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Dynamical properties of single species stage structured model with Michaelis-Menten type harvesting on adult population and linear harvesting on juvenile population

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ABSTRACT

A very common and effective way for investigating future demographics is the study of stage structured models. The focus of this article is to propose a modified model to study the impact of population harvesting on their juvenile and adult stages, and analyze the dynamical properties from both qualitative and numerical perspective. It studies single species stage structured model with linear harvesting on juvenile group and Michaelis-Menten type harvesting on adult group. We exploit general ideas in mathematical modeling process to study the dynamical properties and their biological, ecological, and economic implications. It discusses that bi-stability phenomena may exist, global asymptotic stability at boundary equilibrium points and internal equilibrium points are investigated from construction of suitable Lyapunov and Dulac functions. It has been observed that a suitable linear harvesting on juvenile population can feasibly be carry out along with Michaelis-Menten type harvesting on adult population without endangering extinction of any group of population.

1. Introduction

Every species goes through numerous stages in their life cycle, such as the juvenile, adult, and old adult phases. At various levels, it displays various traits and the value of various utility. Recent studies on single species age-structured models have produced very insightful findings regarding the sustainability, coexistence, and danger of extinction. Many scholars have studied age structured models finding very useful results regarding extinction, permanence or coexistence, and global stability of models [1]. For example, Lei studied commensalism model for a species with two age structures for two species [2]. This model is as follows :

$$\begin{cases} \frac{dx_1}{dt} = & \alpha x_2 - \beta x_1 - \delta_1 x_1 \\ \frac{dx_2}{dt} = & \beta x_1 - \delta_2 x_2 - \gamma x_2^2 + d x_2 y \\ \frac{dy}{dt} = & y(b_2 - a_2 y) \end{cases} \tag{1.1}$$

where x_1 and x_2 in the system (1.1) denote the juvenile and adult population densities at time t of the first species, y denotes the population density of the second species, and the parameters α, β, δ_1, δ_2, d, a_2 and b_2 are all positive. He studied the stability of the equilibrium points. Their research findings demonstrates that one of the best strategies to stop endangered species from extinction is

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fostering strong inter species cooperation. Since the juvenile species needs time to mature, the following stage-structure-prey-predator model in delay differential equation for two species was then suggested by Ma, Li, et al. [3–6].

$$\begin{cases} \frac{dx_1}{dt} = & r_1(t)x_2(t) - d_{11}x_1(t) - r_1(t-\tau_1)e^{-d_{11}\tau_1 x_2(t-\tau_1)} \\ \frac{dx_2}{dt} = & r_1(t-\tau_1)e^{-d_{11}\tau_1 x_2(t-\tau_1)} - d_{12}x_2(t) - b_1(t)x_2^2(t) - c_1(t)x_2(t)y_2(t) \\ \frac{dy_1}{dt} = & r_2(t)y_2(t) - d_{22}y_1(t) - r_2(t-\tau_2)e^{-d_{22}\tau_2)y_2(t-\tau_2)} \\ \frac{dy_2}{dt} = & r_2(t-\tau_2)e^{-d_{22}\tau_2 y_2(t-\tau_2)} - d_{21}y_2(t) - b_2(t)y_2^2(t) - c_2(t)y_2(t)x_2(t) \end{cases} \tag{1.2}$$

They studied the stability of the equilibrium solution, conditions of extinction, and persistence of the species in the system (1.2). It was observed that in certain conditions on the parameter values, the predator population may survive even at the absence of the prey. i.e., under the right conditions, the system is persistent.

As human being consumes and utilizes a species from the ecosystem, it's necessary to find a balanced harvesting strategy from different age groups of a single species without affecting the ecosystem badly [7–11]. As a matter of fact, models with harvesting strategies have captivated the interest of many researchers in order to ensure the long-term progression of the ecosystem while optimizing economic benefits and sustainability of the ecosystem. There are two prominent harvesting functions proposed and studied by May at al. [12]. Constant and linear harvesting have certain limitations in them [13,14]. Constant harvesting of a population is independent of its density while the linear harvesting is proportional to the population densities. For instance, Lei, and Xiao [13] studied the following single species age structured model (1.3) with linear harvesting strategies for both adult and juvenile populations. It turns out that the dynamics of the model is very simple with only two cases- Globally asymptotically stable boundary $(0,0)$ equilibrium under certain parameters values and globally asymptotically stable internal equilibrium (x^*, y^*) under some others parameters values.

$$\begin{cases} \frac{dx}{dt} = & \alpha y - \beta x - \delta_1 x - q_1 Emx \\ \frac{dy}{dt} = & \beta x - \delta_2 y - \gamma y^2 - q_2 Emy \end{cases} \tag{1.3}$$

To resolve the shortcomings in constant and linear harvesting, the researchers consider harvesting strategies with non-linear functions [15–17].

Among the non-linear harvesting strategies, Michaelis-Menten type harvesting model proved to be more realistic biologically and overall economic perspectives [18]. Michaelis-Menten type harvesting strategies are realized using the non-linear functions of the form $h = \frac{ax}{b+cx}$ for some suitable parameters a, b, c. In comparison to the model with constant and linear harvesting, this non-linear type of harvesting models shows complicated and more realistically important and useful dynamical properties. Hu and Cao studied a prey-predator model with Michaelis-Menten type harvesting strategies exhibiting rich bifurcations [19]. Many scholars studied age structured models with Michaelis-Menten type harvesting strategies. For instance, Liu, Huang, Deng, et al. [20] studied the following amensalism model (1.4) with cover harvesting only from the Juvenile population.

$$\begin{cases} \frac{dx}{dt} = & a_1 x - b_1 x^2 - c_1(1-k)xy - \frac{qE(1-k)x}{m_1 E + m_2(1-k)x} \\ \frac{dy}{dt} = & a_2 y - b_2 y^2 \end{cases} \tag{1.4}$$

The studies show that this model exhibits rich bifurcation phenomenons- saddle and transcritical bifurcations. Under certain conditions of parameters values, it is possible to obtain the threshold value for the optimal harvesting.

Yu, Zhu, and Chen studied single species stage structured model (1.5) with harvesting function in the form of Michaelis-Menten type for the juvenile population [21]. They observed practically useful dynamics.

$$\begin{cases} \frac{dx}{dt} = & \alpha y - \beta x - \delta_1 x - \frac{hEx}{mE+nx} \\ \frac{dy}{dt} = & \beta x - \delta_2 y - \gamma y^2 \end{cases} \tag{1.5}$$

From fisheries production perspective, Zhu Lia et al. [16] proposed and studied the following model (1.6) with harvesting strategies in the form of Michaelis-Menten type function on the adult population, as in most cases only adult population is harvested. They found sufficient conditions on the model parameters for the existence of globally stable boundary and internal equilibrium. They also demonstrated interesting bifurcation phenomena.

$$\begin{cases} \frac{dx}{dt} = & \alpha y - \beta x - \delta_1 x \\ \frac{dy}{dt} = & \beta x - \delta_2 y - \gamma y^2 - \frac{hEy}{mE+ny} \end{cases} \tag{1.6}$$

From the fisheries production perspective, it may also be significantly important to harvest the juvenile population of a fishery for cultivation in other fisheries. For example, in Bangladesh, we harvest the juvenile population of Ruhu Fish (*Labeo rohita*) from the river Halda to cultivate them in lakes and ponds across the country commercially. We harvest the Adult fish from the Halda too. In addition, the higher density of the juvenile population may hinder their growth and the adult population as well necessitating their harvesting. As far as we are informed, no author has studied single species age structured model with harvesting strategies that combines linear harvesting on the juvenile population and Michaelis-Menten type harvesting strategies on the adult population. We propose and investigate the following single species stage structured model (1.7).

$$\begin{cases} \frac{dx}{dt} = & \alpha y - \beta x - \delta_1 x - \delta_3 x \\ \frac{dy}{dt} = & \beta x - \delta_2 y - \gamma y^2 - \frac{hEy}{mE+ny} \end{cases} \tag{1.7}$$

where x denotes the density of the juvenile population, y denotes the density of adult population, α denotes the intrinsic growth rate of juvenile population, β denotes the rate at which juvenile population survive to adulthood, δ_1 denotes the death rate of the juvenile group, δ_2 denotes the death rate of the adult group of the population, δ_3 is the linear harvesting rate of the juvenile population, γ is the intra-specific competition coefficients in the adult population, h is the catchability coefficient, E is the coefficient of combined external effort to harvest the adult and juvenile population. All parameters including m, n are assumed to be non-negative. Consider the following transformations

$$\bar{x} = \frac{\gamma}{\alpha}x,\ \bar{y} = \frac{\gamma}{\delta_2}y,\ \bar{t} = \delta_2 t$$

which transform the system into

$$\begin{cases} \frac{d\bar{x}}{d\bar{t}} = & \bar{y} - a\bar{x} - b\bar{x} \\ \frac{d\bar{y}}{d\bar{t}} = & e\bar{x} - \bar{y}(1+\bar{y}) - \frac{d\bar{y}}{f+\bar{y}} \end{cases} \tag{1.8}$$

where,

$$a = \frac{\beta+\delta_1}{\delta_2},\ d = \frac{hE\gamma}{n\delta_2^2},\ b = \frac{\delta_3}{\delta_2},\ f = \frac{mE\gamma}{n\delta_2},\ e = \frac{\alpha\beta}{\delta_2^2}.$$

For biological feasible consequences, we consider the model in the system in the first quadrant in the $\bar{x}\bar{y}-$plane with any arbitrary initial conditions $\bar{x}(0) = \bar{x}_0 > 0$, and $\bar{y}(0) = \bar{y}_0 > 0$. For notational convenience, we will write $x, y,$ and t instead of $\bar{x}, \bar{y},$ and $\bar{t}$ in the model (1.8).

2. Stability analysis of equilibria

2.1. Boundary equilibria

The model (1.8) has a solitary boundary equilibrium $E_0(0,0)$. Rewrite the model (1.8) in $x, y,$ and t as

$$\begin{cases} \frac{dx}{dt} = & y - ax - bx \\ \frac{dy}{dt} = & ex - y(1+y) - \frac{dy}{f+y} \end{cases} \tag{2.1}$$

The Jacobian matrix for the system (2.1) is

$$J = \begin{pmatrix} -(a+b) & 1 \\ e & -1-2y-\frac{df}{(f+y)^2} \end{pmatrix}.$$

The determinant and trace of the Jacobian matrix are

$$DetJ = -(a+b)\left(-1-2y-\frac{df}{(f+y)^2}\right) - e \tag{2.2}$$

and

$$trJ = -(a+b) + \left(-1-2y-\frac{df}{(f+y)^2}\right) < 0. \tag{2.3}$$

If at an equilibrium point $DetJ \neq 0$ is called an elementary equilibrium. An equilibrium point is saddle when $DetJ < 0$ at the point, and at a degenerate equilibrium point $Det(J) = 0$ [22,23]. The condition at which $J(0,0)$ vanishes is useful for further analysis is given by

$$a + \frac{ad}{f} + b + \frac{bd}{f} = e \tag{2.4}$$

Note that, the trajectories of the juvenile and the adult populations starting in a neighborhood of an elementary equilibrium point are attracted in it, or move away from the equilibrium point. For a saddle, the trajectories move away from the equilibrium point along hyperbolic curves. However, only for certain initial conditions, the trajectories approach the equilibrium point. In case of degenerate equilibria, any initial adult and juvenile populations will remain fixed forever, or either the juvenile or the adult population remains constant while the other goes in or away from the equilibrium point.

Lemma 1. *[24] Consider the following system of differential equations*

$$\begin{cases} \frac{dx}{dt} = & P_2(x,y) \\ \frac{dy}{dt} = & y + Q_2(x,y) \end{cases} \tag{2.5}$$

where P_2 and Q_2 in the system (2.5) are analytic functions of degree at least two in a neighborhood $S_\delta(\mathcal{O})$ of the isolated singular point $\mathcal{O}(0,0)$. For a small enough δ, there exists some analytic function $\phi(x)$ satisfying

$$\phi(x) + P_2(x, \phi(x)) = 0, \text{ for } |\delta| < x.$$

Suppose $\psi(x) = \left(P_2(x), \phi(x)\right) = a_m x^m + [x]_{m+1}$, where $m \geq 2$ and $a_m \neq 0$, then

i. *$\mathcal{O}(0,0)$ is a unstable node if $a_m > 0$ and m is an odd integer.*
ii. *$\mathcal{O}(0,0)$ is a saddle if $a_m < 0$ and m is an odd integer.*
iii. *$\mathcal{O}(0,0)$ is a saddle node if m is an even integer.*

The stability of the boundary equilibrium $E_0(0,0)$ can be determined from the trace and determinant of the Jacobian matrix $J(0,0)$.

Theorem 1. *The boundary equilibrium point $E_0 = (0,0)$ of the model (2.1) is*

i. *a saddle when $a + b + \frac{ad}{f} + \frac{bd}{f} < e$.*
ii. *a stable node when $a + b + \frac{ad}{f} + \frac{bd}{f} > e$.*
iii. *a saddle node when $a + b + \frac{ad}{f} + \frac{bd}{f} - e = 0$.*

Proof. At the boundary equilibrium point $E_0(0,0)$, we get from (2.2),

$$DetJ\left(E_0\right) = a + b + \frac{ad}{f} + \frac{bd}{f} - e.$$

Since $trJ(E_0) < 0$, then if $DetJ\left(E_0\right) > 0$, i.e., $a + b + \frac{ad}{f} + \frac{bd}{f} > e$, E_0 is a stable node, and it is a saddle point when $DetJ\left(E_0\right) < 0$, i.e., $a + b + \frac{ad}{f} + \frac{bd}{f} < e$. Moreover, E_0 is a degenerate equilibrium if $DetJ\left(E_0\right) = 0$, i.e., $a + b + \frac{ad}{f} + \frac{bd}{f} = e$ (equation (2.4)). To explain the characteristic of E_0, we use the following transformation $d\tau = \frac{dt}{f+y}$. From (2.1), we have

$$\frac{dx}{d\tau} = -afx - bfx + fy - axy - bxy + y^2$$
$$\frac{dy}{d\tau} = exf + exy - fy - fy^2 - y^2 - y^3 - dy.$$

We make a transformation as follows

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{\alpha\kappa - \beta\gamma}\begin{bmatrix} \kappa x - \beta y \\ -\gamma x + \alpha y \end{bmatrix}. \tag{2.6}$$

Then from the system of equations (2.6), we obtain $u = \frac{1}{p}(\kappa x - \beta y)$ and $v = \frac{1}{p}(-\gamma x + \alpha y)$, where $p = \alpha\kappa - \beta\gamma$ and the values of α, β, γ and κ are as follows.

$$\alpha = constant,$$
$$\kappa = -\frac{d+f}{(d + (1 + a + b)f)^2\alpha}, \tag{2.7}$$
$$\gamma = (a+b)\alpha, \text{ and} \tag{2.8}$$
$$\beta = \frac{(a+b)}{G_1} \tag{2.9}$$

where,

$$G_1 = de + ((a + a^3 + b + 3a^2b + 3ab^2 + b^3 + 2ae)\alpha +$$
$$(2be + \frac{ad}{f} + \frac{bd}{f})\alpha.$$

Differentiating u and v, we get

$$u' = \frac{1}{p}(\kappa x' - \beta y'), \; v' = \frac{1}{p}(-\gamma x' + \alpha y')$$

We compute $\frac{du}{d\tau}$ and $\frac{dv}{d\tau}$ in terms of u and v by substituting the values of $e = a + \frac{ad}{f}$ and κ, β and γ from equations (2.7), (2.8), and (2.9), and make the transformation $\frac{ds}{d\tau} = \alpha\kappa - \gamma\beta$. We get,

$$\frac{du}{ds} = s_{01}u^3 + s_{02}u^2 + s_{03}uv + s_{04}v^2 + r_1(u,v), \tag{2.10}$$

where

$$
\begin{aligned}
s_{01} &= \frac{b^4\alpha^2+4ab^3\alpha^2+6a^2b^2\alpha^2+4a^3b\alpha^2+a^4\alpha^2}{G_2},\\
s_{02} &= \frac{b^3f\alpha+3ab^2f\alpha+3a^2bf\alpha+a^3f\alpha}{G_3}+\frac{b^3d\alpha+3ab^2d\alpha+3a^2bd\alpha+a^3d\alpha}{G_4},\\
G_2 =& de((a+a^3+b+3a^2b+3ab^2+b^3+2ae)f+\\
&(2be+\frac{ad}{f}+\frac{bd}{f})f,\\
G_3 =& de+(a+a^3+b+3a^2b+3ab^2+b^3+2ae)f+\\
&(2be+\frac{ad}{f}+\frac{bd}{f})f, \text{ and}\\
G_4 =& fde+f^2(a+a^3+b+3a^2b+3ab^2+b^3+2ae)+\\
&(2be+\frac{ad}{f}+\frac{bd}{f})f^2.
\end{aligned}
$$

We omit s_{03} and s_{04} as it is not used further. Next, the value of $\frac{dv}{ds}$ is

$$
\frac{dv}{ds}=t_{01}v+t_{02}u^2+t_{03}v^2+t_{04}uv+t_{05}u^3+r_2(u,v), \tag{2.11}
$$

where,

$$
\begin{aligned}
t_{01} &= \frac{d^2+2df+adf+bdf+f^2+af^2+bf^2}{(d+(1+a+b)f)^2}+\frac{G_5}{G_6},\\
t_{02} &= \frac{a^2d\alpha^3+2abd\alpha^3-b^2d\alpha^3}{f}-a^2f\alpha^3-2abf\alpha^3-b^2f\alpha^3,\\
G_5 &= da^2+db^2+2abd+a^2f+a^3f+2abf+3a^2bf+b^2f+3ab^2f+b^3f, \text{ and}\\
G_6 =& de+((a+a^3+b+3a^2b+3ab^2+b^3+2ae)f+\\
&(2be+\frac{ad}{f}+\frac{bd}{f})f.
\end{aligned}
$$

We omit t_{03}, t_{04}, and t_{05} as they are not used further.
Define $H(u,v)=t_{01}v+t_{02}u^2+t_{03}v^2+t_{04}uv+t_{05}u^3+r_2(u,v)$ from equation (2.11). Since $\frac{\partial H}{\partial v}\neq 0$, by implicit function theorem, there exists a function $v=w_1(u)$, such that $w(0)=0$ and $q(u,w(u))=0$. The function $v=w(u)$ may be approximated iteratively as follows

$$
\begin{aligned}
v_1 &= w_1(u)=0,\\
v_2 &= -t_{02}u^2-t_{05}u^3,\\
v_3 &= v_2-H(u,v_2)\\
&= -t_{02}u^2-t_{05}u^3-(-t_{01}t_{02}u^2-t_{01}t_{05}u^3+t_{02}u^2)+t_{03}(-t_{02}u^2-t_{05}u^3)^2\\
&= t_{01}t_{02}u^2+(t_{01}t_{05}-t_{05})u^3.
\end{aligned}
$$

Therefore, we have

$$
v=w_1(u)=t_{01}t_{02}u^2+(t_{01}t_{05}-t_{05})u^3+\dots. \tag{2.12}
$$

We substitute this value of v from equation (2.12) into the equation (2.10) to get

$$
\begin{aligned}
\frac{du}{ds} &= s_{01}u^3+s_{02}u^2+s_{03}uv+s_{04}v^2+r_1(u,v)\\
&= s_{01}u^3+s_{02}u^2+s_{03}u(t_{01}t_{02}u^2+(t_{01}t_{05}-t_{05})u^3)\\
&= s_{01}u^3+s_{02}u^2+s_{03}t_{01}t_{02}u^3\\
&= s_{02}u^2+(s_{03}t_{01}t_{02}+s_{01})u^3.
\end{aligned}
$$

From Lemma 1, since $s_{01}\neq 0$, i.e., $\frac{b^4\alpha^2+4ab^3\alpha^2+6a^2b^2\alpha^2+4a^3b\alpha^2+a^4\alpha^2}{G_2}\neq 0$ and $m=2$, E_0 is a saddle node. However, due to the transformation, $ds=(\alpha\kappa-\gamma\beta)\,d\tau$, it turns into a stable node when $\alpha\kappa-\gamma\beta<0$. This completes the proof. □

2.2. Internal equilibria

Our next target is to analyze the internal equilibrium of the system (2.1). Stationary points are the solution of the system

$$
\begin{cases} y-ax-bx= & 0\\ ex-y(1+y)-\frac{dy}{f+y}= & 0\end{cases} \tag{2.13}
$$

From (2.13), we get from the first equation $x = \frac{y}{a+b}$. Substituting x in the second equation of (2.13), and since $y \neq 0$, we get $(ad + bd + af + bf - ef + ay + by - ey + afy + bfy + ay^2 + by^2) = 0$. The real positive roots of the equation are given by

$$y_1 = \frac{1}{2(a+b)}(-a-b+e-af-bf) - \frac{1}{2(a+b)}\left(\sqrt{(a+b-e+af+bf)^2 - 4(a+b)(ad+bd+af+bf-ef)}\right)$$
$$y_2 = \frac{1}{2(a+b)}(-a-b+e-af-bf) + \frac{1}{2(a+b)}\left(\sqrt{(a+b-e+af+bf)^2 - 4(a+b)(ad+bd+af+bf-ef)}\right)$$

We consider the following two cases:

Case-I. It can be shown that y_1 and y_2 are positive real when $a > 0$, $b > 0$, $e > a+b$, $0 < f < \frac{-(a+b-e)}{a+b}$ and $\frac{-(a+b-e)f}{a+b} < d < \frac{(-a-b+e+af+bf)^2}{4(a+b)^2}$. In this case, we have two internal equilibria.

We substitute y_1, under the conditions in Case-I, into the equations (2.2) and (2.3) for the determinant and trace of the Jacobian matrix at (x_1, y_1). It can be shown that $det(J)(x_1, y_1)$ and $tr(J)(x_1, y_1)$ are both less than zero. Therefore the internal equilibrium $E_1 = (x_1, y_1)$ is a saddle point.

We substitute y_2, under the conditions in Case-I, into the equations (2.2) and (2.3) for the determinant and trace of the Jacobian matrix at (x_2, y_2). It can be shown that $det(J)(x_2, y_2) > 0$ and $tr(J)(x_2, y_2) < 0$. Therefore the internal equilibrium $E_2 = (x_2, y_2)$ is a stable point.

Case-II. It can also be shown that y_2 is the only positive solution when $a > 0$, $b > 0$, $e > a+b$, $0 < d < \frac{-(a+b-e)f}{a+b}$, $f \neq \frac{-(a+b-e)}{a+b}$. In this case, we have only one internal equilibrium.

We substitute y_2, under the conditions in Case-II, into the equations (2.2) and (2.3) for the determinant and trace of the Jacobian matrix at (x_2, y_2). It can be shown that $det(J)(x_2, y_2) > 0$ and $tr(J)(x_2, y_2) < 0$. Therefore the only internal equilibrium $E = (x_2, y_2)$ is a stable point. Moreover, there is no degenerate equilibrium point.

2.3. Global stability analysis

Theorem 2. *If $a + b > e$ or, equivalently $\alpha < \left(\frac{\delta_1+\delta_3}{\beta} + 1\right)\delta_2$, then the boundary equilibrium $E_0(0,0)$ is asymptotically stable globally.*

Proof. We recall our proposed model (2.1)

$$\begin{cases} \frac{dx}{dt} = & y - ax - bx \\ \frac{dy}{dt} = & ex - y(1+y) - \frac{dy}{f+y}. \end{cases}$$

Consider $L(x, y) = x + y$ as a Lyapunov function. The function L clearly vanishes at $E_0(0,0)$, and L is strictly greater than zero when $x > 0$ and $y > 0$. Then

$$\begin{aligned} D^+L(x,y) &= x' + y' \\ &= y - ax - bx + ex - y(1+y) - \frac{dy}{f+y} \\ &= -(a+b-e)x - \frac{dy}{f+y} - y^2. \end{aligned}$$

Notice, when $a + b > e$ then $D^+L(x, y) \leq 0$, and $D^+L(x, y) = 0$ iff $(x, y) = (0, 0)$. Therefore, $L(x, y)$ is a suitable function that satisfies the asymptotic stability theorem of Lyapunov and implies the global asymptotic stability of $E_0(0,0)$. □

Therefore, when the intrinsic growth rate of the juvenile population is sufficiently small, i.e. $\alpha < \left(\frac{\delta_1+\delta_3}{\beta} + 1\right)\delta_2$, both juvenile and adult population will extinct eventually.

Theorem 3. *When $a + b + \frac{ad}{f} + \frac{bd}{f} < e$ and $e > a + b$ holds, or equivalently $\alpha > \left(\frac{h}{m} + \delta_2\right)\left(\frac{\delta_1+\delta_3}{\beta} + 1\right)\delta_2$, the internal equilibrium point E_1 is globally asymptotically stable.*

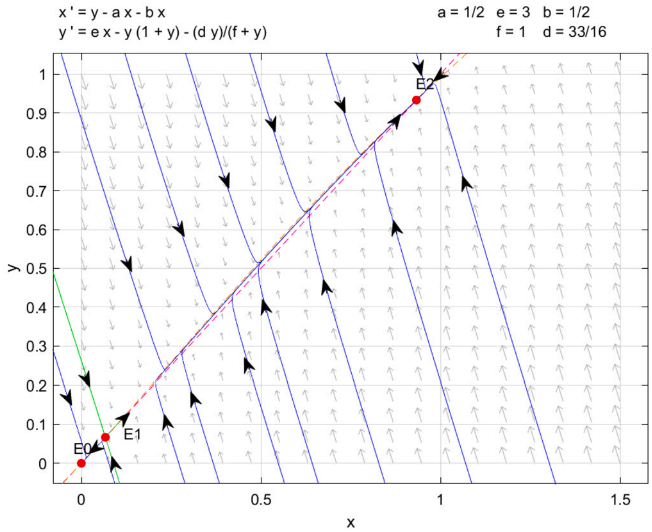


Fig. 1. The coexistence of juvenile and adult species for stable point for case-I.

Proof. To prove this, suppose $D(x, y) = 1$. Let $P(x, y) = y - ax - bx$, and $Q(x, y) = ex - y(1 + y) - \frac{dy}{f+y}$. We have

$$\begin{aligned}\frac{\partial(DP)}{\partial x} + \frac{\partial(DQ)}{\partial y} &= \frac{\partial}{\partial x}(y - ax - bx) + \frac{\partial}{\partial y}(ex - y(1+y) - \frac{dy}{f+y}) \\ &= -a - b - 1 - 2y - \frac{(fd + yd - dy)}{(f+y)^2} \\ &= -a - b - 1 - 2y - \frac{fd}{f+y} < 0.\end{aligned}$$

According to Bendixson-Dulac discriminant [25], when $x > 0$ and $y > 0$, the model (2.1) does not possess any limit cycle. And when $a + b < e$, $E_0(0, 0)$ is a saddle. Therefore, the Poincare-Bendixson theorem implies global asymptotic stability of the internal equilibrium point E_1. □

Therefore, when the intrinsic growth rate of the juvenile population is sufficiently large, i.e. $\alpha > \left(\frac{h}{m} + \delta_2\right)\left(\frac{\delta_1+\delta_3}{\beta} + 1\right)\delta_2$, both juvenile and adult population will coexist despite their combined harvesting.

3. Graphical and numerical analysis

To demonstrate the result of case-I, we choose the parameter values $a = \frac{1}{2}$, $d = \frac{33}{16}$, $e = 3$, $f = 1$, $b = \frac{1}{2}$ so that the conditions in case-I hold. Then we have, boundary equilibrium point $E_0 = (0, 0)$, two internal equilibrium points $E1(0.066987, 0.066987)$ and $E2(0.93301, 0.93301)$ which are saddle and stable (Fig. 1) respectively. Notice that the trajectories move in the direction of the internal equilibrium point $E2$, which implies that adult and juvenile population will stably coexist at the point $E2$.

Now for conditions in case-II, we choose parameter values $a = \frac{1}{2}$, $d = \frac{1}{2}$, $e = 2$, $f = \frac{3}{4}$, $b = \frac{1}{2}$ then we obtain a boundary equilibrium point E_0 and only one internal equilibrium point $E(0.64039, 0.64039)$ which is stable (Fig. 2). Therefore, adult and juvenile population will coexist stably at the point E. The stabilities of the equilibrium points $E2$ in case I and E in case II have also been presented in Figure 3 and Figure 4 respectively from numerical simulations.

4. Conclusion

We have considered a stage structured model for a single species with a linear harvesting strategy on the Juvenile population and Michaelis-Menten form of harvesting on the adult population of the species. The study shows that if the growth rate of the juvenile population is sufficiently small i.e. $\alpha < \left(1 + \frac{\delta_1+\delta_3}{\beta}\right)\delta_2$, the unique boundary equilibrium is globally asymptotically stable. In the case with no linear harvesting term in the system (1.6), this happens when $\alpha < \left(1 + \frac{\delta_1}{\beta}\right)\delta_2$. Therefore harvesting the juvenile population enhances the possibility of the extinction of the both groups of population. When $\alpha > \left(\frac{h}{m} + \delta_2\right)\left(1 + \frac{\delta_1+\delta_3}{\beta}\right)\delta_2$, there exists unique globally asymptotically stable internal equilibrium. In the case with no linear harvesting term in the system (1.6), this happens when $\alpha > \left(\frac{h}{m} + \delta_2\right)\left(1 + \frac{\delta_1}{\beta}\right)\delta_2$. Therefore the persistence of the both groups of population is achieved for larger growthrate of the juvenile population. Harvesting the juvenile population linearly may be the possibility of the extinction of the both groups of population.

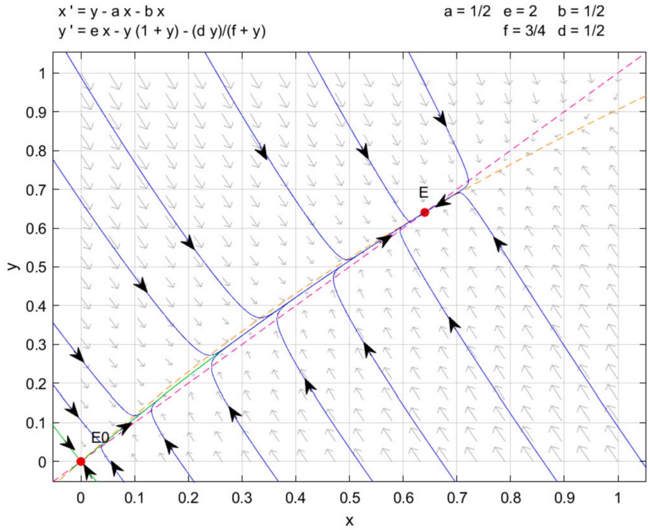
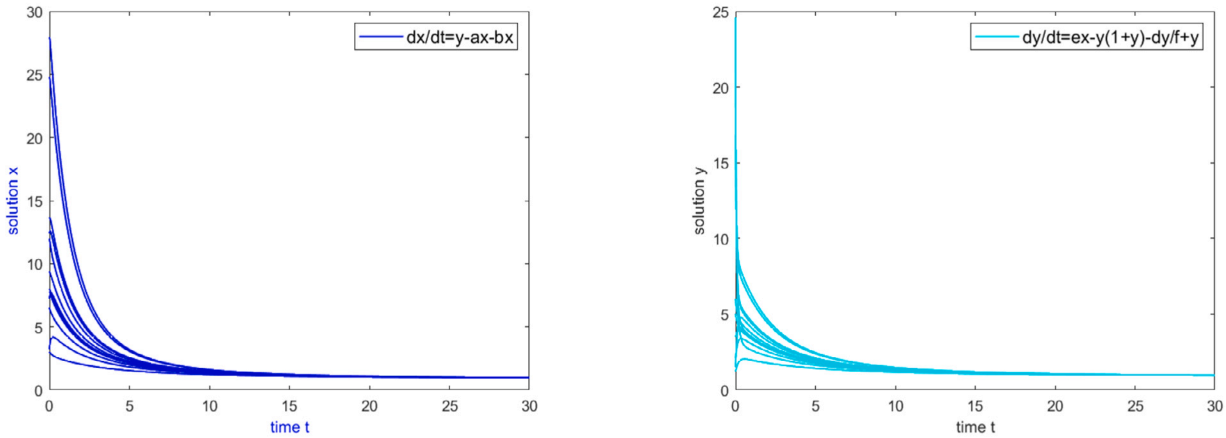
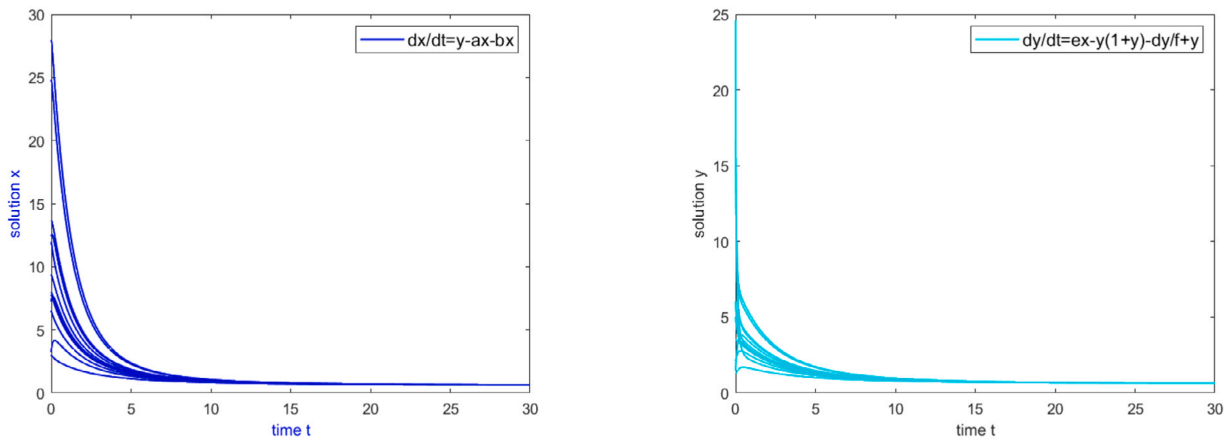


Fig. 2. The coexistence of juvenile and adult species for stable point for case-II.



(a) Juvenile Species (b) Adult Species

Fig. 3. Numerical simulation of juvenile species x and adult species y at the stable node E_2 for Case-I.



(a) Juvenile Species (b) Adult Species

Fig. 4. Numerical simulation of juvenile species x and adult species y at the stable node E for Case-II.

Overall, introduction of a linear harvesting strategy in the juvenile population keeps the dynamics of the system more or less similar in comparison to the dynamics of the model with Michaelis-Menten harvesting on adult group and no harvesting in the juvenile group in the system (1.6). It may be economically beneficial to transfer over production of juvenile group to other habitats and grow them there, or they may be used to supplement as a source of protein. Our study shows that it is possible to harvest both stage groups without endangering any of them to extinction. Therefore the introduction of a linear harvesting term on the juvenile population made the dynamics of the system (1.6) simple and realistic where both stage structured population coexist.

CRediT authorship contribution statement

Saima Akter: conceived and designed the experiments; performed the experiments; contributed reagents, materials, analysis tools or data; wrote the paper.
Md. Shariful Islam: conceived and designed the experiments; analyzed and interpreted the data; contributed reagents, materials, analysis tools or data; wrote the paper.
Touhid Hossain: performed the experiments; analyzed and interpreted the data; contributed reagents, materials, analysis tools or data; wrote the paper.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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