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Interactive multi-criteria group decision-making with probabilistic linguistic information for emergency assistance of COVID-19

Shu-Ping Wan [a,*], Wen-Bo Huang Cheng [a], Jiu-Ying Dong [b]

[a] School of Information Technology, Jiangxi University of Finance and Economics, Nanchang 330013, China
[b] School of Statistics, Jiangxi University of Finance and Economics, Nanchang 330013, China

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ABSTRACT

This paper develops a new method for interactive multi-criteria group decision-making (MCGDM) with probabilistic linguistic information and applies to the emergency assistance area selection of COVID-19 for Wuhan. First, a new possibility degree for PLTSs is defined and a new possibility degree algorithm is devised to rank a series of probabilistic linguistic term sets (PLTSs). Second, some new operational laws of PLTSs based on the Archimedean copulas and co-copulas are defined. A generalized probabilistic linguistic Choquet (GPLC) operator and a generalized probabilistic linguistic hybrid Choquet (GPLHC) operator are developed and their desirable properties are discussed in details. Third, a tri-objective nonlinear programming model is constructed to determine the weights of DMs. This model is transformed into a linear programming model to solve. The fuzzy measures of criterion subsets are derived objectively by establishing a goal programming model. Fourth, using the probabilistic linguistic Gumbel weighted average (PLGWA) operator, the collective normalized decision matrix is obtained by aggregating all individual normalized decision matrices. The overall evaluation values of alternatives are derived by the probabilistic linguistic Gumbel hybrid Choquet (PLGHC) operator. The ranking order of alternatives is generated. Finally, an emergency assistance example is illustrated to validate the proposed method of this paper.

1. Introduction

In 2020, a new coronavirus COVID-19 broke out all over the world. Wuhan in China was also suffering COVID-19. As an important transportation hub with large population flow, Wuhan has great difficulty to prevent and control the epidemic. To win the battle of the epidemic earlier and better, a lot of national medical support teams provide emergency assistance to the hospitals of Wuhan. How to select an appropriate area to assist is an urgent issue. Recently, lots of emergency events become more serious and urgent. Some methods [1–3] have been proposed to solve emergency decision-making, which can be ascribed as a type of the multi-criteria group decision-making (MCGDM) problems. Since the emergency assistance area selection of COVID-19 can be evaluated from different aspects, the emergency event of assistance area selection can also be regarded as a MCGDM problem.

MCGDM is an important component of decision science. Because of the uncertainty and ambiguity of human thinking, it is difficult for decision makers (DMs) to deliver accurate information for complex decision-making problems. Zadeh [4] introduced fuzzy sets (FSs) to express the uncertain information. FSs have been extensively applied to MCGDM. However, it is hard for DMs to quantify the information characters of MCGDM problems directly in some specified situations. DMs tended to express their preferences in vague qualitative linguistic terms such as "excellent", "good", "bad" rather than precise quantitative numerical values. Currently, some extension models of linguistic terms have been proposed, such as 2-tuple fuzzy linguistic representation model [5], uncertain linguistic model [6], intuitionistic fuzzy linguistic model [7] and hesitant fuzzy linguistic model [8]. Xu [9] proposed a subscript-symmetric additive linguistic term set (LTS). However, due to the discreteness of information features, it is not convenient to calculate and analyze many information features. Then, Xu [10] further extended the discrete LTS to a continuous LTS. Rodriguez et al. [11] proposed a conception of hesitant fuzzy linguistic term set (HFLTS). Different with traditional LTS, HFLTS uses several linguistic terms provided

* Correspondence to: School of Information Technology, Jiangxi University of Finance and Economics, No. 169, East Shuang-gang Road, Economic and Technological Development district, Nanchang 330013, China.
E-mail address: shupingwan@163.com (S.-P. Wan).

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by DMs to describe the evaluation information taking the hesitant degree of DMs into account. For a HFLTS, the weights of linguistic terms provided by DMs are equal. However, it is impractical to require DMs to provide linguistic terms with equal weight. HFLTSs may not be suitable to express linguistic terms with different weights. Consequently, Pang et al. [12] proposed the concept of probabilistic linguistic term set (PLTS) in 2016. PLTS can not only contain more than one possible linguistic term, but also reflect the corresponding probability information, which can avoid the information loss. To reflect the DM's cognitive certainty and uncertainty in the group decision making (GDM) process, the probabilistic linguistic preference relations [13,14] have received great attention.

Up to now, the investigations of PLTSs have made fruitful achievements. The existing progresses on the PLTSs mainly contain the following four aspects:

(1) Ranking of PLTSs. Pang et al. [12] compared the PLTSs by the score function and deviation degree of PLTSs. However, the ranking order of two PLTSs obtained by Pang et al. [12] may turn to be the opposite when giving a small perturbation of the probability information. In order to gain robustness, Bai et al. [15] ranked the PLTSs by using lower and upper bounds of the linguistic terms and corresponding probability information. Wu & Liao [16] ranked the PLTSs by gained and lost dominance score functions, which considered the individual regret values and the group utility values of alternatives. Feng et al. [17] ranked PLTSs by using the new probability formula obtained from the main structure of QUALIFLEX (Qualitative flexible) multiple-criteria method. Zhang et al. [18] built a probabilistic linguistic-based deviation model to identify the decision results in multi-expert multi-criteria decision making (MEMCDM).

(2) Information measures for PLTSs. Lin et al. [19] proposed the distance measure of PLTSs and used the probability degree to rank the alternatives for MCGDM. To enrich the calculation of PLTSs, Lin et al. [20] developed a novel distance measure for PLTSs and proposed an entropy measure to measure the uncertainty degree of PLTSs. Wu et al. [21] advanced three kinds of probabilistic linguistic distance measures reflecting on the difference of linguistic terms and probabilities, which do not need to add linguistic terms to the smaller one with the improved Borda rule. To improve the accuracy and visualization of comparison, Xian et al. [22] compared multiple PLTSs by calculating the novel similarity measure based on the RRD (relative repetition degree) and DD (diversity degree). Tang et al. [23] proposed inclusion measure for PLTSs and presented the relationships among the distance, similarity, entropy and inclusion measures.

(3) Operational laws and aggregation operators for PLTSs. Mao et al. [24] defined some new operational laws for PLTSs by using Archimedean triangular norm (t-norm) and triangular conorm (t-conorm or s-norm) and then defined a generalized probabilistic linguistic Hamacher ordered weighted averaging (GPLHOWA) operator. Liu & Teng [25] combined Muirhead mean aggregation operators with PLTSs based on the Archimedean t-conorm and t-norm and linguistic scale functions. Liu & Li [26] proposed a probabilistic linguistic-dependent weighted average (PLDWA) operator based on the prospect theory. Liu & Li [27] extended the generalized Maclaurin symmetric mean (GMSM) operator into probabilistic linguistic information and proposed four new GMSM operators for PLTSs for multi-criteria decision making (MCDM).

(4) Methods for probabilistic linguistic decision-making method. Pang et al. [12] extended TOPSIS method to the probabilistic linguistic group decision environment. Liu & Teng [28] extended PL-TODIM method to evaluate alternative products through consumer opinions regarding product performance. Ahmad et al. [29] proposed a MCGDM method based on VIKOR. Liao et al. [30] introduced the distillation process and Borda rule into the algorithm of the probabilistic linguistic ELECTRE III (PL-ELECTRE III) method. Wu & Liao [31] extended the quality function deployment (QFD) into the probabilistic linguistic context to get the fuzzy weights of design requirements. Furthermore, Wu & Liao [31] developed a probabilistic linguistic ORESTE (organísation, rangement et Synthèse de données relarionnelles, in French) method to obtain the preference, indifference and incomparability relations between the alternatives.

Although the above methods are valid for solving MCGDM with PLTSs, none of them considered the interactions among criteria. However, there are many interactions among criteria in some real decision situations. Consider the emergency assistance area selection based on supply medical support capacity, medical supply delivery speed and other criteria. A stronger supply medical support capacity often needs faster medical supply delivery speed. It is easily seen that supply medical support capacity and medical supply delivery speed are complementary interaction. Additionally, the supply medical support capacity is a qualitative criterion. The evaluation of the supply medical support capacity can be represented by a PLTS $\{s_2(0.2), s_3(0.3), s_4(0.2)\}$, which means that the possible linguistic term of the evaluation of supply medical support capacity may be s_2 or s_3 or s_4. Meanwhile, 0.2, 0.3 and 0.2 are the corresponding probabilities of the linguistic terms s_2, s_3 and s_4. Therefore, the emergency assistance area selection may be ascribed to a kind of the interactive MCGDM with PLTSs. The Choquet integral is suitable to describe the interactions among criteria. It is necessary to develop some Choquet integral operators of PLTSs and investigate some new methods for solving such problems. To achieve this goal, this paper first proposes a new possibility degree algorithm for ranking a series of PLTSs. Then, some new operational laws of PLTSs based on the Archimedean copulas and co-copulas are defined. Considering the interactions among criteria, we develop a probabilistic linguistic Gumbel weighted average (PLGWA) operator, a generalized probabilistic linguistic Choquet (GPLC) operator and a probabilistic linguistic Gumbel hybrid Choquet (PLGHC) operator. Finally, a new method for the interactive MCGDM with PLTSs is put forward and applied to the emergency assistance area selection of COVID-19 for Wuhan. The main contributions of this paper are clarified as follows:

(1) A new possibility degree of PLTSs is defined and then a new possibility degree algorithm is proposed to rank a series of PLTSs. A new similarity degree of PLTSs is defined considering the linguistic scale function of linguistic terms.

(2) Some new operational laws of PLTSs based on the Archimedean copulas and co-copulas are defined. Considering the interactions among criteria, the generalized probabilistic linguistic Choquet (GPLC) operator and generalized probabilistic linguistic hybrid Choquet (GPLHC) operator are developed. Some desirable properties for these operators are discussed in details.

(3) Motivated by TOPSIS, a tri-objective nonlinear programming model is constructed to determine the weights of DMs. This model is transformed into a linear programming model to solve. To derive the fuzzy measures of criteria subsets, an optimization model is built and transformed into a goal programming model for resolution.

(4) Using the PLGWA operator, the collective normalized decision matrix is obtained by aggregating all individual normalized decision matrices. The overall evaluation values of alternatives are derived by the PLGHC operator. The ranking order of alternatives is then generated by the proposed possibility degree algorithm of PLTSs. Thereby, a new method for the interactive MCGDM with PLTSs is put forward.

Section 2 briefly reviews some concepts of PLTSs. Section 3 presents a new possibility degree algorithm to rank PLTSs and defines a new similarity degree of PLTSs. Section 4 defines some new operational laws of PLTSs based on the Archimedean copulas. Section 5 develops some generalized Choquet integral operators of PLTSs. Section 6 proposes a new method for interactive MCGDM with probabilistic linguistic information. Section 7 provides an emergency assistance area selection of COVID-19 to illustrate the validity of the method proposed in this paper. Section 8 draws some conclusions and ends this paper.

2. Preliminaries

In this section, some preliminaries about LTS, PLTS, Archimedean copulas and co-copulas are briefly reviewed to facilitate the discussions.

2.1. Linguistic term set

Definition 1 ([9]). Let $S = \{s_\alpha | \alpha = 0, 1, \ldots, 2\tau\}$ be a finite and totally ordered discrete LTS, where s_α represents a possible value for a linguistic term, and τ is a positive integer. Especially, s_0 and $s_{2\tau}$ denote the lower and the upper limits of linguistic terms, respectively. Furthermore, any two linguistic terms $s_\alpha, s_\beta \in S$ satisfy that $s_\alpha > s_\beta$ if and only if $\alpha > \beta$.

To preserve all given linguistic information, Xu [10] extended the discrete LTS S to a continuous LTS $\bar{S} = \{s_\alpha | \alpha \in [0, 2q]\}$, where $q(q > \tau)$ is a sufficiently large positive integer. If $s_\alpha \in S$, then s_α is called an original linguistic term. If $s_\alpha \in \bar{S}$, then s_α is called a virtual linguistic term.

2.2. Probabilistic linguistic term set

Definition 2 ([12]). Let $S = \{s_\alpha | \alpha = 0, 1, \ldots, 2\tau\}$ be a LTS, a PLTS is defined as:

$$L(p) = \{L^{(k)}(p^{(k)}) | L^{(k)} \in S, p^{(k)} \geq 0, k = 1, 2, \ldots, \#L(p), \sum_{k=1}^{\#L(p)} p^{(k)} \leq 1\} \quad (1)$$

where $L^{(k)}(p^{(k)})$ represents the linguistic term $L^{(k)}$ associated with the probability $p^{(k)}$, and $\#L(p)$ is the number of all different linguistic terms in $L(p)$.

Definition 3 ([12]). Given a PLTS $L(p)$ with $\sum_{k=1}^{\#L(p)} p^{(k)} < 1$, the normalized PLTS $\dot{L}(p)$ is defined as:

$$\dot{L}(p) = \{\dot{L}^{(k)}(\dot{p}^{(k)}) | k = 1, 2, \ldots, \#\dot{L}(p)\} \quad (2)$$

where $\dot{p}^{(k)} = p^{(k)} / \sum_{k=1}^{\#L(p)} p^{(k)}$ for all $k = 1, 2, \ldots, \#L(p)$.

Definition 4 ([12]). Let $L_1(p) = \{L_1^{(k_1)}(p_1^{(k_1)}) | k_1 = 1, 2, \ldots, \#L_1(p)\}$ and $L_2(p) = \{L_2^{(k_2)}(p_2^{(k_2)}) | k_2 = 1, 2, \ldots, \#L_2(p)\}$ be two PLTSs, where $\#L_1(p)$ and $\#L_2(p)$ are the numbers of linguistic terms in $L_1(p)$ and $L_2(p)$ respectively. If $\#L_1(p) > \#L_2(p)$, then add $\#L_1(p) - \#L_2(p)$ linguistic terms to $L_2(p)$. Moreover, the added linguistic terms are the smallest linguistic term in $L_2(p)$ and the probabilities of added linguistic terms are zero.

Definition 5 ([32]). Given a PLTS $L(p) = \{L^{(k)}(p^{(k)}) | L^{(k)} \in S, p^{(k)} \geq 0, k = 1, 2, \ldots, \#L(p), \sum_{k=1}^{\#L(p)} p^{(k)} \leq 1\}$, where $r^{(k)}$ is the subscript of $L^{(k)}$. An ascending ordered normalized PLTS can be obtained in the following:

(1) If all elements in a PLTS $L(p)$ are with different values of $r^{(k)}p^{(k)} (k = 1, 2, \ldots, \#L(p))$, then all the elements are arranged according to the values of $r^{(k)}p^{(k)}$ in ascending order.

(2) If there are two or more elements with equal value of $r^{(k)}p^{(k)}$, then

(i) When the subscripts $r^{(k)} (k = 1, 2, \ldots, \#L(p))$ are unequal, such the elements are arranged according to the values of $r^{(k)} (k = 1, 2, \ldots, \#L(p))$ in ascending order;

(ii) When the subscripts $r^{(k)} (k = 1, 2, \ldots, \#L(p))$ are equal, such the elements are arranged according to the values of $p^{(k)} (k = 1, 2, \ldots, \#L(p))$ in ascending order.

According to above, after all elements of $L(p)$ is ordered by Definition 5, a PLTS $L(p) = \{L^{(k)}(p^{(k)}) | k = 1, 2, \ldots, \#L(p)\}$ is transformed into an ascending ordered normalized PLTS $\tilde{L}(p) = \{\tilde{L}^{(k)}(\tilde{p}^{(k)}) | k = 1, 2, \ldots, \#\tilde{L}(p)\}$.

Definition 6 ([33]). Let $S = \{s_\alpha | \alpha = 0, 1, \ldots, 2\tau\}$ be a LTS, if $\upsilon_i \in (0, 1)$ is a numeric value, then the linguistic scale function g is mapped from s_i to $\upsilon_i (i = 0, 1, \ldots, 2\tau)$, which is represented as follows:

$$g : s_i \rightarrow \upsilon_i \quad (3)$$

where υ_i reflects the preference of the DMs and $0 < \upsilon_0 < \upsilon_1 < \upsilon_2 < \cdots < \upsilon_{2\tau} < 1$.

Wang et al. [33] presented three different forms of linguistic scale functions.

(Form 1) The evaluation scale of the linguistic information is divided on average:

$$g(s_i) = \upsilon_i = \frac{i}{2\tau} \quad (4)$$

(Form 2) The absolute deviation between adjacent linguistic subscripts increases from the middle of the linguistic term set to both ends (The value of a can be determined using a subjective method):

$$g(s_i) = \upsilon_i = \begin{cases} \frac{a^\tau - a^{\tau-i}}{2a^\tau - 2} & (i = 0, 1, 2, \ldots, \tau) \\ \frac{a^\tau + a^{i-\tau} - 2}{2a^\tau - 2} & (i = \tau + 1, \tau + 2, \ldots, 2\tau) \end{cases} \quad (5)$$

The linguistic scale varies with the value of a. The value of a can be obtained experimentally or subjectively [34]. On the one hand, through large number of experimental research data [35], it can be concluded that a is most likely to be obtained within the interval [1.36,1.4]. On the other hand, a can also be determined by using a subjective method. Assuming that indicator A is far more important than indicator B, the importance ratio is m, then $a^k = m$ (k represents the scale level) and $a = \sqrt[k]{m}$. At present, vast majority of researchers believe that $m = 9$ is the upper limit of the importance ratio [35]. In general, if the scale level is 7, then $a = \sqrt[7]{9} \approx 1.37$ [34].

(Form 3) The absolute deviation between adjacent linguistic subscripts decreases from the middle of the linguistic term set to both ends:

$$g(s_i) = \upsilon_i = \begin{cases} \frac{\tau^\alpha - (\tau - i)^\alpha}{2\tau^\alpha} & (i = 0, 1, 2, \ldots, \tau) \\ \frac{\tau^\beta + (i - \tau)^\beta}{2\tau^\beta} & (i = \tau + 1, \tau + 2, \ldots, 2\tau) \end{cases} \tag{6}$$

where $\alpha, \beta \in (0, 1]$. If $\alpha = \beta = 1$, then Form 3 is reduced to Form 1.

Let $\dot{L}_1(p) = \{\dot{L}_1^{(k)}(\dot{p}_1^{(k)})|k_1 = 1, 2, \ldots, \#\dot{L}_1(p)\}$ and $\dot{L}_2(p) = \{\dot{L}_2^{(k)}(\dot{p}_2^{(k)})|k_2 = 1, 2, \ldots, \#\dot{L}_2(p)\}$ be two normalized PLTSs. By using Definitions 4 and 5, we can turn $\dot{L}_1(p)$ and $\dot{L}_2(p)$ into two ascending ordered normalized PLTSs $\tilde{L}_1(P)$ and $\tilde{L}_2(P)$ with equal number of linguistic terms. For simplicity, we still denote the two ascending ordered normalized PLTSs by $\tilde{L}_1(p)$ and $\tilde{L}_2(p)$.

Definition 7 (*[32]*)**.** The distance between two ascending ordered normalized PLTSs $\tilde{L}_1(p)$ and $\tilde{L}_2(p)$ is defined as

$$d(\tilde{L}_1(p), \tilde{L}_2(p)) = \sum_{k=1}^{\#\tilde{L}_1(p)} p(\tilde{r}_1^{(k)}, \tilde{r}_2^{(k)}) d(\tilde{r}_1^{(k)}, \tilde{r}_2^{(k)}) \tag{7}$$

where $\#\tilde{L}_1(p) = \#\tilde{L}_2(p)$, $d(\tilde{r}_1^{(k)}, \tilde{r}_2^{(k)}) = (\tilde{r}_1^{(k)} - \tilde{r}_2^{(k)})/T$ and $p(\tilde{r}_1^{(k)}, \tilde{r}_2^{(k)}) = p(\tilde{r}_1^{(k)})p(\tilde{r}_2^{(k)}) = \tilde{p}_1^{(k)}\tilde{p}_2^{(k)}$, T is the number of linguistic terms in the LTS S.

Based on Definition 7, we define Minkowski distance between $\tilde{L}_1(p)$ and $\tilde{L}_2(p)$ as follows:

$$d(\tilde{L}_1(p), \tilde{L}_2(p)) = \left[\sum_{k=1}^{\#\tilde{L}_1(p)} \tilde{p}_1^{(k)}\tilde{p}_2^{(k)} (|(g(\tilde{r}_1^{(k)}) - g(\tilde{r}_2^{(k)}))/T|)^\rho \right]^{1/\rho} \tag{8}$$

where $\rho > 0$ is a parameter. If $\rho = 1$, Eq. (8) is reduced to Hamming distance; if $\rho = 2$, Eq. (8) is reduced to Euclidean distance; if $\rho \to \infty$, Eq. (8) is reduced to Chebyshev distance.

2.3. Archimedean copulas and co-copulas

Definition 8 (*[36]*)**.** A copula Cp is named as an Archimedean copula, which is denoted by $Cp(x_1, x_2) = \psi[Ge(x_1), Ge(x_2)]$, $\forall (x_1, x_2) \in [0, 1]^2$, if the generated functions Ge and ψ satisfy the following conditions:

(1) The generated function Ge is strictly decreasing and continuous function from $[0, 1]$ to $[0, +\infty]$ with $Ge(1) = 0$;

(2) The function ψ from $[0, +\infty]$ to $[0, 1]$ is defined as follows:

$$\psi(s) = \begin{cases} Ge^{-1}(x), & x \in [0, Ge(0)] \\ 0\ , & x \in [Ge(0), +\infty] \end{cases} \tag{9}$$

Considering the special situation where Cp is a strictly increasing function on $[0, 1]^2$, $Ge(0) = +\infty$ and $\psi = Ge^{-1}$ on $[0, +\infty]$, Genest & Mackay [37] proposed a special Archimedean copula as follows:

$$Cp(x_1, x_2) = Ge^{-1}[Ge(x_1) + Ge(x_2)] \tag{10}$$

Definition 9 (*[38]*)**.** Let Cp be a copula. The co-copula is defined as:

$$Cp^*(x_1, x_2) = 1 - Cp(1 - x_1, 1 - x_2) \tag{11}$$

3. Possibility degree and similarity degree of PLTSs

This section develops a new possibility degree algorithm to rank PLTSs and defines a new similarity degree of PLTSs.

3.1. Existing ranking methods of PLTSs

Definition 10 (*[24]*)**.** Let $\dot{L}_1(p) = \{\dot{L}_1^{(k_1)}(\dot{p}_1^{(k_1)})|k_1 = 1, 2, \ldots, \#\dot{L}_1(p)\}$ and $\dot{L}_2(p) = \{\dot{L}_2^{(k_2)}(\dot{p}_2^{(k_2)})|k_2 = 1, 2, \ldots, \#\dot{L}_2(p)\}$ be two normalized PLTSs. A binary relation $B(\dot{L}_1^{(k_1)}, \dot{L}_2^{(k_2)})$ between $\dot{L}_1^{(k_1)}$ and $\dot{L}_2^{(k_2)}$ is defined as follows:

$$B(\dot{L}_1^{(k_1)}, \dot{L}_2^{(k_2)}) = \begin{cases} \dot{p}_1^{(k_1)}\dot{p}_2^{(k_2)}, & \text{if } \dot{L}_1^{(k_1)} > \dot{L}_2^{(k_2)} \\ \frac{1}{2}\dot{p}_1^{(k_1)}\dot{p}_2^{(k_2)}, & \text{if } \dot{L}_1^{(k_1)} = \dot{L}_2^{(k_2)} \\ 0, & \text{if } \dot{L}_1^{(k_1)} < \dot{L}_2^{(k_2)} \end{cases} \tag{12}$$

The possibility degree $P(\dot{L}_1(p) \geq \dot{L}_2(p))$ is defined as follows [24]:

$$P(\dot{L}_1(p) \geq \dot{L}_2(p)) = \sum_{k_1=1}^{\#\dot{L}_1(p)} \sum_{k_2=1}^{\#\dot{L}_2(p)} B(\dot{L}_1^{(k_1)}, \dot{L}_2^{(k_2)}) \tag{13}$$

Let $r^{(k)}$ be the subscript of $\dot{L}^{(k)}$ in a normalized PLTS $\dot{L}(p) = \{\dot{L}^{(k)}(\dot{p}^{(k)})|k = 1, 2, \ldots, \#\dot{L}(p)\}$, $r^- = \min_k\{r^{(k)}\}$ and $r^+ = \max_k\{r^{(k)}\}$ are the lower and upper bounds of the subscripts of $\dot{L}^{(k)}(k = 1, 2, \ldots, \#\dot{L}(p))$, $\dot{p}^-$ and $\dot{p}^+$ are the corresponding probabilities, respectively. The range value $R(\dot{L}(p))$ is defined as [24]:

$$R(\dot{L}(p)) = r^+\dot{p}^+ - r^-\dot{p}^- \tag{14}$$

Using Eqs. (12) and (13), a preorder of normalized PLTSs $\dot{L}_1(p)$ and $\dot{L}_2(p)$ is presented as follows [24]:
(i) If $P(\dot{L}_1(p) \geq \dot{L}_2(p)) > 0.5$, then $\dot{L}_1(p)$ is bigger than $\dot{L}_2(p)$, denoted by $\dot{L}_1(p) > \dot{L}_2(p)$;
(ii) If $P(\dot{L}_1(p) \geq \dot{L}_2(p)) = 0.5$, then
(a) if $R(\dot{L}_1(p)) < R(\dot{L}_2(p))$, then $\dot{L}_1(p) > \dot{L}_2(p)$;
(b) if $R(\dot{L}_1(p)) = R(\dot{L}_2(p))$, then $\dot{L}_1(p)$ is indifferent to $\dot{L}_2(p)$, denoted by $\dot{L}_1(p) \sim \dot{L}_2(p)$.

Example 1. Let $S = \{s_\alpha|\alpha = 0, 1, \ldots, 6\}$ be a LTS, $\dot{L}_1(p) = \{s_1(0.5), s_2(0.5)\}$, $\dot{L}_2(p) = \{s_0(0.6), s_1(0.4)\}$, $\dot{L}'_1(p) = \{s_1(1)\}$, $\dot{L}'_2(p) = \{s_0(0.8), s_1(0.2)\}$ be four normalized PLTSs.

By Eq. (13), the probability degrees of $\dot{L}_1(p) \geq \dot{L}_2(p)$ and $\dot{L}'_1(p) \geq \dot{L}'_2(p)$ are calculated as:

$$P(\dot{L}_1(p) \geq \dot{L}_2(p)) = 0.9, P(\dot{L}'_1(p) \geq \dot{L}'_2(p)) = 0.9.$$

It is easy to see that the probability degree of Eq. (13) cannot distinguish the difference between $\dot{L}_1(p) \geq \dot{L}_2(p)$ and $\dot{L}'_1(p) \geq \dot{L}'_2(p)$.

3.2. A new possibility degree algorithm for ranking PLTSs

As in Example 1, there are still some different PLTSs that cannot be distinguished. Similar to Eq. (12), a new binary relation $B'(\dot{L}_1^{(k_1)}, \dot{L}_2^{(k_2)})$ between $\dot{L}_1^{(k_1)}$ and $\dot{L}_2^{(k_2)}$ is defined as follows:

$$B'(\dot{L}_1^{(k_1)}, \dot{L}_2^{(k_2)}) = \begin{cases} \dot{p}_1^{(k_1)}\dot{p}_2^{(k_2)}, & \text{if } \dot{L}_1^{(k_1)} > \dot{L}_2^{(k_2)} \\ \frac{1}{2} \times \frac{\dot{p}_1^{(k_1)}}{\dot{p}_1^{(k_1)}+\dot{p}_2^{(k_2)}}\dot{p}_1^{(k_1)}\dot{p}_2^{(k_2)}, & \text{if } \dot{L}_1^{(k_1)} = \dot{L}_2^{(k_2)} \\ 0, & \text{if } \dot{L}_1^{(k_1)} < \dot{L}_2^{(k_2)} \end{cases} \tag{15}$$

To get the precise ranking result, a new possibility degree algorithm is designed in the following.

Definition 11. Let $\dot{L}_1(p) = \{\dot{L}_1^{(k_1)}(\dot{p}_1^{(k_1)})|k_1 = 1, 2, \ldots, \#\dot{L}_1(p)\}$ and $\dot{L}_2(p) = \{\dot{L}_2^{(k_2)}(\dot{p}_2^{(k_2)})|k_2 = 1, 2, \ldots, \#\dot{L}_2(p)\}$ be two normalized PLTSs. A new possibility degree $P'(\dot{L}_1(p) \geq \dot{L}_2(p))$ ($\dot{L}_1(p)$ is not inferior to $\dot{L}_2(p)$) is defined as follows:

$$P'(\dot{L}_1(p) \geq \dot{L}_2(p)) = \frac{\sum_{k_1=1}^{\#\dot{L}_1(p)}\sum_{k_2=1}^{\#\dot{L}_2(p)} B'(\dot{L}_1^{(k_1)}, \dot{L}_2^{(k_2)})}{\sum_{k_1=1}^{\#\dot{L}_1(p)}\sum_{k_2=1}^{\#\dot{L}_2(p)} B'(\dot{L}_1^{(k_1)}, \dot{L}_2^{(k_2)}) + \sum_{k_1=1}^{\#\dot{L}_1(p)}\sum_{k_2=1}^{\#\dot{L}_2(p)} B'(\dot{L}_2^{(k_2)}, \dot{L}_1^{(k_1)})} \tag{16}$$

Property 1. *Let $\dot{L}_1(p) = \{\dot{L}_1^{(k_1)}(\dot{p}_1^{(k_1)})|k_1 = 1, 2, \ldots, \#\dot{L}_1(p)\}$ and $\dot{L}_2(p) = \{\dot{L}_2^{(k_2)}(\dot{p}_2^{(k_2)})|k_2 = 1, 2, \ldots, \#\dot{L}_2(p)\}$ be two normalized PLTSs, where $\#\dot{L}_1(P) = \#\dot{L}_2(P)$. The desirable properties of the new possibility degree in Definition 11 are presented as follows:*
(i) (Normalization) $0 \leq P'(\dot{L}_1(p) \geq \dot{L}_2(p)) \leq 1$.
(ii) (Visualization) If $\dot{L}_1^{(k_1)-} > \dot{L}_2^{(k_2)+}$, then $P'(\dot{L}_1(p) \geq \dot{L}_2(p)) = 1$; if $\dot{L}_1^{(k_1)+} < \dot{L}_2^{(k_2)-}$, then $P'(\dot{L}_1(p) \geq \dot{L}_2(p)) = 0$ ($\dot{L}_i^{(k_i)-}$ and $\dot{L}_i^{(k_i)+}$ are the lower and upper bounds of the linguistic terms $\dot{L}_i^{(k_i)}(k_i = 1, 2, \ldots, \#\dot{L}_i(p))$.
(iii) (Complementarity) $P'(\dot{L}_1(p) \geq \dot{L}_2(p)) + P'(\dot{L}_2(p) \geq \dot{L}_1(p)) = 1$.
(iv) (Transitivity) If $P'(\dot{L}_1(p) \geq \dot{L}_2(p)) \leq 0.5$ and $P'(\dot{L}_2(p) \geq \dot{L}_3(p)) \leq 0.5$, then $P'(\dot{L}_1(p) \geq \dot{L}_3(p)) \leq 0.5$.

It is easy to prove the properties (i), (iii), and (iv) in Property 1 by Eq. (15) and (16). For the property (ii) of visualization, consider two PLTSs $\dot{L}_1(p) = \{\dot{L}_1^{(1)}(\dot{p}_1^{(1)}), \dot{L}_1^{(2)}(\dot{p}_1^{(2)}), \ldots, \dot{L}_1^{(\#\dot{L}_1(p))}(\dot{p}_1^{(\#\dot{L}_1(p))})\}$ and $\dot{L}_2(p) = \{\dot{L}_2^{(1)}(\dot{p}_2^{(1)}), \dot{L}_2^{(2)}(\dot{p}_2^{(2)}), \ldots, \dot{L}_2^{(\#\dot{L}_2(p))}(\dot{p}_2^{(\#\dot{L}_2(p))})\}$. Similar to the comparison between interval numbers, if $\dot{L}_1^{(k_1)+} = \dot{L}_1^{(\#\dot{L}_1(p))} < \dot{L}_2^{(k_2)-} = \dot{L}_2^{(1)}$, then $\dot{L}_1(p) \leq \dot{L}_2(p)$, namely $P'(\dot{L}_1(p) \geq \dot{L}_2(p)) = 0$; if $\dot{L}_1^{(k_1)-} = \dot{L}_1^{(1)} > \dot{L}_2^{(k_2)+} = \dot{L}_2^{(\#\dot{L}_2(p))}$, then $\dot{L}_1(p) \geq \dot{L}_2(p)$, namely $P'(\dot{L}_1(p) \geq \dot{L}_2(p)) = 1$.

If $P'(\dot{L}_1(p) \geq \dot{L}_2(p)) = 0.5$, it is hard to distinguish $\dot{L}_1(p)$ and $\dot{L}_2(p)$. According to the statistical method, when the mean values of two sets of numbers are equal, the variance values can be used to further compare two sets of numbers. Then, a new range value of PLTSs is defined in the sequel.

Definition 12. For a normalized PLTS $\dot{L}(p) = \{\dot{L}^{(k)}(\dot{p}^{(k)})|k = 1, 2, \ldots, \#\dot{L}(p)\}$, the new range value $R'(\dot{L}(p))$ is defined as:

$$R'(\dot{L}(p)) = g^+(\dot{L}^{(k)})\dot{p}^{+(k)} - g^-(\dot{L}^{(k)})\dot{p}^{-(k)} \tag{17}$$

where $g(\dot{L}^{(k)})$ is the linguistic scale function value of linguistic term $\dot{L}^{(k)}$. Let $g^-(\dot{L}^{(k)}) = \min_k\{g(\dot{L}^{(k)})\}$ and $g^+(\dot{L}^{(k)}) = \max_k\{g(\dot{L}^{(k)})\}$ be the lower and upper bounds of $g(\dot{L}^{(k)})$, where $\dot{p}^{+(k)}$ and $\dot{p}^{-(k)}$ are the corresponding probabilities, respectively.

Definition 13. A preorder of two normalized PLTSs $\dot{L}_1(p)$ and $\dot{L}_2(p)$ is defined as follows:
(i) If $P'(\dot{L}_1(p) \geq \dot{L}_2(p)) > 0.5$, then $\dot{L}_1(p)$ is bigger than $\dot{L}_2(p)$, namely $\dot{L}_1(p) > \dot{L}_2(p)$;
(ii) If $P'(\dot{L}_1(p) \geq \dot{L}_2(p)) = 0.5$, then
(a) If $R'(\dot{L}_1(p)) < R'(\dot{L}_2(p))$, then $\dot{L}_1(p)$ is bigger than $\dot{L}_2(p)$, namely $\dot{L}_1(p) > \dot{L}_2(p)$;
(b) If $R'(\dot{L}_1(p)) = R'(\dot{L}_2(p))$, then $\dot{L}_1(p)$ is indifferent to $\dot{L}_1(p)$, namely $\dot{L}_1(p) \sim \dot{L}_2(p)$.
The possibility degree algorithm for ranking a series of PLTSs $\dot{L}_i(p)(i = 1, 2, \ldots, n)$ is designed as:

Step 1. Calculate the possibility degree $P'_{ij} = P'(\dot{L}_i(p) \geq \dot{L}_j(p))(i, j = 1, 2, \ldots, n)$ by Eq. (16) and the range value $R'(\dot{L}_i(p))$ by Eq. (17) respectively.

Step 2. Aggregate P'_{ij} into the ranking value P'_i as follows:

$$P'_i = \sum_{j=1}^{n} P'_{ij}(i, j = 1, 2, \ldots, n) \tag{18}$$

Step 3. Rank PLTSs by the ranking values $P'_i(i = 1, 2, \ldots, n)$ in descending order. If the ranking of some PLTSs is equal, reorder these PLTSs by the range value $R'(\dot{L}_i(p))$ in ascending order.

Example 2. Continue to consider the four normalized PLTSs in Example 1, namely, $\dot{L}_1(p) = \{s_1(0.5), s_2(0.5)\}$, $\dot{L}_2(p) = \{s_0(0.6), s_1(0.4)\}$, $\dot{L}'_1(p) = \{s_1(1)\}$, $\dot{L}'_2(p) = \{s_0(0.8), s_1(0.2)\}$.

By Eq. (13), the possibility degree in Definition 10 is

$$P(\dot{L}'_1(p) \geq \dot{L}'_2(p)) = P(\dot{L}_1(p) \geq \dot{L}_2(p)) = 0.9.$$

By Eq. (16), the new probability degrees are calculated as follows:

$$P'(\dot{L}_1(p) \geq \dot{L}_2(p)) = 0.9506, P'(\dot{L}'_1(p) \geq \dot{L}'_2(p)) = 0.9815.$$

It has $P'(\dot{L}_1(p) \geq \dot{L}_2(p)) < P'(\dot{L}'_1(p) \geq \dot{L}'_2(p))$ by the new probability degrees, which shows that $\dot{L}_1(p) \geq \dot{L}_2(p)$ is different from $\dot{L}'_1(p) \geq \dot{L}'_2(p)$.

Hence, Eq. (16) can identify the difference between $\dot{L}_1(p) \geq \dot{L}_2(p)$ and $\dot{L}'_1(p) \geq \dot{L}'_2(p)$ rationally and validly. Additionally, the intensiveness of $\dot{L}'_1(p) \geq \dot{L}'_2(p)$ is superior to $\dot{L}_1(p) \geq \dot{L}_2(p)$. Hence, the distinguishing power of the new possibility degree in Definition 11 is stronger than that of the possibility degree in Definition 10.

To compare with the Mao et al.'s possibility degree method in [24], the new possibility degree method proposed in this paper is used to solve Examples 3–5 in [24].

Example 3. Consider three PLTSs $L_1(p) = \{s_1(0.4), s_2(0.6)\}$, $L_2(p) = \{s_0(0.2), s_2(0.8)\}$ and $L_3(p) = \{s_1(0.3999), s_2(0.6001)\}$ of Example 3 in [24].

By Eq. (16), one has $P'(L_2(p) \geq L_1(p)) = 0.60150$ and $P'(L_2(p) \geq L_3(p)) = 0.60145$, thus $L_2(p) \geq L_1(p)$ and $L_2(p) \geq L_3(p)$, which are the same as the ranking results of Example 3 in [24]. Thus, the possibility degree defined in this paper still ensures that the order between $L_1(p)$ and $L_2(p)$ remains unchanged with respect to small disturbances of $L_1(p)$, which verifies the robustness of the proposed possibility degree of this paper.

Example 4. Consider two PLTSs $L_1(p) = \{s_0(0.4), s_1(0.4), s_3(0.2)\}$ and $L_2(p) = \{s_0(0.3), s_1(0.7)\}$ of Example 4 in [24].

By Eq. (16), it yields that $P'(L_1(p) \geq L_2(p)) = 0.5065$. Thus, the ranking order is $L_1(p) \geq L_2(p)$, which is same as the result of Example 4 in [24].

Example 5. Consider four PLTSs $L_1(p) = \{s_{-3}(0.1), s_{-2}(0.4), s_1(0.3), s_2(0.2)\}$, $L_2(p) = \{s_{-1}(0.5), s_0(0.5)\}$, $L_3(p) = \{s_3(0.2), s_4(0.8)\}$ and $L_4(p) = \{s_3(0.4), s_4(0.6)\}$ based on $S = \{s_\alpha | \alpha = -4, \ldots - 1, 0, 1 \cdots, 4\}$ of Example 5 in [24].

Step 1. Calculate the possibility degree $P'_{ij} = P'(\dot{L}_i(p) \geq \dot{L}_j(p))(i, j = 1, 2, \ldots, n)$ by Eq. (16) as follows:

$$P'_{11} = 0.50,\ \ P'_{12} = 0.50,\ \ P'_{13} = 0, P'_{14} = 0,\ \ P'_{21} = 0.50,\ P'_{22} = 0.50,\ P'_{23} = 0,\ P'_{24} = 0,$$
$$P'_{31} = 1,\ \ P'_{32} = 1,\ \ P'_{33} = 0.50,\ \ P'_{34} = 0.6534,\ \ P'_{41} = 1,\ \ P'_{42} = 1,\ P'_{43} = 0.3466,\ \ P'_{44} = 0.50.$$

Calculate the range value $R'(\dot{L}_i(p))$ by Eq. (17) based on Form 1 (i.e., Eq. (4)) as:

$$R'(\dot{L}_1(p)) = 0.1375,\ \ R'(\dot{L}_2(p)) = 0.0625,\ \ R'(\dot{L}_3(p)) = 0.6250,\ \ R'(\dot{L}_4(p)) = -0.8000$$

Calculate the range value $R'(\dot{L}_i(p))$ by Eq. (17) based on Form 2 (i.e., Eq. (5)) (Let $a = \sqrt[8]{9} \approx 1.3160$) as:

$$R'(\dot{L}_1(p)) = 0.2186,\ \ R'(\dot{L}_2(p)) = 0.0395,\ \ R'(\dot{L}_3(p)) = 0.9361,\ \ R'(\dot{L}_4(p)) = 0.3721$$

Calculate the range value $R'(\dot{L}_i(p))$ by Eq. (17) based on Form 3 (i.e., Eq. (6)) (Let $\alpha = \beta = 0.8$)as:

$$R'(\dot{L}_1(p)) = 1.3523,\ \ R'(\dot{L}_2(p)) = 0.7579,\ \ R'(\dot{L}_3(p)) = 5.7027,\ \ R'(\dot{L}_4(p)) = 2.2158$$

Step 2. Aggregate P'_{ij} into the ranking value $P_i^{'}$ as follows:

$$P'_1 = 1,\ \ P'_2 = 1,\ \ P'_3 = 3.15,\ \ P'_4 = 2.85$$

Step 3. Rank PLTSs by $P_i^{'}(i = 1, 2, \ldots, n)$ in descending order.

The ranking order based on Form 1 (i.e., Eq. (4)) is generated as $L_3(p) \succ L_4(p) \succ L_2(p) \succ L_1(p)$.

The ranking order based on Form 2 (i.e., Eq. (5)) is generated as $L_3(p) \succ L_4(p) \succ L_2(p) \succ L_1(p)$.

The ranking order based on Form 3 (i.e., Eq. (6)) is generated as $L_3(p) \succ L_4(p) \succ L_2(p) \succ L_1(p)$.

By Eqs. (13) and (14), one has $P_1 = 1,\ \ P_2 = 1,\ \ P_3 = 3.1,\ \ P_4 = 2.9$, $R(\dot{L}_1(p)) = 0.7$, $R(\dot{L}_2(p)) = 0.5$, $R(\dot{L}_3(p)) = 2.6$ and $R(\dot{L}_4(p)) = 1.2$. The ranking order obtained by method [24] is $L_3(p) \succ L_4(p) \succ L_2(p) \succ L_1(p)$, which is the same as that obtained by the proposed new possibility degree algorithm of this paper. Although the ranking result between $L_1(p)$ and $L_2(p)$ obtained by this proposed method is same as that obtained by method [24], the range values $R'(\dot{L}_1(p))$ and $R'(\dot{L}_2(p))$ are different from $R(\dot{L}_1(p)) = 0.7$ and $R(\dot{L}_2(p)) = 0.5$ obtained by [24].

From the above examples, it is easily seen that the proposed possibility degree algorithm can take DM's different preferences of linguistic scale functions into account, which is more robust and more in accordance with real situations.

3.3. Similarity degree of PLTSs

Xian et al. [22] defined the RRD (relative repetition degree) of linguistic terms between PLTSs $L_1(p)$ and $L_2(p)$ as

$$RRD(L_1(p), L_2(p)) = (2\tau + 1)\frac{\alpha_1^T\alpha_2}{\max\{\#L_1(p), \#L_2(p)\}},$$

where $\alpha = (a_1, a_2, \ldots, a_{2\tau+1})^{\mathrm{T}}$ is the linguistic term vector, satisfying $a_i = \begin{cases} 1, & i-(\tau+1) \in r^{(k)} \\ 0, & i-(\tau+1) \notin r^{(k)} \end{cases}$.

The DD (diversity degree) of probability between PLTSs $L_1(p)$ and $L_2(p)$ is defined as [22]:

$$DD(L_1(p), L_2(p)) = \frac{1}{2}\left\|\beta_1 - \beta_2\right\| = \frac{1}{2}\sqrt{(\beta_1-\beta_2)^T(\beta_1-\beta_2)},$$

where $\beta = (b_1, b_2, \ldots, b_{2\tau+1})^{\mathrm{T}}$ is the linguistic term vector, satisfying $b_j = \begin{cases} p^{(k)}, & j-(\tau+1) \in r^{(k)} \\ 0, & j-(\tau+1) \notin r^{(k)} \end{cases}$.

The similarity degree between PLTSs $L_1(p)$ and $L_2(p)$ is defined as [22]:

$$SI(L_1(p), L_2(p)) = \begin{cases} 1 - \frac{DD(L_1(p),L_2(p))}{RRD(L_1(p),L_2(p))}, & \alpha_1^{\mathrm{T}}\alpha_2 \neq 0 \\ 0, & \alpha_1^{\mathrm{T}}\alpha_2 = 0 \end{cases} \tag{19}$$

Example 6. Consider three PLTSs $L_1(p) = \{s_1(0.5), s_2(0.5)\}$, $L_2(p) = \{s_1(0.5), s_3(0.5)\}$ and $L_3(p) = \{s_0(0.4), s_4(0.6)\}$ based on LTS $S = \{s_\alpha | \alpha = -4, \ldots, -1, 0, 1, \ldots, 4\}$. By Eq. (19), the similarity degree $SI(L_1(p), L_3(p)) = SI(L_2(p), L_3(p)) = 0$. However, the similarity degree between $L_1(p)$ and $L_3(p)$ is remarkably different from that between $L_2(p)$ and $L_3(p)$ intuitively. Thus, the similarity degree $SI(L_1(p), L_3(p)) = SI(L_2(p), L_3(p)) = 0$ may be a little unreasonable. To overcome this drawback, a new similarity degree is given below.

Definition 14. Let $S = \{s_\alpha | \alpha = 0, 1, \ldots, 2\tau\}$ be a LTS, $L_1(p) = \{L_1^{(k_1)}(p_1^{(k_1)}) | k_1 = 1, 2, \ldots, \#L_1(p)\}$ and $L_2(p) = \{L_2^{(k_2)}(p_2^{(k_2)}) | k_2 = 1, 2, \ldots, \#L_2(p)\}$ be two PLTSs, where $\#L_1(P) = \#L_2(P)$ and $g(L_i^{(k_i)})$ be the linguistic scale function value of linguistic term $L_i^{(k_i)}$. A similarity degree between $L_1(p)$ and $L_2(p)$ is defined as follows:

$$S(L_1(p), L_2(p)) = \frac{\sum_{k_1=1}^{\#L_1(p)}(p_1^{(k_1)}g(L_1^{(k_1)}))\cdot(p_2^{(k_2)}g(L_2^{(k_2)}))}{\sqrt{\sum_{k_1=1}^{\#L_1(p)}(p_1^{(k_1)}g(L_1^{(k_1)}))^2}\sqrt{\sum_{k_2=1}^{\#L_2(p)}(p_2^{(k_2)}g(L_2^{(k_2)}))^2}} \tag{20}$$

Property 2. *The similarity degree between $L_1(p)$ and $L_2(p)$ satisfies:*
(i) $0 \le S(L_1(p), L_2(p)) \le 1$; (ii) $S(L_1(p), L_2(p)) = S(L_2(p), L_1(p))$;
(iii) If $L_1(p) = L_2(p)$, then $S(L_1(p), L_2(p)) = 1$.

Proof. $L_1(p)$ and $L_2(p)$ can be regarded as two vectors $\varsigma = (g(L_1^{(1)})(p_1^{(1)}), g(L_1^{(2)})(p_1^{(2)}), \ldots, g(L_1^{(\#L_1(p))})(p_1^{(\#L_1(p))}))$ and $\xi = (g(L_2^{(1)})(p_2^{(1)}), g(L_2^{(2)})(p_2^{(2)}), \ldots, g(L_2^{(\#L_2(p))})(p_2^{(\#L_2(p))}))$, respectively. For property (i) in Property 2, the similarity degree between $L_1(p)$ and $L_1(p)$ is equivalent to the cosine between ς and ξ, namely $S(L_1(p), L_2(p)) = \cos(\alpha, \beta)$. Properties (i) and (ii) denote the boundedness and symmetric respectively. For property (iii), if $L_1(p) = L_2(p)$, then $\varsigma = \xi$, namely the angle between ς and ξ is 0. Therefore, $S(L_1(p), L_2(p)) = \cos 0 = 1$, which completes the proof of property (iii).

Example 7. Consider three PLTSs $L_1(p) = \{s_1(0.5), s_2(0.5)\}$, $L_2(p) = \{s_1(0.5), s_3(0.5)\}$ and $L_3(p) = \{s_0(0.4), s_4(0.6)\}$ in Example 6. By Eq. (20), the similarity degrees are obtained as $S(L_1(p), L_3(p)) = 0.89$ and $S(L_2(p), L_3(p)) = 0.95$, which are significantly different from that obtained by Eq. (19). Thus, Eq. (20) can overcome this deficiency of Eq. (19) and get a reasonable result.

4. New operational laws of PLTSs based on the Archimedean copulas and co-copulas

This section defines some new operational laws of PLTSs based on the Archimedean copulas and co-copulas. Some desirable properties of the new operational laws are discussed. Then, some specific cases are presented with respect to four different generated functions.

4.1. New operational laws of PLTSs based on the Archimedean copulas and co-copulas

This subsection defines some new operational laws of PLTSs based on the Archimedean copulas and co-copulas. Some desirable properties of the new operational laws are discussed.

Definition 15. Let $S = \{s_\alpha | \alpha = 0, 1, \ldots, 2\tau\}$ be a LTS, $L(p) = \{L^{(k)}(p^{(k)}) | k = 1, 2, \ldots, \#L(p)\}$, $L_1(p) = \{L_1^{(k_1)}(p_1^{(k_1)}) | k_1 = 1, 2, \ldots, \#L_1(p)\}$ and $L_2(p) = \{L_2^{(k_2)}(p_2^{(k_2)}) | k_2 = 1, 2, \ldots, \#L_2(p)\}$ be three PLTSs. The inverse function of Ge is denoted by Ge^{-1}. $\lambda \in [0, +\infty]$. The new operational laws of PLTSs based on Archimedean copulas and co-copulas are defined as follows:

(1) Additive operation:

$$L_1(p) \oplus_{Cp} L_2(p) = \bigcup_{\substack{k_1=1,2,\ldots,\#L_1(p) \\ k_2=1,2,\ldots,\#L_2(p)}} \left\{ g^{-1}(Cp^*(g(L_1^{(k_1)}), g(L_2^{(k_2)})))((p_1^{(k_1)} + p_2^{(k_2)} - p_1^{(k_1)}p_2^{(k_2)})/L) \right\}$$

Table 1
Four different types of common Archimedean copulas.

Types	Function	Copulas	Parameter
Gumbel	$Ge(x) = (-\ln x)^{\varepsilon}$	$Cp(x_1, x_2) = \exp\{-[(-\ln x_1)^{\varepsilon}] + [(-\ln x_2)^{\varepsilon}]^{1/\varepsilon}\}$	$\varepsilon \geq 1$
Clayton	$Ge(x) = x^{-\varepsilon} - 1$	$Cp(x_1, x_2) = (x_1^{-\varepsilon} + x_2^{-\varepsilon} - 1)^{1/\varepsilon}$	$\varepsilon \geq -1, \varepsilon \neq 0$
Frank	$Ge(x) = -\ln \frac{e^{-\varepsilon x}-1}{e^{-\varepsilon}-1}$	$Cp(x_1, x_2) = -\frac{1}{\varepsilon}\ln(1 + \frac{(e^{-\varepsilon x_1}-1)(e^{-\varepsilon x_2}-1)}{e^{-\varepsilon}-1})$	$\varepsilon \neq 0$
Joe	$Ge(x) = -\ln[1-(1-x)^{\varepsilon}]$	$Cp(x_1, x_2) = 1 - [(1-x_1)^{\varepsilon} + (1-x_2)^{\varepsilon} - (1-x_1)^{\varepsilon}(1-x_2)^{\varepsilon}]^{1/\varepsilon}$	$\varepsilon \geq 1$

Note: In Table 1, $Ge(x)$ and $Cp(x_1, x_2)$ are defined in Definition 8, and ε is the parameter of the function $Ge(x)$.

$$= \bigcup_{\substack{k_1=1,2,\ldots,\#L_1(p)\\ k_2=1,2,\ldots,\#L_2(p)}} \left\{g^{-1}[1 - Ge^{-1}(Ge(1-g(L_1^{(k_1)})) + Ge(1-g(L_2^{(k_2)})))]((p_1^{(k_1)} + p_2^{(k_2)} - p_1^{(k_1)}p_2^{(k_2)})/L)\right\};$$

where $L = \#L_1(p) + \#L_2(p) - 1$.

(2) Multiplication:

$$L_1(p) \otimes_{Cp} L_2(p) = \bigcup_{\substack{k_1=1,2,\ldots,\#L_1(p)\\ k_2=1,2,\ldots,\#L_2(p)}} \left\{g^{-1}(Cp(g(L_1^{(k_1)}), g(L_2^{(k_2)})))(p_1^{(k_1)}p_2^{(k_2)})\right\}$$

$$= \bigcup_{\substack{k_1=1,2,\ldots,\#L_1(p)\\ k_2=1,2,\ldots,\#L_2(p)}} \left\{g^{-1}[Ge^{-1}(Ge(g(L_1^{(k_1)})) + Ge(g(L_2^{(k_2)})))](p_1^{(k_1)}p_2^{(k_2)})\right\};$$

(3) Scalar-multiplication: $\lambda \odot_{Cp} L(p) = \bigcup_{k=1,2,\ldots,\#L(p)} \left\{g^{-1}[1 - Ge^{-1}(\lambda Ge(1-g(L^{(k)})))](p^{(k)})\right\}$;

(4) Power operation: $(L(p))^{\lambda} = \bigcup_{k=1,2,\ldots,\#L(p)} \left\{g^{-1}[Ge^{-1}(\lambda Ge(g(L^{(k)})))](p^{(k)})\right\}$.

Theorem 1. *The linguistic terms of PLTSs based on Archimedean copulas and co-copulas still belong to the LTS and thus the operation results of PLTSs with these new operational laws in Definition 15 are still PLTSs.*

Proof. For any $g(L_1^{(k_1)}), g(L_2^{(k_2)}) \in [0, 1]$, it holds that $Cp(g(L_1^{(k_1)}), g(L_2^{(k_2)})) \in [0, +\infty]$ and $g^{-1}(Cp(g(L_1^{(k_1)}), g(L_2^{(k_2)}))) \in [s_0, s_{2\tau}](k_1 = 1, 2, \ldots, \#L_1(p);\ k_2 = 1, 2, \ldots, \#L_2(p))$ when $Cp : [0, 1]^2 \to [0, +\infty]$ and $g^{-1} : [0, +\infty] \to [s_0, s_{2\tau}]$. Hence, the linguistic terms $g^{-1}(Cp(g(L_1^{(k_1)}), g(L_2^{(k_2)})))$ still belong to the LTS.

By Definition 15, it is easily seen that the operation results of PLTSs with these new operational laws are still PLTSs.

Theorem 2. *Let $L(p) = \{L^{(k)}(p^{(k)})|k = 1, 2, \ldots, \#L(p)\}$, $L_1(p) = \{L_1^{(k_1)}(p_1^{(k_1)})|k_1 = 1, 2, \ldots, \#L_1(p)\}$ and $L_2(p) = \{L_2^{(k_2)}(p_2^{(k_2)})|k_2 = 1, 2, \ldots, \#L_2(p)\}$ be any three PLTSs, $\lambda, \lambda_1, \lambda_2 \in [0, +\infty]$ be the positive real numbers. Some desirable properties of the new operational laws in Definition 15 are satisfied as follows:* (1)$L_1(p) \oplus_{Cp} L_2(p) = L_2(p) \oplus_{Cp} L_1(p)$; (2)$L_1(p) \otimes_{Cp} L_2(p) = L_2(p) \otimes_{Cp} L_1(p)$; (3)$\lambda \odot_{Cp} (L_1(p) \oplus_{Cp} L_2(p)) = (\lambda L_1(p)) \oplus_{Cp} (\lambda L_2(p))$; (4)$(L_1(p) \otimes_{Cp} L_2(p))^{\lambda} = (L_1(p))^{\lambda} \otimes_{Cp} (L_2(p))^{\lambda}$; (5)$(\lambda_1 + \lambda_2) \odot_{Cp} L(p) = (\lambda_1 L(p)) \oplus_{Cp} (\lambda_2 L(p))$; (6)$(L(p))^{\lambda_1} \otimes_{Cp} (L(p))^{\lambda_2} = (L(p))^{\lambda_1+\lambda_2}$; (7)$((L(p))^{\lambda_1})^{\lambda_2} = (L(p))^{\lambda_1\lambda_2}$; (8)$L_1(p) \oplus_{Cp} (L_2(p) \oplus_{Cp} L(p)) = (L_1(p) \oplus_{Cp} L_2(p)) \oplus_{Cp} L(p)$; (9)$L_1(p) \otimes_{Cp} (L_2(p) \otimes_{Cp} L(p)) = (L_1(p) \otimes_{Cp} L_2(p)) \otimes_{Cp} L(p)$.

The proof of Theorem 2 is presented in Appendix A.

4.2. Some different types of operational laws for PLTSs based on common Archimedean copulas

Wang et al. [39] summarized four different types of common Archimedean copulas including Gumbel copula, Clayton copula, Frank copula and Joe copula, which are shown in Table 1.

In the following, some different types of operational laws for PLTSs based on common Archimedean copulas can be obtained by Definition 15 and are listed in Table 2.

4.3. Comparison with Mao et al.'s operational laws

Example 8. Let $S = \{s_{\alpha}|\alpha = 0, 1, \ldots, 4\}$ be a LTS, Consider two PLTSs $L_1(p) = \{s_1(0.4), s_2(0.6)\}$ and $L_2(p) = \{s_1(0.3), s_3(0.7)\}$. To simplify the calculation process, let $Ge(x) = (-\ln x)^{\varepsilon}(\varepsilon = 1)$ (i.e., Gumbel type) and $\lambda = 2$.

The computation results based on Mao et al.'s operational laws and the proposed operational laws of this paper can be obtained and presented in Table 3.

As shown in Table 3, the computation results obtained by Mao et al.'s operational laws [24] are just as same as those obtained by the Gumbel operational laws (when $\varepsilon = 1$) and the linguistic scale function Form 1 (i.e., Eq. (4)). Therefore, the proposed operational laws based on Archimedean copulas and co-copulas greatly generalize Mao et al.'s operational laws based on Archimedean t-corm and t-conorm. The proposed operational laws use three different forms of linguistic scale functions to obtain the calculation results, whereas Mao et al. [24] only used Form 1 (i.e., Eq. (4)) to get the calculation results. In addition, DMs can select different linguistic scale functions and different types of Archimedean copulas according to their preference, which greatly enhances the flexibility of decision.

5. New aggregation operators of PLTSs based on the Archimedean copulas

This section develops a generalized probabilistic linguistic Choquet (GPLC) operator and a generalized probabilistic linguistic hybrid Choquet (GPLHC) operator. Some attractive properties of the proposed operators are discussed in details.

Table 2
Some different types of operational laws for PLTSs based on common Archimedean copulas.

Type	Function	New operational law
Gumbel	$Ge(x) = (-\ln x)^{\varepsilon}$ $(\varepsilon \geq 1)$	$L_1(p) \oplus_{Cp} L_2(p) = \bigcup_{\substack{k_1=1,2,\ldots,\#L_1(p)\\ k_2=1,2,\ldots,\#L_2(p)}} \left\{ g^{-1}\ [1-\exp(-((-\ln(1-g(L_1^{(k_1)})))^{\varepsilon}+(-\ln(1-g(L_2^{(k_2)})))^{\varepsilon})^{1/\varepsilon})](\frac{(p_1^{(k_1)}+p_2^{(k_2)}-p_1^{(k_1)}p_2^{(k_2)})}{L}) \right\}$ $L_1(p) \otimes_{Cp} L_2(p) = \bigcup_{\substack{k_1=1,2,\ldots,\#L_1(p)\\ k_2=1,2,\ldots,\#L_2(p)}} \left\{ g^{-1}[\exp(-((-\ln(g(L_1^{(k_1)})))^{\varepsilon}+(-\ln(g(L_2^{(k_2)})))^{\varepsilon})^{1/\varepsilon})](p_1^{(k_1)}p_2^{(k_2)}) \right\}$ $\lambda \odot_{Cp} L(p) = \bigcup_{k=1,2,\ldots,\#L(p)} \{g^{-1}[1-\exp(-(\lambda(-\ln(1-g(L^{(k_1)})))^{\varepsilon})^{1/\varepsilon})](p^{(k)})\}$ $(L(p))^{\lambda} = \bigcup_{k=1,2,\ldots,\#L(p)} \{g^{-1}[\exp(-(\lambda(-\ln(g(L^{(k)})))^{\varepsilon})^{1/\varepsilon})](p^{(k)})\}$
Clayton	$Ge(x) = x^{-\varepsilon} - 1$ $(\varepsilon \geq -1, \varepsilon \neq 0)$	$L_1(p) \oplus_{Cp} L_2(p) = \bigcup_{\substack{k_1=1,2,\ldots,\#L_1(p)\\ k_2=1,2,\ldots,\#L_2(p)}} \left\{ g^{-1}[1-((1-g(L_1^{(k_1)}))^{-\varepsilon}+(1-g(L_2^{(k_2)}))^{-\varepsilon}-1)^{-1/\varepsilon}](\frac{(p_1^{(k_1)}+p_2^{(k_2)}-p_1^{(k_1)}p_2^{(k_2)})}{L}) \right\}$ $L_1(p) \otimes_{Cp} L_2(p) = \bigcup_{\substack{k_1=1,2,\ldots,\#L_1(p)\\ k_2=1,2,\ldots,\#L_2(p)}} \left\{ g^{-1}[(g(L_1^{(k_1)})^{-\varepsilon}+g(L_2^{(k_2)})^{-\varepsilon}-1)^{-1/\varepsilon}](p_1^{(k_1)}p_2^{(k_2)}) \right\}$ $\lambda \odot_{Cp} L(p) = \bigcup_{k=1,2,\ldots,\#L(p)} \{g^{-1}[1-(\lambda((1-g(L^{(k)}))^{-\varepsilon}-1)+1)^{-1/\varepsilon}](p^{(k)})\}$ $(L(p))^{\lambda} = \bigcup_{k=1,2,\ldots,\#L(p)} \{g^{-1}[(\lambda((g(L^{(k)}))^{-\varepsilon}-1)+1)^{-1/\varepsilon}](p^{(k)})\}$
Frank	$Ge(x) = -\ln \frac{e^{-\varepsilon x}-1}{e^{-\varepsilon}-1}$ $(\varepsilon \neq 0)$	$L_1(p) \oplus_{Cp} L_2(p) = \bigcup_{\substack{k_1=1,2,\ldots,\#L_1(p)\\ k_2=1,2,\ldots,\#L_2(p)}} \left\{ g^{-1}[1+\frac{1}{\varepsilon}\ln(1+\frac{(\exp(-\varepsilon(1-g(L_1^{(k_1)})))-1)(\exp(-\varepsilon(1-g(L_2^{(k_2)})))-1)}{\exp(-\varepsilon)-1})](\frac{(p_1^{(k_1)}+p_2^{(k_2)}-p_1^{(k_1)}p_2^{(k_2)})}{L}) \right\}$ $L_1(p) \otimes_{Cp} L_2(p) = \bigcup_{\substack{k_1=1,2,\ldots,\#L_1(p)\\ k_2=1,2,\ldots,\#L_2(p)}} \left\{ g^{-1}[-\frac{1}{\varepsilon}\ln(1+\frac{(\exp(-\varepsilon g(L_1^{(k_1)}))-1)(\exp(-\varepsilon g(L_2^{(k_2)}))-1)}{\exp(-\varepsilon)-1})](p_1^{(k_1)}p_2^{(k_2)}) \right\}$ $\lambda \odot_{Cp} L(p) = \bigcup_{k=1,2,\ldots,\#L(p)} \left\{ g^{-1}[1+\frac{1}{\varepsilon}\ln((\frac{\exp(-\varepsilon(1-g(L^{(k)})))-1}{\exp(-\varepsilon)-1})^{\lambda}+1)](p^{(k)}) \right\}$ $(L(p))^{\lambda} = \bigcup_{k=1,2,\ldots,\#L(p)} \left\{ g^{-1}[-\frac{1}{\varepsilon}\ln((\frac{\exp(-\varepsilon(1-g(L^{(k)})))-1}{\exp(-\varepsilon)-1})^{\lambda}+1)](p^{(k)}) \right\}$
Joe	$Ge(x) = -\ln[1-(1-x)^{\varepsilon}]$ $(\varepsilon \geq 1)$	$L_1(p) \oplus_{Cp} L_2(p) = \bigcup_{\substack{k_1=1,2,\ldots,\#L_1(p)\\ k_2=1,2,\ldots,\#L_2(p)}} \left\{ g^{-1}[(g(L_1^{(k_1)})^{\varepsilon}+g(L_2^{(k_2)})^{\varepsilon}-g(L_1^{(k_1)})^{\varepsilon}g(L_2^{(k_2)})^{\varepsilon})^{1/\varepsilon}](\frac{(p_1^{(k_1)}+p_2^{(k_2)}-p_1^{(k_1)}p_2^{(k_2)})}{L}) \right\}$ $L_1(p) \otimes_{Cp} L_2(p) = \bigcup_{\substack{k_1=1,2,\ldots,\#L_1(p)\\ k_2=1,2,\ldots,\#L_2(p)}} \left\{ g^{-1}[1-((1-g(L_1^{(k_1)}))^{\varepsilon}+(1-g(L_2^{(k_2)}))^{\varepsilon}-(1-g(L_1^{(k_1)}))^{\varepsilon}(1-g(L_2^{(k_2)}))^{\varepsilon})^{1/\varepsilon}](p_1^{(k_1)}p_2^{(k_2)}) \right\}$ $\lambda \odot_{Cp} L(p) = \bigcup_{k=1,2,\ldots,\#L(p)} \{g^{-1}[(1-(1-g(L^{(k_1)})^{\varepsilon})^{\lambda})^{1/\varepsilon}](p^{(k)})\}$ $(L(p))^{\lambda} = \bigcup_{k=1,2,\ldots,\#L(p)} \{g^{-1}[1-(1-(1-(1-g(L^{(k_1)}))^{\varepsilon})^{\lambda})^{1/\varepsilon}](p^{(k)})\}$

Note: In Table 2, $L(p)$, $L_1(p)$, $L_2(p)$ and λ, λ_1, λ_2 are defined in Theorem 2.

Table 3
Computation results.

Operational laws	Results
Mao et al. [24]	$L_1(p) \oplus_A L_2(p) = \{s_{1.75}(0.19), s_{2.5}(0.24), s_{3.25}(0.27), s_{3.5}(0.30)\}$ $L_1(p) \otimes_A L_2(p) = \{s_{0.25}(0.12), s_{0.5}(0.18), s_{0.75}(0.28), s_{1.5}(0.42)\}$ $2 \odot_A L_1(p) = \{s_{1.75}(0.4), s_3(0.6)\}$ $(L_1(p))^2 = \{s_{0.25}(0.4), s_1(0.6)\}$
This paper with Form 1 of linguistic scale function	$L_1(p) \oplus_{Cp} L_2(p) = \{s_{1.75}(0.19), s_{2.5}(0.24), s_{3.25}(0.27), s_{3.5}(0.30)\}$ $L_1(p) \otimes_{Cp} L_2(p) = \{s_{0.25}(0.12), s_{0.5}(0.18), s_{0.75}(0.28), s_{1.5}(0.42)\}$ $2 \odot_{Cp} L_1(p) = \{s_{1.75}(0.4), s_3(0.6)\}$ $(L_1(p))^2 = \{s_{0.25}(0.4), s_1(0.6)\}$
This paper with Form 2 of linguistic scale function ($a = \sqrt[4]{9} \approx 1.73$)	$L_1(p) \oplus_{Cp} L_2(p) = \{s_{2.23}(0.19), s_{2.89}(0.24), s_{3.38}(0.27), s_{3.57}(0.30)\}$ $L_1(p) \otimes_{Cp} L_2(p) = \{s_{0.26}(0.12), s_{0.43}(0.42), s_{0.62}(0.28), s_{1.11}(0.18)\}$ $2 \odot_{Cp} L_1(p) = \{s_{2.23}(0.4), s_{3.26}(0.6)\}$ $(L_1(p))^2 = \{s_{0.26}(0.4), s_{0.74}(0.6)\}$
This paper with Form 3 of linguistic scale function ($\alpha = \beta = 0.8$)	$L_1(p) \oplus_{Cp} L_2(p) = \{s_{1.67}(0.19), s_{2.29}(0.24), s_{3.20}(0.27), s_{3.48}(0.30)\}$ $L_1(p) \otimes_{Cp} L_2(p) = \{s_{0.22}(0.12), , s_{0.52}(0.18)s_{0.80}(0.28), s_{1.71}(0.42)\}$ $2 \odot_{Cp} L_1(p) = \{s_{1.67}(0.4), s_{2.84}(0.6)\}$ $(L_1(p))^2 = \{s_{0.22}(0.4), s_{1.16}(0.6)\}$

5.1. Probabilistic linguistic weighted averaging operator

Definition 16 ([12]). Let $I = \{L_j(p) \big| j = 1, 2, \ldots, n\}$ be a set of PLTSs. Then, the probabilistic linguistic weighted average (PLWA) operator is defined as:

$$PLWA_{\omega}(L_1(p), L_2(p), \ldots, L_n(p)) = \sum_{j=1}^{n} \omega_j(L_j(p)) \tag{21}$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^{\mathrm{T}}$ is the weight vector of $L_j(p)(j = 1, 2, \ldots, n)$, satisfying $\omega_j \geq 0(j = 1, 2, \ldots, n)$, $\sum_{j=1}^{n} \omega_j = 1$. Especially, if $\omega = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})^{\mathrm{T}}$, then the PLWA operator reduces to the probabilistic linguistic average (PLA) operator.

According to the new operational laws of Definition 15, the PLWA operator can be converted to different operators. Take Gumbel copula function as an example, when $Ge(x) = (-\ln x)^{\varepsilon}(\varepsilon \geq 1)$, the PLWA operator is converted to a probabilistic linguistic Gumbel weighted average (PLGWA) operator as follows:

$$PLGWA_{\omega}(L_1(p), L_2(p), \dots, L_n(p)) = \sum_{j=1}^{n} \omega_j \odot_{Cp} (L_j(p))$$

$$= \bigcup_{\substack{k_j=1,2,\dots,\#L_j(p) \\ j=1,2,\dots,n}} \left\{ g^{-1}[1-\exp(-(\sum_{j=1}^{n} \omega_j(-\ln(1-A_j^{(k_j)}))^{\varepsilon})^{1/\varepsilon})](R_K) \right\} \quad (22)$$

where $A_j^{(k_j)} = g(L_j^{(k_j)})$, $R_K = \frac{\sum_{j=1}^{n} p_j^{(k_j)} - \sum_{1\le j<i\le n} p_j^{(k_j)} p_i^{(k_i)} + \sum_{1\le j<i<l\le n} p_j^{(k_j)} p_i^{(k_i)} p_l^{(k_l)} + \dots + (-1)^{n-1} p_1^{(k_1)} p_2^{(k_2)} \dots p_n^{(k_n)}}{\sum_{j=1}^{n}(\#L_j(p))-1}$ is the probability associated with the linguistic term $g^{-1}[1-\exp(-(\sum_{j=1}^{n} \omega_j(-\ln(1-A_j^{(k_j)}))^{\varepsilon})^{1/\varepsilon})]$, $K = 1, 2, \dots \#L_1(p) \times \#L_2(p) \times \dots \times \#L_n(p)$.

5.2. Generalized probabilistic linguistic Choquet operator

Definition 17 ([40]). Let X be a set of criteria. The set of function $\Gamma : P(X) \to [0, 1]$ is fuzzy measure on X if the following conditions are satisfied:

(1) (Boundary conditions) $\Gamma(\emptyset) = 0$ and $\Gamma(X) = 1$;

(2) (Monotonicity) If $A, B \in P(X)$ and $A \subseteq B$, then $\Gamma(A) \le \Gamma(B)$, where $P(X)$ is the power set of X.

In the MCGDM problem, the properties of interactions among criteria can be represented by $\Gamma(C_{(j)})(j = 1, 2, \cdots, n)$. Consider $\Gamma(\{C_1, C_2, \dots, C_n\})$ as the standard of subjective importance of criteria set $\{C_1, C_2, \dots, C_n\}$. For any pair of criteria subsets $A, B \in P(X)$ with $A \bigcap B = \emptyset$, three types of the properties are described as follows:

(1) (Simple additive measure) If $\Gamma(A \bigcup B) = \Gamma(A) + \Gamma(B)$, A and B are independent.

(2) (Super additive measure) If $\Gamma(A \bigcup B) > \Gamma(A) + \Gamma(B)$, A and B are positive interaction.

(3) (Sub additive measure) If $\Gamma(A \bigcup B) < \Gamma(A) + \Gamma(B)$, A and B are negative interaction.

Definition 18 ([41]). Let η be a positive real function on X and Γ be a fuzzy measure on X. Then, the discrete Choquet integral of η on Γ is defined as follows:

$$CI_{\Gamma} = \sum_{j=1}^{n} \eta(x_j)[\Gamma(S_{(j)}) - \Gamma(S_{(j-1)})] \quad (23)$$

where $\eta(x_1) \le \eta(x_2) \le \dots \le \eta(x_n)$, $S_{(j)} = \{x_{(j)}, x_{(j+1)}, \dots, x_{(n)}\}$ and $x_{(0)} = \emptyset$.

Let X be the set of n fuzzy numbers, which is denoted as $a_{(i)}(i = 1, 2, \dots, n)$. The discrete Choquet integral of $a_{(i)}$ on Γ can be obtained as follows:

$$CI_{\Gamma}(a_1, a_2, \dots, a_n) = \sum_{j=1}^{n} a_{(i)}(\Gamma(S_{(j)}) - \Gamma(S_{(j-1)})) \quad (24)$$

Inspired by the probabilistic linguistic Choquet integral operator in [42], a GPLC operator is proposed below.

Definition 19. Let $I = \{L_j(p) \big| j = 1, 2, \dots, n\}$ be a set of PLTSs. The GPLC operator is defined as:

$$GPLC_{\Gamma}(L_1(p), L_2(p), \dots, L_n(p)) = (\sum_{j=1}^{n}(\Gamma(S_{(j)}) - \Gamma(S_{(j-1)}))\odot_{Cp}(L_{(j)}(p))^{\lambda})^{1/\lambda} \quad (25)$$

where $\lambda \in (0, +\infty)$, $((1), (2) \cdots, (n))$ is a permutation of $(1, 2, \dots, n)$ such that $L_{(1)}(p) \ge L_{(2)}(p) \ge \dots \ge L_{(n)}(p)$ according to the proposed possibility ranking algorithm in Section 3.2, $S_{(j)} = \{x_{(j)}, x_{(j+1)}, \dots, x_{(n)}\}$ and $x_{(0)} = \emptyset$.

Theorem 3. *Let $I = \{L_j(p) \big| j = 1, 2, \dots, n\}$ be a set of PLTSs. The result by using the GPLC operator is obtained as:*

$$GPLC_{\Gamma}(L_1(p), L_2(p), \dots, L_n(p))$$

$$= \bigcup_{\substack{k_j=1,2,\dots,\#L_{(j)}(p) \\ j=1,2,\dots,n}} \left\{ g^{-1}[(1-Ge^{-1}(\sum_{j=1}^{n}(\Gamma(S_{(j)}) - \Gamma(S_{(j-1)}))(Ge(1-A_{(j)}^{(k_j)}))^{\varepsilon})^{1/\varepsilon})^{1/\lambda}](R_K) \right\} \quad (26)$$

where $A_{(j)}^{(k_j)} = Ge^{-1}\left((\lambda(Ge(g(L_{(j)}^{(k_j)})))^{\varepsilon})^{1/\varepsilon}\right)$, $R_K = \frac{\sum_{j=1}^{n} p_{(j)}^{(k_j)} - \sum_{1\le j<i\le n} p_{(j)}^{(k_j)} p_{(i)}^{(k_i)} + \sum_{1\le j<i<l\le n} p_{(j)}^{(k_j)} p_{(i)}^{(k_i)} p_{(l)}^{(k_l)} + \dots + (-1)^{n-1} p_{(1)}^{(k_1)} p_{(2)}^{(k_2)} \dots p_{(n)}^{(k_n)}}{\sum_{j=1}^{n}(\#L_{(j)}(p))-1}$, $K = 1, 2, \dots, \#L_{(1)}(p) \times \#L_{(2)}(p) \times \dots \times \#L_{(n)}(p)$.

The proof of Theorem 3 is shown in Appendix B.

Property 3 (*Idempotency*). *Let $I = \{L_j(p) \big| j = 1, 2, \dots, n\}$ be a set of PLTSs. If $L_j(p) = L(p)(j = 1, 2, \dots, n)$, then $GPLC_{\Gamma}(L_1(p), L_2(p), \dots, L_n(p)) = L(p)$.*

Property 4 (*Monotonicity*). *Let $I = \{L_j(p) \big| j = 1, 2, \ldots, n\}$ and $I' = \{L'_j(p) \big| j = 1, 2, \ldots, n\}$ be two sets of PLTSs. If $L_j(p) \geq L'_j(p) (j = 1, 2, \ldots, n)$, then*

$$GPLC_{\Gamma}(L_1(p), L_2(p), \ldots, L_n(p)) \geq GPLC_{\Gamma}(L'_1(p), L'_2(p), \ldots, L'_n(p)).$$

Property 5 (*Boundedness*). *Let $I = \{L_j(p) \big| j = 1, 2, \ldots, n\}$ be a set of PLTSs, where $L_j(p) = \{L_j^{(k_j)} | k_j = 1, 2, \ldots, \#L_j(p)\}$. $L'_j(p) = \{L_j^{-}(1)\}$ and $L''_j = \{L_j^{+}(1)\} (j = 1, 2, \ldots, n)$ are two special PLTSs, where L_j^- and L_j^+ are the minimal and maximal linguistic terms of $L_j^{(k_j)}$ in $L_j(p)$, respectively.*

Then, one has $GPLC_{\Gamma}(V) \leq GPLC_{\Gamma}(L_1(p), L_2(p), \ldots, L_n(p)) \leq GPLC_{\Gamma}(U)$, where $V = (L'_1(p), L'_2(p), \ldots, L'_n(p))$ and $U = (L''_1(p), L''_2(p), \ldots, L''_n(p))$.

The proofs of Properties 3–5 are shown in Appendix C.

Property 6. *Let $I = \{L_j(p) \big| j = 1, 2, \ldots, n\}$ be a set of PLTSs.*

(1) When $\lambda \to 0$, it easily follows from Eq. (25) that

$$GPLC_{\Gamma}(L_1(p), L_2(p), \ldots, L_n(p)) = \prod_{j=1}^{n} (L_{(j)}(p))^{\Gamma(S_{(j)}) - \Gamma(S_{(j-1)})}$$

which is degenerated to a probabilistic linguistic geometric ordered weighted Choquet (PLGOWC) operator;

(2) When $\lambda = 1$, it easily follows from Eq. (25) that

$$GPLC_{\Gamma}(L_1(p), L_2(p), \ldots, L_n(p)) = \sum_{j=1}^{n} (\Gamma(S_{(j)}) - \Gamma(S_{(j-1)}))(L_{(j)}(p))$$

which is degenerated to a probabilistic linguistic Choquet integral (PLC) operator;

(3) When $\lambda \to +\infty$, it easily follows from Eq. (25) that

$$GPLC_{\Gamma}(L_1(p), L_2(p), \ldots, L_n(p)) = \max\{L_j(p) \big| j = 1, 2, \ldots, n\}$$

which is degenerated to a max operator of PLTSs $L_j(p) (j = 1, 2, \ldots, n)$.

5.3. Generalized probabilistic linguistic hybrid Choquet operator

Definition 20. Let $I = \{L_j(p) \big| j = 1, 2, \ldots, n\}$ be a set of PLTSs. The GPLHC operator is defined as:

$$GPLHC_{\Gamma}(L_1(p), L_2(p), \ldots, L_n(p)) = (\sum_{j=1}^{n} (\Gamma(S_{(j)}) - \Gamma(S_{(j-1)})) \odot_{Cp} (L^*_{(j)}(p))^{\lambda})^{1/\lambda} \tag{27}$$

where $\lambda \in [0, +\infty]$, $L^*_j(p)$ is obtained by weighting the PLTS $L_j(p)$, i.e., $L^*_j(p) = n\omega_j L_j(p)$, $\omega_j \in [0, 1]$ is the weight of $L_j(p)$ satisfying $\sum_{j=1}^{n} \omega_j = 1$, $L^*_{(j)}(p)$ is the j-th largest PLTS of $L^*_1(p), L^*_2(p), \ldots, L^*_n(p)$ according to the proposed possibility ranking algorithm in Section 3.2, $S_{(j)} = \{x_{(j)}, x_{(j+1)}, \ldots, x_{(n)}\}$ and $x_{(0)} = \emptyset$.

Theorem 4. *Let $I = \{L_j(p) \big| j = 1, 2, \ldots, n\}$ be a set of PLTSs. The result by using the GPLHC operator is obtained as*

$$GPLHC_{\Gamma}(L_1(p), L_2(p), \ldots, L_n(p)) = \bigcup_{\substack{k_j = 1, 2, \ldots, \#L^*_{(j)}(p) \\ j = 1, 2, \ldots, n}} \left\{ g^{-1}[(1 - Ge^{-1}(\sum_{j=1}^{n} (\Gamma(S_{(j)}) - \Gamma(S_{(j-1)}))(Ge(1 - A_{(j)}^{(k_j)}))^{\varepsilon})^{1/\varepsilon})^{1/\lambda}](R_K) \right\} \tag{28}$$

where $A_j^{(k_j)} = Ge^{-1}((\lambda(Ge(g(L_{(j)}^{*(k_j)})))^{\varepsilon})^{1/\varepsilon})$, $R_K = \frac{\sum_{j=1}^{n} p_{(j)}^{(k_j)} - \sum_{1 \leq j < i \leq n} p_{(j)}^{(k_{(j)})} p_{(i)}^{(k_i)} + \sum_{1 \leq j < i < l \leq n} p_{(j)}^{(k_j)} p_{(i)}^{(k_i)} p_{(l)}^{(k_l)} + \cdots + (-1)^{n-1} p_{(1)}^{(k_1)} p_{(2)}^{(k_2)} \cdots p_{(n)}^{(k_n)}}{\sum_{j=1}^{n} (\#L^*_{(j)}(p)) - 1}$, $K = 1, 2, \ldots, \#L^*_{(1)}(p) \times \#L^*_{(2)}(p) \times \cdots \times \#L^*_{(n)}(p)$.

Similar to Theorem 3, it is easy to prove Theorem 4.

The GPLHC operator has similar properties to the GPLC operator as follows:

(1) When $\lambda \to 0$, it easily follows from Eq. (27) that

$$GPLHC_{\Gamma}(L_1(p), L_2(p), \ldots, L_n(p)) = \prod_{j=1}^{n} (L^*_{(j)}(p))^{\Gamma(S_{(j)}) - \Gamma(S_{(j-1)})}$$

which is degenerated to a geometric probabilistic linguistic hybrid Choquet integral (GPLHCI) operator;

(2) When $\lambda = 1$, it easily follows from Eq. (27) that

$$GPLHC_{\Gamma}(L_1(p), L_2(p), \ldots, L_n(p)) = \sum_{j=1}^{n} (\Gamma(S_{(j)}) - \Gamma(S_{(j-1)}))(L^*_{(j)}(p))$$

which is degenerated to a probabilistic linguistic hybrid Choquet integral (PLHC) operator;

(3) When $\lambda \to +\infty$, it easily follows from Eq. (27) that

$$GPLHC_{\Gamma}(L_1(p), L_2(p), \ldots, L_n(p)) = \max\{L_j^*(p) \big| j = 1, 2, \ldots, n\}$$

which is degenerated to a max operator of PLTSs $L_j^*(p)(j = 1, 2, \ldots, n)$.

In terms of four different types of common Archimedean copulas functions, the GPLHC operator can be converted into different forms.

Case 1. (Gumbel type) When $Ge(x) = (-\ln x)^{\varepsilon}(\varepsilon \geq 1)$, the GPLHC operator is called a generalized probabilistic linguistic Gumbel hybrid Choquet (GPLGHC) operator as follows:

$$\begin{aligned} &GPLGHC_{\Gamma}(L_1(p), L_2(p), \ldots, L_n(p)) \\ &= \bigcup_{\substack{k_j=1,2,\ldots,\#L_{(j)}^*(p) \\ j=1,2,\ldots,n}} \left\{ g^{-1}[(1 - \exp(-(\sum_{j=1}^{n}(\Gamma(S_{(j)}) - \Gamma(S_{(j-1)}))(-\ln(1 - A_{(j)}^{(k_j)}))^{\varepsilon})^{1/\varepsilon}))^{1/\lambda}](R_K) \right\} \end{aligned} \tag{29}$$

where $A_{(j)}^{(k_j)} = \exp(-(\lambda(-\ln(g(L_{(j)}^{*(k_j)})))^{\varepsilon})^{1/\varepsilon})$, $R_K = \frac{\sum_{j=1}^{n} p_{(j)}^{(k_j)} - \sum_{1 \leq j < i \leq n} p_{(j)}^{(k_j)} p_{(i)}^{(k_i)} + \sum_{1 \leq j < i < l \leq n} p_{(j)}^{(k_j)} p_{(i)}^{(k_i)} p_{(l)}^{(k_l)} + \cdots + (-1)^{n-1} p_{(1)}^{(k_1)} p_{(2)}^{(k_2)} \cdots p_{(n)}^{(k_n)}}{\sum_{j=1}^{n}(\#L_{(j)}^*(p)) - 1}$.

Especially, when $\lambda = 1$, GPLGHC operator is degenerated to a PLGHC operator as

$$\begin{aligned} &PLGHC_{\Gamma}(L_1(p), L_2(p), \ldots, L_n(p)) \\ &= \bigcup_{\substack{k_j=1,2,\ldots,\#L_{(j)}^*(p) \\ j=1,2,\ldots,n}} \left\{ g^{-1}[1 - \exp(-(\sum_{j=1}^{n}(\Gamma(S_{(j)}) - \Gamma(S_{(j-1)}))(-\ln(1 - A_{(j)}^{(k_j)}))^{\varepsilon})^{1/\varepsilon})](R_K) \right\} \end{aligned} \tag{30}$$

where $A_{(j)}^{(k_j)} = g(L_{(j)}^{*(k_j)})$, $R_K = \frac{\sum_{j=1}^{n} p_{(j)}^{(k_j)} - \sum_{1 \leq j < i \leq n} p_{(j)}^{(k_j)} p_{(i)}^{(k_i)} + \sum_{1 \leq j < i < l \leq n} p_{(j)}^{(k_j)} p_{(i)}^{(k_i)} p_{(l)}^{(k_l)} + \cdots + (-1)^{n-1} p_{(1)}^{(k_1)} p_{(2)}^{(k_2)} \cdots p_{(n)}^{(k_n)}}{\sum_{j=1}^{n}(\#L_{(j)}^*(p)) - 1}$.

Case 2. (Clayton type) When $Ge(x) = x^{-\varepsilon} - 1(\varepsilon \geq -1, \varepsilon \neq 0)$, the GPLHC operator is called a generalized probabilistic linguistic Clayton hybrid Choquet (GPLCHC) operator as follows:

$$\begin{aligned} &GPLCHC_{\Gamma}(L_1(p), L_2(p), \ldots, L_n(p)) \\ &= \bigcup_{\substack{k_j=1,2,\ldots,\#L_{(j)}^*(p) \\ j=1,2,\ldots,n}} \left\{ g^{-1}[(1 - (\sum_{j=1}^{n}(\Gamma(S_{(j)}) - \Gamma(S_{(j-1)}))((1 - A_{(j)}^{(k_j)})^{-\varepsilon} - 1) + 1)^{-1/\varepsilon})^{1/\lambda}](R_K) \right\} \end{aligned} \tag{31}$$

where $A_{(j)}^{(k_j)} = (\lambda((g(L_{(j)}^{*(k_j)}))^{-\varepsilon} - 1) + 1)^{-1/\varepsilon}$, $R_K = \frac{\sum_{j=1}^{n} p_{(j)}^{(k_j)} - \sum_{1 \leq j < i \leq n} p_{(j)}^{(k_j)} p_{(i)}^{(k_i)} + \sum_{1 \leq j < i < l \leq n} p_{(j)}^{(k_j)} p_{(i)}^{(k_i)} p_{(l)}^{(k_l)} + \cdots + (-1)^{n-1} p_{(1)}^{(k_1)} p_{(2)}^{(k_2)} \cdots p_{(n)}^{(k_n)}}{\sum_{j=1}^{n}(\#L_{(j)}^*(p)) - 1}$.

Case 3. (Frank type) When $Ge(x) = -\ln \frac{e^{-\varepsilon x} - 1}{e^{-\varepsilon} - 1}(\varepsilon \neq 0)$, the GPLHC operator is called a generalized probabilistic linguistic Frank hybrid Choquet (GPLHC) operator as follows:

$$\begin{aligned} &GPLFHC_{\Gamma}(L_1(p), L_2(p), \ldots, L_n(p)) \\ &= \bigcup_{\substack{k_j=1,2,\ldots,\#L_{(j)}^*(p) \\ j=1,2,\ldots,n}} \left\{ g^{-1}[(1 + \frac{1}{\varepsilon}\ln(1 + \prod_{j=1}^{n}(\frac{\exp(-\varepsilon(1 - A_{(j)}^{(k_j)})) - 1}{\exp(-\varepsilon) - 1})^{\Gamma(S_{(j)}) - \Gamma(S_{(j-1)})}))^{1/\lambda}](R_K) \right\} \end{aligned} \tag{32}$$

where $A_{(j)}^{(k_j)} = -\frac{1}{\varepsilon}\ln(1 + (\frac{\exp(-\varepsilon(1 - g(L_{(j)}^{*(k_j)}))) - 1}{\exp(-\varepsilon) - 1})^{\lambda})$, $R_K = \frac{\sum_{j=1}^{n} p_{(j)}^{(k_j)} - \sum_{1 \leq j < i \leq n} p_{(j)}^{(k_j)} p_{(i)}^{(k_i)} + \sum_{1 \leq j < i < l \leq n} p_{(j)}^{(k_j)} p_{(i)}^{(k_i)} p_{(l)}^{(k_l)} + \cdots + (-1)^{n-1} p_{(1)}^{(k_1)} p_{(2)}^{(k_2)} \cdots p_{(n)}^{(k_n)}}{\sum_{j=1}^{n}(\#L_{(j)}^*(p)) - 1}$.

Case 4. (Joe type) When $Ge(x) = -\ln[1 - (1 - x)^{\varepsilon}](\varepsilon \geq 1)$, the GPLHC operator is called a generalized probabilistic linguistic Joe hybrid Choquet (GPLJHC) operator as follows:

$$\begin{aligned} &GPLJHC_{\Gamma}(L_1(p), L_2(p), \ldots, L_n(p)) \\ &= \bigcup_{\substack{k_j=1,2,\ldots,\#L_{(j)}^*(p) \\ j=1,2,\ldots,n}} \left\{ g^{-1}[((1 - \sum_{j=1}^{n}(1 - (A_{(j)}^{(k_j)})^{\varepsilon})^{\Gamma(S_{(j)}) - \Gamma(S_{(j-1)})})^{1/\varepsilon})^{1/\lambda}](R_K) \right\} \end{aligned} \tag{33}$$

where $A_{(j)}^{(k_j)} = 1 - (1 - (1 - (1 - g(L_{(j)}^{*(k_j)}))^{\varepsilon})^{\lambda})^{1/\varepsilon}$, $R_K = \frac{\sum_{j=1}^{n} p_{(j)}^{(k_j)} - \sum_{1 \leq j < i \leq n} p_{(j)}^{(k_j)} p_{(i)}^{(k_i)} + \sum_{1 \leq j < i < l \leq n} p_{(j)}^{(k_j)} p_{(i)}^{(k_i)} p_{(l)}^{(k_l)} + \cdots + (-1)^{n-1} p_{(1)}^{(k_1)} p_{(2)}^{(k_2)} \cdots p_{(n)}^{(k_n)}}{\sum_{j=1}^{n}(\#L_{(j)}^*(p)) - 1}$.

6. A new method for interactive MCGDM with probabilistic linguistic information

In this section, a new method for interactive MCGDM with PLTSs is proposed.

6.1. Problem description

The notation clarifications of Probabilistic linguistic MCGDM problem are shown below:

$A = \{A_1, A_2, \ldots, A_m\}$ is a set of alternatives, where $A_i (i = 1, 2, \ldots, m)$ denotes the i-th alternative.

$C = \{C_1, C_2, \ldots, C_n\}$ is a set of criteria, where $C_j (j = 1, 2, \ldots, n)$ denotes the j-th criterion. There exist interactions among criteria.

$E = \{E_1, E_2, \ldots, E_g\}$ is a set of DMs, where $E_q (q = 1, 2, \ldots, g)$ denotes the q-th DM.

$\omega = (\omega^1, \omega^2, \ldots, \omega^g)^{\mathrm{T}}$ is the weight vector of DMs, where ω^q denotes the weight of DM E_q, satisfying $0 \leq \omega^q \leq 1 (q = 1, 2, \ldots, g)$ and $\sum_{q=1}^{g} \omega^q = 1$.

$\Gamma = (\Gamma_1, \Gamma_2, \ldots, \Gamma_n)$ is the fuzzy measure of criteria, where $\Gamma_j = \Gamma(S_{(j)})$ denotes the fuzzy measure of criterion subset $S_{(j)} = \{C_{(j)}, C_{(j+1)}, \ldots, C_{(n)}\}$ and $C_{(0)} = \emptyset$.

$U_q = [L_{ij}^q(p)]_{m\times n}$ is an individual decision matrix given by DM E_q, where $L_{ij}^q(p) = \{(L_{ij}^q)^{(k)}((p_{ij}^q)^{(k)}) | k = 1, 2, \ldots, \#L_{ij}^q(p)\}$ is a PLTS and denotes the evaluation of alternative A_i on criterion C_j provided by DM E_q.

$\tilde{U}_q = [\tilde{L}_{ij}^q(p)]_{m\times n}$ is an individual ascending ordered normalized decision matrix given by DM E_q, where $\tilde{L}_{ij}^q(p) = \{(\tilde{L}_{ij}^q)^{(k)}((\tilde{p}_{ij}^q)^{(k)}) | k = 1, 2, \ldots, \#\tilde{L}_{ij}^q(p)\}$ is an ascending ordered normalized PLTS of $L_{ij}^q(p)$.

$\hat{U} = [\hat{L}_{ij}(p)]_{m\times n}$ is a collective normalized decision matrix, where $\hat{L}_{ij}(p) = \{\hat{L}_{ij}^{(k)}(\hat{p}_{ij}^{(k)}) | \hat{L}_{ij}^{(k)} \in S, \hat{p}_{ij}^{(k)} \geq 0, k = 1, 2, \ldots, \#\hat{L}_{ij}(p)\}$ is a collective evaluation of alternative A_i on criterion C_j.

$\bar{L}_i(p) = \{\bar{L}_i^{(k)}(\bar{p}_i^{(k)}) | \bar{L}_i^{(k)} \in S, \bar{p}_i^{(k)} \geq 0, k = 1, 2, \ldots, \#\bar{L}_i(p)\}$ is a PLTS and denotes the collective comprehensive value of alternative A_i.

6.2. Incomplete information structure

This subsection depicts the incomplete information structure of DMs' weights and criteria fuzzy measures.

6.2.1. Incomplete information structure of DMs' weights

Due to the complexity of realistic decision-making problems and the incomprehensive experience and knowledge of DMs, it is hard to determine the weight vector of DMs. Therefore, the information of the DMs' weights $\omega = (\omega^1, \omega^2, \ldots, \omega^g)^{\mathrm{T}}$ is incomplete. Let Λ be the incomplete information structure of DMs' weights, which may consist of several basic forms [43] (please see [43] for more details):

(Form 1) A weak ranking: $\{\omega^q \geq \omega^l\}$;

(Form 2) A strict ranking: $\{\pi_2 \geq \omega^q - \omega^l \geq \pi_1\} (\pi_2 \geq \pi_1 > 0)$;

(Form 3) A ranking of differences: $\{\omega^q - \omega^l \geq \omega^f - \omega^d\} (l \neq f \neq d)$;

(Form 4) A ranking with multiples: $\{\omega^q \geq \varphi\omega^l\} (0 \leq \varphi \leq 1)$;

(Form 5) A interval ranking: $\{\gamma_1 \geq \omega^q \geq \gamma_2\} (\gamma_1 \geq \gamma_2 \geq 0)$.

6.2.2. Incomplete information structure of criteria fuzzy measures

In some real decision situations, DMs tend to specify their preferences on criteria fuzzy measures according to their knowledge and judgment. Therefore, the information of the criteria fuzzy measures is incomplete [44]. Let $\Theta(\Gamma)$ be the incomplete information of criteria fuzzy measures, which may consist of several forms [44] (please see [44] for more details).

6.3. Determination of the weights of DMs

Inspired by Yue [45], this subsection introduces the TOPSIS method to obtain the weights of DMs.

(1) Determine the individual ascending ordered normalized probabilistic linguistic decision matrix $\tilde{U}_q$.

Normalize probabilistic linguistic decision matrix $U_q = [L_{ij}^q(p)]_{m\times n}$ to corresponding ascending ordered normalized probabilistic linguistic decision matrix $\tilde{U}_q = [\tilde{L}_{ij}^q(p)]_{m\times n}$, where $\tilde{L}_{ij}^q(p)$ is an ascending ordered normalized PLTS defined in Definition 5.

(2) Determine the ascending ordered normalized positive ideal decision matrix $\tilde{U}^+ = [\tilde{L}_{ij}^+(p)]_{m\times n}$, where

$$\tilde{L}_{ij}^+(p) = \{(\tilde{L}_{ij}^{+(k)})((\tilde{p}_{ij}^+)^{(k)}) | k = 1, 2, \ldots, \#\tilde{L}_{ij}(p)\}, (\tilde{L}_{ij}^+)^{(k)} = \frac{1}{g}\sum_{q=1}^{g}(\tilde{L}_{ij}^q)^{(k)}, (\tilde{p}_{ij}^+)^{(k)} = \frac{1}{g}\sum_{q=1}^{g}(\tilde{p}_{ij}^q)^{(k)} \tag{34}$$

The ascending ordered normalized negative ideal decision matrix is divided into left ascending ordered normalized negative ideal decision matrix $\tilde{U}^{L-} = [\tilde{L}_{ij}^{L-}(p)]_{m\times n}$ and right ascending ordered normalized negative ideal decision matrix $\tilde{U}^{R-} = [\tilde{L}_{ij}^{R-}(p)]_{m\times n}$.

(3) Determine the left ascending ordered normalized negative ideal decision matrix $\tilde{U}^{L-}$, where

$$\tilde{L}_{ij}^{L-}(p) = \{(\tilde{L}_{ij}^{(k)})^{L-}((\tilde{p}_{ij}^{L-})^{(k)}) | k = 1, 2, \ldots, \#\tilde{L}_{ij}(p)\}, (\tilde{L}_{ij}^{(k)})^{L-} = \min_{q=1,2,\ldots,g}\{(\tilde{L}_{ij}^q)^{(k)}\} \tag{35}$$

$(\tilde{p}_{ij}^{L-})^{(k)}$ is the corresponding probability value of $\min_{q=1,2,\ldots,g}\{(\tilde{L}_{ij}^q)^{(k)}\}$.

(4) Determine the right ascending ordered normalized negative ideal decision matrix $\tilde{U}^{R-}$, where

$$\tilde{L}_{ij}^{R-}(p) = \{(\tilde{L}_{ij}^{(k)})^{R-}((\tilde{p}_{ij}^{R-})^{(k)}) | k = 1, 2, \ldots, \#\tilde{L}_{ij}(p)\}, (\tilde{L}_{ij}^{(k)})^{R-} = \max_{q=1,2,\ldots,g}\{(\tilde{L}_{ij}^q)^{(k)}\} \tag{36}$$

$(\tilde{p}_{ij}^{R-})^{(k)}$ is the corresponding probability value of $\max_{q=1,2,\ldots,g}\{(\tilde{L}_{ij}^q)^{(k)}\}$.

By Eq. (34), the similarity degree sim^+ between $\tilde{U}_q$ and $\tilde{U}^+$ is defined as follows:

$$sim^+ = \frac{\sum_{k=1}^{\#\tilde{L}_{ij}(p)}((\tilde{p}_{ij}^q)^{(k)} g((\tilde{L}_{ij}^q)^{(k)})) \cdot ((\tilde{p}_{ij}^+)^{(k)} g^+(\tilde{L}_{ij}^{(k)}))}{\sqrt{\sum_{k=1}^{\#\tilde{L}_{ij}(p)}((\tilde{p}_{ij}^q)^{(k)} g((\tilde{L}_{ij}^q)^{(k)}))^2}\sqrt{\sum_{k=1}^{\#\tilde{L}_{ij}(p)}((\tilde{p}_{ij}^+)^{(k)} g^+(\tilde{L}_{ij}^{(k)}))^2}} \tag{37}$$

where $g^{+}(\tilde{L}_{ij}^{(k)})=\frac{1}{g}\sum_{q=1}^{g}g((\tilde{L}_{ij}^{q})^{(k)})$.

By Eq. (35), the similarity degree sim^{L-} between $\tilde{U}_q$ and $\tilde{U}^{L-}$ is defined as follows:

$$sim^{L-}=\frac{\sum_{k=1}^{\#\tilde{L}_{ij}(p)}((\tilde{p}_{ij}^{q})^{(k)}g((\tilde{L}_{ij}^{q})^{(k)}))\cdot((\tilde{p}_{ij}^{L-})^{(k)}g^{L-}(\tilde{L}_{ij}^{(k)}))}{\sqrt{\sum_{k=1}^{\#\tilde{L}_{ij}(p)}((\tilde{p}_{ij}^{q})^{(k)}g((\tilde{L}_{ij}^{q})^{(k)}))^{2}}\sqrt{\sum_{k=1}^{\#\tilde{L}_{ij}(p)}((\tilde{p}_{ij}^{L-})^{(k)}g^{L-}(\tilde{L}_{ij}^{(k)}))^{2}}} \tag{38}$$

where $g^{L-}(\tilde{L}_{ij}^{(k)})=\min_{q=1,2,\ldots,g}\{g((\tilde{L}_{ij}^{q})^{(k)})\}$.

By Eq. (36), the similarity degree sim^{R-} between $\tilde{U}_q$ and $\tilde{U}^{R-}$ is defined as follows:

$$sim^{R-}=\frac{\sum_{k=1}^{\#\tilde{L}_{ij}(p)}((\tilde{p}_{ij}^{q})^{(k)}g((\tilde{L}_{ij}^{q})^{(k)}))\cdot((\tilde{p}_{ij}^{R-})^{(k)}g^{R-}(\tilde{L}_{ij}^{(k)}))}{\sqrt{\sum_{k=1}^{\#\tilde{L}_{ij}(p)}((\tilde{p}_{ij}^{q})^{(k)}g((\tilde{L}_{ij}^{q})^{(k)}))^{2}}\sqrt{\sum_{k=1}^{\#\tilde{L}_{ij}(p)}((\tilde{p}_{ij}^{R-})^{(k)}g^{R-}(\tilde{L}_{ij}^{(k)}))^{2}}} \tag{39}$$

where $g^{R-}(\tilde{L}_{ij}^{(k)})=\max_{q=1,2,\ldots,g}\{g((\tilde{L}_{ij}^{q})^{(k)})\}$.

Then, the individual relative closeness degree RC_{ij}^{q} of alternative A_i on criterion C_j for DM E_q is defined as follows:

$$RC_{ij}^{q}=\frac{sim^{+}}{sim^{+}+sim^{L-}+sim^{R-}} \tag{40}$$

The global relative closeness degree RC_{ij} of alternative A_i on criterion C_j is defined as follows:

$$RC_{ij}=\sum_{q=1}^{g}\omega^{q}RC_{ij}^{q} \tag{41}$$

where ω^q denotes the weight of DM E_q.

By Eq. (40), the deviation between the individual relative closeness degree and other individual relative closeness degrees on criterion C_j for DM E_q can be calculated as follows:

$$V_q(\omega)=\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{t=i+1}^{m}\omega^{q}\left|RC_{ij}^{q}-RC_{tj}^{q}\right| \tag{42}$$

By Eqs. (40) and (41), the deviation between the individual relative closeness degree and global relative closeness degree on criterion C_j for DM E_q can be calculated as follows:

$$H_q(\omega)=\sum_{i=1}^{m}\sum_{j=1}^{n}\left|RC_{ij}^{q}-RC_{ij}\right| \tag{43}$$

By Eq. (40), the deviation between the individual relative closeness degrees on the criterion C_j for DM E_q and other DMs can be calculated as follows:

$$T_q(\omega)=\sum_{i=1}^{m}\sum_{r=1}^{g}\sum_{j=1}^{n}\omega^{q}\left|RC_{ij}^{q}-RC_{ij}^{r}\right| \tag{44}$$

To determine DMs' weight $\omega=(\omega^1,\omega^2,\ldots,\omega^g)^{\mathrm{T}}$, it is reasonable to maximize the deviation between the individual relative closeness degree and other individual relative closeness degrees on criterion C_j for DM E_q, while minimize the deviation between the individual relative closeness degree and global relative closeness degree on criterion C_j for DM E_q, and minimize the deviation between the individual relative closeness degrees on the criterion C_j for DM E_q and other DMs. Hence, a tri-objective nonlinear programming model can be constructed as follows:

$$(\text{Mod 1})\ \begin{cases}\max V(\omega)=\sum_{q=1}^{g}\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{t=i+1}^{m}\omega^{q}\left|RC_{ij}^{q}-RC_{tj}^{q}\right|\\ \min H(\omega)=\sum_{q=1}^{g}\sum_{j=1}^{n}\sum_{i=1}^{m}\left|RC_{ij}^{q}-RC_{ij}\right|\\ \min T(\omega)=\sum_{q=1}^{g}\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{r=1}^{g}\omega^{q}\left|RC_{ij}^{q}-RC_{ij}^{r}\right|\end{cases}\\ s.t.\ \ \omega\in\Lambda \tag{45}$$

Then, (Mod 1) is transformed into a single objective nonlinear programming model as follows:

(Mod 2)

$$\min\ H=\sum_{q=1}^{g}\sum_{j=1}^{n}\sum_{i=1}^{m}\left|RC_{ij}^{q}-RC_{ij}\right|+\sum_{q=1}^{g}\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{r=1}^{g}\omega^{q}\left|RC_{ij}^{q}-RC_{ij}^{r}\right|-\sum_{q=1}^{g}\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{t=i+1}^{m}\omega^{q}\left|RC_{ij}^{q}-RC_{tj}^{q}\right|\\ s.t.\ \ \omega\in\Lambda \tag{46}$$

To solve (Mod 2), let

$$\vartheta_{ij}^{q+}=\left(\left|RC_{ij}^{q}-RC_{ij}\right|+(RC_{ij}^{q}-RC_{ij})\right)/2=\left(\left|RC_{ij}^{q}-\sum_{q=1}^{g}\omega^{q}RC_{ij}^{q}\right|+(RC_{ij}^{q}-\sum_{q=1}^{g}\omega^{q}RC_{ij}^{q})\right)/2,$$

$$\vartheta_{ij}^{q-} = \left(\left|RC_{ij}^{q} - RC_{ij}\right| - (RC_{ij}^{q} - RC_{ij})\right)/2 = \left(\left|RC_{ij}^{q} - \sum_{q=1}^{g}\omega^{q}RC_{ij}^{q}\right| - (RC_{ij}^{q} - \sum_{q=1}^{g}\omega^{q}RC_{ij}^{q})\right)/2.$$

Thus, (Mod 2) is transformed into a linear programming model as follows:
(Mod 3)

$$\begin{array}{l}\min\ H = \sum_{q=1}^{g}\sum_{j=1}^{n}\sum_{i=1}^{m}(\vartheta_{ij}^{q+} + \vartheta_{ij}^{q-}) + \sum_{q=1}^{g}\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{r=1}^{g}\omega^{q}\left|RC_{ij}^{q} - RC_{ij}^{r}\right| - \sum_{q=1}^{g}\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{t=i+1}^{m}\omega^{q}\left|RC_{ij}^{q} - RC_{tj}^{q}\right| \\ s.t.\left\{\begin{array}{l}\vartheta_{ij}^{q+} - \vartheta_{ij}^{q-} = RC_{ij}^{q} - \sum_{q=1}^{g}\omega^{q}RC_{ij}^{q};\ \vartheta_{ij}^{q+} \geq 0;\ \vartheta_{ij}^{q-} \geq 0;\ (i = 1, 2, \dots, m;\ j = 1, 2, \dots, n;\ q = 1, 2, \dots, g) \\ \omega \in \Lambda\end{array}\right.\end{array} \tag{47}$$

The weight vector of DMs $\omega = (\omega^1, \omega^2, \dots, \omega^g)^{\mathrm{T}}$ can be derived by solving (Mod 3).

6.4. A new method for interactive MCGDM with probabilistic linguistic information

Based on the above analyses, a new method for interactive MCGDM with probabilistic linguistic information is summarized as follows:

Step 1. Elicit the individual probabilistic linguistic matrix $U_q = [L_{ij}^{q}(p)]_{m\times n}$ $(q = 1, 2, \dots, g)$.

Step 2. Acquire the individual ascending ordered normalized probabilistic linguistic decision matrix $\tilde{U}_q = [\tilde{L}_{ij}^{q}(p)]_{m\times n}$ $(q = 1, 2, \dots, g)$ by Definition 5.

Step 3. Determine the weight of DMs $\omega = (\omega^1, \omega^2, \dots, \omega^g)^{\mathrm{T}}$ by solving (Mod 3).

Step 4. Compute the collective normalized decision matrix $\hat{U} = [\hat{L}_{ij}(p)]_{m\times n}$.

Based on the weight vector of DMs $\omega = (\omega^1, \omega^2, \dots, \omega^g)^{\mathrm{T}}$, aggregate all individual ascending ordered normalized probabilistic linguistic decision matrices $\tilde{U}_q = [\tilde{L}_{ij}^{q}(p)]_{m\times n}$ $(q = 1, 2, \dots, g)$ into a collective normalized decision matrix $\hat{U} = [\hat{L}_{ij}(p)]_{m\times n}$ by PLGWA operator (i.e., Eq. (22)).

$$\begin{aligned}\hat{L}_{ij}(p) &= PLGWA_{\omega}(\tilde{L}_{ij}^{1}(p), \tilde{L}_{ij}^{2}(p), \dots, \tilde{L}_{ij}^{g}(p)) = \sum_{q=1}^{g}\omega^{q}\odot_{Cp}(\tilde{L}_{ij}^{q}(p)) \\ &= \bigcup_{\substack{k_{ij}^{q}=1,2,\dots,\tilde{L}_{ij}^{q}(p) \\ q=1,2,\dots,g}}\left\{g^{-1}[1 - \exp(-(\sum_{q=1}^{g}\omega^{q}(-\ln(1 - (A_{ij}^{q})^{(k_{ij}^{q})}))^{\varepsilon})^{1/\varepsilon})](R_K)\right\}\end{aligned} \tag{48}$$

where $(A_{ij}^{q})^{(k_{ij}^{q})} = g((\tilde{L}_{ij}^{q})^{(k_{ij}^{q})})$, $R_K = \frac{\sum_{q=1}^{g}(\tilde{p}_{ij}^{q})^{(k_{ij})} - \sum_{1\leq q<r\leq g}(\tilde{p}_{ij}^{q})^{(k_{ij})}(\tilde{p}_{ij}^{r})^{(k)} + \sum_{1\leq q<r<l\leq g}(\tilde{p}_{ij}^{q})^{(k_{ij}^{q})}(\tilde{p}_{ij}^{r})^{(k_{ij}^{r})}(\tilde{p}_{ij}^{l})^{(k_{ij}^{l})} + \dots + (-1)^{n-1}(\tilde{p}_{ij}^{1})^{(k_{ij}^{1})}(\tilde{p}_{ij}^{2})^{(k_{ij}^{2})}\dots(\tilde{p}_{ij}^{g})^{(k_{ij}^{g})}}{\sum_{q=1}^{g}(\#\tilde{L}_{ij}^{q}(p)) - 1}$.

Step 5. Calculate the collective comprehensive value $\bar{L}_i(p)$ of alternative A_i.

Based on the fuzzy measures of criteria subsets, aggregate the ith line elements of collective normalized probabilistic linguistic decision matrix $\hat{U} = [\hat{L}_{ij}(p)]_{m\times n}$ by PLGHC operator (i.e., Eq. (30)) to obtain $\bar{L}_i(p)$ as:

$$\begin{aligned}\bar{L}_i(p) &= PLGHC_{\Gamma}(\hat{L}_{i1}(p), \hat{L}_{i2}(p), \dots, \hat{L}_{in}(p)) \\ &= \bigcup_{\substack{k_{i(j)}=1,2,\dots,\#L_{i(j)}^{*}(p) \\ j=1,2,\dots,n}}\left\{g^{-1}[1 - \exp(-(\sum_{j=1}^{n}(\Gamma(S_{(j)}) - \Gamma(S_{(j-1)}))(-\ln(1 - A_{i(j)}^{k_{i(j)}}))^{\varepsilon})^{1/\varepsilon})](R_K)\right\}\end{aligned} \tag{49}$$

where $A_{i(j)}^{k_{i(j)}} = g(\hat{L}_{i(j)}^{*(k_{i(j)})})$, $R_K = \frac{\sum_{j=1}^{n} p_{i(j)}^{(k_{i(j)})} - \sum_{1\leq j<h\leq n} p_{i(j)}^{(k_{i(j)})} p_{i(h)}^{(k_{i(h)})} + \sum_{1\leq j<i<f\leq n} p_{i(j)}^{(k_{i(j)})} p_{i(h)}^{(k_{i(h)})} p_{i(f)}^{(k_{i(f)})} + \dots + (-1)^{n-1} p_{i(1)}^{(k_{i(1)})} p_{i(2)}^{(k_{i(2)})} \dots p_{i(n)}^{(k_{i(n)})}}{\sum_{j=1}^{n}(\#\hat{L}_{i(j)}^{*}(p)) - 1}$, $\hat{L}_{ij}^{*}(p) = n\varpi_j\hat{L}_{ij}(p)$, $\varpi = \{\varpi_1, \varpi_2, \dots, \varpi_n\}^{\mathrm{T}}$ is the weight vector of criteria, $\hat{L}_{i(j)}^{*}(p)$ is the jth largest PLTS of $\hat{L}_{ij}^{*}(p)$ $(j = 1, 2, \dots, n)$ according to the proposed possibility ranking algorithm in Section 3.2.

Step 6. Define the positive ideal solution (PIS).

The positive ideal solution (PIS) L^+ can be defined as $L^+ = \{s_{2\tau}(1)\}$. The Hamming distance between alternative A_i and PIS can be calculated by Eq. (8) with $\rho = 1$ as follows:

$$d(\bar{L}_i(p), L^+) = \sum_{\substack{k_{i(j)}=1,2,\dots,\#\hat{L}_{i(j)}^{*}(p) \\ j=1,2,\dots,n}} R_K \times \left|[1 - \exp(-(\sum_{j=1}^{n}(\Gamma(S_{(j)}) - \Gamma(S_{(j-1)}))(-\ln(1 - A_{i(j)}^{k_{i(j)}}))^{\varepsilon})^{1/\varepsilon}) - g(s_{2\tau})]/T\right|$$

where $A_{i(j)}^{k_{i(j)}} = g(\hat{L}_{i(j)}^{*(k_{i(j)})})$.

Step 7. Determine the fuzzy measures of criteria subsets.

To determine the fuzzy measures of criteria subsets, it is reasonable to minimize the distance between each alternative A_i and PIS L^+. Then, an optimization model is constructed as follows:

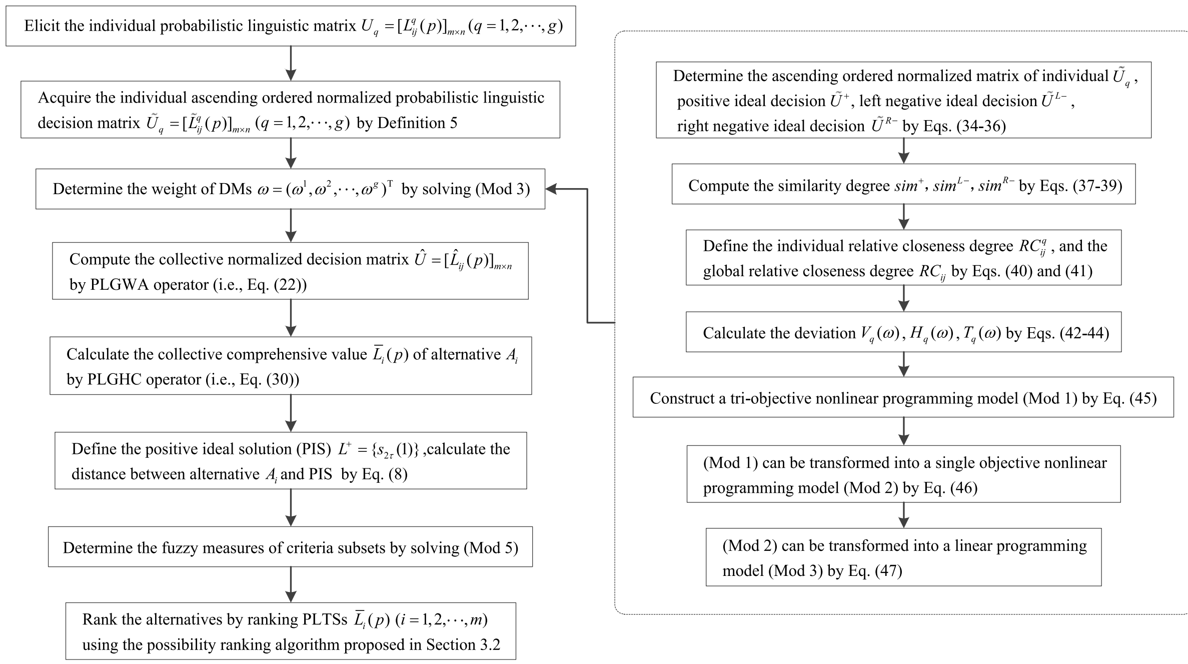

Fig. 1. Decision-making flowchart of the new proposed method.

(Mod 4)

$$\min \sum_{i=1}^{m} d(\bar{L}_i(p), L^+)$$
$$s.t. \begin{cases} \Gamma(A) \le \Gamma(B), \forall A \subseteq B \in P(C) \\ \Gamma(\phi) = 0, \Gamma(C) = 1, \Gamma \in \Theta(\Gamma) \end{cases} \tag{50}$$

To solve (Mod 4), let

$$d^+_{\bar{L}_i(p),L^+} = \frac{1}{2}\left[\sum_{\substack{k_{i(j)}=1,2,\ldots,\#\hat{L}^*_{i(j)}(p)\\ j=1,2,\ldots,n}} \frac{R_K}{T} \times \left|1 - \exp(-(\textstyle\sum_{j=1}^{n}(\Gamma(S_{(j)}) - \Gamma(S_{(j-1)}))(-\ln(1 - A_{i(j)}^{(k_{i(j)})}))^{\varepsilon})^{1/\varepsilon}) - g(s_{2\tau})\right| + \sum_{\substack{k_{i(j)}=1,2,\ldots,\#\hat{L}^*_{i(j)}(p)\\ j=1,2,\ldots,n}} \frac{R_K}{T} \times (1 - \exp(-(\textstyle\sum_{j=1}^{n}(\Gamma(S_{(j)}) - \Gamma(S_{(j-1)}))(-\ln(1 - A_{i(j)}^{(k_{i(j)})}))^{\varepsilon})^{1/\varepsilon}) - g(s_{2\tau}))\right],$$

$$d^-_{\bar{L}_i(p),L^+} = \frac{1}{2}\left[\sum_{\substack{k_{i(j)}=1,2,\ldots,\#\hat{L}^*_{i(j)}(p)\\ j=1,2,\ldots,n}} \frac{R_K}{T} \times \left|1 - \exp(-(\textstyle\sum_{j=1}^{n}(\Gamma(S_{(j)}) - \Gamma(S_{(j-1)}))(-\ln(1 - A_{i(j)}^{(k_{i(j)})}))^{\varepsilon})^{1/\varepsilon}) - g(s_{2\tau})\right| - \sum_{\substack{k_{i(j)}=1,2,\ldots,\#\hat{L}^*_{i(j)}(p)\\ j=1,2,\ldots,n}} \frac{R_K}{T} \times \left(1 - \exp(-(\textstyle\sum_{j=1}^{n}(\Gamma(S_{(j)}) - \Gamma(S_{(j-1)}))(-\ln(1 - A_{i(j)}^{(k_{i(j)})}))^{\varepsilon})^{1/\varepsilon}) - g(s_{2\tau})\right)\right].$$

Thus, (Mod 4) is converted into a goal programming model (Mod 5).
(Mod 5)

$$\min \sum_{i=1}^{m} (d^+_{\bar{L}_i(p),L^+} + d^-_{\bar{L}_i(p),L^+})$$
$$s.t. \begin{cases} \Gamma(A) \le \Gamma(B),\ \forall A \subseteq B \in P(C); \\ \Gamma(\phi) = 0,\ \Gamma(C) = 1,\ \Gamma \in \Theta(\Gamma); \\ d^+_{\bar{L}_i(p),L^+} - d^-_{\bar{L}_i(p),L^+} = \sum_{\substack{k_{i(j)}=1,2,\ldots,\#\hat{L}^*_{i(j)}(p)\\ j=1,2,\ldots,n}} \frac{R_K}{T} \times \left((1 - \exp(-(\sum_{j=1}^{n}(\Gamma(S_{(j)}) - \Gamma(S_{(j-1)}))(-\ln(1 - A_{i(j)}^{(k_{i(j)})}))^{\varepsilon})^{1/\varepsilon})) - g(s_{2\tau})\right); \\ d^+_{\bar{L}_i(p),L^+} \ge 0;\ d^-_{\bar{L}_i(p),L^+} \ge 0;\ \varepsilon \ge 1\ (k = 1, 2, \ldots, \#L^*_{i(j)}(p);\ i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n) \end{cases} \tag{51}$$

The fuzzy measures of criteria subsets can be obtained by solving (Mod 5).
Step 8. Rank the alternatives by ranking PLTSs $\bar{L}_i(p)(i = 1, 2, \ldots, m)$ using the possibility ranking algorithm proposed in Section 3.2. The decision-making flowchart of the new proposed method can be depicted in Fig. 1.

Remark 1. Once we have proposed a new method for solving a MCGDM problem, the complexity effort of the new method should be analyzed by the number of bits in the input and the dimension of the problem (in terms of the O-notation), since the requirement of time in emergency decision making is very important. In this paper, (Mod 3) and (Mod 5) determine the complexity degree of the proposed method. The numbers of decision variables in (Mod 3) and (Mod 5) are m and $2^n - 1$ respectively, where m is the number

Table 4
Decision matrices U_1, U_2, U_3 and U_4 given by four medical support teams.

		C_1	C_2	C_3	C_4
E_1	A_1	$\{s_3(0.1), s_4(0.3), s_5(0.4)\}$	$\{s_2(0.4), s_3(0.3)\}$	$\{s_4(0.1), s_5(0.2), s_6(0.4)\}$	$\{s_3(0.2), s_4(0.4), s_5(0.2)\}$
	A_2	$\{s_3(1)\}$	$\{s_2(0.3), s_3(0.3), s_4(0.4)\}$	$\{s_4(1)\}$	$\{s_2(0.1), s_3(0.2), s_4(0.2)\}$
	A_3	$\{s_2(0.4), s_3(0.6)\}$	$\{s_2(0.7), s_3(0.3)\}$	$\{s_3(0.1), s_4(0.4), s_5(0.2)\}$	$\{s_3(0.4), s_6(0.2)\}$
	A_4	$\{s_3(0.4), s_4(0.5)\}$	$\{s_2(0.3), s_3(0.3)\}$	$\{s_4(0.3), s_5(0.4)\}$	$\{s_2(0.1), s_3(0.4)\}$
	A_5	$\{s_1(0.1), s_2(0.3), s_3(0.3)\}$	$\{s_2(0.4), s_3(0.2)\}$	$\{s_4(0.3), s_5(0.4)\}$	$\{s_2(1)\}$
E_2	A_1	$\{s_5(0.5), s_6(0.1)\}$	$\{s_3(0.1), s_4(0.2), s_5(0.3)\}$	$\{s_5(0.3), s_6(0.5)\}$	$\{s_3(0.2), s_4(0.3), s_5(0.2)\}$
	A_2	$\{s_4(0.1), s_5(0.3), s_6(0.4)\}$	$\{s_3(0.1), s_4(0.4), s_5(0.2)\}$	$\{s_5(0.4), s_6(0.1), s_7(0.1)\}$	$\{s_3(0.3), s_4(0.6)\}$
	A_3	$\{s_4(0.2), s_5(0.1), s_6(0.4)\}$	$\{s_3(0.8), s_4(0.1)\}$	$\{s_5(0.4), s_6(0.2)\}$	$\{s_1(0.2), s_2(0.4), s_3(0.4)\}$
	A_4	$\{s_5(0.3), s_6(0.2)\}$	$\{s_2(0.4), s_3(0.5)\}$	$\{s_4(0.1), s_5(0.1), s_6(0.4)\}$	$\{s_2(0.4), s_3(0.5)\}$
	A_5	$\{s_5(0.4), s_6(0.4)\}$	$\{s_2(0.2), s_3(0.3), s_4(0.2)\}$	$\{s_5(0.7), s_6(0.1)\}$	$\{s_2(0.2), s_3(0.5), s_4(0.2)\}$
E_3	A_1	$\{s_4(0.2), s_5(0.4), s_6(0.3)\}$	$\{s_3(1)\}$	$\{s_4(0.2), s_5(0.3), s_6(0.5)\}$	$\{s_4(0.1), s_5(0.4)\}$
	A_2	$\{s_4(0.7), s_5(0.1)\}$	$\{s_5(0.4), s_6(0.1)\}$	$\{s_5(0.2), s_6(0.1), s_7(0.5)\}$	$\{s_4(0.1), s_5(0.3), s_6(0.1)\}$
	A_3	$\{s_3(0.1), s_4(0.4), s_5(0.3)\}$	$\{s_3(0.5), s_4(0.4)\}$	$\{s_5(0.1), s_6(0.5)\}$	$\{s_3(1)\}$
	A_4	$\{s_3(0.3), s_4(0.3)\}$	$\{s_4(0.3), s_5(0.2), s_6(0.3)\}$	$\{s_4(1)\}$	$\{s_2(0.1), s_3(0.3), s_4(0.1)\}$
	A_5	$\{s_4(0.4), s_5(0.3)\}$	$\{s_6(0.1), s_7(0.5)\}$	$\{s_4(0.1), s_5(0.4), s_6(0.2)\}$	$\{s_1(0.3), s_2(0.2), s_3(0.3)\}$
E_4	A_1	$\{s_5(0.1), s_6(0.6), s_7(0.1)\}$	$\{s_3(0.1), s_4(0.3), s_5(0.1)\}$	$\{s_3(0.1), s_4(0.3), s_5(0.3)\}$	$\{s_5(0.4), s_6(0.1)\}$
	A_2	$\{s_3(0.3), s_4(0.3), s_5(0.3)\}$	$\{s_3(0.1), s_4(0.4), s_5(0.1)\}$	$\{s_4(0.1), s_5(0.1), s_6(0.4)\}$	$\{s_4(1)\}$
	A_3	$\{s_4(0.4), s_5(0.2)\}$	$\{s_2(0.5), s_3(0.4)\}$	$\{s_4(0.3), s_5(0.5), s_6(0.1)\}$	$\{s_3(0.4), s_4(0.4), s_5(0.1)\}$
	A_4	$\{s_3(0.3), s_4(0.4)\}$	$\{s_3(0.2), s_4(0.4)\}$	$\{s_4(0.1), s_5(0.6)\}$	$\{s_3(0.3), s_4(0.3)\}$
	A_5	$\{s_3(0.3), s_4(0.6)\}$	$\{s_2(0.1), s_3(0.5), s_4(0.3)\}$	$\{s_4(0.4), s_5(0.4)\}$	$\{s_3(0.1), s_4(0.3), s_5(0.1)\}$

of DMs and n is the number of criteria. Using Karmarkar's algorithm [46], the time complexity of (Mod 3) is calculated as $O(m^{3.5}L^2)$, where L denotes the number of bits in the input. Similarly, the time complexity of (Mod 5) is $O(2^{3.5n}L^2)$. Hence, the time complexity of the proposed method is $\max\{O(2^{3.5n}L^2), O(m^{3.5}L^2)\}$. Despite the time complexity is a little high, (Mod 3) and (Mod 5) only consume very little computational time by using some mature software packages (e.g., LINGO and MATLAB).

7. Emergency assistance case study for COVID-19

In this section, an emergency assistance case is presented to demonstrate the rationality and robustness of the proposed method. Sensitivity analysis and comparative analysis are conducted to measure and compare the evaluation results of different type generated functions.

7.1. Emergency assistance area selection of COVID-19 for Wuhan

In 2020, a new coronavirus COVID-19 broke out all over the world. Wuhan in China was also suffering COVID-19. How to select an appropriate area to assist is an urgent issue. Taking five Hardest-hit Wuhan areas $A = \{A_1, A_2, A_3, A_4, A_5\}$ into account, whose confirmed and suspected cases of pneumonia were in the top five. There exist four criteria that affect the best and optimal assistance, such as supply medical support capacity C_1, medical supply delivery speed C_2, living material support capacity C_3, medical personnel transport capacity C_4. Preset LTS $S = \{s_0 = \textit{very bad}, s_1 = \textit{bad}, s_2 = \textit{a little bad}, s_3 = \textit{slightly bad}, s_4 = \textit{medium}, s_5 = \textit{slightly good}, s_6 = \textit{a little good}, s_7 = \textit{good}, s_8 = \textit{very good}\}$. The first aid comes from four national medical support teams, including medical support team E_1, E_2, E_3 and E_4. The weights of criteria are determined as $\varpi = (0.3, 0.3, 0.2, 0.2)^{\mathrm{T}}$ after discussion and negotiation. Based on the four criteria, four probabilistic linguistic evaluation matrices are constructed by these four medical support teams. The incomplete information structure of DMs' weights is provided by all medical support teams as follows:

$$\Lambda = \left\{ \omega \in \Lambda_0 \middle| \begin{array}{l} \omega^1 + \omega^3 \geq \omega^2 + \omega^4;\ 0.05 \leq \omega^1 \leq 0.35;\ 0.05 \leq \omega^2 \leq 0.35; \\ 0.05 \leq \omega^3 \leq 0.35;\ 0.05 \leq \omega^4 \leq 0.35;\ \omega^1 + \omega^2 + \omega^3 + \omega^4 = 1 \end{array} \right\}.$$

The incomplete information of fuzzy measures of criteria subsets is given as

$$\Theta(\Gamma) = \left\{ \Gamma \in \Theta(\Gamma) \middle| \begin{array}{l} \Gamma(A) \leq \Gamma(B),\ \forall A \subseteq B \in P(C);\ \Gamma(\phi) = 0,\ \Gamma(C) = 1; \\ 0 \leq \Gamma_1 \leq 1;\ 0 \leq \Gamma_2 \leq 1;\ 0 \leq \Gamma_3 \leq 1;\ 0 \leq \Gamma_4 \leq 1;\ 0 \leq \Gamma_{12} \leq 1;\ 0 \leq \Gamma_{13} \leq 1;\ 0 \leq \Gamma_{14} \leq 1;\ 0 \leq \Gamma_{23} \leq 1; \\ 0 \leq \Gamma_{24} \leq 1;\ 0 \leq \Gamma_{34} \leq 1;\ 0 \leq \Gamma_{123} \leq 1;\ 0 \leq \Gamma_{124} \leq 1;\ 0 \leq \Gamma_{134} \leq 1,\ 0 \leq \Gamma_{234} \leq 1; \\ \Gamma_{12} - (\Gamma_{123} + \Gamma_{124})/2 \geq 0;\ \Gamma_{24} - (\Gamma_{124} + \Gamma_{234})/2 \geq 0;\ \Gamma_{23} - (\Gamma_{123} + \Gamma_{234})/2 \geq 0; \\ \Gamma_1 - (\Gamma_{12} + \Gamma_{13} + \Gamma_{14})/2 + (\Gamma_{123} + \Gamma_{124} + \Gamma_{134})/3 > \Gamma_4 - (\Gamma_{14} + \Gamma_{24} + \Gamma_{34})/2 + (\Gamma_{124} + \Gamma_{134} + \Gamma_{234})/3; \\ \Gamma_2 - (\Gamma_{12} + \Gamma_{23} + \Gamma_{24})/2 + (\Gamma_{123} + \Gamma_{124} + \Gamma_{234})/3 > \Gamma_3 - (\Gamma_{13} + \Gamma_{23} + \Gamma_{34})/2 + (\Gamma_{123} + \Gamma_{234} + \Gamma_{134})/3 \end{array} \right\}.$$

Next, the proposed method of this paper is employed to solve this example.
Step 1. The decision matrices U_1, U_2, U_3 and U_4 are constructed in Table 4.
Step 2. The corresponding ascending ordered normalized matrices $\tilde{U}_1$, $\tilde{U}_2$, $\tilde{U}_3$ and $\tilde{U}_4$ are obtained in Table 5.
Step 3. Determine the weights of DMs.

Table 5
Ascending ordered normalized decision matrices $\tilde{U}_1$, $\tilde{U}_2$, $\tilde{U}_3$ and $\tilde{U}_4$.

		C_1	C_2	C_3	C_4
E_1	A_1	$\{s_3(1/8), s_4(3/8), s_5(1/2)\}$	$\{s_2(0), s_2(4/7), s_3(3/7)\}$	$\{s_4(1/7), s_5(2/7), s_6(4/7)\}$	$\{s_3(1/4), s_4(1/2), s_5(1/4)\}$
	A_2	$\{s_3(0), s_3(0), s_3(1)\}$	$\{s_2(3/10), s_3(3/10), s_4(2/5)\}$	$\{s_4(0), s_4(0), s_4(1)\}$	$\{s_2(1/5), s_3(2/5), s_4(2/5)\}$
	A_3	$\{s_2(0), s_2(2/5), s_3(3/5)\}$	$\{s_2(0), s_2(7/10), s_3(3/10)\}$	$\{s_3(1/7), s_4(4/7), s_5(2/7)\}$	$\{s_3(0), s_3(2/3), s_6(1/3)\}$
	A_4	$\{s_3(0), s_3(4/9), s_4(5/9)\}$	$\{s_2(0), s_2(1/2), s_3(1/2)\}$	$\{s_4(0), s_4(3/7), s_5(4/7)\}$	$\{s_2(0), s_2(1/5), s_3(4/5)\}$
	A_5	$\{s_1(1/7), s_2(3/7), s_3(3/7)\}$	$\{s_2(0), s_2(2/3), s_3(1/3)\}$	$\{s_4(0), s_4(3/7), s_5(4/7)\}$	$\{s_2(0), s_2(0), s_2(1)\}$
E_2	A_1	$\{s_5(0), s_5(5/6), s_6(1/6)\}$	$\{s_3(1/6), s_4(1/3), s_5(1/2)\}$	$\{s_5(0), s_5(3/8), s_6(5/8)\}$	$\{s_3(2/7), s_4(3/7), s_5(2/7)\}$
	A_2	$\{s_4(1/8), s_5(3/8), s_6(1/2)\}$	$\{s_3(1/7), s_4(4/7), s_5(2/7)\}$	$\{s_5(2/3), s_6(1/6), s_7(1/6)\}$	$\{s_3(0), s_3(1/3), s_4(2/3)\}$
	A_3	$\{s_4(2/7), s_5(1/7), s_6(4/7)\}$	$\{s_3(0), s_3(8/9), s_4(1/9)\}$	.$\{s_5(0), s_5(2/3), s_6(1/3)\}$.	$\{s_1(1/5), s_2(2/5), s_3(2/5)\}$
	A_4	$\{s_5(0), s_5(3/5), s_6(2/5)\}$	$\{s_2(0), s_2(4/9), s_3(5/9)\}$	$\{s_4(1/6), s_5(1/6), s_6(2/3)\}$	$\{s_2(0), s_2(4/9), s_3(5/9)\}$
	A_5	$\{s_5(0), s_5(1/2), s_6(1/2)\}$	$\{s_2(2/7), s_3(3/7), s_4(2/7)\}$	$\{s_5(0), s_5(7/8), s_6(1/8)\}$	$\{s_2(2/9), s_3(5/9), s_4(2/9)\}$
E_3	A_1	$\{s_4(2/9), s_5(4/9), s_6(1/3)\}$	$\{s_3(0), s_3(0), s_3(1)\}$	$\{s_4(1/5), s_5(3/10), s_6(1/2)\}$	$\{s_4(0), s_4(1/5), s_5(4/5)\}$
	A_2	$\{s_4(0), s_4(7/8), s_5(1/8)\}$	$\{s_5(0), s_5(4/5), s_6(1/5)\}$	$\{s_5(1/4), s_6(1/8), s_7(5/8)\}$	$\{s_4(1/5), s_5(3/5), s_6(1/5)\}$
	A_3	$\{s_3(1/8), s_4(1/2), s_5(3/8)\}$	$\{s_3(0), s_3(5/9), s_4(4/9)\}$	$\{s_5(0), s_5(1/6), s_6(5/6)\}$	$\{s_3(0), s_3(0), s_3(1)\}$
	A_4	$\{s_3(0), s_3(1/2), s_4(1/2)\}$	$\{s_4(3/8), s_5(1/4), s_6(3/8)\}$	$\{s_4(0), s_4(0), s_4(1)\}$	$\{s_2(1/5), s_3(3/5), s_4(1/5)\}$
	A_5	$\{s_4(0), s_4(4/7), s_5(3/7)\}$	$\{s_6(0), s_6(1/6), s_7(5/6)\}$	$\{s_4(1/7), s_5(4/7), s_6(2/7)\}$	$\{s_1(3/8), s_2(1/4), s_3(3/8)\}$
E_4	A_1	$\{s_5(1/8), s_6(3/4), s_7(1/8)\}$	$\{s_3(1/5), s_4(3/5), s_5(1/5)\}$	$\{s_3(1/7), s_4(3/7), s_5(3/7)\}$	$\{s_5(0), s_5(4/5), s_6(1/5)\}$
	A_2	$\{s_3(1/3), s_4(1/3), s_5(1/3)\}$	$\{s_3(1/6), s_4(2/3), s_5(1/6)\}$	$\{s_4(1/6), s_5(1/6), s_6(2/3)\}$	$\{s_4(0), s_4(0), s_4(1)\}$
	A_3	$\{s_4(0), s_4(2/3), s_5(1/3)\}$	$\{s_2(0), s_2(5/9), s_3(4/9)\}$	$\{s_4(1/3), s_5(5/9), s_6(1/9)\}$	$\{s_3(4/9), s_4(4/9), s_5(1/9)\}$
	A_4	$\{s_3(0), s_3(3/7), s_4(4/7)\}$	$\{s_3(0), s_3(1/3), s_4(2/3)\}$	$\{s_4(0), s_4(1/7), s_5(6/7)\}$	$\{s_3(0), s_3(1/2), s_4(1/2)\}$
	A_5	$\{s_3(0), s_3(1/3), s_4(2/3)\}$	$\{s_2(1/9), s_3(5/9), s_4(1/3)\}$	$\{s_4(0), s_4(1/2), s_5(1/2)\}$	$\{s_3(1/5), s_4(3/5), s_5(1/5)\}$

Table 6
A collective normalized decision matrix $\hat{U}$.

	C_1	C_2	C_3	C_4
A_1	$\{s_{4.20}(0.19), s_{5.12}(0.46), s_{6.19}(0.35)\}$	$\{s_{2.67}(0.15), s_{3.18}(0.40), s_{4.03}(0.45)\}$	$\{s_{3.80}(0.19), s_{4.68}(0.37), s_{5.70}(0.44)\}$	$\{s_{4.00}(0.20), s_{4.38}(0.41), s_{5.40}(0.39)\}$
A_2	$\{s_{3.32}(0.18), s_{3.80}(0.40), s_{4.56}(0.42)\}$	$\{s_{3.19}(0.23), s_{3.92}(0.45), s_{4.94}(0.32)\}$	$\{s_{4.33}(0.36), s_{5.06}(0.18), s_{5.93}(0.46)\}$	$\{s_{3.29}(0.16), s_{3.82}(0.38), s_{4.52}(0.46)\}$
A_3	$\{s_{3.18}(0.17), s_{3.52}(0.41), s_{4.56}(0.42)\}$	$\{s_{2.32}(0), s_{2.32}(0.55), s_{3.32}(0.45)\}$	$\{s_{4.03}(0.19), s_{4.68}(0.41), s_{5.70}(0.40)\}$	$\{s_{2.83}(0.23), s_{3.29}(0.36), s_{4.13}(0.41)\}$
A_4	$\{s_{3.25}(0), s_{3.25}(0.50), s_{4.27}(0.50)\}$	$\{s_{2.81}(0.17), s_{3.10}(0.39), s_{4.15}(0.44)\}$	$\{s_4(0.09), s_{4.11}(0.34), s_{4.95}(0.57)\}$	$\{s_{2.37}(0.10), s_{2.57}(0.44), s_{3.57}(0.46)\}$
A_5	$\{s_{3.89}(0.07), s_{3.16}(0.46), s_{4.19}(0.47)\}$	$\{s_{3.18}(0.16), s_{3.56}(0.42), s_{4.72}(0.42)\}$	$\{s_{4.11}(0.07), s_{4.33}(0.49), s_{5.34}(0.44)\}$	$\{s_{2.19}(0.25), s_{2.89}(0.35), s_{3.64}(0.40)\}$

According to (Mod 3), a linear programming model of the DMs' weights is built as:

$$\min\ H=\sum_{q=1}^{4}\sum_{j=1}^{4}\sum_{i=1}^{5}(\vartheta_{ij}^{q+}+\vartheta_{ij}^{q-})+\sum_{q=1}^{4}\sum_{j=1}^{4}\sum_{i=1}^{5}\sum_{r=1}^{4}\omega^{q}\left|RC_{ij}^{q}-RC_{ij}^{r}\right|-\sum_{q=1}^{4}\sum_{j=1}^{4}\sum_{i=1}^{5}\sum_{t=i+1}^{5}\omega^{q}\left|RC_{ij}^{q}-RC_{tj}^{q}\right|$$
$$s.t.\begin{cases}\vartheta_{ij}^{q+}-\vartheta_{ij}^{q-}=RC_{ij}^{q}-\sum_{q=1}^{g}\omega^{q}RC_{ij}^{q};\ (i=1,2,3,4,5;\ j=1,2,3,4;\ q=1,2,3,4)\\ \vartheta_{ij}^{q+}\geq 0;\ \vartheta_{ij}^{q-}\geq 0;\ \ (i=1,2,3,4,5;\ j=1,2,3,4;\ q=1,2,3,4)\\ \omega^{1}+\omega^{3}\geq\omega^{2}+\omega^{4};\ 0.05\leq\omega^{1}\leq 0.35;\ 0.05\leq\omega^{2}\leq 0.35;\ 0.05\leq\omega^{3}\leq 0.35;\\ 0.05\leq\omega^{4}\leq 0.35;\ \omega^{1}+\omega^{2}+\omega^{3}+\omega^{4}=1\end{cases}\tag{52}$$

Then, consider the Form 1 (i.e., Eq. (4)), the DMs' weights are obtained by solving Eq. (52) as follows:

$$\omega^{1}=0.35,\ \omega^{2}=0.10,\ \omega^{3}=0.20,\ \omega^{4}=0.35.$$

Step 4. Compute the collective normalized decision matrix $\hat{U}$. To simplify the calculation process, set $\varepsilon=1$. By Eq. (48), the collective normalized decision matrix is obtained and listed in Table 6.

Step 5. Combine the weights of criteria $\varpi=(0.3,0.3,0.2,0.2)^{\mathrm{T}}$ to calculate the collective comprehensive value $\bar{L}_i(p)$ of alternative A_i by Eq. (49) as follows:

$$\bar{L}_1(p)=\bigcup_{\substack{k_{1(j)}=1,2,\ldots,\#L_{1(j)}^{*}(p)\\ j=1,2,3,4}}\left\{g^{-1}\left[\begin{array}{l}1-\exp\big(-((1-\Gamma_{234})(-\ln(1-g(4\varpi_1\hat{L}_{1(1)}^{(k_{1(1)})})))^{\varepsilon}+(\Gamma_{234}-\Gamma_{24})(-\ln(1-g(4\varpi_3\hat{L}_{1(3)}^{(k_{1(3)})})))^{\varepsilon}\\ +(\Gamma_{24}-\Gamma_{2})(-\ln(1-g(4\varpi_4\hat{L}_{1(4)}^{(k_{1(4)})})))^{\varepsilon}+\Gamma_{2}(-\ln(1-g(4\varpi_2\hat{L}_{1(2)}^{(k_{1(2)})})))^{\varepsilon})^{1/\varepsilon}\big)\end{array}\right](R_{K_1})\right\}$$

$$\begin{aligned}R_{K_1}=&(p_{1(1)}^{(k_{1(1)})}+p_{1(2)}^{(k_{1(2)})}+p_{1(3)}^{(k_{1(3)})}+p_{1(4)}^{(k_{1(4)})}-p_{1(1)}^{(k_{1(1)})}p_{1(2)}^{(k_{1(2)})}-p_{1(1)}^{(k_{1(1)})}p_{1(3)}^{(k_{1(3)})}-p_{1(1)}^{(k_{1(1)})}p_{1(4)}^{(k_{1(4)})}-p_{1(2)}^{(k_{1(2)})}p_{1(3)}^{(k_{1(3)})}-p_{1(2)}^{(k_{1(2)})}p_{1(4)}^{(k_{1(4)})}\\&-p_{1(3)}^{(k_{1(3)})}p_{1(4)}^{(k_{1(4)})}+p_{1(1)}^{(k_{1(1)})}p_{1(2)}^{(k_{1(2)})}p_{1(3)}^{(k_{1(3)})}+p_{1(1)}^{(k_{1(1)})}p_{1(2)}^{(k_{1(2)})}p_{1(4)}^{(k_{1(4)})}+p_{1(1)}^{(k_{1(1)})}p_{1(3)}^{(k_{1(3)})}p_{1(4)}^{(k_{1(4)})}+p_{1(2)}^{(k_{1(2)})}p_{1(3)}^{(k_{1(3)})}p_{1(4)}^{(k_{1(4)})}\\&-p_{1(1)}^{(k_{1(1)})}p_{1(2)}^{(k_{1(2)})}p_{1(3)}^{(k_{1(3)})}p_{1(4)}^{(k_{1(4)})})/(\#\hat{L}_{1(1)}^{*}(p)+\#\hat{L}_{1(2)}^{*}(p)+\#\hat{L}_{1(3)}^{*}(p)+\#\hat{L}_{1(4)}^{*}(p)-1);\end{aligned}$$

$$\bar{L}_2(p)=\bigcup_{\substack{k_{2(j)}=1,2,\ldots,\#L_{2(j)}^{*}(p)\\ j=1,2,3,4}}\left\{g^{-1}\left[\begin{array}{l}1-\exp\big(-((1-\Gamma_{124})(-\ln(1-g(4\varpi_3\hat{L}_{2(3)}^{(k_{2(3)})})))^{\varepsilon}+(\Gamma_{124}-\Gamma_{24})(-\ln(1-g(4\varpi_1\hat{L}_{2(1)}^{(k_{2(1)})})))^{\varepsilon}\\ +(\Gamma_{24}-\Gamma_{4})(-\ln(1-g(4\varpi_2\hat{L}_{2(2)}^{(k_{2(2)})})))^{\varepsilon}+\Gamma_{4}(-\ln(1-g(4\varpi_4\hat{L}_{2(4)}^{(k_{2(4)})})))^{\varepsilon})^{1/\varepsilon}\ \big)\end{array}\right](R_{K_2})\right\}$$

$$\begin{aligned}R_{K_2}=&(p_{2(1)}^{(k_{2(1)})}+p_{2(2)}^{(k_{2(2)})}+p_{2(3)}^{(k_{2(3)})}+p_{2(4)}^{(k_{2(4)})}-p_{2(1)}^{(k_{2(1)})}p_{2(2)}^{(k_{2(2)})}-p_{2(1)}^{(k_{2(1)})}p_{2(3)}^{(k_{2(3)})}-p_{2(1)}^{(k_{2(1)})}p_{2(4)}^{(k_{2(4)})}-p_{2(2)}^{(k_{2(2)})}p_{2(3)}^{(k_{2(3)})}-p_{2(2)}^{(k_{2(2)})}p_{2(4)}^{(k_{2(4)})}\\&-p_{2(3)}^{(k_{2(3)})}p_{2(4)}^{(k_{2(4)})}+p_{2(1)}^{(k_{2(1)})}p_{2(2)}^{(k_{2(2)})}p_{2(3)}^{(k_{2(3)})}+p_{2(1)}^{(k_{2(1)})}p_{2(2)}^{(k_{2(2)})}p_{2(4)}^{(k_{2(4)})}+p_{2(1)}^{(k_{2(1)})}p_{2(3)}^{(k_{2(3)})}p_{2(4)}^{(k_{2(4)})}+p_{2(2)}^{(k_{2(2)})}p_{2(3)}^{(k_{2(3)})}p_{2(4)}^{(k_{2(4)})}\\&-p_{2(1)}^{(k_{2(1)})}p_{2(2)}^{(k_{2(2)})}p_{2(3)}^{(k_{2(3)})}p_{2(4)}^{(k_{2(4)})})/(\#\hat{L}_{2(1)}^{*}(p)+\#\hat{L}_{2(2)}^{*}(p)+\#\hat{L}_{2(3)}^{*}(p)+\#\hat{L}_{2(4)}^{*}(p)-1);\end{aligned}$$

Table 7
Fuzzy measures of criteria subsets.

Subsets	Γ	Subsets	Γ	Subsets	Γ	Subsets	Γ	Subsets	Γ
C_1	1.0000	C_4	1.0000	C_1, C_4	1.0000	C_3, C_4	1.0000	C_1, C_3, C_4	0.0000
C_2	1.0000	C_1, C_2	0.7273	C_2, C_3	1.0000	C_1, C_2, C_3	0.1818	C_2, C_3, C_4	1.0000
C_3	0.0000	C_1, C_3	0.7273	C_2, C_4	1.0000	C_1, C_2, C_4	1.0000	C_1, C_2, C_3, C_4	1.0000

$$\bar{L}_3(p)=\bigcup_{\substack{k_{3(j)}=1,2,\ldots,\#L^*_{3(j)}(p)\\ j=1,2,3,4}}\left\{g^{-1}\left[\begin{array}{l}1-\exp\left(-((1-\Gamma_{124})(-\ln(1-g(4\varpi_3\hat{L}_{3(3)}^{(k_{3(3)})})))^{\varepsilon}+(\Gamma_{124}-\Gamma_{24})(-\ln(1-g(4\varpi_1\hat{L}_{3(1)}^{(k_{3(1)})})))^{\varepsilon}\right.\\ \left.+(\Gamma_{24}-\Gamma_2)(-\ln(1-g(4\varpi_4\hat{L}_{3(4)}^{(k_{3(4)})})))^{\varepsilon}+\Gamma_2(-\ln(1-g(4\varpi_2\hat{L}_{3(2)}^{(k_{3(2)})})))^{\varepsilon})^{1/\varepsilon}\right)\end{array}\right](R_{K_3})\right\}$$

$$\begin{aligned}R_{K_3}=\;&(p_{3(1)}^{(k_{3(1)})}+p_{3(2)}^{(k_{3(2)})}+p_{3(3)}^{(k_{3(3)})}+p_{3(4)}^{(k_{3(4)})}-p_{3(1)}^{(k_{3(1)})}p_{3(2)}^{(k_{3(2)})}-p_{3(1)}^{(k_{3(1)})}p_{3(3)}^{(k_{3(3)})}-p_{3(1)}^{(k_{3(1)})}p_{3(4)}^{(k_{3(4)})}-p_{3(2)}^{(k_{3(2)})}p_{3(3)}^{(k_{3(3)})}-p_{3(2)}^{(k_{3(2)})}p_{3(4)}^{(k_{3(4)})}\\&-p_{3(3)}^{(k_{3(3)})}p_{3(4)}^{(k_{3(4)})}+p_{3(1)}^{(k_{3(1)})}p_{3(2)}^{(k_{3(2)})}p_{3(3)}^{(k_{3(3)})}+p_{3(1)}^{(k_{3(1)})}p_{3(2)}^{(k_{3(2)})}p_{3(4)}^{(k_{3(4)})}+p_{3(1)}^{(k_{3(1)})}p_{3(3)}^{(k_{3(3)})}p_{3(4)}^{(k_{3(4)})}+p_{3(2)}^{(k_{3(2)})}p_{3(3)}^{(k_{3(3)})}p_{3(4)}^{(k_{3(4)})}\\&-p_{3(1)}^{(k_{3(1)})}p_{3(2)}^{(k_{3(2)})}p_{3(3)}^{(k_{3(3)})}p_{3(4)}^{(k_{3(4)})})/(\#\hat{L}^*_{3(1)}(p)+\#\hat{L}^*_{3(2)}(p)+\#\hat{L}^*_{3(3)}(p)+\#\hat{L}^*_{3(4)}(p)-1);\end{aligned}$$

$$\bar{L}_4(p)=\bigcup_{\substack{k_{4(j)}=1,2,\ldots,\#L^*_{4(j)}(p)\\ j=1,2,3,4}}\left\{g^{-1}\left[\begin{array}{l}1-\exp\left(-((1-\Gamma_{124})(-\ln(1-g(4\varpi_3\hat{L}_{4(3)}^{(k_{4(3)})})))^{\varepsilon}+(\Gamma_{124}-\Gamma_{24})(-\ln(1-g(4\varpi_1\hat{L}_{4(1)}^{(k_{4(1)})})))^{\varepsilon}\right.\\ \left.+(\Gamma_{24}-\Gamma_4)(-\ln(1-g(4\varpi_2\hat{L}_{4(2)}^{(k_{4(2)})})))^{\varepsilon}+\Gamma_4(-\ln(1-g(4\varpi_4\hat{L}_{4(4)}^{(k_{4(4)})})))^{\varepsilon})^{1/\varepsilon}\right)\end{array}\right](R_{K_4})\right\}$$

$$\begin{aligned}R_{K_4}=\;&(p_{4(1)}^{(k_{4(1)})}+p_{4(2)}^{(k_{4(2)})}+p_{4(3)}^{(k_{4(3)})}+p_{4(4)}^{(k_{4(4)})}-p_{4(1)}^{(k_{4(1)})}p_{4(2)}^{(k_{4(2)})}-p_{4(1)}^{(k_{4(1)})}p_{4(3)}^{(k_{4(3)})}-p_{4(1)}^{(k_{4(1)})}p_{4(4)}^{(k_{4(4)})}-p_{4(2)}^{(k_{4(2)})}p_{4(3)}^{(k_{4(3)})}-p_{4(2)}^{(k_{4(2)})}p_{4(4)}^{(k_{4(4)})}\\&-p_{4(3)}^{(k_{4(3)})}p_{4(4)}^{(k_{4(4)})}+p_{4(1)}^{(k_{4(1)})}p_{4(2)}^{(k_{4(2)})}p_{4(3)}^{(k_{4(3)})}+p_{4(1)}^{(k_{4(1)})}p_{4(2)}^{(k_{4(2)})}p_{4(4)}^{(k_{4(4)})}+p_{4(1)}^{(k_{4(1)})}p_{4(3)}^{(k_{4(3)})}p_{4(4)}^{(k_{4(4)})}+p_{4(2)}^{(k_{4(2)})}p_{4(3)}^{(k_{4(3)})}p_{4(4)}^{(k_{4(4)})}\\&-p_{4(1)}^{(k_{4(1)})}p_{4(2)}^{(k_{4(2)})}p_{4(3)}^{(k_{4(3)})}p_{4(4)}^{(k_{4(4)})})/(\#\hat{L}^*_{4(1)}(p)+\#\hat{L}^*_{4(2)}(p)+\#\hat{L}^*_{4(3)}(p)+\#\hat{L}^*_{4(4)}(p)-1);\end{aligned}$$

$$\bar{L}_5(p)=\bigcup_{\substack{k_{5(j)}=1,2,\ldots,\#L^*_{5(j)}(p)\\ j=1,2,3,4}}\left\{g^{-1}\left[\begin{array}{l}1-\exp\left(-((1-\Gamma_{124})(-\ln(1-g(4\varpi_3\hat{L}_{5(3)}^{(k_{5(3)})})))^{\varepsilon}+(\Gamma_{124}-\Gamma_{14})(-\ln(1-g(4\varpi_2\hat{L}_{5(2)}^{(k_{5(2)})})))^{\varepsilon}\right.\\ \left.+(\Gamma_{14}-\Gamma_4)(-\ln(1-g(4\varpi_1\hat{L}_{5(1)}^{(k_{5(1)})})))^{\varepsilon}+\Gamma_4(-\ln(1-g(4\varpi_4\hat{L}_{5(4)}^{(k_{5(4)})})))^{\varepsilon})^{1/\varepsilon}\right)\end{array}\right](R_{K_5})\right\}$$

$$\begin{aligned}R_{K_5}=\;&(p_{5(1)}^{(k_{5(1)})}+p_{5(2)}^{(k_{5(2)})}+p_{5(3)}^{(k_{5(3)})}+p_{5(4)}^{(k_{5(4)})}-p_{5(1)}^{(k_{5(1)})}p_{5(2)}^{(k_{5(2)})}-p_{5(1)}^{(k_{5(1)})}p_{5(3)}^{(k_{5(3)})}-p_{5(1)}^{(k_{5(1)})}p_{5(4)}^{(k_{5(4)})}-p_{5(2)}^{(k_{5(2)})}p_{5(3)}^{(k_{5(3)})}-p_{5(2)}^{(k_{5(2)})}p_{5(4)}^{(k_{5(4)})}\\&-p_{5(3)}^{(k_{5(3)})}p_{5(4)}^{(k_{5(4)})}+p_{5(1)}^{(k_{5(1)})}p_{5(2)}^{(k_{5(2)})}p_{5(3)}^{(k_{5(3)})}+p_{5(1)}^{(k_{5(1)})}p_{5(2)}^{(k_{5(2)})}p_{5(4)}^{(k_{5(4)})}+p_{5(1)}^{(k_{5(1)})}p_{5(3)}^{(k_{5(3)})}p_{5(4)}^{(k_{5(4)})}+p_{5(2)}^{(k_{5(2)})}p_{5(3)}^{(k_{5(3)})}p_{5(4)}^{(k_{5(4)})}\\&-p_{5(1)}^{(k_{5(1)})}p_{5(2)}^{(k_{5(2)})}p_{5(3)}^{(k_{5(3)})}p_{5(4)}^{(k_{5(4)})})/(\#\hat{L}^*_{5(1)}(p)+\#\hat{L}^*_{5(2)}(p)+\#\hat{L}^*_{5(3)}(p)+\#\hat{L}^*_{5(4)}(p)-1).\end{aligned}$$

Step 6. Define the PIS $L^+=\{s_8(1)\}$.

Step 7. Determine the fuzzy measures of criteria subsets.

For simplicity, denote the fuzzy measure of criterion subset by $\Gamma_{jf\cdots h}=\Gamma(\{C_j,C_f,\ldots,C_h\})$ $(1\le j<f<h\le n)$. For example, $\Gamma_1=\Gamma(\{C_1\})$, $\Gamma_{12}=\Gamma(\{C_1,C_2\})$. Set $\varepsilon=1$. According to (Mod 5), a goal programming model of the fuzzy measures is built as follows:
(Mod 6)

$$\min\ \sum_{i=1}^{5}(d^+_{\bar{L}_i(p),L^+}+d^-_{\bar{L}_i(p),L^+})$$

$$s.t.\begin{cases}\Gamma(A)\le\Gamma(B),\ \forall A\subseteq B\in P(C);\ \Gamma(\phi)=0,\ \Gamma(C)=1;\\ 0\le\Gamma_1\le1;\ 0\le\Gamma_2\le1;\ 0\le\Gamma_3\le1;\ 0\le\Gamma_4\le1;0\le\Gamma_{12}\le1;\ 0\le\Gamma_{13}\le1;\ 0\le\Gamma_{14}\le1;\ 0\le\Gamma_{23}\le1;\\ 0\le\Gamma_{24}\le1;\ 0\le\Gamma_{34}\le1;\ 0\le\Gamma_{123}\le1;\ 0\le\Gamma_{124}\le1;\ 0\le\Gamma_{134}\le1;\ 0\le\Gamma_{234}\le1;\\ \Gamma_{12}-(\Gamma_{123}+\Gamma_{124})/2\ge0;\ \Gamma_{24}-(\Gamma_{124}+\Gamma_{234})/2\ge0;\ \Gamma_{23}-(\Gamma_{123}+\Gamma_{234})/2\ge0;\\ \Gamma_1-(\Gamma_{12}+\Gamma_{13}+\Gamma_{14})/2+(\Gamma_{123}+\Gamma_{124}+\Gamma_{134})/3>\Gamma_4-(\Gamma_{14}+\Gamma_{24}+\Gamma_{34})/2+(\Gamma_{124}+\Gamma_{134}+\Gamma_{234})/3;\\ \Gamma_2-(\Gamma_{12}+\Gamma_{23}+\Gamma_{24})/2+(\Gamma_{123}+\Gamma_{124}+\Gamma_{234})/3>\Gamma_3-(\Gamma_{13}+\Gamma_{23}+\Gamma_{34})/2+(\Gamma_{123}+\Gamma_{234}+\Gamma_{134})/3;\\ d^+_{\bar{L}_1(p),L^+}-d^-_{\bar{L}_1(p),L^+}=\sum_{\substack{k_{1(j)}=1,2,\ldots,\#\hat{L}^*_{1(j)}(p)\\ j=1,2,3,4}}\left[\frac{R_{K_1}}{T}\times\left(\begin{array}{l}1-\exp(-((1-\Gamma_{234})(-\ln(1-A_{11}^{k_{11}}))^{\varepsilon}+(\Gamma_{234}-\Gamma_{24})(-\ln(1-A_{13}^{k_{13}}))^{\varepsilon}+\\(\Gamma_{24}-\Gamma_2)(-\ln(1-A_{14}^{k_{14}}))^{\varepsilon}+\Gamma_2(-\ln(1-A_{12}^{k_{12}}))^{\varepsilon})^{1/\varepsilon})-g(s_8)\end{array}\right)\right];\\ d^+_{\bar{L}_2(p),L^+}-d^-_{\bar{L}_2(p),L^+}=\sum_{\substack{k_{2(j)}=1,2,\ldots,\#\hat{L}^*_{2(j)}(p)\\ j=1,2,3,4}}\left[\frac{R_{K_2}}{T}\times\left(\begin{array}{l}1-\exp(-((1-\Gamma_{124})(-\ln(1-A_{23}^{k_{23}}))^{\varepsilon}+(\Gamma_{124}-\Gamma_{24})(-\ln(1-A_{21}^{k_{21}}))^{\varepsilon}+\\(\Gamma_{24}-\Gamma_4)(-\ln(1-A_{22}^{k_{22}}))^{\varepsilon}+\Gamma_4(-\ln(1-A_{24}^{k_{24}}))^{\varepsilon})^{1/\varepsilon})-g(s_8)\end{array}\right)\right];\\ d^+_{\bar{L}_3(p),L^+}-d^-_{\bar{L}_3(p),L^+}=\sum_{\substack{k_{3(j)}=1,2,\ldots,\#\hat{L}^*_{3(j)}(p)\\ j=1,2,3,4}}\left[\frac{R_{K_3}}{T}\times\left(\begin{array}{l}1-\exp(-((1-\Gamma_{124})(-\ln(1-A_{33}^{k_{33}}))^{\varepsilon}+(\Gamma_{124}-\Gamma_{24})(-\ln(1-A_{31}^{k_{31}}))^{\varepsilon}+\\(\Gamma_{24}-\Gamma_2)(-\ln(1-A_{34}^{k_{34}}))^{\varepsilon}+\Gamma_2(-\ln(1-A_{32}^{k_{32}}))^{\varepsilon})^{1/\varepsilon})-g(s_8)\end{array}\right)\right];\\ d^+_{\bar{L}_4(p),L^+}-d^-_{\bar{L}_4(p),L^+}=\sum_{\substack{k_{4(j)}=1,2,\ldots,\#\hat{L}^*_{4(j)}(p)\\ j=1,2,3,4}}\left[\frac{R_{K_4}}{T}\times\left(\begin{array}{l}1-\exp(-((1-\Gamma_{124})(-\ln(1-A_{43}^{k_{43}}))^{\varepsilon}+(\Gamma_{124}-\Gamma_{24})(-\ln(1-A_{41}^{k_{41}}))^{\varepsilon}+\\(\Gamma_{24}-\Gamma_4)(-\ln(1-A_{42}^{k_{42}}))^{\varepsilon}+\Gamma_4(-\ln(1-A_{44}^{k_{44}}))^{\varepsilon})^{1/\varepsilon})-g(s_8)\end{array}\right)\right];\\ d^+_{\bar{L}_5(p),L^+}-d^-_{\bar{L}_5(p),L^+}=\sum_{\substack{k_{5(j)}=1,2,\ldots,\#\hat{L}^*_{5(j)}(p)\\ j=1,2,3,4}}\left[\frac{R_{K_5}}{T}\times\left(\begin{array}{l}1-\exp(-((1-\Gamma_{124})(-\ln(1-A_{53}^{k_{53}}))^{\varepsilon}+(\Gamma_{124}-\Gamma_{24})(-\ln(1-A_{51}^{k_{51}}))^{\varepsilon}+\\(\Gamma_{24}-\Gamma_4)(-\ln(1-A_{52}^{k_{52}}))^{\varepsilon}+\Gamma_4(-\ln(1-A_{54}^{k_{54}}))^{\varepsilon})^{1/\varepsilon})-g(s_8)\end{array}\right)\right];\\ d^+_{\bar{L}_i(p),L^+}\ge0;\ d^-_{\bar{L}_i(p),L^+}\ge0\ (i=1,2,3,4,5)\end{cases}$$

By solving (Mod 6), the fuzzy measures of criteria subsets are obtained and presented in Table 7.

Table 8
Collective comprehensive value of each alternative.

A_1	A_2	A_3	A_4	A_5
$\{s_{3.20}(0.24), s_{3.51}(0.38), s_{4.32}(0.38)\}$	$\{s_{2.63}(0.28), s_{3.06}(0.35), s_{3.61}(0.37)\}$	$\{s_{2.26}(0.21), s_{2.63}(0.40), s_{3.31}(0.39)\}$	$\{s_{1.90}(0.15), s_{2.06}(0.41), s_{2.86}(0.44)\}$	$\{s_{1.76}(0.20), s_{2.31}(0.40), s_{2.91}(0.40)\}$

Table 9
Possibility degree.

P'_{12}	P'_{13}	P'_{14}	P'_{15}	P'_{21}	P'_{23}	P'_{24}	P'_{25}	P'_{31}	P'_{32}	P'_{34}	P'_{35}	P'_{41}	P'_{42}	P'_{43}	P'_{45}	P'_{51}	P'_{52}	P'_{53}	P'_{54}
0.77	0.91	1	1	0.23	0.64	0.88	0.89	0.09	0.36	0.73	0.67	0	0.12	0.27	0.38	0	0.11	0.33	0.62

Table 10
Ranking results for four different types of common Archimedean copulas of PLTSs.

Type	Function	P'_1	P'_2	P'_3	P'_4	P'_5	Ranking
Gumbel	$Ge(x) = (-\ln x)^{\varepsilon}(\varepsilon{=}1)$	4.18	3.14	2.36	1.26	1.56	$A_1 \succ A_2 \succ A_3 \succ A_5 \succ A_4$
Clayton	$Ge(x) = x^{-\varepsilon} - 1(\varepsilon{=}1)$	4.18	3.25	2.25	1.26	1.56	$A_1 \succ A_2 \succ A_3 \succ A_5 \succ A_4$
Frank	$Ge(x) = -\ln \frac{e^{-\varepsilon x}-1}{e^{-\varepsilon}-1}(\varepsilon{=}1)$	4.18	3.25	2.25	1.26	1.56	$A_1 \succ A_2 \succ A_3 \succ A_5 \succ A_4$
Joe	$Ge(x) = -\ln[1-(1-x)^{\varepsilon}](\varepsilon{=}1)$	4.18	3.14	2.36	1.26	1.56	$A_1 \succ A_2 \succ A_3 \succ A_5 \succ A_4$

Table 11
Ranking results for different values of parameter ε.

ε	P'_1	P'_2	P'_3	P'_4	P'_5	Ranking
1	4.18	3.14	2.36	1.26	1.56	$A_1 \succ A_2 \succ A_3 \succ A_5 \succ A_4$
3	4.18	3.14	2.36	1.26	1.56	$A_1 \succ A_2 \succ A_3 \succ A_5 \succ A_4$
5	4.18	3.14	2.36	1.26	1.56	$A_1 \succ A_2 \succ A_3 \succ A_5 \succ A_4$
15	4.18	3.14	2.36	1.26	1.56	$A_1 \succ A_2 \succ A_3 \succ A_5 \succ A_4$
25	4.18	3.14	2.36	1.26	1.56	$A_1 \succ A_2 \succ A_3 \succ A_5 \succ A_4$
45	4.18	3.14	2.36	1.26	1.56	$A_1 \succ A_2 \succ A_3 \succ A_5 \succ A_4$
95	4.18	3.14	2.36	1.26	1.56	$A_1 \succ A_2 \succ A_3 \succ A_5 \succ A_4$

Then, the collective comprehensive value $\bar{L}_i(p)$ of alternative A_i is obtained, as shown in Table 8.

Step 8. To rank PLTSs $\bar{L}_i(p)(i = 1, 2, 3, 4, 5)$, calculate the possibility degree $P'_{ij} = P'(\bar{L}_i(p) \geq \bar{L}_j(p))$ by Eq. (16). The results are presented in Table 9.

The ranking values are obtained by Eq. (16) as $P'_1 = 4.18$, $P'_2 = 3.14$, $P'_3 = 2.36$, $P'_4 = 1.26$, $P'_5 = 1.56$.

Therefore, the ranking order of alternatives is $A_1 \succ A_2 \succ A_3 \succ A_5 \succ A_4$ and the best alternative is A_1.

To illustrate the influence of the linguistic scale function in this example, rank the alternatives based on the Form 2 (i.e., Eq. (5)) and Form 3 (i.e., Eq. (6)). The ranking values are shown below.

For the Form 2 (i.e., Eq. (5)) (Let $a = \sqrt[8]{9} \approx 1.3160$), it has

$$P'_1 = 4.18,\ P'_2 = 3.14,\ P'_3 = 2.36,\ P'_4 = 1.44,\ P'_5 = 1.39.$$

The ranking order of alternatives is $A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$.

For the Form 3 (i.e., Eq. (6)) (Let $\alpha = \beta = 0.8$), it has

$$P'_1 = 3.42,\ P'_2 = 3.61,\ P'_3 = 1.80,\ P'_4 = 1.23,\ P'_5 = 2.44.$$

The ranking order of alternatives is $A_2 \succ A_1 \succ A_5 \succ A_3 \succ A_4$.

7.2. Sensitivity analyses

To manifest that the ranking result is universal, this subsection analyzes the influences of four different types of generator $Ge(x)$ based on common Archimedean copulas and the parameter ε.

The ranking results with four different types of common Archimedean copulas based on Form 1 (i.e., Eq. (4)) are presented in Table 10.

As can be seen from Table 10, the rank results are the same for four different types of common Archimedean copulas. No matter which generator $Ge(x)$ is, the ranking order of alternatives keeps unchanged. The ranking result of Gumbel copula function is the same as that of Joe copula function. The main reason is that the PLGHC operator is degraded to PLJHC operator when $\varepsilon = 1$. From the above analysis, the ranking results are highly similar for these four different types of common Archimedean copulas.

Then, taking Gumbel generator into consideration, the ranking results for difference values of parameter ε are listed in Table 11 (based on Form 1 (i.e., Eq. (4))). No matter the value of ε is, the ranking order remains unchanged. It is easy to see that any tiny intervention on evaluations would not affect the ranking results.

The aforesaid sensitivity analyses reveal that the method proposed in this paper is robust.

7.3. Comparative analysis

In this subsection, comparative analyses with Pang et al.'s method [12] and Liu et al.'s method [26] are conducted to illustrate the advantages of the proposed method.

Table 12
Collective decision matrix $\hat{U}$.

	C_1	C_2	C_3	C_4
A_1	$\{s_3(0.03), s_4(0.15), s_5(0.48), s_6(0.31), s_7(0.03)\}$	$\{s_2(0.14), s_3(0.45), s_4(0.23), s_5(0.18)\}$	$\{s_3(0.04), s_4(0.19), s_5(0.35), s_6(0.42)\}$	$\{s_3(0.13), s_4(0.28), s_5(0.54), s_6(0.05)\}$
A_2	$\{s_3(0.33), s_4(0.33), s_5(0.21), s_6(0.13)\}$	$\{s_2(0.08), s_3(0.15), s_4(0.41), s_5(0.31), s_6(0.05)\}$	$\{s_4(0.29), s_5(0.27), s_6(0.24), s_7(0.20)\}$	$\{s_2(0.05), s_3(0.18), s_4(0.57), s_5(0.15), s_6(0.05)\}$
A_3	$\{s_2(0.10), s_3(0.18), s_4(0.37), s_5(0.21), s_6(0.14)\}$	$\{s_2(0.31), s_3(0.55), s_4(0.14)\}$	$\{s_3(0.04), s_4(0.22), s_5(0.42), s_6(0.32)\}$	$\{s_1(0.05), s_2(0.10), s_3(0.38), s_4(0.44), s_5(0.03)\}$
A_4	$\{s_3(0.34), s_4(0.41), s_5(0.15), s_6(0.10)\}$	$\{s_2(0.24), s_3(0.35), s_4(0.26), s_5(0.06), s_6(0.09)\}$	$\{s_4(0.18), s_5(0.65), s_6(0.17)\}$	$\{s_2(0.21), s_3(0.61), s_4(0.18)\}$
A_5	$\{s_1(0.04), s_2(0.11), s_3(0.19), s_4(0.31), s_5(0.23), s_6(0.12)\}$	$\{s_2(0.27), s_3(0.33), s_4(0.15), s_6(0.04), s_7(0.21)\}$	$\{s_4(0.27), s_5(0.63), s_6(0.10)\}$	$\{s_1(0.09), s_2(0.37), s_3(0.28), s_4(0.21), s_5(0.05)\}$

7.3.1. Comparison with Pang et al.'s method

Pang et al.'s method [12] is used to solve the example in Section 7.1. The steps are described below.

Step 1. The individual decision matrices U_1, U_2, U_3 and U_4 are aggregated into the collective decision matrix $\hat{U}$, as shown in Table 12.

Step 2. Calculate the weights of criteria by Eq. (26) in [12] as follows:

$$\omega = (0.155, 0.249, 0.313, 0.284)^{\mathrm{T}}.$$

Step 3. Determine the PIS $L(p)^+$ and the NIS $L(p)^-$ by Definitions 17 and 18 in [12] as follows:

$$L(p)^+ = (\{s_{2.378}, s_{1.875}, s_{1.030}, s_{0.857}, s_{0.214}, s_{0.036}\}, \{s_{1.642}, s_{1.638}, s_{1.458}, s_{1.042}, s_{0.532}, s_0\}, \{s_{3.147}, s_{2.545}, s_{1.738}, s_{1.167}, s_0, s_0\}, \{s_{2.267}, s_{1.267}, s_{0.736}, s_{0.7}, s_{0.1}, s_0\}),$$

$$L(p)^- = (\{s_{0.014}, s_{0.012}, s_{0.0007}, s_{0.0005}, s_0, s_0\}, \{s_{0.093}, s_{0.082}, s_{0.065}, s_{0.006}, s_0, s_0\}, \{s_{0.149}, s_{0.066}, s_{0.018}, s_0, s_0, s_0\}, \{s_{0.079}, s_{0.041}, s_{0.003}, s_{0.0008}, s_0, s_0\}).$$

Step 4. Calculate the deviation degrees between each alternative and the PIS (NIS) by Eqs. (28) and (29) in [12], respectively. Then, $d_{\min}(A_i, L(p)^+)$ and $d_{\max}(A_i, L(p)^-)$ can be determined as follows:

$$d(A_1, L(p)^+) = 0.671,\ d(A_2, L(p)^+) = 0.815,\ d(A_3, L(p)^+) = 0.832,\ d(A_4, L(p)^+) = 0.904,\ d(A_5, L(p)^+) = 0.950,$$
$$d(A_1, L(p)^-) = 1.232,\ d(A_2, L(p)^-) = 1.085,\ d(A_3, L(p)^-) = 1.023,\ d(A_4, L(p)^-) = 0.986,\ d(A_5, L(p)^-) = 1.013,$$
$$d_{\min}(A_i, L(p)^+) = 0.671,\ d_{\max}(A_i, L(p)^-) = 1.232.$$

Step 5. Derive the closeness coefficient CI of each alternative by Eq. (32) in [12].

$$CI(A_1) = 0,\ CI(A_2) = -0.33,\ CI(A_3) = -0.41,\ CI(A_4) = -0.55,\ CI(A_5) = -0.59.$$

Step 6. Rank the alternatives according to $CI(A_i)(i = 1, 2, 3, 4, 5)$ as follows:

$$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5.$$

Thus, ranking order obtain by method in [12] is slightly different from the ranking $A_1 \succ A_2 \succ A_3 \succ A_5 \succ A_4$ obtained by the proposed method based on Form 1 (i.e., Eq. (4)), but completely the same as the ranking $A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$ obtained by the proposed method based on Form 2 (i.e., Eq. (5)). The best alternative obtained by method in [12] is the same as that obtained by the proposed method of this paper based on Forms 1 and 2. Compares with Pang et al.'s method [12], the proposed method of this paper has some merits:

(1) This paper considers the different importance among the weights of DMs and the interactions among criteria. The weights of DMs are determined objectively by constructing a tri-objective nonlinear programming model and the fuzzy measure of criteria subsets are obtained by constructing a multi-objective optimization model, which makes the decision results more reasonable. However, Pang et al. [12] considers the DMs' weights and the criteria weights by a simple calculation.

(2) This paper uses linguistic scale function to develop the PLGHC operator, which considers the interactions among criteria. However, Pang et al. [12] used linguistic variables labels rather than linguistic scale function to aggregate probabilistic linguistic terms, which ignored the interactions among criteria and may lose the linguistic evaluation information.

7.3.2. Comparison with Liu et al.'s method

Liu et al.'s method [26] is used to solve the example in Section 7.1. The steps are listed as follows:

Step 1. Calculate the dependent weights ω_{ij}^q for the criterion C_j with respect to the alternative A_i of DMs E_q by PLDWA operator (i.e., Eq. (11)) in [26], where $i = 1, 2, 3, 4, 5, j = 1, 2, 3, 4$ and $q = 1, 2, 3, 4$.

$$\omega_{1j}^q = \begin{bmatrix} 0.220 & 0.277 & 0.302 & 0.312 \\ 0.250 & 0.321 & 0.221 & 0.319 \\ 0.305 & 0.202 & 0.312 & 0.178 \\ 0.225 & 0.199 & 0.165 & 0.191 \end{bmatrix}, \omega_{2j}^q = \begin{bmatrix} 0.224 & 0.199 & 0.280 & 0.291 \\ 0.314 & 0.332 & 0.081 & 0.328 \\ 0.151 & 0.152 & 0.317 & 0.166 \\ 0.311 & 0.317 & 0.322 & 0.215 \end{bmatrix}, \omega_{3j}^q = \begin{bmatrix} 0.273 & 0.317 & 0.316 & 0.276 \\ 0.182 & 0.147 & 0.305 & 0.311 \\ 0.317 & 0.262 & 0.118 & 0.197 \\ 0.228 & 0.274 & 0.262 & 0.216 \end{bmatrix},$$

$$\omega_{4j}^q = \begin{bmatrix} 0.308 & 0.283 & 0.154 & 0.236 \\ 0.071 & 0.304 & 0.297 & 0.325 \\ 0.319 & 0.126 & 0.243 & 0.137 \\ 0.302 & 0.287 & 0.306 & 0.302 \end{bmatrix}, \omega_{5j}^q = \begin{bmatrix} 0.206 & 0.313 & 0.263 & 0.249 \\ 0.227 & 0.318 & 0.118 & 0.283 \\ 0.280 & 0.045 & 0.320 & 0.311 \\ 0.287 & 0.324 & 0.300 & 0.156 \end{bmatrix}.$$

Table 13
Collective decision matrix $\hat{U}$.

	C_1	C_2	C_3	C_4
A_1	$\{s_4(0.12), s_5(0.60), s_6(0.28)\}$	$\{s_3(0.09), s_4(0.34), s_5(0.57)\}$	$\{s_4(0.12), s_5(0.35), s_6(0.53)\}$	$\{s_3(0.13), s_4(0.46), s_5(0.41)\}$
A_2	$\{s_3(0.11), s_4(0.43), s_5(0.46)\}$	$\{s_3(0.14), s_4(0.60), s_5(0.26)\}$	$\{s_4(0.28), s_5(0.12), s_6(0.60)\}$	$\{s_2(0.10), s_3(0.35), s_4(0.54)\}$
A_3	$\{s_3(0.11), s_4(0.43), s_5(0.46)\}$	$\{s_3(0), s_3(0.67), s_4(0.33)\}$	$\{s_4(0.11), s_5(0.47), s_6(0.42)\}$	$\{s_2(0.15), s_3(0.35), s_4(0.50)\}$
A_4	$\{s_3(0), s_3(0.50), s_4(0.50)\}$	$\{s_2(0.12), s_3(0.37), s_4(0.51)\}$	$\{s_4(0), s_4(0.21), s_5(0.79)\}$	$\{s_2(0.06), s_3(0.45), s_4(0.49)\}$
A_5	$\{s_3(0.03), s_4(0.47), s_5(0.50)\}$	$\{s_2(0.10), s_3(0.43), s_4(0.47)\}$	$\{s_4(0.04), s_5(0.60), s_6(0.36)\}$	$\{s_2(0.21), s_3(0.35), s_4(0.44)\}$

Table 14
Ranking results obtained using the PL-PT-MULTIMOORA method.

	PL-PT-ratio system method		PL-PT-reference point method		PL-PT-full multiplicative method	
	TS_i'	Rank	TS_i''	Rank	TS_i'''	Rank
A_1	0.5823	1	0.1251	3	3.5691	2
A_2	0.3861	3	0.1732	1	3.7655	1
A_3	0.4358	2	0.1388	2	3.1437	5
A_4	0.3859	4	0.0934	4	3.3597	3
A_5	0.2813	5	0.0932	5	3.2430	4

The decision matrices U_1, U_2, U_3 and U_4 are transformed into the collective decision matrix $\hat{U}$, as shown in Table 13.

Step 2. Determine the ranking results using the PL-PT-MULTIMOORA method [26]. The evaluation values of the alternatives and the ranking results obtained by the PL-PT-ratio system method, PL-PT-reference point method, and PL-PT-full multiplicative method are presented in Table 14. (Set $\alpha = 0.88$, $\beta = 0.88$, $\lambda = 2.25$, $\gamma = 0.61$, $\delta = 0.69$ and $\kappa = (0.29, 0.22, 0.19, 0.17, 0.13)$).

Step 3. Calculate the final ranking of the alternatives.

(1) The normalized results by using the PL-PT-MULTIMOORA method are obtained in the matrix:

$$G^N = [g_{ie}^N]_{5\times 3} = \begin{pmatrix} 0.075 & 0.016 & 0.462 \\ 0.050 & 0.022 & 0.488 \\ 0.057 & 0.018 & 0.407 \\ 0.050 & 0.012 & 0.435 \\ 0.038 & 0.012 & 0.420 \end{pmatrix}.$$

The weights of the three methods are obtained by Eq. (24) and R_i^{num} are obtained by Eq. (25) in [26].

$\chi_e = (0.269, 0.085, 0.647)$, $R_i^{num} = (0.318, 0.327, 0.277, 0.294, 0.280)$.

(2) The ranking results by using the PL-PT-MULTIMOORA method are presented in matrix

$$K = [k_{ie}]_{5\times 3} = \begin{pmatrix} 1 & 3 & 2 \\ 3 & 1 & 1 \\ 2 & 2 & 5 \\ 4 & 4 & 3 \\ 5 & 5 & 4 \end{pmatrix},$$

By Eq. (26) in [26], the synthesized ranking value of alternative A_i is obtained as

$R_i^{ran} = (4, \frac{13}{3}, \frac{10}{3}, \frac{5}{3}, \frac{5}{3})$.

(3) By Eq. (27) in [26], the final ranking value of alternative A_i is obtained as

$R_i^{final} = (2.326, 1.997, 1.972, 0.980, 0.974)$.

Step 4. Rank the alternatives according to $R_i^{final}(i = 1, 2, 3, 4, 5)$ as follows:

$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$.

The ranking result obtained by method [26] is slightly different from the ranking $A_1 \succ A_2 \succ A_3 \succ A_5 \succ A_4$ obtained by the proposed method based on Form 1 (i.e., Eq. (4)), but completely the same as the ranking $A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$ obtained by the proposed method based on Form 2 (i.e., Eq. (5)). The best alternative obtained by method in [26] is A_1, which is the same as that obtained by the proposed method based on Forms 1 and 2. Compared with Liu et al.'s method in [26], the proposed method of this paper has some merits:

(1) The PLDWA operator in [26] provides the lower weight to the too small or too large evaluations, but the effect of the evaluations is not obvious. This paper considers the different importance among the weights of DMs. The DMs' weights are obtained objectively by constructing a tri-objective nonlinear programming model, which makes the decision results more reasonable.

(2) For the PLDWA operator in [26] and the PLGHC operator in the proposed method, two operators both consider the interactions among criteria. The ranking result of Archimedean copula function of this paper is similar to the ranking result of method in [26]. Besides the above, this paper also considers the interrelation among input arguments, which reflects the robustness and rationality of the proposed method of this paper.

The ranking results obtained by Pang et al.'s method [12], Liu et al.'s method [26], the proposed method with Gumbel type (PLGHC), the proposed method with Clayton type (PLCHC), the proposed method with Frank type (PLFHC) and the proposed method with Joe type (PLJHC) based on Form 1 (i.e., Eq. (4)) are visually plotted in Fig. 2.

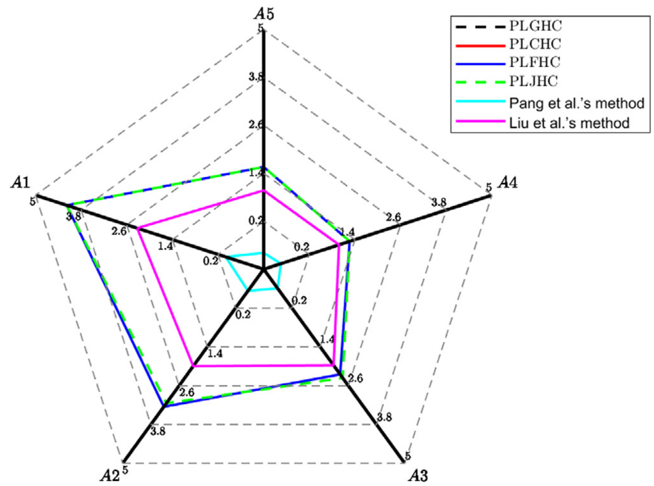

Fig. 2. Ranking results obtained by six different methods.

It can be seen clearly from Fig. 2, A_1 is the best alternative, A_2 is the second best alternative and A_3 is the third best alternative for the above six methods. The ranking results based on common Archimedean copulas are the same, which is $A_1 \succ A_2 \succ A_3 \succ A_5 \succ A_4$. Thus, the ranking results verify the rationality and robustness of the proposed method.

8. Conclusion

To minimize the loss of people's lives and property and to maintain social stability in the disaster areas, it is of great importance to ensure the efficient and orderly emergency assistance after the occurrence of new coronavirus COVID-19. This paper develops a new method for interactive MCGDM with PLTSs and applies to the emergency assistance area selection of COVID-19 for Wuhan. The primary work and features of this paper are outlined as follows:

(1) In this paper, a new possibility degree of PLTSs is defined and then a new possibility degree algorithm is proposed to rank a series of PLTSs. The proposed possibility degree algorithm can take DM's different preferences of linguistic scale functions into account, which is more robust and more in accordance with real situations.

(2) Some new operational laws of PLTSs based on the Archimedean copulas and co-copulas are defined. Archimedean copulas functions are suitable to characterize probability distributions. They have been extended to PLTSs. The proposed operational laws greatly generalize that defined by Mao et al. [24].

(3) Archimedean copulas are monotone non-decreasing, and they can be viewed as aggregation functions on one certain set. Considering the interactions among criteria, the PLGC operator and PLGHC operator are developed. Besides, the properties of these operators are studied, including idempotency, monotonicity and boundedness.

(4) To determine the weights of DMs, a tri-objective nonlinear programming model is constructed and transformed into a linear programming model for resolution. To derive the fuzzy measures of criteria subsets, an optimization model is built and transformed into a goal programming model for resolution. Th DMs' weights and the fuzzy measures of criteria subsets are obtained objectively, which can make the decision results more reasonable and objective.

(5) Use the PLGWA operator to determine the collective normalized decision matrix. Use the PLGHC operator to derive the overall evaluation values of alternatives. The ranking order of alternatives is generated by the proposed possibility degree algorithm of PLTSs. Thereby, a new method for the interactive MCGDM with PLTSs is put forward. The proposed method considers the different importance among the weights of DMs and the interactions among criteria, which is more in accordance with the real decision making situations.

Although the emergence assistance example is provided to illustrate the validity of the proposed method, it can be employed to solve many practical decision-making problems, such as supply chain management, college evaluation, plant siting selection, etc. For future research, based on the defined new operational laws of PLTSs, some new generalized Choquet geometric operators of PLTSs will be developed for MCGDM.

CRediT authorship contribution statement

Shu-Ping Wan: Supervision, Data curation, Writing - original draft, Writing review & editing, Validation, Funding acquisition. **Wen-Bo Huang Cheng:** Conceptualization, Software, Writing - original draft, Writing - review & editing. **Jiu-Ying Dong:** Resources, Investigation, Methodology, Formal analysis, Project administration.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A

Proof of Theorem 2.

(1) $L_1(p) \oplus_{Cp} L_2(p)$

$$= \bigcup_{\substack{k_1=1,2,\ldots,\#L_1(p)\\k_2=1,2,\ldots,\#L_2(p)}} \left\{ g^{-1}(Cp^*(g(L_1^{(k_1)}), g(L_2^{(k_2)})))((p_1^{(k_1)} + p_2^{(k_2)} - p_1^{(k_1)}p_2^{(k_2)})/L) \right\}$$

$$= \bigcup_{\substack{k_1=1,2,\ldots,\#L_1(p)\\k_2=1,2,\ldots,\#L_2(p)}} \left\{ g^{-1}[1 - Ge^{-1}(Ge(1 - g(L_1^{(k_1)})) + Ge(1 - g(L_2^{(k_2)})))]((p_1^{(k_1)} + p_2^{(k_2)} - p_1^{(k_1)}p_2^{(k_2)})/L) \right\}$$

$$= \bigcup_{\substack{k_1=1,2,\ldots,\#L_1(p)\\k_2=1,2,\ldots,\#L_2(p)}} \left\{ g^{-1}(Cp^*(g(L_2^{(k_2)}), g(L_1^{(k_1)})))((p_2^{(k_2)} + p_1^{(k_1)} - p_2^{(k_2)}p_1^{(k_1)})/L) \right\}$$

$= L_2(p) \oplus_{Cp} L_1(p)$;

(2) $L_1(p) \otimes_{Cp} L_2(p)$

$$= \bigcup_{\substack{k_1=1,2,\ldots,\#L_1(p)\\k_2=1,2,\ldots,\#L_2(p)}} \left\{ g^{-1}(Cp(g(L_1^{(k_1)}), g(L_1^{(k_1)})))(p_1^{(k_1)}p_2^{(k_2)}) \right\}$$

$$= \bigcup_{\substack{k_1=1,2,\ldots,\#L_1(p)\\k_2=1,2,\ldots,\#L_2(p)}} \left\{ g^{-1}[Ge^{-1}(Ge(g(L_1^{(k_1)})) + Ge(g(L_2^{(k_2)})))](p_1^{(k_1)}p_2^{(k_2)}) \right\}$$

$$= \bigcup_{\substack{k_1=1,2,\ldots,\#L_1(p)\\k_2=1,2,\ldots,\#L_2(p)}} \left\{ g^{-1}(Cp(g(L_2^{(k_2)}), g(L_1^{(k_1)})))(p_2^{(k_2)}p_1^{(k_1)}) \right\}$$

$= L_2(p) \otimes_{Cp} L_1(p)$;

(3) $\lambda \odot_{Cp}(L_1(p) \oplus_{Cp} L_2(p))$

$$= \lambda \odot_{Cp} \left(\bigcup_{\substack{k_1=1,2,\ldots,\#L_1(p)\\k_2=1,2,\ldots,\#L_2(p)}} \left\{ g^{-1}[1 - Ge^{-1}(Ge(1 - g(L_1^{(k_1)})) + Ge(1 - g(L_2^{(k_2)})))]((p_1^{(k_1)} + p_2^{(k_2)} - p_1^{(k_1)}p_2^{(k_2)})/L) \right\} \right)$$

$$= \bigcup_{\substack{k_1=1,2,\ldots,\#L_1(p)\\k_2=1,2,\ldots,\#L_2(p)}} \left\{ g^{-1}[1 - Ge^{-1}(\lambda Ge(1 - g(L_1^{(k_1)})) + \lambda Ge(1 - g(L_2^{(k_2)})))]((p_1^{(k_1)} + p_2^{(k_2)} - p_1^{(k_1)}p_2^{(k_2)})/L) \right\}$$

$$= \bigcup_{\substack{k_1=1,2,\ldots,\#L_1(p)\\k_2=1,2,\ldots,\#L_2(p)}} \left\{ g^{-1}[1 - Ge^{-1}(\lambda Ge(1 - g(L_2^{(k_2)})) + \lambda Ge(1 - g(L_1^{(k_1)})))]((p_2^{(k_2)} + p_1^{(k_1)} - p_2^{(k_2)}p_1^{(k_1)})/L) \right\}$$

$= \lambda \odot_{Cp}(L_2(p) \oplus_{Cp} L_1(p))$;

(4) $(L_1(p) \otimes_{Cp} L_2(p))^{\lambda}$

$$= \left(\bigcup_{\substack{k_1=1,2,\ldots,\#L_1(p)\\k_2=1,2,\ldots,\#L_2(p)}} \left\{ g^{-1}[Ge^{-1}(Ge(g(L_1^{(k_1)})) + Ge(g(L_2^{(k_2)})))](p_1^{(k_1)}p_2^{(k_2)}) \right\} \right)^{\lambda}$$

$$= \bigcup_{\substack{k_1=1,2,\ldots,\#L_1(p)\\k_2=1,2,\ldots,\#L_2(p)}} \left\{ g^{-1}[Ge^{-1}(\lambda Ge(g(L_1^{(k_1)})))](p_1^{(k_1)}) \right\} \otimes_{Cp} \bigcup_{\substack{k_1=1,2,\ldots,\#L_1(p)\\k_2=1,2,\ldots,\#L_2(p)}} \left\{ g^{-1}[Ge^{-1}(\lambda Ge(g(L_2^{(k_2)})))](p_2^{(k_2)}) \right\}$$

$= (L_1(p))^{\lambda} \otimes_{Cp} (L_2(p))^{\lambda}$;

(5) $(\lambda_1 + \lambda_2)\odot_{Cp} L(p)$

$$= \bigcup_{k=1,2,\ldots,\#L(p)} \left\{ g^{-1}[1 - Ge^{-1}(\lambda_1 Ge(1 - g(L^{(k)})))](p^{(k)}) \right\} \oplus_{Cp} \bigcup_{k=1,2,\ldots,\#L(p)} \left\{ g^{-1}[1 - Ge^{-1}(\lambda_2 Ge(1 - g(L^{(k)})))](p^{(k)}) \right\}$$

$= (\lambda_1 L(p)) \oplus_{Cp} (\lambda_2 L(p))$;

(6) $(L(p))^{\lambda_1} \otimes_{Cp} (L(p))^{\lambda_2}$

$$= \bigcup_{k=1,2,\ldots,\#L(p)} \left\{g^{-1}[Ge^{-1}(\lambda_1 Ge(g(L^{(k)})))](p^{(k)})\right\} \otimes_{Cp} \bigcup_{k=1,2,\ldots,\#L(p)} \left\{g^{-1}[Ge^{-1}(\lambda_2 Ge(g(L^{(k)})))](p^{(k)})\right\}$$
$$= \bigcup_{k=1,2,\ldots,\#L(p)} \left\{g^{-1}[Ge^{-1}((\lambda_1+\lambda_2) Ge(g(L^{(k)})))](p^{(k)})\right\}$$
$$= (L(p))^{\lambda_1+\lambda_2};$$

$$(7) \quad ((L(p))^{\lambda_1})^{\lambda_2} = \left(\bigcup_{k=1,2,\ldots,\#L(p)} \left\{g^{-1}[Ge^{-1}(\lambda_1 Ge(g(L^{(k)})))](p^{(k)})\right\}\right)^{\lambda_2}$$
$$= \bigcup_{k=1,2,\ldots,\#L(p)} \left\{g^{-1}[Ge^{-1}(\lambda_1\lambda_2 Ge(g(L^{(k)})))](p^{(k)})\right\} = (L(p))^{\lambda_1\lambda_2}.$$

It is easy to prove formula (8) and formula (9), therefore, they were omitted.

Appendix B

Proof of Theorem 3. Theorem 3 is proved by mathematical induction in the following.

For $n = 1$, Eq. (26) is right.

Suppose Eq. (26) holds for $n = t$, namely,

$GPLC_{\Gamma}(L_1(p), L_2(p), \ldots, L_t(p)) = \bigcup_{\substack{k_j=1,2,\ldots,\#L_{(j)}(p)\\ j=1,2,\ldots,t}} \left\{g^{-1}[(1 - Ge^{-1}(\sum_{j=1}^{t}(\Gamma(S_{(j)}) - \Gamma(S_{(j-1)}))(Ge(1-A_{(j)}^{(k_j)}))^{\varepsilon})^{1/\varepsilon})^{1/\lambda}](R_K)\right\}$ where $A_{(j)}^{(k_j)} = Ge^{-1}((\lambda(Ge(g(L_{(j)}^{(k_j)})))^{\varepsilon})^{1/\varepsilon})$, $R_K = \frac{\sum_{j=1}^{t} p_{(j)}^{(k_j)} - \sum_{1\le j<i\le t} p_{(j)}^{(k_j)} p_{(i)}^{(k_i)} + \sum_{1\le j<i<l\le t} p_{(j)}^{(k_j)} p_{(i)}^{(k_i)} p_{(l)}^{(k_l)} + \cdots + (-1)^{t-1} p_{(1)}^{(k_1)} p_{(2)}^{(k_2)} \cdots p_{(t)}^{(k_t)}}{\sum_{j=1}^{t}(\#L_{(j)}(p)) - 1}$, $K = 1, 2, \ldots, \#L_{(1)}(p) \times \#L_{(2)}(p) \times \cdots \times \#L_{(t)}(p)$.

For $n = t + 1$, the GPLC operator are obtained by operational laws in Definition 19.

$$GPLC_{\Gamma}(L_1(p), L_2(p), \ldots, L_t(p), L_{t+1}(p))$$
$$= \bigcup_{\substack{k_j=1,2,\ldots,\#L_{(j)}(p)\\ j=1,2,\ldots,t}} \left\{g^{-1}[(1 - Ge^{-1}(\sum_{j=1}^{t}(\Gamma(S_{(j)}) - \Gamma(S_{(j-1)}))(Ge(1-A_{(j)}^{(k_j)}))^{\varepsilon})^{1/\varepsilon})^{1/\lambda}](R_K)\right\}$$
$$\oplus_{Cp} \bigcup_{k_{t+1}=1,2,\ldots,\#L_{(t+1)}(p)} \left\{g^{-1}[(1 - Ge^{-1}((\Gamma(S_{(t+1)}) - \Gamma(S_{(t)}))(Ge(1-A_{(t+1)}^{(k_{t+1})}))^{\varepsilon})^{1/\varepsilon})^{1/\lambda}](p_{t+1}^{(k_{t+1})})\right\}$$
$$= \bigcup_{\substack{k_j=1,2,\ldots,\#L_{(j)}(p)\\ j=1,2,\ldots,t+1}} \left\{g^{-1}\left[\left(\begin{array}{l} 1 - Ge^{-1}(\sum_{j=1}^{t}(\Gamma(S_{(j)}) - \Gamma(S_{(j-1)}))(Ge(1-A_{(j)}^{(k_j)}))^{\varepsilon})^{1/\varepsilon} \\ +((\Gamma(S_{(t+1)}) - \Gamma(S_{(t)}))(Ge(1-A_{(t+1)}^{(k_{t+1})}))^{\varepsilon})^{1/\varepsilon} \end{array}\right)^{1/\lambda}\right]\left(\frac{R_K + p_{(t+1)}^{(k_{t+1})} - R_K \cdot p_{(t+1)}^{(k_{t+1})}}{\sum_{j=1}^{t+1}(\#L_{(j)}(p)) - 1}\right)\right\}$$
$$= \bigcup_{\substack{k_j=1,2,\ldots,\#L_{(j)}(p)\\ j=1,2,\ldots,t+1}} \left\{g^{-1}[(1 - Ge^{-1}(\sum_{j=1}^{t+1}(\Gamma(S_{(j)}) - \Gamma(S_{(j-1)}))(Ge(1-A_{(j)}^{(k_j)}))^{\varepsilon})^{1/\varepsilon})^{1/\lambda}](R_{K+1})\right\}$$

where $A_{(j)}^{(k_j)} = Ge^{-1}((\lambda(Ge(g(L_{(j)}^{(k_j)})))^{\varepsilon})^{1/\varepsilon})$, $R_{K+1} = \frac{\sum_{j=1}^{t+1} p_{(j)}^{(k_j)} - \sum_{1\le j<i\le t+1} p_{(j)}^{(k_j)} p_{(i)}^{(k_i)} + \sum_{1\le j<i<l\le t+1} p_{(j)}^{(k_j)} p_{(i)}^{(k_i)} p_{(l)}^{(k_l)} + \cdots + (-1)^{t} p_{(1)}^{(k_1)} p_{(2)}^{(k_2)} \cdots p_{t+1}^{(k_{t+1})}}{\sum_{j=1}^{(t+1)}(\#L_{(j)}(p)) - 1}$, $K + 1 = 1, 2, \ldots, \#L_{(1)}(p) \times \#L_{(2)}(p) \times \cdots \times \#L_{(t+1)}(p)$.

In conclusion, Eq. (26) satisfies $n = t + 1$. Thus, Eq. (26) holds for all n.

Appendix C

Proof of Property 3. For a set of PLTSs $I = \{L_j(p) | j = 1, 2, \ldots, n\}$, if $L_j(p) = L(p)$, then

$$GPLC_{\Gamma}(L_1(p), L_2(p), \ldots, L_n(p)) = GPLC_{\Gamma}(\underbrace{L(p), L(p), \ldots, L(p)}_{n})$$

According to Definition 19, the result is obtained as follows:

$$GPLC_{\Gamma}(\underbrace{L(p), L(p), \ldots, L(p)}_{n}) = (\sum_{j=1}^{n}((\Gamma(S_{(j)}) - \Gamma(S_{(j-1)}))\odot_{Cp}(L(p))^{\lambda}))^{1/\lambda}$$
$$= (((\Gamma(S_{(1)}) - \Gamma(S_{(0)}))\odot_{Cp}(L(p))^{\lambda}) \oplus_{Cp} ((\Gamma(S_{(2)}) - \Gamma(S_{(1)}))\odot_{Cp}(L(p))^{\lambda}) \oplus_{Cp} \cdots \oplus_{Cp} ((\Gamma(S_{(n)}) - \Gamma(S_{(n-1)}))\odot_{Cp}(L(p))^{\lambda}))^{1/\lambda}$$
$$= ((L(p))^{\lambda} \sum_{j=1}^{n}(\Gamma(S_{(j)}) - \Gamma(S_{(j-1)})))^{1/\lambda}$$

Note that

$$\text{Note that} \quad \sum_{j=1}^{n}(\Gamma(S_{(j)}) - \Gamma(S_{(j-1)})) = (\Gamma(S_{(1)}) - \Gamma(S_{(0)})) + (\Gamma(S_{(2)}) - \Gamma(S_{(1)})) + \cdots + (\Gamma(S_{(n)}) - \Gamma(S_{(n-1)}))$$
$$= \Gamma(S_{(n)}) - \Gamma(S_{(0)}) = 1,$$

Thus, $GPLC_{\Gamma}(L_1(p), L_2(p), \ldots, L_n(p)) = ((L(p))^{\lambda})^{1/\lambda} = L(p)$. Then, the proof of $GPLC_{\Gamma}(L_1(p), L_2(p), \ldots, L_n(p)) = L(p)$ with respect to $L_j(p) = L(p)$ in Property 3 is proven.

Proof of Property 4. For two sets of PLTSs $I = \{L_j(p)\big| j = 1, 2, \ldots, n\}$ and $I' = \{L'_j(p)\big| j = 1, 2, \ldots, n\}$. Since $L_j(p) \geq L'_j(p)$,

$$GPLC_{\Gamma}(L_1(p), L_2(p), \ldots, L_n(p)) = (\sum_{j=1}^{n}((\Gamma(S_{(j)}) - \Gamma(S_{(j-1)}))\odot_{Cp}(L_{(j)}(p))^{\lambda}))^{1/\lambda},$$

$$GPLC_{\Gamma}(L'_1(p), L'_2(p), \ldots, L'_n(p)) = (\sum_{j=1}^{n}((\Gamma(S_{(j)}) - \Gamma(S_{(j-1)}))\odot_{Cp}(L'_{(j)}(p))^{\lambda}))^{1/\lambda}.$$

Then $L_{(j)}(p) \geq L'_{(j)}(p)$, further obtain that $(\sum_{j=1}^{n}((\Gamma(S_{(j)}) - \Gamma(S_{(j-1)}))\odot_{Cp}(L_{(j)}(p))^{\lambda}))^{1/\lambda} \geq (\sum_{j=1}^{n}((\Gamma(S_{(j)}) - \Gamma(S_{(j-1)}))\odot_{Cp}(L'_{(j)}(p))^{\lambda}))^{1/\lambda}$, namely

$$GPLC_{\Gamma}(L_1(p), L_2(p), \ldots, L_n(p)) \geq GPLC_{\Gamma}(L'_1(p), L'_2(p), \ldots, L'_n(p)).$$

Then, the proof of $GPLC_{\Gamma}(L_1(p), L_2(p), \ldots, L_n(p)) \geq GPLC_{\Gamma}(L'_1(p), L'_2(p), \ldots, L'_n(p))$ with respect to $L_j(p) \geq L'_j(p)$ in Property 4 is proven.

Proof of Property 5. For a set of PLTSs $I = \{L_j(p)\big| j = 1, 2, \ldots, n\}$, $L'_j(p) = \{L_j^-(1)\}$ and $L''_j(p) = \{L_j^+(1)\}(j = 1, 2, \ldots, n)$ are two special PLTSs, where L_j^- and L_j^+ are the minimal and maximal linguistic terms of $L_j^{(k_j)}$ in $L_j(p)$, respectively.

Since $L_{(j)}^- \leq L_{(j)} \leq L_{(j)}^+$, one has $\sum_{j=1}^{n}((\Gamma(S_{(j)}) - \Gamma(S_{(j-1)}))\odot_{Cp}(L_{(j)}^-(1))^{\lambda})^{1/\lambda} \leq (\sum_{j=1}^{n}((\Gamma(S_{(j)}) - \Gamma(S_{(j-1)}))\odot_{Cp}(L_{(j)}(p))^{\lambda}))^{1/\lambda} \leq (\sum_{j=1}^{n}((\Gamma(S_{(j)}) - \Gamma(S_{(j-1)}))\odot_{Cp}(L_{(j)}^+(1))^{\lambda}))^{1/\lambda}$.

According to the conclusion of Property 4, it can easily obtain that $GPLC_{\Gamma}(V) \leq GPLC_{\Gamma}(L_1(p), L_2(p), \ldots, L_n(p)) \leq GPLC_{\Gamma}(U)$, where $V = (L'_1(p), L'_2(p), \ldots, L'_n(p))$ and $U = (L''_1(p), L''_2(p), \ldots, L''_n(p))$ Thus, Property 5 is proven.

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