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Exact and explicit traveling wave solutions to two nonlinear evolution equations which describe incompressible viscoelastic Kelvin-Voigt fluid

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Abstract

Two nonlinear evolution equations, namely the Kadomtsev-Petviashvili (KP) equation which describes the dynamics of soliton and nonlinear wave in the field of fluid dynamics, plasma physics and the Oskolkov equation which describes the dynamics of an incompressible visco-elastic Kelvin-Voigt fluid are investigated. We deliberate exact traveling wave solutions, specially kink wave, cusp wave, periodic breather waves and periodic wave solutions of the models applying the modified simple equation method. The solutions can be expressed explicitly. The dynamics of obtained wave solutions are analyzed and illustrated in figures by selecting appropriate parameters. The modified simple equation method is reliable treatment for searching essential nonlinear waves that enrich variety of dynamic models arises in engineering fields.

Keywords: Applied mathematics, Plasma physics

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1. Introduction

Numerous complex phenomena in real life are modeled by nonlinear evolution equations (NLEEs). NLEEs are involved in different field of science and engineering such as mathematics, physics, mechanics, biology, ecology, optical fiber, chemical reaction and so on. Soliton theory may play a significant role and has be applied in almost all the nonlinear sciences, in which many dynamical evolution phenomena are modeled by various nonlinear models [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35]. Searching for innovative analytic exact traveling wave solutions of NLEEs has played a substantial role in the study of dynamics of witnessed phenomena. Many scholars planned through NLEEs for construct traveling wave solution by implementing several methods. The procedures that are well established in recent literature such as the Exp-function method [1], Extended tanh method [2, 3] used to find exact compact and noncompact solutions. A new extended (G'/G)-expansion method [4, 5], Jacobian elliptic function method [8], (G'/G)expansion method [6, 7], Inverse engineering method [9], Fokas method [10], transformed rational function method [11], N-fold Darboux transformation [12], Initial boundary value problem [13, 14], sin-cosine method [15], Enhanced (G'/G)-expansion Method [16]. Li et. al. [17] studied analytically on some NLEEs to construct the interaction of some solitary soliton solution.

In this article we study $(3 + 1)$ dimensional kadomtsev-petviashvili (KP) equation of the form [18]

$$(u_t + 6uu_x + u_{xxx})_x + 3u_{zz} + 3u_{yy} = 0. \tag{1}$$

Usually, $(3 + 1)$ dimensional Kadomtsev-Petviashvili (KP) equation is introduced to describe the dynamics of soliton and nonlinear wave in the field of fluid dynamics, plasma physics etc [18, 24, 26]. Chen [24] studied $(3 + 1)$-dimensional KP equation by using the new generalized transformation in homogeneous balance method. Decomposition method [23] is used to find exact solution from Eq. (1). There are several methods [23, 24, 25, 26, 27, 28, 29, 30] are applied to construct exact solution from Eq. (1).

Let us consider the $(1 + 1)$ dimension Oskolkov equation is of the form [19]:

$$U_t - \lambda U_{xxt} - \alpha U_{xx} + UU_x = 0, \tag{2}$$

where λ, $\alpha$are arbitrary constant and $U(x, t)$ is an unknown function. Eq. (2) described the dynamics of an incompressible visco-elastic Kelvin-Voigt fluid and fluid dynamics. Different types of Oskolkov equation are solved by several methods [20, 21, 22] to construct exact solution. The (G'/G)-expansion method [19] are applied to find exact solution from Oskolkov equation.

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The present work is motivated by the desire to employee the modified simple equation method (3 + 1) dimensional Kadomtsev-Petviashvili (KP) and the (1 + 1) dimension Oskolkov equation. MSE method is more effective to find exact solution through free parameter. MSE methods [31, 32, 33, 34, 35] solve different NLEEs to construct exact solution. The main advantage of this method over the existing other methods is that it provides more new exact traveling wave solutions.

2. Methodology

The main steps of the Modified Simple Equation method is provided as follows:

Let us consider the general form of nonlinear evolution equations (NLEEs)

$$H\left(U, U_x, U_t, U_{xt}, U_{yy}\right) = 0, \tag{3}$$

where $U = U(x, y, z, t)$ be unknown function and H is a function of U and its partial derivative in which the linear and nonlinear term of U and its derivative appear. Now we will solve the nonlinear evolution equations by the following steps

Step 1: At first we implement traveling wave transformation to covert the independent variable x, y, z, t into a compound variable ξ by

$$U(x, y, z, t) = U(\xi), \text{ where } \xi = kx + my + nz \pm \omega t, \tag{4}$$

where ω is the wave speed and μ, l, m, n are arbitrary constant. Eq. (4) allow us to write Eq. (3) into the Ordinary Differential Equation (ODE) form

$$R\left(U, U', U'', U''' \cdots \quad \cdots\right) = 0, \tag{5}$$

where u'derivative with respect to ξ. Eq. (5) is intangible. So we integrate Eq. (5) with respect to ξ.

$$R_1(U, U', U'' \cdots \quad \cdots) = 0. \tag{6}$$

Step 2: Let us suppose the solution of the Eq. (6) can be express in the following form.

$$u(\xi) = \sum_{i=0}^{N} a_i \left(\frac{S'(\xi)}{S(\xi)}\right)^i, \tag{7}$$

where $a_i(k = 0, 1, 2, 3, 4, \ldots, N)$ are unknown constant, $S = S(\xi)$ be unknown function to be obtained such that. $a_N \neq 0$.

Step 3: The positive integer Nin Eq. (7) can be determine by taking into consideration of the homogeneous balance between the highest order derivatives and highest order nonlinear terms in Eq. (6).

Step 4: Now we calculate all the necessary derivatives U', U'', U''', ...by using Eq. (7) and substituting in Eq. (6). For this substituting we acquire a polynomial

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of $\frac{S'(\xi)}{S(\xi)}$ and its derivatives. Let the coefficient of $S^{-k}(\xi)(k = 1, 2, 3, ..., N)$ equal to zero then we obtain a system of equation. From those equations we obtain the value of$a_k(k = 1, 2, 3, ..., N)$ and$S(\xi)$. Substituting all values of $a_k(k = 1, 2, 3, ..., N)$ and $\left(\frac{S'(\xi)}{S(\xi)}\right)^k (k = 1, 2, ..., N)$in Eq. (5) then we obtain the required exact wave solution of the general nonlinear evolution equations (NLEEs).

3. Examples

In this section, we apply the Modified simple equation (MSE) method to find exact solitary wave solution of the (3 + 1) Dimensional Kadomtsev-Petviashvili (KP) Equation and the (1 + 1) Dimensional Oskolkov Equation.

3.1. The (3 + 1) dimensional Kadomtsev-Petviashvili (KP) equation

In this subsection, we submit an application of the Modified simple equation (MSE) method to find exact solitary wave solution of (3 + 1) dimensional kadomtsev-petviashvili (KP) equation. The KP equation is most important to explain the different type of nonlinear wave and soliton wave in fluid dynamics and plasma physics. Chen [24] has studied this equation and find out the explicit solitary wave solution.

Let us consider the (3 + 1) dimensional Kadomtsev-Petviashvili equation in the following form

$$(u_t + 6uu_x + u_{xxx})_x + 3u_{xx} + 3u_{yy} = 0, \tag{8}$$

where $u = u(x, y, z, t)$ is an unidentified function.

Using traveling wave variable $\xi = (kx + ny + lz - \omega t)$ to reduce the Eq. (8) in the ODE form

$$k\left(-\omega u' + 6kuu' + k^3 u'''\right)' + 3\left(n^2 + l^2\right)u'' = 0. \tag{9}$$

Eq. (9) is an integrable equation. So integrate two times with respect to ξ and we seek the integrating constant to zero. Then we obtain

$$k^4 u'' + 3k^2 u^2 + \left(3n^2 + 3l^2 - k\omega\right)u = 0, \tag{10}$$

where the nonlinear term is u^2 and the highest order derivative isu''. So the balance number is N = 2.

Then according to the step-2 the solution of the Eq. (10) can be written as

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$$U(\xi) = a_0 + a_1\left(\frac{S'(\xi)}{S(\xi)}\right) + a_2\left(\frac{S'(\xi)}{S(\xi)}\right)^2, \tag{11}$$

where a_0, a_1, a_2 are arbitrary constants to be determined later and $s(\xi)$ be an unknown function.

Differential Eq. (11) with respect to ξ, then we get

$$U' = \frac{a_1 S''}{S} - \frac{a_1 (S')^2}{S^2} + \frac{2a_2 S' S''}{S^2} - \frac{2a_2 (S')^3}{S^3}$$

$$U'' = \frac{a_1 S'''}{S} - \frac{3a_1 S'' S'}{S^2} + \frac{2a_1 (S')^3}{S^3} + \frac{2a_2 (S'')^2}{S^2} - \frac{10a_2 (S')^2 S''}{S^3} + \frac{2a_2 S' S''}{S^2} + \frac{6a_2 (S')^4}{S^4}.$$

Substituting the value of U, U', U'' in the Eq. (10) and equating the coefficient of S^0, $S^{-1}, S^{-2}, S^{-3}, S^{-4}$ are zero, then we get,

$$\begin{aligned}
&3a_0 l^2 - k a_0 \omega + 3k^2 a_0 + 3n^2 a_0 = 0\\
&k^4 a_1 s'' + 3a_1 l^2 s' + a_1 \omega k s' + 3a_1 n^2 s' + 6k^2 a_0 a_1 s' = 0\\
&k^4\left\{ -3a_1 s'' s' + 2a_2\left(s''' s' + (s'')^2\right)\right\} +\\
&\left\{k^2(6a_0 a_2 + 3a_1) + a_2\left(3n^2 + 3l^2 - k\omega\right)\right\}(s')^2 = 0\\
&2k^2 a_1 (s')^3\left(3a_1 + k^2\right) - 10k^4 a_2 s''(s')^2 = 0\\
&\left(3k^2 a_2 + 6k^4 a_2\right)(s')^4 = 0.
\end{aligned}$$

Solving the 1st and last equation of the above system of equation for $a_{0,}a_2$ we attain the given two set solution

$$a_0 = 0, a_2 = -2k^2 \text{ and } a_0 = \frac{k\omega - 3n^2 - 3l^2}{3k^2}, a_2 = -2k^2,$$

where $\omega, l, k, n$are constants.

Case-01: when $a_0 = 0, a_2 = -2k^2$.

Putting these values in the rest three equations and solving them. Finally, we reach three set solutions

$$S(\xi) = c_1 + c_2 \exp\left(\frac{a_1 \xi}{k^2}\right).$$

Set-01: $a_1 = \pm 2\sqrt{k\omega - (3n^2 + 3l^2)}$, where ω, l, k, n are constants.

Set-02: $\omega = \dfrac{(12n^2 + 12l^2 + a_1{}^2)}{4k}$, where ω, l, k, n are constants.

Set-03: $k = \dfrac{(12n^2 + 12l^2 + a_1{}^2)}{4}$, where ω, l, k, n are constants.

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Set-01: When $a_1 = \pm 2\sqrt{k\omega - (3n^2 + 3l^2)}$, substituting the value a_0, a_1, a_2 and $S(\xi)$in Eq. (11), then we achieve.

$$U(\xi) = \frac{2(k\omega - 3(n^2 + l^2))}{k^2}\left\{\frac{c_2 e^{\pm\theta}}{(c_1 + c_2 e^{\pm\theta})} - \frac{c_2^2 (e^{\pm\theta})^2}{(c_1 + c_2 e^{\pm\theta})^2}\right\}, \tag{12}$$

where $\theta = \dfrac{\sqrt{(k\omega - 3(n^2 + l^2))}}{k^2}\xi$ and $\xi = kx + ny + lz - \omega t$.

If we set the condition $k\omega > (3n^2 + 3l^2)$ and $c_1 \neq c_2$ then we reach the hyperbolic solution

$$U(\xi) = \frac{2(k\omega - 3(n^2 + l^2))}{k^2}\frac{2c_1c_2}{2c_1c_2 + (c_1^2 + c_2^2)\cosh\theta - (c_1^2 - c_2^2)\sinh\theta}. \tag{13}$$

If we set $c_1 = c_2$ then the Eq. (13) become

$$U(\xi) = \frac{(k\omega - 3(n^2 + l^2))}{k^2} sceh^2 \frac{\theta}{2}. \tag{14}$$

Again if we set $c_1 = \pm i c_2$ then the Eq. (13) to be

$$U(\xi) = \frac{2(k\omega - 3(n^2 + l^2))}{k^2}\frac{1}{1 \mp i\sinh\theta}. \tag{15}$$

If we set $k\omega < (3n^2 + 3l^2)$ and $c_1 \neq c_2$, then we have an trigonometric form of the Eq. (13).

$$U(\xi) = \frac{2(k\omega - 3(n^2 + l^2))}{k^2}\frac{2c_1c_2}{2c_1c_2 + \cos\theta(c_1^2 + c_2^2) - i\sin\theta(c_1^2 - c_2^2)}. \tag{16}$$

If we set $c_1 = c_2$, then the Eq. (16) become,

$$U(\xi) = \frac{(k\omega - 3(n^2 + l^2))}{k^2} sce^2 \frac{\theta}{2}. \tag{17}$$

Again if we set $c_1 = \pm i c_2$, then the Eq. (16) to be

$$U(\xi) = \frac{2(k\omega - 3(n^2 + l^2))}{k^2}\frac{1}{1 \pm \sin\theta}. \tag{18}$$

Set-02: When $\omega = \dfrac{(12n^2 + 12l^2 + a_1{}^2)}{4k}$.

Substituting the value a_0, a_1, a_2 and $S(\xi)$in Eq. (11), then we archive,

$$U(\xi) = \frac{a_1^2}{2k^2}\left\{\frac{c_2 e^{\theta}}{(c_1 + c_2 e^{\theta})} - \frac{c_2^2 (e^{\theta})^2}{(c_1 + c_2 e^{\theta})^2}\right\}, \tag{19}$$

where $\theta = \dfrac{a_1}{2k^2}\xi$ and $\xi = kx + ny + lz - \dfrac{(12n^2 + 12l^2 + a_1{}^2)}{4k}t$.

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Eq. (16) can be written in turn of hyperbolic function where $c_1 \neq c_2$,

$$U(\xi) = \frac{a_1^2}{2k^2} \frac{2c_1c_2}{2c_1c_2 + (c_1^2 + c_2^2)\cosh\theta - (c_1^2 - c_2^2)\sinh\theta}. \tag{20}$$

If we set $c_1 = c_2$ then Eq. (20) has the form

$$U(\xi) = \frac{a_1^2}{4k^2} sceh^2\frac{\theta}{2}. \tag{21}$$

Again if we set$c_1 = \pm ic_2$, then Eq. (20) reduce to

$$U(\xi) = \frac{a_1^2}{2k^2} \frac{1}{1 \mp i\sinh\theta}. \tag{22}$$

Set-03: When $a_1 = a_1, k = \dfrac{(12n^2 + 12l^2 + a_1{}^2)}{4}$.

Substituting the value a_0, a_1, a_2 and $S(\xi)$in Eq. (9), then we archive.

$$U(\xi) = \frac{8\omega^2 a_1^2}{(a_1^2 + 12n^2 + 12l^2)^2} \left\{ \frac{c_2 e^\theta}{(c_1 + c_2 e^\theta)} - \frac{c_2^2 (e^\theta)^2}{(c_1 + c_2 e^\theta)^2} \right\}, \tag{23}$$

where $\theta = \dfrac{8\omega^2 a_1}{(a_1{}^2 + 12n^2 + 12l^2)^2}\xi$ and $\xi = \dfrac{(12n^2 + 12l^2 + a_1{}^2)}{4k} x + ny + lz - \omega t$

If we set the $c_1 \neq c_2$, then we reach the hyperbolic solution

$$U(\xi) = \frac{8\omega^2 a_1^2}{(a_1^2 + 12n^2 + 12l^2)^2} \frac{2c_1c_2}{2c_1c_2 + (c_1^2 + c_2^2)\cosh\theta - (c_1^2 - c_2^2)\sinh\theta}. \tag{24}$$

If we set $c_1 = c_2$, then Eq. (24) covert into the following form

$$U(\xi) = \frac{4\omega^2 a_1^2}{(a_1^2 + 12n^2 + 12l^2)^2} sceh^2\frac{\theta}{2}. \tag{25}$$

Again if we set $c_1 = \pm ic_2$, then Eq. (24) become

$$U(\xi) = \frac{8\omega^2 a_1^2}{(a_1{}^2 + 12n^2 + 12l^2)^2} \frac{1}{1 \mp i\sinh\theta}. \tag{26}$$

Case-02: When $a_0 = \dfrac{k\omega - 3n^2 - 3l^2 +}{3k^2}, a_2 = -2k^2$.

Substituting these values in the rest three equations of the above system of equation and solving them. We finally reach to three set solutions

Set-01:$a_1 = \pm 2\sqrt{k\omega - (3n^2 + 3l^2)}$, where ω, l, k, n are constants.

Set-02:$\omega = \dfrac{(12n^2 + 12l^2 + a_1{}^2)}{4k}$, where a_1, l, k, n are constants.

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Set-03:$k=\dfrac{(12n^2+12l^2+a_1{}^2)}{4}$. , where a_1, l, n are constants. where $S(\xi)=c_1+c_2\exp\left(\dfrac{a_1\xi}{k^2}\right)$ and $\xi = kx+ny+lz-\omega t$.

Set-01: when $a_1 = \pm 2\sqrt{k\omega-(3n^2+3l^2)}$.

Substituting the value a_0, a_1, a_2 and $S(\xi)$in Eq. (11), then we achieve.

$$U(\xi)=\frac{2\left(k\omega-3\left(n^2+l^2\right)\right)}{3k^2}\left\{\frac{1}{2}+3\left(\frac{c_2e^{\pm\theta}}{\left(c_1+c_2e^{\pm\theta}\right)}-\frac{c_2^2\left(e^{\pm\theta}\right)^2}{\left(c_1+c_2e^{\pm\theta}\right)^2}\right)\right\}, \tag{27}$$

where $\theta=\dfrac{\sqrt{(3(n^2+l^2)-k\omega)}}{k^2}\xi$ and $\xi = kx+ny+lz-\omega t$.

If we set the condition $k\omega<(3n^2+3l^2)$ and $c_1 \neq c_2$. Then we reach the hyperbolic solution

$$U(\xi)=\frac{2(k\omega-3(n^2+l^2))}{3k^2}\left\{\frac{1}{2}+3\frac{2c_1c_2}{2c_1c_2+(c_1^2+c_2^2)\cosh\theta-(c_1^2-c_2^2)\sinh\theta}\right\}. \tag{28}$$

If we set $c_1 = c_2$ then Eq. (28) become

$$U(\xi)=\frac{2(k\omega-3(n^2+l^2))}{3k^2}\left\{\frac{1}{2}+\frac{3}{2}sceh^2\frac{\theta}{2}\right\}. \tag{29}$$

If we set $c_1 = \pm ic_2$, then Eq. (28) to be the following form

$$U(\xi)=\frac{2(k\omega-3(n^2+l^2))}{3k^2}\left\{\frac{1}{2}+\frac{3}{1\mp i\sinh\theta}\right\}. \tag{30}$$

If we set the condition $k\omega>(3n^2+3l^2)$ and $c_1 \neq c_2$ the Eq. (27) has developed in hyperbolic form

$$U(\xi)=\frac{2(k\omega-3(n^2+l^2))}{3k^2}\left\{\frac{1}{2}+3\frac{2c_1c_2}{2c_1c_2+\cos\theta(c_1^2+c_2^2)-i\sin\theta(c_1^2-c_2^2)}\right\}. \tag{31}$$

If we set $c_1 = c_2$, then Eq. (31) become

$$U(\xi)=\frac{2(k\omega-3(n^2+l^2))}{3k^2}\left\{\frac{1}{2}+\frac{3}{2}sce^2\frac{\theta}{2}\right\}. \tag{32}$$

If we set $c_1 = \pm ic_2$, then we get,

$$U(\xi)=\frac{2(k\omega-3(n^2+l^2))}{3k^2}\left\{\frac{1}{2}+\frac{3}{1\pm\sin\theta}\right\}. \tag{33}$$

Set-02: when $\omega = \dfrac{(12n^2+12l^2-a_1{}^2)}{4k}$.

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Substituting the value a_0, a_1, a_2 and $S(\xi)$in Eq. (11), then we archive.

$$U(\xi) = \frac{a_1^2}{12k^2} + \frac{a_1^2}{2k^2}\left\{\frac{c_2 e^{\theta}}{(c_1 + c_2 e^{\theta})} - \frac{c_2^2 (e^{\theta})^2}{(c_1 + c_2 e^{\theta})^2}\right\}, \tag{34}$$

where $\theta = \frac{a_1}{2k^2}\xi$ and $\xi = kx + ny + lz - \omega t$.

Eq. (34) can be written in turn of hyperbolic function where $c_1 \neq c_2$,

$$U(\xi) = \frac{a_1^2}{2k^2}\left(-\frac{1}{6} + \frac{2c_1 c_2}{2c_1 c_2 + (c_1^2 + c_2^2)\cosh\theta - (c_1^2 - c_2^2)\sinh\theta}\right). \tag{35}$$

If we set $c_1 = c_2$ then the Eq. (35) is reduce in

$$U(\xi) = \frac{a_1^2}{2k^2}\left\{-\frac{1}{6} + \frac{1}{2} sceh^2 \frac{\theta}{2}\right\}. \tag{36}$$

If we set $c_1 = \pm i c_2$ in Eq. (35), then we get

$$U(\xi) = \frac{a_1^2}{2k^2}\left\{-\frac{1}{6} + \frac{1}{1 \mp i \sinh\theta}\right\}. \tag{37}$$

Set-03: when $a_1 = a_1, k = \dfrac{(12n^2 + 12l^2 - a_1{}^2)}{4}, n = n, l = l, \omega = \omega$

Substituting the value a_0, a_1, a_2 and $S(\xi)$in Eq. (11), then we archive.

$$U(\xi) = -\frac{4\omega^2 a_1^2}{3(12n^2 + 12l^2 - a_1^2)^2}\left\{1 - 6\left(\frac{c_2 e^{\theta}}{(c_1 + c_2 e^{\theta})} - \frac{c_2^2 (e^{\theta})^2}{(c_1 + c_2 e^{\theta})^2}\right)\right\}, \tag{38}$$

where $\theta = \dfrac{8\omega^2 a_1}{(12n^2 + 12l^2 - a_1^2)^2}\xi$ and $\xi = kx + ny + lz - \omega t$.

If we set $c_1 \neq c_2$, then we reach the hyperbolic solution

$$U(\xi) = -\frac{4\omega^2 a_1^2}{3(12n^2 + 12l^2 - a_1^2)^2}\left\{1 - 6\frac{2c_1 c_2}{2c_1 c_2 + (c_1^2 + c_2^2)\cosh\theta + (c_1^2 - c_2^2)\sinh\theta}\right\}. \tag{39}$$

If we set $c_1 = c_2$, then the Eq. (39) become

$$U(\xi) = -\frac{8\omega^2 a_1^2}{(12n^2 + 12l^2 - a_1^2)^2}\left\{1 - 3 sceh^2 \frac{\theta}{2}\right\}. \tag{40}$$

Again if we set $c_1 = \pm i c_2$ in the Eq. (39) then we get,

$$U(\xi) = -\frac{8\omega^2 a_1^2}{(12n^2 + 12l^2 - a_1^2)^2}\left\{1 - \frac{6}{1 \mp i \sinh\theta}\right\}. \tag{41}$$

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3.2. The (1 + 1) Dimensional Oskolkov equation

In this subsection we implement the modified simple equation method for (1 + 1) Dimensional Oskolkov Equation in the following form

$$U_t - \beta U_{xxt} - \alpha U_{xx} + UU_x = 0 \tag{42}$$

Where β, α are arbitrary constant and $U(x,t)$ is an unknown function. Using the traveling wave variable $U(x,t) = U(\xi)$ and $\xi = kx - \omega t$where $k$is a constant and ω is wave speed. Now we convert the Eq. (42) into the following Ordinary differential equation.

$$2k^2\omega\beta U'' - 2\alpha k^2 U' - 2\omega U + kU^2 = 0, \tag{43}$$

where the prime represent the derivative with respect to ξ.

According to step-3, the balance number between the linear term $U''$and the nonlinear term U^2 is$N = 2$ so the solution of the Eq. (42) is similar to the solution of the Eq. (11)

Now differential Eq. (11) with respect to ξ we get

$$U' = a_1\left(\frac{S'}{S}\right) - a_1\left(\frac{S'}{S}\right)^2 + \frac{2a_2S'S''}{S^2} - 2a_2\left(\frac{S'}{S}\right)^3$$

$$U'' = \frac{a_1S'''}{S} + (2a_1 + 6a_2)\left(\frac{S'}{S}\right)^3 + 2a_2\left(\frac{S''}{S}\right)^2 - 10a_1\left(\frac{S'}{S}\right)^2\left(\frac{S''}{s}\right) - (3a_1 - 2a_2) \times \left(\frac{S'S'''}{S^2}\right).$$

Substituting the value of U, U', U'' in Eq. (43) and equating the coefficients S^0, S^{-1},S^{-2}, S^{-3}, S^{-4} are equal to zero. Then we get system of equation

$$\frac{1}{2}ka_0^2 - \omega a_0 = 0$$

$$\beta\omega k^2 a_1 S''' - \alpha k^2 a_1 S'' + S'(ka_0a_1 - \omega a_1) = 0$$

$$(S')^2\left(-\omega a_2 + \alpha k^2 a_1 + ka_0a_1 + \frac{1}{2}ka_1^2\right) - 3\beta\alpha k^2 a_1 S''S'$$

$$+2\beta\omega k^2 a_2 (S'')^2 - 2\alpha k^2 a_2 S'S'' + 2\beta\omega k^2 a_2 S'S''' = 0$$

$$-10\beta\omega k^2 a_2 S''(S')^2 + \left(ka_2a_1 + 2\beta\omega k^2 a_1 + 2\alpha k^2 a_2\right)(S')^3 = 0$$

$$(S')^4\left(6\beta\omega k^2 a_2 + \frac{1}{2}ka_2^2\right) = 0$$

Now solve the first and last equation of the above system of equation. Then we estimate two set solutions.

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$a_0 = 0, a_2 = -12\beta\omega k$ and $a_0 = \dfrac{2\omega}{k}, a_2 = -12\omega k$

Case-01: when $a_0 = 0, a_2 = -12\beta\omega k$.

Putting these values in the remaining equation of the system of equation. Then we achieve two set solution

$$\omega = \frac{1}{5}\sqrt{\frac{6}{\beta}}\alpha k, a_1 = -\frac{24}{5}\alpha k, S(\xi) = c_1 + c_2 \exp\left(\frac{1}{60}\frac{12\alpha k + 5a_1}{\beta\omega k}\xi\right)$$

and $k = \dfrac{5\omega}{\alpha}\sqrt{-\frac{\beta}{6}}, a_1 = 0, S(\xi) = c_1 + c_2 \exp\left(\dfrac{1}{60}\dfrac{12\alpha k + 5a_1}{\beta\omega k}\xi\right)$

Set-01: when $\omega = \dfrac{1}{5}\sqrt{\frac{6}{\beta}}\alpha k, a_1 = -\frac{24}{5}\alpha k$.

Now we substitute the above value in Eq. (11). Then we achieve

$$U(\xi) = \frac{2}{5}\sqrt{\frac{6}{\beta}}\frac{\alpha c_2 e^{-\theta}}{(c_1 + c_2 e^{-\theta})}\left(2 - \frac{c_2 e^{-\theta}}{c_1 + c_2 e^{-\theta}}\right), \tag{44}$$

where $\theta = \dfrac{1}{6k}\sqrt{\dfrac{6}{\beta}}\xi$ and $\xi = kx - \omega t$.

Now we set $\beta > 0$ and $c_1 \neq c_2$ then the Eq. (44) has developed form

$$U(\xi) = \frac{2\alpha}{5}\sqrt{\frac{6}{\beta}}\frac{2c_1c_2 + c_2^2(\cosh\theta - \sinh\theta)}{2c_1c_2 + \cosh\theta(c_1^2 + c_2^2) + \sinh\theta(c_1^2 - c_2^2)}. \tag{45}$$

Now if we set $c_1 = c_2$, then Eq. (45) to be the following form

$$U(\xi) = \frac{\alpha}{10}\sqrt{\frac{6}{\beta}} sceh\left(\theta/2\right)\{2 + (\cosh\theta - \sinh\theta)\}. \tag{46}$$

Again if we set $c_1 = \pm i c_2$, then Eq. (45) is reduce in

$$U(\xi) = \frac{\alpha}{5}\sqrt{\frac{6}{\beta}}\frac{\{\pm 2 - i(\cosh\theta - \sinh\theta)\}}{1 \pm i\sinh\theta}. \tag{47}$$

Now we set $\beta < 0$ and $c_1 \neq c_2$, then the Eq. (44) has developed form

$$U(\xi) = \frac{2\alpha}{5}\sqrt{\frac{6}{\beta}}\frac{2c_1c_2 + c_2^2(\cos\theta - i\sin\theta)}{2c_1c_2 + \cos\theta(c_1^2 + c_2^2) + i\sin\theta(c_1^2 - c_2^2)}. \tag{48}$$

Now if we set $c_1 = c_2$, then Eq. (48) to be the following form

$$U(\xi) = \frac{\alpha}{10}\sqrt{\frac{6}{\beta}} sce\left(\theta/2\right)\{2 + (\cos\theta - i\sin\theta)\}. \tag{49}$$

Again if we set $c_1 = \pm i c_2$ then Eq. (48) is reduce in

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$$U(\xi) = \frac{\alpha}{5}\sqrt{\frac{6}{\beta}}\frac{\{(\sin\theta - i\cos\theta) \pm 2\}}{1 \mp \sin\theta}. \tag{50}$$

Set-02: when $k = \frac{5\omega}{\alpha}\sqrt{-\frac{\beta}{6}}$and $a_1 = 0$

Substitute the above value of in the Eq. (11). Then we reach the solution

$$U(\xi) = -\frac{2\alpha}{5}\sqrt{-\frac{6}{\beta}}\frac{c_2^2(e^\theta)^2}{(c_1 + c_2 e^\theta)^2}, \tag{51}$$

where $\theta = \frac{\alpha\xi}{5\beta\omega}$ and $\xi = kx - \omega t$.

Now we set $\beta < 0$ and $c_1 \neq c_2$ then the Eq. (51) has developed form

$$U(\xi) = -\frac{2\alpha}{5}\sqrt{-\frac{6}{\beta}}\frac{c_2^2(\cosh\theta + \sinh\theta)}{2c_1c_2 + \cosh\theta(c_1^2 + c_2^2) - \sinh\theta(c_1^2 - c_2^2)}. \tag{52}$$

If we set $c_1 = c_2$, then Eq. (52) can be written in the following form

$$U(\xi) = -\frac{\alpha}{10}\sqrt{-\frac{6}{\beta}} sceh\left(\theta/2\right)(\cosh\theta + \sinh\theta). \tag{53}$$

Again if we set$c_1 = \pm ic_2$, then Eq. (52) attain in the following form

$$U(\xi) = -\frac{\alpha}{5}\sqrt{-\frac{6}{\beta}}\frac{\cosh\theta + \sinh\theta}{1 \mp i\sinh\theta}. \tag{54}$$

Case-02: When $a_0 = \frac{2\omega}{k}, a_2 = -12\beta\omega k$.

Putting these values in the remaining equation of the system and dissolve the obtaining differential equation. We achieve the given the solution

$$S(\xi) = c_1 + c_2\exp\left(\frac{1}{60}\frac{12\alpha k + 5a_1}{\lambda\omega k}\xi\right)$$

Putting the solution in the remaining equation of the above system and solve them. Finally We get the solution

$$S(\xi) = c_1 + c_2\exp\left(\frac{1}{60}\frac{12\alpha k + 5a_1}{\lambda\omega k}\xi\right)$$

$$\omega = \frac{1}{5}\sqrt{-\frac{6}{\beta}}\alpha k, a_1 = -\frac{24}{5}\alpha k \text{ and } k = \frac{5\omega}{\alpha}\sqrt{\frac{\beta}{6}}, a_1 = 0.$$

Set-01: when $\omega = \frac{1}{5}\sqrt{-\frac{6}{\beta}}\alpha k$ and $a_1 = -\frac{24}{5}\alpha k$.

Substitute the above value of in the Eq. (11). Then we reach the solution

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$$U(\xi)=\frac{2\alpha}{5}\sqrt{\frac{6}{\beta}}+\frac{12}{5}\frac{\alpha}{\sqrt{-6\beta}}\frac{c_2e^{\theta}}{c_1+c_2e^{\theta}}\left(2+\frac{c_2e^{\theta}}{c_1+c_2e^{\theta}}\right), \tag{55}$$

where $\theta=-\frac{\xi}{k\sqrt{-6\beta}}$ and $\xi=kx-\omega t$

Now we set $\beta<0$ and $c_1\neq c_2$, then the Eq. (55) is

$$U(\xi)=\frac{2\alpha}{5}\sqrt{-\frac{6}{\beta}}+\frac{12}{5}\frac{\alpha}{\sqrt{-6\beta}}\frac{2c_1c_2+3c_2^2(\cosh\theta+\sinh\theta)}{2c_1c_2+\cosh\theta(c_1^2+c_2^2)-\sinh\theta(c_1^2-c_2^2)}. \tag{56}$$

If set $c_1=c_2$, then Eq. (56) reduce

$$U(\xi)=\frac{2\alpha}{5}\sqrt{\frac{6}{\beta}}+\frac{3}{5}\frac{\alpha}{\sqrt{-6\beta}}sceh(\theta/2)(2-3(\cosh\theta+\sinh\theta)). \tag{57}$$

Also we set $c_1=\pm ic_2$, then Eq. (56) become

$$U(\xi)=\frac{2\alpha}{5}\sqrt{\frac{6}{\beta}}+\frac{12}{5}\frac{\alpha}{\sqrt{-6\beta}}\left\{\frac{1}{1\mp i\sinh\theta}-\frac{3}{2}\frac{\cosh\theta+\sinh\theta}{\sinh\theta\mp i}\right\}. \tag{58}$$

Now we set $\beta>0$ and $c_1\neq c_2$, then the Eq. (55) is

$$U(\xi)=\frac{2\alpha}{5}\sqrt{\frac{6}{\beta}}+\frac{12}{5}\frac{\alpha}{\sqrt{-6\beta}}\frac{2c_1c_2+3c_2^2(\cos\theta+i\sin\theta)}{2c_1c_2+\cos\theta(c_1^2+c_2^2)-i\sin\theta(c_1^2-c_2^2)}. \tag{59}$$

If set $c_1=c_2$, then Eq. (59) to be

$$U(\xi)=\frac{2\alpha}{5}\sqrt{\frac{6}{\beta}}+\frac{3}{5}\frac{\alpha}{\sqrt{-6\beta}}sce(\theta/2)(2+3(\cos\theta+i\sin\theta)). \tag{60}$$

Also we set $c_1=\pm ic_2$, then Eq. (59) become

$$U(\xi)=\frac{2\alpha}{5}\sqrt{\frac{6}{\beta}}+\frac{12}{5}\frac{\alpha}{\sqrt{-6\beta}}\left\{\frac{1}{1\pm\sin\theta}+\frac{3}{2}\frac{\sin\theta-i\cos\theta}{1\mp\sin\theta}\right\}. \tag{61}$$

Set-02: when$k=\frac{1}{5}\sqrt{\frac{\beta}{6}}\alpha k$ and $a_1=0$.

Now we substitute the value in the Eq. (11). Then we obtain the solution

$$U(\xi)=\frac{2\alpha}{5}\sqrt{\frac{6}{\beta}}\left(1-\frac{c_2^2(e^{\theta})^2}{(c_1+c_2e^{\theta})^2}\right), \tag{62}$$

where $\theta=\frac{\alpha\xi}{5\beta\omega}$ and $\xi=kx-\omega t$.

Now we set $\beta<0$ and $c_1\neq c_2$ then the Eq. (62) has developed form

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$$U(\xi) = \frac{2\alpha}{5}\sqrt{\frac{6}{\beta}}\frac{c_1^2(\cosh\theta - \sinh\theta) + 2c_1c_2}{2c_1c_2 + \cosh\theta(c_1^2 + c_2^2) - \sinh\theta(c_1^2 - c_2^2)}. \tag{63}$$

If we set $c_1 = c_2$, then Eq. (63) to be

$$U(\xi) = \frac{\alpha}{10}\sqrt{\frac{6}{\beta}}((\cosh\theta + \sinh\theta) + 2)sceh(\theta/2). \tag{64}$$

Again if we set $c_1 = \pm ic_2$, then Eq. (63) to be

$$U(\xi) = \frac{\alpha}{5}\sqrt{\frac{6}{\beta}}\left(\frac{(2 \pm i(\cosh\theta + \sinh\theta))}{(1 \mp i\sinh\theta)}\right). \tag{65}$$

4. Results & discussion

In this section, we will discuss the physical representation of the obtained exact and solitary wave solution to the (3 + 1) Dimensional KP equation and (1 + 1) Dimensional Oskolkov equation. We represent these solutions in graphical and verify about the kind of solution.

4.1. The solution and discussion of KP equation

In this subsection we implement MSE method to capture of the traveling wave solution Eqs. (13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 39, 40, 41) from the KP equation. The solution Eqs. (13, 14, 15, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 35, 36, 37, 38, 39, 40, 41) are all hyperbolic function solution and Eqs. (16, 17, 18, 31, 32, 33) are all comes from trigonometric function. All solution has periodic shape, Fig. 1(a) cusp wave comes from Eq. (28) for the parametric values $y = z = 0, n = l = c_1 = 1, k = c_2 = 2, \omega = 3$. The shape of the solutions Eqs. (29, 30, 35, 36, 37) gives same type like the Fig. 1 (a). Fig. 1(b) shows bell type soliton solution of the Eq. (21) for parametric values $c_2 = n = l = c_1 = 1, k = 2$ along $y = z = 0$. The solution Eqs. (13, 14, 15, 20, 21, 22, 24, 25, 26) are also represent the bell type solution. On the other hand, the solution Eqs. (16, 17, 18, 31, 32, 33) are all comes from trigonometric function and they progress periodically. Fig. 1(c) shows bright periodic solution of real part of the Eq. (31) for the parametric values $y = z = 0$, $n = l = c_1 = 1, k = 2.5, c_2 = 2, \omega = 3$. The solution Eq. (16) gives same type figure like Fig. 1 (c). The trigonometric solution of the Eqs. (17, 18, 32, 33) are gives the same figure but dark periodic. Fig. 1(d) shows periodic solution of imaginary part of the Eq. (31) for the parametric values $y = z = 0, n = l = c_1 = 1, k = 2.5, c_2 = 2, \omega = 3$. The solution Eq. (16) gives same type figure like Fig. 1 (d). Fig. 1(e) represents the kink type solution comes from Eq. (22) for the parametric values $n = l = c_1 = 1, k = 1, c_2 = 2, \omega = 2$, along $y = z = 0$. Fig. 1(f) shows Bright periodic solution of the

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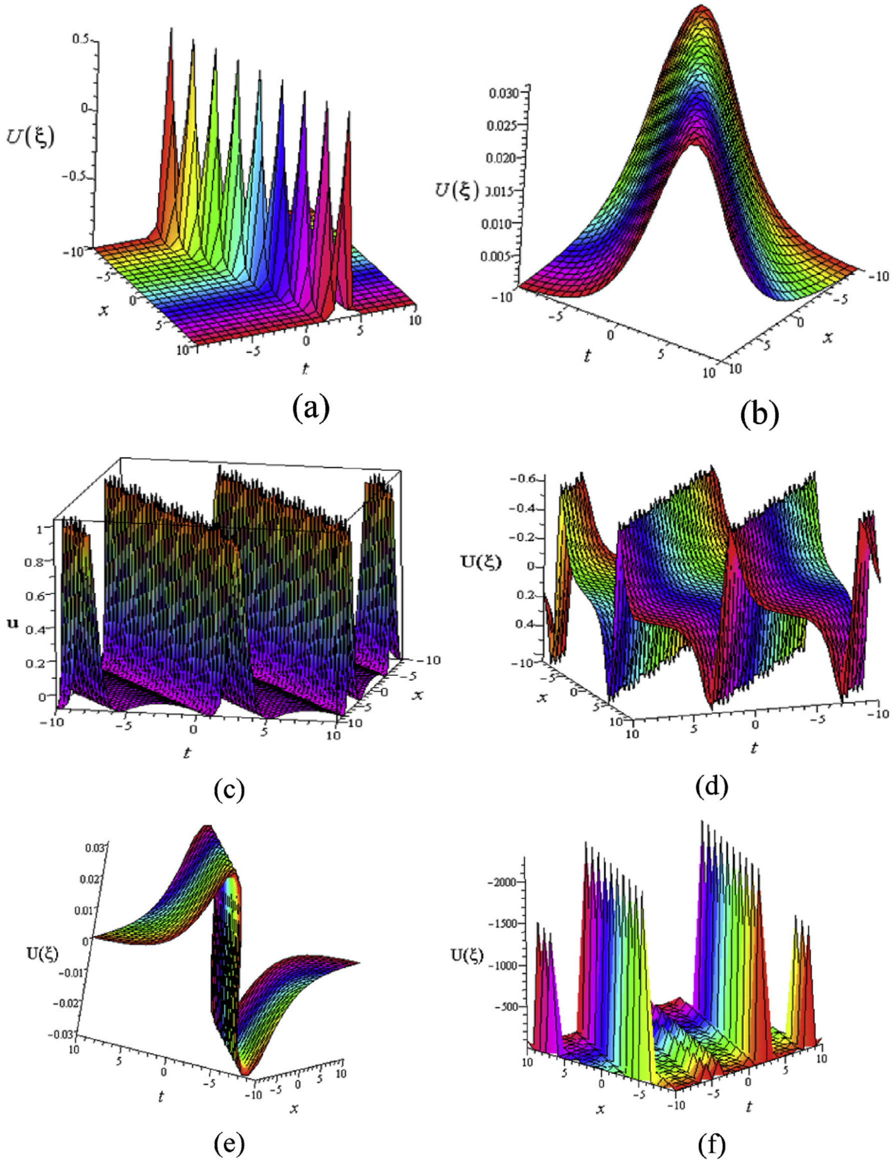

Fig. 1. (a): Shape of cusp wave solution of Eq. (28), (b): Shape of Bell soliton solution of Eq. (21), (c) and (d): Periodic solution of Eq. (31) for $y = z = 0, n = l = c_1 = 1, k = 2.5, c_2 = 2, \omega = 3$, (e): Kinky periodic solution of Eq. (22) and (f): Bright Periodic solution of Eq. (17) for $y = z = 0, n = l = c_1 = 1$, $k = 1, c_2 = 2, \omega = 2$.

Eq. (17) for parametric values $y = z = 0, n = l = c_1 = k = 1, c_2 = \omega = 2$. The solution Eq. (32) gives same type figure like Fig. 1(f).

4.2. (1 + 1) Dimensional Oskolkov equation

In this subsection, we illustrated some obtained solutions achieved via MSE method for the Oskolkov equation. Here we capture the traveling wave solutions of the Eqs. (44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65)

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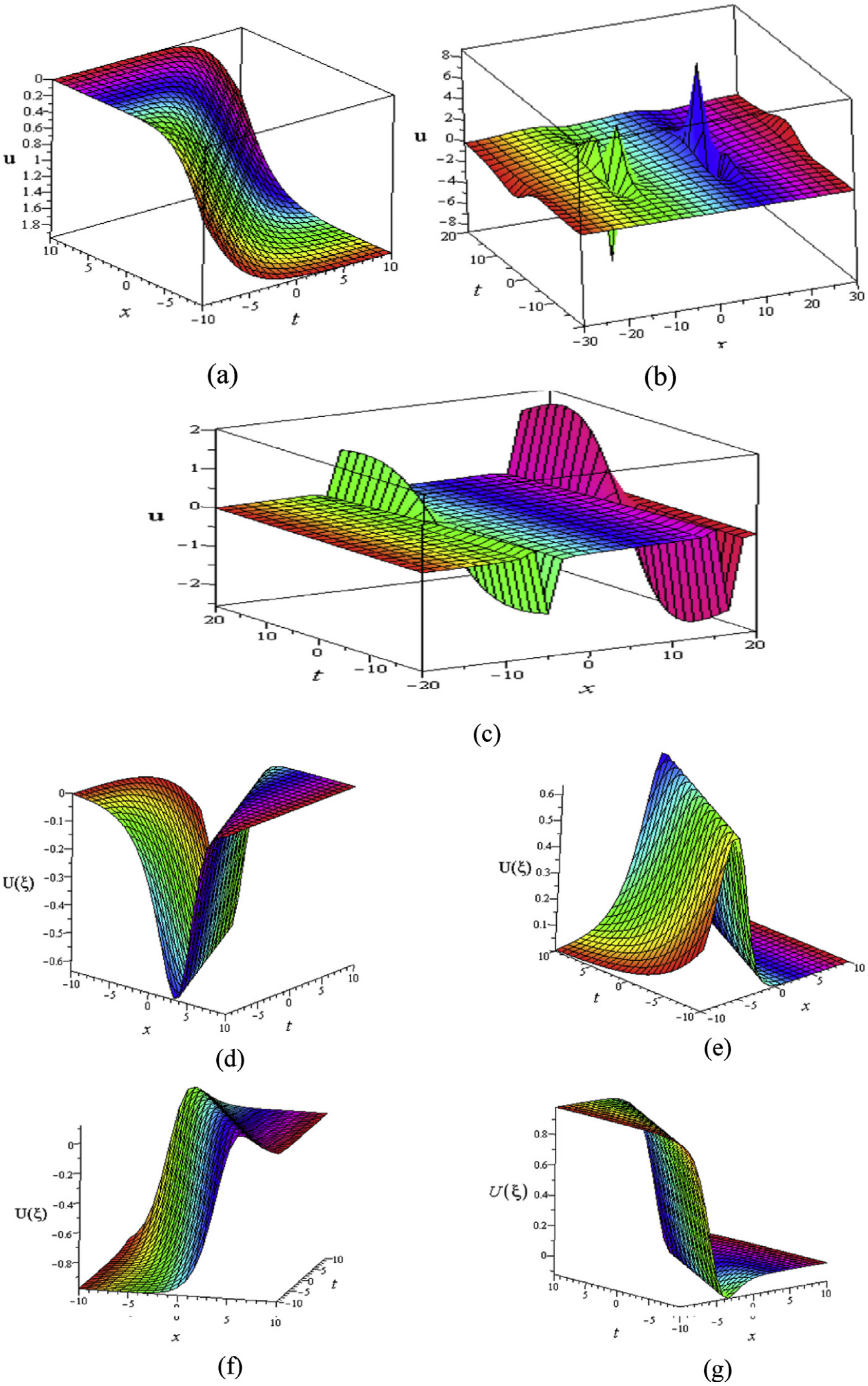

Fig. 2. (a): Kink wave comes from Eq. (52) for $\beta = -1, \alpha = 2, c_1 = 1, c_2 = 2, \omega = o.1$, (b): Periodic Breather wave solutions of the Eq. (64) for $\beta = -2, \alpha = 1, c_1 = c_2 = \omega = 2$ and (c): the real part of Eq. (61) for the parameters $\beta = 2, \alpha = 0.05, c_1 = c_2 = 2, k = -1$, (d): Dark bell shape solution of the Eq. (58) for $\beta = 2, \alpha = k = 1, c_1 = Ic_2, c_2 = 2$, (e): Bright bell shape solution of the Eq. (58) for $\beta = 2, \alpha = -1, k = 1, c_1 = Ic_2, c_2 = 2$, (f): Anti-Kink type solution of the Eq. (58) for $\beta = 2, \alpha = k = 1, c_1 = Ic_2, c_2 = 2$and (g): Kink type solution of the Eq. (58) for $\beta = 2, \alpha = -1, k = 1, c_1 = Ic_2, c_2 = 2$.

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from the Oskolkov equation. The solution Eqs. (44, 45, 46, 51, 52, 53, 54, 55, 56, 57, 58, 63, 64, 65) are comes in terms of hyperbolic function; they exhibits solitonic natures. Fig. 2(a) Kink wave comes from Eq. (52) for the parametric values $\beta = -1$, $\alpha = 2, c_1 = 1, c_2 = 2, \omega = o.1$. The shape of the solutions Eqs. (45, 46, 47, 52, 53, 54, 55, 56, 57, 58) are similar shape like the Fig. 2 (a). On the other hand, the solution Eqs. (48, 49, 50, 59, 60, 61) and for $\beta < 0$Eq. (62) are all comes from trigonometric function and they progress periodically. Fig. 2(b) shows Periodic Breather wave soliton solution of the real part of Eq. (64) for parametric values $\beta = -2, \alpha = 1, c_1 = c_2 = \omega = 2$. The solution Eqs. (48, 50, 59, 60, 61, 63, 64, 65) are also represent the periodic breather wave solution like Fig. 2 (b). Fig. 1(c) shows periodic breather wave solution of Eq. (61) for $\alpha = 0.05, c_1 = c_2 = 2, k = -1\beta = 2$. Fig. 2(d) represent dark bell shape solution of the real part of Eq. (58) for the parametric values $\beta = 2, \alpha = k = 1, c_1 = Ic_2, c_2 = 2$.For $\alpha = -1$ we found bright bell shape solution as show in Fig. 2(e). Fig. 2(f) represent Anti-kink type solution of the imaginary part of Eq. (58) for the parametric values $\beta = 2, \alpha = k = 1, c_1 = Ic_2$, $c_2 = 2$.For $\alpha = -1$ we found Kink type solution as show in Fig. 2(g).

5. Conclusions

In this paper, the main effort is to find, test and analyze the new traveling wave solutions and physical properties of nonlinear the $(3 + 1)$ dimensional KP equation and Oskolkov equation by applying reliable mathematical techniques. The Modified Simple Equation (MSE) method dramas a substantial way to find novel traveling wave solutions in-terms of exponential function from which we can build solitary and periodic wave solutions. This method offers solutions with free parameters that might be important to explain some intricate nonlinear physical phenomena related to kink wave, cusp wave, bell wave, periodic breather wave and periodic wave solutions. In physical science, the solutions of these nonlinear equations have many applications. The methods have been applied directly without requiring linearization, discretization, or perturbation. The obtained results demonstrate the reliability of the algorithm and give it a wider applicability to nonlinear fields. Higher directional waves, multi-waves and even new models of the equations with more nonlinearity are possible which will be our future task.

Declarations

Author contribution statement

Harun-Or-Roshid: Conceived and designed the experiments; Analyzed and interpreted the data.

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Mamunur Roshid: Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

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Competing interest statement

The authors declare no conflict of interest.

Additional information

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References

[1] M.A. Akbar, N.H.M. Ali, Exp-function method for doffing equation and new solutions of (2+1) dimensional dispersive long wave equations, Prog. Appl. Math. 1 (2) (2011) 30–42.

[2] A.M. Wazwaz, The extended tanh method for new compact and noncompact solutions for the KP-BBM and the ZK-BBM equations, Chaos Solit. Fractals 38 (5) (2008) 1505–1516.

[3] E. Yusufoglu, A. Bekir, On the extended tanh method applications of nonlinear equations, Int. J. Nonlinear Sci. 4 (1) (2007) 10–16.

[4] H.O. Roshid, M.F. Hoque, M.A. Akbar, New extended (*G'/G*)-expansion method for traveling wave solutions of nonlinear partial differential equations (NPDEs) in mathematical physics, Italian. J. Pure Appl. Math. 33 (2014) 175–190.

[5] H.O. Roshid, M.N. Alam, M.F. Hoque, M.A. Akbar, A new extended (*G'/G*)-expansion method to find exact traveling wave solutions of nonlinear evolution equations, Math. Stat. 1 (3) (2013) 162–166.

[6] M.L. Wang, J.L. Zhang, X.Z. Li, The (*G'/G*) expansion method and traveling wave solutions of nonlinear evolutions equations in mathematical physics, Phys. Lett. 372 (2008) 417–423.

[7] H. Naher, F.A. Abdullah, Further extension of the generalized and improved (G'/G)-expansion method for nonlinear evolution equation, J. Asso. Arab Unvert. Basic Appl. Sci. 19 (1) (2016) 52–58.

 https://doi.org/10.1016/j.heliyon.2018.e00756

[8] C.Q. Dai, J.F. Zhang, Jacobian elliptic function method for nonlinear Differential-difference equations, Chaos Solut. Fractals 27 (2006) 1042–1049.

[9] S.F. Tian, T.T. Zhang, Long-time asymptotic behavior for the Gerdjikov-Ivanov type of derivative nonlinear Schrödinger equation with time-periodic boundary condition, Proc. Am. Math. Soc. 146 (2018) 1713–1729.

[10] S.F. Tian, Asymptotic behavior of a weakly dissipative modified two-component Dullin–Gottwald–Holm system, Appl. Math. Lett. 83 (2018) 65–72.

[11] W.X. Ma, J.-H. Lee, A transformed rational function method and exact solutions to the (3 + 1)-dimensional Jimbo-Miwa equation, Chaos Solit. Fractals 42 (2009) 1356–1363.

[12] L.L. Feng, T.T. Zhang, Breather wave, rogue wave and solitary wave solutions of a coupled nonlinear Schrödinger equation, Appl. Math. Lett. 78 (2018) 133–140.

[13] Q. Zhou, Analytic study on optical solitons in a Kerr-law medium with an imprinted parity-time-symmetric mixed linear-nonlinear lattice, Proc. Rom. Acad. Ser. A 18 (2017) 223–230.

[14] S.F. Tian, Initial–boundary value problems for the general coupled nonlinear Schrödinger equation on the interval via the Fokas method, J. Differ. Equ. 262 (2017) 506–558.

[15] H. Triki, A.K. Turgut, M. Ekici, A. Sonmezoglu, M. Mirzazadeh, A.H. Kara, T. Aydemir, Some new exact wave solution and conservetion laws of potential KDV, Nonlinear Dynam. 89 (2) (2017) 501–508.

[16] K. Khan, M.A. Akbar, Traveling wave solutions of nonlinear evolution equations via the enhanced (G'/G)-expansion method, J Egypt. Math Soc 22 (2) (2014) 220–226.

[17] L.W. Li, G. Ma, Y. Weitian, Y. Yujia, W. Liu, Analytic study on interaction of some solitary soliton solution, Optic 164 (2018) 132–137.

[18] M. Khalfallah, New exact traveling wave solutions of the (3+1)-dimensional Kadomtsev-Petviashvili (KP) equation, Commun. Nonlinear Sci. Numer. Simulat. 14 (2009) 1169–1175.

[19] S. akcagil, T. aydemir, Omer faruk Gozukizil, Exact solutions of non-linear pseudoparabolic equation by using the (G'/G)-expansion method, NTMSCI 4 (2016) 51–66.

 https://doi.org/10.1016/j.heliyon.2018.e00756

[20] M. Alquran, Bright and dark soliton solutions to the Ostrovsky-Benjamin-Bona-Mahony (OS-BBM) equation, J. Math. Comput. Sci. 2 (2012) 15−22.

[21] K. Khan, M.A. Akbar, M.N. Alam, Traveling wave solutions of the nonlinear Drinfeld-Sokolov-Wilson equation and modified Benjamin-Bona-Mahony equations, J. Egypt. Math. Soc. 21 (2013) 233−240.

[22] G.A. Sviridyuk, A.S. Shipilov, On the stability of solutions of the oskolkov equation on a graph, ISSN 0012-2661, Differ. Equ. 46 (5) (2010) 742−747.

[23] S.M. El-Sayed, D. Kaya, The decomposition method for solving (2+1)-dimensional Boussinesq equation and (3+1) dimensional KP equation, Appl. Math. Comput. 157 (2004) 523−534.

[24] Y. Chen, Z. Yan, H. Zhang, New explicit solitary wave solutions for (2+1)-dimensional Boussinesq equation and (3+1)-dimensional KP equation, Phys. Lett. 307 (2003) 107−113.

[25] F.D. Xie, Z.Y. Yan, Compactons and noncompactons to three-dimensional Kadomtsev-Petviashvili equation with nonlinear dispersion, Chaos Solit. Fractals 36 (2008) 278−282.

[26] A.M. Wazwaz, Multiple-soliton solutions for the KP equation by Hirota's bilinear method and by the tanh-coth method, Appl. Math. Comput. 190 (2007) 633−640.

[27] T. Alagesan, A. Uthayakumar, K. Porsezian, Painlevé analysis and Bäcklund transformation for a three-dimensional Kadomtsev-Petviashvili equation, Chaos Solit. Fractals 8 (1997).

[28] M.A. Akbar, N.H.M. Ali, S.T. Mohyud-Din, Some new exact traveling wave solutions to the (3+1)-dimensional kadomtsev petviashvili equation, World Appl. Sci. J. 16 (11) (2012) 1551−1558.

[29] B. Dorizzi, B. Grammaticos, A. Ramani, P. Winternitz, Are all the equations of the Kadomtsev-Petviashvili hierarchy integrable, J. Math. Phys. 27 (1986) 2848−2852.

[30] H. Zhao, C. Bai, New doubly periodic and multiple soliton solutions of the generalized (3+1)-dimen-sional kadomtsev-Petviashvilli equation with variable coefficients, Chaos Solit. Fractals 30 (2006) 217−226.

[31] M. Mirzazadeh, Modified simple equation method and its applications to nonlinear partial differential equations, Inf. Sci. Lett. 3 (1) (2014) 1−9.

 https://doi.org/10.1016/j.heliyon.2018.e00756

[32] H.A. Arnous, Z.M. Ullah, M.A. Seithuti, P. Moshokoa, Mohammad Mirzazadeh, Anjan Biswas, Milivoj Belic, Nematicons in liquid crystals by modified simple equation method, Nonlinear Dynam. 88 (4) (2017) 2863−2872.

[33] E.M.E. Zayed, A note on the modified simple equation method applied to Sharma-Tasso-Olver equation, Appl. Math. Comput. 218 (7) (2011) 3962−3964.

[34] E.M.E. Zayed, S.A.H. Ibrahim, Exact solutions of nonlinear evolution equations in mathematical physics using the modified simple equation method, Chin. Phys. Lett. 29 (6) (2012).

[35] E.H.M. Zahran, M.M.A. Khater, The modified simple equation method and its applications for solving some nonlinear evolution equations in mathematical physics, Jokull 64 (5) (2014) 297−312.

 https://doi.org/10.1016/j.heliyon.2018.e00756