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Numerical simulation of variable density and magnetohydrodynamics effects on heat generating and dissipating Williamson Sakiadis flow in a porous space: Impact of solar radiation and Joule heating

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ABSTRACT

This study is confined to the numerical evaluation of variable density and magnetohydrodynamics influence on Williamson Sakiadis flow in a porous space. In this study, Joule heating, dissipation, heat generation effect on optically dense gray fluid is encountered. The inclined moving surface as flow geometry is considered to induce the fluid flow. A proposed phenomenon is given a mathematical structure in partial differential equations form. These partial differential equations are then made dimensionless using dimensionless variables. The obtained dimensionless model in partial differential equations is then changed to ordinary differential equations via stream function formulation. A set of transformed equations has been solved with bvp4c solver. The numerical fallout of velocity field, temperature field, skin friction, and heat transfer rate are illustrated in graphs and tables with flow parametric variations. Conclusion is drawn that mounting values of density variation parameter confirm the reduction in velocity field and augmentation in temperature of the fluid. When Williamson fluid parameter enhances, both fluid velocity and temperature are rising correspondingly. Growing magnitudes of the magnetic number, radiation parameter, heat generation, and Eckert number rise the temperature of the fluid. A rise in a porous medium parameter weakens the fluid velocity. Skin friction is reducing as radiation parameter and density variation parameter are increased.

The present solutions are compared to those that have already been published in order to validate the current model. The comparison leads to the conclusion that the two outcomes are in excellent agreement, endorsing the veracity of the current answers.

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1. Introduction

The research community is fascinated by processes of heat and mass transfer in non-Newtonian fluids. Non-Newtonian fluid does not adhere to the viscosity requirements of the Newton law. Sugar solutions, pulps, concentrated juices, jelly, blood, and honey are a few non-Newtonian examples. This non-Newtonian sub-division includes Williamson fluids (pseudo-plastic categories). These fluids have a variety of engineering and industrial fields. Williamson fluids are used in emulsions, adhesives, and blood cells. Non-Newtonian fluids have a crucial role in all aspects of life. There are unavoidable uses for the Williamson fluid, a pseudoplastic kind with non-Newtonian behavior, in human life. A suitable candidate for increasing combustion efficiency and minimizing pollutant production is a porous media. By keeping in view, the above significance, research community paid a lot of attention of the problem that are involving non-Newtonian fluids. El-Bashir et al. [1] explored flow of Williamson fluid past stationary and moving plate. In Ref. [2] study confining to flow of Williamson nano-fluid and heat transportation past sheet stretching non-linearly with porosity impact has been given. Viscous dissipation and heat transportation effects were encountered too. Zehra et al. [3] focused on Williamson fluid flow and heat transfer in inclined channel with pressure-dependent viscosity and porosity. Heat transportation mechanism in Williamson nanofluid with solar radiation and chemical reaction along extending surface implanted in a porous media had been studied by Prasannakumara et al. [4]. Kumar et al. [5] proposed entropy nanomaterial flow of Williamson fluid over a linear stretching sheet. A Joule, magnetic force, dissipation and radiation impact was considered too. Heat and mass transportation processes via Williamson nanofluid fluid model through sheet extending in non-linear manner was discussed by Kho et al. [6] with incorporation of thermal and velocity slip conditions. Mishra with his co-researchers [7] analyzed the phenomena of magneto-hydrodynamic Williamson fluid flow with micro-rotation past expanding sheet. Joule heating, variable thermal conductivity and non-uniform heat generation were considered. Jalili et al. [8] investigated the Williamson nanofluid flow over a non-linear stretching sheet under the impact of inclined magnetic force, variable viscosity and chemical reaction effects. Kataria et al. [9] paid attention to non-Newtonian micropolar fluid flow and heat transportation. Lorentz force, non-linear radiation, Joule effects, velocity slip condition, and flux conditions were considered. Micropolar fluid flow and energy transporatation past stretcging and shrinking surface had been presented in Ref. [10]. Magnetohydrodynamic and solar radiation effects in nonmaterial flow were also considered. Mittal et al. [11] considered micropolar ferro-fluid to study the flow and energy transfer mechanism along sheet extending non-linearly. The influences of Joule heating, mixed convection, and viscous dissipation were involved. Heat-generating, heat-dissipating, and magnetohydrodynamic micropolar fluid flows with slip influence on the stretched surface had been explained by Abbas et al. [12]. The Lorentz force effects on fluid flow of third-grade and energy transport over an inclined exponential stretching sheet with porosity effects with Darcy-Forchheimer were presented in Ref. [13].

In literature review, we have seen there is extensive research involving two-dimensional flow past fixed, extended, and moving geometries under magneto-hydrodynamic and radiation conditions. These studies have real-world applications in nuclear reactors, commercial boilers, thermal insulation, etc. For the first time, Blasius [14] studied the fluid flow on stationary surface placed in a moving stream. Sakiadis [15] gave the idea of fluid flow induced due to continuous moving surface. Cortell [16] investigated Sakiadis flow and energy transfer in under radiation control in numerically way. Pantokratoras [17] examined the Blasius and Sakiadis flow with conductivity and viscosity depending on temperature. Ramesh with his fellow researchers [18] examined flow and heat transportation mechanisms for stationary and moving inclined surfaces under convective boundary conditions. Abbas et al. [19] investigated variable density and radiation influence on the fluid flow and heat transportation on moving inclined surface. Reddy et al. [20] considered natural convection in Casson fluid on periodically moving surface with magnetohydrodynamics and dissipation influence on time-dependent flow. Beg et al. [21] focused on steady heat generating magneto-hydrodynamics flow past tilted surface with heat generation and Soret effects. Mittal [22] proposed the pheromone of three dimensional flow of nanofluid in two rotating and parallel plates and solved the model with homotopy analysis method. Li et al. [23] utilized lattice Boltzman method for numerical evaluation of nanofludi flow and heat transfer in permeable duct with magnetic field effect. In Ref. [24] gave the observation on Soret and Dufour impact on heat generating, MHD, and radiating unsteady flow of nanofluid pas oscillating surface. Investigations on magnetohydrodynamics and porosity influence on gravity-driven flow of nanofluid across periodic surface were carried by Kataria et al. [25]. Patel et al. [26] scrutinized the fractional model of time dependent magnetohydrodynamics nanomaterial flow with porosity influence. Soret effects and heat source effect were considered too. Ramesh et al. [27] took into under their study the problem of Blasius flow and Sakiadis flow using Williamson fluid model equipped with convective boundary conditions. Abbas et al. [28] did computation of flow equations for Newtonian fluid with reduced gravity, solar radiation and Lorentz force effect in porous medium with Darcy-Forchheimer theory about a sphere using finite difference method. Abbas et al. [29] conducted the discussion on Lorentz force and solar radiation effects on reduced gravity flow past non-rotating sphere under porosity impact. Abbas with his researchers [30] did numerical evaluation of reduced gravity driven flow and energy transfer past immobile sphere under the solar radiation and Lorentz force using finite difference method. Abbas et al. [31] proposed Maxwell fluid model with inclined plate and generalized heat and mass transfer laws. Abbas et al. [32] studied bionconvective heat and mass transfer in Williamson nanofluid across moving inclined plate fixed in a porous space under magnetohydrodynamics effects.

Alarmingly large amounts of electricity are being used in homes, businesses, and workplaces around the world. In order to produce energy utilizing thermal or photovoltaic solar energy systems, attention is being devoted to the use of solar/thermal radiation. Sharma et al. [33] investigated a radiating and dissipating nanoparticles flow and heat transfer phenomenon with MHD influence on nonporous sheets. Magnetic field on optical dense gray the hydrothermal flow of behavior of ferro oxid nanoparticles water based nanofluid had been focused by Sheikholeslami and Shamlooei [34]. Sohail et al. [35] explored bioconvective flow in Maxwell nanofluid under homogeneous-heterogeneous reactions using generalized heat transfer rules with entropy generation. Newtonian nanofluid flow and energy transportation with radiation and heat source/sink effects was premeditated by Ali et al. [36]. Radiative fluid flow in viscoelastic nanofluid with buoyant forces and flux conditions were suggested by Waqas et al. [37]. In Ref. [38] authors discussed a phenomenon of fluid and energy transportation via non-Newtonian model of Casson fluid across surface extending exponentially with solar energy impact. Ashraf with his fellow researchers [39] analyzed solar energy transportation in nanofluid flow past sphere and plume region. Kataria et al. [40] examined optimized entropy flow with non-linear solar energy, magnetic field, Joule heating, heat generation, heat absorption and viscous dissipation impact. Mittal et al. [41] discussed the influence of magnetic force on squeezing nanoparticles flow across two equidistant plates. Sheikholeslami [42] did investigation on the process of solar energy based nanoparticles flow with thermal interfacial resistance and micro mixing in suspensions.

Applications for magneto-hydrodynamic Newtonian fluids are expanding in a variety of industries, such as nuclear reactors, chemical engineering, electromagnetic propulsion, etc. The effects of second order slip on electrically conducting liquids flowing across a curved surface were shown by Muhammad et al. [43]. Saleem and Nadeem [44] gave analysis on slip condition and viscous dissipation on fluid flow and temperature transfer along rotating cone. Hayat et al. [45] calculated flow and energy transport phenomenon on with the suspension of nonmaterial on rotating disk under energy generation and dissipation effects. Ferdows et al. [46] examined the mechanism of magnetohydrodynamics heat transfer and fluid flow with variable viscosity in double diffusion, heat generation saturated with porous media. Azam et al. [47] considered solar radiation and magnetohydrodynamics fluid flow above a semi-infinite moving vertical plate under density depending on temperature. Siddiqa et al. [48] paid attention on density depending on temperature effects on natural convective transfer along a horizontal circular disk.

Flows in porous space have concerned researchers' interest because of their weight in engineering fields and industrial sections. Its significance can be found in porous insulation, oil reservoirs, resin transportation model, and fossil fuel beds. Chitra and Kavitha [49] proposed the pulsatile flow in a spherical conduit saturated in porous media with temperature dependent pressure. Gireesha et al. [50] highlighted the magnetohydrodynamics flow and temperature transfer in dusty fluid above an unsteady stretching sheet under porosity control. Pal and Mondal [51] documented the study on magnetohydrodynamics effect on fluid flow process and heat transfer process under varying viscosity and non-uniform heat rise and fall above extending sheet in porous bed. Makinde et al. [52] considered magnetohydrodynamics and variable viscosity effects upon of fluid flow along the heated surface in flux conditions in porous space. Thermophoresis and radiation effects were also considered. Impact of solar radiation and magnetohydrodynamics on dusty fluid flow and energy transmission above stretching cylinder in porous bed under heat generation influence were examined by Manjunatha [53].

Tripathy et al. [54] conducted investigation on chemically reacting electrically conducting fluid over a stirring surface with heat and mass flux conditions in porous media. An investigation of the radiative nanofluid flow on stretched surface in porous bed under magnetic force was done by Hussain and Sheremet [55]. In Ref. [56] authors published semi-analytical fallout for transitory free convection in two concentric vertical cylinders of unlimited lengths with anisotropic porous material and stratified medium. El-Kady [57] conducted an experimental investigation in porous way with two separate heat sources on the bed wall. The major objective was to show how the channel's porous media affects the characteristics of heat transport. Alkhazzan et al. [58] developed and analyzed a new Susceptible-Infected-Recovered-Susceptible (SIRS) model that encounters for infection transporation and three types of noise to examine the role of transport in disease transmission. Khan et al. [59] modeled the problem of waterborne disease in fractional differential equations and solved the equations numerically. Khan et al. [60] focused attention on model that has applications biological

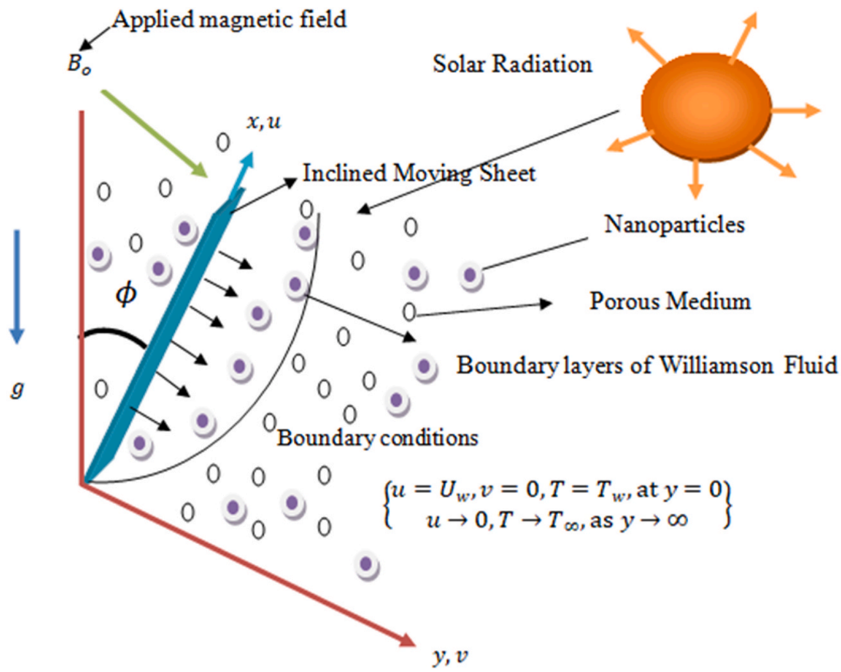

Fig. 1. Flow structure.

field in fractional differential equaiotns and gave their numerical solutions. Siddiqa et al. [61] studies radiation effects on mixed convective flow with variable density effects on porous surface. In Ref. [62] researchers proposed phenomena of fluid flow and heat transportation under the magnetic field and solar radiation effects. Mehta and Kataria [63] focused the attention on solar rays effects on Casson fluid flow and energy transfer on parabolic geometry fixed in a porous medium. Mittal and Kataria [64] examined heat and mass transportation in flow of nanofluid under solar rays influence. Mittal and Patel [65] disclosed the study of two phase model of nanofluid using non-Newtonian Casson fluid considering three dimensional flows under magnetic force, energy generation and non-linear thermal radiation impact.

After evaluating the work that has already been published, a vast amount of work on non-Newtonian fluids on various geometries with various fluid properties has been published. The study on thermal radiation and variable density on magnetohydrodynamics dissipating fluid flow and heat transport with heat generation and Joule heating influence along moving inclined surface (Sakiadis flow) in porous bed is carried out due to the physical value of the aforesaid mechanisms. This study has not been previously published. Coming sections deal with statement of the problem and solution procedure along with the presentation of obtained numerical results.

2. Model formulation

Contemplate two-dimensional, steady flow of Williamson incompressible fluid across inclined moving plate. Motion of inclined plate in motion is constant at U_w and it planted in a porous space. The analysis takes into account the effects of thermal radiation, heat generation, variable density, viscous dissipation, and Joule heating. A magnetic field with a B_o strength is applied at right angles to flow direction. An angle of inclination of geometry is $\xi = \pi/6$. Plate of temperature T_w is and free stream temperature is T_∞ with condition $T_w > T_\infty$. In Fig. 1, a flow structure is shown. Following [5,19], flow equations are provided below:

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\mu\frac{\partial}{\partial y}\left(\frac{1}{\rho}\frac{\partial u}{\partial y}\right)+\sqrt{2}\Gamma\mu\frac{\partial}{\partial y}\left(\frac{1}{\rho}\frac{\partial u}{\partial y}\right).\frac{\partial u}{\partial y}+\frac{g}{\rho}(\rho_\infty-\rho)cos\,\varphi-\frac{\sigma B_o^2 u}{\rho}-\frac{\mu}{\rho K^*}\overline{u}, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{Cp}\frac{\partial}{\partial y}\left(\frac{1}{\rho}\frac{\partial T}{\partial y}\right)-\frac{1}{Cp}\frac{\partial}{\partial y}\left(\frac{1}{\rho}\frac{4\sigma T_\infty^3}{3K_R}\frac{\partial T^4}{\partial y}\right)+\frac{\mu}{\rho C_p}\Gamma\left(\frac{\partial u}{\partial y}\right)^3+\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{B_o^2\sigma u^2}{\rho C_p}+\frac{Q_0(T-T_\infty)}{\rho C_p}. \tag{3}$$

Modeled boundary conditions

$$\begin{aligned} &u=U_w, v=0, T=T_w at\ y=0,\\ &u\to 0, T\to T_\infty \text{ as } y\to\infty. \end{aligned} \tag{4}$$

Here, u, v are velocities in x, y directions. $g, \rho, \mu, Q_0, \sigma$ are acceleration of gravity, density, viscosity, heat generation, and Stefan-Boltzmann constant. $K^*, k, C_P, \alpha,$ and B_o are porosity of medium, thermal conductivity, specific heat, thermal diffusivity, and strength of magnetic field. Designations σ and K_R are Stefan-Boltzmann constant and mean absorption coefficient, respectively.

3. Solution method

The solution mythology is described in full detail in this section. Equations (1)–(4) are made dimensionless first by dimensionless variables given in the following equation (5). The whole solution procedure is outlined below.

3.1. Dimensionless variables

Here, following dimensionless variables given in Ref. [61] are utilized to make Eqs. (1)–(3) with conditions from Eq. (4) dimensionless:

$$\overline{u}=\frac{u}{U_w},\overline{v}=\frac{Lv}{\nu\, Re^{1/2}},\overline{x}=\frac{x}{L},\overline{y}=\frac{Re^{\frac{1}{2}}y}{L},\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\rho=\rho_\infty e^{-\beta(T-T_\infty)},\overline{\rho}=\frac{\rho}{\rho_\infty}. \tag{5}$$

Where $Re=\frac{U_w L}{\nu}$ is Reynolds number. Using Eq. (5) into Eqs. (1)–(4) we have;

$$\begin{gathered} \frac{\partial(\exp(-n\theta)\overline{u})}{\partial\overline{x}}+\frac{\partial(\exp(-n\theta)\overline{v})}{\partial\overline{y}}=0,\\ \overline{u}\frac{\partial\overline{u}}{\partial\overline{x}}+\overline{v}\frac{\partial\overline{v}}{\partial\overline{y}}=\frac{\partial}{\partial\overline{y}}\left(e^{-n\theta}\frac{\partial\overline{u}}{\partial\overline{y}}\right)+We\frac{\partial}{\partial\overline{y}}\left(e^{-n\theta}\frac{\partial\overline{u}}{\partial\overline{y}}\right)\frac{\partial\overline{u}}{\partial\overline{y}}-\alpha\left(\frac{1-e^{n\theta}}{1-e^{-n}}\right)cos\,\xi--e^{-n\theta}K-Me^{-n\theta}\overline{u}, \end{gathered} \tag{6}$$

$$\overline{u}\frac{\partial\theta}{\partial\overline{x}}+\overline{v}\frac{\partial\theta}{\partial\overline{y}}=\frac{1}{Pr}\left(1+\frac{4}{3}R_d\right)\frac{\partial}{\partial\overline{y}}\left(e^{-n\theta}\frac{\partial\theta}{\partial\overline{y}}\right)+\frac{\lambda Ec}{\sqrt{2}}e^{-n\theta}\left(\frac{\partial\overline{u}}{\partial\overline{y}}\right)^3+Ece^{-n\theta}\left(\frac{\partial\overline{u}}{\partial\overline{y}}\right)^2+MEce^{-n\theta}\overline{u}^2+\gamma e^{-n\theta}\theta, \tag{7}$$

Dimensionless boundary conditions

$$\left.\begin{array}{c}\overline{u}=1,\overline{v}=0,\theta=1,\,at\ \overline{y}=0,\\ \overline{u}\to 0,\theta\to 0,\,as\ \overline{y}\to\infty.\end{array}\right\}\tag{8}$$

Where, $K=\frac{\mu}{\rho_\infty K^*}$ is porous medium parameter, $\alpha=\frac{Gr}{Re^2}$ is mixed convection parameter, $M=\frac{\sigma B_o^2 L}{\rho_\infty U_w}$ is magnetic field parameter, $\Pr=\nu/\alpha$ is Prandtl number, $Gr=\frac{g(1-e^{-n})L^3}{\nu^2}$ is Grashof number, $Re=\frac{U_w L}{\nu}$ is Reynolds number, $We=\sqrt{2}\,\Gamma\frac{Re^{\frac{1}{2}}U_w}{\nu L}$ is the Williamson fluid parameter, $\mathrm{Nr}=\frac{4\sigma T_\infty^3}{kK_R}$ is radiation parameter, $n=\beta_T\Delta T$ is variable density parameter, $\mathrm{Ec}=\frac{U_W^2}{C_P\Delta T}$, is Eckert number, Joule heating parameter $J=EcM$, and $Q=\frac{Q_o}{\rho C_P U_w}$ is heat source parameter. Here, ξ is angle of inclination the plate is making with g with vertical axis.

3.2. Stream function formulation

Partial differential equations are challenging to solve directly. Therefore, theses are reduced to ordinary differential equations using proper stream function formulation specified in Eq. (9) utilized by Ref. [61] are employed to transform Eqs. (6) and (7) with Eq. (8) into ordinary differential equations:

$$\psi=x^{-\frac{1}{2}}f(\eta),\eta=x^{-\frac{1}{2}}\int_0^{\overline{y}}\overline{\rho}dy,\theta(\eta)=\theta,\tag{9}$$

The equation of continuity is commonly satisfied by incorporating the stream function formulation from Eq. (9) into Eqs. (6)–(8), then flow equations translated into the following form.

$$(1+Wef'')e^{n\theta}f'''-\left(1-e^{n\theta}\right)n\theta'f''+f'^2-\alpha\left(\frac{1-e^{n\theta}}{1-e^{-n}}\right)\cos\xi-Mf'e^{-n\theta}=0,\tag{10}$$

$$\left(1+\frac{4}{3Nr}\right)\left(e^{-n\theta}\theta''-ne^{-n\theta}\theta'^2\right)+PrEc(f'')^2e^{-n\theta}+\frac{1}{2}Pr\,f\,\theta'+Pr\,Q\theta e^{n\theta}+\frac{1}{\sqrt{2}}Wee^{-n\theta}+Jf'^2e^{-m\theta}=0,\tag{11}$$

Subject to the boundary conditions

$$\left.\begin{array}{c}f(\eta)=0,f'(\eta)=1,\theta(\eta)=1\ at\ \eta=0,\\ f'(\eta)\to 0,\theta(\eta)\to 0\ as\ \eta\to\infty.\end{array}\right\}\tag{12}$$

Prime notation indicates the derivative w.r.t η.

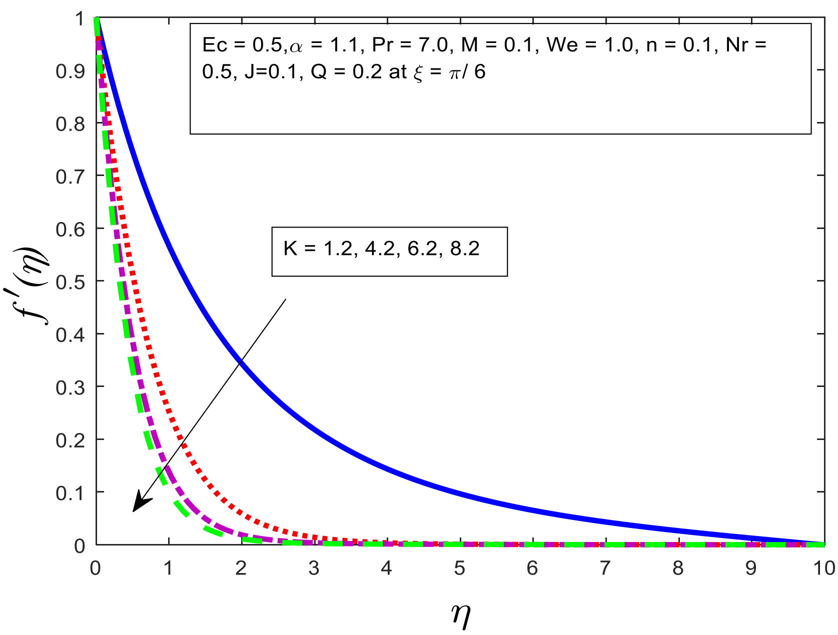

Fig. 2. f' versus K.

3.3. Numerical method

Equations 10 and 11 with boundary conditions (12) are simulated using MATLAB solver bvp4c. Based on the collocation formula, this solver works. Equations 10 and 11 with boundary conditions provided in Equation (12) are converted into a system of first order ordinary differential equations before being used to MATLAB's bvp4c numerical algorithm for numerical output. Solutions are obtained for porous medium parameter K, magnetic field parameter M, heat source parameter Q, radiation parameter Nr, angle of inclination ξ, Eckert number Ec, Williamson fluid parameter We, density variation parameter n, mixed convection number α, Joule heating parameter J, and, Prandtl number Pr. Results are in the next sections presented in graphs and table and discussed in detail with physical reasoning.

4. Results and discussion

The primary motive of this section is to bring out physical outcome of related parameters such as porous medium parameter K, magnetic field parameter M, heat source parameter Q, radiation parameter Nr, angle of inclination ξ, Eckert number Ec, Williamson fluid parameter We, density variation parameter n, mixed convection number α, Joule heating parameter J, and, Prandtl number Pr on velocity of fluid f' and temperature of fluid θ and their gradients skin friction $f''(0)$ and rate of heat transfer $\theta'(0)$. Solutions obtained by the built-in numerical solver bvp4c are presented and discussed in detail.

4.1. Behavior velocity and temperature under sundry parameters

Figs. 2 and 3 are showing the controlling effects of porous medium parameter on f' and θ, respectively. The observation is that velocity is declining and temperature is rising with mounting values of K. By increase in K, porous size are decreased which retards velocity, hence the motion of the fluid slows down rapidly, this is entirely according to the physic of parameter K. A relationship between magnetic field M and velocity is shown graphically in Fig. 4. The outcomes are indicating while magnetic field increases, velocity decreases accordingly. The inverse relationship above demonstrates that when magnetic field is amplified, a strong Lorentz force is created, producing more resistance within the fluid flow and a lower velocity distribution. The controlling impacts of M on temperature distribution are illustrated in Fig. 5. When M enhances the temperature of the fluid rises remarkably. It is due to the fact that increased resistance due to the current generation in boundary layer gives in a rise in fluid temperature. This resistance is induced to increased strength of Lorentz force. Figs. 6 and 7 show the control of the density variation parameter on the distribution of velocity and temperature respectively with specific values of the other dimensionless numbers. It can be seen from the graphical data, when density constraint increases, the velocity of fluid decreases and the temperature profile rise. As per physics of parameter, the temperature distribution rises when n is increased since the temperature difference is substantially increased. A control of Eckert number Ec on velocity distribution and temperature distribution respectively is represented in Figs. 8 and 9. According to Figs. 8 and 9, fluid flow and fluid temperature are increasing when the Eckert number increases. In dissipation of viscous force, the thermal energy is converted into mechanical energy, to dissipate the energy an addition force is applied that is acquired by increasing values of Ec. In this

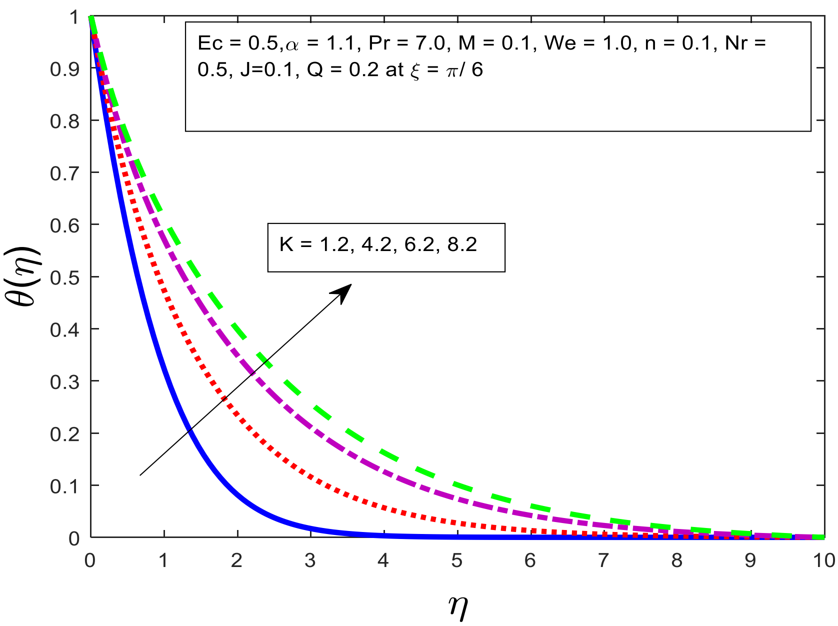

Fig. 3. θ versus K.

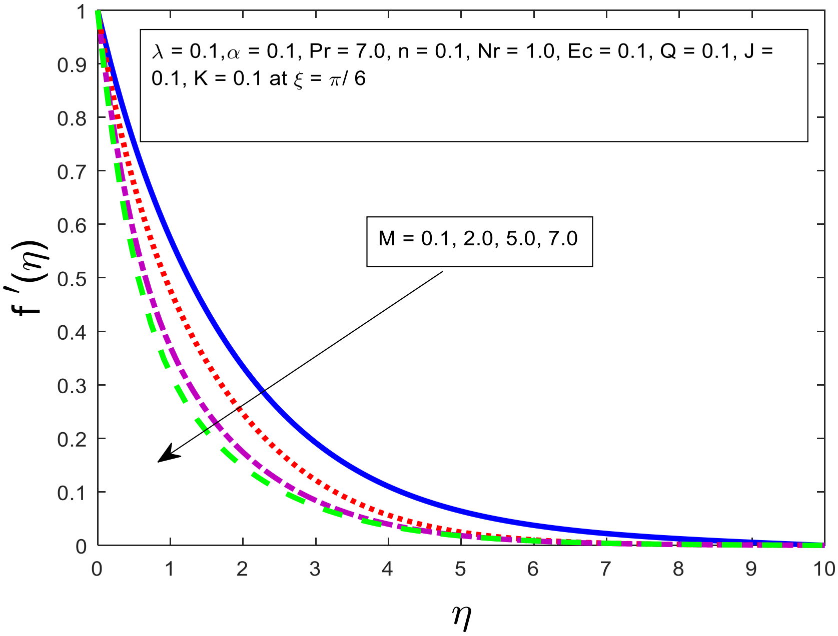

Fig. 4. f' versus M.

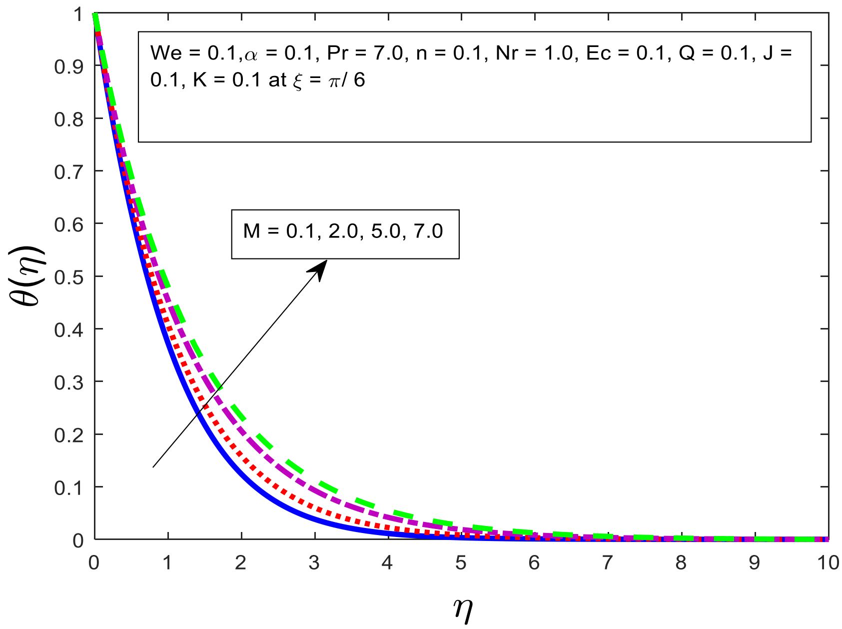

Fig. 5. θ versus M.

viscosity of the fluid are dissipated and so cohesive forces become week allowing the fluid to flow too easily and climb in temperature of fluid. Figs. 10 and 11 are depicting force of Williamson fluid parameter on velocity field and temperature field respectively. When *We* enhances, both velocity and temperature fields are rising correspondingly. It is visible that the viscosity of the fluid decreases to let the particle to move freely as Williamson parameter *We* increases; hence, after raising the value of *We*, we can view from the graphs that the temperature profile and velocity profile are rising. Figs. 12 and 13 are showing that how does the radiation parameter Nr affect f' and θ. Increasing magnitude of Nr is leading to increase the curves for velocity shown in Fig. 12. Similarly, the augmentation in Nr leads to boost up the temperature field as shown in Fig. 13. Physical evidence indicates that solar rays' inclusion in fluid flow causes the temperature to rise, and that this rise in temperature directly affects the reduction of viscosity, leading to an increase in velocity. The velocity distribution f' and temperature distribution θ are shown in Figs. 14 and 15, respective, under the influence of different values

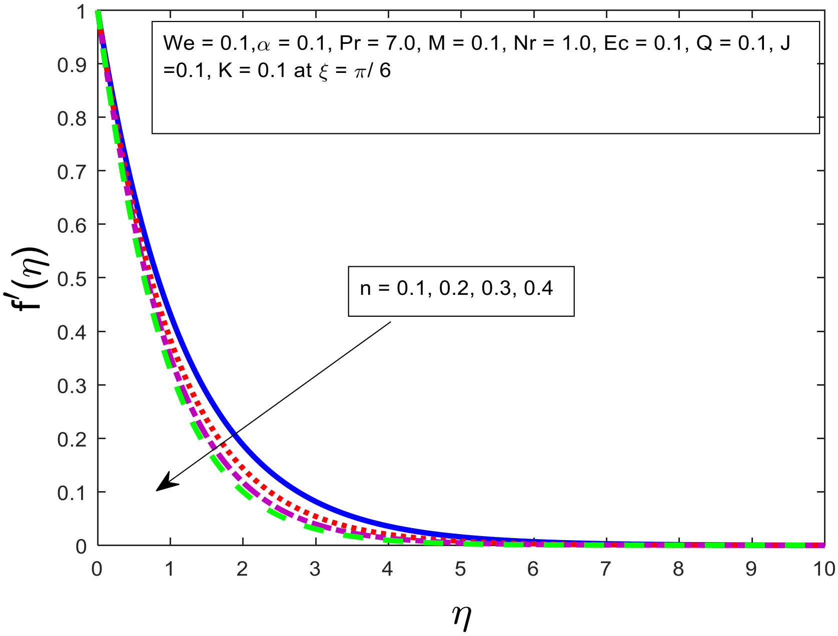

Fig. 6. f' versus n.

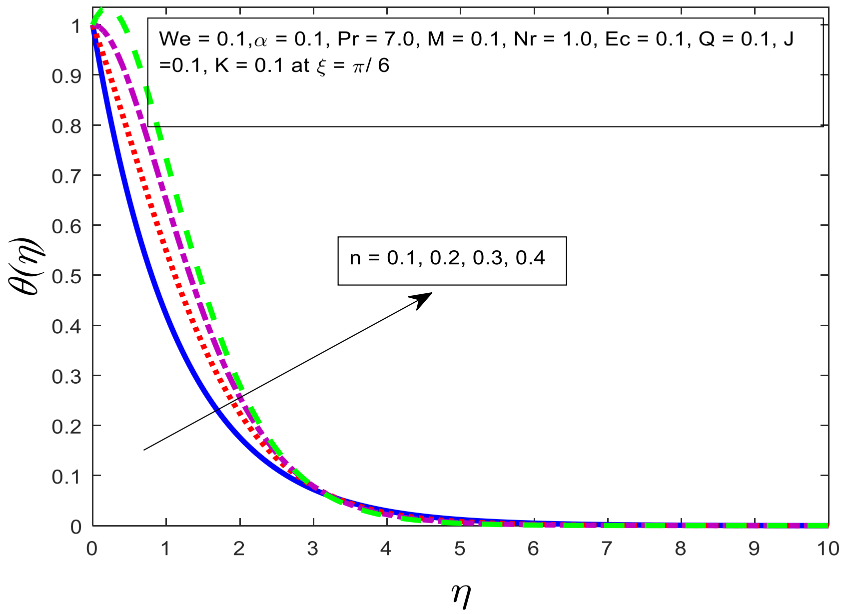

Fig. 7. θ versus n.

of Prandtl number, while the remaining parameters are kept at their predetermined values. Graphs show that the values of f' decrease for increasing Prandtl number Pr. The temperature of fluid falls down against escalating values of Pr. Results can be compared to the physical meaning of the Prandlt number, which is that by increasing the value of Pr, the viscous force that lowers the velocity field is increased. Similar to temperature filed, when the value of Pr is increased, thermal diffusion causes heat transmission to decrease. As a result, temperature distribution decreases as Prandtl number Pr is enriched. Heat generation parameter impact on f' and θ are seen in Figs. 16 and 17, respectively. Figs. 16 and 17 show that the relationship between velocity profile f' and temperature profile θ and the heat source parameter is proportional. Both are rising along with the temperature of heat source. The heat generation strengthens a temperature of fluid flow domain according to its physics.

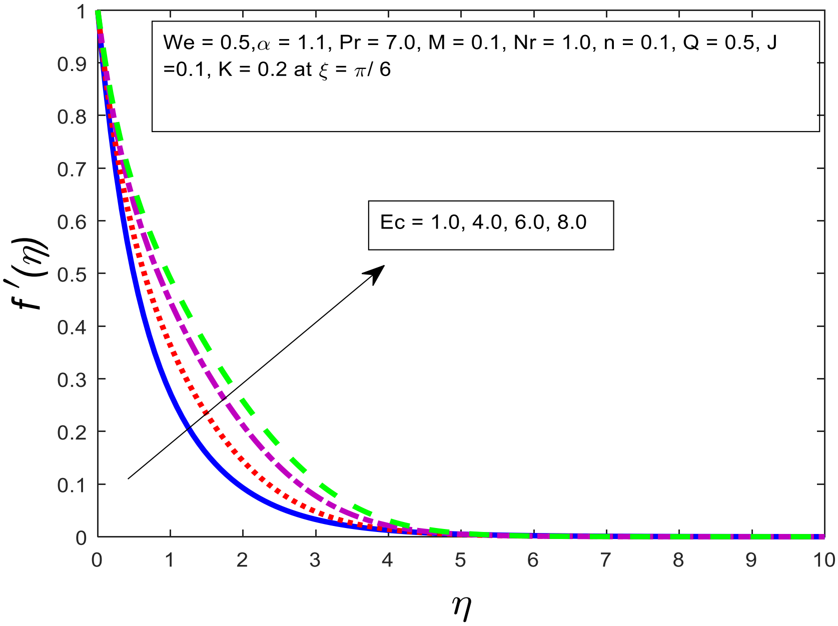

Fig. 8. f' against *Ec*.

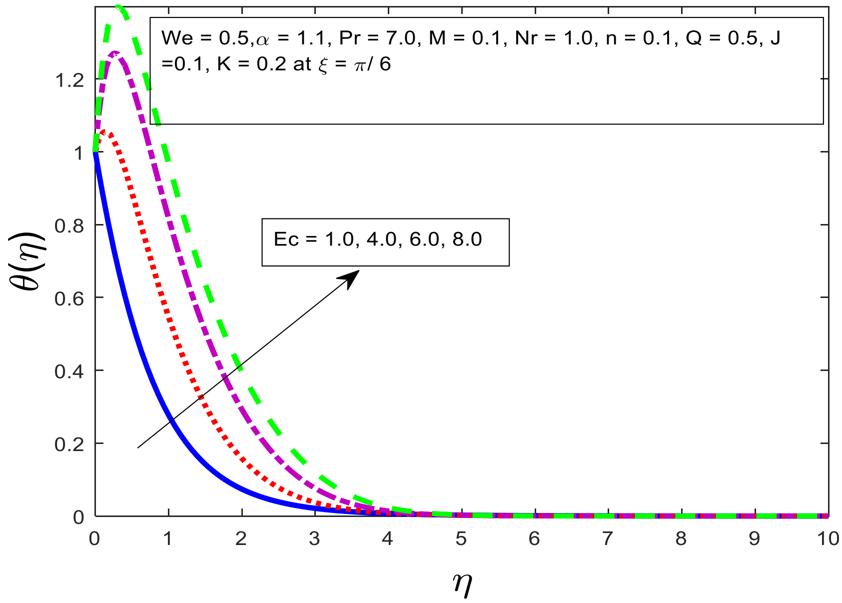

Fig. 9. θ versus *Ec*.

4.2. Effects of sundary parameters on skin friction and rate of heat transfer

Table 1 compares current results with previously published solutions, demonstrating a high degree of agreement between both of them. By this agreement, the current solutions' validation is ensured. Table 2 displays how radiation affects skin friction and rate of heat transfer. Skin friction is decreasing as a result, and heat transfer is accelerated when radiation number is increased. Table 3 portrays numerical solutions for the aforementioned attributes together with different magnitudes of the density variation parameter. It is significant to observe that both qualities are increasing as the density variation parameter is raised.

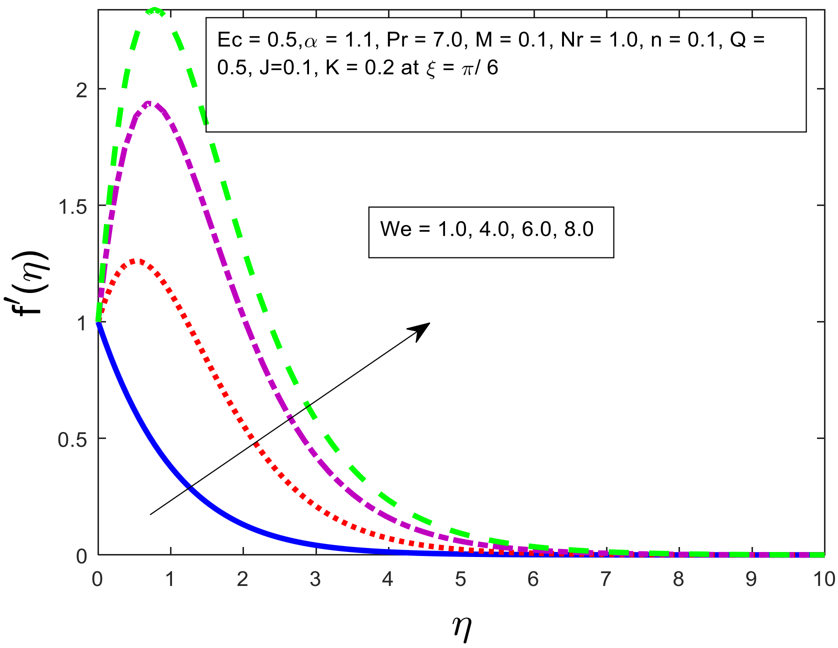

Fig. 10. f' versus *We*.

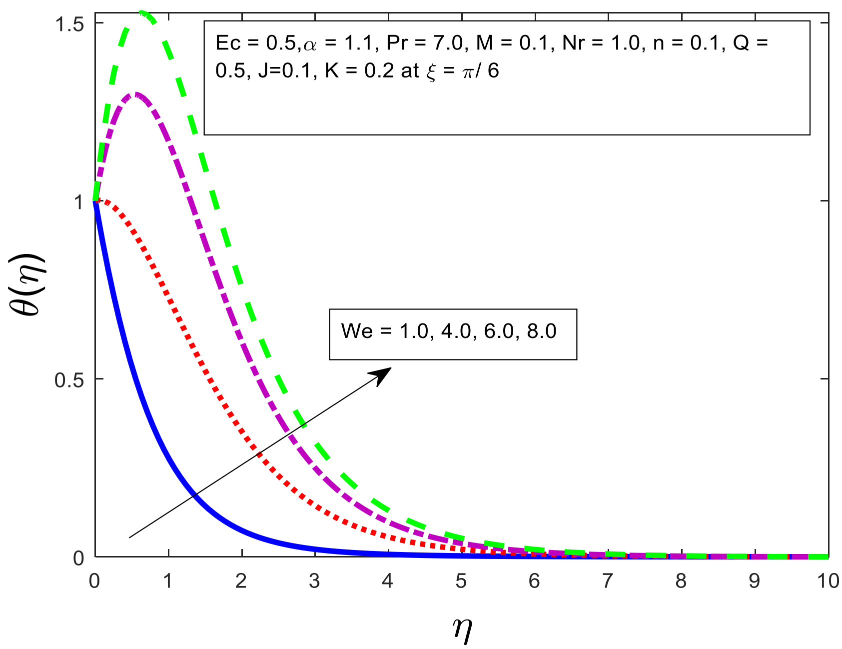

Fig. 11. θ versus *We*.

5. Conclusion

The focus of the current research is on the influence of variable density, viscous dissipation, heat generation, Joule heating, thermal radiation, and magnetohydrodynamics on flow of Williamson fluid past inclined moving plate (Sakiadis flow) implanted in a porous media. The obtained model is solved, and the results of physical quantities under the pertinent parameters are summarized below:

- An increase in K results in reduction in pore sizes which retards velocity, hence the motion of the fluid slows down rapidly, this is entirely according to the physic of the porous medium parameter.

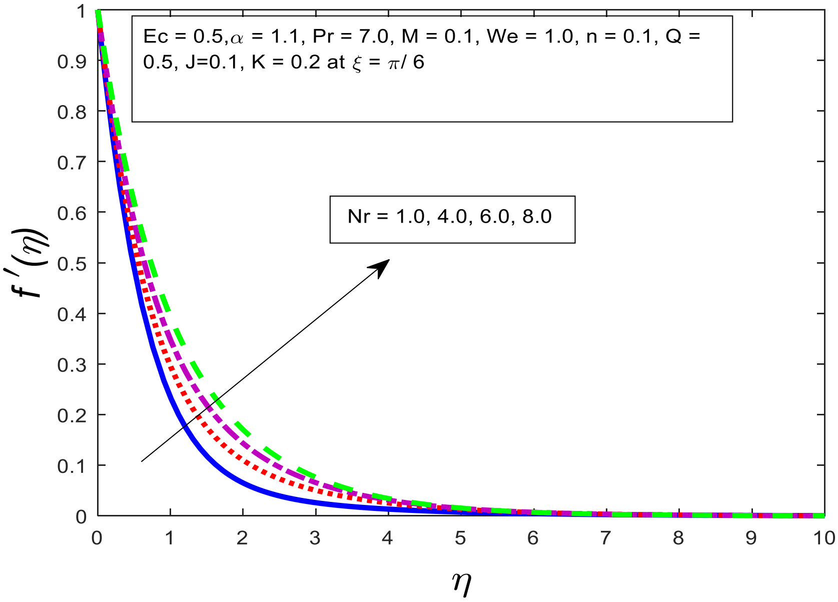

Fig. 12. f' versus *Nr*.

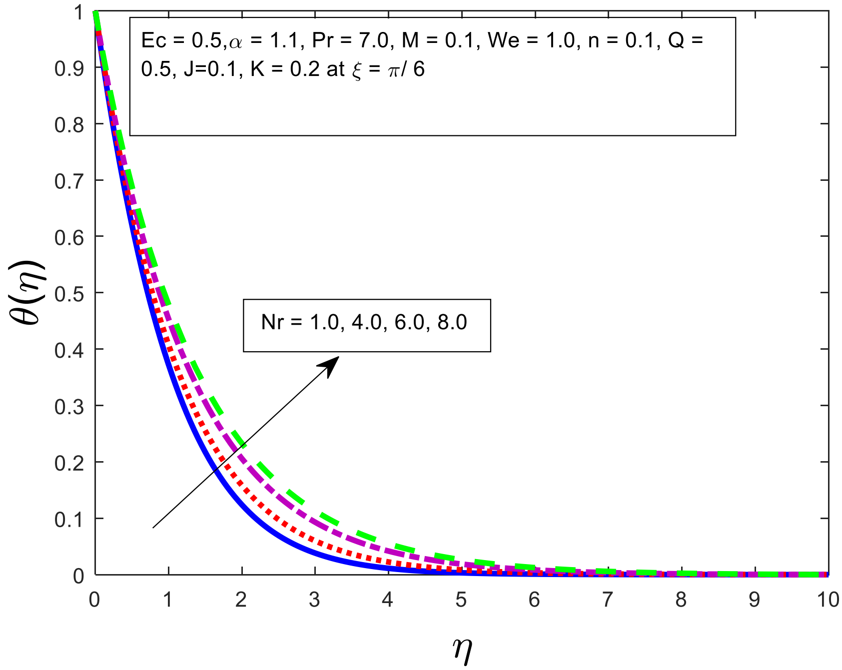

Fig. 13. θ versus *Rd*.

- An opposite relationship between velocity and magnetic field parameter. When magnetic field is amplified, a strong Lorentz force is created, producing more resistance within fluid flow and hence delineates velocity distribution.
- It can be seen from the graphical results that when density variation parameter increases, velocity of fluid decreases and temperature profile rise. According to the physics of this parameter, temperature distribution rises when density variation parameter is increased since the temperature difference is substantially increased.
- When Williamson fluid parameter enhances, both velocity and temperature fields are rising correspondingly. It is visible that the viscosity of the fluid decreases to let the particle to move freely as Williamson fluid parameter increases; hence, after raising the value of Williamson fluid parameter, we can view from the graphs that the temperature profile and velocity profile are rising.
- Escalating value of Prandlt number leads to fall in velocity and increase in temperature of the fluid.

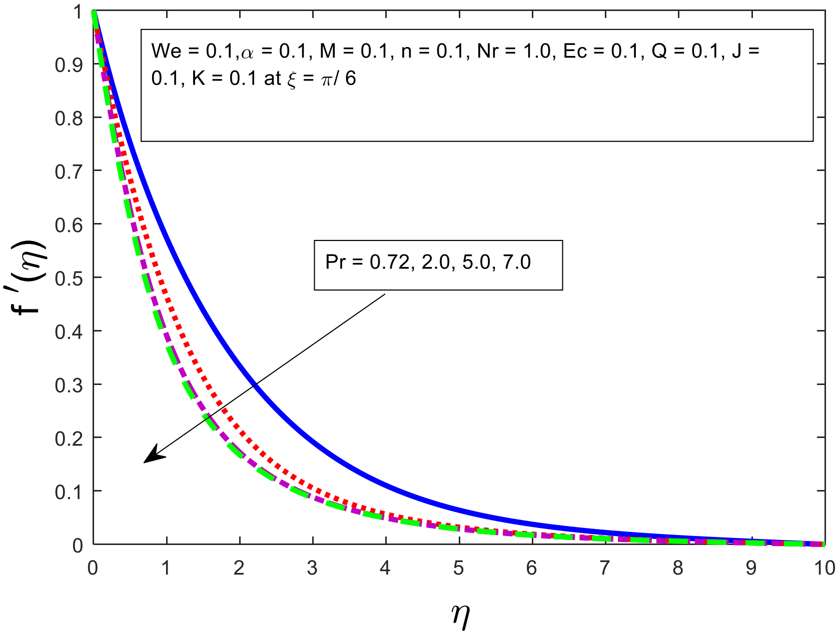

Fig. 14. f' versus *Pr*.

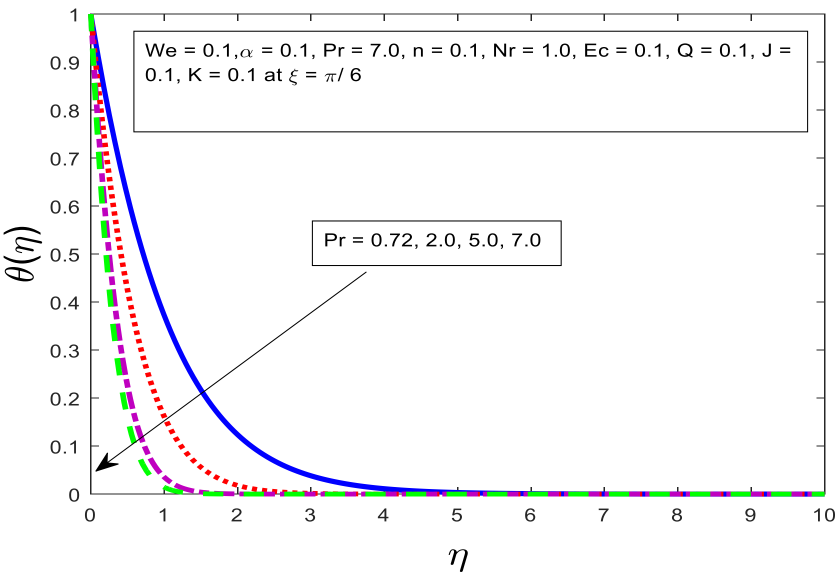

Fig. 15. θ versus *Pr*.

- Increasing magnitudes of Ecker number, heat generation parameter and radiation parameter give rise to augmentation in temperature distribution.
- Skin friction is reducing as radiation parameter and density variation parameter are increased.
- As radiation and density variation parameters increase, rate of heat transfer also boosts.
- The accuracy of the current results is ensured by the fact that all outcomes satisfy their boundary requirements.
- The comparison leads to the conclusion that the two outcomes are in excellent agreement, endorsing the veracity of the current answers and this agreement validates the current results.

Data availability statement

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

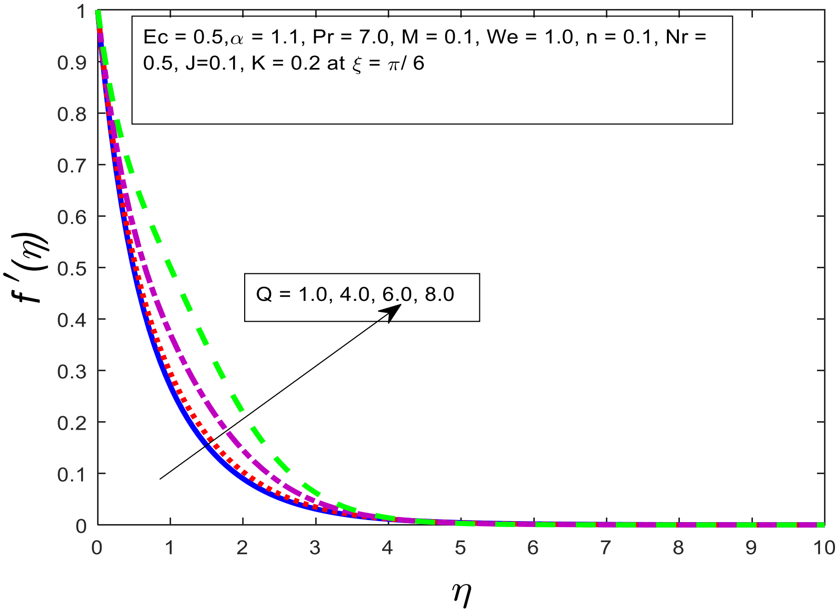

Fig. 16. f' versus Q.

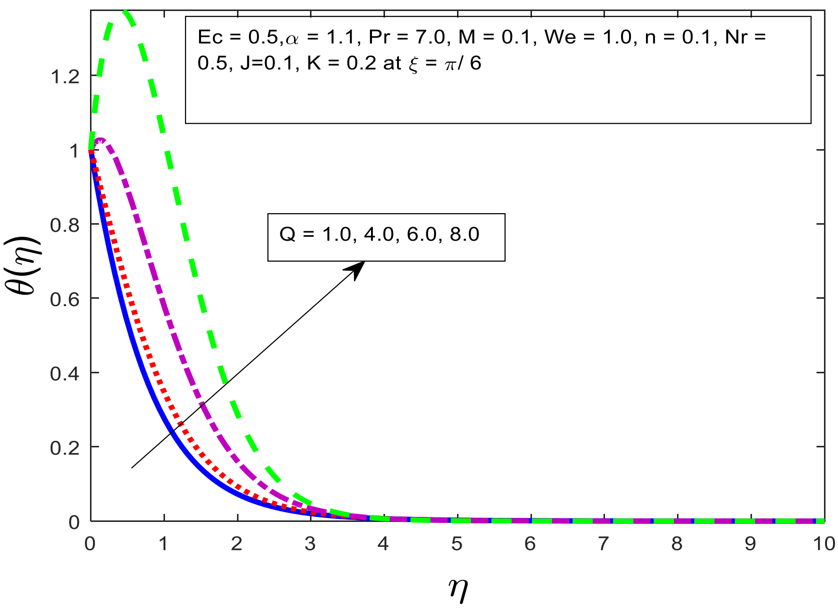

Fig. 17. θ versus Q.

Table 1
The current numerical values for skin friction are compared to previously reported for $We = 0$, $\alpha = 0, Ec = 0, M = 0, Q = 0, Nr = 0, n = 0, Pr = 0.7$ at angle $\xi = \pi/2$.

Similarity variable	Present	Cortell [16]
η	$-f''(0)$	$-f''(0)$
0.0	0.4437482	0.4437473
0.1	0.4426566	0.4426557
0.2	0.4394647	0.4394617
0.3	0.4343060	0.4343068
0.4	0.4273542	0.4273539

Table 2
Effects of Rd on a) $f''(0)$ b) $-\theta(0)$.

Rd	$f''(0)$	$-\theta(0)$
0.1	1.961411	0.223475
0.4	1.877955	1.971770
0.6	1.422415	7.610080
0.8	1.035478	9.212090

Table 3
Effects of Rd on a) $f''(0)$ b) $-\theta(0)$.

n	$f''(0)$	$-\theta(0)$
1.0	1.479067	0.451286
3.0	2.435383	0.095350
5.0	3.360102	0.788717
7.0	4.601139	0.871428

Additional information

No additional information is available for this paper.

CRediT authorship contribution statement

Amir Abbas: Writing – review & editing, Writing – original draft, Supervision, Software, Formal analysis, Conceptualization. **Aziz Khan:** Validation, Methodology, Investigation, Funding acquisition. **Thabet Abdeljawad:** Supervision, Resources, Project administration, Investigation, Funding acquisition. **Muhammad Aslam:** Writing – review & editing, Visualization, Validation, Resources.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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