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RESEARCH PAPER

Effect of Fear, Treatment, and Hunting Cooperation on an Eco-Epidemiological Model: Memory Effect in Terms of Fractional Derivative

Uttam Ghosh[1] · Ashraf Adnan Thirthar[2] · Bapin Mondal[1] · Prahlad Majumdar[1]

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Abstract

In this paper, we have studied a fractional-order eco-epidemiological model incorporating fear, treatment, and hunting cooperation effects to explore the memory effect in the ecological system through Caputo-type fractional-order derivative. We have studied the behavior of different equilibrium points with the memory effect. The proposed system undergoes through Hopf bifurcation with respect to the memory parameter as the bifurcation parameter. We perform numerical simulations for different values of the memory parameter and some of model parameters. In the numerical results, it appears that the system is exhibiting a stable behavior from a period or chaotic nature with the increase in the memory effect. The system also exhibits two transcritical bifurcations with respect to the growth rate of the prey. At low values of prey's growth, all species go to extinction, at moderate values of prey's growth, only preys (susceptible and infected) can survive, and at higher values of prey's growth, all species survive simultaneously. The paper ended with some recommendations.

Keywords Eco-epidemic model · Fear effect · Hunting cooperation · Caputo fractional-order derivative · Transcritical bifurcation · Hopf bifurcation

1 Introduction

In population dynamics, ecological interactions such as competition, mutualism, and predation, play an essential role. However, parasite infection also affects the size of populations. Thus, prey–predator interactions should not ignore this issue. There have been multiple field studies demonstrating parasitic infections in prey and predators. Parasites can reduce the ability of infected organisms to survive and reproduce by affecting their internal mechanisms. Therefore, we ought to be concerned about predator–prey systems in which both populations are infected. An eco-epidemiological approach focuses on infectious diseases in populations and communities. The step-by-step process of analyzing a problem from a molecular, social, and demographic perspective is considered eco-epidemiology. A system's dynamics are affected by infection in any part of the population or both populations. In recent years, infectious disease has emerged as a significant factor. Researchers are increasingly studying predator and prey with infectious diseases.

In many studies, predator–prey models have been investigated only with a disease in the prey. Hethcote et al. (2004) presented a predator–prey model in which SIS parasitic infection in prey led to higher rates of predation on infected prey before predator predation. In Sinha et al. (2010), authors explore prey–predator interactions in the context of environmental toxicants and disease. According to their study, the toxicants affect the population, while the infected prey is much more vulnerable to the toxicants as well as predators than sound prey. Shaikh et al. (2021) investigates the dynamics of an eco-epidemic predator–prey system in which disease is spread to prey species and alternative food is provided to predators. Moustafa et al.

✉ Bapin Mondal
bapinmondal1@gmail.com

1 Department of Applied Mathematics, University of Calcutta, Kolkata 700009, India

2 Department of Studies and Planning, University of Fallujah, Anbar, Iraq

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(2020) analyzes a fractional-order eco-epidemiological model with disease in the prey population and showed that the order of fractional derivative stabilizes the coexistence equilibrium. According to Meng et al. (2018), the predator population can survive in a predator–prey system with prey harvesting and disease spreading among prey species. Recently, Sk et al. (2022) have studied a prey–predator model that incorporated infection in prey in both deterministic and stochastic environments, and found that high levels of fear and low levels of refuge can eliminate the disease from the system.

Some infectious diseases can influence the dynamics of predator–prey systems when they enter either the predator body or the prey body (Djilali and Ghanbari 2021) through some pathogens. Pathogens include germs, viruses, fungi, parasites, etc. The objects can spread by direct or indirect contact with animals or some other way such as water and air (Van Seventer and Hochberg 2017). If the infection is not treated, it may be harmful. According to the Food and Agriculture Organization of the United Nations (FAO) (Romain et al. 2020), livestock accounts for (40%) of the total agricultural production in developed countries, and for (20%) in developing countries. The infection agencies may directly infect the prey or the predator, or the predator may become infected after consumption of the infected prey (Andrew et al. 2016).

An important topic in ecological systems with epidemics is the dynamic relationship between predators and their prey. Eco-epidemiological models are investigated the ecological system with infection (Mukherjee 2010; Chakraborty et al. 2011; Chattopadhyay et al. 2002, 1999). The necessity of conserving wild animals has led many ecologists and eco-epidemiologists to become familiar with eco-epidemiology. Eco-epidemiological models discuss the prey–predator relationships when some of the species are infected (Juneja and Agnihotri 2018). One of the main objectives of the investigation of the eco-epidemic model is to control the spreading of diseases when disease and treatment both coexist simultaneously. Many studies have been published on ecological and epidemiological models with the disease either in prey (Meng et al. 2018; Mortoja et al. 2018) or in predator (Rana et al. 2016; Juneja and Agnihotri 2018) or in both prey and predator (Agnihotri and Juneja 2015; Hsieh and Hsiao 2008) and reference therein.

Several field survey data and experimental results on terrestrial vertebrates showed that the fear of predators would cause a substantial variability of prey density (Sarkar and Khajanchi 2020). In Mukherjee (2020); Zhang et al. (2019), the authors have studied some of these types of models in the presence of fear effect and competitor for the prey in the predator–prey model with prey refuge. Treatment of the infected prey populations restores the prey to its previous situation; as a result, availability of susceptible prey becomes plenty to the predator, and dynamics of the system may be more complex compared to other situations (Adnan Thirthar 2020).

There are ample information in the existing literature on predator–prey interactions, which utilize the diversity of functional responses of both prey and predator populations. In the literature, direct assassinations of prey populations by predator populations have been the focus. Several studies have examined predator and prey behavior as well as antipredator activities (Wang et al. 2016; Wang and Zou 2017). Wang et al. (2016) and Zanette et al. (2011) constructed mathematical models for the predator–prey system by including the cost of fear for prey species due to predators, where the cost of fear determines the birth rate of prey species. They showed that the presence of predator-defeating activities or a major cost of fear can eliminate periodic behaviors, excluding the paradox of the enrichment scenario. Moreover, they demonstrated that fear can stabilize the system by eliminating population oscillations. Additionally, oscillations emerge from either supercritical or subcritical Hopf bifurcation under comparatively low cost for fear (Wang et al. 2016). Therefore, the fear effect can produce multi-stability in the predator–prey system. Mondal et al. (2022) showed the existence of saddle-node bifurcation, Hopf bifurcation, and Bogdanov–Takens bifurcation in an imprecise predator–prey system with fear effect and nonlinear harvesting of predators in an uncertain environment.

The authors in Danane et al. (2021) studied a mathematical model that described COVID-19 dynamics with the effects of governmental action and individual risk awareness to reduce the spread of infection effectively. In addition, the authors showed that the infection converges more quickly to its steady state when the fractional derivative is used. Kumar et al. (2022) evaluated the mathematical model of the COVID-19 epidemic using Caputo–Fabrizio fractional derivatives. Boudaoui et al. (2021) employed the Caputo–Fabrizio derivative to model the novel Coronavirus disease COVID-19 and found that fractional-order epidemic models provide more insight into the disease. Using the Caputo operator, Gao and Baskonus (2022) developed a modified epidemiological Susceptible-Infected-Removed model. In their study, Tanriverdi et al. (2021) examined the dynamics and mathematics of the fractional-order atmosphere–soil–land plant carbon cycle system involving the time-dependent variable of carbon flux in the atmosphere. An experimental study on fractional models of complex permittivity of conductor media with relaxation is conducted by Ciancio et al. (2022). Zamir et al. (2021) examined mathematical modeling of the eradication of the COVID-19 infection with the help of almost non-pharmaceutical interventions (NPIs). Danane et al. (2021) investigated the dynamics of a COVID-19 stochastic model with an isolation strategy using Lévy jump perturbations incorporating toward all compartments of the suggested model. Hama et al. (2022) examined the behavior of a SEIS stochastic model which is performed by a Lévy process. In the research articles (Danane et al. 2021; Kumar et al. 2022; Boudaoui et al. 2021; Gao and Baskonus 2022; Tanriverdi et al. 2021; Ciancio et al. 2022; Zamir et al. 2021; Danane et al. 2021; Hama et al. 2022), the epidemic model with fractional-order derivatives mainly appears. Moustafa et al. (2020) studied a fractional-order eco-epidemiological model with disease in the prey population. But they have not considered any biological effects such as fear and hunting cooperation. In Sk and Pal (2022), authors studied a prey–predator model with infection in a prey population. They have also considered the effects of fear, refuge, and harvesting in their proposed model. But they did not consider any treatment for infected individuals or memory effect. In Moustafa et al. (2022), authors studied a fractional-order prey–predator model with infection in predator and prey harvesting. They did not consider the effects of fear, cooperation, or any treatment on the infected populations. Yousef et al. (2022) studied a fractional-order eco-epidemiological model with fear and hunting cooperation but they did not consider the effect of treatment. Motivating from the above-said articles, we think that there may be some research gap in the memory-dependent eco-epidemic model with hunting cooperation, fear, and treatment of infected prey simultaneously. To fill the research gap in this paper, we have studied the combined effects of fear and hunting cooperation. Furthermore, we include a treatment term for infected prey and also we have analyzed our model with memory effects.

To formulate the model, we assume that the total prey population is composed of two compartments: One is the class of the susceptible prey, and the other is the class of the infected prey, their population density is denoted by $S(t)$ and $I(t)$, respectively, and the density of predator populations is denoted by $y(t)$. We assume that only susceptible prey is capable of reproducing, the disease is spread among the prey population only and the disease is not genetically inherited. We also assume that infected prey does not compete for the resource of being weak due to disease infection. We consider that the predator eats only the infected prey. Let the rate of incidence of disease transmission be βSI and the treatment function $\frac{\rho I}{\sigma + I}$, in which $\rho > 0$ represents the maximum medical resource supplied for treatment, while $\frac{1}{\sigma} > 0$ stands for the saturation factor that measure the effect of the delay in treatment for the infected individuals. Introducing the effect of cooperation of predators are following functional form $(p + by)y$, where y is the predator density, $p > 0$ is the attack rate of the predator on the prey, and $b \geq 0$ describes the predator cooperation in hunting (Mondal et al. 2022, 2022). Here, we consider the fear function in the form $\eta + \frac{v(1-\eta)}{v+y}$, where v represents the fear level and $\eta \in [0, 1]$ represents the minimum cost of fear, which is extensively described in Sarkar and Khajanchi (2020). Based on all the above-described situations, the three-dimensional eco-epidemic model can be formulated as follows:

$$\begin{cases} \frac{dS}{dt} = \left(\eta + \frac{v(1-\eta)}{v+y}\right) rS - (d + \beta I)S + \frac{\rho I}{\sigma + I} \\ \frac{dI}{dt} = \left[\beta S - \delta - (p + by)y - \frac{\rho}{\sigma + I}\right] I \\ \frac{dy}{dt} = [c(p + by)I - m]y \end{cases} . \qquad (1)$$

Table 1 A description of biological symbols along with their meanings and values

Parameter	Environmental Interpretation	Values
η	Minimum cost of fear	0.0011
v	Level of fear	0.12
r	Intrinsic growth rate of prey	0.112
d	Natural death rate of the susceptible prey	0.12
β	Disease transmission rate	0.32
ρ	Maximum medical resource supplied for treatment	0.9
$\frac{1}{\sigma}$	Stands for the saturation factor that measure the effect of the delay in treatment for the infected individuals	$\frac{1}{0.8}$
δ	Death rate of the infected prey	0.12
p	The attack rate of the predator on the infected prey	0.29
b	Predator cooperation in hunting	0.26
c	Efficiency with which predators convert consumed infected prey into new predators	0.34
m	Natural death rate of the predator	0.39

with initial conditions $S(0) > 0, I(0) \geq 0, y(0) \geq 0$ and all the model parameters are nonnegative and their biological consequence are described in Table 1.

Fractional-order derivative is the generalization of integer-order derivative to an arbitrary order (Petráš 2011). Ordinary derivatives are memoryless, i.e., they possess local properties. As a fractional-order derivative, it has non-local properties; namely, it carries not only the current circumstance but also its prior historical states (Al-Khaled and Alquran 2014; Das 2008). In the last few years, a lot of research articles have appeared in different scientific journals studying biological, inventory models, or physical problems (Das et al. 2019; Pakhira et al. 2020). Fractal-order differential equations have gained a lot of attention and appreciation recently due to their ability to find accurate descriptions of various nonlinear phenomena. In recent years, dynamical system research has increasingly utilized models based on fractal order (Li et al. 2015; Hegazi et al. 2013) and references therein. Biological systems carry memory effects in searching for food, locating a safe place to live, and finding a mate. The memory effect is included in biological systems by using fractional-order derivatives and in most cases, it has been observed that the memory effect has a stabilization property. The fundamental results of fractional derivative are given in Kilbas et al. (2006); Podlubny (1998); El-Sayed (1996). The authors in Li et al. (2015); Hegazi et al. (2013); El-Misiery and Ahmed (2006) used fractional-order derivative to study the physical problems like earthquakes, network problems, Liu system, etc.

The reduction in the fractional-order model from the integer-order model was logically developed through the introduction of the kernel function in Saeedian et al. (2017); Ghosh et al. (2021). Following the methodology developed in Saeedian et al. (2017); Ghosh et al. (2021), the system (1) easily can be reduced into a fractional-order model in the following form:

$$\begin{cases} D_t^{\in} S(t) = \left(\eta + \dfrac{v(1-\eta)}{v+y}\right) rS - (d+\beta I)S + \dfrac{\rho I}{\sigma + I}, \\ D_t^{\in} I(t) = \left[\beta S - \delta - (p+by)y - \dfrac{\rho}{\sigma+I}\right] I, \\ D_t^{\in} y(t) = [c(p+by)I - m]y, \end{cases} \quad . \tag{2}$$

with initial conditions $S(0) > 0, I(0) \geq 0, y(0) \geq 0$, $0 < \epsilon < 1$ is the order of fractional derivative, D_t^{ϵ} is the Caputo type of fractional-order derivative (CFD), and t_0 is the initial time which is equal or greater than zero. Defination and properties of CFD with reduction of ODE into FDE are described in Sect. 2.

This manuscript is organized as follows: Sect. 2 discusses the definition and some basic properties of CFD with basic mathematical analysis such as the existence of solutions and their positivity and bounds. Section 3 discusses equilibrium points and their stability. Section 4 summarizes numerical results with memory effects. Finally, Sect. 5 concludes with some conclusions.

2 Definition and Some Basic Properties of CFD with Basic Mathematical Analysis

Here, we shall describe some basic definition and properties of fractional derivative. The commonly use definitions are the Riemann–Liouville (RL) and Caputo derivatives which are given below:

1. RL definition of fractional derivative for any $(n-\epsilon)$ time integrable and n times differentiable function $g(t)$ on $[t_0, t]$ is defined as

$$_{t_0}^{RL}D_t^{\epsilon} g(t) = \frac{1}{\Gamma(n-\epsilon)} \left(\frac{d}{dt}\right)^n \int_{t_0}^{t} \frac{g(w)}{(t-w)^{\epsilon-n+1}} dw,$$

where $n-1 < \epsilon \leq n$.

2. The CFD of order ϵ with $n-1 < \epsilon \leq n$ for the function $g \in C^n([t_0, +\infty), R]$ is defined as Boukhouima et al. (2017):

$$D_t^{\epsilon} g(t) = \frac{1}{\Gamma(n-\epsilon)} \int_{t_0}^{t} \frac{g^{(n)}(w)}{(t-w)^{\epsilon-n+1}} dw$$

where $\Gamma(.)$ is the Gamma function, $t \geq t_0$ and n is a positive integer. Particularly, when $0 < \epsilon \leq 1$, the above defination modified as

$$D_t^{\epsilon} g(t) = \frac{1}{\Gamma(1-\epsilon)} \int_{t_0}^{t} \frac{g'(w)}{(t-w)^{\epsilon}} dw.$$

3. In the above two cases, the memory is expressed in terms of a singular kernel. Recently, Caputo–Fabrizio fractional derivative is developed to replace the singular kernel with a non-singular kernel, which is given below:

$$_{t_0}^{CF}D_t^{\epsilon} g(t) = \frac{1}{(1-\epsilon)} \int_{t_0}^{t} g'(w) \exp\left(-\epsilon \frac{t-w}{1-\epsilon}\right) dw$$

where $t_0 < 0$ and $0 < \epsilon \leq 1$.

Again, any system with the differential equation in the form

$$\frac{dx}{dt} = g(x)$$

can be expressed in terms of the memory kernel function in the following form:

$$\frac{dx}{dt} = \int g(\xi)K(\xi - t)d\xi \quad (3)$$

the integration is consider over suitable domain $[t_0, t]$. The function $k(\xi - t)$ is the memory-dependent kernel is used by different authors in Ghosh et al. (2021). For memoryless system $k(\xi - t) = \delta(\xi - t)$, where $\delta(.)$ is the Dirac delta function.

If we consider $k(\xi - t) = \frac{1}{\Gamma(1-\epsilon)}(\xi - t)^{\epsilon-2}$, where $0 < \epsilon \leq 1$ the RHS of (3) can be expressed as

$$\frac{dx(t)}{dt} = -{}_0D_t^{-(\epsilon-1)}g(x(t)).$$

Operating Caputo-type functional derivative of order $(\epsilon - 1)$ on both sides, we get

$${}_0^cD_t^\epsilon(x(t)) = g(x(t))$$

with initial $x(0) = X_0$. Using the above methodology system (1) can be converted to the system (2).

In the following, we shall present some preliminary results that are satisfied by Caputo-type differential equations, which will be used to establish different analytical results (Li et al. 2009; Choi et al. 2014).

Lemma 1 *The linear fractional-order differential equation of the form* $D^\epsilon y(t) = \lambda y(t)$ *has solution of the form* $y(t) = y(t_0)E_\epsilon(\lambda(t - t_0)^\epsilon)$ *where* $0 < \epsilon < 1$, E_ϵ *is the Mittag-Leffler function with parameter* ϵ *in the domain* $[t_0, t]$.

Lemma 2 *The Caputo differential operator is linear, i.e.,* $D_t{}^\epsilon(\lambda x(t) + \mu y(t)) = \lambda D^\epsilon x(t) + \mu D^\epsilon y(t)$.

Lemma 3 *For any real-valued continuous function* $x(t)$ *with* $t \geq t_0$, *the inequality* $\frac{1}{2}D^\epsilon x^2(t) \leq D^\epsilon x(t)$ *holds.*

Lemma 4 *For any real-valued function* $x(t)$ *with* $t \geq t_0$, *real constant* x^* *and* $0 < \epsilon < 1$ *the inequality*

$$D^\epsilon\left(x(t) - x^* - x^* ln\frac{x(t)}{x^*}\right) \leq \left(1 - \frac{x(t)}{x^*}\right)D^\epsilon x(t)$$

holds.

Lemma 5 *For any real-valued function* $x(t)$ *with* $t \geq t_0$, *two arbitrary real constant* λ, d *and* $D^\epsilon x(t) \leq \lambda x(t) + d$ *the following result holds*:

$$x(t) \leq x(t_0)E_\epsilon(\lambda(t - t_0)^\epsilon) + d(t - t_0)^\epsilon E_{\epsilon,\epsilon}(\lambda(t - t_0)^\epsilon), t \geq t_0.$$

Lemma 6 *For the system of m-dimensional linear fractional-order differential equation* $D^\epsilon x(t) = Bx(t), 0 < \epsilon \leq 1$ *with* $x(t) = (x_1(t), x_2(t), \ldots, x_l(t))$ *and* $B = [b_{ij}]_{l\times l}$ *is the coefficient matrix with eigenvalues* $\lambda_1, \lambda_2, \ldots\lambda_l$. *The stability of the above linear system about the trivial equilibrium point can be verified using the following conditions*:

The trivial equilibrium point of the system will be

(*a*) *Locally asymptotically stable if* $|arg(\lambda_j) > \frac{\epsilon\pi}{2}|$ *for* $j = 1, 2, 3, ..l.$

(*b*) *Stable if* $|arg(\lambda_j) \geq \frac{\epsilon\pi}{2}|$ *and algebraic multiplicity will be same as the geometric multiplicity for those* λ_j *for which the sign of equality holds.*

(*c*) *Unstable if for at least one j,* $|arg(\lambda_j) < \frac{\epsilon\pi}{2}|$.

Let us denote $m(\epsilon) = \frac{\epsilon\pi}{2} - min_{1\leq j\leq 1}\{arg(\lambda_j)\}$ then the above system can be easily expressed as: (i) For stable equilibrium point $m(\epsilon) < 0$ and (ii) For unstable equilibrium point $m(\epsilon) > 0$. Oscillatory or time steady behavior is highly dependent on the memory parameter when other parameters are fixed.

Here we shall present the existence uniqueness and positivity boundedness conditions of solutions of the proposed fractional-order prey–predator model (2).

2.1 Existence and Uniqueness

To prove the existence and uniqueness of the solutions, we have adapted the same methodology which is described in Ghosh et al. (2021); Wang et al. (2019). The existence and uniqueness of the solutions of the fractional-order system (2) are studied in the region $\Upsilon \times (0, T]$ where

$$\Upsilon = \{(S, I, y) \in R^3; max(|S|, |I|, |y|) \leq \chi\}$$

for some positive constant χ.

Theorem 1 *There exists a unique solution* $E(t) \in \Upsilon$ *of the fractional-order prey–predator model* (2) *with initial condition* $(S_0, I_0, y_0) \in \Upsilon$, *which is defined for all* $t \geq 0$.

Proof To establish the above statement, we shall follow the approach as used by the authors in Ghosh et al. (2021); Boukhouima et al. (2017); Wang et al. (2019). For this purpose, we consider two solutions $E, \tilde{E} \in \Upsilon$ and denote the function $W(E) = (W_1(E), W_2(E), W_3(E))$ such

$$\begin{cases} W_1(E) &= \left(\eta+\frac{v(1-\eta)}{v+y}\right)rS-(d+\beta I)S+\frac{\rho I}{\sigma+I}, \\ W_2(E) &= \left[\beta S-\delta-(p+by)y-\frac{\rho}{\sigma+I}\right]I, \\ W_2(E) &= [c(p+by)I-m]y. \end{cases}$$

$$\begin{aligned} \text{Now,}\left\|W(E)-W(\tilde{E})\right\| &= \left|W_1(E)-W_1(\tilde{E})\right| \\ &+\left|W_2(E)-W_2(\tilde{E})\right|+\left|W_3(E)-W_3(\tilde{E})\right| \\ &\le \eta r|S-\tilde{S}|+r(1-\eta)|S-\tilde{S}|+\frac{r}{v}(1-\eta)\chi|S-\tilde{S}| \\ &+\frac{r}{v}(1-\eta)\chi|y-\tilde{y}| \\ &+d|S-\tilde{S}|+\beta\chi|S-\tilde{S}|+\beta\chi|I-\tilde{I}| \\ &+\frac{\rho}{\sigma}|I-\tilde{I}|+\beta\chi|S-\tilde{S}| \\ &+\beta\chi|I-\tilde{I}|+\delta|I-\tilde{I}|+p\chi|I-\tilde{I}| \\ &+p\chi|y-\tilde{y}|+pb\chi|I-\tilde{I}| \\ &+pb\chi^2|y-\tilde{y}|+\frac{\rho}{\sigma}|I-\tilde{I}|+cp\chi|I-\tilde{I}| \\ &+cp\chi|y-\tilde{y}|+cb|I-\tilde{I}| \\ &+cb\chi^2|y-\tilde{y}|+m|y-\tilde{y}| \\ &\le(\eta r+r(1-\eta)+\frac{r}{v}(1-\eta)\chi+d+2\beta\chi)|S-\tilde{S}| \\ &+\Big(\frac{r}{v}(1-\eta)\chi+p\chi+pb\chi^2+cp\chi \\ &+cb\chi^2+m\Big)|y-\tilde{y}| \\ &+(2\beta\chi+2\frac{\rho}{\sigma}+\delta+p\chi+pb\chi+cp\chi+cb)|I-\tilde{I}| \\ &\le K|E-\tilde{E}|, \end{aligned} \tag{4}$$

where

$$K=\max\left\{\begin{matrix} \eta r+r(1-\eta)+\frac{r}{v}(1-\eta)\chi+d+2\beta\chi,\frac{r}{v}(1-\eta)\chi+p\chi+pb\chi^2+cp\chi+cb\chi^2+m, \\ 2\beta\chi+2\frac{\rho}{\sigma}+\delta+p\chi+pb\chi+cp\chi+cb \end{matrix}\right\}.$$

Thus, $W(E)$ satisfied the Lipschitz condition, and hence, the proposed fractional-order system has a unique solution in the domain Υ. □

2.2 Positivity and Boundedness of the Solutions

In this part, we shall establish the positiveness of the solutions of (2). From (2), we have the following:

$$\begin{cases} D_t^{\epsilon}S(t)|_{S=0}=\frac{rhoI}{\sigma+I}\ge 0 \\ D_t^{\epsilon}I(t)|_{I=0}=0 \\ D_t^{\epsilon}y(t)|_{y=0}=0 \end{cases}.$$

Thus, by using lemmas 5 and 6 in Boukhouima et al. (2017), one can demand that the solutions of (2) are non-negative. In the next theorem, the boundedness of the solutions of the fractional-order prey–predator model (2) will be established.

Theorem 2 *All solutions of the system (2) with initial conditions in R_+^3 are uniformly bounded and lie in the domain $\Upsilon=\left\{(S,I,y)\in R_+^3, 0\le M(t)\le\frac{r}{d\tau}+\epsilon,\epsilon>0\right\}$, where $M(t)$ is defined in the proof.*

Proof To establish the boundedness of the proposed fractional-order system, we used the method developed in Li et al. (2017). For this purpose, consider the function $M(t)=S(t)+I(t)+\frac{1}{c}y(t)$ then,

$$D_t^{\epsilon}M(t)=D_t^{\epsilon}S(t)+D_t^{\epsilon}I(t)+\frac{1}{c}D_t^{\epsilon}y(t)=\left(\eta+\frac{v(1-\eta)}{v+y}\right)rS$$
$$-dS-\delta I-\frac{m}{c}y.$$

Thus, for any $\tau>0$,

$$\begin{aligned} D_t^{\epsilon}M(t)+\tau M(t) &= \left(\eta+\frac{v(1-\eta)}{v+y}\right)rS-dS-\delta I \\ &-\frac{m}{c}y+\tau S+\tau I+\frac{\tau}{c}y \\ &=\left(\eta+\frac{v(1-\eta)}{v+y}-d+\tau\right)S+(\tau-\delta)I \\ &+\frac{1}{c}(\tau-m)y \end{aligned}$$

Now, we choose $\tau<min\{d,\delta,m\}$, then,

$$D_t^{\epsilon}M(t)+\tau M(t)\leq \frac{r}{d}$$

By using results in lemma 5, we get,

$$0\leq M(t)\leq M(0)E_{\epsilon}(-\tau(t)^{\epsilon})+\frac{r}{d}E_{\epsilon,\epsilon+1}(-\tau(t)^{\epsilon}),$$

where the function E_{ϵ} is the one parameter Mittag-Leffler function. By Lemma 5 and Corollary 6 in Choi et al. (2014), one gets the following expression:

$$0\leq M(t)\leq \frac{r}{d\tau}, \text{ as } t\to\infty.$$

Hence, for the fractional-order prey–predator system (4), all its solutions that started in R_+^3 are uniformly bounded in the region

$$\Upsilon=\left\{(S,I,y)\in R_+^3, M(t)\leq\frac{r}{d\tau}+\epsilon,\epsilon>0\right\}.$$

□

3 Equilibrium Points and Stabilities

In this section, we shall identify the equilibrium points of the proposed fractional-order system and investigate their nature. The proposed fractional-order system (2) has the following equilibrium points:

1. The species free equilibrium point $E_0(0,0,0)$, which always exists.
2. The predator-free equilibrium point $E_1\left(\frac{1}{\beta\left[\frac{\rho}{\sigma+I^*}+\delta\right]},I^*,0\right)$, where I^* is a root of the following second degree polynomial:

   $$a_0I^2+a_1I+a_2=0 \qquad (5)$$

   where $a_0=\beta\delta, a_1=\delta(\beta\sigma+d-r), a_2=-(\rho+\sigma\delta)(r-d)$. Depending on values of the model parameters the polynomial (5) may have: (i) a unique positive root if $a_1>0$; (ii) two positive roots if $a_1<0, a_2>0$ with $a_1^2-4a_2>0$; and (iii) no positive root if $a_1>0, a_2>0$.
3. The coexistence equilibrium point $E^*(S^*,I^*,y^*)$, where

   $$S^*=\frac{1}{\beta}\left[\frac{m(m-cpI^*)}{bc^2I^{*2}}+\frac{\rho}{\sigma+I^*}+\delta^2\right], y^*=\frac{m-cpI^*}{cbI^*}$$

   and I^* is a root of the following polynomial with fifth degree:

   $$a_0^*I^5+a_1^*I^4+a_2^*I^3+a_3^*I^2+a_4^*I+a_5^*=0, \qquad (6)$$

   where $a_j{}^*$ are

$$\begin{aligned}
a_0^*=&-\beta\delta\sigma bpc^3(v-1)\\
a_1^*=&-c^3p\delta\sigma b(v-1)(\beta-r\eta)\\
&-c^2p(v-1)(dc\delta\sigma b-\beta pm)\\
&-c^2\delta\sigma b(\beta m-rv(1-\eta)bc)\\
a_2^*=&-(bc^2[\delta\sigma+\rho]-mcp)(cpd(v-1)+\beta m\\
&+r\eta cp(v-1)-rvbc(1-\eta))\\
&-\beta cpm(v-1)(m-\sigma cp)+r\eta\delta\sigma bc^2\\
a_3^*=&r\eta cpm(v-1)(m-\sigma cp)+r(\eta m+vbc(1-\eta))\\
&(bc^2[\delta\sigma+\rho]-mcp)-\beta cp\sigma(v-1)\\
&-m(cpd(v-1)+\beta m)(m-\sigma cp)\\
a_4^*=&r\eta\sigma cpm^2(v-1)+r\eta m^2(m-\sigma cp)\\
&-\sigma m^2(cpd(v-1)+\beta m)\\
a_5^*=&m^3(1-d)+bcm(rv((1-\eta))m\sigma+\beta cp).
\end{aligned}$$

To find the nature of equilibrium points, we have to find the characteristic roots of the corresponding linear part. For this purpose, we give the transformation about any characteristic equilibrium point $E^*(S^*,I^*,y^*)$ in the form $S=S^*+u, I=I^*+v$ and $y=y^*+w$ where $0<u,v,w<<1$; then, the model (2) reduce to the following form

$$D^{\epsilon}\begin{pmatrix}u\\v\\w\end{pmatrix}=L\begin{pmatrix}u\\v\\w\end{pmatrix}$$

where $L(E^*)=[l_{ij}], i,j=1,2,3$ and $l_{11}=\left(\eta+\frac{v(1-\eta)}{v+y^*}\right)r-(d+\beta I^*), l_{12}=-\beta S^*+\frac{\rho\sigma}{(\sigma+I^*)^2}, l_{13}=-rS^*\left(\frac{v(1-\eta)}{(v+y^*)^2}\right), l_{21}=\beta I^*, l_{22}=\beta S^*-\delta-(p+by^*)y^*-\frac{\rho\sigma}{(\sigma+I^*)^2}, l_{23}=-(p+2by^*)I^*, l_{31}=0, l_{32}=cy^*(p+by^*), l_{33}=c(p+2by^*)I^*-m.$

In order to study the stability of the system, we use the results of Lemma $1-6$, which are established in Li et al. (2009); Choi et al. (2014).

Theorem 3 *The trivial equilibrium point $E_0(0,0,0)$ of the fractional-order model (2) is locally asymptotically stable provided that $d>r$.*

Proof The characteristic matrix of the fractional-order system is

$$L(E_0) = \begin{pmatrix} r-d & \frac{\rho}{\sigma} & 0 \\ 0 & -\delta-\frac{\rho}{\sigma} & 0 \\ 0 & 0 & -m \end{pmatrix}.$$

The associated eigenvalues of the $L(E_0)$ are $\alpha_1 = r - d, \alpha_2 = -\delta - \frac{\rho}{\sigma}$ and $\alpha_3 = -m$. So, by using Matignon criteria for stability of fractional-order differential equations we have $|arg(\alpha_{2,3})| = \pi > \frac{\epsilon\pi}{2}$ and $|arg(\alpha_1) = \pi > \frac{\epsilon\pi}{2}$, provided that $d > r$ and $0 < \epsilon < 1$. Hence, the trivial equilibrium point E_0 is local asymptotically stable if the natural death rate of the susceptible prey is largest than the intrinsic growth rate. □

Biologically the above result is highly significant because if $d < r$ then both the species will disappear from the system. The analysis shows that the instability of the trivial equilibrium point cannot be changed using the memory effect. If the trivial equilibrium point becomes unstable then the predator-free equilibrium point will generate. In the next theorem, we shall investigate the stability of this equilibrium point.

Another important result is that the system may experience the transcritical bifurcation at $r = d$ for the memoryless system.

Theorem 4 *The predator-free equilibrium point $E_1(S^*, I^*, 0)$ of the fractional-order model (2) is locally asymptotically stable provided $cpI^* < m$ and $r + \frac{\rho}{\sigma + I^*}(1 - \frac{\sigma}{\sigma + I^*}) < d + \beta I^*$.*

Proof Here the characteristic matrix is

$$L(E_1) = \begin{pmatrix} r-(d+\beta I^*) & -\beta S^* + \frac{\rho\sigma}{(\sigma+I^*)^2} & -rS^*(\frac{1-\eta}{v}) \\ \beta I^* & \beta S^* - \delta - \frac{\rho\sigma}{(\sigma+I^*)^2} & -pI^* \\ 0 & 0 & cpI^* - m \end{pmatrix}$$

One of the eigenvalue $L(E_1)$ is $\alpha_1 = cpI^* - m$ and other two satisfies the following equation

$$\alpha^2 - b_1\alpha + b_2 = 0 \tag{7}$$

where $b_1 = \left(r - (d + \beta I^*) + \beta S^* - \delta - \frac{\rho\sigma}{(\sigma + I^*)^2}\right)$

$b_2 = \left((r - (d + \beta I^*))\left(\beta S^* - \delta - \frac{\rho\sigma}{(\sigma + I^*)^2}\right) - \beta I^* \left(\beta S^* - \delta - \frac{\rho\sigma}{(\sigma + I^*)^2}\right)\right)$. It is obvious that $|arg(\alpha_1)| = \pi > \frac{\epsilon\pi}{2}$ provided that $cpI^* < m$. On the other hand, other two roots $\alpha_{2,3}$ have negative real part if roots of (7) have negative real part, which will occur if $b_1 < 0, b_2 > 0$ and so, $|arg(\alpha_{2,3})| > \frac{\epsilon\pi}{2}$. □

Finally, stability of interior equilibrium point $E^*(S^*, I^*, y^*)$ is investigated. The characteristic equation corresponding to the Jacobian matrix $L(E^*)$ is given by:

$$\alpha^3 + R_1\alpha^2 + R_2\alpha + R_3 = 0 \tag{8}$$

where $R_1 = -(l_{11} + l_{22} + l_{33}), R_2 = l_{11}(l_{22} + l_{33}) + l_{22}l_{33} - l_{23}l_{32} - l_{12}l_{21}, R_3 = l_{11}l_{23}l_{32} + l_{12}l_{21}l_{33} - l_{11}l_{22}l_{33} - l_{13}l_{21}l_{32}$.

The local stability of the E^* depends on values of R_1, R_2 and R_3. Using Routh–Hurwitz criterion, the sign of real part of the equations can be easily determine (Ahmed et al. 2006). The equation (8) has all negative real roots if $R_1 > 0, R_2 > 0, R_3 > 0$ and $R_1R_2 > R_3$ then $m(\epsilon) < 0$ but if $R_1R_2 \leq R_3$ then the stability–instability can be easily controlled using the memory effect. Using the results as stated in Ahmed et al. (2006)

Theorem 5 *The local stability of persistence equilibrium point E^* is determined if one of the following is hold*:

1. $Q(R_1, R_2, R_3) > 0$, $R_1 >$, $R_3 > 0$ and $R_1R_2 > R_3$.
2. $Q(R_1, R_2, R_3) < 0, R_1 >, R_2 > 0$ and $R_1R_2 = R_3$.

where $0 < \epsilon < 1$ and $Q(R_1, R_2, R_3)$ is the discriminant of (8) which as follows:

$$Q(R_1, R_2, R_3) = 18R_1R_2R_3 + (R_1R_2)^2 - 4R_3R_1^3 - 4R_2^3 - 27R_3^2. \tag{9}$$

Next, we shall investigate the existence of Hopf bifurcation of the fractional-order system considering the memory parameter (ϵ) as the bifurcation parameter. It is obvious that if the conditions stated in Theorem 5 are not satisfied then the system losses stability through the generation of periodic solutions. Since if $R_i > 0, i = 1, 2, 3$ and $R_1R_2 - R_3 < 0$ then roots of the equation (8) have negative real part and two will be complex conjugate with positive real part. In this situation suppose the roots are $\lambda_1 = -\alpha_1$ and $\lambda_{2,3} = \alpha_2 \pm \beta_2$ with $\alpha_{1,2}, \beta_2 > 0$ then $m_1(\epsilon) < 0$ and $m_{2,3}(\epsilon) = \frac{\pi\epsilon}{2} - arg(\lambda_{2,3}) = \frac{\pi\epsilon}{2} - tan^{-1}(\beta_2/\alpha_2)$. From the expression $m_{2,3}$ one can obtain a fixed value $\epsilon^{[H]} = \frac{2}{\pi} tan^{-1}(\beta_2/\alpha_2)$ such that $m_{2,3}(\epsilon) <=> 0$ accordingly as $\epsilon <=> \epsilon^{[H]}$. Thus the system will experiences Hopf bifurcation when it crosses the critical value $\epsilon = \epsilon^{[H]}$ as the transversality condition $\frac{d(m_{2,3}(\epsilon))}{d\epsilon}|_{\epsilon=\epsilon^{[H]}} = \frac{\pi}{2} \neq 0$ is

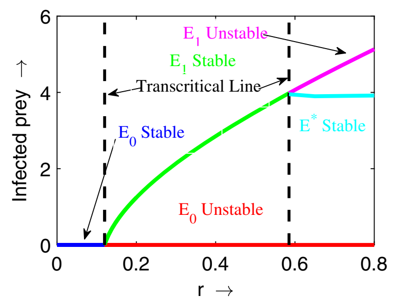


Fig. 1 Complete bifurcation diagram with respect to r and other parameters are given in Table 1

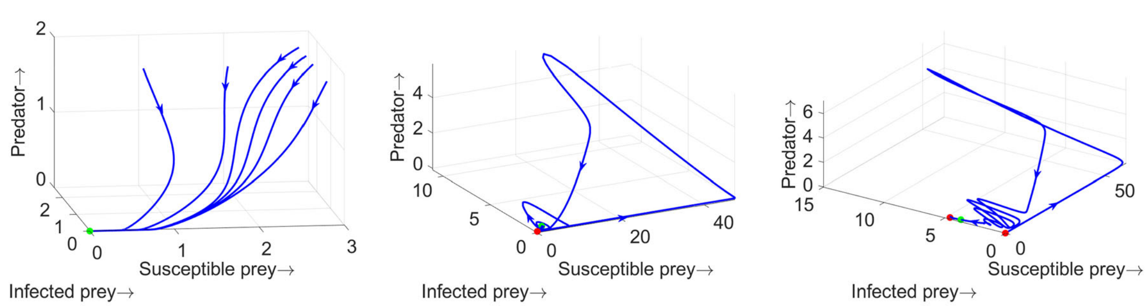
satisfied. These discussions can be summarized in terms of the following theorem:

Theorem 6 *Interior equilibrium point of the system* (2)*will experience Hopf bifurcation for* $R_1 > 0$, $R_2 > 0$, $R_3 > 0$ *and* $R_1R_2 > R_3$ *when the memory parameter* (ϵ) *will cross the critical value* $\epsilon = \epsilon^{[H]}$.

4 Numerical Simulation

In this section, we shall numerically verify the effect of memory parameter (ϵ) and other model parameters on the model dynamics. For this purpose, we have taken the model parameters as shown in Table 1 and different values of $0<\epsilon\leq 1$. To study the dynamics of the ordinary differential equation model, we have drawn a complete bifurcation diagram considering r as the bifurcation diagram (see Fig. 1). It is clear from the figure that the system experiences transcritical bifurcation two times, one for generation of planer equilibrium point E_1 and another for generation of E^* equilibrium point through stability exchange of E_0, E_1, respectively. The full phase portrait corresponding to Fig. 1 is presented in Fig. 2. It is clear from Fig. 2a for lower values of $r(r<0.128)$ all the species go to extinction. Due to the extinction of all species, this situation is harmful to the ecological system. Now, increasing the value of r $(0.128<r<0.585)$, we observe that one stable predator-free equilibrium point arises and E_0 exchange its stability (see Fig. 2b). The situation is also harmful biologically since only prey survives. Again, increasing the value of $r(r>0.585)$, the stable interior equilibrium point E^* generates with E_1 exchange its stability (see Fig. 2c). All the species in the ecosystem survive simultaneously in this situation, which is biologically significant. For these values of the parameters, the memory effect will not affect the stability of the equilibrium points only the time of reaching time to the equilibrium point will increase. To find the periodic solutions of the system, we enhance the prey birth rate as well as the maximum medical resource to $r=0.72$ and $\rho=1.9$; then, we observed that the interior equilibrium point becomes an unstable spiral (stable limit cycle) from stable spiral (see Fig 3).

To study the memory effect, we have drawn phase portraits for different values of memory parameter ϵ (see Fig 3e). Figure 3e shows the limit cycle becomes smaller with the decrease of ϵ. We have verified that the system losses the periodicity and becomes stable for $\epsilon<0.88$ (see Fig. 4). Thus, the memory effect is stabilizing the system when the system is showing unstable behavior. Now, we increased the maximum medical resource ρ to 2.9 form 1.9 and we observed that the system gives two periodic solutions (see Fig. 5). Now, we introduce the memory effect by changing the parameter ϵ. Figure 5e shows, firstly, the two limit cycle collide on a single limit cycle with the decrease of ϵ, and the limit cycle becomes smaller, and finally, the system becomes stable for $\epsilon<0.7$ (see Fig. 6). Thus, the memory effect stabilizes the system. Again, we increased the maximum medical resource ρ to 3.9 from 2.9 and found that the system displayed chaotic behavior (see Fig. 7). Now, we introduce the memory effect changing the parameter ϵ, and the system goes to stable mode from the earlier process but here system is stable for very low values of ϵ (see Fig. 8). Based on the above discussion, a low memory effect will stabilize a system when it shows a periodic solution, but a strong memory effect will stabilize a system when it displays two periods or chaos. Biologically this result is highly sensitive because the system with chaotic behavior (extinction of species) can be stabilized



(a) $r=0.112$ **(b)** $r=0.14$ **(c)** $r=0.72$

Fig. 2 Full phase portrait of the considered system for different value of r and other parameters are given in Table 1

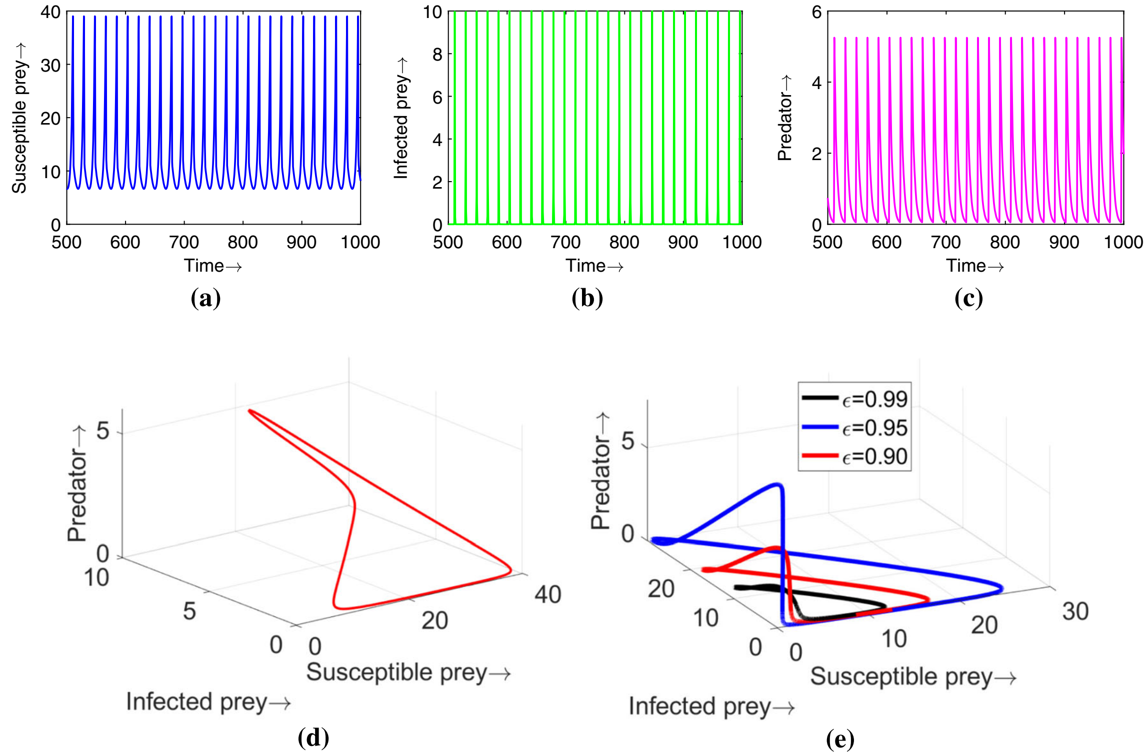
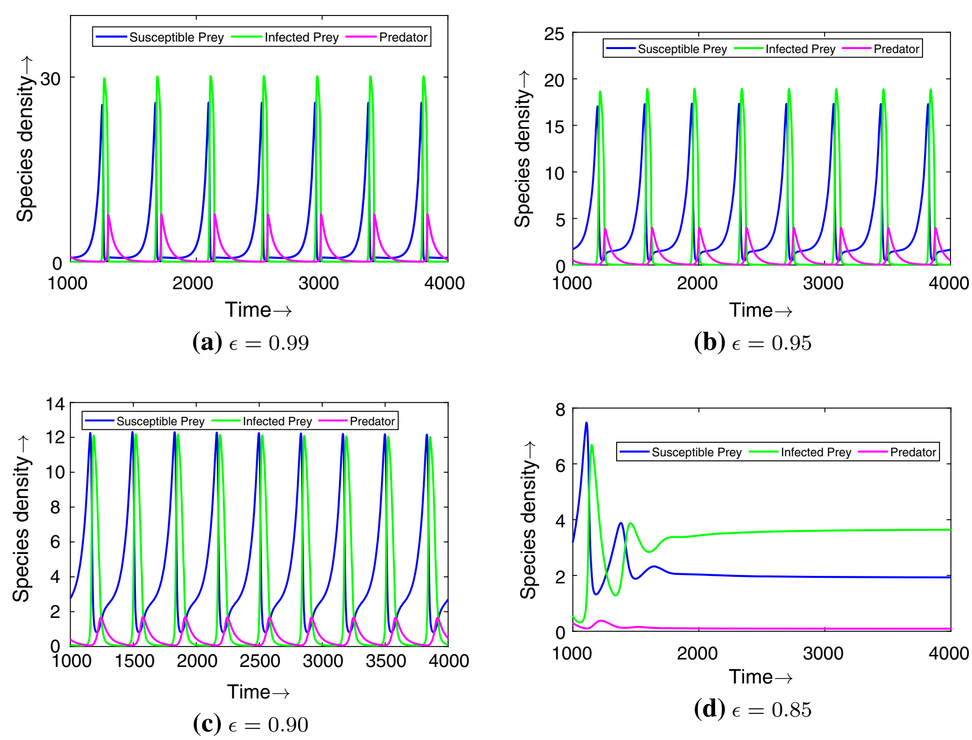


Fig. 3 Time series and phase portrait for $r = 0.72$, $\rho = 1.9$ and other parameters are given in Table 1

Fig. 4 Time series corresponding to the phase portrait Fig. 3(e) for different values of memory parameter (ϵ)



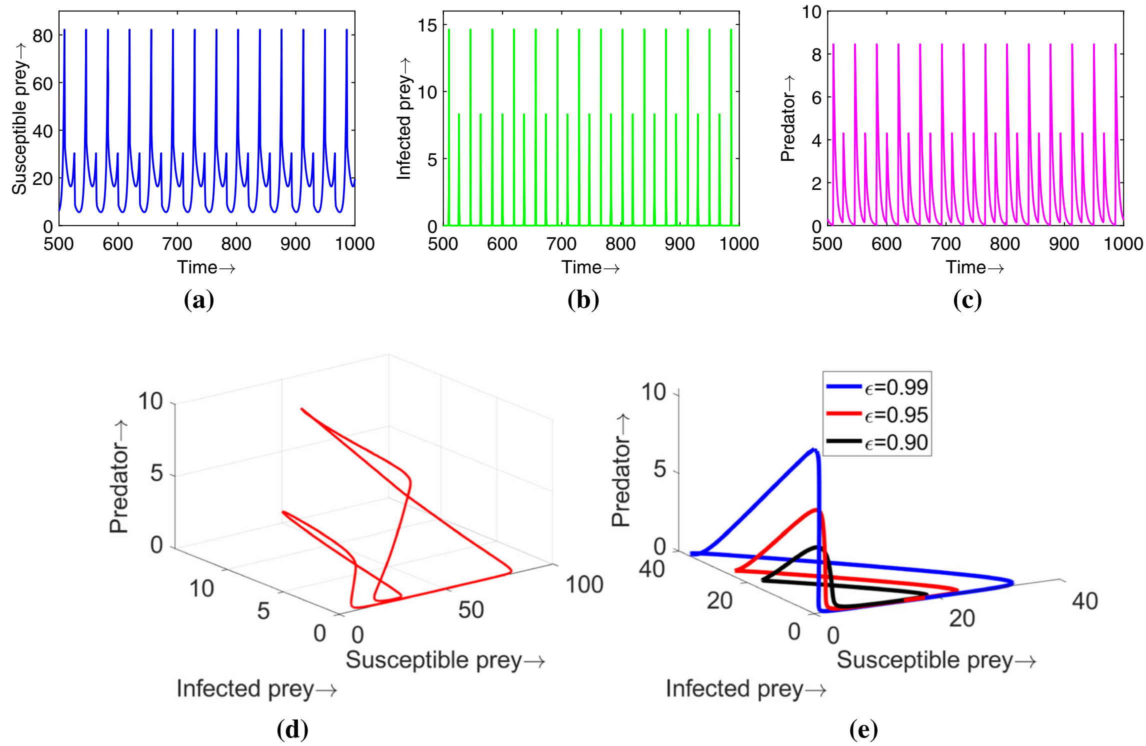
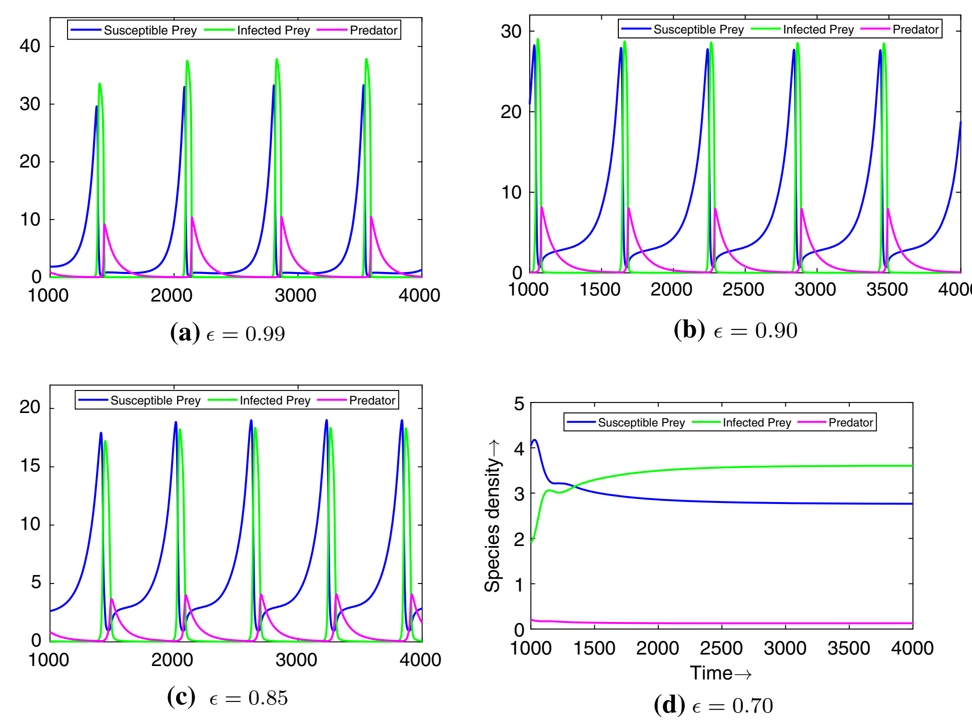


Fig. 5 Time series and phase portrait for $r = 0.72, \rho = 3.9$ other parameters are given in Table 1

Fig. 6 Time series corresponding to the phase portrait Fig. 5(e) for different values of memory parameter (ϵ)



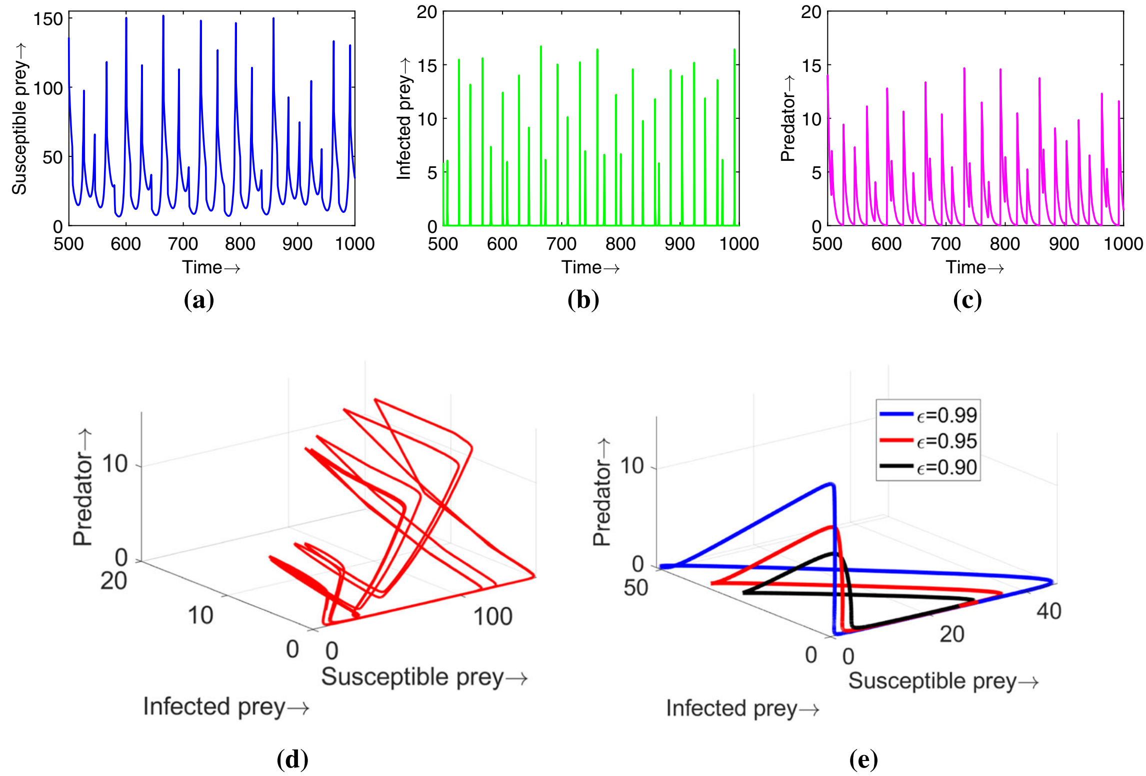
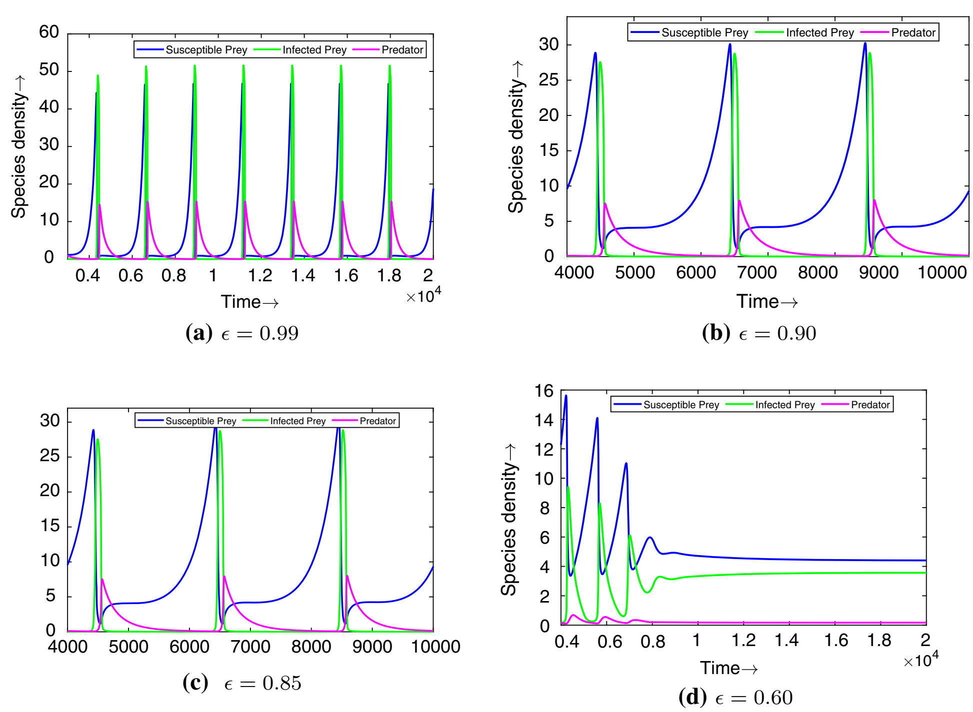


Fig. 7 Time series and phase portrait for $r = 0.72, \rho = 3.9$ other parameters are given in Table 1

Fig. 8 Time series corresponding to the phase portrait Fig. 7(e) for different values of memory parameter (ϵ)



by introducing a high memory effect, i.e., previous memory of the species.

5 Conclusions

In our study, we examined the effect of fear, treatment, and cooperation on assessing an eco-epidemiological model with memory dependence. We have studied the existence, positivity, and boundedness of the solutions of the fractional-order system. The ordinary system experiences transcritical bifurcation considering the prey birth rate as the bifurcation parameter. Transcritical bifurcation is not directly affected by the memory effect. In the presence of the memory effect, the fractional-order system undergoes Hopf bifurcation. The ordinary system exhibits chaotic, period-doubling, or periodic solutions, and memory effects can stabilize these solutions. The numerical results of the simulation show that if the equilibrium point is an unstable node or saddle, the memory effect cannot change its behavior. However, the memory effect can stabilize the system when it is an unstable spiral (including limit cycles or chaotic behavior). The oscillatory behavior (like one or two periodic or chaotic) can be changed to a stable spiral, increasing the memory effect. Fear of predators, hunting cooperation, and treatment of infected prey with memory effects all play a crucial role in preserving biodiversity.

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Data availability All data generated or analyzed during this study are included in this article.

Declarations

Conflict of interest There is no conflict of interest in this research work as declared by authors.

Ethical approval This study does not contain any animal model/experiment and hence does not require any ethical approval.

References

Adnan Thirthar A (2020) Stability and bifurcation of an SIS epidemic model with saturated incidence rate and treatment function. 15(2):129–146

Agnihotri K, Juneja N (2015) An eco-epidemic model with disease in both prey and predator. IJAEEE 4:50–54

Ahmed E, El-Sayed AMA, El-Saka HA (2006) On some Routh-Hurwitz conditions for fractional order differential equations and their applications in Lorenz, Rössler, Chua and Chen systems. Phys Lett A 358(1):1–4

Al-Khaled K, Alquran M (2014) An approximate solution for a fractional model of generalized Harry Dym equation. Math Sci 8(4):125–130

Andrew J, Miguel A, Bret D (2016) The negative effects of pathogen-infected prey on predators: a meta-analysis. Nordic Soc Oikos 000(001–007):2016. https://doi.org/10.1111/oik.03458

Boudaoui A, El Hadj MY, Hammouch Z, Ullah S (2021) A fractional-order model describing the dynamics of the novel coronavirus (COVID-19) with nonsingular kernel. Chaos Solitons Fractals 146:110859

Boukhouima A, Hattaf K, Yousfi N (2017) Dynamics of a fractional order HIV infection model with specific functional response and cure rate. Int J Differ Equ

Chakraborty K, Das S, Kar TK (2011) Optimal control of effort of a stage structured prey-predator fishery model with harvesting. Nonlinear Anal Real World Appl 12(6):3452–3467

Chattopadhyay J, Ghosal G, Chaudhuri KS (1999) Nonselective harvesting of a prey-predator community with infected prey. Korean J Comput Appl Math 6(3):601–616

Chattopadhyay J, Sarkar RR, Ghosal G (2002) Removal of infected prey prevent limit cycle oscillations in an infected prey-predator system-a mathematical study. Ecol Model 156(2–3):113–121

Choi SK, Kang B, Koo N (2014) Stability for Caputo fractional differential systems. In: Abstract and applied analysis, Vol 2014. Hindawi

Ciancio A, Ciancio V, d'Onofrio A, Flora BFF (2022) A fractional model of complex permittivity of conductor media with relaxation: theory vs. experiments. Fractal Fract 6(7):390

Danane J, Allali K, Hammouch Z, Nisar KS (2021) Mathematical analysis and simulation of a stochastic COVID-19 Lévy jump model with isolation strategy. Results Phys 23:103994

Danane J, Hammouch Z, Allali K, Rashid S, Singh J (2021) A fractional-order model of coronavirus disease 2019 (COVID-19) with governmental action and individual reaction. Math Methods Appl Sci

Das S (2008) Functional fractional calculus for system identification and controls. Springer, New York

Das T, Ghosh U, Sarkar S, Das S (2019) Higher-dimensional fractional time-independent Schrödinger equation via fractional derivative with generalised pseudoharmonic potential. Pramana 93(5):1–9

Djilali S, Ghanbari B (2021) The influence of an infectious disease on a prey-predator model equipped with a fractional-order derivative. Adv Differ Equ 2021(1):1–16

El-Misiery AEM, Ahmed E (2006) On a fractional model for earthquakes. Appl Math Comput 178(2):207–211

El-Sayed A (1996) Fractional-order diffusion-wave equation. Int J Theor Phys 35(2):311–322

Gao W, Baskonus HM (2022) Deeper investigation of modified epidemiological computer virus model containing the Caputo operator. Chaos Solitons Fractals 158:112050

Ghosh U, Pal S, Banerjee M (2021) Memory effect on Bazykin's prey-predator model: stability and bifurcation analysis. Chaos Solitons Fractals 143:110531

Hama MF, Rasul RR, Hammouch Z, Rasul KA, Danane J (2022) Analysis of a stochastic SEIS epidemic model with the standard Brownian motion and Lévy jump. Results Physi 37:105477

Hegazi AS, Ahmed E, Matouk AE (2013) On chaos control and synchronization of the commensurate fractional order Liu system. Commun Nonlinear Sci Numer Simul 18(5):1193–1202

Hethcote HW, Wang W, Han L, Ma Z (2004) A predator-prey model with infected prey. Theor Popul Biol 66(3):259–68

Hsieh YH, Hsiao CK (2008) Predator-prey model with disease infection in both populations. Math Med Biol J IMA 25(3):247–266

Juneja N, Agnihotri K (2018) Conservation of a predator species in SIS prey-predator system using optimal taxation policy. Chaos Solitons Fractals 116:86–94
Juneja N, Agnihotri K (2018) Global stability of harvested prey-predator model with infection in predator species. In Information and Decision Sciences (pp 559-568). Springer, Singapore
Kilbas AA, Srivastava HM, Trujillo JJ (2006) Theory and applications of fractional differential equations, vol 204. Elsevier
Kumar A, Prakash A, Mehmet Baskonus H (2022) The epidemic COVID-19 model via Caputo–Fabrizio fractional operator. Waves in Random and Complex Media, pp 1-15
Li Y, Chen Y, Podlubny I (2009) Mittag-Leffler stability of fractional order nonlinear dynamic systems. Automatica 45(8):1965–1969
Li HL, Jiang YL, Wang ZL, Hu C (2015) Global stability problem for feedback control systems of impulsive fractional differential equations on networks. Neurocomputing 161:155–161
Li HL, Zhang L, Hu C, Jiang YL, Teng Z (2017) Dynamical analysis of a fractional-order predator-prey model incorporating a prey refuge. J Appl Math Comput 54(1):435–449
Meng XY, Qin NN, Huo HF (2018) Dynamics analysis of a predator-prey system with harvesting prey and disease in prey species. J Biol Dyn 12(1):342–374
Mondal B, Ghosh U, Rahman MS, Saha P, Sarkar S (2022) Studies of different types of bifurcations analyses of an imprecise two species food chain model with fear effect and non-linear harvesting. Math Comput Simul 192:111–135
Mondal B, Roy S, Ghosh U, Tiwari PK (2022) A systematic study of autonomous and nonautonomous predator-prey models for the combined effects of fear, refuge, cooperation and harvesting. Eur Physi J Plus 137(6):724
Mondal B, Sarkar S, Ghosh U (2022) Complex dynamics of a generalist predator-prey model with hunting cooperation in predator. Eur Phys J Plus 137(1):1–21
Mortoja G, Panja P, Mondal K (2018) Dynamics of a predator-prey model with nonlinear incidence rate, Crowley-Martin type functional response and disease in prey population. Ecol Genet Gen 10:100035
Moustafa M, Mohd MH, Ismail AI, Abdullah FA (2020) Dynamical analysis of a fractional-order eco-epidemiological model with disease in prey population. Adv Differ Equ 2020(1):1–24
Moustafa M, Abdullah FA, Shafie S (2022) Dynamical behavior of a fractional-order prey-predator model with infection and harvesting. J Appl Math Comput, pp 1-18
Mukherjee D (2010) Hopf bifurcation in an eco-epidemic model. Appl Math Comput 217(5):2118–2124
Mukherjee D (2020) Study of fear mechanism in predator-prey system in the presence of competitor for the prey. Ecol Genet Genom 15:100052
Pakhira R, Sarkar S, Ghosh U (2020) Study of memory effect in an inventory model for deteriorating items with partial backlogging. Comput Ind Eng 148:106705
Petráš I (2011) Fractional-order nonlinear systems: modeling, analysis and simulation. Springer Science & Business Media
Podlubny I (1998) Fractional differential equations: an introduction to fractional derivatives, fractional differential equations, to methods of their solution and some of their applications. Elsevier
Rana S, Samanta S, Bhattacharya S (2016) The interplay of Allee effect in an eco-epidemiological system with disease in predator population. Bull Calcutta Math Soc 108:103–122
Romain E, Damian T, Nicolas T, (2020) Infectious diseases and meat production. Nature Public Health Emergency Collection
Saeedian M, Khalighi M, Azimi-Tafreshi N, Jafari GR, Ausloos M (2017) Memory effects on epidemic evolution: the susceptible-infected-recovered epidemic model. Phys Rev E 95(2):022409
Sarkar K, Khajanchi S (2020) Impact of fear effect on the growth of prey in a predator-prey interaction model. Ecol Complex 42:100826
Shaikh AA, Das H, Ali N (2021) Complex dynamics of an eco-epidemic system with disease in prey species. Int J Bifurc Chaos 31(03):2150046
Sinha S, Misra OP, Dhar J (2010) Modelling a predator-prey system with infected prey in polluted environment. Appl Math Model 34(7):1861–1872
Sk N, Pal S (2022) Dynamics of an infected prey-generalist predator system with the effects of fear, refuge and harvesting: deterministic and stochastic approach. Eur Phys J Plus 137(1):138
Tanriverdi T, Baskonus HM, Mahmud AA, Muhamad KA (2021) Explicit solution of fractional order atmosphere-soil-land plant carbon cycle system. Ecol Complex 48:100966
Van Seventer JM, Hochberg NS (2017) Principles of infectious diseases: transmission, diagnosis, prevention, and control. International encyclopedia of public health, p 22
Wang X, Zou X (2017) Modeling the fear effect in predator-prey interactions with adaptive avoidance of predators. Bull Math Biol 79(6):1325–1359
Wang X, Zanette L, Zou X (2016) Modelling the fear effect in predator-prey interactions. J Math Biol 73(5):1179–1204
Wang Z, Xie Y, Lu J, Li Y (2019) Stability and bifurcation of a delayed generalized fractional-order prey-predator model with interspecific competition. Appl Math Comput 347:360–369
Yousef A, Thirthar AA, Alaoui AL, Panja P, Abdeljawad T (2022) The hunting cooperation of a predator under two prey's competition and fear-effect in the prey-predator fractional-order model. AIMS Math 7(4):5463–5479
Zamir M, Nadeem F, Abdeljawad T, Hammouch Z (2021) Threshold condition and non pharmaceutical interventions's control strategies for elimination of COVID-19. Results Phys 20:103698
Zanette LY, White AF, Allen MC, Clinchy M (2011) Perceived predation risk reduces the number of offspring songbirds produce per year. Science 334(6061):1398–1401
Zhang H, Cai Y, Fu S, Wang W (2019) Impact of the fear effect in a prey-predator model incorporating a prey refuge. Appl Math Comput 356:328–337