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Classification of possible solutions regarding business engineering problems by using complex Pythagorean fuzzy rough WASPAS approach

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Business engineering creates novel business solutions using a social and technological system. Business engineering is facing several challenges like (1) complexity management (2) rapid technological advancements (3) Resource constraints (4) Interdisciplinary collaboration etc., and there is a need to classify the solutions for these issues faced by the business community. To cover more advanced data and overcome the chance of data loss, in this article, we have developed the idea of a complex Pythagorean fuzzy rough set based on Tamir's idea of a complex fuzzy set. We have developed basic operational laws for the proposed idea under the notion of Yager's t-norm and t-conorm. Additionally, we have initiated the theory of complex Pythagorean fuzzy rough Yager weighted average and geometric aggregation operators. To discuss the utilization of the initiated work, we have introduced the WASPAS technique that can help us tackle the MADM problems. Moreover, we have provided an illustrative example for the classification of the solutions regarding the problems in business engineering. Also, a comparative analysis of the initiated theory shows the advantages of the introduced work.

Keywords Business engineering, Causes, Effects and solutions, Novel complex Pythagorean fuzzy rough set, WASPAS technique

The creation and application of business approaches, ranging from technology and data systems to business models, company procedures, and organizational frameworks, is known as business engineering. The goal of business engineering is to create novel business solutions using a social and technological systems perspective. Because an enterprise and an airline system or a manufacturing facility are similar in complexity, the social and technological system approach treats an enterprise as a whole system. Engineering, management, and business administration are all combined in the multidisciplinary discipline of business engineering. The objective is to enhance efficiency, effectiveness, and competitiveness through the design and optimization of business processes, structures, and strategies. For contemporary firms, business engineering is essential for several reasons:

- Through the use of business engineering, firms can become more adaptable and successful in dynamic situations by being more sensitive to shifts in the market, technology, and client needs.
- Business engineering helps companies minimize waste, cut expenses, and boost productivity through process optimization, which results in more effective operations.
- By using business engineering concepts to continuously innovate and improve their procedures, goods, and amenities, firms can maintain their competitiveness.

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- To ensure that assets are deployed successfully and that efforts are concentrated on reaching long-term goals, business engineering assists in aligning business procedures and frameworks with the organization's overarching strategic goals.
- Business engineering fosters and controls invention within the company, assisting in the creation of novel goods, services, and business models that promote expansion and uniqueness.
- Business engineering assists in identifying possible risks and developing ways to manage them, assuring company stability and resilience, by carefully examining business procedures and systems.

While business engineering offers numerous benefits, certain issues must be fixed if businesses are to be successful. It can be challenging to identify and create a workforce with specialized knowledge in both engineering and business, however, this is necessary for executing business engineering ideas. (2) Some firms may find it prohibitive to invest in business engineering projects due to the large initial costs associated with technology, training, and process transformation. (3) Gathering, organizing, and interpreting accurate and thorough data is essential for effective business engineering, but organizations may encounter difficulties in doing so. (4) All organizational levels must be committed to, and actively monitor, changes to make sure they are long-lasting. (5) Business engineering initiatives need to be in line with the organization's overarching strategic objectives, which can be difficult to do if they face an abundance of leadership clarity or interaction. (6) The application of business engineering techniques may be impacted by differences in corporate culture, necessitating initiatives to promote an innovative and continually enhanced culture.

Researchers are working and producing their ideas in the field of business engineering. Hengst and Vreede[1] discuss collaborative business process reengineering. The primary goal was to provide ten years of fieldwork related to collaborative business process reengineering. Following the implementation and thorough analysis of nine business process reengineering projects, 87 themes about the project's efficacy and efficiency were found. A strategy-driven approach to business reengineering is proposed by Talwar[2] where he/she has discussed the two primary methods for business re-engineering (1) Process re-engineering and (2) Business re-engineering. The business performance measuring techniques used by engineering organizations in construction were covered by Robinson et al.[3]. Legner et al.[4] talked about the opportunities and challenges facing the field of business and information systems engineering. Buchanan[5] talked about the advantages and disadvantages of business process re-engineering in a politically sensitive workplace environment in 1997. Childe et al.[6] discussed the frameworks for understanding business process re-engineering. Here, Childe et al. talked about a specific method for small- and medium-sized manufacturing enterprises to use when re-engineering their business processes. The current challenges in business process re-engineering were covered by Maull et al.[7]. In their research, Maull et al. expressed dissatisfaction with the findings of empirical research regarding problems faced by 25 businesses undergoing business process re-engineering initiatives. Furthermore, Weerakkody et al.[8] discussed the difficulties, issues, and concerns associated with using application service providers to re-engineer business processes. It is generally acknowledged that the absence of suitable alignment approaches makes maintaining business and information technology alignment a difficult undertaking. Based on this observation Ullah et al.[9] have described a requirements elicitation method that is goal-oriented for the organization. Van Meel and Sol[10] have defined business engineering as the integral design of both data systems and organizational frameworks.

Multiple criteria decision making (MCDM) is a field of decision theory that addresses decision-making when faced with several criteria. In many domains, including engineering, business, healthcare, and public policy, MADM methods are employed to assess and choose the optimal solution from a range of possibilities. Some key aspects of MADM are criteria, alternatives, weights, and decision matrix. MADM follows the following steps (1) Categorize the issues (2) Recognize the choices (3) Determine standards (4) Allocate weights (5) Apply MADM method (6) Make decision based on score values. Many MADM approaches have been made in literature based on fuzzy set theory. Kousar and Kausar[11] utilized the notion of an integrated fuzzy rough set approach and proposed MCDM for sustainable agritourism. Baranidharan et al.[12] utilized the new Fermatean fuzzy score function for assessing the impact of pre-pandemic impact. Jeon et al.[13] proposed an MCDM perspective for selecting flexible packaging bags under the notion of a hesitant fuzzy set. Kumar and Pamucar[14] proposed a comprehensive and systematic review of MCDM to solve decision-making problems.

Different generalized fuzzy structures have utilized MADM like Verma and Alvarez-Miranda[15] used the CRITIC-WASPAS technique with power aggregation operators (AOs) to implement a MAGDM methodology under the environment of two-dimensional linguistic intuitionistic fuzzy sets (IFSs). Generalized Interval neutrosophic Schweizer-Sklar prioritized AOs are the foundation of the MADM that Khan et al.[16] suggested. Extension of probability IF AOs in the fuzzy MADM approach has been developed by Sirbiladze and Sikharulidze[17]. In 1994 Yager[18] proposed Yager t-norm and t-conorm and Dhankhar and Kumar[19] MADM utilized these norms to establish q-rung orthopair fuzzy (q-ROF) Yager power weighted geometric AOs. Akram et al.[20] have utilized complex Pythagorean fuzzy (CPyF) Yager structures and proposed an MCDM model. Akram and Shahzadi[21] established the DM approach under the framework of PyF Yager weighted AOs. Based on q-ROFS, a hybrid DM approach has been produced by Shahzadi et al.[22] based on Yager AOs. Chinram et al.[23] introduced a decision support approach for spherical fuzzy (SF) Yager AOs and utilized it in a wind energy project site. Moreover, Naeem et al.[24] introduced a novel DM method under the structure of spherical hesitant fuzzy Yager AOs. Also, Ali et al.[25] proposed a MADM approach under the environment of q-ROF bipolar soft sets.

Pawlak[26] developed the rough set theory (RST), a mathematical tool for managing ambiguity in data processing. It is employed to approximate sets that are challenging to define with exact bounds. Applications for rough set theory can be found in many domains, such as information discovery, algorithms for learning, and data analysis. Some key aspects of rough set theory are (1) information system (2) Indiscernibility relation (3) Lower and upper approximation (4) Boundary region. Different work is done on the application of RS like Wang

et al.[27] provided a survey on RST and application. Pieta and Szmuc[28] gave an introduction and discussed the uses of RS in large-scale data processing. Using RS data explorer, RST was applied in data mining market evaluation by Jayasuruthi et al.[29]. The use of RST to predict the use of energy in houses experiencing heat modernization was created by Szul and Kokoszka[30]. The current hot issue is fuzzy set theory (FST) combined with RST. Recently much research all over the world has focused on the hybrid notions of FS and RS. Dubois and Prade[31] introduced a hybrid structure of fuzzy rough sets (FRSs) by combining the FS and RS. Many other structures like IF rough set (IFRS)[32], multi-granulation PyF rough set (PyFRS)[33], PyF soft rough set (PyFSRS)[34], q-ROF soft rough set (q-ROFSRS)[35], SF soft rough set (SFSRS)[36] and T-SF soft rough set (T-SFSRS)[37] have been developed which are the evidence of the hybrid structure of RS and generalized fuzzy structures. Moreover, Khan et al.[38] developed the idea of a complex fuzzy rough aggregation operator in polar form. Yi et al.[39] developed the theory of complex fuzzy rough sets in Cartesian form and proposed its application in digital marketing for business growth. Moreover, Tamir et al.[40] looked at a novel interpretation of complex membership grades. In 2016, Ali et al.[41] discussed the complex IF classes. Labassi et al.[42] developed the structure of complex PyF set (CPyFS) theory and its uses in visualization technology.

Notice that the notion of the complex fuzzy set (CFS) proposed by Tamir et al.[40] is a limited structure because there is a possibility of data loss in this arrangement. This proposed structure can never discuss the rough structure that is the upper and lower approximations. The possibility of data loss exists in this structure. Furthermore, this structure is unable to address the non-membership grade (NG). The notion of complex IFS (CIFS) developed by Ali et al.[41] can discuss the membership grade (MG) and NG in Cartesian form. But it is also limited structure because it can never discuss the roughness in its structure. The idea of CPyFS was recently developed by Labassi et al.[42] in Cartesian form. Although this structure is more advanced than that of Ali et al.[41] idea but it can still never discuss the upper and lower approximations. If we discuss the notion of FRS[31], IFRS[32], and multi-granulation PyFRS[33], we can notice that these notions can discuss the roughness in their structures. But if the decision-makers discuss their data in complex form that is if fuzzy MG and NG are in Cartesian form. The above structure lacks these conditions.

As we can see there is a research gap in the literature that can be disused which could be helpful for many decision-making situations. There is a research gap in discussing the membership and non-membership grades in one structure in a Cartesian form that can discuss the upper and lower approximation. To cover this research gap of data loss and all limitation-based that occur in prevailing theories, we have developed the idea of complex Pythagorean fuzzy rough sets (CPyFRSs) in Cartesian form. Moreover, we have discussed Yager's operational laws based on the initiated notion of CPyFRNs. After that, we discussed AOs like CPyFR Yager weighted average (CPyFRYWA) and CPyFR Yager weighted geometric (CPyFRWG) AOs in Cartesian form. To see the application perspectives of the proposed theory for the utilization of the developed theory, we have developed an algorithm for the WASPAS approach for MADM. We have developed an application for the classification of key aspects of business engineering based on the WASPAS technique. Moreover, we have a comparative analysis of the initiated theory to discuss its importance. In the end, we have a conclusion.

The arrangement of the article is given as follows: In Sect. 1 we have discussed the introduction of business engineering. Additionally, we have discussed the literature review and motivation of the initiated theory. Section 2, is devoted to discussing some useful definitions that can help in defining the notion of CPyFRSs. In Sect. 3, we have developed the basic notion of CPyFRS and Yager's operational laws based on the developed notion. Section 4 is developed to discuss the idea of CPyFRYWA and CPyFRYWG AOs. In Sect. 5, we have defined a WASPAS algorithm for the developed theory and provided an application for the classification of possible solutions for business engineering. In Sect. 6, we have compared the initiated work with some existing theories to discuss the importance and superiority of the produced work. Section 7, is about the concluding remarks.

Preliminaries

The concepts of Yager t-norm and t-conorm, RS, and CFRS in Cartesian form are to be covered in this section.

Definition 1[18] For given two real numbers, p and q, the definitions of Yager's t-norm and t-conorm are respectively as follows:

$$Y(p, q) = 1 - min\left(1, \left((1-p)^r + (1-q)^r\right)^{\frac{1}{r}}\right);$$

$$Y^*(p, q) = min\left(1, \left(p^r + q^r\right)^{\frac{1}{r}}\right)$$

Definition 2[26] Let U be a universal set and $R \subseteq U \times U$ be an arbitrary relation on U. Let R^* be a set-valued map given by $R^* : U \to P(U)$ and defined by $R^*(u) = \{v \in U : (u, v) \in R \text{ and } v \in U\}$, then the pair (U, R) is crisp approximation space. The lower and upper approximations of A w.r.t (U, R) are

$$\underline{R}(A) = ((v \in U : R^*(v) \subseteq A))$$

$$\overline{R}(A) = ((v \in U : R^*(v) \cap A \neq \varnothing))$$

The pair $\left(\underline{R}(A), \overline{R}(A)\right)$ is a RS, where $\underline{R}(A) \neq \overline{R}(A)$.

| Development	Reasons
Development of complex Pythagorean fuzzy rough structure	To cover the chance of data loss as it exists in the existing notion of complex intuitionistic fuzzy rough set
Development of Yager's operational laws for complex Pythagorean fuzzy rough numbers	These developed operational rules can help us define the aggregation theory.
Development of complex Pythagorean fuzzy rough Yager weighted average and geometric AOs	These AOs can cover more advanced data and provide more space for decision-makers.
Development of the WASPAS approach	The WASPAS can combine the weighted sum model and the weighted product model and this method can be used in MCDM approaches.
Development of comparative analysis	The comparative analysis of the initiated approach can help to discuss the advantages of the initiated work.

Table 1. Developments of this study.
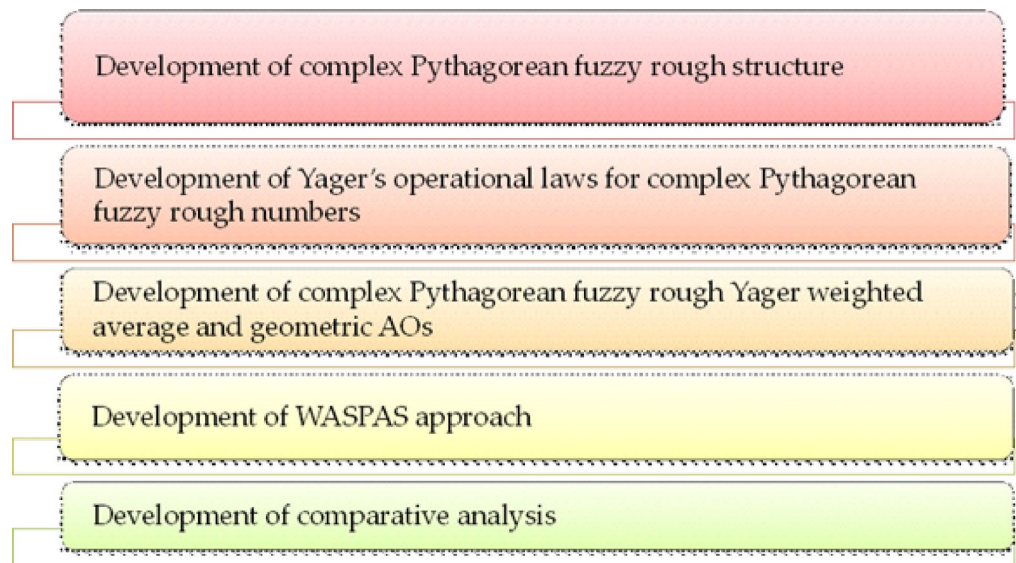
Fig. 1. Flow chart of the developed approach.

Definition 3[39] Let R be a complex fuzzy relation on $\mathbb{U}$ and $(\mathbb{U}, R)$ be a complex fuzzy approximation space. Now for any set $A \in CFS(\mathbb{U})$, $A = \{\mathcal{M}(\mathcal{O}) + \iota \mathcal{M}'(\mathcal{O})\}$. The upper and lower approximations are

$$\overline{R}(A) = \left\{ \left(\mathcal{O}, \mathrm{M}_{\overline{R}}(\mathcal{O}) \right) | \mathcal{O} \in \mathbb{U} \right\}$$

$$\underline{R}(A) = \left\{ \left(\mathcal{O}, \mathrm{M}_{\underline{R}}(\mathcal{O}) \right) | \mathcal{O} \in \mathbb{U} \right\}$$

Where

$$\mathrm{M}_{\overline{R}}(\mathcal{O}) = \vee_{\mathcal{O} \in \mathbb{U}} [a(\mathcal{O}, s) \wedge \mathcal{M}(\mathcal{O})] + \mathrm{i}\vee_{\mathcal{O} \in \mathbb{U}} [b(\mathcal{O}, s) \wedge \mathcal{M}'(\mathcal{O})]$$

$$\mathrm{M}_{\underline{R}}(\mathcal{O}) = \wedge_{\mathcal{O} \in \mathbb{U}} [1 - a(\mathcal{O}, s) \vee \mathcal{M}(\mathcal{O})] + \mathrm{i}\wedge_{\mathcal{O} \in \mathbb{U}} [1 - b(\mathcal{O}, s) \vee \mathcal{M}'(\mathcal{O})]$$

The pair $\left(\overline{R}(A), \underline{R}(A) \right)$ is called CFRS if $\overline{R}(A) \neq \underline{R}(A)$.

The main development of this article is given in (Table 1).

Moreover, the flow chart of the developed approach is given in (Fig. 1).

Complex Pythagorean fuzzy rough set (CPyFRS)
In this section, we initiate the notion of CPyFRS, and for this first, we define the CPyF relation. We denote the CPyF relation by "$\mathcal{R}$" and the universal set by $\mathbb{U}$.

Definition 4 Let $\mathbb{U}$ be the universal set, then any CPyF subset $\mathcal{R}$ is called CPyF relation on $\mathbb{U}$ that is given as $\mathcal{R} = \{(\mathfrak{r}, y), \mathcal{M}(\mathfrak{r}, y), \mathcal{N}(\mathfrak{r}, y) | \mathcal{M}(\mathfrak{r}, y) = a(\mathfrak{r}, y) + \iota b(\mathfrak{r}, y), \mathcal{N}(\mathfrak{r}, y) = c(\mathfrak{r}, y) + \iota d(\mathfrak{r}, y)\}$, where $\mathcal{M}(\mathfrak{r}, y)$ and $\mathcal{N}(\mathfrak{r}, y)$ are called MG and NG respectively and $\mathrm{M}(\mathfrak{r}, y) : \mathbb{U} \times \mathbb{U} \to [0, 1] + \iota[0, 1]$ and $\mathcal{N}(\mathfrak{r}, y) : \mathbb{U} \times \mathbb{U} \to [0, 1] + \iota[0, 1]$ satisfying the following conditions:

$$0 \leq (a(\mathfrak{x},\, y))^2 + (c(\mathfrak{x},\, y))^2 \leq 1$$

$$0 \leq (b(\mathfrak{x},\, y))^2 + (d(\mathfrak{x},\, y))^2 \leq 1$$

Example 1 Let $\mathbb{U} = \{\mathfrak{x}_1,\, \mathfrak{x}_2\}$ then CPyF relation on $\mathbb{U}$ is given by

$$\mathcal{R} = \left\{ \begin{array}{l} ((\mathfrak{x}_1,\, \mathfrak{x}_1),\, (0.24 + \iota\, 0.21,\, 0.31 + \iota\, 0.15)),\; ((\mathfrak{x}_1,\, \mathfrak{x}_2),\, (0.33 + \iota\, 0.43,\, 0.27 + \iota\, 0.19)), \\ ((\mathfrak{x}_2,\, \mathfrak{x}_1),\, (0.15 + \iota\, 0.12,\, 0.13 + \iota\, 0.10)),\; ((\mathfrak{x}_2,\, \mathfrak{x}_2),\, (0.25 + \iota\, 0.32,\, 0.31 + \iota\, 0.41)) \end{array} \right\}$$

Definition 5 Let $\mathbb{U} \neq \varnothing$ finite universe of discourse and $\mathcal{R}$ be a CPyF relation on $\mathbb{U}$, then $(\mathbb{U},\, \mathcal{R})$ define the CPyF approximation space. Let $A = \{(\mathfrak{x},\, \mathcal{M}_A(\mathfrak{x}) + \iota\, \mathcal{M}'_A(\mathfrak{x}),\; \mathcal{N}_A(\mathfrak{x}) + \iota\, \mathcal{N}'_A(\mathfrak{x})) | \mathfrak{x} \in \mathbb{U}\}$ be a CPyFS over $\mathbb{U}$, where $\mathcal{M}_A(\mathfrak{x})$ and $\mathcal{N}_A(\mathfrak{x})$ are real parts of MG and NG, $\mathcal{M}'_A(\mathfrak{x})$ and $\mathcal{N}'_A(\mathfrak{x})$ are imaginary parts of MG and NG. Now upper and lower approximation of a set A is defined and denoted as:

$$\overline{\mathcal{R}}(A) = \left\{ \left(\mathfrak{x},\; M_{\overline{\mathcal{R}}}(\mathfrak{x}),\, N_{\overline{\mathcal{R}}}(\mathfrak{x})\right) | \mathfrak{x} \in \mathbb{U} \right\}$$

$$\underline{R}(A) = \left\{ \left(\mathfrak{x},\; M_{\underline{R}}(\mathfrak{x}),\, N_{\underline{R}}(\mathfrak{x})\right) | \mathfrak{x} \in \mathbb{U} \right\}$$

Where,

$$M_{\overline{\mathcal{R}}}(\mathfrak{x}) = \vee_{y \in \mathbb{U}}\, [a(\mathfrak{x},\, y) \vee \mathcal{M}_A(y)] + \iota\, \vee_{y \in \mathbb{U}}\, [b(\mathfrak{x},\, y) \vee \mathcal{M}\prime_A(y)] = \alpha_{\overline{\mathcal{R}}} + \iota\, \beta_{\overline{\mathcal{R}}}$$

$$N_{\overline{\mathcal{R}}}(\mathfrak{x}) = \wedge_{y \in \mathbb{U}}\, [c(\mathfrak{x},\, y) \wedge \mathcal{N}_A(y)] + \iota\, \wedge_{y \in \mathbb{U}}\, [d(\mathfrak{x},\, y) \wedge \mathcal{N}\prime_A(y)] = \delta_{\overline{\mathcal{R}}} + \iota\, \gamma_{\overline{\mathcal{R}},}$$

With the conditions of $0 \leq \left(\alpha_{\overline{\mathcal{R}}}\right)^2 + \left(\delta_{\overline{\mathcal{R}}}\right)^2 \leq 1$ and $0 \leq \left(\beta_{\overline{\mathcal{R}}}\right)^2 + \left(\gamma_{\overline{\mathcal{R}}}\right)^2 \leq 1$.

$$M_{\underline{R}}(\mathfrak{x}) = \wedge_{y \in \mathbb{U}}\, [a(\mathfrak{x},\, y) \wedge \mathcal{M}_A(y)] + \iota\, \wedge_{y \in \mathbb{U}}\, [b(\mathfrak{x},\, y) \wedge \mathcal{M}\prime_A(y)] = \alpha_{\underline{R}} + \iota\, \beta_{\underline{R}}$$

$$N_{\underline{R}}(\mathfrak{x}) = \vee_{y \in \mathbb{U}}\, [c(\mathfrak{x},\, y) \vee \mathcal{N}_A(y)] + \iota\, \vee_{y \in \mathbb{U}}\, [d(\mathfrak{x},\, y) \vee \mathcal{N}\prime_A(y)] = \delta_{\underline{R}} + \iota\, \gamma_{\underline{R}}$$

With the conditions of $0 \leq \left(\alpha_{\overline{\mathcal{R}}}\right)^2 + \left(\delta_{\overline{\mathcal{R}}}\right)^2 \leq 1$ and $0 \leq \left(\beta_{\underline{R}}\right)^2 + \left(\gamma_{\underline{R}}\right)^2 \leq 1$.

The pair $\mathcal{R}(A) = \left(\overline{\mathcal{R}}(A),\, \underline{R}(A)\right) = \left\{ (\mathfrak{x}, < M_{\overline{\mathcal{R}}}(\mathfrak{x}),\, N_{\overline{\mathcal{R}}}(\mathfrak{x}) >, < M_{\underline{R}}(\mathfrak{x}),\, N_{\underline{R}}(\mathfrak{x}) >) | \mathfrak{x} \in \mathbb{U} \right\}$ is called CPyFRS with respect $(\mathbb{U},\, \mathcal{R})$. For the sake of simplicity, we can say $\mathcal{R}(A) = \left(\overline{\mathcal{R}}(A),\, \underline{R}(A)\right) = \left(\left(\alpha_{\overline{\mathcal{R}}} + \iota\, \beta_{\overline{\mathcal{R}}},\, \delta_{\overline{\mathcal{R}}} + \iota\, \gamma_{\overline{\mathcal{R}}}\right),\, \left(\alpha_{\underline{R}} + \iota\, \beta_{\underline{R}},\, \delta_{\underline{R}} + \iota\, \gamma_{\underline{R}}\right) \right)$ represent the CPyFRN in Cartesian form.

Example 2 Let $\mathbb{U} = \{\mathfrak{x}_1,\, \mathfrak{x}_2\}$ be the universal set and $\mathcal{R} = \left\{ \begin{array}{l} ((\mathfrak{x}_1,\, \mathfrak{x}_1),\, (0.24 + \iota\, 0.21,\, 0.31 + \iota\, 0.15)),\; ((\mathfrak{x}_1,\, \mathfrak{x}_2),\, (0.33 + \iota\, 0.43,\, 0.27 + \iota\, 0.19)), \\ ((\mathfrak{x}_2,\, \mathfrak{x}_1),\, (0.15 + \iota\, 0.12,\, 0.13 + \iota\, 0.10)),\; ((\mathfrak{x}_2,\, \mathfrak{x}_2),\, (0.25 + \iota\, 0.32,\, 0.31 + \iota\, 0.41)) \end{array} \right\}$ be the CPyFR on $\mathbb{U}$ as given in example 1. Now assume that $A = \{(\mathfrak{x}_1,\, 0.23 + \iota\, 0.35,\, 0.45 + \iota\, 0.39),\, (\mathfrak{x}_2,\, 0.21 + \iota\, 0.17,\, 0.16 + \iota\, 0.13)\}$ be CPyFS. Now the upper and lower approximations are given by

$$M_{\overline{\mathcal{R}}}(\mathfrak{x}_1) = 0.33 + \iota\, 0.43,\; N_{\overline{\mathcal{R}}}(\mathfrak{x}_1) = 0.16 + \iota\, 0.13$$

$$M_{\overline{\mathcal{R}}}(\mathfrak{x}_2) = 0.25 + \iota\, 0.35,\; N_{\overline{\mathcal{R}}}(\mathfrak{x}_2) = 0.13 + \iota\, 0.1$$

And

$$M_{\underline{R}}(\mathfrak{x}_1) = 0.21 + \iota\, 0.17,\; N_{\underline{R}}(\mathfrak{x}_1) = 0.45 + \iota\, 0.39$$

$$M_{\underline{R}}(\mathfrak{x}_2) = 0.15 + \iota\, 0.12,\; N_{\underline{R}}(\mathfrak{x}_2) = 0.45 + \iota\, 0.41$$

Hence.

$$\mathcal{R}(A) = \left(\overline{\mathcal{R}}(A),\, \underline{R}(A)\right) = \left\{ \begin{array}{l} \left(\begin{array}{l} \mathfrak{x}_1,\, (0.33 + \iota\, 0.43,\, 0.16 + \iota\, 0.13), \\ (0.21 + \iota\, 0.17,\, 0.45 + \iota\, 0.39) \end{array} \right), \\ \left(\begin{array}{l} \mathfrak{x}_2,\, (0.25 + \iota\, 0.35,\, 0.13 + \iota\, 0.1), \\ (0.15 + \iota\, 0.12,\, 0.45 + \iota\, 0.41) \end{array} \right) \end{array} \right\} \text{ is a CPyFRS.}$$

Definition 6 For the classification of CPyFRNs, there is a need to define the concept of score function and accuracy function. The defied notions are given by

For CPyFRN $CPyFR_1 = \left\{ \left(\left(\alpha_{1\overline{\mathcal{R}}} + \iota\, \beta_{1\overline{\mathcal{R}}},\, \delta_{1\overline{\mathcal{R}}} + \iota\, \gamma_{1\overline{\mathcal{R}}}\right),\, \left(\alpha_{1\underline{R}} + \iota\, \beta_{1\underline{R}},\, \delta_{1\underline{R}} + \iota\, \gamma_{1\underline{R}}\right) \right) \right\}$, the score function is defined by

$$S\left(CPyFR_1\right) = \frac{1}{8}\left(4 + \alpha_{1\overline{\mathcal{R}}} + \alpha_{1\underline{R}} + \beta_{1\overline{\mathcal{R}}} + \beta_{1\underline{R}} - \delta_{1\overline{\mathcal{R}}} - \delta_{1\underline{R}} - \gamma_{1\overline{\mathcal{R}}} - \gamma_{1\underline{R}}\right) \; and \; S\left(CPyFR_1\right) \in [0,\,1]$$

$$Acr.\left(CPyFR_1\right) = \frac{1}{8}\left(4 + \alpha_{1\overline{\mathcal{R}}} + \alpha_{1\underline{R}} + \beta_{1\overline{\mathcal{R}}} + \beta_{1\underline{R}} + \delta_{1\overline{\mathcal{R}}} + \delta_{1\underline{R}} + \gamma_{1\overline{\mathcal{R}}} + \gamma_{1\underline{R}}\right) \; and \; Acr.\left(CPyFR_1\right) \in [0,\,1]$$

Example 3 For any CPyFRNs $CPyFR_1 = \left\{\left(\begin{array}{c}(0.19 + \iota\,0.34,\; 0.13 + \iota\,0.11),\\ (0.12 + \iota\,0.22, 0.34 + \iota\,0.17)\end{array}\right)\right\}$, the score value and accuracy values are calculated by

$$S\left(CPyFR_1\right) = \frac{1}{8}\left(4 + 0.19 + 0.12 + 0.34 + 0.22 - 0.13 - 0.34 - 0.11 - 0.17\right) = 0.515$$

$$Acr.\left(CPyFR_1\right) = \frac{1}{8}\left(4 + 0.19 + 0.12 + 0.34 + 0.22 + 0.13 + 0.34 + 0.11 + 0.17\right) = 0.7025$$

Aggregation theory based on CPyFRSs

Definition 7 Le $CPyFR_1 = \left\{\left(\left(\alpha_{1\overline{\mathcal{R}}} + \iota\,\beta_{1\overline{\mathcal{R}}},\; \delta_{1\overline{\mathcal{R}}} + \iota\,\gamma_{1\overline{\mathcal{R}}}\right),\left(\alpha_{1\underline{R}} + \iota\,\beta_{1\underline{R}}, \delta_{1\underline{R}} + \iota\,\gamma_{1\underline{R}}\right)\right)\right\}$ and $CPyFR_2 = \left\{\left(\left(\alpha_{2\overline{\mathcal{R}}} + \iota\,\beta_{2\overline{\mathcal{R}}},\; \delta_{2\overline{\mathcal{R}}} + \iota\,\gamma_{2\overline{\mathcal{R}}}\right),\left(\alpha_{2\underline{R}} + \iota\,\beta_{2\underline{R}}, \delta_{2\underline{R}} + \iota\,\gamma_{2\underline{R}}\right)\right)\right\}$ be two CPyFRNs and $\mho > 0$ and $q > 0$ The Yager t-norm and t-conorm operations are as follows

1)

$$CPyFR_1 \oplus CPyFR_2 = \left\{\left(\begin{array}{c} \left(\sqrt{\min\left(1,\left(\left(\alpha_{1\overline{\mathcal{R}}}\right)^{2\mho} + \left(\alpha_{2\overline{\mathcal{R}}}\right)^{2\mho}\right)^{\frac{1}{\mho}}\right)} + \iota\sqrt{\min\left(1,\left(\left(\beta_{1\overline{\mathcal{R}}}\right)^{2\mho} + \left(\beta_{2\overline{\mathcal{R}}}\right)^{2\mho}\right)^{\frac{1}{\mho}}\right)},\right. \\ \left.\sqrt{1 - \min\left(1,\left(\left(1-\left(\delta_{1\overline{\mathcal{R}}}\right)^2\right)^{\mho} + \left(1-\left(\delta_{2\overline{\mathcal{R}}}\right)^2\right)^{\mho}\right)^{\frac{1}{\mho}}\right)} + \iota\sqrt{1 - \min\left(1,\left(\left(1-\left(\gamma_{1\overline{\mathcal{R}}}\right)^2\right)^{\mho} + \left(1-\left(\gamma_{2\overline{\mathcal{R}}}\right)^2\right)^{\mho}\right)^{\frac{1}{\mho}}\right)}\right), \\[2em] \left(\sqrt{\min\left(1,\left(\left(\alpha_{1\underline{R}}\right)^{2\mho} + \left(\alpha_{2\underline{R}}\right)^{2\mho}\right)^{\frac{1}{\mho}}\right)} + \iota\sqrt{\min\left(1,\left(\left(\beta_{1\underline{R}}\right)^{2\mho} + \left(\beta_{2\underline{R}}\right)^{2\mho}\right)^{\frac{1}{\mho}}\right)},\right. \\ \left.\sqrt{1 - \min\left(1,\left(\left(1-\left(\delta_{1\underline{R}}\right)^2\right)^{\mho} + \left(1-\left(\delta_{2\underline{R}}\right)^2\right)^{\mho}\right)^{\frac{1}{\mho}}\right)} + \iota\sqrt{1 - \min\left(1,\left(\left(1-\left(\gamma_{1\underline{R}}\right)^2\right)^{\mho} + \left(1-\left(\gamma_{2\underline{R}}\right)^2\right)^{\mho}\right)^{\frac{1}{\mho}}\right)}\right) \end{array}\right)\right\}$$

2)

$$CPyFR_1 \otimes CPyFR_2 = \left\{\left(\begin{array}{c} \left(\sqrt{1 - \min\left(1,\left(\left(1-\left(\alpha_{1\overline{\mathcal{R}}}\right)^2\right)^{\mho} + \left(1-\left(\alpha_{2\overline{\mathcal{R}}}\right)^2\right)^{\mho}\right)^{\frac{1}{\mho}}\right)} + \iota\sqrt{1 - \min\left(1,\left(\left(1-\left(\beta_{1\overline{\mathcal{R}}}\right)^2\right)^{\mho} + \left(1-\left(\beta_{2\overline{\mathcal{R}}}\right)^2\right)^{\mho}\right)^{\frac{1}{\mho}}\right)},\right. \\ \left.\sqrt{\min\left(1,\left(\left(\delta_{1\overline{\mathcal{R}}}\right)^{2\mho} + \left(\delta_{2\overline{\mathcal{R}}}\right)^{2\mho}\right)^{\frac{1}{\mho}}\right)} + \iota\sqrt{\min\left(1,\left(\left(\gamma_{1\overline{\mathcal{R}}}\right)^{2\mho} + \left(\gamma_{2\overline{\mathcal{R}}}\right)^{2\mho}\right)^{\frac{1}{\mho}}\right)}\right), \\[2em] \left(\sqrt{1 - \min\left(1,\left(\left(1-\left(\alpha_{1\underline{R}}\right)^2\right)^{\mho} + \left(1-\left(\alpha_{2\underline{R}}\right)^2\right)^{\mho}\right)^{\frac{1}{\mho}}\right)} + \iota\sqrt{1 - \min\left(1,\left(\left(1-\left(\beta_{1\underline{R}}\right)^2\right)^{\mho} + \left(1-\left(\beta_{2\underline{R}}\right)^2\right)^{\mho}\right)^{\frac{1}{\mho}}\right)},\right. \\ \left.\sqrt{\min\left(1,\left(\left(\delta_{1\underline{R}}\right)^{2\mho} + \left(\delta_{2\underline{R}}\right)^{2\mho}\right)^{\frac{1}{\mho}}\right)} + \iota\sqrt{\min\left(1,\left(\left(\gamma_{1\underline{R}}\right)^{2\mho} + \left(\gamma_{2\underline{R}}\right)^{2\mho}\right)^{\frac{1}{\mho}}\right)}\right) \end{array}\right)\right\}$$

3)

$$\mathscr{g}CPyFR_1 = \left\{ \left(\begin{array}{c} \left(\begin{array}{c} \sqrt{\min\left(1, \left((\mathscr{g}\alpha_{1\overline{\mathcal{R}}})^{2\mathcal{O}}\right)^{\frac{1}{\mathcal{O}}}\right)} + \iota \sqrt{\min\left(1, \left((\mathscr{g}\beta_{1\overline{\mathcal{R}}})^{2\mathcal{O}}\right)^{\frac{1}{\mathcal{O}}}\right)}, \\ \sqrt{1 - \min\left(1, \left(\mathscr{g}\left(1 - (\delta_{1\overline{\mathcal{R}}})^2\right)^{\mathcal{O}}\right)^{\frac{1}{\mathcal{O}}}\right)} + \\ \iota \sqrt{1 - \min\left(1, \left(\mathscr{g}\left(1 - (\gamma_{1\overline{\mathcal{R}}})^2\right)^{\mathcal{O}}\right)^{\frac{1}{\mathcal{O}}}\right)} \end{array} \right), \\ \left(\begin{array}{c} \sqrt{\min\left(1, \left((\mathscr{g}\alpha_{1\underline{R}})^{2\mathcal{O}}\right)^{\frac{1}{\mathcal{O}}}\right)} + \iota \sqrt{\min\left(1, \left((\mathscr{g}\beta_{1\underline{R}})^{2\mathcal{O}}\right)^{\frac{1}{\mathcal{O}}}\right)}, \\ \sqrt{1 - \min\left(1, \left(\mathscr{g}\left(1 - (\delta_{1\underline{R}})^2\right)^{\mathcal{O}}\right)^{\frac{1}{\mathcal{O}}}\right)} + \\ \iota \sqrt{1 - \min\left(1, \left(\mathscr{g}\left(1 - (\gamma_{1\underline{R}})^2\right)^{\mathcal{O}}\right)^{\frac{1}{\mathcal{O}}}\right)} \end{array} \right) \end{array} \right) \right\},$$

4)

$$(CPyFR_1)^{\mathscr{g}} = \left\{ \begin{array}{c} \left(\begin{array}{c} \sqrt{1 - \min\left(1, \left(\mathscr{g}\left(1 - (\alpha_{1\overline{\mathcal{R}}})^2\right)^{\mathcal{O}}\right)^{\frac{1}{\mathcal{O}}}\right)} + \\ \iota \sqrt{1 - \min\left(1, \left(\mathscr{g}\left(1 - (\beta_{1\overline{\mathcal{R}}})^2\right)^{\mathcal{O}}\right)^{\frac{1}{\mathcal{O}}}\right)}, \\ \sqrt{\min\left(1, \left((\mathscr{g}\delta_{1\overline{\mathcal{R}}})^{2\mathcal{O}}\right)^{\frac{1}{\mathcal{O}}}\right)} + \iota \sqrt{\min\left(1, \left((\mathscr{g}\gamma_{1\overline{\mathcal{R}}})^{2\mathcal{O}}\right)^{\frac{1}{\mathcal{O}}}\right)}, \end{array} \right) \\ \left(\begin{array}{c} \sqrt{1 - \min\left(1, \left(\mathscr{g}\left(1 - (\alpha_{1\underline{R}})^2\right)^{\mathcal{O}}\right)^{\frac{1}{\mathcal{O}}}\right)} + \\ \iota \sqrt{1 - \min\left(1, \left(\mathscr{g}\left(1 - (\beta_{1\underline{R}})^2\right)^{\mathcal{O}}\right)^{\frac{1}{\mathcal{O}}}\right)}, \\ \sqrt{\min\left(1, \left((\mathscr{g}\delta_{1\underline{R}})^{2\mathcal{O}}\right)^{\frac{1}{\mathcal{O}}}\right)} + \iota \sqrt{\min\left(1, \left((\mathscr{g}\gamma_{1\underline{R}})^{2\mathcal{O}}\right)^{\frac{1}{\mathcal{O}}}\right)} \end{array} \right) \end{array} \right\}.$$

Theorem 1 *Let* $CPyFR_1 = \left\{ \left(\left(\alpha_{1\overline{\mathcal{R}}} + \iota\beta_{1\overline{\mathcal{R}}}, \delta_{1\overline{\mathcal{R}}} + \iota\gamma_{1\overline{\mathcal{R}}} \right), \left(\alpha_{1\underline{R}} + \iota\beta_{1\underline{R}}, \delta_{1\underline{R}} + \iota\gamma_{1\underline{R}} \right) \right) \right\}$ *and* $CPyFR_2 = \left\{ \left(\left(\alpha_{2\overline{\mathcal{R}}} + \iota\beta_{2\overline{\mathcal{R}}}, \delta_{2\overline{\mathcal{R}}} + \iota\gamma_{2\overline{\mathcal{R}}} \right), \left(\alpha_{2\underline{R}} + \iota\beta_{2\underline{R}}, \delta_{2\underline{R}} + \iota\gamma_{2\underline{R}} \right) \right) \right\}$ *be two CPy-FRNs, then the following properties hold in this case*

1)

$$CPyFR_1 \oplus CPyFR_2 = CPyFR_2 \oplus CPyFR_1$$

2)

$$CPyFR_1 \otimes CPyFR_2 = CPyFR_2 \otimes CPyFR_1$$

3)

$$\mathscr{g}(CPyFR_1 \oplus CPyFR_2) = \mathscr{g}(CPyFR_1) \oplus \mathscr{g}(CPyFR_2)$$

4)

$$\left(\mathscr{I}_1 + \mathscr{I}_2\right) CPyFR_1 = \mathscr{I}_1 CPyFR_1 \oplus \mathscr{I}_2 CPyFR_1$$

5)

$$(CPyFR_1 \otimes CPyFR_2)^{\mathscr{I}} = CPyFR_1{}^{\mathscr{I}} \otimes CPyFR_2{}^{\mathscr{I}} \ for \ \mathscr{I} > 0$$

6)

$$CPyFR_1{}^{\mathscr{I}_1} \otimes CPyFR_1{}^{\mathscr{I}_2} = CPyFR_1{}^{\left(\mathscr{I}_1 + \mathscr{I}_2\right)} \ for \ \mathscr{I}_1, \ \mathscr{I}_2 > 0$$

Now utilizing the operational laws given in def. 7, we can define AOs under the environment of CPyFRNs as follows.

Definition 8 Let $CPyFR = \left\{ \left(\left(\alpha_{i_{\overline{\mathcal{R}}}} + \iota\beta_{i_{\overline{\mathcal{R}}}}, \ \delta_{i_{\overline{\mathcal{R}}}} + \iota\gamma_{i_{\overline{\mathcal{R}}}} \right), \left(\alpha_{i_{\underline{R}}} + \iota\beta_{i_{\underline{R}}}, \delta_{i_{\underline{R}}} + \iota\gamma_{i_{\underline{R}}} \right) \right) \right\}$ $(i = 1, \ 2, \ \ldots, \ m)$ be the collection of CPyFRNs. Then the notion of the CPyFRYWA operator is defined by a mapping $F : A^m \to A$ such that

$$CPyFRYWA_\beth \left(CPyFR_1, \ CPyFR_2, \ CPyFR_3, \ldots, \ CPyFR_m \right) = \oplus_{i=1}^{m} \left(\beth_i CPyFR_i \right)$$

Where $\beth = (\beth_1, \beth_2, \beth_3, \ldots, \beth_m)^{\mathrm{T}}$ is the weight vector (WV) of $CPyFRNs$ and $\sum_{i=1}^{m} \beth_i = 1$.

Theorem 2 Let $CPyFR = \left\{ \left(\left(\alpha_{i_{\overline{\mathcal{R}}}} + \iota\beta_{i_{\overline{\mathcal{R}}}}, \ \delta_{i_{\overline{\mathcal{R}}}} + \iota\gamma_{i_{\overline{\mathcal{R}}}} \right), \left(\alpha_{i_{\underline{R}}} + \iota\beta_{i_{\underline{R}}}, \delta_{i_{\underline{R}}} + \iota\gamma_{i_{\underline{R}}} \right) \right) \right\}$ $(i = 1, \ 2, \ \ldots, \ m)$ be the collection of CPyFRNs. The outcome obtained through the CPyFRYWA operator is again CPyFRN.

$$CPyFRYWA_\beth \left(CPyFR_1, \ CPyFR_2, \ CPyFR_3, \ldots, \ CPyFR_m \right) = \oplus_{i=1}^{m} \left(\beth_i CPyFR_i \right)$$

$$= \left\{ \begin{pmatrix} \left(\begin{matrix} \sqrt{\min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(\alpha_{i\overline{\mathcal{R}}}\right)^{2\mathcal{O}}\right)\right)^{\frac{1}{\mathcal{O}}}\right)} + \iota\sqrt{\min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(\beta_{i\overline{\mathcal{R}}}\right)^{2\mathcal{O}}\right)\right)^{\frac{1}{\mathcal{O}}}\right)}, \\ \sqrt{1 - \min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(1 - \left(\delta_{i\overline{\mathcal{R}}}\right)^2\right)^{\mathcal{O}}\right)\right)^{\frac{1}{\mathcal{O}}}\right)} + \iota\sqrt{1 - \min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(1 - \left(\gamma_{i\overline{\mathcal{R}}}\right)^2\right)^{\mathcal{O}}\right)\right)^{\frac{1}{\mathcal{O}}}\right)} \end{matrix} \right), \\ \left(\begin{matrix} \sqrt{\min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i(\alpha_{i\underline{R}})^{\mathcal{O}}\right)\right)^{\frac{1}{\mathcal{O}}}\right)} + \iota\sqrt{\min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(\beta_{i\underline{R}}\right)^{\mathcal{O}}\right)\right)^{\frac{1}{\mathcal{O}}}\right)} \\ \sqrt{1 - \min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(1 - \delta_{i\underline{R}}{}^{\mathcal{O}}\right)\right)\right)^{\frac{1}{\mathcal{O}}}\right)} + \iota\sqrt{1 - \min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(1 - \gamma_{i\underline{R}}{}^{\mathcal{O}}\right)\right)\right)^{\frac{1}{\mathcal{O}}}\right)} \end{matrix} \right) \end{pmatrix} \right\} \quad (1)$$

Proof Proof of Eq. (1) followed by the use of a mathematical induction technique.

Definition 9 Let $CPyFR = \left\{ \left(\left(\alpha_{i_{\overline{\mathcal{R}}}} + \iota\beta_{i_{\overline{\mathcal{R}}}}, \ \delta_{i_{\overline{\mathcal{R}}}} + \iota\gamma_{i_{\overline{\mathcal{R}}}} \right), \left(\alpha_{i_{\underline{R}}} + \iota\beta_{i_{\underline{R}}}, \delta_{i_{\underline{R}}} + \iota\gamma_{i_{\underline{R}}} \right) \right) \right\}$ $(i = 1, \ 2, \ \ldots, \ m)$ be the collection of CPyFRNs. Then the notion of CPyFRYWG AO is defined by

$$CPyFRYWG \left(CPyFR_1, \ CPyFR_2, \ CPyFR_3, \ldots, \ CPyFR_m \right) = \otimes_{i=1}^{m} \left(CPyFR_i{}^{\beth_i} \right)$$

Where $\beth = (\beth_1, \beth_2, \beth_3, \ldots, \beth_m)^{\mathrm{T}}$ is the WVs of $CPyFR_i$ with condition that $\sum_{i=1}^{m} \beth_i = 1$ and $\beth_i > 0$.

Theorem 3 Let $CPyFR = \left\{ \left(\left(\alpha_{i_{\overline{\mathcal{R}}}} + \iota\beta_{i_{\overline{\mathcal{R}}}}, \ \delta_{i_{\overline{\mathcal{R}}}} + \iota\gamma_{i_{\overline{\mathcal{R}}}} \right), \left(\alpha_{i_{\underline{R}}} + \iota\beta_{i_{\underline{R}}}, \delta_{i_{\underline{R}}} + \iota\gamma_{i_{\underline{R}}} \right) \right) \right\}$ $(i = 1, \ 2, \ \ldots, \ m)$ be the collection of CPyFRNs. The outcome obtained through the CPyFRYWG operator is again CPyFRN.

$$CPyFRYWG_\beth \left(CPyFR_1, \ CPyFR_2, \ CPyFR_3, \ldots, \ CPyFR_m \right) = \otimes_{i=1}^{m} \left(CPyFR_i{}^{\beth_i} \right)$$

$$= \left\{ \begin{matrix} \sqrt{1 - \min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(1 - \left(\alpha_{i\overline{\mathcal{R}}}\right)^2\right)^{\mathcal{O}}\right)\right)^{\frac{1}{\mathcal{O}}}\right)} + \iota\sqrt{1 - \min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(1 - \left(\beta_{i\overline{\mathcal{R}}}\right)^2\right)^{\mathcal{O}}\right)\right)^{\frac{1}{\mathcal{O}}}\right)}, \\ \sqrt{\min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(\delta_{i\overline{\mathcal{R}}}\right)^{2\mathcal{O}}\right)\right)^{\frac{1}{\mathcal{O}}}\right)} + \iota\sqrt{\min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(\gamma_{i\overline{\mathcal{R}}}\right)^{2\mathcal{O}}\right)\right)^{\frac{1}{\mathcal{O}}}\right)}, \\ \sqrt{1 - \min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(1 - \left(\alpha_{i\underline{R}}\right)^2\right)^{\mathcal{O}}\right)\right)^{\frac{1}{\mathcal{O}}}\right)} + \iota\sqrt{1 - \min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(1 - \left(\beta_{i\underline{R}}\right)^2\right)^{\mathcal{O}}\right)\right)^{\frac{1}{\mathcal{O}}}\right)}, \\ \sqrt{\min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(\delta_{i\underline{R}}\right)^{2\mathcal{O}}\right)\right)^{\frac{1}{\mathcal{O}}}\right)} + \iota\sqrt{\min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(\gamma_{i\underline{R}}\right)^{2\mathcal{O}}\right)\right)^{\frac{1}{\mathcal{O}}}\right)} \end{matrix} \right\} \quad (2)$$

Proof Proof of Eq. (2) followed by the use of a mathematical induction technique.

Methods
The Weighted Aggregated Sum Product Assessment (WASPAS) method is a multi-criteria decision-making methodology that combines the advantages of the weighted product model and the weighted sum model.

Algorithm 1
To handle the multicriteria decision-making techniques, many decision-making tools have been developed. Here in this section, we have defined the WASPAS technique. Assume that $A_L = \{A_{L1}, A_{L2}, A_{L3}, \ldots, A_{Lm}\}$ is the collection of m alternatives and $A_T = \{A_{T1}, A_{T2}, A_{T3}, \ldots, A_{Tn}\}$ represents n attributes. Let $\{e_1, e_2, e_3, \ldots, e_\hbar\}$ represents "$\hbar$" decision analyst for each alternative $A_{Li} (i = 1, 2, 3, \ldots, m)$ against attributes $A_{Tj} (j = 1, 2, 3, ., n)$. Let $\beth = (\beth_1, \beth_2, \beth_3, \ldots, \beth_n)^{\mathrm{T}}$ be the WVs for attributes A_{Tj} and $\varpi = (\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_\hbar)^{\mathrm{T}}$ be the WVs for experts $e_z (z = 1, 2, 3, \ldots, \hbar)$ such that $\sum_{j=1}^{n} \beth_j = 1$ and $\sum_{z=1}^{\hbar} \varpi_z = 1$. The algorithm of the WASPAS method is given by

Step 1 Assume decision analysts deliver their evaluation as CPyFRNs corresponding to each attribute. The decision matrix is given by.

$$\mathcal{M}_{m \times n} = \begin{bmatrix} \left\{ \begin{pmatrix} \begin{pmatrix} \alpha_{11_{\overline{\mathcal{R}}}} + \beta_{11_{\overline{\mathcal{R}}}}, \\ \delta_{11_{\overline{\mathcal{R}}}} + \iota\gamma_{11_{\overline{\mathcal{R}}}} \end{pmatrix}, \\ \begin{pmatrix} \alpha_{11_{\underline{R}}} + \iota\beta_{11_{\underline{R}}}, \\ \delta_{11_{\underline{R}}} + \iota\gamma_{11_{\underline{R}}} \end{pmatrix} \end{pmatrix} \right\} & \left\{ \begin{pmatrix} \begin{pmatrix} \alpha_{12_{\overline{\mathcal{R}}}} + \beta_{12_{\overline{\mathcal{R}}}}, \\ \delta_{12_{\overline{\mathcal{R}}}} + \iota\gamma_{12_{\overline{\mathcal{R}}}} \end{pmatrix}, \\ \begin{pmatrix} \alpha_{12_{\underline{R}}} + \iota\beta_{12_{\underline{R}}}, \\ \delta_{12_{\underline{R}}} + \iota\gamma_{12_{\underline{R}}} \end{pmatrix} \end{pmatrix} \right\} & \cdots & \left\{ \begin{pmatrix} \begin{pmatrix} \alpha_{1n_{\overline{\mathcal{R}}}} + \beta_{1n_{\overline{\mathcal{R}}}}, \\ \delta_{1n_{\overline{\mathcal{R}}}} + \iota\gamma_{1n_{\overline{\mathcal{R}}}} \end{pmatrix}, \\ \begin{pmatrix} \alpha_{1n_{\underline{R}}} + \iota\beta_{1n_{\underline{R}}}, \\ \delta_{1n_{\underline{R}}} + \iota\gamma_{1n_{\underline{R}}} \end{pmatrix} \end{pmatrix} \right\} \\ \left\{ \begin{pmatrix} \begin{pmatrix} \alpha_{21_{\overline{\mathcal{R}}}} + \beta_{21_{\overline{\mathcal{R}}}}, \\ \delta_{21_{\overline{\mathcal{R}}}} + \iota\gamma_{21_{\overline{\mathcal{R}}}} \end{pmatrix}, \\ \begin{pmatrix} \alpha_{21_{\underline{R}}} + \iota\beta_{21_{\underline{R}}}, \\ \delta_{21_{\underline{R}}} + \iota\gamma_{21_{\underline{R}}} \end{pmatrix} \end{pmatrix} \right\} & \left\{ \begin{pmatrix} \begin{pmatrix} \alpha_{22_{\overline{\mathcal{R}}}} + \beta_{22_{\overline{\mathcal{R}}}}, \\ \delta_{22_{\overline{\mathcal{R}}}} + \iota\gamma_{22_{\overline{\mathcal{R}}}} \end{pmatrix}, \\ \begin{pmatrix} \alpha_{22_{\underline{R}}} + \iota\beta_{22_{\underline{R}}}, \\ \delta_{22_{\underline{R}}} + \iota\gamma_{22_{\underline{R}}} \end{pmatrix} \end{pmatrix} \right\} & \cdots & \left\{ \begin{pmatrix} \begin{pmatrix} \alpha_{2n_{\overline{\mathcal{R}}}} + \beta_{2n_{\overline{\mathcal{R}}}}, \\ \delta_{2n_{\overline{\mathcal{R}}}} + \iota\gamma_{2n_{\overline{\mathcal{R}}}} \end{pmatrix}, \\ \begin{pmatrix} \alpha_{2n_{\underline{R}}} + \iota\beta_{2n_{\underline{R}}}, \\ \delta_{2n_{\underline{R}}} + \iota\gamma_{2n_{\underline{R}}} \end{pmatrix} \end{pmatrix} \right\} \\ \vdots & \vdots & \vdots \\ \left\{ \begin{pmatrix} \begin{pmatrix} \alpha_{m1_{\overline{\mathcal{R}}}} + \beta_{m1_{\overline{\mathcal{R}}}}, \\ \delta_{m1_{\overline{\mathcal{R}}}} + \iota\gamma_{m1_{\overline{\mathcal{R}}}} \end{pmatrix}, \\ \begin{pmatrix} \alpha_{m1_{\underline{R}}} + \iota\beta_{m1_{\underline{R}}}, \\ \delta_{m1_{\underline{R}}} + \iota\gamma_{m1_{\underline{R}}} \end{pmatrix} \end{pmatrix} \right\} & \left\{ \begin{pmatrix} \begin{pmatrix} \alpha_{m2_{\overline{\mathcal{R}}}} + \beta_{m2_{\overline{\mathcal{R}}}}, \\ \delta_{m2_{\overline{\mathcal{R}}}} + \iota\gamma_{m2_{\overline{\mathcal{R}}}} \end{pmatrix}, \\ \begin{pmatrix} \alpha_{m2_{\underline{R}}} + \iota\beta_{m2_{\underline{R}}}, \\ \delta_{m2_{\underline{R}}} + \iota\gamma_{m2_{\underline{R}}} \end{pmatrix} \end{pmatrix} \right\} & \ddots & \left\{ \begin{pmatrix} \begin{pmatrix} \alpha_{mn_{\overline{\mathcal{R}}}} + \beta_{mn_{\overline{\mathcal{R}}}}, \\ \delta_{mn_{\overline{\mathcal{R}}}} + \iota\gamma_{mn_{\overline{\mathcal{R}}}} \end{pmatrix}, \\ \begin{pmatrix} \alpha_{mn_{\underline{R}}} + \iota\beta_{mn_{\underline{R}}}, \\ \delta_{mn_{\underline{R}}} + \iota\gamma_{mm_{\underline{R}}} \end{pmatrix} \end{pmatrix} \right\} \end{bmatrix}$$

Step 2 Afterward, normalize the provided decision-making matrix in Step 1 utilizing the formula as

$$\mathcal{N}_{ij} = \begin{cases} \left(\begin{pmatrix} \alpha_{ij_{\overline{\mathcal{R}}}} + \beta_{ij_{\overline{\mathcal{R}}}}, \\ \delta_{ij_{\overline{\mathcal{R}}}} + \iota\gamma_{ij_{\overline{\mathcal{R}}}} \end{pmatrix}, \\ \begin{pmatrix} \alpha_{ij_{\underline{R}}} + \iota\beta_{ij_{\underline{R}}}, \\ \delta_{ij_{\underline{R}}} + \iota\gamma_{ij_{\underline{R}}} \end{pmatrix} \right) & \text{for benefit - type criteria} \\ \\ \left(\begin{pmatrix} \delta_{ij_{\overline{\mathcal{R}}}} + \iota\gamma_{ij_{\overline{\mathcal{R}}}}, \\ \alpha_{ij_{\overline{\mathcal{R}}}} + \beta_{ij_{\overline{\mathcal{R}}}} \end{pmatrix}, \\ \begin{pmatrix} \delta_{ij_{\underline{R}}} + \iota\gamma_{ij_{\underline{R}}}, \\ \alpha\delta_{ij_{\underline{R}}} + \iota\beta_{ij_{\underline{R}}} \end{pmatrix} \right) & \text{for cost - type criteria} \end{cases} \quad (3)$$

Where $\left(\begin{pmatrix} \alpha_{ij_{\overline{\mathcal{R}}}} + \beta_{ij_{\overline{\mathcal{R}}}}, \\ \delta_{ij_{\overline{\mathcal{R}}}} + \iota\gamma_{ij_{\overline{\mathcal{R}}}} \end{pmatrix}, \begin{pmatrix} \alpha_{ij_{\underline{R}}} + \iota\beta_{ij_{\underline{R}}}, \\ \delta_{ij_{\underline{R}}} + \iota\gamma_{ij_{\underline{R}}} \end{pmatrix} \right)$ is the complement of $\left(\begin{pmatrix} \delta_{ij_{\overline{\mathcal{R}}}} + \iota\gamma_{ij_{\overline{\mathcal{R}}}}, \\ \alpha_{ij_{\overline{\mathcal{R}}}} + \beta_{ij_{\overline{\mathcal{R}}}} \end{pmatrix}, \begin{pmatrix} \delta_{ij_{\underline{R}}} + \iota\gamma_{ij_{\underline{R}}}, \\ \alpha\delta_{ij_{\underline{R}}} + \iota\beta_{ij_{\underline{R}}} \end{pmatrix} \right)$

Step 3 Compute the relative importance of the alternative by using Eq. (1)

$$B_i^{WS} = \left\{ \begin{pmatrix} \frac{\sqrt{\min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(\alpha_{i\overline{\mathcal{R}}}\right)^{2\mathcal{O}'}\right)\right)^{\frac{1}{\mathcal{O}'}}\right)} + \iota\sqrt{\min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(\beta_{i\overline{\mathcal{R}}}\right)^{2\mathcal{O}'}\right)\right)^{\frac{1}{\mathcal{O}'}}\right)},}{\sqrt{1-\min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(1-\left(\delta_{i\overline{\mathcal{R}}}\right)^2\right)^{\mathcal{O}'}\right)\right)^{\frac{1}{\mathcal{O}'}}\right)} + \iota\sqrt{1-\min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(1-\left(\gamma_{i\overline{\mathcal{R}}}\right)^2\right)^{\mathcal{O}'}\right)\right)^{\frac{1}{\mathcal{O}'}}\right)}} \end{pmatrix}, \\ \begin{pmatrix} \frac{\sqrt{\min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(\alpha_{i\underline{R}}\right)^{2\mathcal{O}'}\right)\right)^{\frac{1}{\mathcal{O}'}}\right)} + \iota\sqrt{\min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(\beta_{i\underline{R}}\right)^{2\mathcal{O}'}\right)\right)^{\frac{1}{\mathcal{O}'}}\right)},}{\sqrt{1-\min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(1-\left(\delta_{i\underline{R}}\right)^2\right)^{\mathcal{O}'}\right)\right)^{\frac{1}{\mathcal{O}'}}\right)} + \iota\sqrt{1-\min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(1-\left(\gamma_{i\underline{R}}\right)^2\right)^{\mathcal{O}'}\right)\right)^{\frac{1}{\mathcal{O}'}}\right)}} \end{pmatrix} \right\}$$

Step 4 Similarly, we have used Eq. (2) to find out the weighted product model given by.

$$
B_i^{WP} = \left\{
\begin{array}{c}
\sqrt{1 - \min\left(1, \left(\sum_{i=1}^m \left(\beth_i\left(1 - \left(\alpha_{i\overline{\mathcal{R}}}\right)^2\right)\right)^{\mho}\right)^{\frac{1}{\mho}}\right)} + \iota \sqrt{1 - \min\left(1, \left(\sum_{i=1}^m \left(\beth_i\left(1 - \left(\beta_{i\overline{\mathcal{R}}}\right)^2\right)\right)^{\mho}\right)^{\frac{1}{\mho}}\right)}, \\
\sqrt{\min\left(1, \left(\sum_{i=1}^m \left(\beth_i\left(\delta_{i\overline{\mathcal{R}}}\right)^{2\mho}\right)\right)^{\frac{1}{\mho}}\right)} + \iota \sqrt{\min\left(1, \left(\sum_{i=1}^m \left(\beth_i\left(\gamma_{i\overline{\mathcal{R}}}\right)^{2\mho}\right)\right)^{\frac{1}{\mho}}\right)}, \\
\sqrt{1 - \min\left(1, \left(\sum_{i=1}^m \left(\beth_i\left(1 - \left(\alpha_{i\underline{R}}\right)^2\right)\right)^{\mho}\right)^{\frac{1}{\mho}}\right)} + \iota \sqrt{1 - \min\left(1, \left(\sum_{i=1}^m \left(\beth_i\left(1 - \left(\beta_{i\underline{R}}\right)^2\right)\right)^{\mho}\right)^{\frac{1}{\mho}}\right)}, \\
\sqrt{\min\left(1, \left(\sum_{i=1}^m \left(\beth_i\left(\delta_{i\underline{R}}\right)^{2\mho}\right)\right)^{\frac{1}{\mho}}\right)} + \iota \sqrt{\min\left(1, \left(\sum_{i=1}^m \left(\beth_i\left(\gamma_{i\underline{R}}\right)^{2\mho}\right)\right)^{\frac{1}{\mho}}\right)}
\end{array}
\right\}
$$

Step 5 Lastly, using WASPAS methods to arrange alternatives, as provided by

$$
B_i = \frac{B_i^{WS} + B_i^{WP}}{2}
$$

Step 6 To obtain score values and access the optimal option, use def. 6.

The flowchart of the proposed WASPAS technique is given in (Fig. 2).

Illustrative example
Discussion
Business engineering is an interdisciplinary profession that combines information technology, engineering concepts, and business administration to create, optimize, and apply creative solutions to problems in the real world. Its main goals are to increase organizational effectiveness, simplify intricate procedures, and improve decision-making by using data-driven approaches. In practice, business engineering uses systematic frameworks, mathematical modeling, and AI-powered technologies to handle issues including supply chain inefficiencies, digital transformation, and risk management. This strategy is especially helpful in sectors where increasing operational resilience and resource optimization are essential for competitive success, such as manufacturing, healthcare, and finance. A multifaceted strategy integrating expertise from administration, engineering, computer science, and business is necessary for business engineering. Experts in this domain typically possess an engineering or business background, together with supplementary instruction in data technology and operation management.

Some key aspects of business engineering are given in tabular form in (Table 2).

Here we have explained the possible solutions for handling the issues regarding business engineering.

Improve process efficiency through six sigma methods and robotic process automation (RPA)   Put lean principles into practice to save waste and boost workflow effectiveness. Utilize six sigma methods to lower process variance and raise quality. Businesses can improve the effectiveness of their internal processes by implementing the six sigma technique, which provides the necessary tools. This increase in efficiency and decrease in process
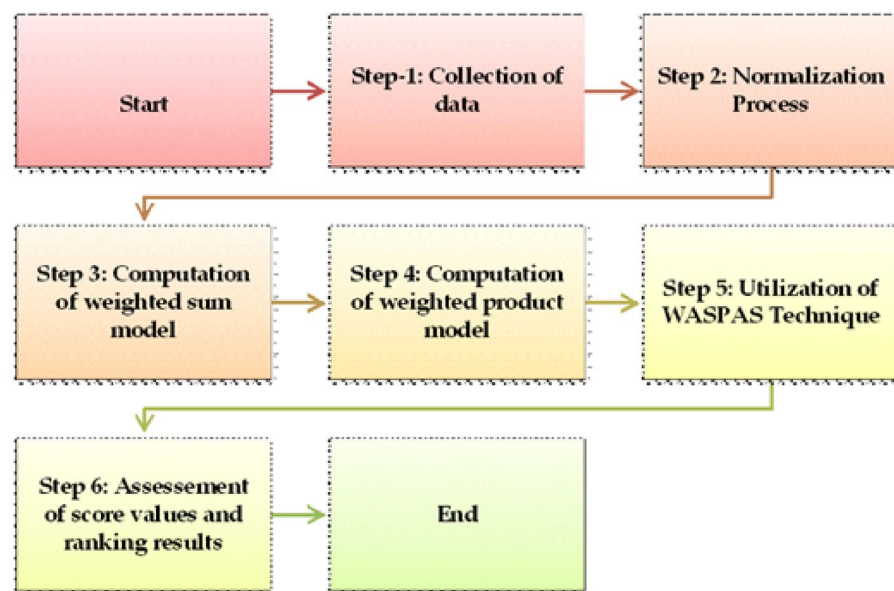

Fig. 2. Flowchart of WASPAS technique.

Key aspects	Explanation
Process optimization	Process optimization examines and enhances corporate procedures to increase their effectiveness and efficiency. Process mapping, lean management, and six sigma approaches are used in this.
System thinking	This method makes it easier to comprehend how adjustments made in one area impact the entire company.
Information technology integration	Use IT to improve and automate corporate procedures. Employing business intelligence and relationships with clients, among other enterprise software, is part of this use of IT to improve and automate corporate procedures.
Innovation and change management	This entails creating new goods, services, and business models in addition to handling organizational change with effectiveness.
Strategic planning	Creating and putting into practice plans to accomplish long-term company objectives. This covers the use of resources, competitive assessment, and market evaluation.
Quality management	Guaranteeing the fulfillment of specific quality criteria by the company's offerings. Methodologies for ongoing enhancement along with quality control are frequently used here.
Performance measurement	Creating measurements and key performance indicators to track and evaluate the effectiveness of different business operations and procedures.

Table 2. Some key aspects of business engineering.

Fig. 3. Aspects of improving process efficiency.

variance leads to improvements in profits, employee morale, and the caliber of the products or services. Defects also reduce as a result. Often referred to as software robotics, RPA leverages intelligent automation technology to carry out data extraction and other tedious office duties that are performed by human workers. To improve efficiency and precision, automate repetitive operations with software solutions like RPA.

Figure 3 shows aspects of improving process efficiency.

Deploy enterprise resource planning (ERP), Implement customer relationship management system (CRMS) and data integration platform (DIP)   ERP is a platform that businesses use to handle and integrate the essential elements of their operations. Businesses need a variety of ERP software systems because they make asset planning easier to adopt by combining all operational processes into one system. To combine several business operations into a single system, the implementation of ERP is necessary for business engineering. Moreover, customer relationship management implementation entails configuring CRM software to gather accurate information that the sales and marketing team may use to direct advertising and boost revenue. CRMs let employees learn more about client interaction paths, identify potential, and expand their company. So the implementation of a CRM system is necessary to enhance customer interaction and data efficiency. IT specialists may combine data from several sources and give a full, precise, and current dataset for business intelligence (BI), data mining, and other applications and procedures by using a data integration platform. To guarantee smooth data transfer between various systems and programs, the deployment of data integration platforms is the best solution in business engineering to overcome issues regarding data analysis.

The graphical representation of ERP, CRMS, and DIP is given in (Fig. 4).

Decide the best quality management principle   In Table 3 below we have elaborated on different quality management principles that help to resolve the issues related to business engineering.
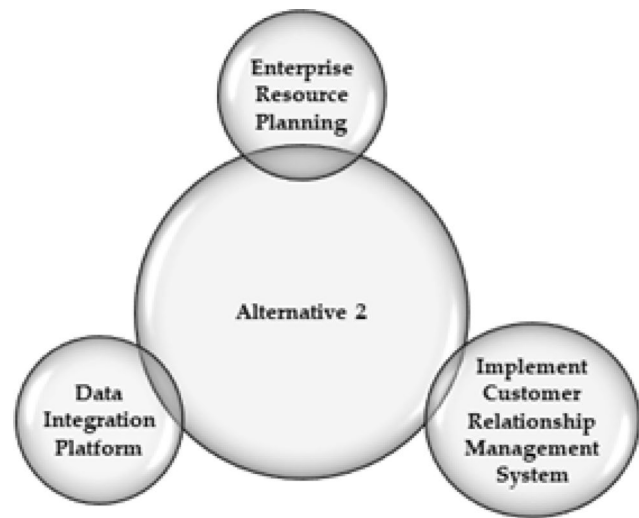
Fig. 4. Graphical representation of ERP, CRMS, and DIP.

Quality management principles	
Total quality management (TQM)	A management approach known as TQM places a strong emphasis on satisfying customers to achieve long-term success. Every worker in an organization takes part in a TQM initiative to improve the products, services, and culture of the business. Adopt total quality management concepts to concentrate on ongoing development across the board for the company.
ISO certification	ISO 9001 is the international standard that describes requirements for a quality management system (QMS). Organizations can demonstrate their ability to consistently provide products and services that meet customer and regulatory requirements by utilizing the standard. It is also the most extensively utilized standard in the ISO 9000 series because it is the only one that permits enterprises to certify. Accreditation with ISO 9001 would help to standardize quality control techniques.
Customer feedback loops	A customer feedback loop is a method of client engagement created to help you continuously improve your product by considering user opinions, suggestions, and assessments. The concept of reciprocal communication between the business and its clients forms the basis of this feedback loop. Create client input loops to collect feedback on quality consistently and take appropriate action.

Table 3. Different quality management principles.

Enhance supply chain visibility   Implement technologies like loT (The network of physical items, or "things," that are implanted across software, hardware, sensors, and other tools to communicate and exchange information with other gadgets and networks over the internet, is known as the Internet of Things) to enhance supply chain visibility. To minimize risks and lessen reliance on a single source, there is a need to diversify the supplier base. To maximize stock levels and guarantee rapid supply, sophisticated inventory management systems must be used in business engineering.

Now based on these alternatives, a team of experts is assigned a task to select the best solution for issues related to business engineering. The committee members provided a theory assessment under the following attributes given in (Table 4).

Results of Algorithm 1

Step 1 Assume that a team of three experts provide their assessment in the form of CPyFRNs in Cartesian form and the overall data against each alternative is given in (Tables 5, 6 and 7).

The combined matrix by utilizing the notion of CPyFRYWA aggregation operators and WVs of experts that $\varpi = (0.42,\ 0.39,\ 0.19)^{\mathrm{T}}$ is given in Table 8 as follows.

Step 2 As all criteria are of benefit type, so we utilize the Eq. (3) to normalize the data given in (Table 8). We can observe that by utilizing the formula in Eq. (3), we get the same values as given in (Table 8). Hence Table 8 is a normalized matrix.

Step 3 Assume that the WVs for the attributes are $(0.23,\ 0.31,\ 0.28,\ 0.18)$. Use CPyFRYWA AOs to compute the relative importance of alternatives by using the formula

Attributes symbols	Attributes name	Explanation of attributes
A_{T1}	Process orientation	Focuses on enhancing the efficacy of business processes through analysis, design, and optimization.
A_{T2}	Innovation and creativity	Promotes original thought and imaginative problem-solving to create new technology and business strategies.
A_{T3}	Project management	Focuses on developing strong project management abilities to organize, carry out, and oversee corporate initiatives.
A_{T4}	Customer focus	Considers the demands and satisfaction of customers when developing and refining company processes.

Table 4. Attributes and their description.

	A_{T1}	A_{T2}	A_{T3}	A_{T4}
A_{L1}	$\left(\begin{pmatrix} 0.23 + \iota\,0.24, \\ 0.24 + \iota\,0.21 \end{pmatrix},\; \begin{pmatrix} 0.25 + \iota\,0.26, \\ 0.11 + \iota\,0.12 \end{pmatrix}\right)$	$\left(\begin{pmatrix} 0.12 + \iota\,0.13, \\ 0.14 + \iota\,0.12 \end{pmatrix},\; \begin{pmatrix} 0.16 + \iota\,0.18, \\ 0.11 + \iota\,0.15 \end{pmatrix}\right)$	$\left(\begin{pmatrix} 0.13 + \iota\,0.14, \\ 0.23 + \iota\,0.15 \end{pmatrix},\; \begin{pmatrix} 0.23 + \iota\,0.14, \\ 0.24 + \iota\,0.16 \end{pmatrix}\right)$	$\left(\begin{pmatrix} 0.13 + \iota\,0.35, \\ 0.18 + \iota\,0.21 \end{pmatrix},\; \begin{pmatrix} 0.14 + \iota\,0.36, \\ 0.11 + \iota\,0.12 \end{pmatrix}\right)$
A_{L2}	$\left(\begin{pmatrix} 0.21 + \iota\,0.2, \\ 0.25 + \iota\,0.27 \end{pmatrix},\; \begin{pmatrix} 0.31 + \iota\,0.23, \\ 0.1 + \iota\,0.18 \end{pmatrix}\right)$	$\left(\begin{pmatrix} 0.17 + \iota\,0.19, \\ 0.23 + \iota\,0.17 \end{pmatrix},\; \begin{pmatrix} 0.19 + \iota\,0.24, \\ 0.24 + \iota\,0.27 \end{pmatrix}\right)$	$\left(\begin{pmatrix} 0.24 + \iota\,0.15, \\ 0.25 + \iota\,0.19 \end{pmatrix},\; \begin{pmatrix} 0.26 + \iota\,0.16, \\ 0.27 + \iota\,0.33 \end{pmatrix}\right)$	$\left(\begin{pmatrix} 0.15 + \iota\,0.21, \\ 0.14 + \iota\,0.14 \end{pmatrix},\; \begin{pmatrix} 0.17 + \iota\,0.22, \\ 0.12 + \iota\,0.13 \end{pmatrix}\right)$
A_{L3}	$\left(\begin{pmatrix} 0.23 + \iota\,0.27, \\ 0.31 + \iota\,0.37 \end{pmatrix},\; \begin{pmatrix} 0.13 + \iota\,0.15, \\ 0.16 + \iota\,0.17 \end{pmatrix}\right)$	$\left(\begin{pmatrix} 0.15 + \iota\,0.24, \\ 0.12 + \iota\,0.28 \end{pmatrix},\; \begin{pmatrix} 0.31 + \iota\,0.15, \\ 0.10 + \iota\,0.23 \end{pmatrix}\right)$	$\left(\begin{pmatrix} 0.27 + \iota\,0.17, \\ 0.28 + \iota\,0.34 \end{pmatrix},\; \begin{pmatrix} 0.28 + \iota\,0.18, \\ 0.29 + \iota\,0.38 \end{pmatrix}\right)$	$\left(\begin{pmatrix} 0.19 + \iota\,0.23, \\ 0.31 + \iota\,0.13 \end{pmatrix},\; \begin{pmatrix} 0.22 + \iota\,0.35, \\ 0.13 + \iota\,0.14 \end{pmatrix}\right)$
A_{L4}	$\left(\begin{pmatrix} 0.32 + \iota\,0.41, \\ 0.11 + \iota\,0.12 \end{pmatrix},\; \begin{pmatrix} 0.21 + \iota\,0.22, \\ 0.17 + \iota\,0.19 \end{pmatrix}\right)$	$\left(\begin{pmatrix} 0.33 + \iota\,0.39, \\ 0.14 + \iota\,0.24 \end{pmatrix},\; \begin{pmatrix} 0.27 + \iota\,0.28, \\ 0.15 + \iota\,0.25 \end{pmatrix}\right)$	$\left(\begin{pmatrix} 0.29 + \iota\,0.19, \\ 0.30 + \iota\,0.34 \end{pmatrix},\; \begin{pmatrix} 0.33 + \iota\,0.20, \\ 0.13 + \iota\,0.12 \end{pmatrix}\right)$	$\left(\begin{pmatrix} 0.24 + \iota\,0.37, \\ 0.17 + \iota\,0.12 \end{pmatrix},\; \begin{pmatrix} 0.35 + \iota\,0.22, \\ 0.17 + \iota\,0.19 \end{pmatrix}\right)$

Table 5. CPyFR data by expert 1.

	A_{T1}	A_{T2}	A_{T3}	A_{T4}
A_{L1}	$\left(\begin{pmatrix} 0.11 + \iota\,0.21, \\ 0.31 + \iota\,0.30 \end{pmatrix},\; \begin{pmatrix} 0.12 + \iota\,0.23, \\ 0.32 + \iota\,0.32 \end{pmatrix}\right)$	$\left(\begin{pmatrix} 0.51 + \iota\,0.24, \\ 0.32 + \iota\,0.36 \end{pmatrix},\; \begin{pmatrix} 0.12 + \iota\,0.26, \\ 0.33 + \iota\,0.37 \end{pmatrix}\right)$	$\left(\begin{pmatrix} 0.21 + \iota\,0.42, \\ 0.14 + \iota\,0.23 \end{pmatrix},\; \begin{pmatrix} 0.24 + \iota\,0.61, \\ 0.11 + \iota\,0.21 \end{pmatrix}\right)$	$\left(\begin{pmatrix} 0.33 + \iota\,0.12, \\ 0.10 + \iota\,0.13 \end{pmatrix},\; \begin{pmatrix} 0.14 + \iota\,0.15, \\ 0.16 + \iota\,0.17 \end{pmatrix}\right)$
A_{L2}	$\left(\begin{pmatrix} 0.13 + \iota\,0.20, \\ 0.33 + \iota\,0.31 \end{pmatrix},\; \begin{pmatrix} 0.21 + \iota\,0.21, \\ 0.34 + \iota\,0.33 \end{pmatrix}\right)$	$\left(\begin{pmatrix} 0.12 + \iota\,0.20, \\ 0.38 + \iota\,0.33 \end{pmatrix},\; \begin{pmatrix} 0.14 + \iota\,0.23, \\ 0.39 + \iota\,0.44 \end{pmatrix}\right)$	$\left(\begin{pmatrix} 0.12 + \iota\,0.27, \\ 0.52 + \iota\,0.25 \end{pmatrix},\; \begin{pmatrix} 0.13 + \iota\,0.29, \\ 0.19 + \iota\,0.15 \end{pmatrix}\right)$	$\left(\begin{pmatrix} 0.13 + \iota\,0.13, \\ 0.15 + \iota\,0.19 \end{pmatrix},\; \begin{pmatrix} 0.21 + \iota\,0.22, \\ 0.23 + \iota\,0.24 \end{pmatrix}\right)$
A_{L3}	$\left(\begin{pmatrix} 0.22 + \iota\,0.19, \\ 0.30 + \iota\,0.34 \end{pmatrix},\; \begin{pmatrix} 0.23 + \iota\,0.17, \\ 0.35 + \iota\,0.35 \end{pmatrix}\right)$	$\left(\begin{pmatrix} 0.17 + \iota\,0.27, \\ 0.40 + \iota\,0.45 \end{pmatrix},\; \begin{pmatrix} 0.19 + \iota\,0.15, \\ 0.41 + \iota\,0.35 \end{pmatrix}\right)$	$\left(\begin{pmatrix} 0.32 + \iota\,0.17, \\ 0.34 + \iota\,0.16 \end{pmatrix},\; \begin{pmatrix} 0.31 + \iota\,0.16, \\ 0.33 + \iota\,0.18 \end{pmatrix}\right)$	$\left(\begin{pmatrix} 0.32 + \iota\,0.33, \\ 0.34 + \iota\,0.36 \end{pmatrix},\; \begin{pmatrix} 0.37 + \iota\,0.38, \\ 0.39 + \iota\,0.40 \end{pmatrix}\right)$
A_{L4}	$\left(\begin{pmatrix} 0.19 + \iota\,0.15, \\ 0.23 + \iota\,0.35 \end{pmatrix},\; \begin{pmatrix} 0.10 + \iota\,0.11, \\ 0.24 + \iota\,0.41 \end{pmatrix}\right)$	$\left(\begin{pmatrix} 0.20 + \iota\,0.41, \\ 0.43 + \iota\,0.21 \end{pmatrix},\; \begin{pmatrix} 0.30 + \iota\,0.22, \\ 0.24 + \iota\,0.18 \end{pmatrix}\right)$	$\left(\begin{pmatrix} 0.10 + \iota\,0.19, \\ 0.32 + \iota\,0.17 \end{pmatrix},\; \begin{pmatrix} 0.12 + \iota\,0.13, \\ 0.18 + \iota\,0.16 \end{pmatrix}\right)$	$\left(\begin{pmatrix} 0.23 + \iota\,0.25, \\ 0.20 + \iota\,0.27 \end{pmatrix},\; \begin{pmatrix} 0.27 + \iota\,0.29, \\ 0.30 + \iota\,0.33 \end{pmatrix}\right)$

Table 6. CPyFR data by expert 2.

$$
B_i^{WS} = \left\{ \begin{pmatrix} \begin{pmatrix} \sqrt{\min\left(1,\left(\sum_{i=1}^m \left(\beth_i\left(\alpha_{i\overline{\mathcal{R}}}\right)^{2\mho}\right)\right)^{\frac{1}{\mho}}\right)} + \iota \sqrt{\min\left(1,\left(\sum_{i=1}^m \left(\beth_i\left(\beta_{i\overline{\mathcal{R}}}\right)^{2\mho}\right)\right)^{\frac{1}{\mho}}\right)}, \\ \sqrt{1-\min\left(1,\left(\sum_{i=1}^m \left(\beth_i\left(1-\left(\delta_{i\overline{\mathcal{R}}}\right)^2\right)^{\mho}\right)\right)^{\frac{1}{\mho}}\right)} + \iota \sqrt{1-\min\left(1,\left(\sum_{i=1}^m \left(\beth_i\left(1-\left(\gamma_{i\overline{\mathcal{R}}}\right)^2\right)^{\mho}\right)\right)^{\frac{1}{\mho}}\right)} \end{pmatrix}, \\ \begin{pmatrix} \sqrt{\min\left(1,\left(\sum_{i=1}^m \left(\beth_i(\alpha_{i\underline{R}})^{2\mho}\right)\right)^{\frac{1}{\mho}}\right)} + \iota \sqrt{\min\left(1,\left(\sum_{i=1}^m \left(\beth_i\left(\beta_{i\underline{R}}\right)^{2\mho}\right)\right)^{\frac{1}{\mho}}\right)}, \\ \sqrt{1-\min\left(1,\left(\sum_{i=1}^m \left(\beth_i\left(1-\left(\delta_{i\underline{R}}\right)^2\right)^{\mho}\right)\right)^{\frac{1}{\mho}}\right)} + \iota \sqrt{1-\min\left(1,\left(\sum_{i=1}^m \left(\beth_i\left(1-\left(\gamma_{i\underline{R}}\right)^2\right)^{\mho}\right)\right)^{\frac{1}{\mho}}\right)} \end{pmatrix} \end{pmatrix} \right\}
$$

Hence Table 9 shows the relative importance of alternatives.

	A_{T1}	A_{T2}	A_{T3}	A_{T4}
A_{L1}	$\left(\left(\begin{smallmatrix}0.11+\iota\,0.22,\\0.13+\iota\,0.33\end{smallmatrix}\right),\left(\begin{smallmatrix}0.12+\iota\,0.23,\\0.14+\iota\,0.34\end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix}0.25+\iota\,0.41,\\0.49+\iota\,0.12\end{smallmatrix}\right),\left(\begin{smallmatrix}0.33+\iota\,0.42,\\0.48+\iota\,0.133\end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix}0.21+\iota\,0.21,\\0.24+\iota\,0.34\end{smallmatrix}\right),\left(\begin{smallmatrix}0.25+\iota\,0.22,\\0.11+\iota\,0.35\end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix}0.23+\iota\,0.24,\\0.24+\iota\,0.20\end{smallmatrix}\right),\left(\begin{smallmatrix}0.25+\iota\,0.14,\\0.13+\iota\,0.22\end{smallmatrix}\right)\right)$
A_{L2}	$\left(\left(\begin{smallmatrix}0.13+\iota\,0.24,\\0.19+\iota\,0.30\end{smallmatrix}\right),\left(\begin{smallmatrix}0.14+\iota\,0.25,\\0.20+\iota\,0.35\end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix}0.34+\iota\,0.43,\\0.47+\iota\,0.14\end{smallmatrix}\right),\left(\begin{smallmatrix}0.35+\iota\,0.44,\\0.46+\iota\,0.19\end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix}0.23+\iota\,0.23,\\0.15+\iota\,0.36\end{smallmatrix}\right),\left(\begin{smallmatrix}0.24+\iota\,0.23,\\0.14+\iota\,0.18\end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix}0.13+\iota\,0.15,\\0.14+\iota\,0.23\end{smallmatrix}\right),\left(\begin{smallmatrix}0.13+\iota\,0.16,\\0.15+\iota\,0.24\end{smallmatrix}\right)\right)$
A_{L3}	$\left(\left(\begin{smallmatrix}0.15+\iota\,0.26,\\0.21+\iota\,0.36\end{smallmatrix}\right),\left(\begin{smallmatrix}0.16+\iota\,0.27,\\0.22+\iota\,0.37\end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix}0.37+\iota\,0.45,\\0.12+\iota\,0.18\end{smallmatrix}\right),\left(\begin{smallmatrix}0.38+\iota\,0.46,\\0.11+\iota\,0.17\end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix}0.25+\iota\,0.27,\\0.15+\iota\,0.29\end{smallmatrix}\right),\left(\begin{smallmatrix}0.17+\iota\,0.15,\\0.13+\iota\,0.18\end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix}0.23+\iota\,0.15,\\0.14+\iota\,0.24\end{smallmatrix}\right),\left(\begin{smallmatrix}0.13+\iota\,0.19,\\0.16+\iota\,0.25\end{smallmatrix}\right)\right)$
A_{L4}	$\left(\left(\begin{smallmatrix}0.17+\iota\,0.28,\\0.14+\iota\,0.38\end{smallmatrix}\right),\left(\begin{smallmatrix}0.18+\iota\,0.29,\\0.15+\iota\,0.39\end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix}0.39+\iota\,0.47,\\0.13+\iota\,0.16\end{smallmatrix}\right),\left(\begin{smallmatrix}0.40+\iota\,0.48,\\0.14+\iota\,0.15\end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix}0.28+\iota\,0.14,\\0.11+\iota\,0.17\end{smallmatrix}\right),\left(\begin{smallmatrix}0.19+\iota\,0.22,\\0.17+\iota\,0.19\end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix}0.15+\iota\,0.19,\\0.22+\iota\,0.26\end{smallmatrix}\right),\left(\begin{smallmatrix}0.12+\iota\,0.21,\\0.23+\iota\,0.19\end{smallmatrix}\right)\right)$

Table 7. CPyFR data by expert 3.

	A_{T1}	A_{T2}	A_{T3}	A_{T4}
A_{L1}	$\left(\left(\begin{smallmatrix}0.2064+\iota\,0.2273,\\0.2520+\iota\,0.2704\end{smallmatrix}\right),\left(\begin{smallmatrix}0.2244+\iota\,0.2457,\\0.2141+\iota\,0.2540\end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix}0.4534+\iota\,0.3343,\\0.2895+\iota\,0.2332\end{smallmatrix}\right),\left(\begin{smallmatrix}0.2683+\iota\,0.3432,\\0.2849+\iota\,0.2470\end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix}0.1965+\iota\,0.3734,\\0.2008+\iota\,0.2242\end{smallmatrix}\right),\left(\begin{smallmatrix}0.2385+\iota\,0.5422,\\0.1744+\iota\,0.2220\end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix}0.2943+\iota\,0.3148,\\0.1673+\iota\,0.1806\end{smallmatrix}\right),\left(\begin{smallmatrix}0.2041+\iota\,0.3230,\\0.1349+\iota\,0.1621\end{smallmatrix}\right)\right)$
A_{L2}	$\left(\left(\begin{smallmatrix}0.1891+\iota\,0.2125,\\0.2755+\iota\,0.2915\end{smallmatrix}\right),\left(\begin{smallmatrix}0.2795+\iota\,0.2293,\\0.2307+\iota\,0.2769\end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix}0.2765+\iota\,0.3497,\\0.3372+\iota\,0.2361\end{smallmatrix}\right),\left(\begin{smallmatrix}0.2850+\iota\,0.3587,\\0.3429+\iota\,0.3253\end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix}0.2230+\iota\,0.2441,\\0.3417+\iota\,0.2497\end{smallmatrix}\right),\left(\begin{smallmatrix}0.2396+\iota\,0.2604,\\0.2183+\iota\,0.2408\end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix}0.1408+\iota\,0.1895,\\0.1439+\iota\,0.1792\end{smallmatrix}\right),\left(\begin{smallmatrix}0.1911+\iota\,0.2147,\\0.1745+\iota\,0.1996\end{smallmatrix}\right)\right)$
A_{L3}	$\left(\left(\begin{smallmatrix}0.2199+\iota\,0.2524,\\0.2884+\iota\,0.3564\end{smallmatrix}\right),\left(\begin{smallmatrix}0.2054+\iota\,0.2212,\\0.2541+\iota\,0.2863\end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix}0.3008+\iota\,0.3678,\\0.2516+\iota\,0.3317\end{smallmatrix}\right),\left(\begin{smallmatrix}0.3232+\iota\,0.3737,\\0.2507+\iota\,0.2703\end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix}0.2951+\iota\,0.2221,\\0.2836+\iota\,0.2679\end{smallmatrix}\right),\left(\begin{smallmatrix}0.2894+\iota\,0.1694,\\0.2813+\iota\,0.2730\end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix}0.2862+\iota\,0.2955,\\0.2951+\iota\,0.2538\end{smallmatrix}\right),\left(\begin{smallmatrix}0.3298+\iota\,0.3571,\\0.2596+\iota\,0.2756\end{smallmatrix}\right)\right)$
A_{L4}	$\left(\left(\begin{smallmatrix}0.2877+\iota\,0.3688,\\0.1699+\iota\,0.2754\end{smallmatrix}\right),\left(\begin{smallmatrix}0.1914+\iota\,0.2421,\\0.1962+\iota\,0.3208\end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix}0.3358+\iota\,0.4210,\\0.2706+\iota\,0.2145\end{smallmatrix}\right),\left(\begin{smallmatrix}0.3358+\iota\,0.3916,\\0.1875+\iota\,0.2066\end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix}0.2699+\iota\,0.1855,\\0.2795+\iota\,0.2498\end{smallmatrix}\right),\left(\begin{smallmatrix}0.2963+\iota\,0.1956,\\0.1585+\iota\,0.1509\end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix}0.2296+\iota\,0.3337,\\0.1919+\iota\,0.2146\end{smallmatrix}\right),\left(\begin{smallmatrix}0.3183+\iota\,0.2624,\\0.2368+\iota\,0.2499\end{smallmatrix}\right)\right)$

Table 8. Combined matrix.

$B_1{}^{WS}$	$B_2{}^{WS}$	$B_3{}^{WS}$	$B_4{}^{WS}$
$\left(\left(\begin{smallmatrix}0.3926+\iota\,0.3386,\\0.2370+\iota\,0.2309\end{smallmatrix}\right),\left(\begin{smallmatrix}0.2456+\iota\,0.4647,\\0.2158+\iota\,0.2280\end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix}0.2460+\iota\,0.3046,\\0.2942+\iota\,0.2441\end{smallmatrix}\right),\left(\begin{smallmatrix}0.2664+\iota\,0.3135,\\0.2562+\iota\,0.2699\end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix}0.2871+\iota\,0.3234,\\0.2771+\iota\,0.3065\end{smallmatrix}\right),\left(\begin{smallmatrix}0.3058+\iota\,0.3371,\\0.2618+\iota\,0.2757\end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix}0.3025+\iota\,0.3775,\\0.2389+\iota\,0.2392\end{smallmatrix}\right),\left(\begin{smallmatrix}0.3103+\iota\,0.3400,\\0.1916+\iota\,0.2305\end{smallmatrix}\right)\right)$

Table 9. Weighted sum values "$B_i{}^{WS}$".

$B_1{}^{PS}$	$B_2{}^{PS}$	$B_3{}^{PS}$	$B_4{}^{PS}$
$\left(\left(\begin{smallmatrix}0.3025+\iota\,0.3189,\\0.2584+\iota\,0.2407\end{smallmatrix}\right),\left(\begin{smallmatrix}0.2390+\iota\,0.3712,\\0.2489+\iota\,0.2371\end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix}0.2236+\iota\,0.2636,\\0.3203+\iota\,0.2576\end{smallmatrix}\right),\left(\begin{smallmatrix}0.2556+\iota\,0.2782,\\0.2983+\iota\,0.2903\end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix}0.2790+\iota\,0.2900,\\0.2802+\iota\,0.3220\end{smallmatrix}\right),\left(\begin{smallmatrix}0.2899+\iota\,0.2855,\\0.2639+\iota\,0.2761\end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix}0.2882+\iota\,0.3324,\\0.2583+\iota\,0.2474\end{smallmatrix}\right),\left(\begin{smallmatrix}0.2912+\iota\,0.2837,\\0.2028+\iota\,0.2716\end{smallmatrix}\right)\right)$

Table 10. Weighted product values "$B_i{}^{WP}$".

B_1	B_2	B_3	B_4
$\left(\left(\begin{array}{c} 0.1992 + \iota\, 0.1798, \\ 0.2478 + \iota\, 0.2358 \end{array}\right),\right.$ $\left.\left(\begin{array}{c} 0.1322 + \iota\, 0.2368, \\ 0.2327 + \iota\, 0.2326 \end{array}\right)\right)$	$\left(\left(\begin{array}{c} 0.1290 + \iota\, 0.1576, \\ 0.3073 + \iota\, 0.2509 \end{array}\right),\right.$ $\left.\left(\begin{array}{c} 0.1425 + \iota\, 0.1632, \\ 0.2776 + \iota\, 0.2802 \end{array}\right)\right)$	$\left(\left(\begin{array}{c} 0.1544 + \iota\, 0.1689, \\ 0.2786 + \iota\, 0.3143 \end{array}\right),\right.$ $\left.\left(\begin{array}{c} 0.1628 + \iota\, 0.1736, \\ 0.2628 + \iota\, 0.2795 \end{array}\right)\right)$	$\left(\left(\begin{array}{c} 0.1614 + \iota\, 0.1961, \\ 0.2487 + \iota\, 0.2433 \end{array}\right),\right.$ $\left.\left(\begin{array}{c} 0.1645 + \iota\, 0.1745, \\ 0.1973 + \iota\, 0.2515 \end{array}\right)\right)$

Table 11. B_i values.

Step 4 Also compute $B_i{}^{WP}$ by utilizing the formula

$$B_i{}^{WP} = \left\{ \begin{array}{c} \sqrt{1 - \min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(1 - \left(\alpha_{i\overline{\mathcal{R}}}\right)^2\right)^{\mathcal{O}'}\right)\right)^{\frac{1}{\mathcal{O}'}}\right)} + \iota\sqrt{1 - \min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(1 - \left(\beta_{i\overline{\mathcal{R}}}\right)^2\right)^{\mathcal{O}'}\right)\right)^{\frac{1}{\mathcal{O}'}}\right)}, \\ \sqrt{\min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(\delta_{i\overline{\mathcal{R}}}\right)^{2\mathcal{O}'}\right)\right)^{\frac{1}{\mathcal{O}'}}\right)} + \iota\sqrt{\min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(\gamma_{i\overline{\mathcal{R}}}\right)^{2\mathcal{O}'}\right)\right)^{\frac{1}{\mathcal{O}'}}\right)}, \\ \sqrt{1 - \min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(1 - \left(\alpha_{i\underline{R}}\right)^2\right)^{\mathcal{O}'}\right)\right)^{\frac{1}{\mathcal{O}'}}\right)} + \iota\sqrt{1 - \min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(1 - \left(\beta_{i\underline{R}}\right)^2\right)^{\mathcal{O}'}\right)\right)^{\frac{1}{\mathcal{O}'}}\right)}, \\ \sqrt{\min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(\delta_{i\underline{R}}\right)^{2\mathcal{O}'}\right)\right)^{\frac{1}{\mathcal{O}'}}\right)} + \iota\sqrt{\min\left(1, \left(\sum_{i=1}^{m}\left(\beth_i\left(\gamma_{i\underline{R}}\right)^{2\mathcal{O}'}\right)\right)^{\frac{1}{\mathcal{O}'}}\right)} \end{array} \right\}$$

Hence $B_i{}^{WP}$ values are given in (Table 10).

Step 5 Finally use WASPAS techniques for ordering alternatives as given by.

$$B_i = \frac{B_i{}^{WS} + B_i{}^{WP}}{2} = 0.5 * B_i{}^{WS} + 0.5 * B_i{}^{WP}$$

Table 11 for B_i values are given below

Step 6 Now we utilize the formula given in def. 6 to compute score values and score values are given by.

$$S(B_1) = 0.47488, \ S(B_2) = 0.43453$$

$$S(B_3) = 0.44099, \ S(B_4) = 0.46946$$

Hence ranking of the alternative is given by.

Algorithm 2
To handle the multicriteria decision-making techniques, many decision-making tools have been developed. Assume that $A_L = \{A_{L1}, A_{L2}, A_{L3}, ,\ldots, A_{Lm}\}$ is the collection of m' alternatives and $A_T = \{A_{T1}, A_{T2}, A_{T3}, \ldots, A_{Tn}\}$ represents n' attributes. Let $\{e_1, e_2, e_3, \ldots, e_\hbar\}$ represents "$\hbar$" decision analyst for each alternative A_{Li} $(i = 1, 2, 3, \ldots, m)$ against attributes A_{Tj} $(j = 1, 2, 3, ., n)$. Let $\beth = (\beth_1, \beth_2, \beth_3, \ldots, \beth_n)^T$ be the WVs for attributes A_{Tj} and $\varpi = (\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_\hbar)^T$ be the WVs for experts e_z $(z = 1, 2, 3, \ldots, \hbar)$ such that $\sum_{j=1}^{n}\beth_j = 1$ and $\sum_{z=1}^{\hbar}\varpi_z = 1$. The step-wise algorithm is given by

Step 1 Assume decision analysts deliver their evaluation as CPyFRNs corresponding to each attribute. The decision matrix is given by.

$$\mathcal{M}_{m \times n} = \left[\begin{array}{cccc} \left\{\left(\begin{array}{c}\alpha_{11\overline{\mathcal{R}}} + \beta_{11\overline{\mathcal{R}}}, \\ \delta_{11\overline{\mathcal{R}}} + \iota\gamma_{11\overline{\mathcal{R}}}\end{array}\right), \left(\begin{array}{c}\alpha_{11\underline{R}} + \iota\beta_{11\underline{R}}, \\ \delta_{11\underline{R}} + \iota\gamma_{11\underline{R}}\end{array}\right)\right\} & \left\{\left(\begin{array}{c}\alpha_{12\overline{\mathcal{R}}} + \beta_{12\overline{\mathcal{R}}}, \\ \delta_{12\overline{\mathcal{R}}} + \iota\gamma_{12\overline{\mathcal{R}}}\end{array}\right), \left(\begin{array}{c}\alpha_{12\underline{R}} + \iota\beta_{12\underline{R}}, \\ \delta_{12\underline{R}} + \iota\gamma_{12\underline{R}}\end{array}\right)\right\} & \cdots & \left\{\left(\begin{array}{c}\alpha_{1n\overline{\mathcal{R}}} + \beta_{1n\overline{\mathcal{R}}}, \\ \delta_{1n\overline{\mathcal{R}}} + \iota\gamma_{1n\overline{\mathcal{R}}}\end{array}\right), \left(\begin{array}{c}\alpha_{1n\underline{R}} + \iota\beta_{1n\underline{R}}, \\ \delta_{1n\underline{R}} + \iota\gamma_{1n\underline{R}}\end{array}\right)\right\} \\ \left\{\left(\begin{array}{c}\alpha_{21\overline{\mathcal{R}}} + \beta_{21\overline{\mathcal{R}}}, \\ \delta_{21\overline{\mathcal{R}}} + \iota\gamma_{21\overline{\mathcal{R}}}\end{array}\right), \left(\begin{array}{c}\alpha_{21\underline{R}} + \iota\beta_{21\underline{R}}, \\ \delta_{21\underline{R}} + \iota\gamma_{21\underline{R}}\end{array}\right)\right\} & \left\{\left(\begin{array}{c}\alpha_{22\overline{\mathcal{R}}} + \beta_{22\overline{\mathcal{R}}}, \\ \delta_{22\overline{\mathcal{R}}} + \iota\gamma_{22\overline{\mathcal{R}}}\end{array}\right), \left(\begin{array}{c}\alpha_{22\underline{R}} + \iota\beta_{22\underline{R}}, \\ \delta_{22\underline{R}} + \iota\gamma_{22\underline{R}}\end{array}\right)\right\} & \cdots & \left\{\left(\begin{array}{c}\alpha_{2n\overline{\mathcal{R}}} + \beta_{2n\overline{\mathcal{R}}}, \\ \delta_{2n\overline{\mathcal{R}}} + \iota\gamma_{2n\overline{\mathcal{R}}}\end{array}\right), \left(\begin{array}{c}\alpha_{2n\underline{R}} + \iota\beta_{2n\underline{R}}, \\ \delta_{2n\underline{R}} + \iota\gamma_{2n\underline{R}}\end{array}\right)\right\} \\ \vdots & \vdots & \ddots & \vdots \\ \left\{\left(\begin{array}{c}\alpha_{m1\overline{\mathcal{R}}} + \beta_{m1\overline{\mathcal{R}}}, \\ \delta_{m1\overline{\mathcal{R}}} + \iota\gamma_{m1\overline{\mathcal{R}}}\end{array}\right), \left(\begin{array}{c}\alpha_{m1\underline{R}} + \iota\beta_{m1\underline{R}}, \\ \delta_{m1\underline{R}} + \iota\gamma_{m1\underline{R}}\end{array}\right)\right\} & \left\{\left(\begin{array}{c}\alpha_{m2\overline{\mathcal{R}}} + \beta_{m2\overline{\mathcal{R}}}, \\ \delta_{m2\overline{\mathcal{R}}} + \iota\gamma_{m2\overline{\mathcal{R}}}\end{array}\right), \left(\begin{array}{c}\alpha_{m2\underline{R}} + \iota\beta_{m2\underline{R}}, \\ \delta_{m2\underline{R}} + \iota\gamma_{m2\underline{R}}\end{array}\right)\right\} & \ddots & \left\{\left(\begin{array}{c}\alpha_{mn\overline{\mathcal{R}}} + \beta_{mn\overline{\mathcal{R}}}, \\ \delta_{mn\overline{\mathcal{R}}} + \iota\gamma_{mn\overline{\mathcal{R}}}\end{array}\right), \left(\begin{array}{c}\alpha_{mn\underline{R}} + \iota\beta_{mn\underline{R}}, \\ \delta_{mn\underline{R}} + \iota\gamma_{mn\underline{R}}\end{array}\right)\right\} \end{array} \right]$$

Step 2 Assessment of combined matrix by using the notion of CPyFRYWA AOs.

Step 3 Normalize the decision-making matrix in Step 1 by using the formula as.

$$
\mathcal{N}_{ij} = \begin{cases}
\left(\begin{pmatrix} \alpha_{ij_{\overline{\mathcal{R}}}} + \beta_{ij_{\overline{\mathcal{R}}}}, \\ \delta_{ij_{\overline{\mathcal{R}}}} + \iota\gamma_{ij_{\overline{\mathcal{R}}}} \end{pmatrix}, \\ \begin{pmatrix} \alpha_{ij_{\underline{R}}} + \iota\beta_{ij_{\underline{R}}}, \\ \delta_{ij_{\underline{R}}} + \iota\gamma_{ij_{\underline{R}}} \end{pmatrix} \right) & \text{for benefit - type criteria} \\[3em]
\left(\begin{pmatrix} \delta_{ij_{\overline{\mathcal{R}}}} + \iota\gamma_{ij_{\overline{\mathcal{R}}}}, \\ \alpha_{ij_{\overline{\mathcal{R}}}} + \beta_{ij_{\overline{\mathcal{R}}}} \end{pmatrix}, \\ \begin{pmatrix} \delta_{ij_{\underline{R}}} + \iota\gamma_{ij_{\underline{R}}}, \\ \alpha\delta_{ij_{\underline{R}}} + \iota\beta_{ij_{\underline{R}}} \end{pmatrix} \right) & \text{for cost - type criteria}
\end{cases}
$$

Where $\left(\begin{pmatrix} \alpha_{ij_{\overline{\mathcal{R}}}} + \beta_{ij_{\overline{\mathcal{R}}}}, \\ \delta_{ij_{\overline{\mathcal{R}}}} + \iota\gamma_{ij_{\overline{\mathcal{R}}}} \end{pmatrix}, \\ \begin{pmatrix} \alpha_{ij_{\underline{R}}} + \iota\beta_{ij_{\underline{R}}}, \\ \delta_{ij_{\underline{R}}} + \iota\gamma_{ij_{\underline{R}}} \end{pmatrix} \right)$ is the complement of $\left(\begin{pmatrix} \delta_{ij_{\overline{\mathcal{R}}}} + \iota\gamma_{ij_{\overline{\mathcal{R}}}}, \\ \alpha_{ij_{\overline{\mathcal{R}}}} + \beta_{ij_{\overline{\mathcal{R}}}} \end{pmatrix}, \\ \begin{pmatrix} \delta_{ij_{\underline{R}}} + \iota\gamma_{ij_{\underline{R}}}, \\ \alpha\delta_{ij_{\underline{R}}} + \iota\beta_{ij_{\underline{R}}} \end{pmatrix} \right)$

Step 4 Utilization of CPyFRYWA or CPyFRYWG AOS and find out the aggregated result.

Step 5 Use the formula in def. (6) to get the score values.

Step 6 Rank the results and find out the best alternative.

Results of Algorithm 2
Utilization of CPyFRYWA AOs
Step 1 Here we use the data of (Tables 5, 6 and 7).

Step 2 Assessment of combined matric by using the notion of CPyFRYWA AOs is given in (Table 8).

Step 3 No need to normalize the combined matrix because all attributes are benefit types.

Step 4 Now we use the notion of CPyFRYWA AOS and find out the aggregated result. The obtained outcomes are given by.

$$
B_1 = \left(\begin{pmatrix} 0.3926 + \iota\,0.3386, \\ 0.2370 + \iota\,0.2309 \end{pmatrix}, \\ \begin{pmatrix} 0.2456 + \iota\,0.4647, \\ 0.2158 + \iota\,0.2280 \end{pmatrix} \right), \quad B_2 = \left(\begin{pmatrix} 0.2460 + \iota\,0.3046, \\ 0.2942 + \iota\,0.2441 \end{pmatrix}, \\ \begin{pmatrix} 0.2664 + \iota\,0.3135, \\ 0.2562 + \iota\,0.2699 \end{pmatrix} \right)
$$

$$
B_3 = \left(\begin{pmatrix} 0.2871 + \iota\,0.3234, \\ 0.2771 + \iota\,0.3065 \end{pmatrix}, \\ \begin{pmatrix} 0.3058 + \iota\,0.3371, \\ 0.2618 + \iota\,0.2757 \end{pmatrix} \right), \quad B_4 = \left(\begin{pmatrix} 0.3025 + \iota\,0.3775, \\ 0.2389 + \iota\,0.2392 \end{pmatrix}, \\ \begin{pmatrix} 0.3103 + \iota\,0.3400, \\ 0.1916 + \iota\,0.2305 \end{pmatrix} \right)
$$

Step 5 Now we compute the score values by using the def. (6). The score values are given by.

$$
S\,(B_1) = 0.5662, \ S\,(B_2) = 0.5095, \ S\,(B_3) = 0.5165, \ S\,(B_4) = 0.5537
$$

Step 6 According to score values the ranking results are given by.

$$
A_{L1} > A_{L4} > A_{L3} > A_{L2}
$$

Utilization of CPyFRYWG AOs
Step 1 Same as above.

Step 2 Same as above.

Step 3 Same as above.

Step 4 Now we use the notion of CPyFRYWG AOS and find out the outcomes. The obtained outcomes are given by.

$$
B_1 = \left(\begin{pmatrix} 0.3025 + \iota\,0.3189, \\ 0.2584 + \iota\,0.2407 \end{pmatrix}, \\ \begin{pmatrix} 0.2390 + \iota\,0.3712, \\ 0.2489 + \iota\,0.2371 \end{pmatrix} \right), \quad B_2 = \left(\begin{pmatrix} 0.2236 + \iota\,0.2636, \\ 0.3203 + \iota\,0.2576 \end{pmatrix}, \\ \begin{pmatrix} 0.2556 + \iota\,0.2782, \\ 0.2983 + \iota\,0.2903 \end{pmatrix} \right)
$$

Structures	Score values	Ranking results	Optimal result
CFS[40]	Cannot discuss CPyFR data	No result	No solution
CIFS[41]	Cannot discuss CPyFR data	No result	No solution
CPyFS[42]	Cannot discuss CPyFR data	No result	No solution
FRS[31]	Cannot discuss CPyFR data	No result	No solution
IFRS[32]	Cannot discuss CPyFR data	No result	No solution
Multi-granulation PyFRS[33]	Cannot discuss CPyFR data	No result	No solution
The WASPAS Method based on IFS[43]	Cannot discuss CPyFR data	No result	No solution
The WASPAS techniques for PyF data[44]	Cannot discuss CPyFR data	No result	No solution
Complex intuitionistic fuzzy rough[45] approach	Cannot discuss CPyFR data	No result	No solution
Hesitant FS[13,14] approaches	Cannot discuss CPyFR data	No result	No solution
The WASPAS method based on the proposed theory	$S(B_1) = 0.47488$, $S(B_2) = 0.43453$ $S(B_3) = 0.44099$, $S(B_4) = 0.46946$	$A_{L1} > A_{L4} > A_{L3} > A_{L2}$	A_{L1}
CPyFRYWA AOs (proposed)	$S(B_1) = 0.5662$, $S(B_2) = 0.5082$ $S(B_3) = 0.5165$, $S(B_4) = 0.5537$	$A_{L1} > A_{L4} > A_{L3} > A_{L2}$	A_{L1}
CPyFRYWG AOs (proposed)	$S(B_1) = 0.5308$, $S(B_2) = 0.4930$ $S(B_3) = 0.50027$, $S(B_4) = 0.5269$	$A_{L1} > A_{L4} > A_{L3} > A_{L2}$	A_{L1}

Table 12. Overall results for comparative analysis.

$$B_3 = \left(\begin{array}{c} \left(\begin{array}{c} 0.2790 + \iota\, 0.2900, \\ 0.2802 + \iota\, 0.3220 \end{array} \right), \\ \left(\begin{array}{c} 0.2899 + \iota\, 0.2855, \\ 0.2639 + \iota\, 0.2761 \end{array} \right) \end{array} \right), \quad B_4 = \left(\begin{array}{c} \left(\begin{array}{c} 0.2882 + \iota\, 0.3324, \\ 0.2583 + \iota\, 0.2474 \end{array} \right), \\ \left(\begin{array}{c} 0.2912 + \iota\, 0.2837, \\ 0.2028 + \iota\, 0.2716 \end{array} \right) \end{array} \right)$$

Step 5 Now we compute the score values by using the def. (6). The score values are given by.

$$S(B_1) = 0.5308, \ S(B_2) = 0.4818, \ S(B_3) = 0.5002, \ S(B_4) = 0.5269$$

Step 6 According to score values the ranking results are given by.

$$A_{L1} > A_{L4} > A_{L3} > A_{L2}$$

Comparative analysis

Here in this section, we have to develop a comparative analysis of the proposed study to depict the advantages of the proposed theory. Moreover, we have to mention the limitations of the previous theories and determine that initiative approaches are more advanced and more suitable for handling complicated data. We will talk about the data in Tables 5, 6 and 7 now.

Tamir et al.[40] presented the idea of a CFS, which has a limited structure due to the potential for data loss. The rough structure, which consists of the upper and lower approximations, cannot be discussed in this proposed framework[40]. It means this structure has some gaps in its structure to address the complete information. Furthermore, this structure is unable to address the non-membership grade as well. The concept of a CIFS, as introduced by Ali et al.[41], allows for the Cartesian discussion of MG and NGs. However, because it is unable to address the roughness in its structure, we can notice that structure is also limited and it can discuss limited information. Labassi et al.[42] recently established the idea of CPyFS in Cartesian form. Despite being a more advanced structure than the notion presented by Ali et al.[42], it is unable to address the upper and lower approximations. The concepts of FRS[31], IFRS[32], and multi-granulation PyFRS[33] can all be used to talk about the roughness of their respective structural formulations. However, if fuzzy MG and NGs are in Cartesian form, and the decision-makers argue their facts in a complex manner. Certain requirements are absent from the structure mentioned above. If we compare our work with the WASPAS technique based on previously limited notions of intuitionist fuzzy set[43], Pythagorean fuzzy data[44]. The data of the proposed theory can never be handled by prevailing theories while introduced work can handle all limitations that exist in previous structures. In this regard, we may state that the current theory is dominated by progressed work. The analysis of the results given in (Table 12), shows that the data covered in Tables 5, 6 and 7 is based on CPyFR information and it can never be handled by all the above existing notions. Table 12 shows the overall results. Recently Mahmood et al.[45] introduced the idea of a complex intuitionistic fuzzy rough set model and it is based on Tamir's idea of CFS. We can notice that this approach uses the condition that the sum of the upper and lower approximation of MG and NMG must belong to [0, 1]. But we can note that if the decision maker provides their assessment in the form of data like $\left(\left(\begin{array}{c} 0.3 + \iota\, 0.5, \\ 0.5 + \iota\, 0.6 \end{array} \right), \left(\begin{array}{c} 0.2 + \iota\, 0.3, \\ 0.4 + \iota\, 0.5 \end{array} \right) \right)$ then we can notice that the basic condition is violated

Different values of the parameter	score values	Ranking results
For $\mho = 5$	$S(B_1) = 0.4745,$ $S(B_2) = 0.4335$ $S(B_3) = 0.4402,$ $S(B_4) = 0.4682$	$A_{L1} > A_{L4} > A_{L3} > A_{L2}$
For $\mho = 6$	$S(B_1) = 0.4746,$ $S(B_2) = 0.4330$ $S(B_3) = 0.4398,$ $S(B_4) = 0.4676$	$A_{L1} > A_{L4} > A_{L3} > A_{L2}$
For $\mho = 7$	$S(B_1) = 0.4748,$ $S(B_2) = 0.4329$ $S(B_3) = 0.4397,$ $S(B_4) = 0.4673$	$A_{L1} > A_{L4} > A_{L3} > A_{L2}$
For $\mho = 8$	$S(B_1) = 0.4750,$ $S(B_2) = 0.4330$ $S(B_3) = 0.4397,$ $S(B_4) = 0.4672$	$A_{L1} > A_{L4} > A_{L3} > A_{L2}$
For $\mho = 9$	$S(B_1) = 0.4752,$ $S(B_2) = 0.4332$ $S(B_3) = 0.4397,$ $S(B_4) = 0.4673$	$A_{L1} > A_{L4} > A_{L3} > A_{L2}$
For $\mho = 10$	$S(B_1) = 0.4755,$ $S(B_2) = 0.4334$ $S(B_3) = 0.4398,$ $S(B_4) = 0.4674$	$A_{L1} > A_{L4} > A_{L3} > A_{L2}$
For $\mho = 11$	$S(B_1) = 0.4757,$ $S(B_2) = 0.4337$ $S(B_3) = 0.4398,$ $S(B_4) = 0.4675$	$A_{L1} > A_{L4} > A_{L3} > A_{L2}$

Table 13. Sensitivity analysis.

Methods	Discussion of fuzzy data	Discussion of fuzzy rough data	Discussion of complex data
CFS[40]	Yes	No	Yes
CIFS[41]	Yes	No	Yes
CPyFS[42]	Yes	No	Yes
FRS[31]	Yes	Yes	No
IFRS[32]	Yes	Yes	No
Multi-granulation PyFRS[33]	Yes	Yes	No
The WASPAS method based on IFS[43]	Yes	No	No
The WASPAS techniques for PyF data[44]	Yes	No	No
Complex intuitionistic fuzzy rough[45] approach	Yes	Yes	Yes
The WASPAS method based on the proposed theory	Yes	Yes	Yes
CPyFRYWA AOs (proposed)	Yes	Yes	Yes
CPyFRYWG AOs (proposed)	Yes	Yes	Yes

Table 14. Characteristic analysis of the proposed theory.

because $(0.5 + 0.6) \notin [0, 1]$. Therefore, we can say that the devised form of CIFRS by Mahmood et al.[45] is limited. The introduced idea can handle this limitation because of using a more advance condition that the sum of the square of upper and lower approximation values of MG and NMG must belong to [0, 1] and in this case, the condition is satisfied. Hence, we can say that the chance of data loss decreases. If we compare our work with hesitant FS[13,14] we can see that the data covered in Tables 5, 6 and 7 can never be covered by idea in[13,14]. This idea can never discuss the upper and lower approximations that show the limitations of the existing work. The developed approach can handle such a situation. The initiated approach can assist in MCDM approaches to handle the more advance information. The developed approach helps in business engineering to handle the issues and it can provide assistance to prioritize the solutions.

Sensitivity analysis of the proposed work
In this subsection, we have analyzed the sensitivity analysis of the proposed work by using different values of parameters. The overall score values and ranking results are given in (Table 13).

From the results of Table 13 we can notice that in all of these cases, the best result in these cases is "A_{L1}" that show the consistency and reliability of the introduced approach. As there is no change in ranking result, we can say that the introduced approach is more reliable and authentic.

Moreover, the characteristic analysis of the developed approach is given in (Table 14).

Full form of the abbreviation/symbols	Short form/symbols meaning
CFS	Complex fuzzy set
CIFS	Complex intuitionistic fuzzy set
CPyFS	Complex Pythagorean fuzzy set
FRS	Fuzzy rough set
IFRS	Intuitionistic fuzzy rough set
PyFRS	Pythagorean fuzzy rough set
CPyFRS	Complex Pythagorean fuzzy rough set
CPyFRN	Complex Pythagorean fuzzy rough number
CPyFRYWA AOs	Complex Pythagorean fuzzy rough Yager weighted average aggregation operators
CPyFRYWG AOs	Complex Pythagorean fuzzy rough Yager weighted geometric aggregation operators
WASPAS	Weighted Aggregated Sum Product Assessment
$\left(\left(\alpha_{\overline{\mathcal{R}}} + \iota\,\beta_{\overline{\mathcal{R}}}, \delta_{\overline{\mathcal{R}}} + \iota\,\gamma_{\overline{\mathcal{R}}}\right), \left(\alpha_{\underline{R}} + \iota\,\beta_{\underline{R}}, \delta_{\underline{R}} + \iota\,\gamma_{\underline{R}}\right)\right)$	$\alpha_{\overline{\mathcal{R}}}=$ Real part of upper approximation of membership grade
	$\beta_{\overline{\mathcal{R}}}=$ Imaginary part of upper approximation of membership grade
	$\delta_{\overline{\mathcal{R}}}=$ Real part of upper approximation of non-membership grade
	$\gamma_{\overline{\mathcal{R}}}=$ Imaginary part of upper approximation of non-membership grade
	$\alpha_{\underline{R}}=$ Real part of lower approximation of membership grade
	$\beta_{\underline{R}}=$ Imaginary part of lower approximation of membership grade
	$\delta_{\underline{R}}=$ Real part of lower approximation of non-membership grade
	$\gamma_{\underline{R}}=$ Imaginary part of lower approximation of non-membership grade
A_L	This symbol is used as an alternative.
A_T	This symbol is used for attributes.

Table 15. Nomenclature.

Conclusion

WASPAS approach is the MCDM approach that can combine the weighted sum and weighted product models to handle decision-making problems. Prioritization of possible solutions regarding business engineering is MADM problem. To discuss this issue, we have developed the idea of a CPyFR set based on Tamir's idea of CFS. To discuss the utilization of this approach to aggregation theory, we have utilized Yager's t-norm and t-conorm. We have developed the idea of CPyFRYWA and CPyFRYWG AOs. The developed approach can help to handle the chance of data loss in the form of upper and lower approximation and more accurate decisions can be made based on initiated work. In this way, the developed theory is superior to many other existing theories like CIFS in polar and Cartesian form, and CPyFS in Cartesian and polar form. Moreover, to discuss the utilization of the delivered approach we have developed the WASPAS approach and discussed the applicability of this approach to business engineering. The comparative analysis of the introduced work depicts the importance and feasibility of the developed theory. The sensitivity analysis for the introduced work shows the reliability of the delivered. We can notice for different values of parameters there is no change in the ranking result and the best alternative is the same in all cases which is evidence of the reliability of the introduced work.

The developed notions are limited because these notions used the condition that $0 \leq \left(\alpha_{\overline{\mathcal{R}}}\right)^2 + \left(\delta_{\overline{\mathcal{R}}}\right)^2 \leq 1$ and $0 \leq \left(\beta_{\overline{\mathcal{R}}}\right)^2 + \left(\gamma_{\overline{\mathcal{R}}}\right)^2 \leq 1$ and $0 \leq \left(\alpha_{\overline{\mathcal{R}}}\right)^2 + \left(\delta_{\overline{\mathcal{R}}}\right)^2 \leq 1$ and $0 \leq \left(\beta_{\underline{R}}\right)^2 + \left(\gamma_{\underline{R}}\right)^2 \leq 1$. We can notice that whenever decision-makers provide theory assessment as $\begin{pmatrix} 0.5 + \iota\,0.9,\ 0.4 + \iota\,0.8, \\ 0.3 + \iota\,0.5,\ 0.2 + \iota\,0.7 \end{pmatrix}$ then we can notice that for this data the main condition fails to cover it because we can see that $sum\left(0.9^2,\ 0.8^2\right) = 1.45 \notin [0,\ 1]$ and the main condition $0 \leq \left(\beta_{\overline{\mathcal{R}}}\right)^2 + \left(\gamma_{\overline{\mathcal{R}}}\right)^2 \leq 1$ is violated. Hence we can say that introduced notions are also limited.

These notions can be extended to complex q-rung orthopair fuzzy rough set theory. Moreover, some new developments can be made on this idea like distance measures as given in[46]. A new algorithm can be developed based on the initiated work as discussed in[47]. Some distance ranking methods can be formulated based on the developed approach as covered in[48]. Additionally, MCDM approach can be developed as proposed in[49].

Moreover, the nomenclature Table is given in (Table 15).

Data availability

The data will be available on reasonable request to corresponding author or anyone can use the data by just citing the article.

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Author contributions

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Declarations

Competing interests

The authors declare no competing interests.

Ethics declaration statement

The authors state that this is their original work and it is neither submitted nor under consideration in any other journal simultaneously.

Human and animal participants

This article does not contain any studies with human participants or animals performed by any of the authors.

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