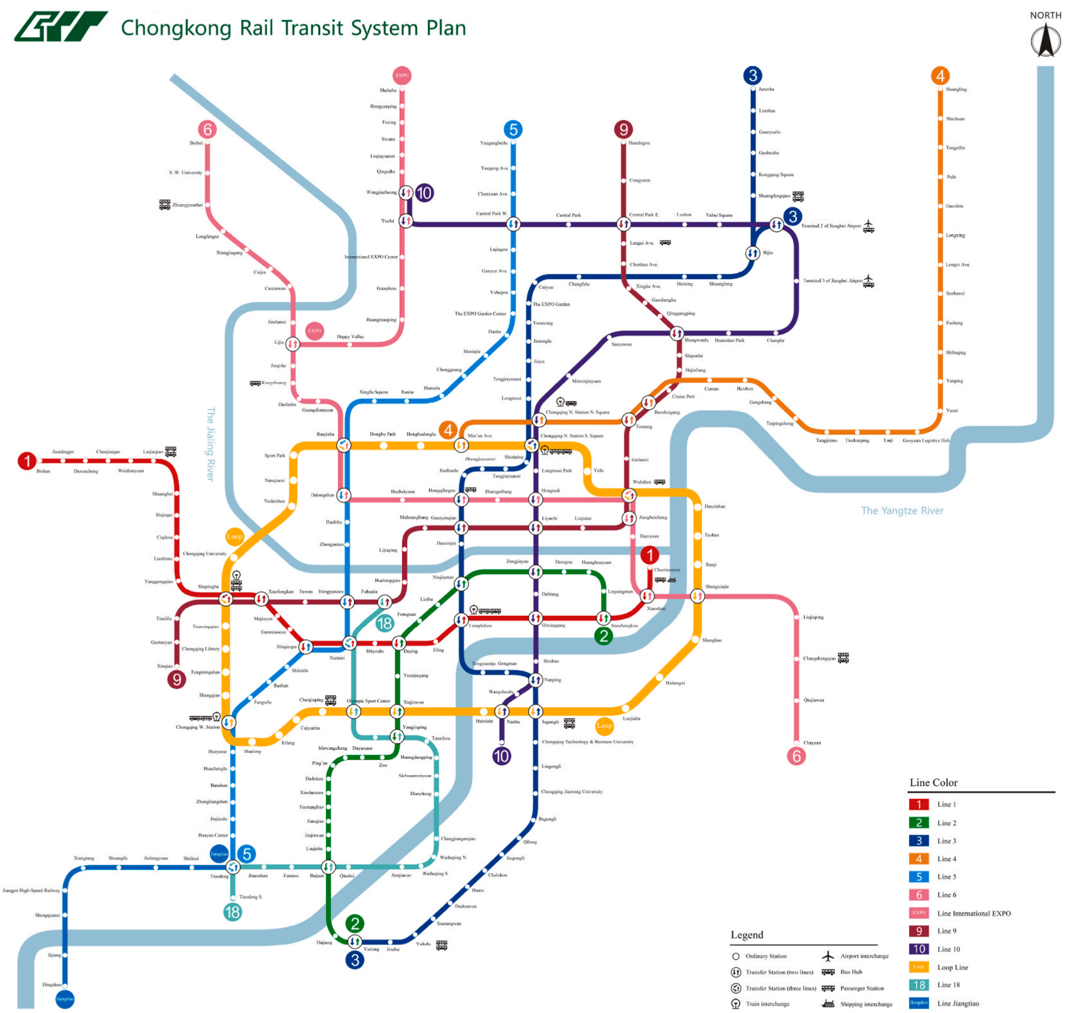

Fig. 2. CRT system plan (Chongqing Rail Transit group, 2023).

2.2. Analysis process

The analysis flow chart is shown in Fig. 4. The research involved the development of a network topology model using the spatial L method, followed by an analysis of the network structure characteristics and the importance ranking of key nodes under different indicators, all based on complex network theory. The nodes within the RTN were classified according to three fundamental topological indicators: degree, betweenness centrality, network efficiency, and PFV, by k-means clustering. Finally, this study proposed specific management strategies tailored to different clusters of nodes.

2.3. Complex network

Complex network theory and its applications have emerged as a significant research focus across various disciplines, including social sciences [38], biological sciences [39], and computer sciences [40]. To portray the fundamental structure of complex networks, existing studies have employed topology models, depicting directed and non-oriented networks comprising nodes and edges connecting them. These models facilitated a clearer analysis of network structure characteristics by incorporating different indicators. Notably, in recent years, numerous scholars have utilized complex network theory to study the topology and structure of RTN [41].

2.3.1. Topology modeling of RTN

Topology modeling involves creating a mathematical representation of a system's network topology [42]. Space P, Space B, Space C and Space L are four common methods for developing topology models of RTN [43]. In the Space P model, nodes represent stations, and a link is established between two stations if they share at least one common route. This feature makes the Space P model ideal for analyzing transfer characteristics in RTN [44]. The Space B model constructs the transportation network based on individual activities or trips, encompassing the entire travel chain, including origins, destinations, and activities at different locations [45]. The Space C model divides the RTN into cells or zones, examining the movement of vehicles or flows between these cells, representing spatial units within the network. Commonly used in dynamic traffic assignment and transportation planning, the Space C model provides insights into the network's spatial dynamics [45]. In the Space L model, stations function as nodes, and links between nodes represent routes connecting two consecutive stations without intermediate stops [46]. The Space L model offers a more intuitive representation of the RTN, closely resembling the actual network and is particularly suitable for studying network structure characteristics and vulnerability. Therefore, this study adopted the Space L method to develop the topology model of the RTN. Based on graph theory, the RTN was defined as a non-oriented network denoted as "G", comprising n nodes and m edges. The topological connection between node i and node j in the graph was denoted by a_{ij}. If there was a straight edge between node i and node j, a_{ij} equaled to 1; otherwise, a_{ij} equaled

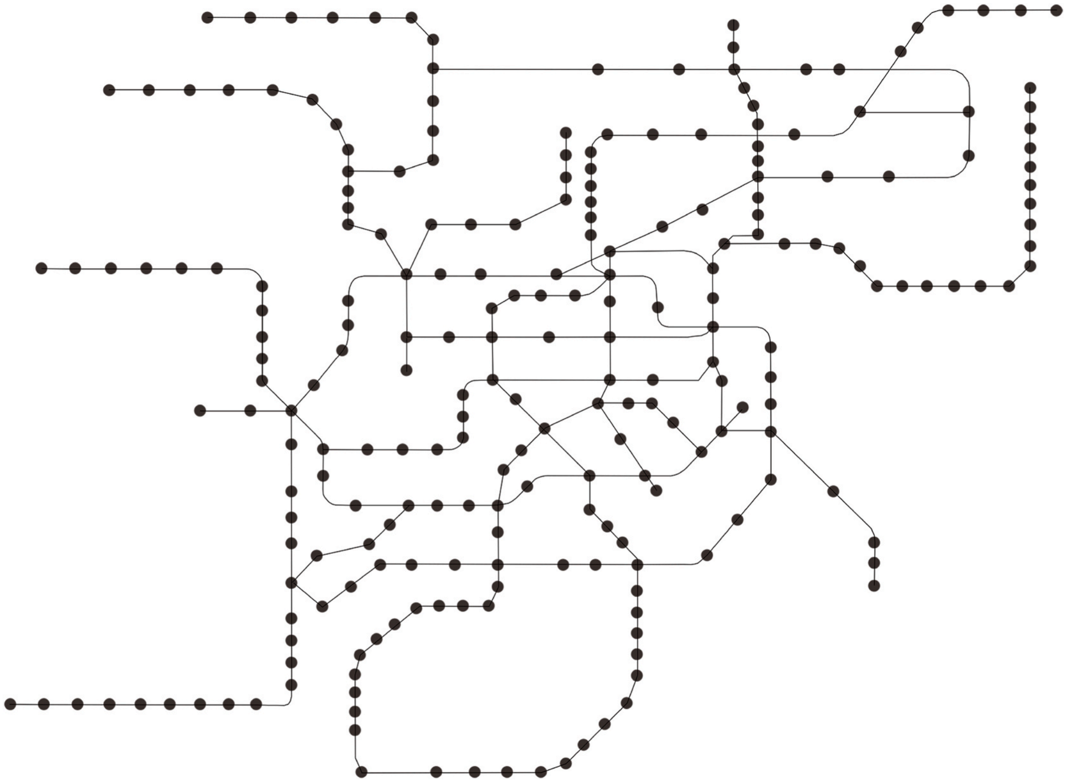

Fig. 3. Network topology model of CRT.

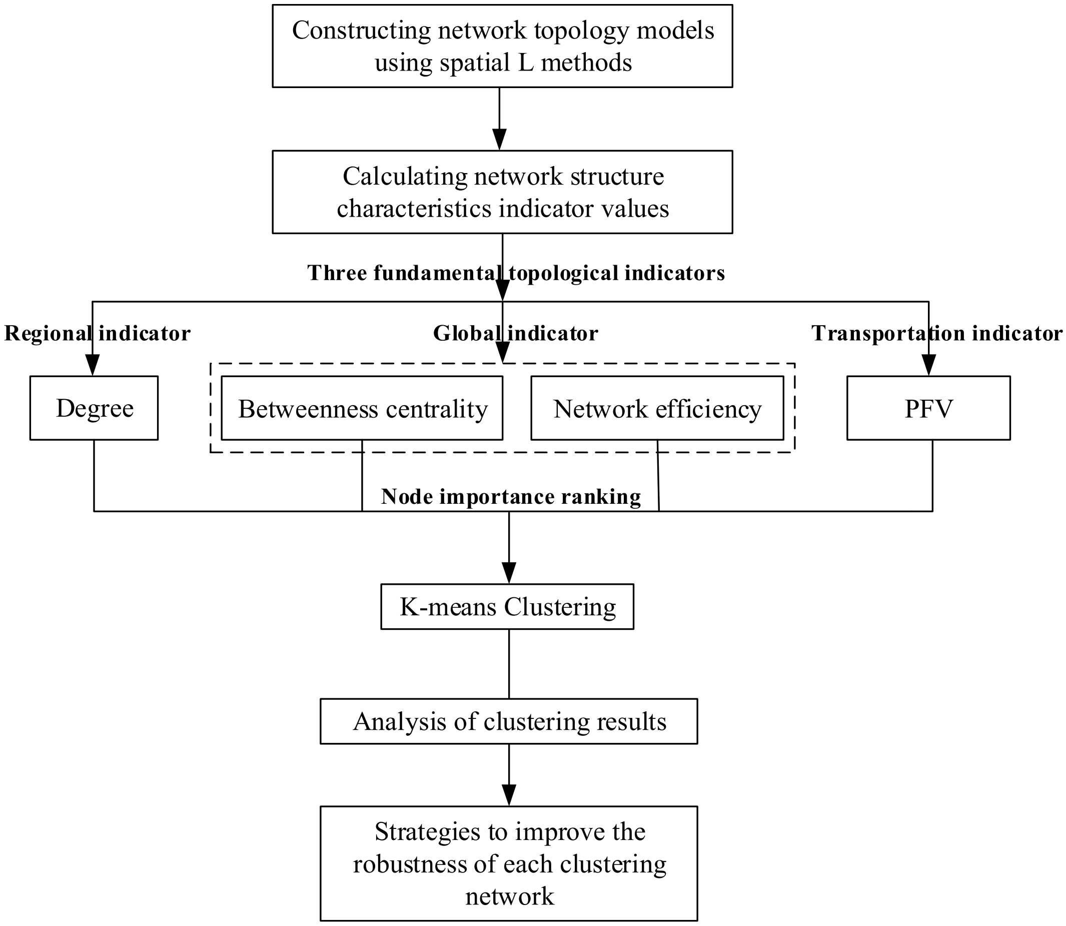

Fig. 4. Analysis flow chart.

to 0. The path consisted of edges between two target nodes, and the path length equaled to the number of edges along the path. If no path exists between the selected pair of nodes, the distance between the two nodes is considered infinite.

2.3.2. Network structure characteristic indicators

To analyze the basic network structure characteristics of CRT, static statistical indicators, such as degree, average degree, degree distribution, average shortest paths and clustering coefficient were calculated.

2.3.3. Degree, average degree and degree distribution

The degree and degree distribution are crucial measures for nodes in a complex network. Specifically, the degree k_i of a node i is defined as the total number of edges connected to that node.

The average of the degree k_i of all nodes i in the network is called the average degree k of the nodes, which can be written as [42]:

$$K = \frac{1}{N}\sum_{i=1}^{N} k_i \tag{1}$$

P (k) the degree distribution, representing the probability of any node having a degree of k. Pc (k) represents the probability distribution of nodes with degree equal to or greater than k_i, which can be written as [42]:

$$p_c(k) = \sum_{k' \geq k} p(k') \tag{2}$$

2.3.4. Average shortest paths

The average shortest paths refer to the average value of the shortest path between any two nodes in the network, which is a global indicator to measure the transmission efficiency of the network. A smaller average path length indicates a more closely connected network. The equation for this measure can be written as [42]:

$$L = \frac{\sum_{i=j} d_{ij}}{n(n-1)/2} \tag{3}$$

where L is the average path length; n is the number of nodes in the network; i, j are nodes; d_{ij} denotes the shortest path length between i and j.

2.3.5. Clustering coefficient

The clustering coefficient C_i is defined as the ratio of the number of edges E_i existing between node i and k_i neighboring nodes in the network to the total number of possible edges $\frac{k_i}{2}(k_i - 1)$, which can be written as [42]:

$$C_i = \frac{2E_i}{k_i(k_i - 1)} \tag{4}$$

The clustering coefficient C of the whole network refers to the average of the clustering coefficients of all nodes in the network, which can be written as [42]:

$$C = \frac{1}{N} \sum_{i=1}^{N} C_i \tag{5}$$

2.4. Key node identification method

In this study, three fundamental topological indicators, namely, degree, betweenness centrality, network efficiency, and PFV, were selected to rank the importance of key nodes. Then, the key nodes were classified by K-means clustering according to three fundamental topological indicators and PFV.

2.4.1. Fundamental topological indicators

(1) Degree

Degree refers to the number of connections a node has with other nodes, indicating its importance. The variable a_{ij} represents the adjacency matrix variable. The equation can be written as follow [42]:

$$k_i = \sum_j a_{ij} \tag{6}$$

(2) Betweenness centrality

Betweenness centrality measures the number of times a node lies on the shortest path between other nodes, indicating its ability to control behavior and information transfer. Normalized betweenness centrality (nbetweenness) is used for comparison with betweenness centrality values in other networks. In the equation, $D_{ij}(i)$ represents the number of shortest paths through node i. D_{kj} represents the number of shortest paths between nodes k and j. The equation can be written as follow [42]:

$$B_i = \sum_{k \neq j \in G} \frac{D_{ij}(i)}{D_{kj}} \tag{7}$$

(3) Network efficiency

The network efficiency of a network serves as a comprehensive measure of its connectivity characteristics and is considered a comprehensive metric. When certain nodes in the network are attacked, the connectivity or average clustering coefficient may decrease, causing some pairs of nodes to become disconnected from each other, leading to an infinite distance between these pairs. The efficiency of any two nodes i and j is expressed as the reciprocal of the distance between the two nodes, namely $\frac{1}{d_{ij}}$. Then the network efficiency of the whole network is expressed as the average efficiency between any two nodes, which can be written as [42]:

$$E = \frac{1}{N(N-1)} \sum_{i \neq j}^{N} \frac{1}{d_{ij}} \tag{8}$$

2.4.2. Fundamental topological indicators

The PFV data of CRT from August 22–28, 2022 was provided by the Automatic Fare Collection System (AFCS). This data was utilized to identify key nodes that play a crucial role in transportation within the RTN.

2.4.3. K-means cluster analysis

K-means clustering analysis method has been widely recognized and extensively utilized in previous studies [47]. It is known for its remarkable computational efficiency and effectiveness in handling large datasets with a moderate number of clusters. K-means algorithm is particularly suitable for grouping similar documents together by clustering data points. By organizing the nodes into groups, it facilitates the identification of outliers or anomalies that deviate from the predominant patterns observed in the majority of nodes. Some researchers adopted K-means to evaluate and classify the surrounding environments of nodes [48]. Other researchers used K-means to identify the major functions of nodes [49]. There were also scholars who utilized GIS, Kernel Density Estimation, and environmental data to create a classification of road accident hotspots using the K-means clustering method [50]. Therefore, the K-means clustering analysis method was chosen in this study for node classification.

It takes k as an input parameter and partitions m object sets into k clusters, maximizing the similarity of items within the same cluster while minimizing the similarity between different clusters. The algorithm aims to minimize the within-cluster sum of squares by dividing the data points into k clusters within certain dimensions [51]. The equation can be written as follows [47]:

$$E = \sum_{i=1}^{k} \sum_{P \epsilon C_i} |p - m_i| \tag{9}$$

Where E refers to the square error-sum of all objects, p denotes the point in space, and m_i denotes the average of cluster C_i. The sum of squares of the distance between each object and its cluster center is calculated [47]:

In this study, we used k-means to classify the key nodes based on three fundamental topological indicators: degree, betweenness centrality, network efficiency, and PFV.

3. Result

3.1. Topology model of RTN

The static statistical indicators of CRT based on complex network theory were calculated in Table 2. The results indicated there were 233 nodes and 254 connected edges in CRT. The average degree was found to be 2.197, signifying that each node was, on average, directly connected to 2.197 other nodes. The degree distribution and cumulative degree distribution are shown in Fig. 5. Notably, the figure showed that the maximum degree of nodes in CRT was 6, with only one node falling into this category. Meanwhile, the majority of nodes (82.85% of all nodes) had a degree of 2.

The average shortest path length was 15.529 nodes; the network efficiency was 9.972%; and the clustering coefficient was only 0.002. Fig. 6 illustrates the shortest path statistics of the CRT network, with the horizontal coordinate d indicating the shortest path length and the vertical coordinate $P(k)$ representing the cumulative probability of the shortest path being less than or equal to d. The analysis of actual data revealed the longest distance between any two nodes in the RTN was 52 nodes. Approximately 49.9% of nodes could be reached within 15 nodes, and an impressive 79.2% or more nodes could be reached within 23 nodes.

Based on Fig. 1, the spatial distribution of nodes in each group was shown in Fig. 7. The distribution of nodes exhibited a noticeable spatial imbalance, with a concentration of nodes in the central group, followed by the northern and southern groups, while fewer nodes were situated in the peripheral groups.

3.2. Network structure characteristics

3.2.1. Degree

Degree represents the number of connections a node has with other nodes, indicating its local aggregation ability and importance. The calculation of the degrees in the network revealed some significant findings. Specifically, the maximum degree observed was 6, which was associated with only one node, accounting for a mere 0.43% of the total. On the other hand, the majority of nodes, a staggering 82.85% of them, had a degree of 2. Additionally, nodes with degrees of 5, 4, 3, and 1 accounted for 0.86%, 7.3%, 4.7%, and 3.86% of the total, respectively. Table 3 presents the ranking of nodes with degrees greater than or equal to 4, while Fig. 8 illustrates their distribution in the network topology model.

3.2.2. Betweenness centrality

The betweenness centrality metric measures the number of pairs of nodes that pass through a specific node in the network,

Table 2
Characteristics of CRT network indicator values.

Characteristic indicators	Numerical value
Number of nodes	233
Number of edges	254
Average degree	2.197
Clustering coefficient	0.002
Average shortest paths	15.529
Network efficiency	9.972%

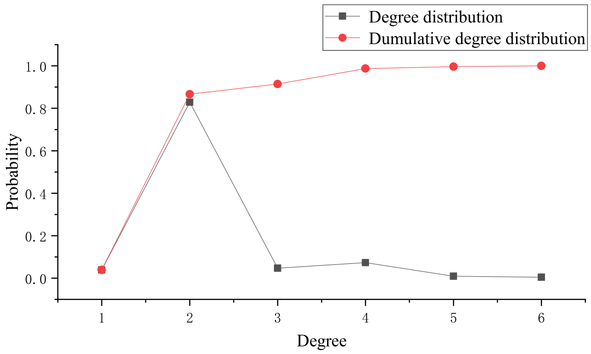

Fig. 5. Scatter plot of degree distribution and cumulative degree distribution.

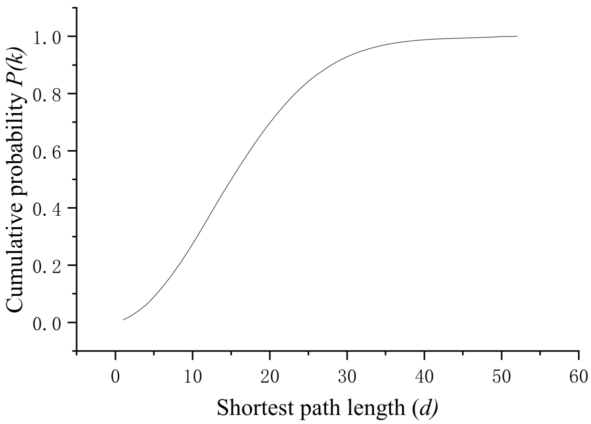

Fig. 6. Cumulative probability of shortest path length.

indicating its ability to control behavior and information delivery by providing alternative paths. The betweenness centrality and relative betweenness centrality of each node in the network were calculated. Out of all the nodes, 13 had a relative intermediate centrality of 0, making up 5.58% of the total. The majority of nodes (176 in total) had intermediate centrality greater than 0 and less than or equal to 10, accounting for 75.5% of the total, which represented the largest proportion. Additionally, there were 41 nodes with intermediate centrality greater than 10 and less than or equal to 20, comprising 17.6% of the total. Only 2 nodes had intermediate centrality greater than 20 and less than or equal to 30, amounting to 8.6% of the total. Moreover, there was just one node with intermediate centrality greater than or equal to 30, making up 4.3% of the total. Table 4 displays the top 20 nodes based on their intermediate centrality, and their distribution in the network topology model is illustrated in Fig. 9.

3.2.3. *Network efficiency*

The network efficiency is directly influenced by the level of connectivity between nodes in the network. A higher level of disconnection among nodes leads to lower network efficiency and increased vulnerability. To assess the clustering degree among nodes in the RTN, a deliberate attack was conducted on 233 nodes of the CRT. The network efficiency was then compared before and after the attack, and the rate of change represented the ability of the nodes to reduce the path length of the entire network. The 10 nodes with the most significant impact are listed in Table 5, and their distribution in the network topology model is illustrated in Fig. 10.

The analysis revealed that several nodes, namely Ranjiaba, Shapingba, Baoshuigang, Cuntan, Heshizi, Chongqing West Station, Taipingchong, Lijia, Tangjiatuo, and Tieshanping, played crucial roles in the network. If any of these nodes were targeted and attacked, the overall network efficiency would decrease by more than 6%. Interestingly, the impact of nodes on the network's efficiency did not always align with their degree or betweenness centrality rankings. For instance, Chongqing North Station North Square had a degree of 4 and ranked third in terms of betweenness centrality, but its removal only resulted in a 3% reduction in network efficiency. Conversely, Cuntan had a degree of 2 and its betweenness centrality was 24th, yet its removal led to an 8% decrease in overall network efficiency.

3.2.4. *PFV*

Fig. 11 displays the average PFV data of the top 30 nodes in CRT from August 22–28. Among these nodes, the top 10 nodes with the highest PFV values were Guanyinqiao, Xiaoshizi, Guangianyuan, Hongqihegou, Lianglukou, Nanping, Jiazhou, Ranjiaba, Huahuiyuan, and Changshengqiao. Their distribution in the network topology model is visualized in Fig. 12.

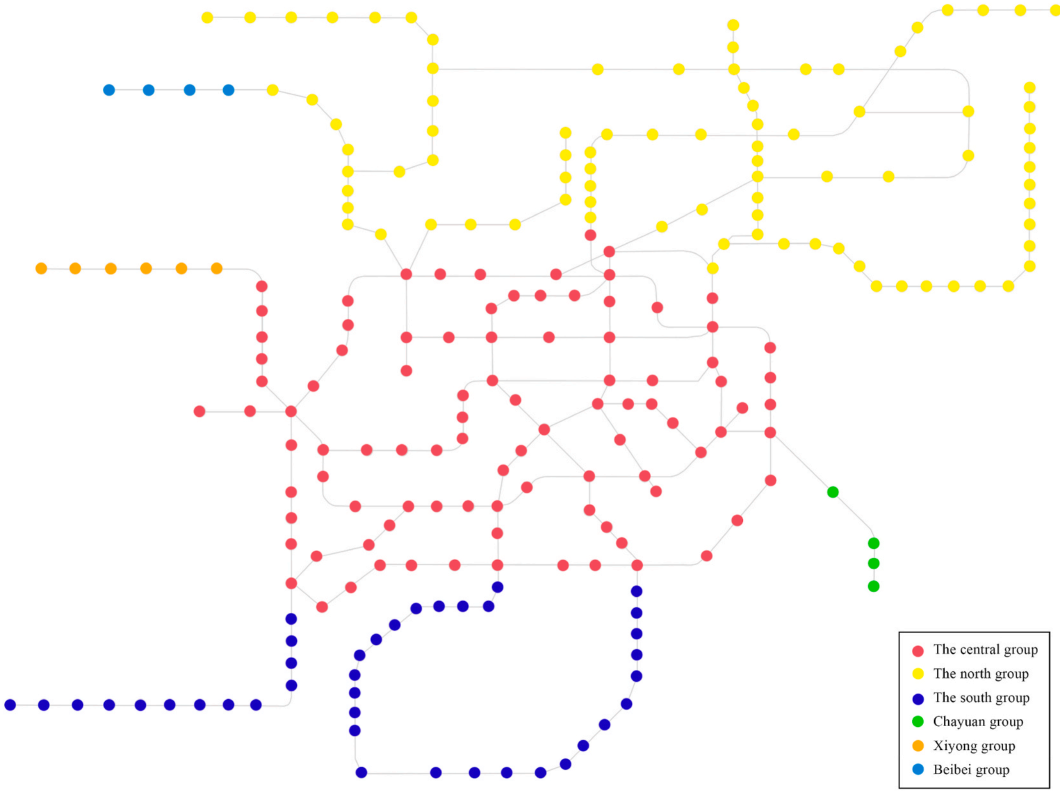

Fig. 7. Spatial distribution of six groups.

Table 3
Node degree ranking.

Name	Degree	Name	Degree
Chongqing N. Station S. Square	6	Hongtudi	4
Ranjiaba	5	Guanyinqiao	4
Wuli dian	5	Sigongli	4
Chongqing N. Station N. Square	4	Chongqing W. Station	4
Daping	4	Zengjiayan	4
Niujiaotuo	4	Hongqihegou	4
Shangwanlu	4	Shangxinjie	4
Xiejiawan	4	Xiaoshizi	4
Liyuchi	4	Central Park E.	4
Lianglukou	4	Qixinggang	4

The results indicated that among the top 10 nodes in terms of PFV, five were interchange nodes, while the rest were ordinary nodes. Most nodes had a degree of either 4 or 2, with only one node among the top 10 having a degree of 5, and two nodes among the top 30. However, apart from Ranjiaba, the betweenness centralities of the other nodes were relatively lower.

3.3. Cluster analysis

The nodes were clustered based on degree, betweenness centrality, network efficiency, and PFV using k-means. The results revealed six clusters of nodes, and the cluster centers of the four indicators for different clusters after dimensionless processing are shown in Table 6. For detailed clusters and corresponding nodes, please refer to the Appendix.

The data in Table 5 and Appendix revealed that the only node in cluster A, Ranjiaba, achieved the top rank in betweenness, degree, and network efficiency, while securing the second position in terms of PFV. This indicated that all four indicators demonstrated a high level for this node.

Cluster B comprised 16 nodes, ranking 2nd in terms of betweenness and degree, and 3rd in terms of network efficiency and PFV. Notably, all four indicators were at a slightly higher level, second only to Cluster A.

Cluster C consisted of 22 nodes, ranking 3rd in terms of betweenness, 5th in terms of degree and network efficiency, and 4th in

Fig. 8. Spatial distribution of node degree.

Table 4
Betweenness centrality ranking.

Name	Betweenness	nBetweenness
Ranjiaba	8886.22	33.16
Shapingba	5966.00	22.26
Chongqing N. Station N. Square	5724.25	21.36
Daping	5085.31	18.98
Niujiaotuo	5039.56	18.81
Toutang	4704.67	17.56
Sports Park	4625.95	17.26
Shangwanlu	4604.00	17.18
Toutang	4556.41	17.00
Chongqing N. Station S. Square	4530.27	16.91
Nanqiaosie	4514.95	16.85
Xiejiawan	4435.91	16.55
Yudaishan	4432.76	16.54
Chongqing University	4377.69	16.34
Wulidian	4324.86	16.14
Liyuchi	4230.00	15.79
Min'an Ave.	4132.45	15.42
Guangdianyuan	4115.18	15.36
Honghudonglu	4067.45	15.18
Dongbu Park	4058.62	15.15

terms of PFV. The results suggested that all four indicators were generally at a low level, with the exception of a moderate level of betweenness.

Cluster D comprised 2 nodes, ranking 4th in terms of betweenness and network efficiency, 3rd in terms of degree, and 1st in terms of PFV. This cluster was distinguished by its highest PFV, while the other three indicators remained at a low level.

Cluster E contained 19 nodes, ranking 5th in terms of betweenness, 4th in terms of degree, 2nd in terms of network efficiency, and 6th in terms of the PFV. This category was characterized by a higher network efficiency, while lower betweenness, degree and PFV.

Cluster F was the largest cluster among the six with a total of 132 nodes. It ranked 6th among all clusters in terms of betweenness

Fig. 9. Spatial distribution of betweenness centrality ranking.

Table 5
Changes in network efficiency after node failure.

Name	Degree	Betweenness centrality ranking	Network efficiency after the attack	Network efficiency change rate/%
Ranjiaba	5	1	0.089	11
Shapingba	5	2	0.091	9
Baoshuigang	3	7	0.091	9
Cuntan	2	23	0.092	8
Heishizi	2	28	0.092	8
Chongqing W. Station	4	34	0.093	7
Taipingchong	2	33	0.093	7
Lijia	3	24	0.094	6
Tangjiatuo	2	38	0.094	6
Tieshanping	2	10	0.094	6

and network efficiency, and 5th in terms of degree and PFV, indicating that all four indicators were at a low level.

4. Discussion

4.1. Topology model of CRT

The results in Table 2 showed that the current average degree of the CRT network was 2.197. Comparatively, the average degree of rail transit networks in some large cities had already reached 2.4 [52], indicating that the mutual crossover among CRT nodes was relatively lower. The scatter diagram of degree distribution and cumulative degree distribution further supported this observation, as there were relatively few crossed lines in CRT. These findings implied that the CRT network required further improvement in its future development. Moreover, the average shortest path length was 15.529 nodes, signifying that the average shortest distance was relatively large, indicating the need to enhance the connectivity of the CRT, which aligned with the fact that the development of RTN in China was at a preliminary stage [53]. The clustering coefficient of CRT is relatively low, indicating a weaker connectivity between rail transit nodes. From Fig. 1, it could be observed that only the central group exhibit higher network density, while there were fewer connections between nodes in the peripheral groups. This suggested that the overall density of the CRT was relatively low, and there was an imbalance in the development among different groups.

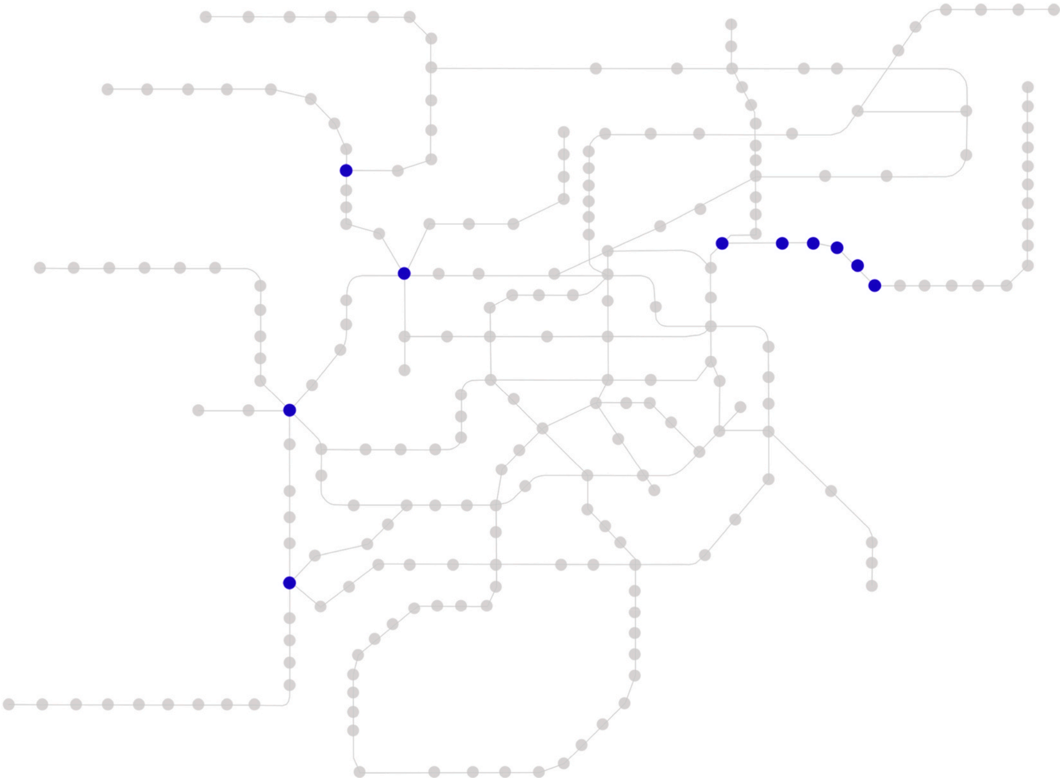

Fig. 10. Spatial distribution of network efficiency change ranking after node failure.

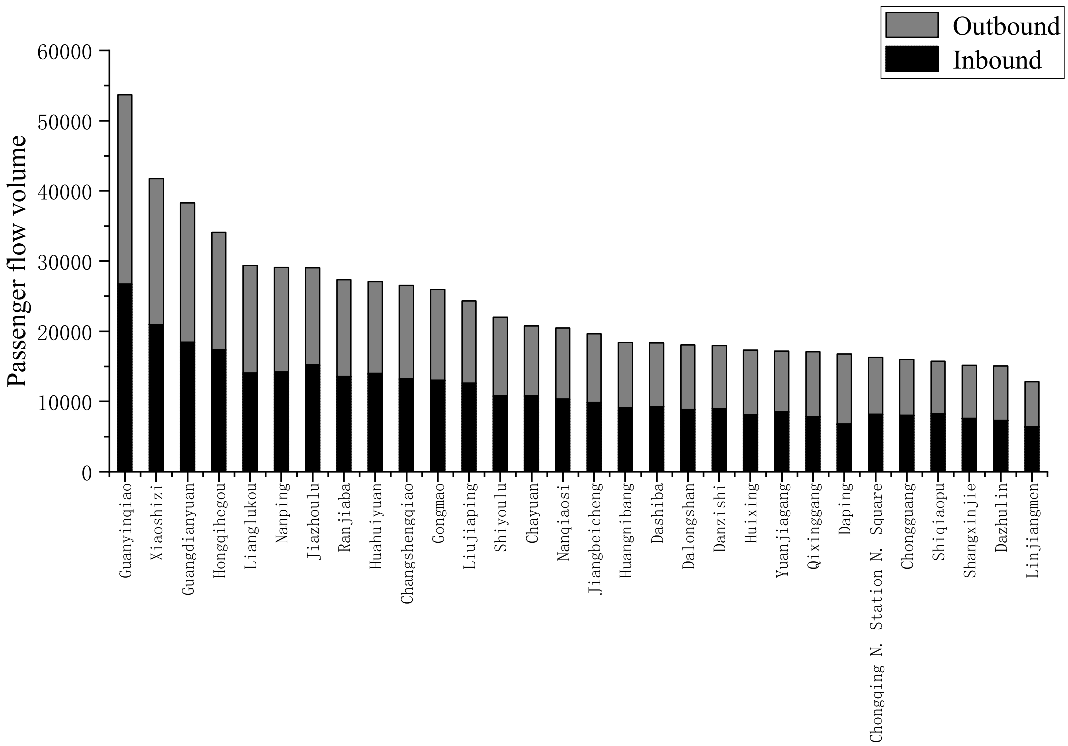

Fig. 11. Average PFV of CRT from August 22–28.

Fig. 12. Spatial distribution of the top 10 nodes in terms of PFV.

Table 6
Cluster centers of different indicators of the six clusters.

Clustering	A	B	C	D	E	F
Betweenness	1	0.43	0.39	0.32	0.32	0.11
Degree	0.8	0.6	0.2	0.6	0.3	0.2
Network efficiency	1	0.26	0.22	0.23	0.51	0.09
PFV	0.37	0.22	0.2	0.86	0.05	0.1

4.2. Network vulnerability analysis under different indicators

Through the comparison of Figs. 8–10 and 12, notable disparities emerged in the ranking of key nodes across different metrics. Notably, there was no strict positive correlation between network efficiency and degree, betweenness centrality, or PFV. Furthermore, The comparison of Figs. 8–10 and 12 with Fig. 7 unveiled distinct distribution patterns of key nodes among groups under varying metrics.

Analyzing Fig. 8 in conjunction with Fig. 7 revealed a notable concentration in the central group, with 90% of nodes exhibiting a degree equal to or greater than 4. This concentration was attributed to the abundance of large-scale transfer nodes within the central group, contributing to an increased network density.

Upon comparing Figs. 9–7, it became apparent that nodes with high betweenness centrality fall into two distinct categories: 1) central nodes or hub nodes within the central group, often serving as intersections for multiple urban rail transit lines; 2) transitional nodes located between the central and northern groups, enhancing connectivity between the two groups with the highest number of nodes.

By comparing Fig. 10 with Fig. 7, it became apparent that nodes significantly influencing network efficiency were mainly situated along transitional lines connecting the central group to the peripheral groups. As per the network efficiency definition, removing central nodes had minimal impact on network efficiency, given the presence of alternative lines in the denser road networks of the central group. On the contrary, removing nodes that connect the central group to the peripheral groups disrupted inter-group connectivity, leading to a substantial change in network efficiency.

Comparing Figs. 12–7 revealed that nodes with high PFV were primarily concentrated within the central group. This concentration could be attributed to the elevated prosperity, the focal point of commercial districts, and the existence of multiple attractions within the central group, attracting a dense influx of people.

In summary, the RTN could be analyzed from four different aspects using the following indicators: degree, which reflected the local aggregation ability of nodes; betweenness centrality, which reflected the ability to provide optional paths; network efficiency, which reflected the ability to reduce the average path of the whole network; and PFV, which reflected the importance of nodes and edges in transportation. Previous studies [17,31,34,36] have also drawn similar conclusions, showing that the ranking of key nodes varied significantly under different indicators. Therefore, it was not reasonable to assess the overall function of a node using just one indicator without considering its global effects. Additionally, solely relying on holistic indicators did not effectively measure the local clustering characteristics of nodes. Furthermore, the impact of PFV carried by nodes and intervals in operation on the RTN could not be ignored. Hence, when assessing the network vulnerability, it was crucial to consider the impact of different aspects to get a comprehensive understanding.

4.3. Strategies to improve the vulnerability of RTN

In order to enhance the response to interference and ensure the safe operation of the system, the corresponding strategies was proposed based on the network structure characteristics, vulnerability analysis, and urban spatial structure.

Nodes of cluster A had the highest network-wide ability to provide alternative routes, the highest ability to reduce rout lengths, the highest local impact and higher transportation capacity. Their influence on the entire network, both in terms of physical topology and PFV, was substantial. Analyzing the characteristics of the node revealed that Ranjiaba was situated in a densely populated central group. As a major interchange hub, it lay along the connecting paths of most nodes, serving as a bridge connecting the central group with the northern and Beibei groups. Additionally, it was surrounded by tourist attractions such as parks and squares. Given its vulnerability as a critical node, comprehensive improvements should be implemented. To enhance the overall network, the end of single-chain lines could be looped to enhance efficiency and network integrity, thereby increasing resistance to damage [54]. On a local level, additional transfer nodes near Ranjiaba would reduce reliance on this central hub [27]. To address PFV concerns, it was essential to increase the frequency of rail train and bus services on this line and improved risk resistance capabilities of the nodes and adjacent nodes. Such measures would enable timely diversion of unexpected interferences, mitigating adverse impacts. Additionally, priority should be given to these nodes when establishing emergency facilities for the RTN [21,55].

Cluster B nodes possessed the second-highest local impact capacity, ability to provide alternative routes, and showed higher efficiency in reducing route length and transportation capacity. An analysis of the nodes characteristics revealed that 87.5% of the nodes were situated in the central group, while 12.5% of the nodes were located in the northern group. Importantly, all of these nodes served as major interchange hubs, with a degree of connectivity equal to or greater than 4. On the periphery of the network, there were fewer interchange nodes, resulting in a lower number of nodes extending outward. The key node identification indicators for this cluster were slightly lower than those of cluster A. Consequently, similar strategies could be employed, considering them as the second priority nodes.

Nodes of cluster C had higher ability to provide alternative routes, but lower local impact and transportation capacity, as well as lower ability to reduce route length. An analysis of the nodes characteristics revealed that these nodes could be divided into two categories: 1) regular nodes located within the central group, and 2) interchange nodes that connected the central group with the southern and northern groups respectively (with a degree of connectivity equal to or less than 3). Consequently, when considering the addition of transfer nodes, priority should be given to this cluster to ensure there were more alternative routes for the normal operation of the network, particularly when the nodes or lines were under attack.

Cluster D nodes exhibited the highest transportation capacity and higher local impact capacity, but their ability to provide alternative routes and reduce route length was only 20%–30% of that seen in cluster A. The network structure characteristics revealed that the two nodes in cluster D were interchange nodes located at the central group, and they possessed significant PFV due to the presence of numerous scenic spots or commercial complexes in their vicinity. Therefore, increasing the frequency of the metro and bus departures on these lines was necessary. Additionally, efforts should be made to shorten the headway and following distance of consecutive trains during peak hours to enhance capacity and accommodate the exceptionally high PFV during busy times [56]. Furthermore, it was crucial to ensure seamless connections to adjacent nodes and enhance the risk resistance of the nodes and their neighbors. Proper maintenance and repair of the railroad system were also of great significance in facilitating timely diversions and minimizing adverse impacts during attacks [57].

Cluster E nodes demonstrated a very high ability to reduce route length, but their local impact capability and ability to provide optional routes were low, and they possessed the lowest transportation capability. An analysis of the node characteristics revealed that nodes of cluster E served as transitional nodes within the northern and southern groups, connecting them individually to the central group. Additionally, the majority of these nodes had a degree of connectivity equal to 2. The network structure characteristics revealed that these nodes were predominantly interchange nodes located at the edge of the network or single-chain nodes that extend outward. To enhance network efficiency and integrity, it was advisable to consider looping the end of such single-chain lines, as this measure could play a crucial role in fortifying the network against damage [54]. Additionally, in the event of an unforeseen disruption, ensuring the uninterrupted operation of the nearby bus system was essential to safeguard the safety of passengers [58].

Cluster F nodes had low global, local, and transportation capacity. Their impact on the whole network was minimal, both in terms of topology and PFV. The network structure characteristics revealed that the majority of nodes in this category were common nodes located at the peripheral groups. Consequently, in the subsequent operation and maintenance, these nodes held a lower priority. Collecting and analyzing the historical accidents related to these nodes, followed by summarizing the vulnerability factors, would aid in promoting safety management [59,60].

4.4. Implication and limitation

This study has put forward optimization schemes and differentiated protection plans for different clusters of nodes in the CRT system, with the aim of effectively preventing accidents and minimizing potential losses. The research findings can serve as valuable references and decision support for RTN planning, line design, and optimization not only in Chongqing but also in other cities, thus laying the foundation for the development of a more secure and reliable RTN.

Additionally, it was worth mentioning that this paper explored the structure of the RTN by examining the distinctive characteristics of mountainous cities through a group-based analysis of its horizontal layout. However, the vertical aspect, specifically the variations in elevation between nodes, was not considered. To ensure a more comprehensive analysis, our future research endeavors would incorporate the elevation values of the nodes, unveiling the three-dimensional spatial structure of RTN influenced by the unique terrain and topography of mountainous cities.

Furthermore, it was crucial to acknowledge that the topology model of the RTN discussed in this study assumed a non-oriented and unweighted network. Practical factors, such as the physical length of tunnels between nodes, the timing and routes of line transfers, the capacity of different lines, and political considerations, have not been incorporated into the model. Exploring the relationship between these factors and the RTN would be essential for conducting a more comprehensive analysis in the future.

5. Conclusion

Taking CRT as an example, this study developed a topology model via spatial L method, and analyzed the network structure characteristics based on complex network theory and urban spatial structure. Then the nodes in the RTN were classified into six clusters according to the three basic topological indicators as well as the PFV. The main conclusions are shown as follows:

1) Based on the complex network theory, the network structure characteristic and performance were analyzed, revealing an average degree of 2.197, an average shortest path length of 15.529, a network efficiency of 9.972%, and a clustering coefficient of 0.002. There existed lack of connection between nodes and topological waste. By conducting a group analysis of nodes based on the topography and urban spatial structure of Chongqing, it could be observed that the distribution of nodes exhibited a noticeable spatial imbalance, indicating an imbalance in the development among different groups.
2) The importance ranking of key nodes based on four different indicators showed significant variations. The analysis resulted in the identification of six clusters of nodes, each representing distinct global capacity, local capacity, and transportation capacity. For instance, nodes in cluster D had a lesser influence on the physical topology but carried more passengers in operation, whereas nodes in cluster A were critical to both the physical topology and specific transportation functions.
3) Considering the characteristics of RTN, it was essential to propose distinct protection and emergency rescue strategies for different clusters of nodes. Among all the nodes, Ranjiaba stood out as the most crucial node, drawing high attention due to its overall performance, local performance, PFV performance, and actual geographical location. Consequently, comprehensive safety protection and emergency rescue measures should be implemented. This included establishing connecting loops, adding extra transit nodes, increasing departure frequency and improving node accessibility by carrying out infrastructural interventions to enhance the network's vulnerability.

RTN plays a pivotal role in urban transportation systems and urban economic development. The key node identification method developed in this study establishes a theoretical and empirical foundation for evaluating network topology and transportation conditions. Furthermore, the results of this method can assist in controlling critical nodes to reduce the occurrence of accidents. Building on this foundation, our future research will explore the following aspects: firstly, we will integrate elevation values as weights for each node to construct a spatially weighted three-dimensional network topology model for RTN. This model will allow us to quantitatively characterize and analyze the spatial structure of nodes, highlighting the uniqueness of RTN in mountainous cities. Secondly, it would be intriguing to introduce new indicators such as rail transit time between nodes, a factor significantly influencing the resilience of the overall network by providing alternative paths [61]. Lastly, exploring the vulnerability of the integrated public transportation system by combining the RTN and the bus network in further studies is also a worthwhile pursuit.

Statement

During the preparation of this work the authors used ChatGPT in order to improve readability and language. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the content of the publication.

Data availability statement

The data associated with our study has not been deposited into a publicly available repository, and authors do not have permission to share data.

CRediT authorship contribution statement

Jinghua Song: Writing – review & editing, Supervision, Resources, Project administration, Methodology, Funding acquisition, Data curation, Conceptualization. **Jianfeng Ding:** Writing – original draft, Software, Investigation, Formal analysis, Data curation. **Xuechen Gui:** Writing – review & editing, Supervision, Methodology, Conceptualization. **Yuyi Zhu:** Investigation, Formal analysis, Data curation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

This work was supported by the National Natural Science Foundation of China [grant number: 50608061] and Wuhan Municipal Urban-Rural Development Bureau [grant number: 202346].

Appendix

Table 7
Nodes classification

Clusters	Name	Number of clusters
A	Ranjiaba	1
B	Chongqing N. Station N. Squear; Daping; Niujiaotuo; Shangwanlu; Chongqing N. Station S. Squear; Xiejiawan; Wulidian; Liyuchi; Lianglukou; Hongtudi; Sigongli; Zengjijayan; Hongqihegou; Shangxinjie; Central Park E.; Qixinggang	16
C	Sports Park; Toutang; Nanqiaosi; Yudaishan; Min'an Ave.; Guangdianyuan; Honghudonglu; Dongbu Park; Dazhulin; Kangzhuang; Minxinjiayuan; Jiuquhe; Sanyawan; Yuanjiagang; Dalongshan; Huahuiyuan; Chongqing Technology & Business University; Shiqiaopu; Terminal 2 of Jiangbei Airport; Shiyoulu; Liugongli; Xietaizi	22
D	Guanyingqiao; Xiaoshizi	2
E	Baoshuigang; Cuntan; Lijia; Heishizi; Taipingchong; Chongqing W. Station; Tangjiatuo; Tieshanping; Luqi; Guoyuan Logistics Hub; Huayansi; Yuzui; Huachenglu; Yuelai; Yanping; Banshan; Zhongliangshan; Bijin; Jinshansi	19
F	Gailanxi; Eling; Huanshan Park; Huaxinjie; Changhe; Happy Valley; Chongqing Jiaotong University; Longtousi Park; Terminal 3 of Jiangbei Airport; Shiheqing; Huangmaoping; Haitangxi; Qinggangping; Gaoyikou; Luajiaba; Fusheng; Jinjianlu; Liziba; Ping'an; Fotuguan; Jiangbeicheng; Baoshenghu; Sanbanxi; Huayan Center; Dajuyuan; Xingke Ave.; Longtousi; Dadukou; Longyi Ave.; Xingfu Squear; Tiaodeng; Caojiawan; Wangjiazhuang; Olympic Sport Center; Tongjiayuanzi; Xinshancun; Nanhu; Haixialu; Central Park; Danzishi; Hejialiang; Tongyuanju; Central Park W.; Chenjiaping; Tushan; Jinyu; Laijiaqiao; Shuangfengqiao; Longxing; Renhe; Caijia; Shilinsi; Qingxihe; Shipanhe; Tiantangbao; Renji; Gongmao; Yulu; Caiyunhu; Erlang; Hualong; Nanping; Mahuangliang; Jintonglu; Jianqiao; Weidianyuan; Konggang Squear; Gaoshita; Hemulu; Xiangjiagang; Jiulongyuan; Liujiayuanzi; Shixinlu; Lijiaping; Yuanyang; Jinjiawan; Bashan; Hualongqiao; Yubei Squear; Chenjiaqiao; Gaobaohu; Pufu; Chongguang; Liujiaping; Longfengxi; Shuangfu; Siyuan; Jiaochangkou; The EXPO Garden; Fengxilu; Fuhualu; Lushan; Liujiaba; Hongyancun; Cuiyun; Daxuecheng; Guanyuelu; Tongxinlin; Huxiajie; Changshengqiao; Zhuangyuanbei; Xiangtang; Huangnibang; Changfulu; Baijusi; Shuanglong; Huixing; Liujiatai; Jiazhoulu; Shiziping; Daxigou; Jiandingpo; Lianhua; Shichuan; Danhe; Qiujiawan; S. W. University; Jiangjin High-Speed Railway; Yudong; Zhengjiayuanzi; Tangjiayuanzi; Huanghuayuan; Linjiangmen; Chaotianmen; Bishan; Jurenba; Huangling; International EXPO Center; Dashiba; Chayuan; Beibei; Shengquansi	132

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