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A new approach for estimating variance of a population employing information obtained from a stratified random sampling

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ABSTRACT

In this article, we suggest an enhanced estimator for the estimation of finite population variance using twofold auxiliary variable under stratified random sampling. The numerical expressions for the bias and MSE are determined up to the first order of approximation. In order to effectively validate the theoretical findings, three actual data sets are included. Additionally, the application of the suggested estimators is demonstrated using a simulation study. Results of an empirical comparison among the suggested and existing estimators were investigated. To determine how good the suggested estimator, in comparison to the preliminary estimators, the MSE criterion is used. The suggested estimator has a smaller MSE and better PRE than existing estimators, according to numerical results utilizing actual data sets and a simulation analysis.

1. Introduction

It is well acknowledged in the theory of survey sampling, that the efficiency of estimators may increase by properly utilizing the auxiliary information. To boost the productivity of estimators with minimal resources, surveys are employed to acquire evidence on population characteristics. This is accomplished by surveying a representative sample of the population. There are many alternative ways that are based on obtaining auxiliary information that can be found in the literature. Traditional estimators provide reliable results for unknown parameters. In some circumstances, simple random sampling accomplishes better results when the population of consideration is homogenous. However, when the population of consideration is varied, stratified random sampling is preferred over simple random sampling. To increase an estimator precision, we need to use a sampling technique that can reduce population heterogeneity. A stratified sampling method is one of these sampling techniques if the population is heterogeneous with regard to the

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attribute being studied. In stratified sampling, the entire population is distributed into a numeral of distinct, non-overlapping classes or subclasses known as strata. Although each of these groups is entirely homogeneous, a sample is occupied from each stratum separately. Because it has been considered that the fundamental purpose of stratification is to raise the accuracy in determining the population mean, the majority of recent studies on stratified random sampling seem to have been carried out within its limitations. Almost each aspect of human existence has some degree of variation. The laws of nature dictate that no two things or individuals can be identical. To prescribe appropriately, a doctor, for instance, needs to have an in-depth understanding of how blood pressure, temperature, and pulse rate vary in different people. Moreover, an agriculturist needs to be aware of the various climate conditions that affect the environment in order to maximize crop yield. The necessity to estimate the population variance of the research variable arises in a wide variety of contexts. In areas like agriculture, health, ecology, and production where we deal with populations that are expected to be asymmetric, the variance estimate for a population is essential. In our daily lives, variations can be observed everywhere, including in economic, genetic, and environmental studies. Because this is a natural law, there should never be two identical sorts of things or people. Through variance estimate, the presence of population variability can be determined and used to inform future surveys, forecasts, or estimates of sample sizes, among other things. There are many authors who have usually challenged the difficulty of measuring the population variance, including [3–10,12–14,16–19,21–23,26,28,29], and [31–40]. In Table 8, we give the conditional values using simulation study. In Table 9, the percentage relative efficiency using simulation study are given.

As was indicated above, a large number of authors have made contributions to the estimation of population variance. In this article, we suggest a more precise estimator and show how auxiliary variables are essential when estimating population variance using a stratified random. A comparison of estimators is done with existing estimators in terms of minimum mean square error. Through the use of the actual data and a simulation study, the application of the propose estimator is highlighted. Our proposed estimator provides a novel and valuable contribution to the field of population variance estimation, and we believe it has the potential to be useful in a wide range of applications.

2. Material and methods

Let a population $\Psi = \{\Psi_1, \Psi_2, \ldots, \Psi_N\}$ of size N, are divided into Ψ strata of size N_h $(h = 1, 2, \ldots, \Psi)$, such that $\sum_{h=1}^{\intercal} N_h = N$. Let y, x and r_x take the values of y_{hi}, x_{hi} and r_{xhi}.

$s_{yh}^2 = \frac{1}{n_h-1}\sum_{i=1}^{n_h}(y_{hi} - \overline{y}_h)^2$, $s_{xh}^2 = \frac{1}{n_h-1}\sum_{i=1}^{n_h}(x_{hi} - \overline{x}_h)^2$, $s_{rxh}^2 = \frac{1}{n_h-1}\sum_{i=1}^{n_h}(r_{xhi} - \overline{r}_{xh})^2$, are the sample variances, compatible with population variances of y, x and r_x, in the h^{th} stratum.

$S_{yh}^2 = \frac{1}{N_h-1}\sum_{i=1}^{N_h}(y_{hi} - \overline{Y}_h)^2$, $S_{xh}^2 = \frac{1}{N_h-1}\sum_{i=1}^{N_h}(x_{hi} - \overline{X}_h)^2$, $S_{rxh}^2 = \frac{1}{N_h-1}\sum_{i=1}^{N_h}(r_{xhi} - \overline{R}_{xh})^2$ be the conforming populace variances of Y, X and R_x.

$$\text{Let}\xi_{oh} = \frac{s_{yh}^2 - S_{yh}^2}{S_{yh}^2}, \xi_{1h} = \frac{s_{xh}^2 - S_{xh}^2}{S_{xh}^2}, \text{and}\xi_{2h} = \frac{s_{rxh}^2 - S_{rxh}^2}{S_{rxh}^2},$$

$$\left[E\left(\xi_{oh}^2\right) = \Lambda_h \lambda_{400h}^*, E\left(\xi_{1h}^2\right) = \Lambda_h \lambda_{040h}^*, E\left(\xi_{2h}^2\right) = \Lambda_h \lambda_{004h}^*\right],$$

$$\left[E(\xi_{oh}\xi_{1h}) = \Lambda_h \lambda_{220h}^*, E(\xi_{0h}\xi_{2h}) = \Lambda_h \lambda_{202h}^*, E(\xi_{1h}\xi_{2h}) = \Lambda_h \lambda_{022h}^*\right],$$

$$\lambda_{tpsh} = \frac{\upsilon_{tpsh}}{\upsilon_{200h}^{\frac{t}{2}}\upsilon_{020h}^{\frac{p}{2}}\upsilon_{002h}^{\frac{s}{2}}}, \upsilon_{tpsh} = \frac{\sum_{i=1}^{N}(y_i - \overline{y}_h)^t (x_i - \overline{x}_h)^p (r_{xhi} - \overline{R}_{xh})^s}{N_h - 1}.$$

$$\lambda_{tpsh}^* = \left(\lambda_{tpsh} - 1\right), \Lambda_h = \left(\frac{1}{n_h} - \frac{1}{N_h}\right),$$

where t, p and s be the positive, and λ_{tpsh} is the moment ratio.

Additionally, the work is designed as:

Section 2 consists of methods and materials. Section 3, included some existing counterparts. Section 4 discusses the suggested estimator and its properties. Efficiency conditions are formulated, and a comparison is made between the suggested estimator and the standard and other considered current estimators in Section 5. In Section 6, a numerical study is drawn. A simulation is obtainable to verify the efficacy of the suggested estimator in section 7. Section 8, present argumentation of the manuscript.

3. Literature review

In this part, we discussed about some existing latest estimators of the variance using stratified random sampling.

(i) The typical unbiased estimator, given in equation (1):

$$\tau_{1(st)} = \sum_{h=1}^{l} W_h^4 \lambda_h S_{yh}^2, \tag{1}$$

The variance of $\tau_{1(st)}$, given in equation (2):

$$\mathrm{Var}\left(\tau_{1(st)}\right) = \sum_{h=1}^{l} W_h^4 \lambda_h^3 S_{yh}^4 \lambda_{400h}^{*} \tag{2}$$

(ii) The [15] recommended ratio estimator, given in equation (3):

$$\tau_{2(st)} = \sum_{h=1}^{l} W_h^4 \lambda_h \, s_{yh}^2 \left(\frac{S_{xh}^2}{s_{xh}^2}\right) \tag{3}$$

The bias of $\tau_{2(st)}$:
Bias $(\tau_{2(st)}) = \sum_{h=1}^{l} W_h^2 \lambda_h^2 S_{yh}^2 \{\lambda_{040h}^{*} - \lambda_{220h}^{*}\}$,
The MSE of $\tau_{2(st)}$, given in equation (4):

$$\mathrm{MSE}\left(\tau_{2(st)}\right) \cong \sum_{h=1}^{l} W_h^4 \lambda_h^3 S_{yh}^4 \{\lambda_{400h}^{*} + \lambda_{040h}^{*} - 2\lambda_{220h}^{*}\} \tag{4}$$

(iii) The difference estimator, given in equation (5):

$$\tau_{3(st)} = \sum_{h=1}^{l} W_h^2 \lambda_h \left\{ s_{yh}^2 + U_{11h}\left(S_{xh}^2 - s_{xh}^2\right) \right\}, \tag{5}$$

where U_{11h} is constant,

$$U_{11h} = \left(\frac{S_{yh}^2 \lambda_{220h}^{*}}{S_{xh}^2 \lambda_{040h}^{*}}\right)$$

The variance using U_{11h}, given in equation (6):

$$\mathrm{Var}\left(\tau_{3(st)}\right) = \sum_{h=1}^{l} W_h^4 \lambda_h^3 S_{yh}^4 \lambda_{400h}^{*} \left(1 - \rho_h^2\right), \tag{6}$$

where.

$\rho_h = \frac{\lambda_{220h}^{*}}{\sqrt{\lambda_{400h}^{*}}\sqrt{\lambda_{040h}^{*}}}$.

(iv) [24] suggested the estimator, given in equation (7):

$$\tau_{4(st)} = \sum_{h=1}^{l} W_h^2 \lambda_h \left\{ U_{12h} s_{yh}^2 + U_{13h}\left(S_{xh}^2 - s_{xh}^2\right) \right\}, \tag{7}$$

as U_{12h} and U_{13h} are the constants:
$U_{12h} = \frac{\lambda_{040h}^{*}}{\lambda_h[\lambda_{400h}^{*}\lambda_{040h}^{*} - \lambda_{220h}^{*2}] + \lambda_{040h}^{*}}$,
$U_{13h} = \frac{S_{yh}^2 \, \lambda_{220h}^{*}}{S_{xh}^2[\lambda_h(\lambda_{400h}^{*}\lambda_{040h}^{*} - \lambda_{220h}^{*2}) + \lambda_{040h}^{*}]}$,
The properties at U_{12h} and U_{13h}, are given by:
Bias $(\tau_{4(st)}) = \sum_{h=1}^{l} W_h^2 \lambda_h^2 S_{yh}^2 \left\{ \frac{\lambda_{040h}^{*}}{\lambda_h\{\lambda_{400h}^{*}\lambda_{040h}^{*} - \lambda_{220h}^{*2}\} + \lambda_{040h}^{*}} - 1 \right\}$,
The MSE of $\tau_{4(st)}$, given in equation (8):

$$\mathrm{MSE}\left(\tau_{4(st)}\right) = \sum_{h=1}^{l} W_h^4 \lambda_h^3 S_{yh}^4 \left(\frac{\lambda_{400h}^{*}\lambda_{040h}^{*} - \lambda_{220h}^{*2}}{\lambda_h\{\lambda_{400h}^{*}\lambda_{040h}^{*} - \lambda_{220h}^{*2}\} + \lambda_{040h}^{*}}\right) \tag{8}$$

(v) The [27] recommended estimator, given in equation (9):

$$Ꚍ_{5(st)} = \sum_{h=1}^{l} W_h^2 \Lambda_h^2 S_{yh}^2 \exp\left\{\frac{S_{xh}^2 - s_{xh}^2}{S_{xh}^2 + s_{xh}^2}\right\} \tag{9}$$

The bias of $Ꚍ_{5(st)}$, given in equation (10):

$$\text{Bias}\left(Ꚍ_{5(st)}\right) = \sum_{h=1}^{l} W_h^2 \Lambda_h^2 S_{yh}^2 \left\{\frac{3}{8}\lambda_{040h}^* - \frac{1}{2}\lambda_{220h}^*\right\}, \tag{10}$$

The MSE of $Ꚍ_{5(st)}$, given in equation (11):

$$\text{MSE}\left(Ꚍ_{5(st)}\right) = \sum_{h=1}^{l} W_h^4 \Lambda_h^3 S_{yh}^4 \left[\lambda_{400h}^* + \frac{1}{4}\lambda_{040h}^* - \lambda_{220h}^*\right] \tag{11}$$

(vi) [25] suggested regression-in-exponential estimator, given in equation (12):

$$Ꚍ_{6(st)} = \sum_{h=1}^{l} W_h^2 \Lambda_h \left[U_{14h} s_{yh}^2 + U_{15h}\left(S_{xh}^2 - s_{xh}^2\right)\right] \exp\left(\frac{S_{xh}^2 - s_{xh}^2}{S_{xh}^2 + s_{xh}^2}\right) \tag{12}$$

As U_{14h} and U_{15h} are constants.

$$U_{14h} = \frac{\Lambda_h \lambda_{040h}^*}{8}\left[\frac{8 - \Lambda_h \lambda_{040h}^*}{\Lambda_h\left(\lambda_{040h}^* + \lambda_{004h}^* \lambda_{040h}^* - \lambda_{220h}^{*2}\right)}\right],$$

and

$$U_{15h} = \frac{S_{yh}^2}{8 S_{xh}^2}\left[\frac{-3\lambda_{040h}^* + 8\lambda_{220h}^{*2} - \lambda_{040h}^* \lambda_{220h}^{*2} + 4\lambda_{004h}^* \lambda_{040h}^* - 4\lambda_{220h}^{*2}}{\left(\lambda_{040h}^* + \lambda_{004h}^* \lambda_{040h}^* - \lambda_{220h}^{*2}\right)}\right]$$

$$Bias\ \left(Ꚍ_{6(st)}\right) = \sum_{h=1}^{l} W_h^2 \Lambda_h^2 \left(-S_{yh}^2 + Ш_{3h} S_{yh}^2 \left\{1 + \frac{3}{8}\lambda_{040h}^* - \frac{1}{2}\lambda_{220h}^*\right\} + \frac{1}{2} Ш_{4h} S_{xh}^2 \Lambda_h \lambda_{040h}^*\right),$$

and.

The MSE of $Ꚍ_{6(st)}$, given in equation (13):

$$MSE\ \left(Ꚍ_{6(st)}\right) = \sum_{h=1}^{l} W_h^4 \Lambda_h^3 S_{yh}^4 \left[\frac{\left[64\left\{\lambda_{400h}^* \lambda_{040h}^* - \lambda_{220h}^{*2}\right\} - \Lambda_h \lambda_{040h}^{*3} - 16\Lambda_h \lambda_{040h}^* \left\{\lambda_{400h}^* \lambda_{040h}^* - \lambda_{220h}^{*2}\right\}\right]}{64\left\{\Lambda_h\left(\lambda_{400h}^* \lambda_{040h}^* - \lambda_{220h}^{*2}\right) + \lambda_{040h}^*\right\}}\right] \tag{13}$$

(vii) The [1] recommended estimator, given in equation (14):

$$Ꚍ_{7(st)} = \sum_{h=1}^{l} W_h^2 \Lambda_h \left[U_{16h} s_{yh}^2 + U_{17h}\left(S_{xh}^2 - s_{xh}^2\right) + U_{18h}\left(S_{rxh}^2 - s_{rxh}^2\right)\right] \exp\left(\frac{S_{xh}^2 - s_{xh}^2}{S_{xh}^2 + s_{xh}^2}\right) \tag{14}$$

$\text{Bias}\ \left(Ꚍ_{7(st)}\right) \cong \sum_{h=1}^{l} W_h^2 \Lambda_h^2 \left[-S_{yh}^2 + U_{16h} S_{yh}^2 \left(1 + \frac{3}{8}\lambda_{040h}^* - \frac{1}{2}\lambda_{220h}^*\right) + \frac{1}{2}\Lambda_h\left(U_{17h} S_{xh}^2 \lambda_{040h}^* + U_{18h} S_{rxh}^2 \lambda_{022h}^*\right)\right],$

The MSE using U_{16h}, U_{17h} and U_{18h}, given in equation (15):

$$\text{MSE}\ \left(Ꚍ_{7(st)}\right) = \sum_{h=1}^{l} W_h^4 \Lambda_h^3 S_{yh}^4 \left[\frac{64\left(\psi_h^* + 1\right) - \Lambda_h\left(\frac{\lambda_{040h}^{*2}}{\lambda_{400h}^*}\right) - 16\Lambda_h \lambda_{040h}^*\left(\psi_h^* + 1\right)}{64\left(\frac{1}{\lambda_{400h}^*} + \Lambda_h\left(\psi_h^* + 1\right)\right)}\right], \tag{15}$$

where.

$\psi_h^* = \frac{2\lambda_{220h}^* \lambda_{202h}^* \lambda_{022h}^* - \lambda_{040h}^* \lambda_{202h}^{*2} - \lambda_{004h}^* \lambda_{220h}^{*2}}{\lambda_{400h}^*\left(\lambda_{040h}^* \lambda_{004h}^* - \lambda_{022h}^{*2}\right)}.$

4. Suggested estimator

It is well-established that exponential-type estimators outperform comparable to usual estimators in terms of minimum MSE when there is a linear relationship among the auxiliary and study variables. The issue arises because it is difficult to obtain values for the parameters which prefer exponential type estimators over more traditional estimators. It would appear that developing novel estimators that are more efficient than the standard exponential and standard estimators is the best strategy for doing so. Taking motivation from Ref. [2], we suggested the following enhanced estimator for estimation of population variance under stratified random sampling using twofold auxiliary variable. We believe our suggested estimator has the potential to be useful in a variety of contexts, and it represents a fresh and valuable contribution to the field of population variance estimation, which given in equation (16):

$$\mho_{prop(st)} = \sum_{h=1}^{l} W_h^2 \Lambda_h \left[U_{19h} s_{yh}^2 + U_{20h}\left(S_{xh}^2 - s_{xh}^2\right) \exp\left(\frac{S_{xh}^2 - s_{xh}^2}{S_{xh}^2 + s_{xh}^2}\right) U_{21h}\left(S_{rxh}^2 - s_{rxh}^2\right) \exp\left(\frac{S_{rxh}^2 - s_{rxh}^2}{S_{rxh}^2 + s_{rxh}^2}\right)\right] \tag{16}$$

Where U_{19h}, U_{20h} and U_{21h} are constants.

After simplifying the above equation, we get equation (17):

$$\mho_{prop(st)} = \sum_{h=1}^{l} W_h^2 \Lambda_h \left[U_{19h} S_{yh}^2 (1 + \xi_{oh}) - U_{20h} S_{xh}^2 \xi_{1h}\left(1 - \frac{1}{2}\xi_{1h} + \frac{3}{8}\xi_{1h}^2\right) - U_{21h} S_{rxh}^2 \xi_{2h}\left(1 - \frac{1}{2}\xi_{2h} + \frac{3}{8}\xi_{2h}^2\right) \right. \tag{17}$$

$$\mho_{prop(st)} - \sum_{h=1}^{l} W_h^2 \Lambda_h^2 S_{yh}^2 = \sum_{h=1}^{l} W_h^2 \Lambda_h \left[(U_{19h}-1) S_{yh}^2 + U_{19h} S_{yh}^2 \xi_{oh} - U_{20h} S_{xh}^2 \left(\xi_{1h} - \frac{1}{2}\xi_{1h}^2\right) - U_{21h} S_{rxh}^2 \left(\xi_{2h} - \frac{1}{2}\xi_{2h}^2\right)\right]$$

$$\mathrm{Bias}\left(\mho_{prop(st)}\right) = \sum_{h=1}^{l} W_h^2 \Lambda_h^2 \left[(U_{19h}-1)\, S_{yh}^2 + \frac{1}{2}\, U_{20h} S_{xh}^2 \Lambda_h\, \lambda_{040}^* + \frac{1}{2}\, U_{21h} S_{rxh}^2 \Lambda_h\, \lambda_{004}^* \right]$$

Simplify the equation before expression of bias, we get equation (18):

$$\mathrm{MSE}\left(\mho_{prop(st)}\right) = (U_{19h}-1)^2 S_{yh}^4 + U_{19}^2\, S_{yh}^4 \xi_{0h}^2 + U_{20}^2\, S_{xh}^4 \xi_{1h}^2 + U_{21}^2\, S_{rxh}^4 \xi_{2h}^2 + 2U_{19h}(U_{19h}-1) S_{yh}^4\, \xi_{oh} - 2(U_{19h}-1) U_{20h} S_{yh}^2 S_{xh}^2 \left(\xi_{1h} - \frac{1}{2}\xi_{1h}^2\right)$$
$$- 2(U_{19h}-1) U_{21h} S_{yh}^2 S_{rxh}^2 \left(\xi_{2h} - \frac{1}{2}\xi_{2h}^2\right) - 2U_{19h} U_{20h}\, S_{yh}^2 S_{xh}^2 (\xi_{0h}\xi_{1h}) - 2U_{19h} U_{21h} S_{yh}^2 S_{rxh}^2 (\xi_0 \xi_{2h}) + 2U_{20h} U_{21h}\, S_{yh}^2 S_{rxh}^2 (\xi_{1h}\xi_{2h})$$

$$\mathrm{MSE}\Big(\mho_{prop(st)} = (U_{19h}-1)^2 S_{yh}^4 + U_{19}^2\, S_{yh}^4\, \Lambda_h \lambda_{400h}^* + U_{20}^2\, S_{xh}^4 \Lambda_h \lambda_{040h}^* + U_{21}^2 S_{rxh}^4\, \Lambda_h \lambda_{004}^* + 2(U_{19h}U_{20h} - U_{20h}) S_{yh}^2 S_{xh}^2 \frac{\Lambda_h\, \lambda_{040h}^*}{2} + 2(U_{19h}U_{21h} -$$
$$U_{21h}) S_{yh}^2 S_{rxh}^2 \frac{\Lambda_h\, \lambda_{004}^*}{2} - 2U_{19h} U_{20h} S_{yh}^2 S_{xh}^2\, \Lambda_h\, \lambda_{220}^* - 2U_{19h} U_{21h} S_{yh}^2 S_{rxh}^2\, \Lambda_h\, \lambda_{202h}^* + 2U_{20h} U_{21h}\, S_{yh}^2 S_{rxh}^4\, \Lambda_h \lambda_{022h}^* \tag{18}$$

The optimum values of U_{19h}, U_{20h} and U_{21h}:

$$U_{19h} = \frac{\Lambda_h \left[N_{1h}/\lambda_{400h}^* - 2(y_{11h} + N_{11h})\right] - 4/\lambda_{400h}^*}{\Lambda_h \left(4\{(\gamma_h^* + 1) + (y_{11h} + N_{11h})\} - N_{11h}/\lambda_{400h}^* \right) + 4/\lambda_{400h}^*},$$

$$U_{20h} = \frac{S_{yh}^2 \left(2\Lambda_h \lambda_{400h}^* \lambda_{004h}^* (\lambda_{040h}^* - \lambda_{022h}^*) + \Lambda_h\, \lambda_{040h}^* \lambda_{202h}^* (\lambda_{004h}^* - 2\lambda_{202h}^*) + \Lambda_h \lambda_{004h}^* \lambda_{220h}^* (-\lambda_{004h}^* + 2\lambda_{202h}^*) + 4(\lambda_{004h}^* \lambda_{220h}^* - \lambda_{202h}^* \lambda_{022h}^*)\right)}{S_{xh}^2 \lambda_{400h}^* (\lambda_{040h}^* \lambda_{004h}^* - \lambda_{022h}^{*2}) \left\{ \Lambda_h \left(4\{(\gamma_h^* + 1) + (y_{11h} + N_{11h})\} - N_{1h}/\lambda_{400h}^* \right) + 4/\lambda_{400h}^* \right\}}$$

$$U_{21h} = \frac{S_{yh}^2 \left(2\Lambda_h \lambda_{400h}^* \lambda_{040h}^* (\lambda_{004h}^* - \lambda_{022h}^*) + \Lambda_h\, \lambda_{040h}^* \lambda_{202h}^* (-\lambda_{040h}^* + 2\lambda_{220h}^*) + \Lambda_h\, \lambda_{004h}^* \lambda_{220h}^* (\lambda_{040h}^* - 2\lambda_{220h}^*) + 4(\lambda_{040h}^* \lambda_{202h}^* - \lambda_{220h}^* \lambda_{022h}^*)\right)}{S_{rxh}^2 \lambda_{400h}^* (\lambda_{040h}^* \lambda_{004h}^* - \lambda_{022h}^{*2}) \left\{ \Lambda_h \left(4\{(\gamma_h^* + 1) + (y_{11h} + N_{11h})\} - N_{1h}/\lambda_{400h}^* \right) + N_{1h}/\lambda_{400h}^* \right\}}$$

where,

$y_{1h} = \frac{\lambda_{202h}^{*2} \lambda_{040h}^{*2} + \lambda_{004h}^{*2} \lambda_{220h}^{*2} - 2\lambda_{040h}^* \lambda_{004h}^* \lambda_{220h}^* \lambda_{202h}^*}{\lambda_{400h(\lambda_{040h}^* \lambda_{004h}^* - \lambda_{022h}^{*2})}^*}$, $N_{1h} = \frac{\lambda_{040h}^* \lambda_{004h}^* (\lambda_{040h}^* + \lambda_{004h}^* - 2\lambda_{022h}^*)}{\lambda_{040h}^* \lambda_{004h}^* - \lambda_{022h}^{*2}}$,

$y_{11h} = \frac{\lambda_{004h}^* \lambda_{220h}^* (\lambda_{040h}^* - \lambda_{022h}^*)}{\lambda_{400h}^* (\lambda_{040h}^* \lambda_{004h}^* - \lambda_{022h}^{*2})}$, $N_{11h} = \frac{\lambda_{040h}^* \lambda_{20h2}^* (\lambda_{004h}^* - \lambda_{022h}^*)}{\lambda_{400h}^* (\lambda_{040h}^* \lambda_{004h}^* - \lambda_{022h}^{*2})}$,

$\gamma_h^* = \frac{2\lambda_{220h}^* \lambda_{202h}^* \lambda_{022h}^* - \lambda_{040h}^* \lambda_{202h}^{*2} - \lambda_{004h}^* \lambda_{220h}^{*2}}{\lambda_{400h}^* (\lambda_{040h}^* \lambda_{004h-}^* \lambda_{022h}^{*2})}$.

The MSE at the ideal values of U_{19h}, U_{20h} and U_{21h}, given in equation (19):

$$\mathrm{MSE}\left(\mho_{prop(st)}\right) = \sum_{h=1}^{l} W_h^4 \Lambda_h^3 S_{yh}^4 \left[\frac{(y_{1h} - N_{1h})\Lambda_h + 4(\gamma_h^* + 1)}{\Lambda_h \left[4\{(\gamma_h^* + 1) + (y_{11h} + N_{11h})\} - N_{1h}/\lambda_{400h}^* \right] + 4/\lambda_{400h}^*} \right] \tag{19}$$

5. Efficiency comparison

The following conditions were generated, when we equate the MSE of the recommended estimator with the existing counterparts.

(i) Using (2) and (19)

$\mathrm{Var}\left(Ʊ_{1(st)}\right)\text{-MSE}\left(Ʊ_{prop(st)}\right) > 0$

Simplify the above expression; we get equation (20):

$$\sum_{h=1}^{l} W_h^4 ʌ_h^3 S_{yh}^4 ƛ^*_{400h}\left(1-\frac{(ƴ_{1h}-Ɲ_{1h})ʌ_h+4\left(Ɣ_h^*+1\right)}{ʌ_h\left[4\{(Ɣ_h^*+1)+(ƴ_{11h}+Ɲ_{11h})\}-Ɲ_{1h}/ƛ^*_{400h}\right]+{}^4/ƛ^*_{400h}}\right) > 0 \tag{20}$$

(ii) Using (4) and (19)

$\text{MSE}\left(Ʊ_{2(st)}\right)\text{-MSE}\left(Ʊ_{prop(st)}\right) > 0$

Simplify the above expression; we get equation (21):

$$\sum_{h=1}^{l} W_h^4 ʌ_h^3 S_{yh}^4\left(\left\{ƛ^*_{400h}+ƛ^*_{040h}-2ƛ^*_{220h}\right\}-\frac{(ƴ_{1h}-Ɲ_{1h})ʌ_h+4\left(Ɣ_h^*+1\right)}{ʌ_h\left[4\{(Ɣ_h^*+1)+(ƴ_{11h}+Ɲ_{11h})\}-Ɲ_{1h}/ƛ^*_{400h}\right]+{}^4/ƛ^*_{400h}}\right) > 0 \tag{21}$$

(iii) By (6) and (19)

$\mathrm{Var}\left(Ʊ_{3(st)}\right)\text{-MSE}\left(Ʊ_{prop(st)}\right) > 0$

Simplify the above expression; we get equation (22):

$$\sum_{h=1}^{l} W_h^4 ʌ_h^3 S_{yh}^4\left(ƛ^*_{400h}\left(1-\rho_h^2\right)-\frac{(ƴ_{1h}-Ɲ_{1h})ʌ_h+4\left(Ɣ_h^*+1\right)}{ʌ_h\left[4\{(Ɣ_h^*+1)+(ƴ_{11h}+Ɲ_{11h})\}-Ɲ_{1h}/ƛ^*_{400h}\right]+{}^4/ƛ^*_{400h}}\right) > 0 \tag{22}$$

(iv) Using (8) and (19)

$\text{MSE}\left(Ʊ_{4(st)}\right)-\text{MSE}\left(Ʊ_{prop(st)}\right) > 0$

Simplify the above expression; we get equation (23):

$$\sum_{h=1}^{l} W_h^4 ʌ_h^3 S_{yh}^4\left(\frac{\left[ƛ^*_{400h}ƛ^*_{040h}-ƛ^{*2}_{220h}\right]}{ʌ_h\left[ƛ^*_{400h}ƛ^*_{040h}-ƛ^{*2}_{220h}\right]+ƛ^*_{040h}}-\frac{(ƴ_{1h}-Ɲ_{1h})ʌ_h+4\left(Ɣ_h^*+1\right)}{ʌ_h\left[4\{(Ɣ_h^*+1)+(ƴ_{11h}+Ɲ_{11h})\}-Ɲ_{1h}/ƛ^*_{400h}\right]+{}^4/ƛ^*_{400h}}\right) > 0 \tag{23}$$

(v) Using (11) and (19)

$\text{MSE}\left(Ʊ_{5(st)}\right)\text{-MSE}\left(Ʊ_{prop(st)}\right) > 0$

Simplify the above expression; we get equation (24):

$$\sum_{h=1}^{l} W_h^4 ʌ_h^3 S_{yh}^4\left(ƛ^*_{400h}+\frac{1}{4}ƛ^*_{040h}-ƛ^*_{220h}-\frac{(ƴ_{1h}-Ɲ_{1h})ʌ_h+4\left(Ɣ_h^*+1\right)}{ʌ_h\left[4\{(Ɣ_h^*+1)+(ƴ_{11h}+Ɲ_{11h})\}-Ɲ_{1h}/ƛ^*_{400h}\right]+{}^4/ƛ^*_{400h}}\right) > 0 \tag{24}$$

(vi) Using (13) and (19)

$\text{MSE}\left(Ʊ_{6(st)}\right)-\text{MSE}\left(Ʊ_{prop(st)}\right) > 0$

Simplify the above expression; we get equation (25):

$$\sum_{h=1}^{l} W_h^4 ʌ_h^3 S_{yh}^4\left(\frac{64\left\{ƛ^*_{400h}ƛ^*_{040h}-ƛ^{*2}_{220h}\right\}-ʌ_hƛ^{*3}_{040h}-16ʌ_hƛ^*_{040h}\left\{ƛ^*_{400h}ƛ^*_{040h}-ƛ^{*2}_{220h}\right\}}{64\left\{ʌ_h\left(ƛ^*_{400h}ƛ^*_{040h}-ƛ^{*2}_{220h}\right)+ƛ^*_{040h}\right\}}-\frac{(ƴ_{1h}-Ɲ_{1h})ʌ_h+4\left(Ɣ_h^*+1\right)}{ʌ_h\left[4\{(Ɣ_h^*+1)+(ƴ_{11h}+Ɲ_{11h})\}-Ɲ_{1h}/ƛ^*_{400h}\right]+{}^4/ƛ^*_{400h}}\right) > 0 \tag{25}$$

Table 1
Data description using Population-I.

H	N_h	n_h	λ_h	$\overline{Y}_h$	$\overline{X}_h$	$\overline{R}_{xh}$
1	19	10	0.04736	2975.26300	64.89400	10
2	32	16	0.03125	4653.28100	140.12500	16
3	14	7	0.07142	6553.28600	402. 07100	7.5
4	15	8	0.05833	782.66700	764.26600	8
λ^*_{400h}	λ^*_{040h}	λ^*_{004h}	λ^*_{220h}	λ^*_{202h}	λ^*_{022h}	S^2_{yh}
2.27121	0.44742	0.68580	0.45340	0.77870	0.52660	583977.5
0.55407	2.09213	0.74210	0.73860	0.57890	0.97010	456563.3
1.23406	0.61043	0.66000	0.80100	0.78490	0.62330	195208.8
1.21886	0.89154	0.67000	1.01320	0.69650	0.68570	437923.5

Population-II [**Source**: [30]]:
Y: Region under wheat in 1974.
X: area under wheat in 1973.
R_x = rank of the X.

Table 2
Data description using Population-II.

h	N_h	n_h	λ_h	$\overline{Y}_h$	$\overline{X}_h$	$\overline{R}_{xh}$
1	9	3	0.2222	253.0000	253.4000	5
2	10	3	0.2333	213.5000	226.8000	5.5
3	15	4	0.1833	157.9000	170.2000	8
λ^*_{400h}	λ^*_{040h}	λ^*_{004h}	λ^*_{220h}	λ^*_{202h}	λ^*_{022h}	S^2_{yh}
1.9286	1.0700	0.5700	1.3800	0.7700	0.6400	31978.2500
0.5110	0.4200	0.5900	0.4200	0.4100	0.4000	37629.3900
1.2000	1.4200	0.6700	1.2800	0.6500	0.7600	6893.0670

Y: FFMC index from FWI system.
X: ISI index from FWI system.
R_x = rank of the X variable.
Population-III: [**Source**: [11]]

Table 3
Data description using Population-III.

h	N_h	n_h	λ_h	$\overline{Y}_h$	$\overline{X}_h$	$\overline{R}_{xh}$
1	50	8	0.0911	9.0680	91.8500	25.5000
2	72	13	0.0630	9.6361	90.7194	36.5000
3	55	10	0.0818	8.5963	90.6709	28.0000
4	90	16	0.0513	8.6122	89.9477	45.5000
5	30	6	0.1333	10.1333	91.8866	15.5000
6	84	15	0.5476	7.9726	89.7904	42.5000
7	60	11	0.0742	10.1783	91.0033	30.5000
λ^*_{400h}	λ^*_{040h}	λ^*_{004h}	λ^*_{220h}	λ^*_{202h}	λ^*_{022h}	S^2_{yh}
2.2235	3.3482	0.7579	0.6485	0.5121	1.1293	3.0478
22.8872	1.8083	0.7732	2.3165	1.5761	1.0616	43.6210
19.0805	2.2830	0.7646	2.2207	1.4512	1.1015	14.4800
52.4910	1.9438	0.7803	2.4870	1.6933	1.1097	73.9340
3.2831	1.5836	0.7364	0.9112	0.8332	0.9180	5.0598
16.6615	3.7723	0.7803	2.1354	1.4839	1.2556	40.0390
4.1525	24.7158	0.7688	0.5705	1.0920	1.5483	8.7932

Table 4
MSE results using actual data.

Estimators	Population-I	Population-II	Population-III
$\mho_{1(st)}$	425574.0	188538.0	0.08164951
$\mho_{2(st)}$	406392.3	26463.3	0.00076849
$\mho_{3(st)}$	262236.5	21238.9	0.00076070
$\mho_{4(st)}$	245259.3	20723.0	0.00024080
$\mho_{5(st)}$	305432.6	74105.5	0.0007805
$\mho_{6(st)}$	242985.2	19524.5	0.0002336
$\mho_{7(st)}$	117496.9	18484.1	0.00023110
$\mho_{prop(st)}$	111406.8	9792.9	0.00022400

Table 5
Conditional values using actual data.

Conditions	Population-I	Population-II	Population-III
1	314167.2	178745.00	0.08142551
2	294985.5	16670.41	0.00054449
3	150829.7	11445.97	0.0005367
4	133852.5	10930.08	1.68e-05
5	194025.8	64312.58	0.0005565
6	131578.4	9731.62	9.6e-06
7	6090.1	8691.15	7.1e-06

Table 6
PRE results using actual data.

Estimators	Population-I	Population-II	Population-III
$\mho_{1(st)}$	100.000	100.000	100.000
$\mho_{2(st)}$	104.724	712.430	106.288
$\mho_{3(st)}$	162.280	887.700	107.333
$\mho_{4(st)}$	173.523	909.799	339.055
$\mho_{5(st)}$	139.333	254.412	104.611
$\mho_{6(st)}$	175.144	965.644	349.488
$\mho_{7(st)}$	362.200	1020.000	353.266
$\mho_{prop(st)}$	382.000	1925.240	363.300

Table 7
MSE using simulation.

Estimators	I	II	III
$\mho_{1(st)}$	1.428034e-06	5.525052e-06	0.0002381502
$\mho_{2(st)}$	9.009566e-07	1.78917e-06	0.0001354334
$\mho_{3(st)}$	7.428279e-07	1.453957e-06	0.0001101128
$\mho_{4(st)}$	4.783174e-07	8.812802e-07	7.834854e-05
$\mho_{5(st)}$	1.001442e-06	2.392016e-06	0.0001555074
$\mho_{6(st)}$	8.00193e-08	1.486207e-07	1.325342e-05
$\mho_{7(st)}$	1.916723e-08	1.024857e-07	2.751263e-06
$\mho_{prop(st)}$	1.772493e-08	6.207924e-08	2.675845e-06

Table 8
conditional values using simulation.

Conditions	Population-I	Population-II	Population-III
1	1.410309e-06	5.462973e-06	0.0002354744
2	8.832317e-07	1.727091e-06	0.0001327576
3	7.25103e-07	1.391878e-06	0.000107437
4	4.605925e-07	8.19201e-07	7.56727e-05
5	9.837171e-07	2.329937e-06	0.0001528316
6	6.229437e-08	8.654146e-08	1.057758e-05
7	1.4423e-09	4.040646e-08	7.5418e-08

Table 9
PRE using Simulation study.

Estimators	I	II	III
$\text{Ʊ}_{1(st)}$	100.000	100.000	100.000
$\text{Ʊ}_{2(st)}$	158.502	308.805	175.843
$\text{Ʊ}_{3(st)}$	192.243	380.001	216.278
$\text{Ʊ}_{4(st)}$	298.553	626.934	303.962
$\text{Ʊ}_{5(st)}$	142.597	230.978	153.144
$\text{Ʊ}_{6(st)}$	1784.613	3717.553	1796.890
$\text{Ʊ}_{7(st)}$	7450.394	5391.046	8656.030
$\text{Ʊ}_{prop(st)}$	8056.640	8899.999	8900.000

(vii) Using (15) and (19)

$$\text{MSE}\left(\text{Ʊ}_{7(st)}\right) - \text{MSE}\left(\text{Ʊ}_{prop(st)}\right) > 0$$

Simplify the above expression; we get equation (26):

$$\sum_{h=1}^{l} W_h^4 \text{Ʌ}_h^3 S_{yh}^4 \left(\frac{64\left(\psi_h^*+1\right) - \text{Ʌ}_h\left(\frac{\text{ƛ}_{040h}^{*2}}{\text{ƛ}_{400h}^*}\right) - 16\text{Ʌ}_h \text{ƛ}_{040h}^*\left(\psi_h^*+1\right)}{64\left(\frac{1}{\text{ƛ}_{400h}^*} + \text{Ʌ}_h\left(\psi_h^*+1\right)\right)} - \frac{\left(\text{ƴ}_{1h} - \text{Ɲ}_{1h}\right)\text{Ʌ}_h + 4\left(\text{ɣ}_h^*+1\right)}{\text{Ʌ}_h\left[4\{(\text{ɣ}_h^*+1) + (\text{ƴ}_{11h} + \text{Ɲ}_{11h})\} - \text{Ɲ}_{1h}/\text{ƛ}_{400h}^*\right] + 4/\text{ƛ}_{400h}^*} \right) > 0 \tag{26}$$

6. Numerical study

In direction to determine the efficiency of the recommended estimator, we yield a mathematical analysis utilizing three actual data sets. We measure our suggested estimator accomplishes in terms of PRE as compared to other exisitng counterparts. Tables 1–3 list the summary statistics. Tables 4 and 6 provide the MSE and PRE. We applied the following equation to determine the PRE. Table 5 presnts the conditional values using actual data sets. In Table 7 we give the MSE using simulation study.

$$\text{PRE} = \frac{Var\left(\text{Ʊ}_{1(st)}\right)}{MSE\left(\text{Ʊ}_{\text{ɖ}(st)}\right)} \times 100,$$

where (ɖ)= (1, 2, 3, 4, 5, 6, 7, 8, *prop*).

Population-I [**Source**: [20]]:
Y: Manufacture of a workshop,
X: number of workers,
R_x = rank of the X

7. Simulation

To assess the performance of the estimators for variance stratified random sampling, a simulation analysis is carried out. Using the R language program, three populations are produced from a normal distribution. With a limited sample size, the first data set is formed for identical strata, the second for not equal strata, and the last data sets include similar strata.

7.1. Population I

$$N_1 = 1200, N_2 = 1200, N_3 = 1200, N_4 = 1200, N_5 = 1200, N_6 = 1200,$$

$$n_1 = 300, n_2 = 300, n_3 = 300, n_4 = 300, n_5 = 300, n_6 = 300,$$

$$X_1 = \text{rn}(1200, 5, 10), U_1 = X_1 + \text{rn}(1200, 0, 1), Y_1 = U_1 + \text{rn}(1200, 1, 3),$$

$$X_2 = \text{rn}(1200, 4, 8), U_2 = X_2 + \text{rn}(1200, 0, 1), Y_2 = U_2 + \text{rn}(1200, 1, 3),$$

$$X_3 = \text{rn}(1200, 4, 9), U_3 = X_3 + \text{rn}(1200, 0, 1), Y_3 = U_3 + \text{rn}(1200, 1, 3),$$

$$X_4 = \text{rn}(1200, 3, 7), U_4 = X_4 + \text{rn}(1200, 0, 1), Y_5 = U_5 + \text{rn}(1200, 1, 3),$$

$$X_5 = \text{rn}(1200, 3, 8), U_5 = X_5 + \text{rn}(1200, 0, 1), Y_5 = U_5 + \text{rn}(1200, 1, 3),$$

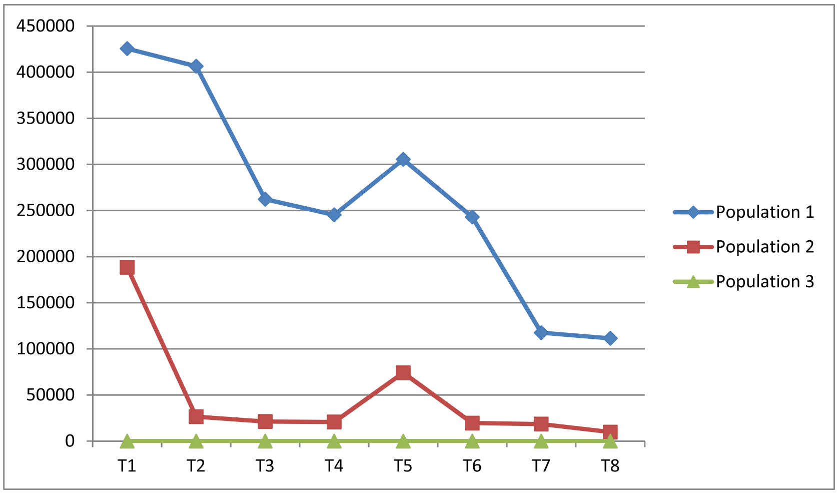

Fig. 1. MSE of the suggested and existing estimators using actual data sets.

$X_6 = \text{rn}(1200, 2, 5), U_6 = X_6 + \text{rn}(1200, 0, 1), Y_6 = U_6 + \text{rn}(1200, 1, 3)$

7.2. Population II

$N_1 = 1500, N_2 = 1000, N_3 = 1600, N_4 = 1200, N_5 = 1400, N_6 = 900,$

$n_1 = 340, n_2 = 260, n_3 = 240, n_4 = 230, n_5 = 200, n_6 = 120,$

$X_1 = \text{rn}(1500, 5, 10), U_1 = X_1 + \text{rn}(1500, 0, 1), Y_1 = U_1 + \text{rn}(1500, 1, 3),$

$X_2 = \text{rn}(1000, 4, 8), U_2 = X_2 + \text{rn}(1000, 0, 1), Y_2 = U_2 + \text{rn}(1000, 1, 3),$

$X_3 = \text{rn}(1600, 4, 9), U_3 = X_3 + \text{rn}(1600, 0, 1), Y_3 = U_3 + \text{rn}(1600, 1, 3),$

$X_4 = \text{rn}(1200, 3, 7), U_4 = X_4 + \text{rn}(1200, 0, 1), Y_5 = U_5 + \text{rn}(1200, 1, 3),$

$X_5 = \text{rn}(1400, 3, 8), U_5 = X_5 + \text{rn}(1400, 0, 1), Y_5 = U_5 + \text{rn}(1400, 1, 3),$

$X_6 = \text{rn}(900, 2, 5), U_6 = X_6 + \text{rn}(900, 0, 1), Y_6 = U_6 + \text{rn}(900, 1, 3)$

7.3. Population III

$N_1 = 400, N_2 = 400, N_3 = 400, N_4 = 400, N_5 = 400, N_6 = 400,$

$n_1 = 80, n_2 = 80, n_3 = 80, n_4 = 80, n_5 = 80, n_6 = 80,$

$X_1 = \text{r}(400, 5, 10), U_1 = X_1 + \text{r}(400, 0, 1), Y_1 = U_1 + \text{r}(400, 1, 3),$

$X_2 = \text{r}(400, 4, 8), U_2 = X_2 + \text{r}(400, 0, 1), Y_2 = U_2 + \text{r}(400, 1, 3),$

$X_3 = \text{r}(400, 4, 9), U_3 = X_3 + \text{r}(400, 0, 1), Y_3 = U_3 + \text{r}(400, 1, 3),$

$X_4 = \text{r}(400, 3, 7), U_4 = X_4 + \text{r}(400, 0, 1), Y_5 = U_5 + \text{r}(400, 1, 3),$

$X_5 = \text{r}(400, 3, 8), U_5 = X_5 + \text{r}(400, 0, 1), Y_5 = U_5 + \text{r}(400, 1, 3),$

$X_6 = \text{r}(400, 2, 5), U_6 = X_6 + \text{r}(400, 0, 1), Y_6 = U_6 + \text{r}(400, 1, 3)$

The PRE is computed as given below:

$$PRE = \frac{Var(\tau_{0(st)})}{MSE(\tau_{u(st)})} \times 100,$$

where $u = 1,2,3,4,5,6,7, prop(st)$.

8. Conclusion

In this study, we have suggested an enhanced estimator for population variance by making use of a twofold auxiliary variable under stratified random sampling. To determine the MSE and PRE of each estimator under the variance stratified sampling, we used three actual data sets. To further demonstrate the effectiveness of the proposed estimator in comparison to its contemporaries, a simulation investigation is also performed. Tables 1–3 contain details of the data based on actual datasets. The outcome in terms of MSE and PRE of simulation and actual data sets will be used for visualization. Fig. 1, shows the MSE of the suggested and existing estimator using actual data sets. Fig. 2, shows the PRE of the existing and suggested estimator using actual data sets. On the same way Fig. 3, shows MSE of the existing and suggested estimator using simulation study. Fig. 4, show PRE of the existing and suggested estimator using simulation study. A bars length is proportional to the estimator's accuracy. When measuring MSE, a narrower line bar indicates a more precise estimate. Alternatively, it is true for PRE; the longer the line bar, the more precise the estimate. Based on the theoretical and numerical investigations, it is observed that the suggested estimator is more efficient than the exisitng counterparts, for all populations which are used here. The current work can be extended to non-response and measurement error. The suggested estimator could also be used to estimate the population variance of a sensitive variable when only non-sensitive supporting data is available.

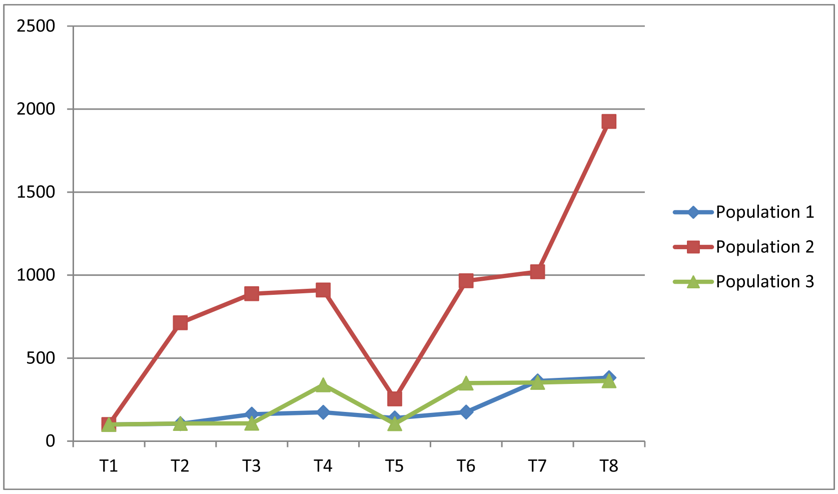

Fig. 2. PRE of the suggested and existing estimators using actual data sets.

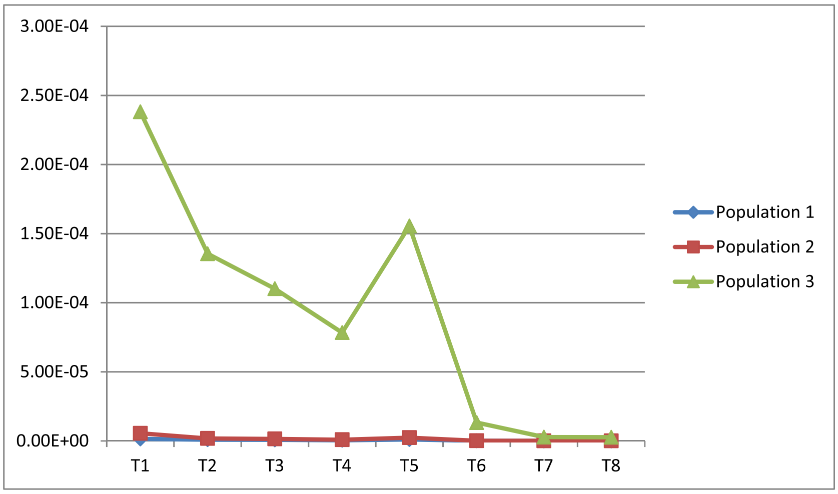

Fig. 3. MSE of the suggested and existing estimators using simulation study.

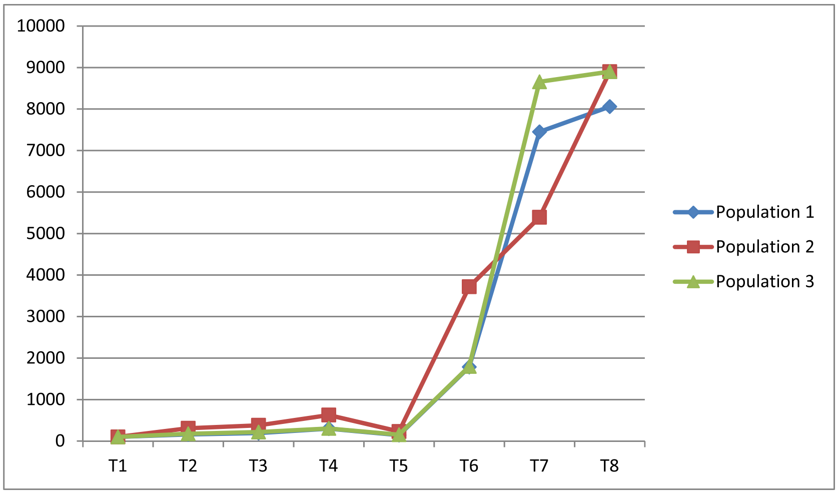

Fig. 4. PRE of the suggested and existing estimators using simulation study.

Data availability statement

Data will be made available on request.

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CRediT authorship contribution statement

Sohaib Ahmad: Writing – original draft. **Aned Al Mutairi:** Data curation. **Said G. Nassr:** Formal analysis. **Hassan Alsuhabi:** Investigation. **Mustafa Kamal:** Validation. **Masood Ur Rehman:** Methodology.

Declaration of competing interest

The authors declare no conflict of interest.

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