

Evaluation on the Risk of Water Inrush Due to Roof Bed Separation Based on Improved Set Pair Analysis–Variable Fuzzy Sets

Xiao Li, Wenquan Zhang,* Xin Wang, Zaiyong Wang,* and Chuming Pang



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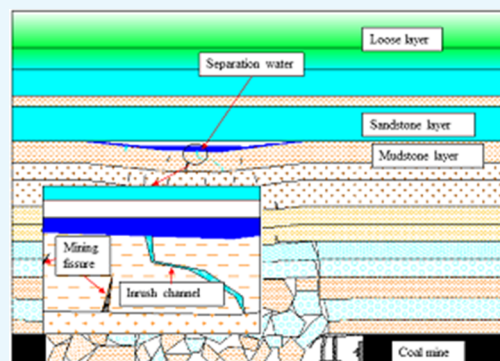
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ABSTRACT: To study and prevent bed separation water inrush accidents in coal mines, it is necessary to evaluate the risk according to the limited geological data correctly. In this work, based on hydrogeological and mining conditions, we established a risk evaluation model and selected seven important factors, including the aquifer thickness, aquifer water abundance, hydraulic pressure of the aquifer, effective aquifuge thickness, mining failure ratio, mining height of the working face, and advancing distance as evaluation indexes. The intuitionistic fuzzy analytic hierarchy process (IFAHP) and entropy weight method (EWM) were used to analyze the weights of the original data, and the minimum information entropy principle was used to further integrate the abovementioned calculation results. With the weight results, set pair analysis–variable fuzzy set (SPA-VFS) theory was applied to determine the risk grade of each working face, which provided scientific guidance for the safe mining of coal mines. For the working face where water inrush may occur, the risk of bed separation water inrush can be reduced by optimizing the parameters or changing the mining conditions through the model analysis.



1. INTRODUCTION

In recent 20 years, water inrush accidents due to roof bed separation, as a new type of roof water disaster, has brought a great threat to coal mine safety production. Field research found that the disaster has features of unobvious signs, recurrence, and large-scale and great destructiveness. In China, coal mines with bed separation water inrush are mainly distributed in 13 provinces such as Shandong, Anhui, Shaanxi, Liaoning, Chongqing, etc. The unpredictable disaster has caused serious accidents such as flooding in coal mine areas and working faces and casualties of miners, which has attracted the attention of coal mining enterprises and many experts.^{1–5} However, due to the difference in the sedimentary rock formation, the mechanism of water inrush is complex and varies from region to region.^{6–11} The schematic diagram of roof bed separation water inrush is shown in Figure 1.

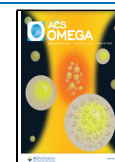
To better prevent the occurrence of the water inrush disaster, many scholars have researched the mechanism of bed separation water inrush and its prevention measures. Dong et al.¹² initially analyzed the uncertainty of the water inrush in coal mines and the calculation of water inflow, discussed the mechanism of water inrush disaster due to bed separation, and proposed new measures to prevent the water-induced disaster. Hu et al.¹³ developed a trapezoid platform model suitable for fracture in the overlying strata, which can analyze the evolution process of the bed separation space and obtain the geometric form of the bed separation space. Plate theory was used to reveal the relationship between the fracture in the overlying strata and the water inrush

from the bed separation. Wu et al.¹⁴ proposed a rock plate method to determine the development position and maximum development height of the bed separation space in the overlying strata. This method was used in the Yuanzigou Coal Mine, and its prediction was consistent with the results obtained from a field borehole video. Li et al.¹⁵ explored the development position of roof bed separation in Hongliu Coal Mine through field drilling and color TV and detected the accumulation process of groundwater in the bed separation space through the underground whole-space transient electromagnetic method. Lu et al.¹⁶ constructed a risk evaluation system of separation water inrush using the fuzzy analytic hierarchy process and entropy weight method (EWM), in which the hard rock thickness, coal seam thickness, aquifuge thickness, aquifer thickness, and hydrostatic head were selected as evaluation indexes. The method was adopted to evaluate the risk of water inrush in Yangliu Coal Mine. On the other hand, a similar simulation test experiment in the laboratory is also an important means to study the mechanism of water inrush due to bed separation.¹⁷ Wang et al.¹⁸ developed a new similar material containing river sand,

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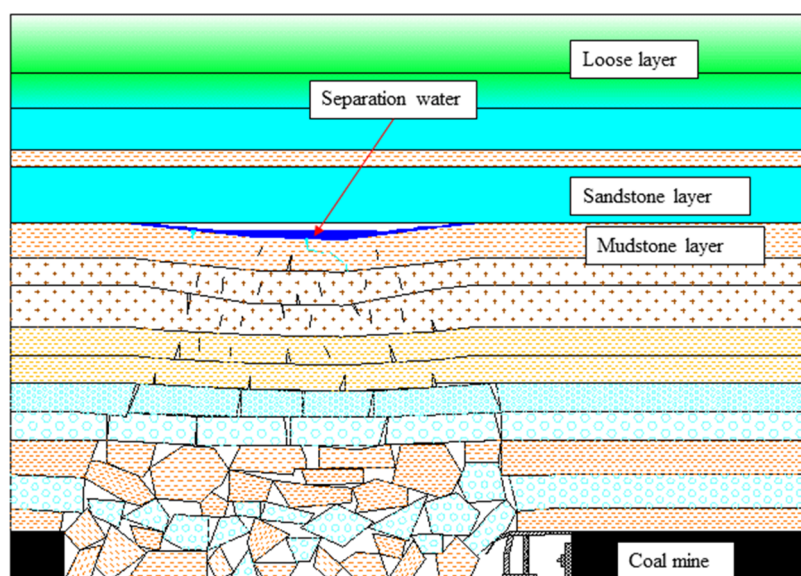


Figure 1. Schematic diagram of water inrush due to roof bed separation.

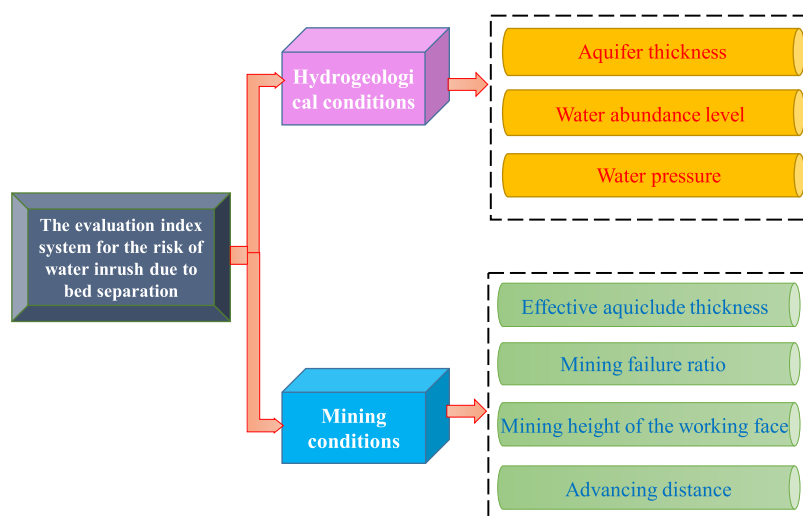


Figure 2. Evaluation indexes of the risk evaluation of separation water inrush.

nano-calcium carbonate, gypsum, and emulsified paraffin. During the test, different lithologic strata were simulated by adjusting the proportions of material components.

The results of the abovementioned scholars have enriched the research in the fields of the development location of the bed separation space, the mechanism of water inrush due to roof bed separation, and simulation experiments in the laboratory. However, the risk evaluation of separation water inrush, based on the governing factors of mining conditions at the working face and the water abundance level, was rarely reported.

The research goal of this work is to build a reliable evaluation system of the water inrush due to roof bed separation which takes the mining conditions and factors affecting the formation of separation water and the separation water inrush as the evaluation indexes. In the system, the intuitionistic fuzzy analytic hierarchy process (IFAHF) and the entropy weight method (EWM) were introduced to quantify the weights of the evaluation indexes, and their results were coupled using the minimum information entropy principle. Set pair analysis–variable fuzzy set (SPA-VFS) theory was used to grade a group

of working faces that had accidents and find the correlation of various evaluation indexes. The findings of this paper are useful in predicting the risk of water inrush due to roof bed separation and can help reduce the risk by locating the critical parameters during the mining process.

2. MODEL PREPARATION

2.1. Determination of the Evaluation Index and Evaluation Standard. In recent years, a large number of bed separation water inrush accidents have occurred in China, which seriously affected the safety production of the working face in the coal mines. Many scholars have researched this issue thoroughly, mainly on the roof bed separation mechanism, layer of water inrush occurrence, bed separation water drainage, and other related problems.^{40,41} We comprehensively considered the reported research results, theoretical analysis, and practical results of relevant experts and selected scientific and effective evaluation indexes.^{16,42} From the perspective of hydrogeological and mining conditions, seven evaluation indexes were selected: aquifer thickness, water abundance level, water pressure, effective aquiclude thickness, mining failure ratio, mining height of the working face, and advancing distance.

effective aquiclude thickness, mining failure ratio, mining height of the working face, and advancing distance. The evaluation index system for the risk evaluation of water inrush due to bed separation has been established, as shown in Figure 2.

- (1) The accumulated water in the separation space comes from the upper sandstone layer. In general, the greater the thickness of the aquifer, the more the pore and fissure water stored in the aquifer. The amount of water accumulated in the separation space can greatly increase due to the upper layer, leading to water inrush risk.⁴³
- (2) The water abundance level indicates the amount of water stored in the aquifer.³⁹ According to the previous field experience, it has been found that the water abundance level of the upper aquifer at the working face where the bed separation water inrush occurs is not high. A

Table 1. Type and Assignment of the Water Abundance Level in the Aquifer

water abundance level	extremely weak	extremely weak ~ weak	weak	weak ~ medium	medium
assignment	1	3	5	7	9

quantitative analysis of the water abundance level of the aquifer was required, and the result is shown in Table 1.

- (3) After the water in the aquifer is connected with the accumulated water in the separation space, the water pressures are the same in these two places. This will cause the water pressure to affect the stability of the lower aquiclude.
- (4) The integrity of the aquiclude is conducive to the accumulation of water in the separation space, which is a precondition for the separation water inrush.⁴⁴ In some coal mines in western China, excavation at the working face forms few microcracks in the aquiclude below the separation space. In the process of water accumulation in the separation space above the aquiclude, physical and chemical reactions between groundwater and the rock of the aquiclude occur, and the microcracks were closed. For this reason, the water-resisting effect of the aquiclude was enhanced, and the corresponding rock thickness is defined as the effective aquiclude thickness.
- (5) The mining failure ratio is the ratio of the height of the water-conducting fractured zone to the height of the separation space. When other conditions of the working face are consistent, the greater the mining failure ratio, the higher the probability of separation water inrush is.

- (6) The mining height of the working face is an important factor that affects the caving state of overlying strata in the vertical direction. High mining height exerts a vast influence on the overlying strata and makes the mining fissure occur near the separation space.
- (7) The advancing distance indicates the advancing speed of the working face. The larger the advancing distance, the smaller the development height of the separation space is. The reason is that a high advancing speed of the working face can delay the change of rock strata in the upper separation space.

2.2. Classification of Evaluation Indexes. Based on the collected engineering geological data and the research results of relevant scholars, the risk of separation water inrush is divided into five levels. The first level is a safe state, and there is no water inrush on the working face. The second level is a relatively safe state, with the water inrush occurring on the working face and the maximum water inflow $q < 125 \text{ m}^3/\text{h}$. The third level is a critical state. At this level, the maximum water inflow satisfies $125 < q \leq 500 \text{ m}^3/\text{h}$. The fourth level is a dangerous state. At this level, the working face is flooded, and the maximum water inflow satisfies $500 < q \leq 1000 \text{ m}^3/\text{h}$. The fifth level is extremely dangerous, where the working face and even the mining area are flooded, with the maximum water inflow $q > 1000 \text{ m}^3/\text{h}$.

Based on the classification of water inrush levels, a quantitative analysis of each evaluation index is also required, as shown in Table 2.

2.3. Background of the Study Case. To better guide the field workers to prevent water inrush accidents due to roof bed separation, the water inrush accidents occurred at 24 working faces in 19 coal mines were surveyed. These coal mines belong to Shandong Province, Anhui Province, Liaoning Province, Shaanxi Province, Ningxia Hui Autonomous Region, Inner Mongolia Autonomous Region, and Xinjiang Uygur Autonomous Region.⁴⁵ Figure 3 is a map showing the distribution of these selected coal mines. It can be seen from Figure 3 that these coal mines are widely distributed and the geological conditions are greatly different, causing water inrush prevention to be very difficult.

3. RESULTS AND DISCUSSION

3.1. Calculation Results. **3.1.1. Weight Calculation by IFAHP.** Based on the opinions of experts with experience in the management of separation water disasters, the IF judgment matrix R was obtained by making pairwise comparisons among the evaluation indexes.

Table 2. Interval Value of Evaluation Index of the Risk of Water Inrush due to Bed Separation

indexes	the risk level of separation water inrush				
	I	II	III	IV	V
aquifer thickness/(m)	[0, 8.1)	[8.1, 16.1)	[16.1~24.1)	[24.1~32.1)	[32.1~40.1]
water abundance level/(-)	[0, 2)	[2, 4)	[4, 6)	[6, 8)	[8, 10]
water pressure/(MPa)	[0, 1.11)	[1.11, 1.41)	[1.41, 1.71)	[1.71, 2.01)	[2.01, 2.31]
effective aquiclude thickness/(m)	[50.1, 40.1)	[40.1, 30.1)	[30.1, 20.1)	[20.1, 10.1)	[10.1, 0]
mining failure ratio/(-)	[0, 0.51)	[0.51, 0.61)	[0.61, 0.71)	[0.71, 0.81)	[0.81, 1]
the mining height of working face/(m)	[0, 2.51)	[2.51, 5.01)	[5.01, 7.51)	[7.51, 10.01)	[10.01, 12.51]
the advancing distance/(m)	[1, 0.8)	[0.8, 0.6)	[0.6, 0.4)	[0.4, 0.2)	[0.2, 0]

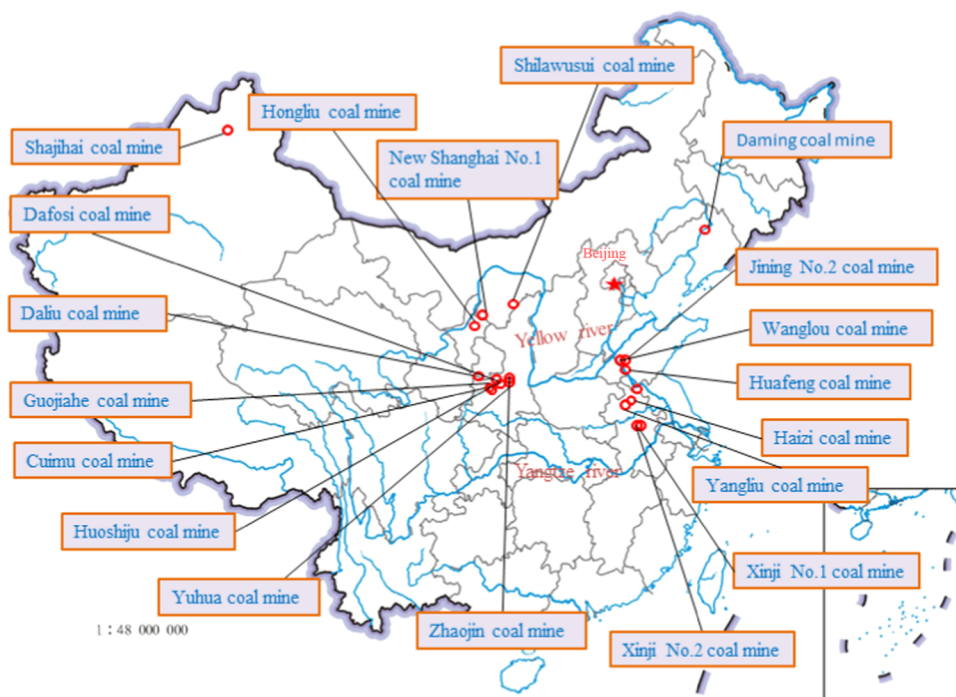


Figure 3. Location maps of the selected coal mines.

$$R = \begin{bmatrix} (0.5, 0.5) & (0.55, 0.4) & (0.6, 0.35) & (0.45, 0.45) & (0.75, 0.2) & (0.45, 0.5) & (0.6, 0.4) \\ (0.4, 0.55) & (0.5, 0.5) & (0.5, 0.5) & (0.6, 0.25) & (0.4, 0.5) & (0.35, 0.55) & (0.5, 0.45) \\ (0.35, 0.6) & (0.5, 0.5) & (0.5, 0.5) & (0.5, 0.45) & (0.45, 0.5) & (0.7, 0.3) & (0.8, 0.15) \\ (0.45, 0.45) & (0.25, 0.6) & (0.45, 0.5) & (0.5, 0.5) & (0.45, 0.55) & (0.45, 0.5) & (0.5, 0.4) \\ (0.2, 0.75) & (0.5, 0.4) & (0.5, 0.45) & (0.55, 0.45) & (0.5, 0.5) & (0.6, 0.3) & (0.55, 0.45) \\ (0.5, 0.45) & (0.55, 0.35) & (0.3, 0.7) & (0.5, 0.45) & (0.3, 0.6) & (0.5, 0.5) & (0.4, 0.55) \\ (0.4, 0.6) & (0.45, 0.5) & (0.15, 0.8) & (0.4, 0.5) & (0.45, 0.55) & (0.55, 0.4) & (0.5, 0.5) \end{bmatrix}$$

According to eqs 2–5, the IF consistency judgment matrix \bar{R} is as follows

$$R = \begin{bmatrix} (0.5000, 0.50000) & (0.5500, 0.4000) & (0.5500, 0.4000) & (0.5505, 0.3989) & (0.4666, 0.5000) & (0.5929, 0.3639) & (0.4697, 0.5100) \\ (0.4000, 0.5500) & (0.5000, 0.5000) & (0.5000, 0.5000) & (0.5000, 0.4500) & (0.5256, 0.3896) & (0.5000, 0.4299) & (0.4363, 0.5251) \\ (0.4495, 0.5251) & (0.5000, 0.5000) & (0.5000, 0.5000) & (0.5000, 0.4500) & (0.4500, 0.5000) & (0.5256, 0.3956) & (0.5368, 0.4463) \\ (0.4321, 0.5337) & (0.4749, 0.5000) & (0.4500, 0.5000) & (0.5000, 0.5000) & (0.4500, 0.5000) & (0.5510, 0.3438) & (0.4248, 0.5251) \\ (0.5000, 0.4749) & (0.4257, 0.5171) & (0.5000, 0.4749) & (0.5500, 0.4500) & (0.5000, 0.5000) & (0.6000, 0.3000) & (0.5000, 0.3438) \\ (0.3901, 0.5746) & (0.4472, 0.5000) & (0.4299, 0.5171) & (0.4199, 0.5256) & (0.3000, 0.6000) & (0.5000, 0.5000) & (0.4000, 0.5500) \\ (0.5084, 0.4748) & (0.5201, 0.4490) & (0.4597, 0.5276) & (0.5167, 0.4497) & (0.4199, 0.5000) & (0.5500, 0.4000) & (0.5000, 0.5000) \end{bmatrix}$$

The modified IF matrix R' is as follows

Table 3. Standardized Value of Each Index Data

number	cases	aquifer thickness	water abundance level	water pressure	effective aquifuge thickness	mining failure ratio	mining height of working face	advancing distance
1	21301 working face of Cuimu coal mine	0.9223	0.6667	0.8077	0.5171	0.9487	0.8990	0.7500
2	21302 working face of Cuimu coal mine	0.2913	0.6667	0.5513	0.1935	0.3077	0.4949	0.5000
3	21303 working face of Cuimu coal mine	0.4854	0.6667	0.7372	0.7742	0.6154	0.7980	0.5000
4	1412 working face of Yuhua coal mine	0.3883	0.3333	0.7115	0.9300	1.0000	0.7980	1.0000
5	1418 working face of Yuhua coal mine	0.1563	0.3333	0.6859	0.6774	0.9231	1.0000	0.5000
6	118 working face of Zhaojin coal mine	0.8738	0.6667	0.9167	0.7742	0.9487	0.6970	0.7500
7	41103 working face of Dafosi coal mine	0.4854	0.6667	0.7436	0.6452	0.8718	0.0909	0.5000
8	41104 working face of Dafosi coal mine	0.3398	0.6667	0.5897	0.1935	0.8462	0.0707	0.2500
9	8506 working face of Huoshiju coal mine	0.6214	0.3333	0.9295	0.1065	0.0513	0.4859	1.0000
10	1306 working face of Guojiahe coal mine	0.1155	0.6667	1.0000	0.4516	0.1026	0.3535	0.0000
11	1121 working face of Hongliu coal mine	1.0000	0.3333	0.4167	0.6774	1.0000	0.5535	0.7500
12	1401 working face of Daliu coal mine	0.5995	0.3333	0.2500	0.5774	0.1026	0.2424	0.5000
13	B1003w01g working face of Shajihai coal mine	0.2660	0.3333	0.3205	0.7742	0.5641	0.4949	0.0000
14	103A working face of Shilawusui coal mine	0.1966	0.0000	1.0000	0.7706	0.5385	0.7980	0.5000
15	111084 working face of New Shanghai no. 1 coal mine	0.9806	1.0000	0.3974	1.0000	0.7179	0.4343	0.7500
16	745 working face of Haizi coal mine	0.8252	1.0000	0.0000	0.5161	0.8718	0.5808	1.0000
17	10414 working face of Yangliu coal mine	0.5272	0.3333	0.4167	0.0129	0.6154	0.0202	0.5000
18	1307 working face of Xinji no. 1 coal mine	0.5825	0.3333	0.1154	0.1774	0.5641	0.4737	0.5000
19	1113104 working face of Xinji no. 2 coal mine	0.0000	0.0000	0.1154	0.5161	0.2821	0.1919	0.0000
20	11305 working face of Jining no. 2 coal mine	0.5825	0.3333	0.1538	0.9939	0.3590	0.3859	0.2500
21	1409 working face of Huafeng coal mine	0.8252	0.0000	0.5833	0.3226	0.6154	0.4141	0.7500
22	11305 working face of Wanglou coal mine	0.8738	0.6667	0.3462	0.1935	0.0000	0.0000	0.7500
23	13301 working face of Wanglou coal mine	0.8738	0.6667	0.3590	0.6194	0.8205	0.3293	0.5000
24	EW416 working face of Daming coal mine	0.6796	0.6667	0.5513	0.0000	0.4615	0.1657	0.2500

$$R' = \begin{pmatrix} (0.5000, 0.5000) & (0.5500, 0.4000) & (0.5641, 0.3857) & (0.5224, 0.4131) & (0.5526, 0.4042) & (0.5534, 0.4008) & (0.5066, 0.4789) \\ (0.4000, 0.5500) & (0.5000, 0.5000) & (0.5000, 0.5000) & (0.5284, 0.3889) & (0.4901, 0.4199) & (0.4568, 0.4633) & (0.4540, 0.5040) \\ (0.4208, 0.5463) & (0.5000, 0.5000) & (0.5000, 0.5000) & (0.5000, 0.4500) & (0.4500, 0.5000) & (0.5771, 0.3677) & (0.6211, 0.3450) \\ (0.4371, 0.5103) & (0.4062, 0.5284) & (0.4500, 0.5000) & (0.5000, 0.5000) & (0.4500, 0.5500) & (0.5228, 0.3857) & (0.4457, 0.4897) \\ (0.4042, 0.5586) & (0.4463, 0.4839) & (0.5000, 0.4679) & (0.5500, 0.4500) & (0.5000, 0.5000) & (0.6000, 0.3000) & (0.5140, 0.3724) \\ (0.4203, 0.5400) & (0.4759, 0.4568) & (0.3916, 0.5711) & (0.4421, 0.5044) & (0.3000, 0.6000) & (0.5000, 0.5000) & (0.4000, 0.5500) \\ (0.4777, 0.5102) & (0.5004, 0.4632) & (0.3539, 0.6148) & (0.4837, 0.4637) & (0.4283, 0.5140) & (0.5500, 0.4000) & (0.5000, 0.5000) \end{pmatrix}$$

This paper took $\Gamma = 0.1$, and the distance measure $d \langle R, \bar{R} \rangle$ between R and \bar{R} obtained from eq 6 was 0.1424. Because $0.1424 > 0.1$, the consistency of R was not good. The IF matrix was modified by eq 7~9, and the correction factor was determined as $\delta = 0.72$, with which $d \langle R, \bar{R} \rangle = 0.0999$, indicating that R had satisfactory consistency.

According to eq 10, the IF weight can be written as

$$\omega'' = [(0.1191, 0.8309) \quad (0.1315, 0.8446) \quad (0.1268, 0.8395) \\ (0.1372, 0.8509) \quad (0.1234, 0.8357) \quad (0.1463, 0.8610) \\ (0.1339, 0.8473)]$$

According to eqs 11 and 12, the weight based on IFAHP was found to be

$$\omega_1 = [0.1336 \quad 0.1431 \quad 0.1396 \quad 0.1475 \quad 0.1369 \quad 0.1544 \quad 0.1449]$$

3.1.2. Weight Calculation by the EWM. The EWM was used to calculate seven evaluation indexes and then determine the main control factors affecting the roof bed separation water inrush.

It is necessary to determine the value of each evaluation index, for which the positive index value is 1 and the negative index value is -1 . Except for the mining failure ratio and the effective aquiclude thickness, all the others have a positive index. To eliminate the influence of the index unit on the evaluation results, the collected data were standardized. The calculation results obtained from eqs 14 and 15 are shown in Table 3.

The eqs 16 and 18 were programmed using MATLAB software. The standardized values of each index were input into the program for calculation. Finally, the entropy value Z_j and entropy weight ξ_j of each index were obtained; see Table 4.

Table 4. Weight and Sorting of Each Evaluation Index

evaluation index	the index value	Z_j	ξ_j	importance ranking
aquifer thickness	1	0.9494	0.1178	7
water abundance level	1	0.9348	0.1519	3
water pressure	1	0.9453	0.1274	6
effective aquiclude thickness	-1	0.9344	0.1529	2
mining failure ratio	1	0.9423	0.1346	5
mining height of working face	1	0.9291	0.1653	1
advancing distance	-1	0.9356	0.1502	4

3.1.3. Weight Calculation by the Comprehensive Weighting Method (CWM). Substituting the weight results from the IFAHP and EWM in eqs 19–21 gives $\omega = [0.1252 \ 0.1471 \ 0.1331 \ 0.1499 \ 0.1355 \ 0.1594 \ 0.1498]$, for which the corresponding indexes are the aquifer thickness, the mining height of the working face, the effective aquiclude thickness, the advancing distance, the water abundance level, the mining failure ratio, and the hydraulic pressure of the aquifer. See Figure 3 for more details.

Figure 4 shows that the weight value obtained from the CWM is in between that of the IFAHP and EWM. The results obtained by the CWM are more objective because this method can

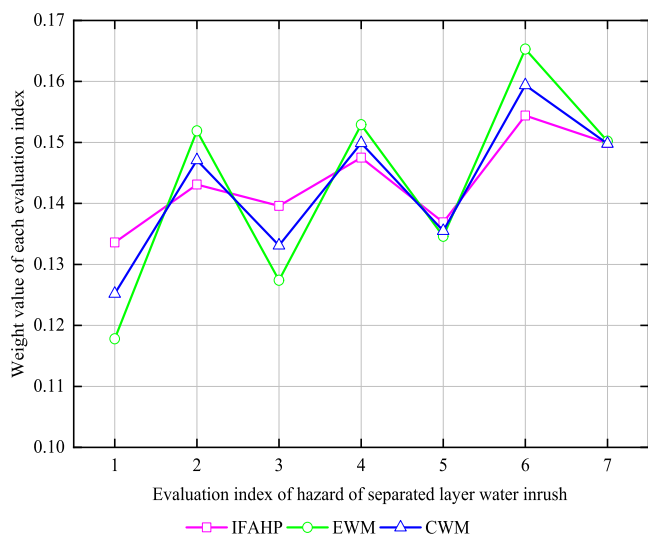


Figure 4. Comparison diagram of subjective and objective weights.

effectively balance the inherent differences of the other two methods. Except for the aquifer thickness, the total weight of the other six evaluation indexes is 0.8748, which contains most of the information and can effectively evaluate the risk of bed separation water inrush. With further research, it can provide some reference ideas for reducing or even preventing the occurrence of bed separation water inrush accidents.

3.2. Discussion. Taking the 21301 working face in Cuimu Coal Mine as an example, the relevant data in Table 2 and Table 3 were substituted in eqs 24 and 25, and the connection degree of the single indexes was obtained. The specific results are shown in Table 5.

Table 5. Single Index Connection Degree of the 21301 Working Face in Cuimu Coal Mine

evaluation index	I	II	III	IV	V
aquifer thickness	-0.4541	-0.3080	-0.0431	0.4302	1
water abundance level	-0.2857	0	0.5000	1	0.8333
water pressure	0.2474	-0.0506	0.4184	1	0.8659
effective aquiclude thickness	0.4978	0.7452	0.9936	1	0.2586
mining failure ratio	0.1505	-0.4048	-0.2188	0.1818	1
mining height of working face	-0.4300	-0.2638	0.0434	0.5745	1
advancing distance	0.4286	0.6000	0.8333	1	0.5000

The relative membership degrees calculated by eq 26 and the weight values of the evaluation indexes calculated by the CWM were combined with eqs 27–29 to calculate the comprehensive membership vectors, as shown in Table 6.

Similarly, the risk evaluation grades of bed separation water inrush in other cases were calculated, and the results of the IFAHP and EWM were introduced to calculate the evaluation grades of all cases. In Table 7, we compare the calculation results of SPA-VFS with those of the three methods.

In the results of the SPA-VFS and CWM, 21 of the 24 grades were correctly evaluated, and the accuracy was 87.5%. In contrast, both the SPA-VFS and IFAHP and SPA-VFS and EWM had 20 correct evaluations, with an accuracy of 83.33%. Compared with the other two models, the SPA-VFS and CWM model has higher accuracy and can better reflect the risk evaluation degree of bed separation water inrush.

In the model, the mining height of the working face is the most important index, which can directly affect the formation of separation water. Hence, limiting the mining height is an important mining strategy that can effectively solve the risk of separation water inrush.⁴⁶ If it is too high, the fracture caused by water flowing can connect the separation space, causing water accumulation to be difficult. In contrast, if the mining height is low, it can not only increase the aquiclude thickness but also reduce the maximum development height of the separation space. In this case, the separation water is too little to cause accidents. The effective aquiclude thickness is key in accumulation of separation water and the occurrence of water inrush.

In the model, its weight is only second to the mining height of the working face and is equivalent to the advancing distance. The advancing distance affects the time of water accumulation in the separation space. In theory, the greater the distance each day,

Table 6. Vectors of Comprehensive Membership and Calculation Results of the 21301 Working Face in Cuimu Coal Mine

parameters	comprehensive membership vector					evaluation level	actual level	water inflow (m ⁻³ ·h)
	V ₁	V ₂	V ₃	V ₄	V ₅			
d = 1, p = 1	0.1468	0.1518	0.1970	0.2507	0.2538	V	V	1100
d = 1, p = 2	0.1524	0.1581	0.1970	0.2445	0.2478	V		
d = 2, p = 1	0.1354	0.1443	0.2138	0.2528	0.2538	V		
d = 2, p = 2	0.1379	0.1480	0.2089	0.2518	0.2534	V		
results						V		

Table 7. Risk Evaluation Result of Separation Water Inrush in the Coal Mining Face

number	parameters				CWM	IFAHP	EWM	actual gradactual grade	water inflow (m ⁻³ ·h)
	d = 1, p = 1	d = 1, p = 2	d = 2, p = 1	d = 2, p = 2					
1	[0.1468, 0.1518, 0.1970, 0.2507, 0.2538]	[0.1524, 0.1581, 0.1970, 0.2445, 0.2478]	[0.1354, 0.1443, 0.2138, 0.2528, 0.2538]	[0.1379, 0.1480, 0.2089, 0.2518, 0.2534]	V	V	V	V	1100
2	[0.1611, 0.1888, 0.2428 , 0.2213, 0.1860]	[0.1636, 0.1909, 0.2409 , 0.2196, 0.1851]	[0.1676, 0.1987, 0.2207 , 0.2168, 0.1963]	[0.1648, 0.1982, 0.2259 , 0.2188, 0.1923]	III	III	III	III	500
3	[0.1466, 0.1563, 0.2095, 0.2563 , 0.2313]	[0.1509, 0.1610, 0.2118, 0.2521 , 0.2242]	[0.1437, 0.1602, 0.2230, 0.2390 , 0.2340]	[0.1433, 0.1602, 0.2229, 0.2421 , 0.2314]	IV	IV	IV	IV	570
4	[0.1444, 0.1550, 0.2058, 0.2428, 0.2520]	[0.1491, 0.1597, 0.2019, 0.2361, 0.2532]	[0.1399, 0.1583, 0.2217, 0.2393, 0.2408]	[0.1409, 0.1591, 0.2160, 0.2394, 0.2447]	V	V	V	V	2000
5	[0.1556, 0.1720, 0.2224, 0.2272 , 0.2227]	[0.1622, 0.1760, 0.2105, 0.2282 , 0.2232]	[0.1508, 0.1757, 0.2237, 0.2260 , 0.2239]	[0.1529, 0.1744, 0.2162, 0.2299 , 0.2266]	IV	IV	IV	IV	600
6	[0.1439, 0.1477, 0.1943, 0.2537, 0.2605]	[0.1483, 0.1534, 0.1955, 0.2449, 0.2580]	[0.1337, 0.1407, 0.2140, 0.2551, 0.2565]	[0.1355, 0.1446, 0.2108, 0.2522, 0.2569]	V	V	V	V	2000
7	[0.1674, 0.1772, 0.2124, 0.2319 , 0.2111]	[0.1726, 0.1791, 0.2119, 0.2282 , 0.2083]	[0.1695, 0.1823, 0.2135, 0.2219 , 0.2128]	[0.1714, 0.1803, 0.2137, 0.2236 , 0.2110]	IV	IV	IV	IV	600
8	[0.1863, 0.1995, 0.2185 , 0.2057, 0.1899]	[0.1900, 0.1975, 0.2184 , 0.2061, 0.1879]	[0.1886, 0.2008, 0.2130 , 0.2054, 0.1922]	[0.1901, 0.1985, 0.2168 , 0.2069, 0.1876]	III	III	III	IV	500
9	[0.1610, 0.1862, 0.2175, 0.2225 , 0.2128]	[0.1655, 0.1874, 0.2184, 0.2223 , 0.2063]	[0.1620, 0.1930, 0.2150, 0.2171 , 0.2128]	[0.1621, 0.1910, 0.2177, 0.2199 , 0.2093]	IV	IV	IV	IV	1000
10	[0.1769, 0.1954, 0.2184 , 0.2144, 0.1949]	[0.1817, 0.1985, 0.2137 , 0.2123, 0.1938]	[0.1782, 0.1980, 0.2143 , 0.2121, 0.1975]	[0.1793, 0.1996, 0.2139 , 0.2128, 0.1944]	III	III	III	III	200
11	[0.1551, 0.1762, 0.2151, 0.2297 , 0.2239]	[0.1600, 0.1779, 0.2076, 0.2271, 0.2274]	[0.1520, 0.1825, 0.2181, 0.2248 , 0.2225]	[0.1532, 0.1799, 0.2132, 0.2267, 0.2269]	IV/V	IV/V	IV	V	3000
12	[0.1731, 0.2089, 0.2259 , 0.2135, 0.1787]	[0.1729, 0.2049, 0.2256 , 0.2157, 0.1809]	[0.1823, 0.2078, 0.2126 , 0.2095, 0.1879]	[0.1783, 0.2067, 0.2157 , 0.2121, 0.1873]	III	III	III	III	430
13	[0.1755, 0.1868, 0.2208 , 0.2206, 0.1962]	[0.1788, 0.1946, 0.2187 , 0.2122, 0.1956]	[0.1797, 0.1918, 0.2142 , 0.2142, 0.2000]	[0.1778, 0.1964, 0.2164 , 0.2120, 0.1974]	III	III	III/ IV	III	300
14	[0.1600, 0.1743, 0.2115, 0.2338 , 0.2204]	[0.1650, 0.1752, 0.2089, 0.2344 , 0.2164]	[0.1608, 0.1807, 0.2149, 0.2242 , 0.2195]	[0.1618, 0.1767, 0.2136, 0.2287 , 0.2191]	IV	IV	IV	IV	921.4
15	[0.1543, 0.1694, 0.2021, 0.2305, 0.2437]	[0.1572, 0.1706, 0.1993, 0.2335, 0.2394]	[0.1498, 0.1735, 0.2120, 0.2302, 0.2345]	[0.1495, 0.1710, 0.2083, 0.2343, 0.2369]	V	V	V	V	2000
16	[0.1587, 0.1643, 0.1981, 0.2333, 0.2457]	[0.1612, 0.1683, 0.1989, 0.2337, 0.2380]	[0.1540, 0.1631, 0.2085, 0.2348, 0.2396]	[0.1522, 0.1638, 0.2069, 0.2373, 0.2398]	V	V	V	V	3887
17	[0.1907, 0.2063, 0.2164 , 0.2070, 0.1796]	[0.1918, 0.2029, 0.2201 , 0.2060, 0.1792]	[0.1945, 0.2054, 0.2103 , 0.2058, 0.1839]	[0.1939, 0.2041, 0.2157 , 0.2066, 0.1797]	III	III	III	III	500
18	[0.1729, 0.1929, 0.2285 , 0.2183, 0.1873]	[0.1740, 0.1941, 0.2283 , 0.2175, 0.1861]	[0.1773, 0.1981, 0.2176 , 0.2140, 0.1931]	[0.1743, 0.1980, 0.2220 , 0.2164, 0.1893]	III	III	III	III	400
19	[0.2008, 0.2186 , 0.2185, 0.1966, 0.1654]	[0.2038, 0.2228 , 0.2122, 0.1930, 0.1682]	[0.2039, 0.2122 , 0.2121, 0.2012, 0.1706]	[0.2060, 0.2168 , 0.2115, 0.1971, 0.1686]	II	II/III	II	II	85
20	[0.1724, 0.1938, 0.2205 , 0.2153, 0.1980]	[0.1737, 0.1978, 0.2226 , 0.2117, 0.1942]	[0.1769, 0.1981, 0.2129 , 0.2109, 0.2012]	[0.1741, 0.2005, 0.2171 , 0.2110, 0.1972]	III	III	III	III	356
21	[0.1657, 0.1875, 0.2220, 0.2247 , 0.2000]	[0.1702, 0.1873, 0.2172, 0.2270 , 0.1982]	[0.1700, 0.1944, 0.2152, 0.2161 , 0.2043]	[0.1710, 0.1918, 0.2153, 0.2198 , 0.2022]	IV	IV	IV	IV	720
22	[0.1873, 0.2055, 0.2127 , 0.2065, 0.1881]	[0.1839, 0.1992, 0.2162 , 0.2109, 0.1899]	[0.1914, 0.2042, 0.2077 , 0.2047, 0.1921]	[0.1855, 0.2006, 0.2126 , 0.2094, 0.1919]	III	III	III	III	450
23	[0.1643, 0.1870, 0.2143, 0.2241 , 0.2104]	[0.1680, 0.1854, 0.2106, 0.2279 , 0.2081]	[0.1636, 0.1920, 0.2139, 0.2188 , 0.2116]	[0.1639, 0.1877, 0.2131, 0.2243 , 0.2110]	IV	IV	IV	IV	790
24	[0.1834, 0.2036, 0.2117, 0.2128 , 0.1884]	[0.1866, 0.2017, 0.2198 , 0.2070, 0.1849]	[0.1857, 0.2040, 0.2094, 0.2100 , 0.1909]	[0.1865, 0.2031, 0.2180 , 0.2080, 0.1844]	III/ IV	III/IV	III/ IV	III	185

the lower the risk. The water abundance level is an important prerequisite for the formation of separation water, and its weight is also high in the model.³⁸

Combined with the data in Tables 7 and 3, it can be found that the abovementioned indexes are large for multiple working faces with the evaluation level of V and are small in the 1113104

working face in Xinji No. 2 coal mine, indicating that they are closely related. Notably, the main purpose of this model is to predict the risk of water disaster in the working face and provide help for the later construction process. From the data mentioned above, the risk of water inrush in the working face can be reduced by changing these indexes. Importantly, this con-

struction scheme selection needs to consider the cost and construction difficulty. For example, for a coal mining face whose risk level is high, when the aquifuge thickness is small, its risk can be reduced by limiting the mining height, reducing the advancing distance and other mining conditions. For the working face whose water inrush risk level is greater than that of IV, when the aquifer is highly water-rich, it is a good solution to use hydraulic fracturing technology to cut the upper layer in the separation space,⁴⁷ which prevents the separation space from formation and interrupts the space conditions of the separation water accumulation. These measures have positive engineering value to prevent the bed separation water inrush, especially in the working face with high risk.

4. CONCLUSIONS AND FURTHER WORK

4.1. Conclusions.

- (1) The CWM was used to optimize the weights of evaluation indexes obtained by the IFAHP and EWM, which eliminated the adverse effects of using either method alone. From high to low, the influence degrees of the evaluation indexes are in the order mining height of the working face, effective aquifuge thickness, advancing distance, aquifer water abundance, mining failure ratio, hydraulic pressure of the aquifer, and aquifer thickness.
- (2) A model for risk evaluation of bed separation water inrush based on SPA-VFS was established. By setting four parameters, four types of models were obtained, and their comprehensive membership vectors were calculated. The new model better solved the adjacent problem and provided more scientific and accurate evaluation results.
- (3) Compared to the SPA-VFS and EWM and SPA-VFS and IFAHP models, the SPA-VFS and CWM model has higher accuracy and can reflect the risk evaluation degree of bed separation water inrush better. This model has better handled the fusion of data and grades.

4.2. Further Work. In this paper, problems of superposition mining and fractures existing in multiple coal seams are not considered. In the later research, it is necessary to consider the actual influence of separation space volume on water inrush to improve the accuracy of the model.

5. RESEARCH METHODOLOGY

5.1. Intuitionistic Fuzzy Analytic Hierarchy Process.

The intuitionistic fuzzy analytic hierarchy process (IFAHP) is a comprehensive evaluation coupling intuitionistic fuzzy (IF) set and analytic hierarchy process (AHP). This method introduces the idea of membership degree, nonmembership degree, and hesitancy degree in the IF set into the initial AHP, which has solved some uncertain situations effectively and made the judgment conclusion more in line with experts' expectations.^{19–21}

5.1.1. Establishment of the IF Judgment Matrix. The evaluation indexes affecting the bed separation water inrush need to be selected first, and the degree of relative importance of each evaluation index can be determined by experts in this field through pairwise comparisons among the indexes of the same index layer.²² Afterward, a quantitative analysis should be conducted on the degree of the relative importance of indexes, and the reference values are shown in Table 8.

The IF values required in the judgement matrix R are calculated from the degrees of relative importance; see below.

Table 8. Evaluation Grade and Score Values

evaluation grade	score values
index i is absolutely important	0.9
index i is strongly important	0.8
index i is essential important	0.7
index i is slightly important	0.6
index i and j are equally important	0.5
index j is slightly important	0.4
index j is essential important	0.3
index j is strongly important	0.2
index j is absolutely important	0.1

$$R = (r_{ij})_{n \times n} = (\mu_{ij}, v_{ij})_{n \times n} = \begin{bmatrix} (\mu_{11}, v_{11}) & (\mu_{12}, v_{12}) & \cdots & (\mu_{1n}, v_{1n}) \\ (\mu_{21}, v_{21}) & (\mu_{22}, v_{22}) & \cdots & (\mu_{2n}, v_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (\mu_{n1}, v_{n1}) & (\mu_{n2}, v_{n2}) & \cdots & (\mu_{nn}, v_{nn}) \end{bmatrix} \quad (1)$$

- (1) $\mu_{ij} - v_{ij} \in [0, 1]$.
- (2) n refers to the number of evaluation indexes.
- (3) μ_{ij} refers to the membership degree of the IF value. The index i should be more important than index j .
- (4) v_{ij} refers to the nonmembership degree of the IF value. The index i should be more important than index j , where index j is more important than index i .
- (5) $\pi_{ij} = 1 - \mu_{ij} - v_{ij}$ refers to the hesitation degree of the IF value obtained by pairwise comparisons of experts.

5.1.2. Consistency Checking and Correction Methods of the IF Judgment Matrix. The consistency checking of the IF judgment matrix is to further analyze the compatibility of the degree of each index's relative importance and obtain a more reasonable index weight. The detailed steps are as follows:²³

- (1) The construction of the IF consistency judgment matrix $\bar{R} = (\bar{r}_{ij})_{n \times n}$ based on the R matrix:
If $j > i + 1$, $\bar{r}_{ij} = (\bar{\mu}_{ij}, \bar{v}_{ij})$, and the calculation formulas are as follows

$$\bar{\mu}_{ij} = \frac{j^{-i-1} \sqrt{\prod_{t=i+1}^{j-1} \mu_{it} \mu_{tj}}}{j^{-i-1} \sqrt{\prod_{t=i+1}^{j-1} \mu_{it} \mu_{tj}} + j^{-i-1} \sqrt{\prod_{t=i+1}^{j-1} (1 - \mu_{it})(1 - \mu_{tj})}} \quad (2)$$

$$\bar{v}_{ij} = \frac{j^{-i-1} \sqrt{\prod_{t=i+1}^{j-1} v_{it} v_{tj}}}{j^{-i-1} \sqrt{\prod_{t=i+1}^{j-1} v_{it} v_{tj}} + j^{-i-1} \sqrt{\prod_{t=i+1}^{j-1} (1 - v_{it})(1 - v_{tj})}} \quad (3)$$

- If $j = i + 1$ or $j = i$, $\bar{r}_{ij} = r_{ij} = (\mu_{ij}, v_{ij})$;
If $j < i$, $\bar{r}_{ij} = (\bar{\mu}_{ij}, \bar{v}_{ij})$, and the calculation formulas are as follows:

$$\bar{\mu}_{ij} = \frac{i^{-j} \sqrt{\prod_{t=j}^{i-1} \mu_{tj} \mu_{it}}}{i^{-j} \sqrt{\prod_{t=j}^{i-1} \mu_{tj} \mu_{it}} + i^{-j} \sqrt{\prod_{t=j}^{i-1} (1 - \mu_{tj})(1 - \mu_{it})}} \quad (4)$$

$$\bar{v}_{ij} = \frac{i^{-j} \sqrt{\prod_{t=j}^{i-1} v_{ij} v_{it}}}{i^{-j} \sqrt{\prod_{t=j}^{i-1} v_{ij} v_{it}} + i^{-j} \sqrt{\prod_{t=j}^{i-1} (1 - v_{ij})(1 - v_{it})}} \quad (5)$$

- (2) Definition of the consistency threshold coefficient Γ and comparison of distance measures between R and \bar{R} : If $d\langle R, \bar{R} \rangle < \Gamma$ matrix, R is considered to meet the consistency inspection rules. The distance measure between R and \bar{R} is

$$d\langle R, \bar{R} \rangle = \frac{1}{2(n-1)(n-2)} \sum_{i=1}^n \sum_{j=1}^n (|\bar{\mu}_{ij} - \mu_{ij}| + |\bar{v}_{ij} - v_{ij}| + |\bar{\pi}_{ij} - \pi_{ij}|) \quad (6)$$

If $d\langle R, \bar{R} \rangle > \Gamma$, a correction factor $\delta \in [0, 1]$ is introduced.²⁴ The matrix R is modified by changing the value of δ from large to low to make it comply with the consistency inspection rules. In such a case, the IF judgment matrix R' with the correction factor is

$$R' = (r'_{ij})_{n \times n} = (\mu'_{ij}, v'_{ij})_{n \times n} \quad (7)$$

$$\mu'_{ij} = \frac{(\mu_{ij})^{1-\delta} (\bar{\mu}_{ij})^\delta}{(\mu_{ij})^{1-\delta} (\bar{\mu}_{ij})^\delta + (1 - \mu_{ij})^{1-\delta} (1 - \bar{\mu}_{ij})^\delta} \quad (8)$$

$$v'_{ij} = \frac{(v_{ij})^{1-\delta} (\bar{v}_{ij})^\delta}{(v_{ij})^{1-\delta} (\bar{v}_{ij})^\delta + (1 - v_{ij})^{1-\delta} (1 - \bar{v}_{ij})^\delta} \quad (9)$$

5.1.3. Determination of Weight. The subjective weight of each evaluation index can be calculated based on the IF judgment matrix, where the IF weight is²⁵

$$\omega_j'' = \left\langle \frac{\sum_{i=1}^n \mu_{ij}}{\sum_{i=1}^n \sum_{j=1}^n (1 - v_{ij})}, 1 - \frac{\sum_{i=1}^n (1 - \mu_{ij})}{\sum_{i=1}^n \sum_{j=1}^n v_{ij}} \right\rangle \quad (10)$$

The actual weight of each evaluation index can be calculated using eqs 11 and 12.

$$\Omega(j) = \mu_j'' + \pi_j'' \left(\frac{\mu_j''}{\mu_j'' + v_j''} \right) \quad (11)$$

$$\omega_{1j} = \frac{\Omega(j)}{\sum_{i=1}^n \Omega(j)} \quad (12)$$

where $\Omega(j)$ refers to the fuzzy transfer function, μ_j'' is the membership of w_j'' , v_j'' is the nonmembership of w_j'' , π_j'' is the hesitancy degree of w_j'' , and ω_{1j} refers to the subjective weight of index j .

5.2. Entropy Weight Method. The entropy weight method (EWM) is an analysis method for calculating the weight of each evaluation index through field-measured data. Its result mainly depends on the variation degree among evaluation indexes. A great degree of variation of an index combining a small information entropy gives a great weight value. This method produces more accurate results by evaluating the actual values of the influencing factors in the model, and it is widely used in multiattribute decision-making problems.^{26,27}

- (1) Establishment of the initial matrix: if there are m evaluation samples and n evaluation indexes, the established initial matrix is

$$B_{m \times n} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \quad (13)$$

- (2) Standardization of indexes:
Standardization of positive indexes is

$$\bar{x}_{ij} = \frac{x_{ij} - \min_{1 \leq i \leq m} x_{ij}}{\max_{1 \leq i \leq m} x_{ij} - \min_{1 \leq i \leq m} x_{ij}} > 0 \quad (14)$$

Standardization of negative indexes is

$$\bar{x}_{ij} = \frac{\max_{1 \leq i \leq m} x_{ij} - x_{ij}}{\max_{1 \leq i \leq m} x_{ij} - \min_{1 \leq i \leq m} x_{ij}} > 0 \quad (15)$$

- (3) Weight distribution of each index in the matrix is

$$Y_{ij} = \frac{\bar{x}_{ij}}{\sum_{i=1}^m \bar{x}_{ij}} \quad (16)$$

where Y_{ij} refers to the weight of the j th evaluation index of the i th evaluation sample in the matrix and $0 \ll Y_{ij} \ll 1$, $\sum_{i=1}^m Y_{ij} = 1$.

- (4) Entropy of the j th index.

$$Z_j = -\frac{1}{\log m} \sum_{i=1}^m Y_{ij} \log Y_{ij} \quad (17)$$

- (5) Entropy weight of each index.

$$\xi_j = \frac{1 - Z_j}{n - \sum_{j=1}^n Z_j} \geq 0 \quad (18)$$

where $\sum_{j=1}^n \xi_j = 1$.

It can be seen that the greater the entropy weight, the greater the importance of the evaluation index.

5.3. Comprehensive Weighting Method. The comprehensive weighting method (CWM) is based on the principle of information entropy, which combines the subjective weight method and objective weight method to allocate a more reasonable comprehensive weight. The calculation formulas are as follows^{28,29}

$$\min Z = \sum_j \omega_j (\ln \omega_j - \ln \omega_{1j}) + \sum_j \omega_j (\ln \omega_j - \ln \xi_j) \quad (19)$$

$$\omega_j = \frac{(\omega_{1j} \times \xi_j)^{1/2}}{\sum_{j=1}^n (\omega_{1j} \times \xi_j)^{1/2}} \quad (20)$$

$$\sum_{j=1}^n \omega_j = 1 \quad (\omega_j > 0) \quad (21)$$

where ω_j refers to the value of the comprehensive weight, ω_{1j} refers to the value of the subjective weight, and ξ_j refers to the value of the objective weight.

5.4. Coupling Evaluation Method Based on SPA-VFS.

In 1989, Zhao Keqin, a Chinese scholar, proposed the set pair analysis (SPA) method based on the complexity of uncertain problems, which divided objectives into an uncertain system and analyzed the internal relations of the system from aspects of similarity, difference, and opposition, and the main tool adopted in the analysis was “connection degree”.^{30–32}

In 2008, with the aim to better analyze the relativity and dynamic variability of fuzzy research objects, Chen et al. established variable fuzzy set (VFS) theory based on engineering fuzzy set theory, which was a theoretical fuzzy set model that combined fuzzy clustering, identification, and optimization.³³

Based on the coupling analysis of the SPA and VFS, the “connection degree” of SPA is considered the relative difference degree of VFS, which helps establish a comprehensive evaluation model that can reduce the loss of information and make the evaluation results more scientific and effective.^{34,35}

The evaluation steps are as follows:

- (1) Selection of evaluation indexes and establishment of evaluation grade standards. The influencing factors of bed-separated water inrush were sorted into a Q set (a set named Q), and a set of the evaluation standard (W) was established according to the evaluation results (including five grades). Combining these two sets gave a set $B = (Q, W)$.

$$Q = (q_{11}, q_{21}, \dots, q_{m1}, q_{12}, q_{22}, \dots, q_{m2} \dots q_{1n}, q_{2n}, \dots, q_{mn})$$

$$W = (x_{01}, x_{02}, \dots, x_{0n}, x_{11}, x_{12}, \dots, x_{1n} \dots x_{m1}, x_{m2}, \dots, x_{mn})$$

(22)

where q_{mn} represents the m th evaluation result under the n th index in the Q set and x_{mn} denotes the threshold value of the evaluation interval corresponding to the n th evaluation index in the W set.

- (2) In addition, the 3-element (difference degree, opposition degree, and identical degree) connection degree needs to be extended. The difference degree in SPA was divided into excellence degree and inferiority degree, which was expressed as $b = b_1 + b_2$. The opposition degree contained a superior degree and inferior degree and was expressed as $c = c_1 + c_2$. The multivariate connection degree formula is as follows³⁶

$$\begin{aligned} \mu &= a + (b_1 + b_2)i + (c_1 + c_2)j \\ &= a + b_1i_1 + b_2i_2 + c_1j_1 + c_2j_2 \end{aligned} \tag{23}$$

According to eq 23, one can conclude that (a) when the index evaluation value q_n is in the mid of the third grade, the index parameters are identical, that is, $a = 1, b_1 = b_2 = c_1 = c_2 = 0$; (b) when q_n is located on both sides of the third grade, the index evaluation result can be divided into excellence and inferiority, with the former being marked as b_1 and the latter being marked as b_2 . The closer the q_n is to the third grade, the greater the a is and the smaller the b_1 becomes and vice versa; (c) when q_n is located in the first and fifth grade, the evaluation result is divided into superior and inferior. The superior side is marked as c_1 , and when q_n is closer to the third grade, a and b_1 are larger and c_1 is smaller. The other side is marked as c_2 , and when

q_n is closer to the third grade interval, a and b_2 are larger and c_2 is smaller.

Considering the actual situation and the equal division principle of difference coefficient and opposition coefficient, one can take $i_1 = 0.5$ and $i_2 = -0.5$, and make $j_1 = 0$ and $j_2 = -1$ according to the special coefficient value method.

When the evaluation index is negative, the smaller the evaluation result of the index, the lower the harmfulness is.

It is expressed as eq 24.^{37,38}

$$\begin{aligned} \mu_{1n} &= \begin{cases} 1 & q_n \in [x_{0n}, x_{1n}) \\ \frac{x_{1n} - q_n - x_{1n}}{q_n - 2q_n} & q_n \in [x_{1n}, x_{2n}) \\ \frac{x_{1n} - x_{2n} - x_{1n} - \frac{q_n - x_{2n}}{q_n}}{2q_n} & q_n \in [x_{2n}, x_{3n}] \end{cases} \\ \mu_{2n} &= \begin{cases} \frac{x_{2n} - x_{1n} + \frac{x_{1n} - q_n}{2(x_{2n} - q_n)}}{x_{2n} - q_n} & q_n \in [x_{0n}, x_{1n}) \\ 1 & q_n \in [x_{1n}, x_{2n}) \\ \frac{x_{2n} - x_{1n} - \frac{q_n - x_{2n}}{2(q_n - x_{1n})}}{q_n - q_n} & q_n \in [x_{2n}, x_{3n}) \\ \frac{x_{2n} - x_{1n} - \frac{x_{3n} - x_{2n}}{2(q_n - x_{1n})} - \frac{q_n - x_{3n}}{q_n - x_{1n}}}{q_n - q_{1n}} & q_n \in [x_{3n}, x_{5n}] \end{cases} \\ \mu_{3n} &= \begin{cases} \frac{x_{3n} - x_{2n} + \frac{x_{2n} - x_{1n}}{2(x_{3n} - q_n)}}{x_{3n} - q_n} & q_n \in [x_{0n}, x_{1n}) \\ \frac{x_{3n} - x_{2n} + \frac{x_{2n} - q_n}{2(x_{3n} - q_n)}}{x_{3n} - q_n} & q_n \in [x_{1n}, x_{2n}) \\ 1 & q_n \in [x_{2n}, x_{3n}) \\ \frac{x_{3n} - x_{2n} - \frac{q_n - x_{3n}}{2(q_n - x_{2n})}}{q_n - x_{2n}} & q_n \in [x_{3n}, x_{4n}) \\ \frac{x_{3n} - x_{2n} - \frac{x_{4n} - x_{3n}}{2(q_n - x_{2n})}}{q_n - x_{2n}} - \frac{q_n - x_{4n}}{q_n - x_{2n}} & q_n \in [x_{4n}, x_{5n}] \end{cases} \\ \mu_{4n} &= \begin{cases} \frac{x_{4n} - x_{3n} + \frac{x_{3n} - x_{2n}}{2(x_{4n} - q_n)}}{x_{4n} - q_n} & q_n \in [x_{0n}, x_{2n}) \\ \frac{x_{4n} - x_{3n} + \frac{x_{3n} - q_n}{2(x_{4n} - q_n)}}{x_{4n} - q_n} & q_n \in [x_{2n}, x_{3n}) \\ 1 & q_n \in [x_{3n}, x_{4n}) \\ \frac{x_{4n} - x_{3n} - \frac{q_n - x_{4n}}{2(q_n - x_{3n})}}{q_n - x_{3n}} & q_n \in [x_{4n}, x_{5n}] \end{cases} \\ \mu_{5n} &= \begin{cases} \frac{x_{5n} - x_{4n} + \frac{x_{4n} - x_{3n}}{2(x_{5n} - q_n)}}{x_{5n} - q_n} & q_n \in [x_{0n}, x_{3n}) \\ \frac{x_{5n} - x_{4n} + \frac{x_{4n} - q_n}{2(x_{5n} - q_n)}}{x_{5n} - q_n} & q_n \in [x_{3n}, x_{4n}) \\ 1 & q_n \in [x_{4n}, x_{5n}] \end{cases} \end{aligned} \tag{24}$$

When the evaluation index is positive, the larger the evaluation result of the index, the lower the harmfulness is.

It is expressed as eq 25.

$$\mu_{5n} = \begin{cases} 1 & q_n \in [x_{5n}, x_{4n}) \\ \frac{x_{4n} - q_n - x_{4n}}{q_n - 2q_n} & q_n \in [x_{4n}, x_{3n}) \\ \frac{x_{4n} - x_{3n} - x_{4n} - \frac{q_n - x_{3n}}{q_n}}{q_n - 2q_n} & q_n \in [x_{3n}, x_{0n}] \end{cases}$$

$$\mu_{4n} = \begin{cases} \frac{x_{3n} - x_{4n} + \frac{x_{4n} - q_n}{2(x_{3n} - q_n)}}{x_{3n} - q_n} & q_n \in [x_{5n}, x_{4n}) \\ 1 & q_n \in [x_{4n}, x_{3n}) \\ \frac{x_{3n} - x_{4n} - \frac{q_n - x_{3n}}{2(q_n - x_{4n})}}{q_n - q_{4n}} & q_n \in [x_{3n}, x_{2n}) \\ \frac{x_{3n} - x_{4n} - \frac{x_{2n} - x_{3n}}{2(q_n - x_{4n})} - \frac{q_n - x_{2n}}{q_n - x_{4n}}}{q_n - x_{4n}} & q_n \in [x_{2n}, x_{0n}] \end{cases}$$

$$\mu_{3n} = \begin{cases} \frac{x_{2n} - x_{3n} + \frac{x_{3n} - x_{4n}}{2(x_{2n} - q_n)}}{x_{2n} - q_n} & q_n \in [x_{5n}, x_{4n}) \\ \frac{x_{2n} - x_{3n} + \frac{x_{3n} - q_n}{2(x_{2n} - q_n)}}{x_{2n} - q_n} & q_n \in [x_{4n}, x_{3n}) \\ 1 & q_n \in [x_{3n}, x_{2n}) \\ \frac{x_{2n} - x_{3n} - \frac{q_n - x_{2n}}{2(q_n - x_{3n})}}{q_n - x_{3n}} & q_n \in [x_{2n}, x_{1n}) \\ \frac{x_{2n} - x_{3n} - \frac{x_{1n} - x_{2n}}{2(q_n - x_{3n})}}{q_n - x_{3n} - \frac{q_n - x_{1n}}{q_n - x_{3n}}} & q_n \in [x_{1n}, x_{0n}] \end{cases}$$

$$\mu_{2n} = \begin{cases} \frac{x_{1n} - x_{2n} + \frac{x_{2n} - x_{3n}}{2(x_{1n} - q_n)}}{x_{1n} - q_n} & q_n \in [x_{5n}, x_{3n}) \\ \frac{x_{1n} - x_{2n} + \frac{x_{2n} - q_n}{2(x_{1n} - q_n)}}{x_{1n} - q_n} & q_n \in [x_{3n}, x_{2n}) \\ 1 & q_n \in [x_{2n}, x_{1n}) \\ \frac{x_{1n} - x_{2n} - \frac{q_n - x_{1n}}{2(q_n - x_{2n})}}{q_n - x_{2n}} & q_n \in [x_{1n}, x_{0n}] \end{cases}$$

$$\mu_{1n} = \begin{cases} \frac{x_{0n} - x_{1n} + \frac{x_{1n} - x_{2n}}{2(x_{0n} - q_n)}}{x_{0n} - q_n} & q_n \in [x_{5n}, x_{2n}) \\ \frac{x_{0n} - x_{1n} + \frac{x_{1n} - q_n}{2(x_{0n} - q_n)}}{x_{0n} - q_n} & q_n \in [x_{2n}, x_{1n}) \\ 1 & q_n \in [x_{1n}, x_{0n}] \end{cases}$$

$$v_k = \frac{1}{1 + \eta_k}$$

$$\eta_k = \left[\frac{\sum_{n=1}^N [\omega_n(1 - \xi_{kn})]^p}{\sum_{n=1}^N (\omega_n \xi_{kn})^p} \right]^{d/p}$$

where d is the optimization criterion parameter, $d \in [0, 1]$; $p = 1$ is the Hamming distance, and $p = 2$ is the Euclidean distance. Combining d and p gives four comprehensive membership results, which are (1) $d = 1, p = 1$; (2) $d = 1, p = 2$; (3) $d = 2, p = 1$; and (4) $d = 2, p = 2$.

(6) Determination of the grade characteristic value and evaluation grade. Four groups of comprehensive membership vectors were obtained with the four combinations mentioned above, which were then normalized to obtain the dimensionless comprehensive membership vector V for any evaluation grade.

$$V_k = \frac{v_k}{\sum_{k=1}^5 v_k}$$

where k refers to the evaluation grade, and $k = 1, 2, \dots, 5$.

Thus, the comprehensive membership V_k of the risk of bed separation water inrush was finally obtained, whose maximum vector can be used to determine the risk grade of bed separation water inrush.

AUTHOR INFORMATION

Corresponding Authors

Wenquan Zhang – College of Energy and Mining Engineering, Shandong University of Science and Technology, Qingdao 266590, China; State Key Laboratory of Strata Intelligent Control and Green Mining Co-founded by Shandong Province and the Ministry of Science and Technology, Qingdao 266590, China; Email: wenquanzhang415@163.com

Zaiyong Wang – Key Laboratory of In-situ Property Improving Mining of Ministry of Education, Taiyuan University of Technology, Taiyuan 030024, China; orcid.org/0000-0003-1027-6257; Email: zaiyongwang@163.com

Authors

Xiao Li – College of Energy and Mining Engineering, Shandong University of Science and Technology, Qingdao 266590, China; State Key Laboratory of Strata Intelligent Control and Green Mining Co-founded by Shandong Province and the Ministry of Science and Technology, Qingdao 266590, China

Xin Wang – College of Civil Engineering and Hydraulic Engineering, Shandong University, Jinan 250100, China

Chuming Pang – College of Energy and Mining Engineering, Shandong University of Science and Technology, Qingdao 266590, China; State Key Laboratory of Strata Intelligent Control and Green Mining Co-founded by Shandong Province and the Ministry of Science and Technology, Qingdao 266590, China

Complete contact information is available at:

<https://pubs.acs.org/10.1021/acsomega.1c06700>

Notes

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where μ_{kn} is the connection degree between the parameter value under the n th evaluation index and the k th evaluation grade.

(3) Calculation of the relative membership degree:

the relative membership degree of the fuzzy evaluation grade k is calculated from eqs 24 and 25.

$$\xi_{kn} = \frac{1 + \mu_{kn}}{2}$$

(4) Determination of the weight of each evaluation index:

The subjective weight and objective weight of each evaluation index was calculated using the IFAHP and EWM, respectively. The final weight of each evaluation index was determined by the CWM based on the cooperative game.

(5) Calculation of the comprehensive membership is as follows³⁹

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