

Article Joint Selection of Influential Users and Locations under Target Region in Location-Based Social Networks

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Abstract: Influence Maximization problem, selection of a set of users in a social network to maximize the influence spread, has received ample research attention in the social network analysis domain due to its practical applications. Although the problem has been extensively studied, existing works have neglected the location's popularity and importance along with influential users for product promotion at a particular region in Location-based Social Networks. Real-world marketing companies are more interested in finding suitable locations and influential users in a city to promote their product and attract as many users as possible. In this work, we study the joint selection of influential users and locations within a target region from two complementary perspectives; general and specific location type selection perspectives. The first is to find influential users and locations at a specified region irrespective of location type or category. The second perspective is to recommend locations matching location preference in addition to the target region for product promotion. To address general and specific location recommendations and influential users, we propose heuristic-based methods that effectively find influential users and locations for product promotion. Our experimental results show that it is not always an optimal choice to recommend locations with the highest popularity values, such as ratings, check-ins, and so, which may not be a true indicator of location popularity to be considered for marketing. Our results show that not only influential users are helpful for product promotion, but suitable influential locations can also assist in promoting products in the target region.

Keywords: recommendation system; influence maximization; social network analysis; viral marketing; location-based social networks

1. Introduction

Influence Maximization (IM) problem, which finds k users from a social network to activate a maximum number of users, is a widely studied research topic [1–6]. Domingos and Richardson [7,8] are the first to study the problem as an algorithmic problem and modeled the problem using Markov random fields. The IM problem is formulated as a discrete stochastic optimization problem by Kempe et al. [1] and used two models, i.e., Independent Cascade (IC) model and Linear Threshold model, to describe the influence propagation over social networks.

In addition, researchers have analyzed the location-based social networks (LBSN) [9,10]. LBSN provides a platform to users to "check-in" and share the location information with their online friends. Considering the popularity of LBSNs, the IM problem has also been studied on location-based social networks (LBSN) [11–14]. Li et al. [11] have formulated the location-aware influence maximization problem. Given a query region and users' locations, the problem is to find influential users that could maximize the influence spread in the query region. Bouros et al. [12] have tried to solve a similar problem as proposed in [11] of finding influential users at query region, but he ranked the influential users



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rather than finding a group of users which collectively maximize the influence spread in the query region. Finally, a very recent work by Wang et al. [14] on IM with similar context to our work tries to find regionally influential users considering the distance between seed set and influenced users in a target region. However, none of the previous work has considered a selection of influential users and locations simultaneously for product promotion. Evidently, in a real-world marketing scenario, business companies are as keen on finding suitable venues as selecting a set of key users that could provide maximum profit.

Due to the popularity of smartphones and location-based social networks (LBSNs), users can check-in at locations or venues and share their check-in information with their friends and family. This free word-of-mouth marketing phenomenon facilitates companies to promote their products by exploring users' check-in sharing history to attract more users to visit. However, location popularity values, such as check-ins and ratings, are not always available in each location-based social network, except that we are provided only the user social network and set of locations. Thus, it is challenging to find influential users and locations in the target region simultaneously. Further, it is also not sure that the selected locations based on popularity values are always good choices for product promotion. We discuss this issue using the example in Figure 1. In this example, we have selected the top-20 locations across San Francisco from Gwalla dataset based on the number of users and check-in frequency. It can be observed that some locations have the highest check-ins, but these are visited by only few users (marked in red font). So It can be misleading to consider location popularity values only for product promotion. To address this problem, we find influential users and locations which can provide a solid platform for product promotion irrespective of location popularity values.

#checkins	loc_id	#users	#check	ins	loc_id		#us
1713 41059	b00f964a520850b1fe3	682	1713	41059	b00f964	la520850b1fe	36
919 4bd217	7d046076b055357371	409	919	4bd21	77d0460	76b05535737	1 4
364 4a5824	e5f964a5206db71fe3	210	440	49d01	698f964	a520fd5a1fe3	16
323 4ab595	e1f964a520877520e3	167	364	4a582	4e5f964a	a5206db71fe3	21
440 49d016	98f964a520fd5a1fe3	161	333	4a71e	4cff964a	520ccd91fe3	133
333 4a71e4	cff964a520ccd91fe3 1	133	323	4ab59	5e1f964a	a520877520e	3 1
171 49cc41	3df964a5205a591fe3	123	193	49f796	68df964a	520bf6c1fe3	36
169 430672	80f964a52023271fe3	119	175	4acfad	eef964a	52049d520e3	11
128 42cc70	80f964a520e9251fe3	104	171	49cc41	L3df964a	a5205a591fe3	12
101 40c3b0	00f964a520e7001fe3	82	169	43067	280f964	a52023271fe3	3 11
106 409ad1	80f964a520eef21ee3	82	161	40ca4	780f964a	a52008011fe3	47
123 49fdea1	19f964a5204a6f1fe3	80	153	40b7d	280f964	a52093001fe3	6
100 40943a	00f964a520e5f21ee3	78	150	4c2f9c	1d6f1fef	3b2c94eb3d	1
108 452b81	ddf964a520393b1fe3	74	150	42853	f80f964a	5200c231fe3	49
82 409d748	80f964a520f2f21ee3	73	146	4ab81	d6ff964a	5203d7c20e3	7
102 4249ec	00f964a5208f201fe3	73	143	455f77	abf964a	520903d1fe3	62
89 40bbc70	00f964a520b1001fe3	73	130	4a4aa	5a7f964a	a5200bac1fe3	2
93 4bb9db	c353649c74364e48fb	72	128	42cc70)80f964a	a520e9251fe3	10
107 444df33	33f964a5208d321fe3	72	125	43598	100f964a	a520f7281fe3	7
89 3fd6620	0f964a52041ee1ee3	70	123	49fdea	19f964a	5204a6f1fe3	80

With highest user frequency

With highest checkin frequency

Figure 1. Top-20 locations across San Francisco from Gwalla Dataset.

In short, we answer the following research questions:

- 1. Is the location check-in frequency always an optimal choice for product promotion in LBSN?
- 2. How much percent influenced users belong to the query region, and should we consider influenced users out of the region for product promotion?
- 3. What users and locations to suggest for product promotion matching query region and query topic without considering location popularity information?

To address the aforementioned challenge, we propose two main approaches that select influential users and locations irrespective of location popularity values at the target region. The first approach referred as Region-Aware Influential users and locations selection (IUL), consists of algorithms which select influential users and locations within a target region *R* for product promotion. In-Region (IR Selection) algorithm selects *k* users and *m* locations which are under target region *R*. Region-Free (RF Selection) algorithm considers users who are out of region but connected with users inside target region for finding *k* influential users and *m* influential locations. Greedy-In Region (G-IR Selection) algorithm selects *k*-influential users and *m*-influential locations by considering check-in frequency. It is worth noting that we are not provided the boundary area of the target region except in some LBSNs such as **Foursquare** and so. So we have extracted all the locations which are under a certain distance from the target region. Initially, we find *k* influential users, with highest overall users frequency.

The second approach, named topic-aware influential users and locations selection (t-IUL), selects influential users and locations matching the query region and query topic. The algorithms proposed for this approach are similar to the ones proposed in the first approach but with the additional constraint of a specific type of location selection. To sum it all, we make the following contributions.

- We formally define the influential seed and location selection problem over locationbased social networks.
- We formally define the topic-aware influential seed and location selection problem over location-based social networks.
- We propose heuristic-based algorithms for influential users and location selection simultaneously for product promotion.

The rest of the paper is organized as follows. Section 2 reviews the related work. Section 3 describes the problem of identifying influential users and locations at a query region in LBSNs. We discuss the proposed frameworks in Section 4. Section 5 show our experimental results, followed by discussion and conclusion in Section 6.

2. Literature Review

2.1. Influence Maximization in Social Network

The influence maximization (IM) problem has been extensively studied in literature [1–5,7,8,11,15]. The IM problem was proposed by Domingos et al. in their seminal work [7]. The authors proposed probabilistic methods to address the IM problem. Kempe et al. [1] utilized two models, Independent cascade (IC) and Linear threshold (LT), to solve the IM problem. Futher, he proved the monotonic and the submodular property of the influence spread function and the problem's hardness (i.e. NP-hard). As the problem is NP-hard, the authors proposed the greedy algorithms with the approximation ratio of $(1-\frac{1}{\rho})$. Further, research is carried out to improve the efficiency and scalability of the greedy algorithm while maintaining the approximation ratio. Leskovec et al. [15] proposed a lazy forward approach which achieves 700 times better efficiency performance than the algorithm proposed in [1]. Chen et al. [2] proved that it is #P-hard to compute the influence spread. Thus, many algorithms rely on heuristic strategies to enhance performance. Chen et al. [2] utilized the degree discount heuristic to compute the influence spread for influence maximization problem. Recently, a near-linear time approach is proposed by Borgs et al. [4], which utilizes the Monte-Carlo Simulations to compute the influence spread. Further, Ohsaka et al. [5] proposed a pruning technique for reducing Monte-Carlo simulations time complexity to achieve better results with theoretical guarantees.

2.2. Influence Maximization in Geo-Social Network

With the increase in the use of location-enabled devices, the location factor carries an important role in social network analysis. Zhu et al. [13] consider a geo-social network where each user is associated with multiple check-ins. The authors proposed algorithms to compute the propagation probability distribution based on user check-in history in a promoted location. The works mostly related to our work are [11,12,14]. Li et al. [11] selected a seed set which globally maximizes the influence spread in a query region. Bouros et al. [12]

used the Dijkstra algorithm to compute the regional influence of all users at a specific region and rank the influential users accordingly. Wang et al. [14], have considered a distance between seed set and promoted location for assigning edge weight. They have assigned different weights to users based on the distance between a user and the promoted location.

All of the aforementioned works have considered the location popularity and tried to find the seed set, which can globally maximize the influence spread. Our work mostly resembles to the work proposed by Bouros et al. [12], but we recommend influential locations along with an influential seed set for product promotion at a target region in location-based social networks.

2.3. Location Recommendation in LBSNs

Researchers have investigated the location-based social networks for analysis as well [9,10,16]. Authors in [9] analyzed the LBSN, its properties, and possible issues related to it. While, Ahmed et al. [16] discussed the importance of different centrality measures for identifying a node in geo-based social networks. Besides, Venue or location recommendation matching user query in LBSN has been well studied in literature [17–22]. Ye et al. [21] used collaborative filtering recommendation system by developing *friend-based collaborative filtering* system to recommend locations to users. Levandoski et al. [18] used the user ratings to locations for venue recommendation. Yuan et al. [17] proposed a *time-aware POI recommendation problem*, which suggests a list of venues for a user to visit at a given time. A recent work by Zhang et al. [23] recommends locations based on users preferences learning from the community-contributed data. The work done by Chang et al. [23] is similar to our *t-IUL* problem, but we recommend venues to merchants which satisfy merchant's query topic and target region by considering influential users in mind.

3. Problem Formulation

A LBSN $\langle G, L \rangle$ consists of a social network G = (U, E), where U is the set of users, E is the set of edges and the set L represents the set of locations with users check-in history as $\{(u, l_1, t)\}$, where (u, l_1, t) represents a check-in record of user u at location l_1 at time t, and $l_1 \in L$. A location l_1 consists of latitude, longitude and location id. For *t*-*IUL* problem, we assume that each location has category type which defines the type of location.

Definition 1. *Influential Users: Influential users are a set of key users selected by companies that can propagate companies product information to a large number of users in a social network.*

Definition 2. *Influential Locations: Influential locations are locations that help companies spread their product information to a large volume of users even without investing in seed users set.*

Now, we formally define the joint selection of influential users and locations problem (IUL) as following.

Definition 3. *IUL Problem: Given a LBSN* $\langle G, L \rangle$, *a query region R, constants k and m, the IUL problem is to select a set of users S, where S* \in *U*, |S| = k, *and set of locations V, where V* \in *L*, |V| = m, which both can help promote products at target region R and attract as many users as possible to adopt products.

We illustrate the importance of the joint selection of influential users and locations in the following example.

Example 1. Figure 2 shows locations with users check-in frequency. We suppose all locations lie under a target query region. Under this query region, top-2 locations (m = 2) would be locations A and location C having the most users and highest check-ins, respectively. Although top-2 locations are locations A and C, in fact, we do not know the actual visited users and check-ins



at the target region. So, it is not easy to find influential users and locations under a query region for product promotion.

Figure 2. Example Figure.

Next, we define the Topic-Aware selection of influential users and locations (*t-IUL*) problem as follows.

Definition 4. *t*-IUL Problem: Given a LBSN $\langle G, L \rangle$, a target region R, a constant m and a location category C, the t-IUL Problem is to select a set of seeds S, where $S \in U$, |S| = k, and set of locations V, where $V \in L$, |V| = m, matching a query category C, which can help companies to promote product at target region R and spread their product information to large number of users in a target region.

4. Methodology

Section 4.1 discusses the approaches for solving *IUL* problem followed by strategies for solving *t-IUL* problem in Section 4.2.

4.1. IUL Approaches

Given a LBSN $\langle G, L \rangle$, a target region *R*, constants *k* and *m*, the problem is to select *k* influential users and *m* influential locations that can help to promote products. Initially, we extract all the locations which are within target query region *R* followed by social sub-graph construction of users who have checked-in at extracted locations. Then influential *k*-seeds are found by using degree discount proposed by Chen et al. [2]. It should be noted that we can find influential *k*-seeds using other approaches too, such as proposed by Ohsaka et al. [5], and Borgs et al. [4]. Finally, we select all locations visited by *k*-seeds and select top-*m* locations from these locations based on highest number of users visiting those locations.

We propose three heuristic-based methods, In-Region Selection *IR Selection*, Region-Free Selection *RF Selection* and Greedy In-Region Selection *G-IR*, to address the *IUL* problem.

IR Selection. In this method, we consider all those users who have checked-in at least once within a query region boundary for influential seeds as well as the targeted users who can be influenced by product promotion. This approach is suitable when the marketing

company wants to target the users who have frequently visited the places within a query region boundary and have a social connection with each other.

The Algorithm 1 selects the influential users and locations within a target query region *R* using the IR Selection approach.

Algorithm 1 IR influential users and locations selection.

Input: LBSN: $\langle G, L \rangle$; Query: (R, k, m)Output: S-seeds, V-locations 1: $S \leftarrow \emptyset, V \leftarrow \emptyset$ 2: $\theta \leftarrow 150$ miles \triangleright The boundary area for target region *R* 3: $L' \leftarrow \emptyset$ ▷ Location set 4: *I* ← 10,000 ▷ Number of Simulations 5: for all locations $l \in L$ do if distance(l, R) $\leq \theta$ then 6: $L' = L' \cup \{l\}$ 7: 8: Extract Users checked in at L'9: $G' = (U', E') \mid \{U' \subseteq U \land U' \exists L'\}$ > Subgraph of users who checked in at locations lying under target region 10: $S \leftarrow DegreeDiscount(G', k, I)$ 11: $L'' \subseteq L' \mid \{S \exists L''\}$ ▷ extract locations visited by top S seed users 12: **for** $i \leftarrow 1 : m$ **do** for all locations $l \in L''$ do 13: $L_{UF} = ComputeLocationUserFreq(l)$ 14: $V = V \cup \{\arg\max_{l \in L_{UF} \setminus V} \{L_{UF}\}\}$ 15: L'' = L'' - V16: 17: Return S, V

In this Algorithm 1, we consider users within a target query region *R*. Line 2 computes the boundary area of a target region *R*. Here, we assume the boundary area of each target city as 150 miles. It means we extract all locations which lie under 150 miles of a target region/city (i.e., New York). L' and *I* represent the locations within target region and number of simulations to run for influence at line 3 and 5 respectively. We extract all the locations which lie under target region, a social sub-graph, G', is constructed of users who checked-in at those locations, i.e., line 8 selects users who checked in at locations L' which lie under a target query region. We extract top-k seeds at line 9 using the degree discount method. Line 10 of this algorithm extracts the locations, L'', visited by top-k influential seeds. From line 14 to 15, we compute the location's user visiting frequency, L'', and each location is stored along with the number of users (not only top-k seeds but all users) who checked-in at these locations. Top-m locations are selected by extracting locations with highest user frequency from L'' at line 17 to 19.

RF Selection: In the region-free selection approach, users who have checked-in at least once at query region *R* boundary are selected along with the inclusion of their children who can be within target region or out of target region boundary. As we are not provided the user's home location, so, we consider users having social links with users inside the target region boundary. This approach is beneficial when we do not know the actual home location of users but can target them if they have any social ties with users inside the target region.

The Algorithm 2 discusses the selection of influential users and locations using the RF selection approach. In this algorithm, line 1 extracts all those users who have checked-in at least once within a target query region, *R*, boundary first. Next, we add the child nodes of users who were selected earlier (at line 1) from lines 3 to 6. Further, we select the *k*-influential users and *m* influential locations from line 7 to line 13 in the same way as discussed in Algorithm 1.

Algorithm 2 RF Selection of influential users and locations.

Input: LBSN: $\langle G, L \rangle$; Query: (R, k, m)Output: S-seeds, V-locations Initialization and locations extraction, L', are same as in Algorithm 1. 2: Extract Users checked in at L' $G' = (U', E') \mid \{ U' \subseteq U \land U' \exists L' \}$ > Subgraph of users who checked in at locations lying under target region 4: $G'' \leftarrow G'$ for all users $u \in U'$ do for each neighbor *v* of *u* do 6: **if** (*v* not in *G*") **then** $G'' = G'' \cup v$ 8: $S \leftarrow DegreeDiscount(G'', k, I)$ 10: $L'' \subseteq L'$ $\{S \exists L''\}$ ▷ extract locations visited by top S seed users for $i \leftarrow 1 : m$ do for all locations $l \in L''$ do 12: $L_{UF} = ComputeLocationUserFreq(l)$ $V = V \cup \{\arg\max_{l \in L_{UF} \setminus V} \{L_{UF}\}\}$ 14: L'' = L'' - V16: Return S, V

G-IR Selection. The Greedy in-region selection approach selects top-*k* influential users and *m*-locations with highest number of check-ins. It is assumed that locations and users with the highest number of check-ins are suitable for product promotion in the target region and can spread product information to many users in a region.

The Algorithm 3 discusses the greedy approach for finding top-k influential users and m-influential locations. Line 1 extracts the users who have checked-in at target query region R. From lines 2 to 6, we select k-influential users considering the user's highest number of check-ins. Next, we select top m-influential locations with the most number of visits, from line 7 to 13.

Each proposed heuristic-based methods carries pros and cons with it. For instance, the IR-Selection approach focuses only on users who are within the target region boundary but lacks users consideration who are indirectly inside the target region boundary. The RF-Selection approach considers both users, i.e., users directly inside region boundary and indirectly within region boundary, for product promotion.

Algorithm 3 G-IR Selection of influential users and locations.

Input: LBSN: $\langle G, L \rangle$; Query: (R, k, m)Output: S-seeds, V-locations Initialization and locations extraction, L', are same as in Algorithm 1. Extract Users checked in at L'3: $G' = (U', E') \mid \{U' \subseteq U \land U' \exists L'\}$ ▷ Subgraph of users who checked in at location lying under target region for $i \leftarrow 1 : k$ do for all users $u \in U'$ do $U'_{CF} = ComputeUserCheckinFreq(u)$ 6: $S = S \cup \{ \arg \max_{u \in U'_{CF} \setminus U'} \{ U'_{UF} \} \}$ U' = U' - S9: for $i \leftarrow 1 : m$ do for all locations $l \in L'$ do $L_{UF} = ComputeLocationUserFreq(l)$ $V = V \cup \{ \arg \max_{l \in L_{UF} \setminus V} \{ L_{UF} \} \}$ 12: L' = L' - VReturn S, V

Further, the effectiveness of each approach, i.e., In-Region Selection (IR Selection), Region-Free Selection (RF Selection), and Greedy In-Region Selection (G-IR), is discussed in Section 5.

4.2. t-IUL Approaches

t-IUL problem is to select influential users and locations attarget query region matching a specific query topic.

We propose two heuristic-based approaches to solve *t-IUL* problem; topic-aware Region-Free Selection (t-RF selection) and topic-aware Greedy In-Region Selection (t-GIR selection).

t-RF Selection. Topic-Aware Region free selection approach is similar to the **RF-Selection** approach of *IUL* problem with one more constraint in location selection. Here, we consider locations lying within a target query region boundary and matching the query topic that can be the merchant's own interest or preference for product promotion.

The Algorithm 4 illustrates our topic-aware region free selection approach. We compare the location type/category with a given query type. We have designed this algorithm considering *Foursquare* dataset, which have location city and category information. It can be extended to any other LBSNs dataset, provided that every location has a category or topic field. Lines 2 to 5 of this algorithm extracts those locations which match the target query region and query topic. The remaining part of the algorithm is the same as the RF-Selection algorithm and discussed earlier in *IUL approach*.

Algorithm 4 t-RF Selection of influential users and locations.

```
Input: LBSN: \langle G, L \rangle; Query: (R, k, m, q)
    Output: S-seeds, V-locations
    I \leftarrow 10.000
                                                                                                                       ▷ Number of Simulations
    for all locations l \in L do
        if (l.city == R) and (l.category == q) then
             L' = L' \cup \{l\}
 4:
    G' = (U', E') \mid \{ U' \subseteq U \land U' \exists L' \}
                                                         > Subgraph of users who checked in at location lying under target region
    G'' \leftarrow G'
    for all users u \in U' do
        for each neighbor v of u do
 8:
            if (v \text{ not in } G'') then
                 G'' = G'' \cup (v)
    S \leftarrow DegreeDiscount(G'', k, I)
12: L'' \subseteq L' \mid \{S \exists L''\}
    for i \leftarrow 1 : m do
        for all locations l \in L'' do
             L_{UF} = ComputeLocationUserFreq(l)
        V = V \cup \{\arg\max_{l \in L_{UF} \setminus V} \{L_{UF}\}\}
16:
        L' = L' - V
    Return S, V
```

t-GIR Selection. Topic-Aware Greedy In-Region approach resembles the **G-IR Selection** approach of *IUL* problem with additional constraint in location selection. Here, we consider locations belonging to the target query region and matching the merchant's preferred query topic for product promotion.

The Algorithm 5 presents the greedy approach for selecting influential users and locations matching user query topic. It is worth noting that these both Algorithms 4 and 5, can easily be used to find *m*-influential locations to recommend to users for visiting (from consumer's perspective).

Algorithm 5 t-GIR Selection of influential users and locations.

Input: LBSN: $\langle G, L \rangle$; Query: (R, k, m, q)**Output:** S-seeds, V-locations 1: Initialization and locations extraction, L', are same as in Algorithm 4. 2: $G' = (U', E') \mid \{U' \subseteq U \land U' \exists L'\}$ 3: for $i \leftarrow 1 : k$ do for all users $u \in U'$ do 4: $U'_{CF} = ComputeUserCheckinFreq(u)$ 5: $S = S \cup \{ \arg \max_{u \in U'_{CT} \setminus U'} \{ U'_{UF} \} \}$ 6: U' = U' - S7: for $i \leftarrow 1 : m$ do 8: for all locations $l \in L'$ do 9: $L_{UF} = ComputeLocationUserFreq(l)$ 10: $V = V \cup \{\arg\max_{l \in L_{UF} \setminus V} \{L_{UF}\}\}$ 11: L' = L' - V12: 13: Return S, V

4.3. Time Complexity

Time Complexity: Let *L* denotes the total number of locations in LBSN, L' represents number of locations in target region, L'' number of locations visited by influential seeds. And *n* represents the total number of users, n'- number of users checked-in at target region, *m*-number of edges.

Algorithms 1 and 2 use the degree discount [2] heuristic-based method. As mentioned by Chen et al. [2], the running time of it is $O(k \log n' + m)$. So the running time of Algorithm 1 is $O(L + nL' + k \log n + m)$ and the running time of Algorithm 2 is $O(L + nL' + nn' + k \log n + m)$. Though the Algorithm 1 is faster than the Algorithm 2 but it lacks user selection who have directly social connection with users inside target region and could be useful for marketing the company products.

The running time of Algorithm 3 is O(L + nL' + kn' + mL'). The greedy In-Region selection approach is faster than the other two approaches. However, it does not provide users and locations which are beneficial for product promotion as discussed in Section 5.

The time complexity of Algorithms 4 and 5 is the same as the running time complexity of Algorithm 2 and 3, respectively.

Further, we discuss the effectiveness of our proposed algorithms in results Section 5.

5. Experiments

We conducted experiments of our proposed algorithms on three real-world locationbased social networks, as discussed in Table 1. The objective is to find which algorithms can provide better and effective results for product promotion under certain constraints, such as within target region R and location type q. Besides, we analyze the recommended users and locations for product promotions about how and why these results would be helpful for product promotion in a target region.

5.1. Experimental Setup

Datasets. We used three real world datasets **Gowolla**, **Brighkite**, **Foursquare**. Gowalla and Brighkite data sets are publicly available at stanford website [24]. While the Foursquare date set (http://net.pku.edu.cn/daim/yinhongzhi/index.html), used by Wang et al. in [25], was downloaded from his website (publicly made available for download). The three datasets are directed graphs and details of each data set is discussed in Table 1.

Datasets	#Vertices	#Edges	#Check-ins
Gowalla	196,591	950,327	6.4 M
Brighkite	58,228	214,078	4.49 M
Foursquare	4163	32,512	483,813

Table 1. Datasets.

Comparison Methods. We compare our heuristic-based algorithms (Algorithms 1, 2 and 4) with greedy approaches (Algorithms 3 and 5). For the heuristic methods, we used the degree discount approach [2] to find the seed set and compute the influence spread. Finding the influential users and their influence spread can easily be done by other state-of-the-art approaches, such as [4–6].

Target Query Region. We have selected the big modern cities of the U.S., such as New York, Chicago, San Francisco, Washington, Seattle, Las Angeles, as our target query region and computed the influential users and locations selection within a specified target region. For each city, we extracted its geo-coordinates, i.e., latitude and longitude, and then selected all those check-in venues which lie under the target region. Locations within the target region are extracted using Vincenty distance formula. Since the Foursquare dataset contains city information, so we extracted all locations matching the query region.

Propagation Probability. We randomly selected each edge's propagation probability from {0.1, 0.01, 0.5, 0.001} as mentioned in [3].

Parameters. We measure the effectiveness of the proposed approaches for reporting region-aware influence spread, comparing selected influential users and locations. The seed set size *k* varies from 10 to 50 and set m-locations as 20.

The influence spread of heuristic approaches is obtained by running 10,000 simulations for each set and took the average of the influence spread that matches the settings in [3]. All the proposed approaches are implemented in Python. Experiments are conducted on a PC with Intel Core(TM) i7 3.6 GHz and 8 GB memory using Windows 10.

5.2. Experimental Results

We evaluate the effectiveness of heuristic-based algorithms with greedy-based algorithms proposed in *IUL* and *t-IUL* approaches by varying seed set size *k*. We discuss the experimental results of *IUL* and *t-IUL* approaches in Sections 5.2.1 and 5.2.2 respectively.

5.2.1. IUL Approaches Result Summary

First, we discuss the influence spread result achieved by IR, RF, and G-IR selection algorithms. As all the algorithms showed similar behavior on the Brightkite dataset as they performed on the Gowalla dataset, so we have excluded the Brightkite analysis result summary here.

Figure 3 shows the influence spread within or out of target cities where locations lie under 150 miles of the target city on the Gowalla dataset. It can be seen that there is a huge difference in influence spread within and out of the target city achieved by RF, IR, and G-IR algorithms. Like in Figure 3a, when k = 20, the number of influenced users achieved by G-IR, IR, and RF algorithms are around 100, 180, and 650, respectively. There is a significant percentage difference in influence spread within and out of the target region. Similar difference is observed in other target cities as shown in Figure 3b–d. We conducted the same experiment on other cities of U.S., such as Seattle, California, Las Angles, and Los Vegas, and found the same influence spread difference in these target cities as well.



Figure 3. Influence spread on various regions on Gwalla dataset.

Figure 4 shows the results of influence spread of big cities of U.S. on Foursquare dataset. Since we do not have a considerable number of users and location check-in information of the Foursquare dataset, so it looks like that there is not much significant difference in influence spread achieved by all these algorithms. But we can observe the influence spread difference once the provided social network is large and consists of a vast amount of locations with check-in information.



Figure 4. Influence spread on various regions on Foursquare dataset.

It is evident from Figures 3 and 4 that the RF Selection algorithm achieved better results than IR Selection and G-IR Selection algorithms. IR and G-IR algorithms achieved similar influence spread in most of the cities on Foursquare datasets except in some cities where IR-algorithm performed better than G-IR.

From these experimental results, we believe that the top k-influential seed set who are local to the target city and have friends across the region can influence users across the target city and motivate them to visit famous locations under target cities.

Next, we discuss the effectiveness of a joint selection of influential users and locations selected by IR, RF, and G-IR algorithms.

Figure 5 shows the top-k (k = 20) influential users selected by G-IR, IR and FR algorithms respectively. It is observed that each algorithm produce a different k-influential seed set, so it is better to select the seed set which could maximize the product promotion. Further, we discuss the influential locations selected by each algorithm.

[148698, 130377, 145008, 145007, 104417, 148699, 68677, 56539, 83397, 19574, 41, 5524, 11275, 53473, 109503, 4487, 19216, 137957, 29984, 63837]

G-IR Algorithm

[148698, 38416, 145007, 13368, 106255, 18542, 33916, 25275, 114993, 12604, 11103, 56539, 11275, 34050, 107022, 88597, 541, 3102, 3105, 191011]

IR Algorithm

[8091, 41, 34496, 467, 373, 5524, 499, 25275, 4487, 6598, 12604, 7301, 57007, 33839, 3978, 23616, 11103, 3855, 4150, 13080]

RF Algorithm

Figure 5. Top-20 influential Users across San Francisco on Gwalla dataset.

Figure 6 shows the top-m (m = 20), influential locations selected by IR and FR selection algorithms with varying k-seeds. Figure 7 shows the same number of locations selected by the G-IR selection algorithm. It can be observed that when we increase the number of k-seeds, our proposed approach achieves better and similar influential locations as selected by a greedy approach with the highest users. However, the greedy approach (G-IR), which considers only check-in frequency distribution, yields poor results in influential locations selected by G-IR highlighted in red color shows the venues with less number of user visiting frequency than heuristic-based approaches.

Further, we conducted the same experiment for selecting influential locations on Foursquare and Gwalla datasets as well. Figures 8 and 9 show the results obtained by RF and GR-Selection algorithms on Foursquare dataset. Locations selected by G-IR highlighted in red color shows the venues with less number of users visiting frequency than our approach.

loc_Id #users	loc_id Users	loc_Id #users	loc_id #users	loc_id #users	loc_id #users
23261.0 1048	23261.0 1048	23261.0 1048	23261.0 1048	23261.0 1048	23261.0 1048
12505.0 923	12505.0 923	12505.0 923	12505.0 923	12505.0 923	12505.0 923
14128.0 888	14128.0 888	14128.0 888	14128.0 888	14128.0 888	14128.0 888
11844.0 819	11844.0 819	11844.0 819	11844.0 819	11844.0 819	11844.0 819
24963.0 746	24963.0 746	24963.0 746	24963.0 746	24963.0 746	24963.0 746
16399.0 689	16399.0 689	16399.0 689	16399.0 689	16399.0 689	16399.0 689
13022.0 453	13022.0 453	12535.0 635	12535.0 635	12535.0 635	12535.0 635
349982.0 443	349982.0 443	13022.0 453	13022.0 453	24767.0 627	24767.0 627
623366.0 416	623366.0 416	349982.0 443	349982.0 443	13022.0 453	13022.0 453
11875.0 350	11875.0 350	623366.0 416	623366.0 416	349982.0 443	349982.0 443
12313.0 335	12313.0 335	11875.0 350	11875.0 350	623366.0 416	623366.0 416
14151.0 302	14151.0 302	12164.0 345	12164.0 345	11875.0 350	11875.0 350
11794.0 296	11834.0 257	12313.0 335	12313.0 335	12164.0 345	12164.0 345
11834.0 257	17417.0 236	12525.0 332	12525.0 332	12313.0 335	12313.0 335
17417.0 236	16907.0 228	14151.0 302	14151.0 302	12525.0 332	12525.0 332
106840.0 232	14148.0 214	11794.0 296	11794.0 296	14151.0 302	14151.0 302
16907.0 228	130586.0 187	11834.0 257	11834.0 257	11794.0 296	11794.0 296
14148.0 214	11797.0 153	12171.0 244	12171.0 244	11834.0 257	11834.0 257
130586.0 187	60450.0 152	17417.0 236	17417.0 236	12171.0 244	12171.0 244
17710.0 179	16310.0 145	12174.0 236	12174.0 236	11720.0 239	11720.0 239

(a) K=10, IR / FR

(b) K=20, IR / FR

(c) K=40, IR / FR



#checkins location	s #users	#checkin	s location	s #users
1654.0 23261.0	1048	1680.0	12505.0	923
1680.0 12505.0	923	1654.0	23261.0	1048
1583.0 14128.0	888	1583.0	14128.0	888
1089.0 11844.0	819	1093.0	16399.0	689
1089.0 24963.0	746	1089.0	11844.0	819
1093.0 16399.0	689	1089.0	24963.0	746
838.0 12535.0	635	1021.0	24767.0	627
1021.0 24767.0	627	1005.0	13022.0	453
1005.0 13022.0	453	838.0	12535.0	635
567.0 349982.0	443	754.0	623366.0	416
754.0 623366.0	416	567.0	349982.0	443
430.0 11875.0	350	430.0	11875.0	350
405.0 12164.0	345	405.0	12164.0	345
390.0 12313.0	335	394.0	11794.0	296
392.0 12525.0	332	392.0	12525.0	332
353.0 14151.0	302	390.0	12313.0	335
394.0 11794.0	296	376.0	11720.0	239
364.0 11834.0	257	370.0	101149.0	15
250.0 12171.0	244	364.0	11834.0	257
376.0 11720.0	239	363.0	42503.0	37

With highest user frequency

With highest checkin frequency

Figure 7. Top-20 influential locations across New York using G-IR algorithm.

loc_id	#users	loc_id	#users	loc_id	#users
41059b00f964a520	0850b1fe3 682	41059b00f964a520	850b1fe3 682	41059b00f964a5	20850b1fe3 682
4bd2177d046076b	055357371 409	4bd2177d046076b0	55357371 409	4bd2177d04607	5b055357371 409
49d01698f964a520	0fd5a1fe3 161	49d01698f964a520	fd5a1fe3 161	4a5824e5f964a5	206db71fe3 210
4a71e4cff964a520	ccd91fe3 133	4a71e4cff964a520c	cd91fe3 133	4ab595e1f964a5	20877520e3 167
43067280f964a520	023271fe3 119	43067280f964a520	23271fe3 119	49d01698f964a5	20fd5a1fe3 161
49fdea19f964a520	4a6f1fe3 80	42cc7080f964a520e	e9251fe3 104	4a71e4cff964a52	20ccd91fe3 133
40bbc700f964a520	0b1001fe3 73	40c3b000f964a520	e7001fe3 82	49cc413df964a5	205a591fe3 123
409d7480f964a520	0f2f21ee3 73	49fdea19f964a5204	a6f1fe3 80	43067280f964a5	2023271fe3 119
414e1d80f964a520	0ed1c1fe3 64	40bbc700f964a520	b1001fe3 73	42cc7080f964a5	20e9251fe3 104
40b7d280f964a520	093001fe3 61	409d7480f964a520	f2f21ee3 73	40c3b000f964a5	20e7001fe3 82
40f71a80f964a520	af0a1fe3 60	414e1d80f964a520	ed1c1fe3 64	49fdea19f964a5	204a6f1fe3 80
4390a026f964a520	04d2b1fe3 58	40b7d280f964a520	93001fe3 61	40943a00f964a5	20e5f21ee3 78
49ca8f4df964a520	b9581fe3 55	40f71a80f964a520a	f0a1fe3 60	409d7480f964a5	20f2f21ee3 73
49d68c61f964a520	0e65c1fe3 54	4390a026f964a520	4d2b1fe3 58	40bbc700f964a5	20b1001fe3 73
4468f484f964a520	4f331fe3 53	49ca8f4df964a520b	9581fe3 55	4249ec00f964a5	208f201fe3 73
44646408f964a520	026331fe3 53	49d68c61f964a520	e65c1fe3 54	4bb9dbc353649d	74364e48fb 72
3fd66200f964a520	aOec1ee3 51	40e4a580f964a520f	f5091fe3 53	444df333f964a5	208d321fe3 72
42814b00f964a520	002221fe3 50	4468f484f964a5204	4f331fe3 53	454cf014f964a5	20c53c1fe3 65
4aa48566f964a520	0024720e3 45	44646408f964a520	26331fe3 53	414e1d80f964a5	20ed1c1fe3 64
42bf4180f964a520	b1251fe3 44	3fd66200f964a520a	Oeclee3 51	43728d80f964a5	20102a1fe3 63
K-10		K=20			K=40
K-10		N=20			N-19

K=10

K=40

Figure 8. Top-20 influential locations across San Francisco using RF-algorithm on Foursquare.

			_					
checkins lo	c_id	#users] [#check	ins	loc_id		#users
713 41059b00f964	a520850b1fe3	682		1713	41059	b00f964a	520850b1fe3	3 682
919 4bd2177d0460	76b05535737:	1 409		919	4bd217	77d04607	6b05535737	1 409
364 4a5824e5f964a	5206db71fe3	210		440	49d016	598f964a	520fd5a1fe3	161
323 4ab595e1f964a	520877520e3	167		364	4a5824	4e5f964a	5206db71fe3	210
440 49d01698f964a	a520fd5a1fe3	161		333	4a71e4	lcff964a5	20ccd91fe3	133
333 4a71e4cff964a	520ccd91fe3	133		323	4ab595	5e1f964a	520877520e3	167
171 49cc413df964a	5205a591fe3	123		193	49f796	8df964a5	20bf6c1fe3	36
169 43067280f964a	a52023271fe3	119		175	4acfad	eef964a5	2049d520e3	11
128 42cc7080f964a	520e9251fe3	104		171	49cc41	.3df964a5	5205a591fe3	123
101 40c3b000f964a	520e7001fe3	82		169	430672	280f964a	52023271fe3	119
106 409ad180f964a	a520eef21ee3	82		161	40ca47	80f964a	52008011fe3	47
123 49fdea19f964a	5204a6f1fe3	80		153	40b7d2	280f964a	52093001fe3	61
100 40943a00f964a	a520e5f21ee3	78		150	4c2f9c	1d6f1fef3	b2c94eb3d	1
108 452b81ddf964a	a520393b1fe3	74		150	42853f	80f964a5	200c231fe3	49
82 409d7480f964a	520f2f21ee3	73		146	4ab81d	16ff964a5	203d7c20e3	7
102 4249ec00f964a	5208f201fe3	73		143	455f77	abf964a5	20903d1fe3	62
89 40bbc700f964a	520b1001fe3	73		130	4a4aa5	5a7f964a	5200bac1fe3	2
93 4bb9dbc353649	c74364e48fb	72		128	42cc70	80f964a	520e9251fe3	104
107 444df333f964a	5208d321fe3	72		125	435981	L00f964a	520f7281fe3	7
89 3fd66200f964a	52041ee1ee3	70		123	49fdea	19f964a5	204a6f1fe3	80
			J					

With highest user frequency

With highest checkin frequency



5.2.2. t-IUL Approaches Result Summary

We conducted the experiments for *t-IUL* problem on *Foursquare* dataset only because it have location category information while *Gowolla* and *Brighkite* does not have any location category information. We selected the target region, *R*, as *Log Angeles* and category topic, *q*, as *food*. We have considered "food" as a location category type as users are found to share favorite food locations with their friends. Here, we discuss the influential locations selected by t-RF and t-GIR algorithms and see the results difference produced by both algorithms. Influential seeds, k, selected by t-RF and t-GIR algorithms are almost the same for the Foursquare dataset, so we have not shown that result in this paper.

Figure 10 shows the top-10 influential locations selected by the t-RF algorithm. We found around 5.3 k check-ins matching query category and target city. Besides, we set k-seeds as 50 because there are not many users in *Foursquare* dataset. Influential users visit the 95 unique venues. Further, Figures 11 and 12 show the venues selected by *t*-*GIR* Selection algorithm considering highest users with check-in frequency. The venues highlighted in red color shows the drawback of *t*-*GIR* algorithm because it returns locations with the highest check-ins, but only a few users have visited those places

ven_name	latitude	longitude	category	#users
bottega louie	34.047075	-118.256568	nightlife spot,food,	94
wurstk?che	34.045623	-118.236102	nightlife spot,food,	75
father's office	34.030345 -	118.384606	nightlife spot,food,	72
yard house	34.045081-	118.265784	nightlife spot,food,	63
pinktaco 3	34.059042 -1	18.420429	nightlife spot,food,	56
the happy ending bar &	restaurant 3	4.097796-118.	342876 nightlife	spot,food, 47
the bazaar by jos? a	ndr?s 34.06	9809-118.3765	74 nightlife spot,	food, 44
the parlor	34.083197 -1	18.347731	nightlife spot,food,	41
supperclub	34.101931 -	118.335666	nightlife spot,food,	39
jw marriott la liv	e 34.045308	3-118.266588 r	nightlife spot,travel & transp	ort,food, 37

Figure 10. Top-10 influential locations in Log Angles using t-RF algorithm.

ven_name	latitude longitude	category	#users	#checkins
bottega lo	uie 34.047075-118.256568	nightlife spot,food,	94 12	20
wurstk?c	the 34.045623-118.236102	nightlife spot,food,	75 10	08
father's off	ice 34.030345-118.384606	nightlife spot,food,	72 10	4
yard hou	ise 34.045081-118.265784	nightlife spot,food,	63 89	9
pinktac	o 34.059042-118.420429	nightlife spot,food,	56 70	
el rey thea	tre 34.062698-118.348880 mu	sic venue,food,arts & entert	tainment,	55 70
canter's delica	tessen 34.078780-118.361716	nightlife spot,foo	d, 53	69
the happy ending bar	& restaurant 34.097796-118.3	42876 nightlifes	pot,food,	47 63
kogi bbq tr	uck 34.045832-118.256836	travel & transport,foo	d, 47	56
the bazaar by jos	?andr?s 34.069809-118.37657	4 nightlife spot, f	ood, 44	61

Figure 11. Top-10 influential locations in Log Angles using t-GIR algorithm.

ven_name	latitude	longitude	category	#users	# checkins
te??kil	a 34.101782-1	18.332483	nightlife spot,food,	6 12	21
bottega l	ouie 34.04707	5-118.256568	nightlife spot,food	, 94	120
wurstk	che 34.045623	3-118.236102	nightlife spot,food	, 75	108
father's of	fice 34.030345	-118.384606	nightlife spot,food,	72	104
yard ho	use 34.045081	-118.265784	nightlife spot,food,	63	89
little next o	loor 34.072222	2-118.367370	shop & service,foo	d, 31	81
supper	lub 34.101931	-118.335666	nightlife spot,food,	39	81
hollywood farm	er's market 34.	099762-118.328362	shop & service, for	d & drir	nkshop, 31 77
pink ta	co 34.059042 -	118.420429	nightlife spot,food,	56	70
el rey the	atre 34.062698	8-118.348880 music	venue,food,arts & ente	ertainme	ent, 55 70

Figure 12. Top-10 influential locations in Log Angles using t-GIR algorithm considering check-ins.

5.3. Discussion

The experimental results provides the following analysis.

- There is a significant percentage difference in influence spread within and out of the target region.
- RF-algorithms achieved better performance as they considered users having connections across the target city for promotion. The top k-influential seed set who are local to the target city and have friends across the region can influence users to visit famous locations under target city.
- Third, the locations selected by considering check-in information only are not always an optimal choice for production promotion. Since, few users can have checked-in location hundreds or thousands times but it may not denote its real popularity.
- When we increase the number of k-seeds, our proposed approach achieves better influential locations as selected by a greedy approach with the highest users. However, the greedy approach (G-IR), which considers only check-in frequency distribution, yields poor results in influential locations selection.

6. Conclusions

In this paper, we have studied the problem of finding influential users and locations jointly at the target query region for product promotion. Further, we studied the selection of influential users and locations matching the specific location category. As location popularity values, such as the number of check-ins, are not always available, so we have proposed heuristic-based methods that can find influential users and locations in the target region. Experimental results showed that our heuristic-based methods achieved better performance than the greedy-based methods. Further, our proposed algorithms can easily be extended by adding more constraints such as recommending top *M* locations, by considering the distance between each location, i.e., we can suggest influential locations for product promotion which should be distant apart from each other. In future work, we would like to find a co-relation between *k*-influential users and *m*-influential locations under target region in location-based social networks.

Author Contributions: K.A. and C.-T.L. formulated the problem and proposed a methodology. C.-T.L. provided an insightful analysis about results and guided in the proposed methodology. K.A. implemented the proposed method and prepared the initial paper draft. Y.-S.C. contributed in manuscript draft and provided suggestions in results analysis. All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

MDPI	Multidisciplinary Digital Publishing Institute
IM	Influence Maximization
LBSNs	Location-based Social Networks
IUL	Influential Users and Locations
t-IUL	Topic-aware Influential Users and Locations
IR	In-Region
RF	Region-Free
G-IR	Greedy In-Region
IC	Independent cascade
LT	Linear threshold
t-RF	Topic-aware Region-Free

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