An Analytical-empirical Calculation of Linear Attenuation Coefficient of Megavoltage Photon Beams

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ABSTRACT

Background: In this study, a method for linear attenuation coefficient calculation was introduced.

Methods: Linear attenuation coefficient was calculated with a new method that base on the physics of interaction of photon with matter, mathematical calculation and x-ray spectrum consideration. The calculation was done for Cerrobend as a common radiotherapy modifier and Mercury.

Results: The values of calculated linear attenuation coefficient with this new method are in acceptable range. Also, the linear attenuation coefficient decreases slightly as the thickness of attenuating filter (Cerrobend or mercury) increased, so the procedure of linear attenuation coefficient variation is in agreement with other documents. The results showed that the attenuation ability of mercury was about 1.44 times more than Cerrobend.

Conclusion: The method that was introduced in this study for linear attenuation coefficient calculation is general enough to treat beam modifiers with any shape or material by using the same formalism; however, calculating was made only for mercury and Cerrobend attenuator. On the other hand, it seems that this method is suitable for high energy shields or protector designing.

Keywords

Cerrobend, Linear Attenuation Coefficient, Energy Spectral

Introduction

In radiation therapy, beam modifiers such as shield, wedge, compensator and MLC (Multi-Leaf Collimator) are used in order to deliver enough doses to a tumor and protect (Organs at Risk) OARs with achieving desirable dose distribution. In (Intensity Modulated Radiation Therapy) IMRT techniques including MLC base or compensator base, the linear attenuation coefficient is a very important factor to choose benefit beam modifier material and thickness and the quality of IM produced depends on the linear attenuation coefficient [1-6]. In narrow and monoenergy beam conditions, source and detector are assumed to be collimated and the measurement made at a short distance, so the attenuation takes place with just a simple exponential reduction law. In real therapy conditions, polychromatic beams passing though the material with different thicknesses by multiple scattering, it must be entered in treatment planning systems [7-11]. Since X-ray beam of the medical accelerators is broad energy spectrum, it must be considered in attenuation calculations and treatment planning systems [12]. In this study, a new

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method was introduced to calculate the linear attenuation coefficient. In this method, linear attenuation coefficient was calculated from integration equation considering the incident energy spectra and changing that happened in beam quality because of various thickness of absorber. Moreover, this calculation will be useful for designing radiation protector materials.

Material and Methods

Linear Attenuation Coefficient Calculation

Ideally, linear attenuation coefficient must be measured in narrow beam conditions but in real procedures or broad beams, scattered photons such as Compton photons cannot be ignored. In this paper, linear attenuation coefficient for various thicknesses of Cerrobend and mercury were calculated with the base of experimental dosimetry, mathematical calculations, physics of interaction of photon with matter and with new functional forms for Varian Linac photon spectra [13].

In this research, Cartesian coordinates system has been used because collimator of Varian as like most Linacs are rectangular. As shown in Figure 1, when primary photon beams (I_0) passes through the attenuator with thickness t_0 (mercury or Cerrobend in this research) the photon beams that receive the detector (I_1) have two components: scattered radiation (I_s) and primaryradiation (I_p). So,

$$I_1 = I_p + I_s \quad (1)$$

When there is not any attenuator filter, the primary photon beams (I_0) reduce only because of inverse square law, so in this situation the photon beams that receive the detector (I_2) is equal to:

$$I_2 = \frac{I_0}{D^2} \quad (2)$$

Reduction of beams in (I_p) is because of attenuator with t_0 thickness and inverse square law, thus (I_p) is equal to:

$$I_p = \frac{I_0 e^{-\mu t}}{D^2} \quad (3)$$

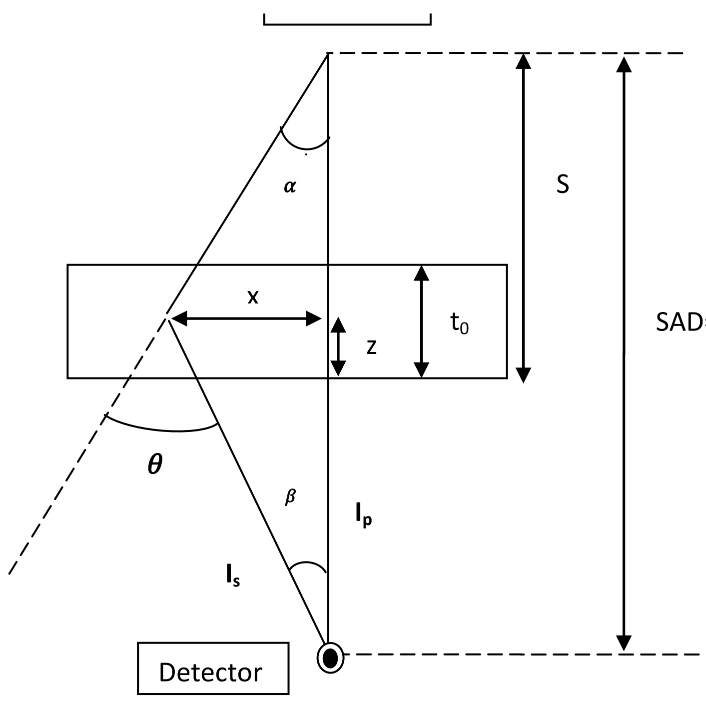

Figure 1: schematic diagram of scatter calculation geometry

For calculation of scattered radiation that receives the detector (I_s), volume element with dx, dy and dz dimensions were considered. Since the spectrum of a beam is not monoenergetic, the energy is the other variable. We used 6MV Varian spectrum (E) determined with a technique based on the effective spectrum method described in Ref [13].

Thus, the scattered intensity (I_s) has been calculated by calculating scatter due to an elemental volume and energy spectrum and integrating over four variables: the thickness of absorber, entire field size and energy spectrum, so we have quadruplet integral. Thus, (I_s) is equal to:

$$I_s = \frac{4I_0 \int_0^{t_0}\int_0^{\frac{y_f}{2}}\int_0^{\frac{x_f}{2}}\int_0^{E_e} N_e \frac{e^{-\mu t_1}}{r_1} \times \frac{e^{-\mu t_2}}{r_2} \left(\frac{d\sigma_0}{d\Omega} F_{KN}\right) \Psi(E)\, EdEdxdydz}{\int_{E=0}^{E_e} \Psi(E)\, EdE} \qquad (4)$$

In order to simplify equation (4), one can write:

$$I_s = I_0\, Int \qquad (5)$$

If the ratio of chamber readings with and without attenuator consider as (r), from equations (1 to 5):

$$r = \frac{I_1}{I_2} = \frac{\frac{I_0 e^{-\mu t}}{D^2} + I_0 Int}{I_0 / D^2} = e^{-\mu t} + D^2 Int \qquad (6)$$

Therefore, with measuring readings ratio for different attenuator thicknesses and solving the integration equation, the linear attenuation coefficient can be calculated for attenuator with arbitrary material or thickness.

Experimental Dosimetry

The measurement of readings ratio was made on Varian 2100C/D accelerator (Ahvaz Golestan Hospital) at 6 MV photon beams by using absolute dosimetry with Farmer ionization chamber (FC65) and Dose1 electrometer (scanditronix-wellhöfer) at 110cm distance from source (SSD=110 cm) (Figures 2-4).

Readings ratios were recorded for 6×6, 8×8, 10×10, 12×12 and 14×14 cm^2 field sizes modulated by mercury (z=80, ρ=13.53) with thicknesses (t=0, 0.65, 1, 1.5, 2, 3, 4, 5 and 6 cm) and Cerrobend (z=70.8, ρ=9.38) [14] with thicknesses (t=0, 1.35, 2.7, 4.05, 5.4, 6.75, 8.1 and 9.45cm) were placed on the Perspex shielding tray. Chen et al. [15] showed scatter from phantom is significant. In this research, the measurements were performed in air with related build-up cap according to IAEA TRS398 protocol [16]. Since reading ratio is reading of Farmer chamber in the presence of mercury (or Cerrobend) divided by reading of Farmer chamber without any attenuator at reference dose in water phantom (d=10 cm). The photon energy for all experiments was 6MV. The Cerrobend attenuators have cut with electronic block cutter system that user can have attenuator with desirable thicknesses.

For solving the integration equation, some parameters in equation (4) must be replaced as fallow: r_0 is the classical electron radius that is equal to 2.8182×10^{-13} cm, α is the ratio of photon energy to the electron rest mass energy, so $\alpha = \frac{h\upsilon}{m_0 c^2} = \frac{E}{0.511(Mev)}$ that E is integration variable and Ne is the number of electrons/cm^3 that is equal to 32.49×10^{23} for mercury and 22.92×10^{23} for Cerrobend. The other parameter $(\frac{d\sigma_0}{d\Omega} F_{KN})$ is differential of incoherent scattering cross section per electron and per unit solid angle in direction θ that is known as Klein_Nishina formula and is given by [17].

According to Figure 1 scattering angle, θ is equal to:

$$\frac{d\sigma_0}{d\Omega}F_{KN}=\frac{r_0^2}{2}\left(1+\cos^2\theta\right)F_{KN}=\frac{r_0^2}{2}\left(1+\cos^2\theta\right)\left\{\frac{1}{1+\alpha(1-\cos\theta)}\right\}^2\left\{1+\frac{\alpha^2(1-\cos\theta)^2}{\left[1+\alpha(1-\cos\theta)\right](1+\cos^2\theta)}\right\} \quad (7)$$

$$\theta=\alpha+\beta \quad (8)$$

$$\theta=\tan^{-1}\frac{D\left(\sqrt{x^2+y^2}\right)}{(S-Z)(D-S+Z)-x^2-y^2} \quad (9)$$

And also t_1, t_2, r_1 and r_2 in Equation 4 must be replaced by:

$$t_1=\frac{t_0-z}{S-z}r_1 \text{ where } r_1=\sqrt{x^2+y^2+(D-z)^2} \quad (10)$$

$$t_2=\frac{z}{D-s+z}r_2 \text{ where } r_2=\sqrt{x^2+y^2+(D-S+z)^2} \quad (11)$$

As mentioned before, a typical therapy beam is not monoenergetic, we replaced (E) in integration equation with Varian Linac 6MV photon spectra functional form that is specified by Ali et al. [16].

$$\psi(E)=\left[1+C_3+\left(\frac{E}{E_0}\right)^2\right]\left[ln\left(\frac{E_e(E_e-E)}{E}+1.65\right)-0.5\right]exp\left(-\mu_W(E)C_1^2-\mu_{Al}(E)C_2^2\right) \quad (12)$$

For Varian 6MV photons C_1=1.222, C_2=5.147, C_3=-1.186 and E_e=5.76 Mev

Parameterization of μ_W and μ_{Al} is extracted from table 3 of Ali's article. At the end, with replace all necessary parameters and equations and readings ratio from dosimetry in Equation (6); we have four variables (three for coordinates and one for energy) in integration equation. This is a complicated integration equation because attenuation coefficient is related to the energy and also is a part of integrand, so there is not simple or direct method to solve this integrated equation consisting of quadruplet integral. We solved this by Gaussian integration method in Matlab software and calculated attenuation coefficients for different thicknesses of mercury and Cerrobend.

Results

Linear Attenuation Coefficient Calculation

Linear attenuation coefficient of variable attenuator and field sizes calculated for 6MV photon beam from new method by solving a complicated integration equation consist of quadruplet integral described before. The values of Reading ratio and calculated linear attenuation coefficient for different Cerrobend and mercury thickness were tabulated in Tables 1 and 2.

The linear attenuation coefficient values of different Cerrobend and mercury thicknesses that were calculated for 6×6, 8×8, 10×10, 12×12 and 14×14 cm^2 field sizes are shown in Tables 1 and 2.

Figure 2: Farmer Ionization chamber (FC65)

Table 1: Reading ratio and calculated linear attenuation coefficient for different Cerrobend thickness

Cerrobend thickness (cm±0.01)	6×6 cm²		8×8 cm²		10×10 cm²		12×12 cm²		14×14 cm²	
	r	mu	r	mu	R	mu	r	mu	r	mu
1.35	48.5	0.514	48.8	0.491	49	0.472	49.3	0.447	51.2	0.395
2.7	26	0.477	26.3	0.457	27	0.427	26.9	0.405	28	0.364
4.05	14.6	0.453	14.8	0.433	15	0.409	15.3	0.378	16	0.337
5.4	8.3	0.439	8.4	0.419	8.6	0.392	8.8	0.359	9.2	0.315
6.75	4.7	0.431	4.8	0.409	4.9	0.382	5	0.346	5.3	0.293
8.1	2.7	0.423	2.8	0.4	2.9	0.369	2.9	0.331	3.1	0.28
9.45	1.3	0.42	1.4	0.4	1.4	0.368	1.5	0.33	1.6	0.27

Table 2: Reading ratio and calculated linear attenuation coefficient for different Mercury thickness

Mercury thickness (cm±0.01)	6×6 cm²		8×8 cm²		10×10 cm²		12×12 cm²		14×14 cm²	
	R	mu	r	mu	R	mu	R	mu	r	Mu
0.65	58.6	0.792	59	0.759	59	0.733	58.8	0.709	60.9	0.623
1	46.9	0.726	47	0.702	47.5	0.665	47.8	0.628	49.5	0.56
1.5	34	0.688	34.6	0.654	35	0.619	35.1	0.585	36.4	0.525
2	24.7	0.668	25	0.639	25.3	0.605	25.5	0.567	26.6	0.508
3	13.6	0.634	13.8	0.605	14	0.569	14.2	0.527	14.9	0.467
4	7	0.633	7.5	0.591	7.7	0.551	7.9	0.503	8.3	0.437
5	4	0.612	4.3	0.571	4.4	0.531	4.5	0.478	4.8	0.397
6	2	0.619	2.4	0.562	2.5	0.516	2.6	0.452	2.7	0.38

Figure 3: Dose 1 Electrometer

Discussion

When a beam modifier (compensator or shield) is placed in an x-ray beam, the characteristics of the beam change because of scattered photon produced in the beam modifier. This work studied the influence of scattered photons from Cerrobend and mercury modifiers on the transmission and the quality changes in beam. In addition, we introduced a method for linear attenuation coefficient calculations.

The exponential model is usually used for modifiers attenuation calculation [18]; however, this model introduces unnecessary errors

Figure 4: Varian Linac and water phantom

for polyenergetic beams. On the other hand, the linear attenuation coefficient is strongly energy dependent. In general, lower energetic X-ray photons have a higher interaction probability. Since an X-ray device produces photons in a wide energy range, the transmission should actually be considered for the whole energy range. Alles et al. [12] suggested the model for linear attenuation coefficient calculations for different discretizing thicknesses and energies (up to 150 keV). The method that was introduced in this paper was based on Equation 6 considered all energies and thickness continuously. The procedure of calculated linear attenuation coefficient from method of this study versus attenuator thickness is in agreement with other studies such as du Plessis et al [3] and Sasaki et al. [10]. As shown in Figure 5 and 6, the linear attenuation coefficient decreases slightly as the thickness of the attenuating filter (Cerrobend or mercury) increases, reflecting the progressive hardening of X-ray beam. These data indicate that linear attenuation coefficients are approximately linear with modifier thickness increment up to about 4cm for cerrobend and 2.8 cm for mercury (this reported about 3cm for MCP96 alloy) [8] which shows hardening effect happened at lower energy but for much thicker modifiers, the change in linear attenuation coefficient did not occur. It is also seen that the value of linear attenuation coefficient decreases as the field size increases, because the scatter component increases at large field sizes and can reach the detector. It is seen that the curves are parallel to each other. On the other hand, the difference between the linear attenuation coefficient for 6×6 cm^2 and 14×14 cm^2 is approximately constant for all Cerrobend and mercury thicknesses.

Conclusion

The method introduced in this study for linear attenuation coefficient calculation is general enough to treat compensatory filters, wedges, shields and any intensity modulator or radiation absorber with any shape or ma-

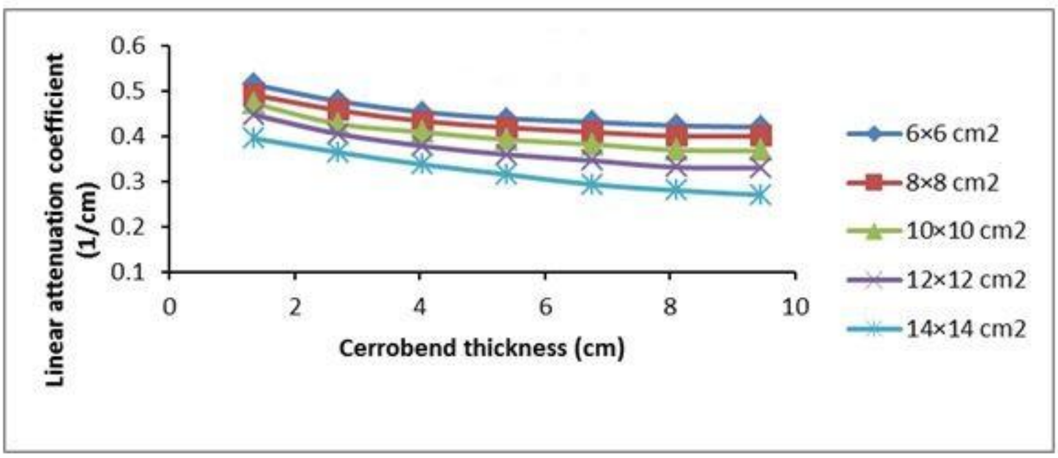

Figure 5: Linear attenuation coefficient values v.s mercury attenuator thicknesses for different field sizes.

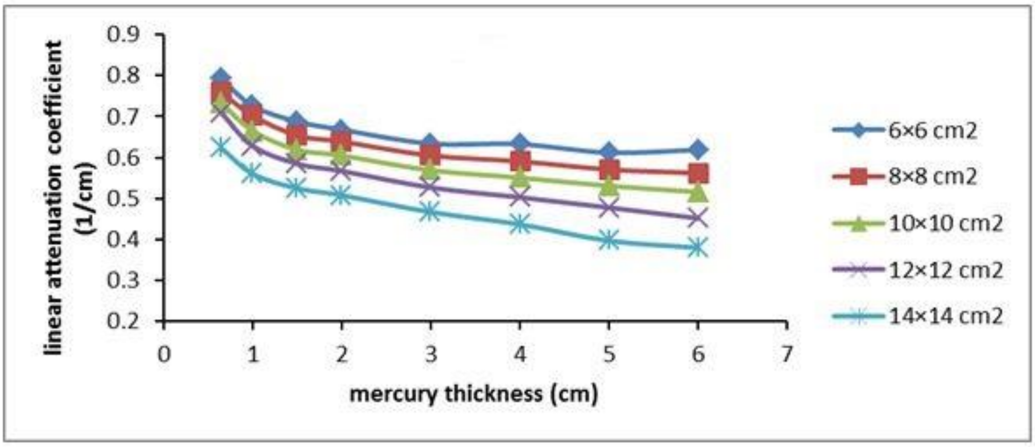

Figure 6: linear attenuation coefficient for different thickness of mercury attenuator and field sizes.

terial by using the same formalism, although calculating was made only for mercury and Cerrobend attenuator. On the other hand, it seems that this method is suitable for high energy shields or protector designing.

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Conflict of Interest

None

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