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Milky Way could invalidate the hypothesis of exotic matter and favor a gravitomagnetic solution to explain dark matter

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We demonstrate a very general mathematical and physical expression of the rotation speed at the end of the galaxy (far from the vast majority of the galaxy's baryonic mass) obtained from General Relativity without non-baryonic matter. We show the excellent agreement with measurements obtained for the Milky Way published in a recent article which confirms a significantly faster decline in the circular velocity curve at outer galactic radii up to 30 kpc compared to the inner parts. This relation comes from Linearized General Relativity (GRL). Some papers argue that the GRL solution cannot explain dark matter (DM). We demonstrate that this conclusion is too premature because they only consider mass currents of the galaxies which is not the most general theoretical solution. And because this GRL explanation suffers from the same defects as exotic matter, only direct measurement of the Lense-Thirring effect can objectively reject this solution. Current experiments are not yet precise enough to test this solution. But meanwhile, if the relevance of this expression were confirmed for most galaxies, this would strongly challenge exotic matter to explain DM and could drastically change the point of view on the DM component. Two known physical fields (contrary to an exotic matter) which are until now neglected or rather underestimated would then explain DM. The DM mystery would then consist for theory in understanding how the values of these fields can be larger than expected and for observation in being able to measure these two fields with sufficient precision. In addition, these fields allow obtaining the TULLY-FISHER relation and the MOND theory.

Keywords Gravitation, Galaxies: fundamental parameters, Galaxies: halos, Dark matter

Since the 1960s, it has been observed that the rotation speed of galaxies is greater than expected by the gravitational theory[1]. A similar problem was observed in the 1930s for galaxy clusters[2]. One explanation for these speeds is to assume the existence of unknown non-baryonic matter. This solution of exotic matter is called Dark Matter (DM) because invisible, undetectable and insensitive to Electromagnetism (EM). This is the most studied assumption to date, with growing observational evidence confirming this DM problem but still no particle detection.

To try to explain this dark component within the framework of general relativity (GR), several avenues have been opened. To put it simply, we can roughly define four theoretical frameworks for gravitational interaction which all correspond to different levels of approximation of the GR. In order of increasing precision we first have the Newtonian approximation which corresponds to the Newtonian mechanics and which corresponds to a degraded form of the linearization of the GR (GRL) by neglecting the frame dragging and therefore only contains the Newtonian component of the gravitational interaction. Secondly and third, there is the GRL which is the exact linearization of GR. It enriches the Newtonian component with a 2nd component, the frame dragging, similar to the magnetic field in EM. The equations of GRL lead to two types of solutions for the frame dragging term, a non-homogeneous solution (NHS) and a homogeneous solution (HS).GRL is also called GravitoElectroMagnetism (GEM)[3]. Therefore, GRL comes in two theoretical frameworks, one with only NHS and the other with both NHS and HS also called "strong gravitomagnetism". To simplify, we will now call GEM the solutions without the HS term and GRL the solutions with the HS term. Finally, fourth there is the GR. The problem with applying GR directly is the complexity of solving its non-linear equations in many cases. Chronologically, the first attempts to explain the rotation curves of galaxies were made using the Newtonian approximation. It is within this theoretical framework that we made, on the one hand, the hypothesis of the

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existence of an exotic matter (because invisible and insensitive to EM) and, on the other hand, the hypothesis that its distribution extends very far to the end of the galaxy (to explain the flat curve at the end of the galaxy). Subsequently, faced with this problem, some authors attempted to directly use the GR[4–9], followed by other attempts to use the GEM [10,12,13] and more recently the GRL[11,14,15].

Our article belongs to the GRL category. While most GEM articles simply reduce the amount of DM, the GRL articles[11,14,15] entirely explain the DM component without exotic matter. But more recently, some articles[16–18] demonstrate that the corrections of GEM are about $\mathcal{O}(10^{-6})$ smaller than the standard Newtonian effects, rejecting then this GEM solution. In most of these previous articles except[11,14,15] the GRL solution (GEM with both NHS and HS terms) is never studied. Explicitly, they do not take into account the possibility of a uniform field (similar to the field generated by magnets in EM, as proposed in our article). This field corresponds to a particular HS term. It was already proposed a few years ago in[11] and its relevance is confirmed by[15]. In EM, such uniform fields in particle accelerators are essential to maintain particles on high-speed trajectories. This situation in particle accelerators is very similar to the trajectories of matter at the ends of the galaxies maintained at high speed. We can add that[11] also already demonstrated the impossibility of the GEM solution to explain entirely the DM from the distribution of the known visible matter because the rotation curve should anyway necessarily decrease at the end of the galaxies. In this latter work, it was the justification to study this particular HS term. We refer to this pioneering work on GRL solution that the recent articles[16–18] therefore confirm.

We investigate this more general solution of GRL with a recent publication[19] which obtained the most accurate measurement of the rotation speeds of the Milky Way (MW). In particular, they find a significantly faster decline in the circular velocity curve at outer galactic radii up to 30 kpc compared to the inner parts. On the one hand, we demonstrate the excellent agreement of this relation of GRL with the measurements of[19], enough accurate to also strongly challenge exotic matter assumption, and on the other hand we confirm nevertheless the smallness of the value expected by the theory for these fields, around $\mathcal{O}(10^{-6})$. But unlike[16–18], we argue that it is too early to rule out this solution without more direct observational evidences because, in EM, Maxwell equations and the charge currents cannot either more explain the magnetic field values of magnets. Quantum Mechanics (QM) must be invoked for EM to explain these fields. With the GEM solution presented in[16–18] we are in exactly the same situation (without QM). In the EM context, their modeling would lead to the impossibility of the existence of ferromagnetic materials, which nevertheless exist. Their conclusion should therefore rather be that mass currents cannot indeed explain these fields and that if these field values were confirmed by observations, they would have to be generated by another physical process (non-linearity of the GR[14], Quantum Gravity or something else). Moreover, as we will see in this article, the solution of an exotic matter suffers from similar defects to the GRL solution, which should therefore not discredit this GRL solution.

In this article we tackle the GRL solution in a different manner. In terms of method, we start from measurements of the rotation curve for the MW and we look for the value of the gravitational fields which allow us to best adjust this curve. Then we analyze the value of these fields within the framework of known physics. In theoretical terms, our GRL solution essentially differs from other GRL solutions by its simplification making it valid far from the center of the galaxy (in addition to take in account a uniform gravitomagnetic field, called gravitic field, equivalent to that integrated in EM by QM to explain the magnetic field of magnets). The interest in studying the velocity curve far from the center of the galaxy, by focusing where the DM problem appears, is to limit the number of hypotheses, in particular on the thickness of the galactic disc or on the mass distribution (potential and surface density), which is considered as a central mass in our modeling. By this way, we hope to be as close as possible to the very essence of the DM problem, the counterpart is that this idealization will not be valid in the inner parts of the galaxy where the mass distribution must be taken into account. But it is precisely this simplification on the mass distribution which makes this solution very constraining for the exotic matter assumption. Surprisingly, this "galaxy modeled as point-particle" constitutes its strong point.

Our study brings several novelties. It proposes to use the homogeneous solution of GRL as the keystone of the explanation of the DM component. It goes further than[15] by proposing a concrete physical interpretation of this homogeneous solution (the uniform gravitic field k_0 in this study). We will see that this relation could also be a good approximation of the virial theorem in the context of GRL. We demonstrate that the neighboring clusters could be the origin of this uniform k_0 term. We show that the existence of this gravitic field from the clusters would make it possible to obtain the rotation curves at the ends of the galaxies without exotic matter, it would then totally explain the DM component. We also show that the intensity of this field is sufficiently weak to explain why it is not detected at our scale even though it is dominant at the ends of the galaxies (a logic that exotic matter does not follow). The value of this field also makes it possible to find several observed relations, in particular the Tully-Fisher relation. The value of this field also makes it possible to find the MOND modeling, it means that MOND theory appears as an approximation of this GRL solution. Furthermore, this solution, by its simplicity, strongly challenges the hypothesis of exotic matter because it does not imply any notable distribution of matter at the ends of the galaxies. Since this expression is a general solution of GR far from the central mass of the galaxy, it should be able to describe the rotation speed at the ends of a large number of other galaxies but also for a large number of dwarf satellite galaxies. If its relevance were confirmed for most galaxies and dwarf satellite galaxies, it would strongly challenge the exotic matter explaining DM because this expression cannot be obtained with a wide distribution of matter but only far from the central mass. And finally, our very concrete physical model allows us to propose several predictions, for example the planar movement of dwarf satellite galaxies around their host.

To summarize, our study provides a physical interpretation of the homogeneous solution of GRL, obtains the intensity of the physical gravitic field expected to totally explain DM, consolidates this intensity by other relations making this explanation very self consistent, challenges the assumption of exotic matter and proposes several testable predictions.

Expression of the rotation speed at the end of the galaxies

To simplify our idealization, we assume that the galaxy exhibits axisymmetry and we will evaluate the physical quantities on the galactic plane $z = 0$ without velocity dispersion. To use the GRL equations (3), we assume that the galaxy is a stationary system. The traditional computation of rotation speeds of galaxies consists in obtaining the force equilibrium from the three following components: the disk, the bulge and the halo of DM. More precisely, one has[20]:

$$\frac{v^2(r)}{r} = \frac{\partial \phi(r)}{\partial r} \quad with\ \phi = \phi_{disk} + \phi_{bulge} + \phi_{halo} \tag{1}$$

The total speed squared can be written as the sum of squares of each of the three speed components:

$$\begin{aligned} v^2(r) &= r\frac{\partial \phi_{disk}(r)}{\partial r} + r\frac{\partial \phi_{bulge}(r)}{\partial r} + r\frac{\partial \phi_{halo}(r)}{\partial r} \\ &= v^2_{disk}(r) + v^2_{bulge}(r) + v^2_{halo}(r) \end{aligned} \tag{2}$$

The assumption we are going to make which impacts the form of the expression that we are going to obtain is that far from the center of the galaxy, the gravitational field is weak enough to allow the use of the GRL. This approximation of a weak gravitational field will be justified later.

The linearization of the field equations of the GR leads to the Einstein-Maxwell equations[21–23] similar to the Maxwell equations of EM:

$$\begin{aligned} &\vec{g} = -\vec{\nabla}\phi \ ; && \vec{k} = \vec{\nabla} \wedge \vec{H} \\ &\vec{\nabla} \wedge \vec{g} = 0 \ ; && \vec{\nabla} \wedge \vec{k} = -4\pi\tfrac{G}{c^2}\rho\vec{v} \\ &\vec{\nabla}.\vec{g} = -4\pi G\rho \ ; && \vec{\nabla}.\vec{k} = 0 \end{aligned} \tag{3}$$

Where $\vec{k}$ is the 2nd component of GRL (component that we will call "gravitic field") at the origin of the Lense-Thirring effect observed in the Gravity Probe B mission[24–26] and $\vec{H}$ is the vector potential of the gravitic field. ρ is the mass density. The parametrized Post-Newtonian (PPN) formalism can be retrieved with $\vec{k} = \vec{B_g}/4$, $\vec{B_g}$ being the gravitomagnetic field[27] and $\vec{H} = \vec{A_g}/4$, $\vec{A_g}$ being the gravitomagnetic vector potential[21]. The interest of our notation is that the field equations are strictly equivalent to Maxwell idealization (with a wave speed of *c*). Only the equations of motion are different with a factor "4" (see hereafter), but once again with $\vec{k} = \vec{B_g}/4$, we retrieve the equations of motion in the PPN formalism.

The linearization of the geodesic equations of the GR, combined with the Einstein-Maxwell equations (3) leads to the following equations of motion[21,23,28] for a particle of mass m_p:

$$\frac{\vec{dp}}{dt} = m_p[\vec{g} + 4\vec{v} \wedge \vec{k}] \tag{4}$$

According to the Einstein-Maxwell equations (3) and the equations of motion (4), $\vec{k}$ is equivalent to the magnetic field in EM. Far from the center of the galaxy of mass M_{Gal} and with $\vec{v} \perp \vec{k}$, one then has with (4):

$$\begin{aligned} &\frac{\partial(\phi_{disk}(r) + \phi_{bulge}(r))}{\partial r} \approx \frac{GM_{Gal}}{r^2} \\ &\frac{\partial \phi_{halo}(r)}{\partial r} \approx 4k(r)v(r) \end{aligned} \tag{5}$$

The perpendicularity $\vec{v} \perp \vec{k}$ (with $\vec{k}$ perpendicular to the disc) is justified in[23]. It is the more likely and natural equilibrium state with time. Furthermore, this perpendicularity is not fundamentally required to obtain the following expression because it doesn't impact the form of the expression that we are going to obtain, it only impacts the value of *k*(*r*) because $\vec{v} \wedge \vec{k} \propto v(r)k(r)$ whatever the angle between $\vec{k}$ and $\vec{v}$. We can specify that in our solution, there is no exotic matter. Relation (5) allows us to see that in our solution the halo contribution is all captured in the field $\vec{k}(r)$. At equilibrium and at the end of the galaxy, GR implies with (2) and (5) that :

$$\frac{v^2}{r} = \frac{GM_{Gal}}{r^2} + 4k(r)v(r) \iff v^2 - [4rk(r)]v(r) - \frac{GM_{Gal}}{r} = 0 \tag{6}$$

This equation leads to the general solutions:

$$v_{\pm} = 2rk(r) \pm \sqrt{4(rk(r))^2 + \frac{GM_{Gal}}{r}} \tag{7}$$

$$= 2rk(r)\left(1 \pm \sqrt{1 + \frac{\frac{GM_{Gal}}{r}}{4(rk(r))^2}}\right) \tag{8}$$

We can go further by expressing explicitly $k(r)$ in a more general way. Indeed, GR in its linearized form strongly constrains the expression of the gravitic field $k(r)$. Not all curves of $k(r)$ are experimentally allowed. As said before, the gravitic field is similar to the magnetic field. The Einstein-Maxwell equations (3) lead then to two possible idealizations far from the source of the fields. The first one concerns the gravitic field of the galaxy itself $k_{Gal}(r)$. Thanks to the Poisson's equations[21] in weak field (i.e. far from the center of the galaxy and from the vast majority of the galaxy's baryonic mass), one must have $k_{Gal}(r) \approx \frac{K_1}{r^2}$ (like $\vec{g}$) where K_1 characterizes the own gravitic field of the galaxy[23]. But more generally, a second field can be idealized. We can have a uniform gravitic field k_0, just like in a particle accelerator a uniform magnetic field can hold high-speed particles in an orbit. Unlike $\vec{g}$ field (and electric $\vec{E}$ field), the mathematical expression of $\vec{k}$ field (like magnetic $\vec{B}$ field) allows the existence of uniform fields at large scale (like atomic spins for magnets generate permanent magnetization of the material). Our general $k(r)$ expression allowed by GR in weak field is then:

$$k(r) \approx \frac{K_1}{r^2} + k_0 \tag{9}$$

This uniform field k_0 is the term that is not taken into account in previous GEM articles[12,16–18]. We do not deduce it from mass currents. This term is obtained with the specific potential vector $\vec{H_0} \propto \vec{k_0} \wedge \vec{r}$, just as in EM[29,30] for a uniform magnetic field. This expression of $\vec{H_0}$ verified the Einstein-Maxwell equations (3) and thus the introduction of the term $\vec{k_0}$ is mathematically justified in RG. From this expression, we can remark that H_r and H_z are negligible in our solution because of the perpendicularity $\vec{v} \perp \vec{k}$[23] as in[18]. Without $\vec{k_0}$ we find the known GEM solution[12,16–18].

Returning to the general solutions (8), we focus our study on the positive speed. Our general solution of GR for rotation speed at the end of the galaxy depends on three parameters K_1, k_0, M_{Gal} (without exotic matter) and is:

$$v_+ = 2(\frac{K_1}{r} + k_0 r) + \sqrt{4(\frac{K_1}{r} + k_0 r)^2 + \frac{GM_{Gal}}{r}} \tag{10}$$

The left member of our physical expression (6) is to be compared to the mathematical expression (25) and (26) of[15]. Therefore, our relation (10) provides a direct physical interpretation of these relations of[15].

It is important to understand that for this expression (10), it is necessary to be in a weak field and far from the vast majority of the galaxy's baryonic mass (or any kind of mass in fact). In the inner parts of the galaxy, both $\frac{K_1}{r^2}$ and $\frac{GM_{Gal}}{r^2}$ diverge because the mass distribution must be taken into account and K_1 and M_{Gal} can no longer be considered constant. But in the outer part of the galaxy, we will verify that we have a weak gravitational field (with the vast majority of the mass in the inner parts). Consequently, this relation (10) is then a very general expression allowed mathematically and physically at the end of the galaxies by GRL.

We can add this interesting point that, as we will show, this expression (10) obtained from GRL can retrieve the rotational speed of MW, i.e. without taking into account the non-linearity of GR. But this expression will also confirm the very high values of the gravitic fields. Taking these non-linear terms into account could help explain these high values.

In this solution without exotic matter, and because this relation is valid far from the center of the galaxy (idealized as a central point mass), the mass of the galaxy M_{Gal} is known, therefore only K_1 and k_0 are unknown parameters. Moreover K_1 and k_0 are constant parameters which makes this relation extremely constrained. Two measurement points are enough to freeze the entire curve. Despite this strong constraint, we will see the excellent accuracy of this relation with all the MW measurement points for $r > 16$ kpc. One can add that 90% of the mass of MW stars lies within a radius less than 3 kpc and in our proposed explanation of DM, there is no exotic matter. The fact that this expression (10) is valid for $r > 16$ kpc justifies the approximation which considers the Milky Way as a point particle, as a central point mass.

Applications

To determine the two constants K_1 and k_0, we fit our relation (10) to the end of the galaxy rotation speed curve and maximize the area towards the center to fit as close as possible to the rotation speed curve.

Application to the Milky Way

Concretely, we applied this expression (10) to the measurements of[19] and obtained the blue curve of Fig. 1. Figure 1 takes up the graph of[19] in which we added our blue curve obtained with the following parameters:

$$K_1 \approx 10^{25.21}\, m^2\, s^{-1}; k_0 \approx 10^{-16.65}\, s^{-1} \tag{11}$$

$$M_{Gal} \approx 8 \times 10^{40}\, kg(\,only\, baryonic\, matter) \tag{12}$$

As a reminder on the notation, $\frac{K_1}{r^2}$ (and not K_1) has the same physical unit as k_0. We have differentiated K_1 in capital letters from k_0, just as we traditionally distinguish the gravitational constant G in capital letters from the gravitational acceleration g.

The blue curve is in excellent agreement with measurements[19] from 16 kpc and until the end of the Milky Way. On this part of the curve, our modeling without exotic matter is as good as the red curve (Baryon+DM

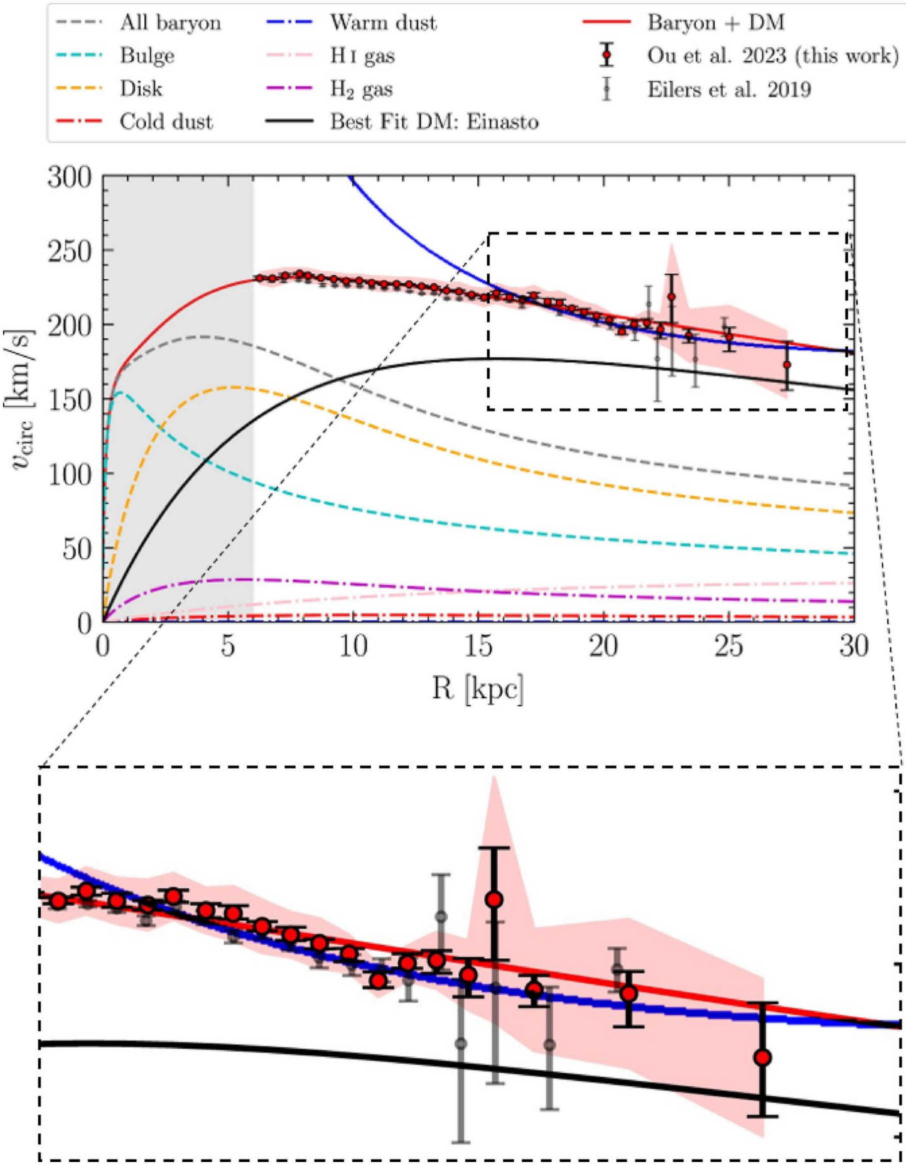

Fig. 1. Superimposition of the measurements (red points) from[19] and our GR expression (10) (blue curve) obtained with $K_1 \approx 10^{25.21}\ \mathrm{m}^2\,\mathrm{s}^{-1}$; $k_0 \approx 10^{-16.65}\ \mathrm{s}^{-1}$; $M_{Gal} \approx 8 \times 10^{40}$ kg (baryonic matter). Bottom graph is a zoom on the area beyond 16 kpc which allows to see the excellent agreement of the GR expression (10) with measurements of the rotation speed of the Milky Way.

model) of[19]. We insist that only two measurement points completely define this curve (unlike the hypothesis of exotic matter for which the DM mass distribution can be adjusted for each position *r*). This means that it is impossible to constrain this curve to pass through the other 16 measurement points. And yet the curve actually passes through all these measurement points (which are spread over more than 10 kpc) except for one which presents the greatest uncertainty, but the curve is nevertheless very close. Taking into account the uncertainty bars around the measurement points, we can define two limit curves, “maximum” ($K_1 \approx 10^{25.14}$, $k_0 \approx 10^{-16.55}$) and “minimum” ($K_1 \approx 10^{25.28}$, $k_0 \approx 10^{-16.85}$) which define the possible limit values of K_1 and k_0. Beyond these values, our GRL solution does not reflect physical reality. It is also important to note that without the uniform field k_0 (specificicity of our idealization compared to GEM solutions), our solution would not work either. The field k_0 and its relative uniformity along the end of the galaxy are the keystone to explain DM without exotic matter.

We can now justify the use of GRL by checking that the gravitational field is weak enough. Let’s compute the values of the acceleration fields due to the three gravitational fields (gravity field of the Milky Way, “internal” gravitic field of the Milky Way $k_{Gal}(r)$ and “external” uniform gravitic field k_0). At $r \approx 16.24$ kpc $\approx 5 \times 10^{20}$ m, where our expression starts being in excellent agreement with the measurement, the rotation speed is $v(r) \approx 218\ \mathrm{km\,s}^{-1}$. The values of the acceleration fields are:

$$a_G = \frac{GM_{Gal}}{r^2} \approx \frac{6 \times 10^{-11} \times 8 \times 10^{40}}{(5 \times 10^{20})^2} \approx 10^{-10.72}\, m\, s^{-2} \tag{13}$$

$$k_{Gal}(r) = \frac{K_1}{r^2} \approx \frac{10^{25.21}}{r^2}\, s^{-1} \Rightarrow a_{K_1} = 4k_{Gal}(r)v(r) \approx 10^{-10.24}\, m\, s^{-2} \tag{14}$$

$$k_0 \approx 10^{-16.65}\, s^{-1} \Rightarrow a_{k_0} = 4k_0 v(r) \approx 10^{-10.71}\, m\, s^{-2} \tag{15}$$

We can also compute the values around the end of the measurements, for instance at $r \approx 25.02$ kpc $\approx 8 \times 10^{20}$ m for which the rotation speed is $v(r) \approx 191$ km s^{-1}:

$$a_G = \frac{GM_{Gal}}{r^2} \approx \frac{6 \times 10^{-11} \times 8 \times 10^{40}}{(8 \times 10^{20})^2} \approx 10^{-11.12}\, m\, s^{-2} \tag{16}$$

$$k_{Gal}(r) = \frac{K_1}{r^2} \approx \frac{10^{25.21}}{r^2}\, s^{-1} \Rightarrow a_{K_1} = 4k_{Gal}(r)v(r) \approx 10^{-10.71}\, m\, s^{-2} \tag{17}$$

$$k_0 \approx 10^{-16.65}\, s^{-1} \Rightarrow a_{k_0} = 4k_0 v(r) \approx 10^{-10.77}\, m\, s^{-2} \tag{18}$$

We can then confirm that we are effectively in a weak gravitational field at the end of the galaxy. Another way is to verify that one has $|h_{\mu\nu}| << 1$ from the traditional decomposition of the GR metric $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with $\eta_{\mu\nu}$ the Minkowski metric. GRL gives[21] $h_{00} = h_{11} = h_{22} = h_{33} = \frac{2}{c^2}\phi$. In our case, at $r \approx 16.24$ kpc $\approx 5 \times 10^{20}$ m it gives:

$$|h_{00}| = |h_{11}| = |h_{22}| = |h_{33}| = \frac{2}{c^2}\frac{GM_{Gal}}{r} \approx 10^{-6.67} << 1 \tag{19}$$

It is interesting to note that, with our solution, four zones seem to stand out. For the first zone, in the inner part of the galaxy, we cannot use our relation (10) because a_G and a_{K_1} depend on the mass distribution. But thanks to Fig. 1, we can see that around $r \approx 9$ kpc the black curve of DM intersects the curve of the baryonic component. This means that $a_{K_1} \approx a_G$ (in this zone the effect of the uniform gravitic field a_{k_0} is clearly negligible). The other three zones can be defined with the previous calculations. We can then define the following four zones for MW (and certainly for many other galaxies):

At $r \approx 9$ kpc, we have $a_{K_1} \approx a_G$. The area $r < 9$ kpc represents the zone where a_G dominates but with a_{K_1} which takes on proportionally more importance despite the reduction of these 2 terms which characterize the own gravitational field of the galaxy, until reaching the maximum rotation speed.

At $r \approx 15$ kpc, we have $a_{k_0} \approx a_G$. The area 9 kpc$< r < 15$ kpc represents the zone where a_{K_1} dominates. a_{k_0}, which slightly increases, compensates for the decrease of the 2 terms of the own gravitational field of the galaxy. This curve is relatively constant.

At the end of the galaxy $r \approx 25$ kpc, we have $a_{k_0} \approx a_{K_1}$. The area 15 kpc$< r < 25$ kpc represents the zone where a_{k_0} is no longer negligible until it dominates while the 2 terms of the own gravitational field of the galaxy become negligible. This marks a slight decrease in the rotation speed curve and also marks the end of the galaxy.

The fourth area is at $r > 25$ kpc. It represents the zone where a_{k_0} dominates. In this zone, some objects can indirectly turn around the galaxy such as dwarf satellite galaxies, indirectly because thanks to the uniform gravitic field k_0. In this area, the expression (10) can be further simplified, leading then to:

$$v_+ = 4k_0 r \tag{20}$$

For instance, at 100 kpc with the previous value of k_0, (20) gives $v_+ \approx 270$ km s^{-1}. The speed can be even greater with a greater distance in agreement with[31,32].

One can add that in the first two inner zones, the rotation of the galaxy must occur naturally in a plane perpendicular to the own gravitic field of the galaxy "$\frac{\vec{K_1}}{r^2}$" and in the last outer zone the rotation of the galaxy must occur in a plane perpendicular to $\vec{k_0}$. If $\vec{k_0}$ and "$\frac{\vec{K_1}}{r^2}$" are not collinear, the galaxy must have a warping which should begin in the third zone, transition zone between the zone of domination of "$||\frac{\vec{K_1}}{r^2}||$" and the zone of domination of $||\vec{k_0}||$.

Application to the precession of the galactic disks warp

As seen previously, our idealization exhibits an axisymmetry with in particular the ideal situation of the following perpendicularities, $\vec{v} \perp \vec{k}$ and $\vec{k}$ perpendicular to the galactic disk. From (9), we know that the vector $\vec{k}$ is composed of the galactic gravitic field "$\frac{\vec{K_1}}{r^2}$" and of the external gravitic field $\vec{k_0}$. Because $\vec{k_0}$ is imposed by the environment and not by the galaxy, the history of the galaxy may put it in a situation where $\vec{k_0}$ is no longer perpendicular to the galactic disk of the inner part (cf. section 4.4). It is then interesting to study the consequences of this break in axisymmetry. This should generate disk warping due to $\vec{k_0}$ in the outer areas where k_0 dominates. The deformation of the disk should then undergo precession.

In a previous article[33], we studied this precession (also known as the Lense-Thirring effect) of k_0 (i.e. due to DM in our interpretation) inside galaxies. The expression of this precession can be deduced from the Einstein-Maxwell equations (3) and the order of magnitude of the value of this precession in a galaxy is expected to be[33]:

$$||\vec{\Omega}_{LTDM}|| = 2||\vec{k}_0|| \approx 0.6 \times 10^{-16}\, s^{-1} \tag{21}$$

A recent article[34] measured the precession of the Galactic disk warp, and their obcervations gave the following result for the outer disk over the Galactocentric radius [7.5, 25] kpc:

$$|\omega| = 2.1 \pm 0.5(stat.) \pm 0.6(syst.)\, km\, s^{-1}\, kpc^{-1} \approx 0.68 \times 10^{-16}\, s^{-1} \tag{22}$$

The measured value is in excellent agreement with our precession prediction[33]. Our relation (21) is calculated from a mean value of $k_0 = 10^{-16.5}$ s^{-1} obtained from a sample of 16 galaxies[33]. For the MW, case of the relation (22), we have just seen that we should have $k_0 = 10^{-16.65}$ s^{-1}. The precession should then be (which is always in agreement with (22)):

$$||\vec{\Omega}_{LTDM}|| = 2||\vec{k}_0|| \approx 0.45 \times 10^{-16}\, s^{-1} \tag{23}$$

Application to the Coma Cluster (CC)

Given our approximation, the relation (10) applies to ends of systems where the vast majority of mass is at the center. The clusters with their brightest cluster galaxy (BCG) constitute in a first approximation such a system. Due to the much smaller number of measurement points, our application to CC is much less constrained than for the MW which is the main result of this article. But it is nevertheless interesting because it shows that the relation (10) can also make sense for clusters and especially because these results will allow us to understand the origin of the k_0 field. In[2], the missing mass problem is revealed with the application of the virial theorem on the Coma cluster. With a cluster mass of $M_{Cl} \approx 1.6 \times 10^{42}$ kg used in[2] and a cluster radius of $r_{Cl} \approx 350$ kpc $\approx 10^{22}$ m, the author obtained:

$$v_{Cl} = \sqrt{\alpha \frac{GM_{Cl}}{r_{Cl}}} \approx 80\, km\, s^{-1} with\ \alpha = \frac{3}{5} \tag{24}$$

But the velocity in the Coma cluster should be of 1000 km s^{-1} or more, leading to the conclusion that the mass should be at least 400 times larger than the luminous matter. Since then, the baryonic mass of CC has been revised upwards and today it is estimated at $M_{Cl} \approx 6 \times 10^{43}$ kg, which gives $v_{Cl} \approx 400$ km s^{-1}. It still implies a mass which should be 6 times larger than luminous matter.

Let's apply our relation (10) to the Coma cluster. First, one can verify that our relation agrees with the virial theorem used in[2]. In this context, we take into account only the gravity field (i.e. $K_1 = 0$ and $k_0 = 0$) and expression (10) becomes:

$$v_{+\_Cl} = \sqrt{\frac{GM_{Cl}}{r_{Cl}}} \tag{25}$$

This reduced relation (25) is in good agreement with (24). This allows then to observe the same mass problem and in the same proportion. But if we take into account the gravitic fields, the problem can be solved. Our hypothesis is that k_0 would be generated by the own gravitic field of the clusters "$\frac{K_{1\_Cl}}{r^2}$". As a first approximation, we can therefore neglect the k_0 component for the clusters. In Fig. 2, we test our relation (10) for the case of the Coma cluster with $M_{Cl} \approx 6 \times 10^{43}$ kg. The black curve is obtained with $k_0 = 0$ and the value $K_{1\_Cl} \approx 10^{27.24}$ to reach the speed of 1000 km s^{-1} at $r_{Cl} \approx 10^{22}$ m ≈ 350 kpc, specifications of the expression (24). The green curve is obtained with a residual $k_0 \approx 10^{-17.25}$ s^{-1} due to the neighboring clusters. We have represented this green curve to show that the value of K_{1_Cl} remains in the same order of magnitude even in taking into account a residual k_0 which must exist everywhere in the universe. The value of k_0 taken into account corresponds to the value obtained with the relation (33) for an average distance of 1.5 Mpc between neighboring clusters. The value of k_0 is less than the value in MW because the parameter *r* in (33) for this case represents the distance between neighboring clusters which is larger than the distance between galaxies and neighboring clusters since the galaxies are located between and inside the clusters.

Not only did expression (10) account for Milky Way but this same expression in a quite different context makes it possible to obtain the range of speeds of the Coma cluster without exotic matter. This expression could therefore also be a good approximation of the virial theorem in the context of GRL. In section 4.1, we will demonstrate that the values of K_{1_Cl} and k_0 are physically self consistent.

Once again, we remind that for the study of CC, because we can fix the value of k_0 (thanks to our previous calculation section 3.1), only one measurement point is required to define univoquely the value of K_{1_Cl}. We have no freedom to adjust its value. Again let us recall that in the case of MW, in order to fix k_0 it was enough to have a second measurement point to fix its value. And these are 18 measurement points over nearly 15kpc that the expression (10) made it possible to account for.

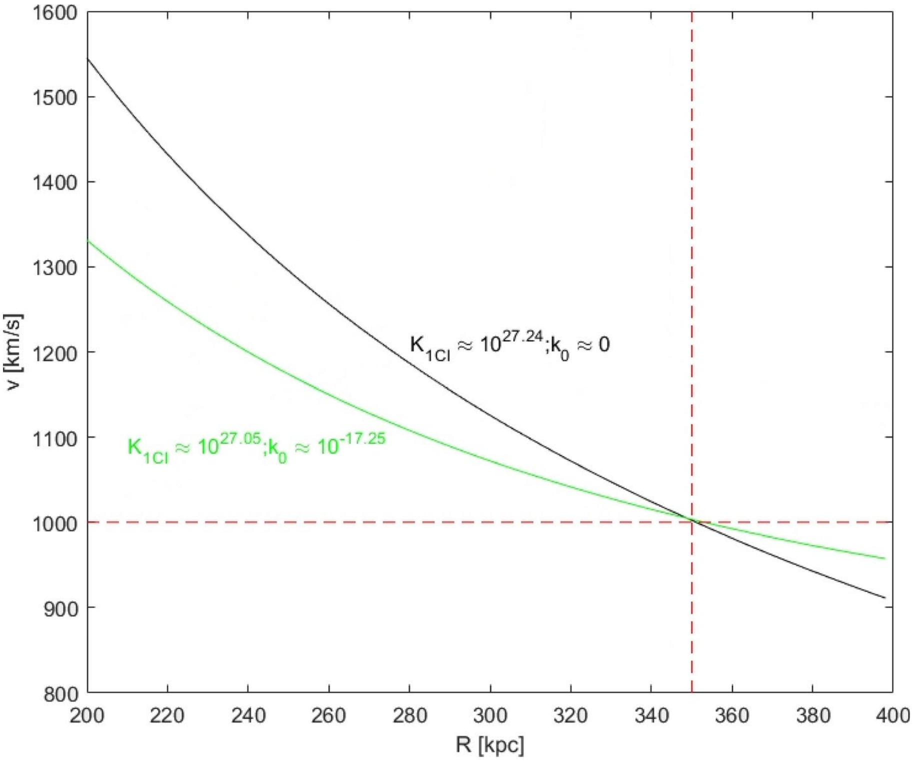

Fig. 2. Application of our GR expression (10) to the Coma cluster studied in[2] at the origin of the missing mass problem. The dotted red lines at $r_{Cl} \approx 350$ kpc and $v_{+\_Cl} \approx 1000$ km s^{-1} indicate the speed to obtain. The black curve defines then the value of $K_{1\_Cl} \approx 10^{27.24}$ if we neglect k_0. The green curve defines the value of $K_{1\_Cl} \approx 10^{27.05}$ if we take into account a residual $k_0 \approx 10^{-17.25}$.

Application to the Radial Acceleration Relation (RAR)

In[35], the authors report a correlation (RAR) between the radial acceleration traced by rotation curves and that predicted by the observed distribution of baryons followed by 153 galaxies with very different morphologies, masses, sizes, and gas fractions. In the limit of the low acceleration, RAR is written:

$$g_{obs}(r) \approx \sqrt{g_{bar}(r) g_{\dagger}} \tag{26}$$

Where $g_{obs}(r)$ is the observed centripetal acceleration obtained from the rotation curve at a given radius *r*, and $g_{bar}(r)$ is the acceleration due to baryons alone, given by the first relation of (5). Observations and statistical study give:

$$g_{\dagger} \approx 1.2 \times 10^{-10} \text{ m s}^{-2} \tag{27}$$

In our GRL solution, far from the center of the galaxy where the contribution of $\frac{\partial(\phi_{disk}(r)+\phi_{bulge}(r))}{\partial r}$ is negligible, the observed acceleration is then given by only the second relation of (5):

$$g_{obs-GRL}(r) \approx 4k(r)v(r) \tag{28}$$

Where *k*(*r*) is given by our theoretical relation (9) and *v*(*r*) is given by our theoretical relation (10). Our values (11) of K_1 and k_0 deduced from observation allow confirming the RAR (26) with its value of $g_{\dagger}$ (27). Indeed, in Fig. 3, we can observe that the brown curve corresponding to $g_{obs-GRL}(r)$ is close to the blue curve corresponding to $g_{obs}(r)$ for $R >> 20$ kpc.

Application to MOND theory

It has already been demonstrated that this solution of DM with these values of k_0 allow obtaining the MOND idealization[36]. Briefly, from the relations (2), one can obtain the expression of the MOND parameter a_0 far from the center of the galaxy, which therefore depends on k_0[36]:

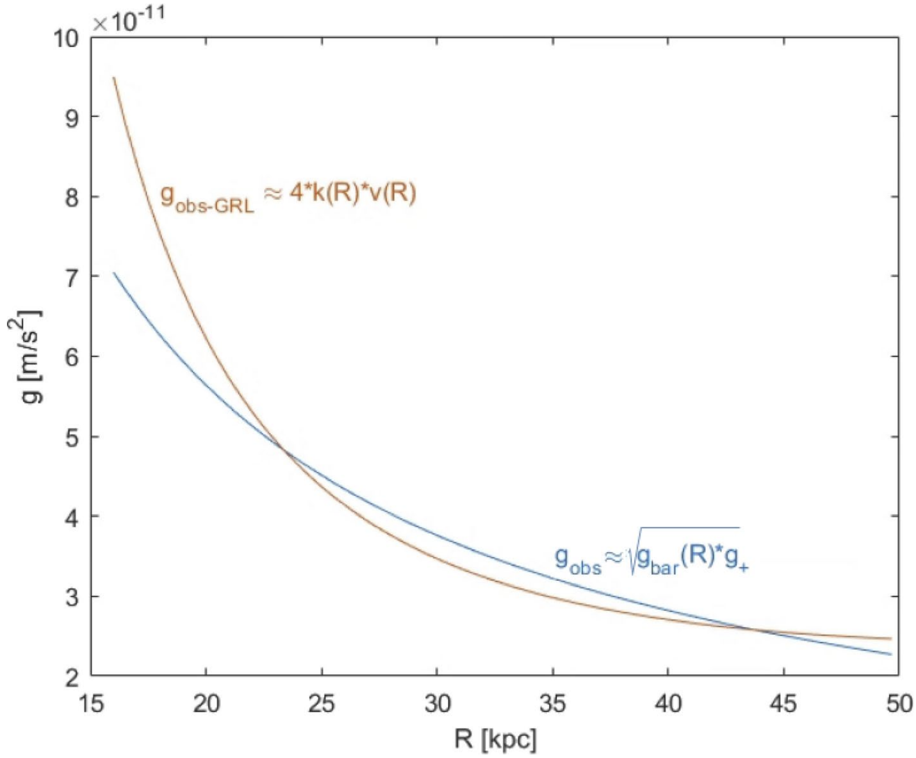

Fig. 3. The Radial Acceleration Relation (RAR) obtained from our GRL solution for MW (brown curve) confirms the RAR (26) with in particular its value $g_{\dagger} \approx 1.2 \times 10^{-10}$ m s^{-2} (blue curve) obtained in[35] for a statistical sample of 153 galaxies.

$$a_0 \approx \frac{GM_{Gal}}{r^2}\left(1 + \frac{4k_0 v(r) r^2}{GM_{Gal}}\right)^2 \tag{29}$$

For example, at $r \approx 30$ kpc $\approx 9.26 \times 10^{20}$ m, one has $v \approx 180$ km s^{-1} and with $k_0 \approx 10^{-16.65}$ s^{-1}; $M_{Gal} \approx 8 \times 10^{40}$ kg (baryonic matter), the expression (29) gives:

$$a_0 \approx 0.85 \times 10^{-10}\, m\, s^{-2} \tag{30}$$

Application to the Baryonic TULLY-FISHER relation

It has also already been demonstrated that this solution of DM with these values of k_0 allow obtaining the baryonic TULLY-FISHER relation[37] with in addition a very interesting result. Indeed in[38] the authors observe a break from the baryonic Tully-Fisher relation (BTFR) for super spirals of baryonic mass $M_b > 10^{11.5}\ M_{\odot}$ and rotation speeds $v > 340$ km s^{-1}. In[37] we obtain the BTFR and also the break at the rotation speed and the mass observed.

Once again, from the relations (2), we can obtain the following relation[37] with F_{Seuil} a threshold value of the intensity of the force associated with the characteristic speed of baryonic TULLY-FISHER relation:

$$M(v) \approx \frac{1}{GF_{Seuil}}\left(v^4 - \frac{4k_0}{F_{Seuil}} v^5\right) \tag{31}$$

This relation (31) gives the red curve of Fig. 4.

Application to the gravitational lensing or CMB

We can add that in[23], a promising first attempt has been made to idealize the light deviation and to retrieve some results from CMB in the frame of GRL and with the expected values of k_0. But more in-depth studies will be necessary.

Discussion

How to explain the values of K_1 and k_0

The previous values of K_1 and k_0 were obtained graphically to fit the rotation speed measurements. How can the theory obtain k_0? In EM, the permanent magnetic fields of ferromagnetic materials are explained by the sum of

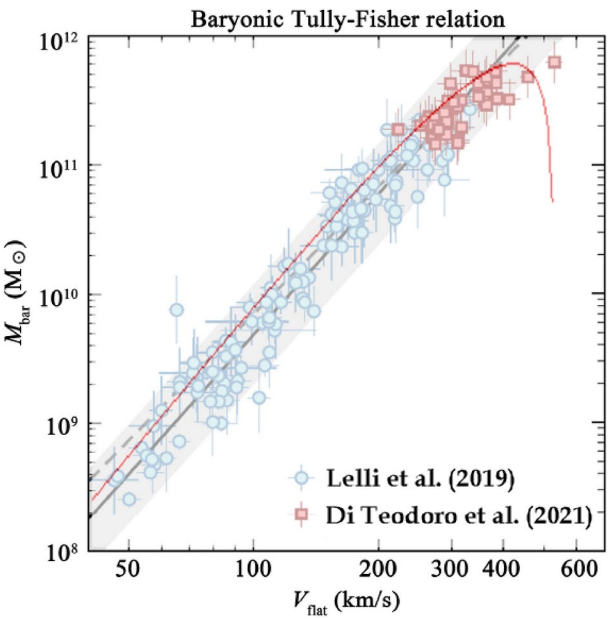

Fig. 4. The red curve represents the relation (31) obtained from GRL superposed on the graph from[39].

the atomic magnetic fields of neighbors. It turns out that, in our solution, the permanent gravitic field k_0 can be explained by the sum of the cluster fields K_{1_Cl} of the neighbors. Indeed, if we suppose an average distance of 1.5 Mpc between two neighboring clusters, we obtain at $r = 0.75$ Mpc $\approx 10^{22.36}$ m (in the middle of neighboring clusters) with the estimated value previously obtained for the Coma cluster:

$$\frac{K_{1\_Cl}}{r^2} \approx \frac{10^{27.24}}{(10^{22.36})^2} \approx 10^{-17.48}\, s^{-1} \tag{32}$$

Unlike $\vec{g}$ (and like $\vec{B}$ for ferromagnetic materials) the gravitic field of neighboring structures can be added. With 7 neighboring clusters, one can retrieve the value of k_0 at the intersection of these 7 clusters:

$$k_0 \approx 7\frac{K_{1\_Cl}}{r^2} \approx 10^{-16.64}\, s^{-1} \tag{33}$$

And this value can be maintained over large distances. Indeed, by getting closer to the center of a cluster, its internal gravitic field ($\frac{K_{1\_Cl}}{r^2}$) increases while the influence of the neighbors decreases. This makes it possible to maintain a relatively uniform gravitic field over large distances. For instance, with the values of the Coma cluster, at $r \approx 250$ kpc, we still have $k_0 \approx 10^{-16.4}$ with its neighboring clusters. k_0 is then relatively uniform over approximately two thirds of the cluster (and consequently of the Universe, by covering). This explanation of k_0 from the value of K_{1_Cl} demonstrates a great self-consistency of the expression (10). Nothing in our hypotheses could suggest this. It is even the graphical adjustment which makes it possible to show that the gravitic fields of neighboring galaxies are not powerful enough to explain k_0 but that those of the clusters are. To summarize, our solution therefore implies that most likely the clusters are the origin of k_0 and that this addition procedure with neighboring clusters not only explains the value of k_0 but also its uniformity over long distances. Let us also add that according to Kepler's conjecture on spatial arrangements, we could not have had more than 12 neighbors. The addition was therefore very limited. It should be noted again the remarkable self-consistency of this solution. k_0 would then not be due to mass currents unlike the only cases studied in GEM articles. k_0 would be similar in EM to the permanent magnetic field of magnets or more generally of ferromagnetic materials.

What about K_1, the galaxy's own gravitic field (K_{1_Gal}) and the cluster's own gravitic field (K_{1_Cl})? From Einstein-Maxwell equations (3), we can obtain the theoretical gravitic field generated by a particle of mass M (as the Biot-Savart law in EM for $\vec{B}$):

$$||\vec{k}_{Th}|| = \frac{G}{c^2} M \frac{||\vec{v} \wedge \vec{u}||}{r^2} \tag{34}$$

In a first approximation, we can expect that $||\vec{k}_{Th}|| \approx \frac{K_1}{r^2}$. We deduce that $K_1 \approx \frac{G}{c^2} M v$. For the Milky Way with a typical value of $v \approx 200$ km s^{-1}, it gives $K_1 \approx 10^{19} \approx 10^{-6} K_{1\_Gal}$. For the Coma cluster, with $v \approx 1500$ km s^{-1} it gives $K_1 \approx 10^{22.8} \approx 10^{-4.5} K_{1\_Cl}$. The values of K_1 obtained graphically is clearly too large to be explained by the expression (34). These ratios confirm the results obtained in[16–18] of a GEM correction of the order of

10^{-6} for the galaxies and validate in a certain way our idealization "far from the central point mass". We are therefore led to make the same observation as the GEM articles[16–18]. Mass currents, i.e. expression (34), cannot explain these gravitic fields. But we dispute their conclusion which rejects GEM solutions based on these results. With the hypothesis of exotic matter, we are in the same situation. Indeed, first, the theory fails to explain the DM component with known matter, as in our case with known gravitic fields (i.e. with known mass currents). Second, the quantity of matter required to explain DM component is enormous compared to the known matter, such as in our case the values of the gravitic fields. And we must pay attention to the consequences of these quantities. The factor 5 or 6 for exotic matter, even if it is not of the same order of magnitude as the factor 10^6 for gravitic fields, enormously modifies the quantity of the main parameter of gravitation, the mass, and strangely without physical consequences on our scale, in the solar system, which is intriguing. While k_0, even with its value 10^6 greater than expected, modifies a term which is originally even more enormously negligible, which has the consequence that this term still remains negligible on our scale and largely under our capacity of measurement ([33] or section 4.4), which explains the non-detection of DM on our scale. Third, the matter explaining the DM component does not follow known physical theories (it is insensitive to EM), so it is a new theoretical entity called exotic (or dark) matter leading to several specific alternative theories (WIMPS, Axions, Tachyons, ...). So why, for GEM solutions, leave no chance[16–18] for a specific physical process giving greater values to gravitic fields (which nevertheless still remains negligible on our scale and under the current detection capability of our experiments) when this does not pose a problem for a hypothetical exotic matter which would dominate the Universe (present on our scale and which has never been directly detected) ?

Several avenues can be imagined to explain these high values of the gravitic field. And this list is obviously not exhaustive. The first avenue would be to take into account the non-linear features of GR or the strong frame dragging (not taken into account in our idealization). Indeed, several articles suggest that they could play a key role in DM phenomena[5–9,14]. Another avenue would be to continue the explanation of k_0 seen previously. Indeed, in EM, the values of the magnetic field of ferromagnetic materials (e.g. magnets) cannot be explained by charge currents. If we applied the solution studied by[16–18] in the EM context, we would be led to wrongly conclude that the magnetic fields of magnets are not possible. But compliant with EM, an explanation other than charge currents can explain the large magnetic fields of magnets, the spin, physical concept specific to QM. Moreover, it is interesting to take up the theoretical framework of ferromagnetic materials that the k_0 field inspired us. These permanent fields are essentially based on two theoretical principles, the sum of the magnetic moments of the neighbors and the existence of an intrinsic magnetic moment, the spin, without which it would not be possible to have such intense fields from only the charge currents. For the 1st principle, we have just seen that k_0 can exactly be explained by this sum of the neighborhood like the magnetic fields of ferromagnetic materials. It is then tempting to continue the parallel and to apply the 2nd principle. Because QM is necessary to explain permanent magnetic fields in the EM context and because GRL is similar to EM, we therefore cannot exclude that although permanent gravitic fields are not explainable by mass currents, they could be explained by a quantum gravitation.

This situation is no worse than the exotic matter hypothesis. Indeed, when in physics we assume the existence of a missing mass (Le Verrier for Neptune or nowadays the hypothetical planet Nine) it is because there is a small deviation from the theory and that the missing mass is a normal mass. With exotic matter, the only known baryonic matter becomes a minority. In a normal logical sequence, exotic matter should be a minority. Given the situation, baryonic matter should then be the missing mass or the theory should be called into question, that's the way proposed for example by MOND. Furthermore, exotic matter is not a normal matter because it is insensitive to EM. The assumption of exotic matter is therefore unprecedented and without equivalent in physics. While in the GRL solution, gravitic fields are normal physical entities (known and measured) and the hypothesis of a field more powerful than those obtained by only mass currents, is a physical situation already tested and verified in the field of EM. Thus, even if to date, we cannot explain the value of K_1, only direct measurements will be able to confirm the physical value of K_1. Just as for exotic matter, only direct detections of exotic particles will be able to definitively confirm this hypothesis. We can also note that among the explicit parameters of the GR, there is only this gravitic field (at the origin of the Lense-Thirring effect) which has not yet been measured with sufficient precision. It is therefore not unreasonable to think that this parameter could contain the soultion to this component of DM.

About the agreement of rotation curve and expression (10)

The article[19] find a significantly faster decline in the circular velocity curve at outer galactic radii up to 30 kpc compared to the inner parts. Expression (10) of GR makes it precisely possible to obtain this characteristic with great accuracy. This accuracy is not a trivial fact because only two measurement points completely define the curve and nevertheless, despite this very strong constraint, at 16 points among the 18 measurement points from 16 kpc to 30 kpc (red points in Fig. 1), the blue curve differs by less than 5 km s^{-1} with the speed measurements which are of the order of 200 km s^{-1}. The accuracy is such that it could even justify a posteriori the use of GRL, and consequently justify that the gravitational fields must be weak at the end of the Milky Way and that the vast majority of the mass must be in the inner part. Our relation (10) goes against the presence of a large quantity of any kind of matter in this distant area. With exotic matter, the mass is distributed throughout the galaxy in large quantities, even at the ends. The approximation used to obtain expression (10) would then make no sense. This solution cannot therefore cohabit with any kind of exotic matter. And its accuracy really challenge the solution of an exotic matter.

DM history and expression (10)

This general expression (10) of GR allows retrieving and understanding the evolution of the DM history over time. Initially, the case $k_0 = 0$ and $K_1 = 0$ was the expected Newtonian solution. The failure of this solution is at the origin of the DM mystery. The case $k_0 = 0$ and $K_1 \neq 0$ (GEM solution) is the case where we consider only the gravitic field of the galaxy itself. The 2nd component of GR (at the origin of the Lense-Thirring effect) is generally considered negligible. Several publications[11,40–42] have already demonstrated that the own Lense-Thirring effect of the galaxy was unable to explain the DM component. And effectively expression (10) combined with expression (34) also confirms that this effect of the galaxy's own field is not sufficient to obtain the measurements of the rotation speeds and with the same order of magnitude as in[16–18]. This expression (10) shows that there is a third physically acceptable case $k_0 \neq 0$ and $K_1 \neq 0$ (GRL solution) as in[11,14,15]. In our study, we demonstrate that these two terms together are necessary to be able to explain DM. The extremely small term k_0 would finally become essential at the scale of the galaxy, especially at its ends, due to its mathematic expressions such as (5) and (20), with large $v(r)$ and large r. The hypothesis of a uniform field k_0 embedding the galaxies (and therefore the Universe on large scale) would thus make it possible to fully explain the DM component. One can also note that this extremely small value of k_0 would explain why at our scale DM is undetectable ($v(r)$ and r being too small).

Possible steps to validate this solution of DM without exotic matter

A first step would be to ensure that this expression makes it possible to retrieve most of the rotation curves of galaxies, and ideally with values of k_0 and K_1 of the same order of magnitude for most of the galaxies. More precisely, for galaxies of the same mass and size, the values of K_1 should be close (because K_1 characterizes the galaxy), and for neighboring galaxies, the values of k_0 should be close (because k_0 characterizes the environment of galaxies). These characteristics are not trivial and constitute a first prediction of our GRL solution.

Several other predictions can be proposed. Because k_0 is uniform on the scale of a galaxy and because at the end of the galaxy k_0 dominates, a host's dwarf satellites galaxies should move in a planar distribution.

Because k_0 is expected to evolve slightly between two neighboring galaxies, a statistical study could reveal a tendency of collinearity for two neighboring galaxies. We write "could" because we will see that this prediction is finally not necessary. But, if we observed such a characteristic it would be a strong point for our solution which would again strongly constrain the hypothesis of exotic matter.

These two previous predictions require clarifications on the term "neighborhood" and on our approximation of a relatively uniform k_0. By "neighborhood", we mean the first neighbors, that is to say that no other galaxy (respectively cluster) is found between two neighboring galaxies (respectively neighboring clusters). And about our approximation of a relatively uniform k_0, it should be kept in mind that the value of k_0 remains very small compared to the Newtonian component. This means that on the one hand k_0 needs time to leave its mark on the geometry of the galaxy (essentially the planar distribution) and on the other hand if two galaxies have interacted "recently", the Newtonian component will certainly have erased this geometric mark of k_0. Furthermore, the relatively uniformity of k_0 along a galaxy does not necessarily mean the same constant value and the same direction of $\vec{k_0}$ throughout the Universe. In our explanation, we made the simplifying assumption of having neighboring clusters with directions of $\vec{K_{1\_Cl}}$ (whose sum defines $\vec{k_0}$) approximately collinear, and with values of K_{1_Cl} of approximately the same order of magnitude. But actually what is required is that the vectorial addition of neighboring $\vec{K_{1\_Cl}}$ gives the expected $\vec{k_0}$ along the galaxy and this can be done mathematically with very different directions and values of neighboring K_{1_Cl}. But we can expect that with a too random arrangement, it will be difficult to obtain a vectorial sum that gives a sufficient value of k_0 (not too close to zero). We can then expect that physically, the neighboring $\vec{K_{1\_Cl}}$ are not in a too random arrangement. It should be added that it is not only the direction and magnitude of the individual spins of neighboring clusters but also the number of neighboring clusters which influence the collinearity requirement. For example, with our calculation, we needed 7 neighboring clusters of the same magnitude and direction. According to Kepler's sphere packing theorem, we can go up to 13, we would then double the value of k0. Consequently, to achieve the expected lower value, one way to reduce it would be to relax the collinearity requirement. An angle of almost 60° would be required (relative to the final direction of the sum, i.e. $\vec{k_0}$) to divide by 2 (and compensate for the excess due to the addition of the 6 other clusters). In other words, the requirement for collinearity between neighboring clusters would be ultimately quite low. This is why the latter prediction would be quite difficult to reveal and perhaps not even necessary. From these clarifications, we can then understand that k_0 can change from galaxy to galaxy and that on the scale of several galaxies, the $\vec{k_0}$ can become very different, explaining that on the scale of the Universe the angular momentum distribution appears totally isotropic.

Thus, to observe the statistical tendency of collinearity, if it exists, of two neighboring galaxies (our latter prediciton), the sample of galaxies must be chosen carefully. It should not contain galaxies with a disc warp because in their history an event oriented K_{1_Gal} differently from the surrounding k_0 (or the k_0 to take in account for these galaxies should be the direction of the warp) and it must not contain neighboring galaxies which have been in gravitational interaction because, in this case, the Newtonian component of the gravitational interaction will have erased the negligible influence of k_0 and will misalign the two galaxies. It is, however, not certain that it is necessary to exclude these latter neighboring galaxies because in this case it could perhaps reveal a characteristic statistical distance below which neighboring galaxies are statistically under Newtonian gravitational influence and above which the k_0 of two neighboring galaxies become statistically too different. We can then expect this characteristic statistical distance to be between 100 kpc (distance of a few galaxies) and 1000 kpc (a little less than the typical size of a cluster).

With our previous discussion on the non-necessity of collinearity of spins of neighboring clusters, a prediction is then probable, statistically 3 neighboring clusters which follow each other should have their spins included in the same half-space because with our values of k0 and K$\vec{1}$ Cl we saw that the discrepancy between two cluster spins could go up to around 60°. This latter angle therefore also suggests that this "local" anisotropy could even concern 4 neighboring clusters which follow one another. On the scale of approximately a diameter of 3 or 4 clusters, the Universe would then be slightly anisotropic but would become isotropic beyond that. It is important to specify that this is a soft anisotropy because at this scale of 5Mpc it is only a half-space which is favored and not a precise direction. We can also add that our solution predicts at least two ways of obtaining galaxies without DM, if the galaxy is isolated (far from clusters) or if a specific configuration orients the $\vec{K_{1\_Cl}}$ of the neighboring clusters such that their addition cancels out.

A second step would be to apply these gravitic field values to explain the other phenomena in which the DM appears (gravitational lens, CMB, ...). A first study demonstrates that gravitational lens and CMB can be explained in this theoretical frame and with the expected values of k_0 ([23]). For the other phenomena, the work remains to be done. But many elements have already been seen previously in this article demonstrating great consistency (same value of k_0) and effectiveness of this solution.

A third step, certainly the most important, would be to be able to directly measure these fields k_0 and K_1 to confirm or not the conclusion of[16–18]. In section 3.2, we saw that the k_0 field must produce a precession effect (the Lense-Thirring effect) of the rate measured in[34]. This observation could constitute the "most direct" measurement to date of this k_0 field (DM component in our solution). But no attempt has really been made to directly observe these two fields. More precisely, measurements of the terrestrial Lense-Thirring effect (produced by the terrestrial gravitic field) were carried out[24,25]. They made it possible to confirm the existence of this Lense-Thirring effect. The k_0 and K_1 fields are present everywhere, measurements of the terrestrial Lense-Thirring effect must therefore also integrate their effects. But this requires a precision that experiments cannot yet achieve[33]. The GINGER experiment could achieve the required precision[26]. Our study also makes it possible to refine the measurements expected from such an experiment (GINGER). In[33], the value of the Lense-Thirring effect of DM Ω_{LTDM} was calculated for the gravitic field k_0 only. But with our results, we must also consider the gravitic field of our own Galaxy. Indeed, at $R \approx 17$ kpc the gravitic field of the MW "$\frac{K_1}{R^2}$" is approximately of the same order of magnitude as k_0. However, the position of the solar system in the MW is around 8.5 kpc $\approx \frac{R}{2}$. Consequently, it is very likely that this MW field in the solar system is at least of the same order of magnitude as k_0 but also certainly less than $4k_0$ because our approximation in "$\frac{K_1}{r^2}$" is no more valid at $\frac{R}{2}$ and overestimates its value (because in particular it diverges at the center of MW). According to[33], we had for $k_0 \approx 10^{-16.5}$, $\Omega_{LTDM} \approx 0.4 milliarcsecond.year^{-1}$. For $k_0 \approx 10^{-16.65}$ (result of our study), we would then rather have $\Omega_{LTDM} \approx 0.3 milliarcsecond.year^{-1}$ and adding the MW gravitic field of $4k_0$ maximum, we can expect to finally measure a maximum value of $\Omega_{LTDM} \approx 1.5 milliarcsecond.year^{-1}$, so a little larger than what was calculated in[33]. To confirm our solution of DM, the measurement of the terrestrial Lense-Thirring effect Ω_{LTGPB}, instead of the only purely terrestrial effect $\Omega_{LTGPB} \approx 39 milliarcsecond.year^{-1}$, would rather be in the interval $39.3 milliarcsecond.year^{-1} < \Omega_{LTDM} + \Omega_{LTGPB} < 40.5 milliarcsecond.year^{-1}$. This represents at most only 4% in addition to the value of the purely terrestrial effect. The effect of the DM remains very weak compared to the terrestrial effect, yet it is the necessary and sufficient value to fully explain the DM. This solution is very coherent with an appearance of DM only on large scales. One more thing about a direct measure of the DM component. It is surprising to note that the acceleration anomaly of the Pioneer probes, which has been explained by thermal effects, is of the same order of magnitude as what would be due to the gravitic field of the galaxy, relation (14), $a_{K_1} \approx 10^{-10.24}$ m s^{-2}. Especially if we adapt this calculation to the position of the solar system which is roughly at a distance 2 times smaller than for relation (14) and a rotation speed of 250 km s^{-1} , the calculation (14) then gives 2.8×10^{-10} m s^{-2}. This would represent almost 35% of this deceleration of Pioneer (measured around 8.7×10^{-10} m s^{-2}). Therefore, we can imagine an experiment which would consist of sending a small object which would follow a trajectory similar to that of Pioneer and on which we would be sure that there is no thermal effect. The measurement of a deceleration of the object of the order of 10^{-10} would then be a direct measurement of dark matter, i.e. of the gravitic field explaining the DM component.

A fourth step would be to find the source of this k_0 field. As seen in section 4.1, it seems very likely that the neighboring clusters of galaxies can generate this field together, like assemblies of spin fields generate magnets for ferromagnetic materials.

A fifth step, certainly the most complicated, would be to justify the value of K_1 with a physical process (e.g. the non-linear effects of GR or a quantum gravitation).

About self-consistency of this GRL solution

Throughout our study, we noted several points of self-consistency of this GRL solution. We remind them because it seems to us that they are not anecdotal but that they make this solution very structured and reliable:

Although the relation (10) is completely defined with only two measurement points, all other measurement points for $r > 16$ kpc follow this curve with formidable precision. That the GRL solution is very poorly adaptable means that the probability of it being wrong while still conforming to the observation is low.

The curve fitting was carried out in a purely graphical manner. It is therefore graphically that we obtained that our relation begins to fit the measurement points at the value $r \approx 16$ kpc. It turns out that this start of a good fit is about four times the distance from the maximum speed of the baryonic component (which is about $r \approx 4$ kpc, be careful this is not the maximum of all the components). The main gravitational forces (which evolve in r^{-2}) are therefore weaker by a little more than an order of magnitude compared to this maximum. But it is also about twice the distance where the baryonic and DM components are similar (around 9 kpc). The forces are then weaker by a little less than an order of magnitude compared to this characteristic point. The start of the adjustment which is deduced graphically thus seems consistent with our weak field approximation "far from the central point mass" since we are approximately an order of magnitude weaker from where the Newtonian force dominates and where it begins to be strongly competed by the gravitic force.

The k_0 field is a uniform field. In EM, inside a ferromagnetic material, the way to explain the uniform magnetic field is by the sum with its neighbors. The fact that we can obtain k_0 with the $K_{1_{Cl}}$ fields of the first neighboring clusters is a very interesting and unexpected result which makes this GRL solution very self-consistent, because these two k_0 and $K_{1_{Cl}}$ fields are obtained independently.

The value of k_0 field and its mathematical expressions (5) and (20) allow explaining two facts of the DM problem which seem counter-intuitive in the assumption of exotic matter. The DM problem demonstrates that we are surrounded by an enormous quantity of this DM component (larger than baryonic matter) and yet on our scale it is completely invisible. Second, the DM component only appears for large astrophysical structures, starting with galaxies. In the GRL solution, the value of k_0 is so small that it is currently undetectable by the insufficient precision of our experiments (see previous remarks on the GINGER experiment). This logically explains why this DM component (despite its omnipresence) is invisible on our scale. Second, with its mathematical expression (20), the effects of k_0 field can only appear with large speeds and large distances, at least of the order of magnitude of the values of speeds and radii at the ends of galaxies. Once again, the GRL solution offers great self-consistency.

Another fact is counter-intuitive in the assumption of an exotic matter. DM is not sensitive to EM. In the GRL solution, this fact is naturally explained because gravitic fields are purely gravitational components.

Conclusions

In this article, we obtained a very general expression of the rotation speed at the end of the galaxies (far from the vast majority of the galaxy's baryonic mass) in the frame of GR without exotic matter. This expression depends on two parameters, k_0 (relatively constant along a galaxy) and K_1 a constant which characterize the 2nd component of the gravitational field in GR "$\frac{K_1}{r^2}$" (at the origin of the Lense-Thirring effect). K_1 defines the own "internal" gravitic field of the galaxy and k_0 defines a uniform "external" field:

$$v_+ = 2(\frac{K_1}{r} + k_0 r) + \sqrt{4(\frac{K_1}{r} + k_0 r)^2 + \frac{GM_{Gal}}{r}} \tag{35}$$

$$= 2(\frac{K_1}{r} + k_0 r)\left(1 + \sqrt{1 + \frac{\frac{GM_{Gal}}{r}}{4(\frac{K_1}{r} + k_0 r)^2}}\right) \tag{36}$$

We have demonstrated that this expression can retrieve, with an excellent agreement, the recent measurements of the rotation speed of the Milky Way[19] from 16 kpc and until the end of our Galaxy, with, in particular, the significantly faster decline they observe in the circular velocity curve compared to the inner parts (section 3.1). We also demonstrated that this expression can retrieve the velocity range of the Coma cluster[2] (section 3.3). This relation could therefore also be a good approximation of the virial theorem in the context of GRL. And with the same values of k_0, we obtain the precession measured for the galactic disk warp (section 3.2), the radial acceleration relation (section 3.4), the TULLY-FISHER relation (section 3.6) and the MOND theory (section 3.5). Unlike some articles[16–18] which do not study such a general solution (36) of the Einstein-Maxwell equations (3), we demonstrate that in theoretical terms this GRL solution is no worse than the solution of a hypothetical exotic matter. Briefly, in the hypothesis of the existence of an exotic matter the problem of dark matter is materialized by a greater value of matter than predicted by the theory, in our hypothesis of the GRL, the problem of dark matter is materialized by a greater value of the Lense-Thirring effect than predicted by theory. Furthermore, the DM component explained by the gravitic field makes it possible to naturally account for the insensitivity to the EM because it is a component of gravitation, as well as its undetectability despite its omnipresence because the intensity of k_0 is too weak to be detected by current instruments. While the component of DM explained by exotic matter requires the creation of a new matter (unprecedented in physics because it is insensitive to EM) and strangely omnipresent but not to be taken into account locally where gravitation is extremely well verified (locally without any influence and dominant on a large scale). We argue that it is currently premature to reject the GRL solution. In terms of observation, very few experiments have been carried out to date and they are not yet precise enough to test the GRL solution but with our results the values of what they should observe are known. And the range of possible values is quite limited. This solution is very constrained and well defined for what must be observed.

We cannot exclude that a happy coincidence made it possible to adjust our curve on all the measurement points for $r > 16$ kpc, but given its very low adaptability (2 measurement points freeze the curve), this probability is very low. If the curve fits as well for other galaxies, it will be difficult to argue that its ability to account for DM is fortuitous. This expression should now be tested for many other galaxies. And if its relevance were confirmed, it would be an advancement in the understanding of DM. Indeed, it could invalidate the assumption of an exotic matter because the expression (36) makes no sense with an extended distribution of matter. Furthermore, k_0 and K_1 are known physical fields (contrary to an unknown exotic matter) to explain DM. Until now, they were neglected or rather underestimated. The DM mystery would then consist in understanding how these fields can be greater than expected (this could even give clues to a quantum gravity) and in being able to measure these two fields with sufficient precision (to confirm or not this explanation).

Data availability

All data generated or analysed during this study are included in this published article.

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Author contributions

S.L.C. wrote the main manuscript text and prepared all the figures.

Declarations

Competing interests

The authors declare no competing interests.

Additional information

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