PNAS COMMENTARY

Putting memories on paper

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Consider a mechanical kitchen scale with a spring that compresses in proportion to the weight placed on it. Removal of the weight makes the scale return to its initial position. Reversibility is an essential feature for this device to function. Now, assume that instead of the kitchen scale you use a sandbag. A weight placed on the sandbag will compress its surface in proportion to it; however, the sandbag lacks the feature of reversibility. Not only will removal of the weight fail to cause a reversion of the bag to its initial shape, but subsequent weights put on the bag will change the current deformation state little, unless you place a heavier weight on it. However, the sandbag does measure something: the largest deformation it has been subjected to in its past. Thus, while the kitchen scale measures a state, the shape of the sandbag provides information about its deformation history. It is a conceptually simple example for memory formation in matter (1) and demonstrates that the disorder of the grains, as well as the dissipation of energy when the sandbag is deformed, play an essential role in its irreversible behavior. On the other hand, it is precisely this irreversibility which permits one to go from measuring states to measuring histories and thereby encoding memory. In PNAS, Shohat et al. (2) investigate memory formation in crumpled sheets of paper. Exploring the complex interplay of the geometry and mechanics of wrinkled paper, they demonstrate how memory emerges from the interactions of bistable mechanical instabilities formed by its folds and creases. The findings of Shohat et al. (2) are remarkable, since they demonstrate in an explicit and experimentally accessible manner how memory can emerge in everyday systems such as a crumpled sheet of paper, thereby contributing toward our understanding of memory formation in driven disordered systems.

Memory-recording mechanical devices can be constructed from bistable elements (3) such as origami bellows (4), corrugated elastic sheets (5), and, more generally, mechanical metamaterials (6). Under a compression, a bistable element remains in one of its stable states, call it "0," until the force rises above a threshold level F^+, upon which a fast relaxation snap event causes a transition into the other state, "1," as shown in Fig. 1 *A*, *Top Left*. Subsequently reducing the forcing, a transition back into state "0" will occur when the force falls below a threshold level F^-. Hysteresis emerges when the two switching fields $F^\pm$ are such that $F^- < F^+$, so that for values between these two thresholds the actual state of the bistable element depends on the history of the forcing. Such elementary units of hysteresis are called Preisach hysterons (7–9).

Consider now a collection of N hysterons, each with a pair of threshold forces $F_i^\pm$, and each coupling independently to the applied force F. One can construct a history-recording device by ordering the threshold forces as $F_1^+ < F_2^+ < \ldots < F_N^+$ and $F_1^- > F_2^- > \ldots > F_N^-$, as sketched in Fig. 1*A*, *Lower Right*. Initializing the device in a configuration where each hysteron is in its "0" state, upon increasing the force they will change states one by one in the order $1, 2, \ldots, N$. Due to the choice of ordering the F_i^-, this will be also the same order in which the elements return from their "1" to their "0" state. A state transition graph of this system for the case $N = 4$ is shown in Fig. 1*A*. Here, gray/black (orange/red) arrows indicate transitions when the force has just increased (decreased) enough so that one element becomes unstable and changes state.

Note the nesting of cycles in Fig. 1*A*: Starting from the state 0010, a sequence of gray/black arrows followed by orange/red arrows leads via 1110 back to 0010, forming a graph cycle, which in turn is nested within the cycle between states 0000 and 1110, etc. Observe also that from any state in the graph, by lowering the forcing sufficiently we can "reset" the system by bringing it back to the state 0000. The hierarchical nesting of cycles within cycles is a characteristic feature associated with return-point memory (RPM) (10), which emerges in the hysteretical magnetization response of magnets to an applied magnetic field. In the transition graph setting, the nesting feature is a topological property of the transitions between hysteron states, which is referred to as loop RPM (11). Assume now that starting in state 0000 a sequence of forcings leads the system to state 0101. The hierarchical nesting of cycles constrains the possible trajectories that the system must have followed in order to reach 0101 from the zero state: In particular, in the recent past the force must have risen above the value of F_3^+, subsequently fallen, but not below F_3^-, then risen again to a value between F_2^+ and F_3^+, etc. The shortest trajectory from 0000 to 0101 has been highlighted by the red and black arrows in Fig. 1*A*.

Remarkably, mechanical instabilities, which give rise to hysteresis in the form of near-perfect RPM with a hierarchy of nested hysteresis cycles, are rather ubiquitous. They are found in the athermal quasi-static response of various driven disordered systems, such as simulations of amorphous solids under shear strain (12) and experiments on sheared colloidal suspensions (13) as well as jammed granular particles under uniaxial compression (14). In all these systems thermal effects are negligible, and the mechanical instabilities are due to spatially localized plastic events, called soft spots (15–17), which effectively respond as

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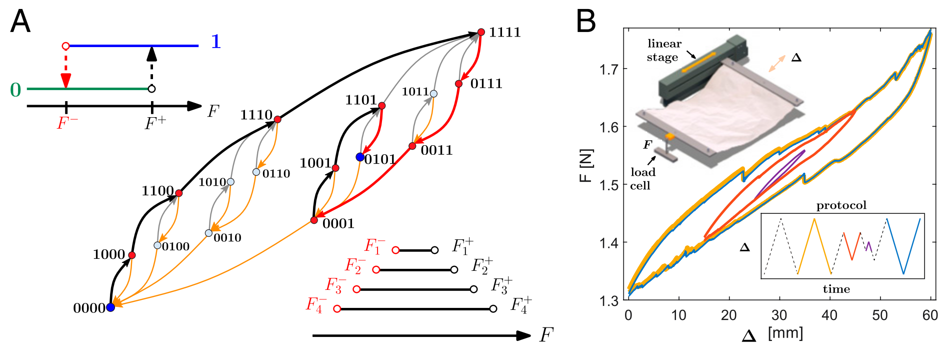

Fig. 1. (*A*, *Top Left*) Elementary hysteresis cycle of a Preisach hysteron resulting from a bistable mechanical element with states 0 and 1 under force *F*. The hysteron can be in state 1 for $F > F^-$ and state 0 for $F < F^+$. In the interval $F^- < F < F^+$ the state depends on the history of the forcing. (*A*, *Center*) The state transition graph of four hysterons each coupling independently to the driving *F*. (*A*, *Lower Right*) The ordering of the switching fields $F_i^{\pm}$ is as indicated. Black/gray (red/orange) arrows indicate state changes of the corresponding hysteron *i* when *F* is just above (below) F_i^+ (F_i^+). A possible trajectory from 0000 to the state 0101 is shown by the black and red arrows. (*B*, *Top Left*) The experimental set up of ref. 2 for a crumpled sheet of paper driven by a variable strain Δ. (*B*, *Center*) The force–strain response under cyclic straining, once a periodic cycle has set in (yellow cycle). A subsequent driving protocol of the form shown in the lower right inset results in the subcycles shown. Adapted from Shohat et al. (2). The transition graph (*A*) and the cyclically sheared paper response (*B*) reveal a hierarchy of nested cycles characteristic of RPM.

hysterons. However, in contrast to the collection of noninteracting Preisach hysterons, these soft spots interact with each other via long-range elastic deformations. As a result, the state change of one soft spot can alter the switching fields of the other, but it can also give rise to new soft spots or deactivate existing ones by disabling their switching behavior. Overall, these interactions can give rise to highly complex dynamics, including avalanches where the instability of one soft spot triggers instabilities of multiple others, and more generally dynamically critical phenomena, such as yielding. The Preisach model, due to the absence of interactions, lacks all this complexity. Nevertheless, it is a conceptually simple model to illustrate key mechanisms of memory formation, such as RPM, serving a role similar to the ideal gas in statistical mechanics.

The interactions between hysterons turn out to be both a blessing and a curse. They are a curse since in principle interactions can destroy the hierarchical RPM-type nesting of hysteresis cycles and hence adversely affect the capability of memory formation. They are a blessing since the interactions permit a degree of "programmability": Cyclically driving the system until a periodic response has set in is tantamount to selecting a set of interacting soft spots that "play nice" with each other, i.e., they only moderately influence or impede each other's switching behavior so that an RPM-like hierarchy of nested cycles remains largely intact.

Surprisingly, many of the driven disordered systems mentioned above achieve both the programmability and nesting of hysteresis cycles, and the reasons for this are still not sufficiently understood. Now the paper by Shohat et al. (2) adds one more such system to the fray: a thin sheet of crumpled paper subject to driving via stretching and unstretching it (Fig. 1*B*). The relation between the complex geometry of folds and creases of a crumpled sheet of paper and its mechanical response to external driving has so far been largely unexplored. By combining cyclic stretching protocols with three-dimensional (3D) imaging, Shohat et al. (2) are able to correlate the mechanical response of the sheet to its local geometrical transformation.

Shohat et al. (2) find that when the sheet is cyclically strained it settles into a near-perfect periodic force–strain hysteresis cycle dotted with a large number of abrupt force jumps. Using 3D imaging, Shohat et al. (2) are able to trace the force jumps to localized plastic events in the sheet, originating from the snapping of vertices formed by the crossings of ridges (18). Shohat et al. (2) demonstrate that these mechanical instabilities behave as hysterons that interact with each other. As in the sheared amorphous solids, depending on the locations and states of the other hysterons, the interactions can facilitate or impede each other's switching behavior. The sheet of paper has become a mechanical realization of a spin glass, an archetypical model for understanding the emergence of complexity in disordered systems (19).

These findings set the stage for a thorough investigation of memory formation. First, Shohat et al. (2) "program" their sheet of paper by applying cyclic strain over a strain range $[\Delta_0^{\min}, \Delta_0^{\max}]$ until a periodic response is attained in the force–strain response of the sheet (Fig. 1*B*). The response to cyclic deformation has all the hallmarks of RPM. Once a periodic response has set in, a subsequent reduction of $\Delta_0^{\max}$ to $\Delta_1^{\max}$ results in force-versus-strain subcycles of the original hysteresis cycle, as depicted in Fig. 1*B*. In fact, the amplitude $\Delta_1^{\max}$ can be lowered and raised as long as $\Delta_1^{\max} \leq \Delta_0^{\max}$, i.e., it does not exceed the value at which the original hysteresis cycle was obtained. Raising $\Delta_1^{\max}$ back to $\Delta_0^{\max}$, the original cycle is recovered. Thus, applying oscillatory shear to the crumpled paper has led to the selection of an interacting hysteron system that supports a hierarchy of cycles and subcycles, as shown in Fig. 1*B*. As in simulations of sheared amorphous solids (12), the emergence of the interacting hysteron system is at the heart of the memory that is imprinted into the disordered system as a result of cyclic loading. In fact, raising $\Delta_1^{\max}$ to a value above $\Delta_0^{\max}$ and then back to $\Delta_0^{\max}$, Shohat et al. (2) show that this memory is erased.

The relative ease with which the interacting hysterons of a driven disordered system establish a hierarchy of nested hysteresis cycles suggests that this may not have to be a consequence of the cyclic driving. The approximate rate independence of the response implies that the duration of the driving cycle is not important, while the hierarchy of nested cycles suggests that the driving need not be periodic, as long as it is confined to some range, e.g., the interval $[\Delta_0^{min}, \Delta_0^{max}]$ in the case of the crumpled sheet. It is therefore conceivable that a random driving protocol confined to a range of values may produce a similar hierarchy of nested cycles and perhaps even enforce that the emerging set of soft spots "play nice." If so, this implies that memory formation in athermal systems is more abundant in nature then one might have suspected, since the interactions of these systems with their fluctuating environment may suffice. One could thus regard these systems as simple and ad hoc sensors of their environment with the hysteron configurations serving as its representation. Seen in this way, the kitchen scale has a rather narrow representation of its environment in terms of the instantaneous displacement of its plate. On the other hand, a driven disordered system whose response to driving is capable of self-organizing into a hierarchy of nested hysteresis cycles establishes a complex representation of its environment and captures thereby certain features of the environment's history.

We tend to associate memory formation with elaborate processes of engineering design or long-term evolution. The work of Shohat et al. (2) provides a rather elegant and accessible table-top demonstration that this need not be the case. Memory formation in matter—at least in its most basic form—seems to be abundant, emerging rather easily. So, next time you crumple a piece of paper and throw it away, be aware. You may have created a memory.

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