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Review article

A method to test weak-form market efficiency from sectoral indices of the WAEMU stock exchange: A wavelet analysis

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ABSTRACT

This study assesses the efficiency of the West African Economic and Monetary Union (WAEMU) regional stock exchange using daily data on its seven (7) sectoral indices from December 31, 2013, to January 4, 2019. To this end, we analyze the market structure and calculate the generalized Hurst index by using the discrete wavelet transformation (DWT) and wavelet leader transformation (WLT) approaches. Our conclusions can be summarized as follows: first, this study highlights the multifractal nature of the WAEMU stock market. Second, the Hurst generalized index reveals a persistent or nonpersistent process depending on the sector, according to the q chosen or the method used (DWT or WLT). The dynamics of the indices reveal the characteristics of short memory or, in some cases, long memory, and the efficient market hypothesis is rejected.

1. Introduction

Africa's financial sphere has never ceased developing, and the number of stock exchanges grew from approximately ten in the 1990s to 28 in 2017 (source: African Securities Exchange Association, 2017). Thus, the focus on these markets is becoming increasingly accentuated. However, it is difficult to discuss modern finance without mentioning the informational efficient market hypothesis, which has been the basis of most financial theory since the late 1950s and the subject of many studies. (Fama, 1965) defines an efficient market as a market in which the prices of securities fully reflect all available information such that it is impossible to predict future costs based on past price history. This theory has made enormous progress among academics and professionals thanks to the various proposed approaches stemming from it. However, the majority of the studies conducted to date concern developed stock markets. Moreover, the frequency of financial crises (stock market crashes of 1929, October 1987 and the second half of 1997, the bubble of 2000, and the subprime crisis of 2008) calls this hypothesis into question. Thus, the problem of efficiency has resurfaced and revived the debate.

This paper analyzes the weak-form informational efficiency of the West African Economic and Monetary Union (WAEMU) stock exchange.[1] Following a first paper (Diallo and Mendy, 2019) analyzing the Bourse Régionale des Valeurs Mobilières (BRVM) through the BRVM10 index,[2] we were able to demonstrate that the BRVM does not verify the efficient market hypothesis. Thus, this paper conducts a more detailed, sector-by-sector analysis using the seven sectoral indices. The choice of this exchange is motivated by the role it occupies for its member states. At present, with the advent of phenomena such as globalization and financial integration, the focus is increasingly on emerging or frontier markets that are generally perceived as the "next generation" of emerging markets with a strong potential to generate attractive long-term returns (Perroud, 2013). Thus, this stock exchange, which was created in the 1990s by WAEMU authorities through the union's central bank, can play a leveraging role in supporting member states for which investment financing is an important issue. This stock exchange is young, and thus, it would be desirable to determine whether it has developed into an efficient market.

Financial series have been the subject of much research, which has led to improvements in modeling. The first stock price models date back to the pioneering work of (Bachelier, 1900), who analyzed the dynamics of stock prices in the Paris Stock Exchange with Brownian motion (BM). Thus, most of the work that followed started from BM (Samuelson, 1965). However, more recent studies have demonstrated the existence of specific facts or phenomena (price discontinuity, nonnormality of returns, nonstationarity), calling into question the use of

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[1] The WAEMU stock exchange includes eight WAEMU countries: Benin, Burkina Faso, Ivory Coast, Guinea, Mali, Niger, Senegal, and Togo.

[2] BRVM10 is the BRVM's index and consists of the ten most active securities.

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BM for modeling financial series. BM can only model phenomena that exhibit constant regularity over time. (Mandelbrot, 1963) was the first to detect this issue for cotton prices. (Peters, 1991) and (Lo, 1997) also came to the same conclusion, especially since with the weak-form efficient market hypothesis, the objective is generally to verify random behavior in the distribution of prices.

Fractional Brownian motion (FBM), which is a generalization of the standard BM, was introduced by (Kolmogorov, 1940) and made famous by (Mandelbrot and Van Ness, 1968). It is a stochastic process parameterized by H, called the Hurst exponent. This parameter both allows one to characterize the long memory of series and can be interpreted as a measure of the degree of imperfection or inefficiency of the market. There are different methods for estimating this parameter. In this paper, we use the approach based on wavelet analysis (Daubechies, 1992; Mallat, 1999; Percival and Walden, 2000). The advantage of this approach is that it takes into account the limitations of the Brownian model mentioned above. It accounts for the time/frequency aspect and allows us to analyze the behavior of stock prices at different time scales. This is important because financial markets include a large number of operators with different investment horizons. This paper contributes to the existing literature by analyzing the efficiency of the WAEMU stock exchange, which has not yet been done, to promote the visibility of this market and attract investors.

Although our results reveal that the seven indices are multifractal in nature, we also observe that the dynamics of this variation are not constant and can vary in direction and scale. Moreover, a long memory process is also observable in the fluctuations, a result that contradicts the efficient market hypothesis.

The remainder of this paper is structured as follows. Section 2 reviews the existing literature. Section 3 presents the methodology employed. Section 4 reviews the data and discusses the results. Section 5 presents the conclusions.

2. Literature review

Since the work of (Fama, 1970), we distinguish two currents among academics, who have generated a large and varied literature on the efficient market hypothesis.

On the one hand, we have a group of scholars who hold that the efficient market hypothesis remains reliable and have produced results favorable to the efficient market hypothesis. (Working, 1934) demonstrates that the variation of stock prices is random. (Kendall and Debreu, 1953) analyzed 19 different groups of British financial and industrial companies (1928-1938). He also studied weekly averages of the price of wheat on the Chicago market in the period 1883-1934 (excluding the period 1915-1920) and cotton on the New York Stock Exchange from 1816 to 1951. He reports that the memory of a market is at most one week. (Osborne, 1959) and (Malkiel, 1995) obtain the same findings as (Ryoo and Smith, 2002) and (Jefferis and Smith, 2005) from studying the South American market. All these authors generally reach the same conclusion, and they find a notable absence of a trend in forecasting returns, owing to the price changes being random, and hence the impossibility of forecasting future prices, a favorable result for the efficient market hypothesis.

Next, there is another current consisting of (Grossman and Stiglitz, 1980), Thaler, (Shiler, 1981a,b), and (Lo and MacKinlay, 1988), which holds that the efficient hypothesis is contestable. They argue that stock price variations have a particular dependency. (Alexander, 1961) used the filter technique to demonstrate the possibility of realizing abnormally high profits and the existence of a trend in the movement of prices. (Fama, 1965), based on an analysis of 23 of the 30 Dow Jones stocks, found that daily returns show the presence of a trend. (Mobarek and Keasey, 2000), through a study of the indices and recoveries of 30 companies actively traded on the Bangladesh stock exchange (1988 1997), used both parametric techniques such as autocorrelation, autoregression and integrated moving average (ARIMA) as well as the nonparametric run test. Independent of the test and sample considered, the authors find evidence that the Bangladesh stock market, for the period under review, is not efficient. (Akbar and Baig, 2010) find the same results for the Pakistani market (2004-2007). (Mlambo and Biekpe, 2007) conducted a comparative study on low efficiency in 11 African stock markets (Egypt, Kenya, Zimbabwe, Mauritius, Morocco, Tunisia, Ghana, Namibia, Botswana, and BVRM). They used daily data from 1997 to 2002. Their study shows that most markets do not exhibit market efficiency, with the exceptions being Kenya and Zimbabwe over the period 1997-2002, and are all characterized by low trading volume. In more recent studies, (Peters, 1991; Lardic and Mignon, 2006; Lahmiri, 2017) demonstrate the presence of a persistent trend thanks to the long memory observed in time series. For many of these authors, the presence of anomalies or certain stylized facts leads to the rejection or at least questioning of the efficient market hypothesis.

Finally, the various studies on the efficient market hypothesis show that it is undoubtedly one of the most controversial assumptions in finance. It has been the subject of much research, sometimes leading to contradictory results. These mixed results sometimes stem from the specification of the model used and not from market inefficiency. Recent developments have brought to light stylized facts that call into question previous work. Indeed, these earlier works are limited to testing only the predictability of stock market returns with BM questioned by authors such as (Mandelbrot, 1963; Peters, 1991; Lo, 1997). In our analysis, we use MBF, which is a stochastic process parameterized by $H \in [0,1]$, called the Hurst exponent. This parameter allows us to study financial series behavior, particularly the long memory phenomenon.

3. Methodology

Since Box-Jenkins' 1970 article, the conventional structure for time series modeling has continued to evolve, particularly in economics. In statistics, ARMA models (autoregressive and moving average models), also called Box-Jenkins models, are the main time series models. The ARMA process is a tool adapted to the analysis of short memory. It is characterized by an autocorrelation function and an impulse function that geometrically decrease exponentially. However, financial series are often characterized by ARCH and GARCH processes that allow for the description of the nonlinear dynamics of volatility that induce long-term dependence.

Long memory is characterized by an autocorrelation function that disintegrates at hyperbolic rate or, equivalently, an infinite spectrum at zero frequency. The precursor of this concept of long memory is Hurst (1951), who was the first to show that time series have a long-term dependence through his study of the Nile water level using rescaled scaling (R/S) analysis. Mathematical modeling of fractals was proposed and introduced in financial markets by (Mandelbrot, 1963) and (Mandelbrot and Van Ness, 1968). Their results employ the notion of multifractality observed in other domains, such as meteorology and complex socioeconomic systems, including finance...

Multifractal analysis allows us to understand the complex nonlinear nature of time and financial series and to account for the decline in scale properties often observed in empirical data (Ciuciu et al., 2008). The use of such analysis is necessary for the study of market efficiency and risk management.

Given the limitations of the previous methods cited above, the wavelet method was developed. The literature on wavelet analysis modeling has mainly studied long memory processes. The advantage of this approach is that it addresses the limitations of the Brownian model mentioned above and is also suitable for the analysis of both stationary and nonstationary series. It also accounts for the time/frequency aspect and allows us to analyze the behavior of stock prices at different time scales. This motivates its use because financial markets combine a large number of operators with different investment horizons.

Hurst estimation. The slow decay of the autocovariance function of the long-memory process is such that its spectral density function has a pole equal to zero, which makes the covariance matrix dense. This property of the spectral density makes it difficult to estimate the coefficient of long memory by the maximum likelihood method. To overcome this difficulty, many other approaches have been proposed: the conditional sum approach (Li and McLeod, 1986; Robinson, 1994; Hualde and Robinson, 2011; Nielsen, 2015), the frequency-domain approach based on log-periodogram regression (Geweke and Porter-Hudak, 1983; Robinson, 1995) and local Whittle estimation (Robinson, 1995; Shimotsu and Phillips, 2005).

(Tewfik and Kim, 1992), (Flandrin, 1992), and (McCoy and Walden, 1996) derived the property of the long memory coefficient estimator by using wavelets; the approximate maximum likelihood estimate was proposed by (Jensen, 2000) and (Percival and Walden, 2000); regression at logarithmic scale was developed by (Abry and Veitch, 1998) and (Jensen, 1999); and the asymptotic properties of these estimators were given by (Bardet, 2000), (Bardet et al., 2000), (Moulines et al., 2008) and (Roueff and Taqqu, 2009). (Teyssière and Abry, 2007) conducted their analysis under the white noise property of the wavelet coefficients of long memory processes. (Jensen, 1999) and (Craigmile et al., 2005) proposed a white noise hypothesis for these coefficients and an approach based on the correlations of these coefficients to improve the estimator. (Knight et al., 2017) propose a new method for estimating the Hurst exponent that naturally accounts for the irregularity of the data sampling.

(Khamis et al., 2018), (Jiang et al., 2019) and (Diallo and Mendy, 2019) use multivariate analysis to estimate the generalized Hurst coefficient $H(q)$ (Ciuciu et al., 2008) and concluded that the approach based on wavelet leader (WL) coefficients gives a better estimate of $H(q)$.

3.1. Wavelet analysis

In this subsection, we briefly present the wavelet method used to assess the weak form of the efficient market hypothesis. A wavelet is simply a small localized wave. We distinguish wavelet analysis based on discrete wavelet coefficients and WL coefficients. (Daubechies, 1992) defines a mother wavelet as follows: let $\psi_0(t)$ be an oscillating function of $L_2(\mathbb{R})$ with null average and finite support of order m if the following properties are respected:

$$\int_{\mathbb{R}} \psi_0(t)dt = 0 \tag{1}$$

This function of time t must follow two basic rules:

i) condition of admissibility: $C_\psi = \int_{\mathbb{R}} \Psi(f)df < \infty$
ii) condition of regularity: $\int_{\mathbb{R}} t^k \psi(t)dt \equiv 0 \quad \forall k = 0,\dots,N-1$

where $\Psi(f)$ represents the Fourier transform of ψ. The first condition ensures rapid convergence of $\Psi(f)$ to 0 when $f \longrightarrow 0$ (Grossman and Morlet, 1984; Mallat, 1999).

3.1.1. The wavelet leaders

The discrete wavelet coefficients often take values close to 0 and cannot be raised to negative powers. In this case, the corresponding structure functions $S(j,q)$ become unstable according to (Ciuciu et al., 2008). To overcome this problem, they propose using WLs for better estimations.

Now assume that $\psi_0(t)$ has compact time support, and define the dyadic intervals as follows:

$$\lambda \equiv \lambda_{j,k} = [k2^j, (k+1)2^j]. \tag{2}$$

We denote by 3λ the union of the λ interval with its two adjacent dyadic intervals such that:

$$3\lambda_{(j,k)} = \lambda_{(j,k-1)} \cup \lambda_{(j,k)} \cup \lambda_{(j,k+1)} \tag{3}$$

According to (Jaffard et al., 2005), the WL coefficients are defined as follows:

$$L_X(j,k) \equiv L_\lambda = \sup_{\lambda' \subset 3\lambda_{(j,k)}} |d_{\lambda'}| \tag{4}$$

$L_X(j,k)$ is then constituted by the supremum of the coefficients $d_X(j',k')$ calculated at all the finest scales $2^{j'} \le 2^j$ in a restricted temporal neighborhood $(k-1)2^j \le 2^{j'}k' < (k+1)2^j$.

3.1.2. The structure function

The structure function is defined by (Ciuciu et al., 2008) for fixed scales $a = 2^j$, and the time averages (q-th powers) of $d_X(j,k)$ are called structure functions (with n_j being the number of WL coefficients available at scale 2^j).

$$S^d_{(j,q)} = \frac{1}{n_j}\sum_{k=1}^{n_j} |d_X(j,k)|^q \tag{5}$$

The WLs exactly reproduce the regularity of Hölder (cf. (Parisi and Frisch, 1985; Jaffard, 2004; Jaffard et al., 2006)) under a slight condition of uniform regularity of Hölder on $X(t)$.

$$S^d_{(j,q)} = Fq2^{j\zeta(q)} \tag{6}$$

The long-term dependence may be limited to describing the second-order statistic ($q = 2$) of the data. X is a self-similar process when it satisfies $\forall c > 0, \{X(t), t \in \mathbb{R}\} \overset{fdd}{=} \{c^H X(t/c), t \in \mathbb{R}\}$ or $\overset{fdd}{=}$ means the equality of all finite dimensional distributions (see (Samorodnitsky and Taqqu, 1994)) For self-similar processes of finite variance and stationary increment, the self-similarity parameter H is restricted to $H \in [0,1]$ (see (Abry et al., 2000; Jaffard et al., 2006) for a demonstration)

$$S^d_{(j,q)} = C_q 2^{jqH}, \ \forall 2^j, \ q \in (-1, +\infty) \tag{7}$$

Self-similarity implies respecting certain restrictions on the empirical data; first, the scale property must be valid for all scales, and second, the parameter H controls all the statistical properties of the data. These restrictions have led to the use of multifractal (MF) models. MF processes are generally considered extensions of scale invariance. They allow us to account for a decline in scale properties often verified in empirical data. For a q-order range and a scale range 2^j, the structure functions of MF processes exhibit power law behavior with respect to scales. For a q-order range $q \in \mathbb{Z}$ and a scale range 2^j , $j \in \mathbb{R}^{+*}$, the structure functions of MF processes show power law behavior with respect to scales.

$$S^d_{(j,q)} = C^d_q 2^{j\zeta(q)}, \quad q \in [0, q^+_*] \tag{8}$$

where $\zeta(q)$ is the scale exponent or MF exponent, with a concave shape that thus deviates from linear behavior qH, considered an autosimilarity (equation (7)).

3.1.3. The log-cumulants

The scale exponent measure or MF exponent $\zeta(q)$ from the data theoretically implies making an estimate for every q (Castaing et al., 1993; Arneodo et al., 2002; Wendt et al., 2007) that has been proposed to consider a polynomial expansion of $\zeta(q)$. Equation (6) consists of time means, which can be read as sample mean estimators for the ensemble means $Ed_X(j,.)^q$. This heuristic analysis proposed for the first time with the increments in (Castaing et al., 1993) was first developed for the continuous wavelet coefficients in (Delour et al., 2001) before the extension for the WLs. Therefore, equation (7) is rewritten as follows:

$$\mathbb{E}L_X(j,.)^q = Fq2^{j\zeta(q)} \tag{9}$$

When $\mathbb{E}L_X(j,.)^q$ is finite, an expansion of the standard generator function yields:

$$\ln E e^{q \ln L_X(j,.)} = \sum_{p=1}^{\infty} c^L(j,p) \frac{q^p}{p!} \tag{10}$$

where $c^L(j,p)$ represents the cumulants of order $p \geq 1$ of $\ln L_X(j,.)$.

The combination of the above equations yields:

$$\zeta(q) = \sum_{p \geq 1} c_p \frac{q^p}{p!} \tag{11}$$

where the c_p coefficients can be related to the cumulants of order p and satisfy $\forall p \geq 1 \;\; c_p(j) = c_{0,p} + c_p \ln 2^j$

We note the following:

i) Using only the first two cumulants, we arrive at a good approximation of $\zeta(q)$.

$$\zeta(q) \simeq c_1(q) + \frac{1}{2} c_2(q)^2 \quad \text{and} \quad D(h) \simeq 1 - \frac{(h - c_1)^2}{2c_2}$$

ii) The definition of cumulants implies that for $p' > p$ if $c_p = 0$, $c_{p'} = 0$ (for further details, see (Jaffard et al., 2006; Wendt et al., 2009)).

The triplet (c_1, c_2, c_3) represents the main characterization of self-similarity, the measure of the deviation from pure self-similarity and asymmetry, hence the impact of the MF component of the data. If $\zeta(q)$ is a linear function, we have a monofractal process. If $\zeta(q)$ is a nonlinear function, the process is MF.

3.2. Testing the efficient market hypothesis

The generalized Hurst exponent in the case of $q = 2$, i.e., $H(2)$, is identical to the standard Hurst exponent that can be used to test the long memory property of time series to conclude whether we have market efficiency. We apply a one-tailed test to test the efficient market hypothesis: H_0: $H(2) = \frac{1}{2}$ versus H_1: $H(2) \neq \frac{1}{2}$. A failure to reject the null hypothesis means an absence of memory. We would therefore conclude that we have a weak form of market efficiency. On the other hand, if we reject H_0, we would have either persistent or antipersistent processes. In the case of a persistent process ($H(2) > \frac{1}{2}$), we have long memory, the autocorrelation function of which decreases very slowly. In the case of an antipersistent process ($H(2) < \frac{1}{2}$), we are in the presence of short memory characterized by a negative correlation; phases of increase tend to be followed by phases of decrease (clustering phenomenon). Following the work of (Mandelbrot and Taqqu, 1979), this antipersistence can be seen as a special case of long memory, which they call "a form of antipersistence of long-term dependence". According to them, the closer H is to zero, the more the process will have a systematic tendency to offset a large variation with another large variation of the opposite sign. Like persistent processes, antipersistent processes are a means to predict future return trends.

4. Data and empirical results

4.1. Data

We use daily return data for seven sector indices of the WAEMU Stock Exchange[3] from December 31, 2013, through January 4, 2019 (the data are extracted from the BRVM database). The daily returns are computed by taking the difference in the logarithms of two consecutive prices as follows: $r_t = \ln(P_t/P_{t-1})$, where P_t and P_{t-1} represent returns in t and $t-1$, respectively.

Fig. 1 displays the trajectory of daily level series of the seven indices. We observe a period that covers several episodes of considerable instability. Stock prices experienced sharp declines that can be explained by periods of political crises in the Ivory Coast and the drastic decrease in international cocoa prices that plunged the Ivory Coast (which hosts 35 of the 44 companies listed on the WAEMU stock exchange) into crisis, as well as financial crises that affected the WAEMU stock exchange through contagion.

The statistical properties of the return series of the seven indices are presented in Table 1. The average returns are generally negative. The skewness values $S < 0$ of the indices for the distribution, industry, transportation and other sectors indicate that the returns are negatively skewed (characterized by small regular gains and little excess loss), while the indices for the agriculture, finance, and public sectors have skewness ($S > 0$), meaning the returns are positively skewed (characterized by frequent small losses with less chance of excessive gains). Kurtosis values of $K < 3$ are observed for the majority of sectors (agriculture, distribution, finance, industry, and transport), and the return series are platykurtic, which means that assets in these sectors are less likely to produce extreme results, and these sectors will attract more risk-averse investors. Only the public and other sectors show a leptokurtic distribution ($K > 3$). This means that the probability of significant losses or gains is greater than would be expected if returns were on a normal curve.

4.2. Empirical results

Table 2 reports the slopes of the generalized Hurst exponent $H(q)$ across different scales ranging from [−5.5]. Our results show that $H(q)$ coefficient values are not stable and are dependent on q, which indicates strong evidence of MF behavior in all seven return series. We observe antipersistence $\left(H \in \left]\frac{1}{2}\right[\right)$, persistence $\left(H \in \left]\frac{1}{2}, 1\right]\right)$, and $H > 1$ in different cases. According to (Qu, 2011) and (Smith, 2005), the observation of $H > 1$ may result from a structural change, bubble behavior or nonstationarity of the series (NB: wavelet analysis is suitable for both stationary and nonstationary series). Wavelet leader transformation (WLT) produces the best estimates (Ciuciu et al., 2008).

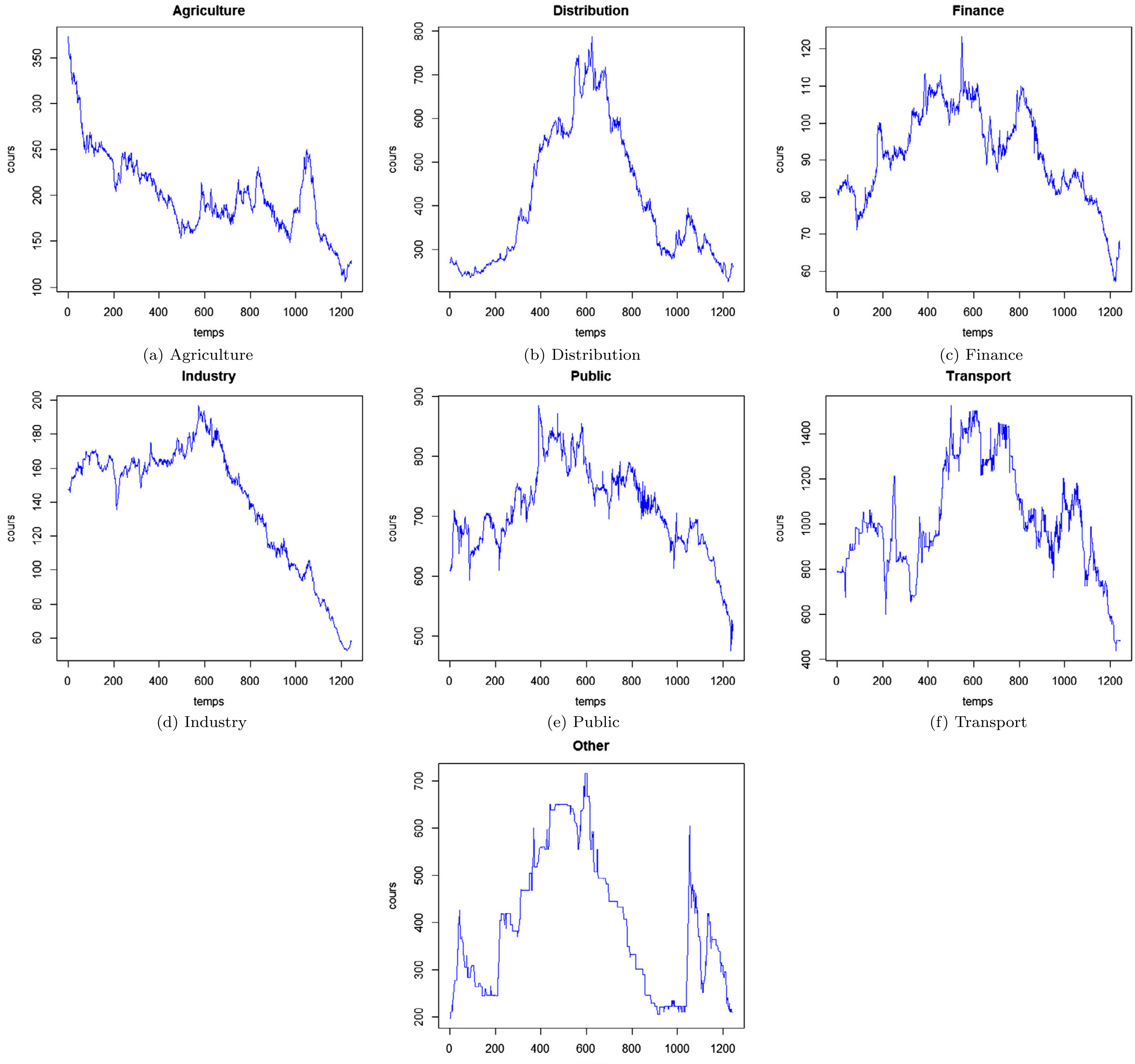
Table 3 presents the triplet estimate (c_1, c_2, c_3). Scaling exponents $\zeta(q)$, the MF spectrum $D(h)$ and the log-cumulants c_p are theoretically equivalent in accounting for the scaling content of the analyzed data, and we concentrate on the log-cumulants c_1 and c_2 only: $c1$ essentially amounts to the self-similarity characterization, while c_2 measures the deviation from pure self-similarity and hence the impact of the MF component of the data. We observe that the parameter c_1 takes values in the range $0.05 \leq c_1 \leq 1.7$, thus confirming the relevance of long-term dependency characterizing time series. The precise estimate of c_2 is also very useful to conclude whether we have a monofractal ($\forall p \leq 2 : c_p \equiv 0$) or MF ($c_2 \neq 0$) process. Therefore, we conclude that we have multifractality because c_2 is generally $\neq 0$.

Figures (see Appendix A) graphically represent the log-cumulants. The boxplots are obtained from c_p and show the lower and upper quartile, median, and support of their empirical distributions. All bootstrap confidence intervals (in red) are obtained with the percentile method ($\alpha = 0.05$). Therefore, Tables 2 and 3 and Figure reference (Appendix A) support the MF nature of the market.

Table 4 reports the test results of the null hypothesis of an efficient market ($H(2) = \frac{1}{2}$). The results reject the null hypothesis for all return series except agriculture. Therefore, with the exception of agriculture, future returns from other sectoral indices are predictable.

Figures (see Appendix 2) report estimates (in solid black) of the scaling exponent $\zeta(q)$ and MF spectrum $D(h)$. The red lines indicate the 95% confidence interval based on the (Wendt et al., 2007) bootstrap. We can see that $\zeta(h)$ deviates from the linear behavior of a monofractal process. In both cases (discrete wavelet transformation (DWT) and WLT), $\zeta(h)$ shows the downward concavity characteristic of MF processes. Moreover, $D(h)$ shows broad support that is not reduced to a single point, as indicated by the confidence intervals of the end points, which, although broad, do not overlap. We find that the WLTs produce the best results.

[3] The WAEMU stock exchange includes 7 sector indices: agriculture, distribution, finance, industry, transport, public and other sectors.



(a) Agriculture (b) Distribution (c) Finance (d) Industry (e) Public (f) Transport (g) Other sectors

Fig. 1. Dynamics of WAEMU sector indices.

Table 1. Descriptive statistics.

Variables	Observations	Mean	Minimum	Maximum	St-dev	Skewness	Kurtosis	Jarque-Bera
Agriculture	1245	-0.0008	-0.0611	0.0652	0.0170	0.0570	0.6105	19.977***
Distribution	1245	-1.6046	-0.0536	0.0448	0.0146	-0.2386	0.6372	32.826***
Finance	1245	-0.0002	-0.0648	0.0643	0.0121	0.0898	2.1539	241.950***
Industry	1245	-0.0007	-0.0447	0.0484	0.0113	-0.1151	1.9515	199.990***
Transport	1245	-0.0004	-0.1249	0.0723	0.0283	-0.0595	1.8139	171.140***
Public	1245	-0.0001	-0.0644	0.0648	0.0126	0.1779	6.5784	2247.900***
Other sectors	1245	4.8329	-0.0780	0.0723	0.0247	-0.0176	4.2878	952.290***

5. Conclusion and policy implications

The WAEMU stock exchange is the first example of a regional stock exchange and has been the subject of particular attention since its creation. The behavior of this exchange in relation to other African and global financial markets is important for investors, portfolio managers and academics. This paper examined the structure and efficiency of the market through seven WAEMU sectoral indices. Our results can be summarized as follows: first, according to our application of wavelet analysis, there is evidence of multifractality features for all seven indices. Second, with the exception of agriculture, all sector indices reject the null hypothesis of the weak form of an efficient market. These sec-

Table 2. Generalized Hurst exponent.

DWT							
c_p	Agriculture	Distribution	Finance	Industry	Public	Transport	Other sectors
-5	1.7000	0.3760	0.2860	1	1.7360	0.0980	0.7200
-4	1.7000	0.3460	0.2760	1	1.7370	0.0740	0.7140
-3	1.7000	0.2990	0.2410	1	1.7200	0.0470	0.6870
-2	1.7000	0.2470	0	0.9630	1.6150	0.0080	0.5610
-1	1.7000	0.1970	-0.5590	0.4190	1.1580	-0.0110	0.0500
-0.5	1.7000	0.1160	-0.2060	0.1050	0.6690	0.0760	0.2000
0	1.7030	0.1570	0.2240	0	0.2860	0.1550	0.2510
0.5	1.4170	0.2230	0.2920	0	0.1410	0.1770	0.1480
1	0.6770	0.2600	0.2750	0.1420	0.1100	0.1830	0.0430
2	0.4560	0.2760	0.2310	0.2070	0.1470	0.2070	-0.1150
3	0.4310	0.2670	0.1890	0.1310	0.1960	0.2010	-0.1880
4	0.4270	0.2500	0.1430	0	0.2170	0.1720	-0.2130
5	0.4260	0.2290	0	-0.1410	0.2150	0.1470	-0.2190
WLT							
c_p	Agriculture	Distribution	Finance	Industry	Public	Transport	Other sectors
-5	1.7320	0.2430	0.5790	0.5750	1.7980	0.5970	0.9170
-4	1.7320	0.3070	0.5850	0.5310	1.7970	0.5850	0.8570
-3	1.7320	0.3760	0.5810	0.4860	1.7420	0.5590	0.7840
-2	1.7320	0.4040	0.5490	0.4450	1.4210	0.5150	0.7080
-1	1.7320	0.3800	0.4860	0.4130	0.7360	0.4520	0.6190
-0.5	1.7310	0.3350	0.4460	0.4020	0.4980	0.4170	0.5590
0	1.7350	0.3580	0.4050	0.3930	0.3630	0.3810	0.4870
0.5	1.4320	0.3120	0.3640	0.3860	0.2890	0.3440	0.4040
1	0.6780	0.2920	0.3260	0.3780	0.2470	0.3090	0.3190
2	0.4570	0.2590	0.2610	0.3500	0.2120	0.2450	0.1780
3	0.4310	0.2360	0.2120	0.2950	0.2080	0.1880	0.0970
4	0.4270	0.2190	0.1770	0.2230	0.2170	0.1380	0.0580
5	0.4270	0.2080	0.1540	0.1580	0.2280	0	0

Table 3. Log-cumulants.

DWT							
c_p	Agriculture	Distribution	Finance	Industry	Public	Transport	Other sectors
c_1	1.7030**	0.1570	0.2240*	0.0510	0.2860**	0.1550**	0.1340**
	(0.2580)	(0.1530)	(0.1320)	(0.0890)	(0.0910)	(0.0700)	(0.0680)
c_2	0.0270	0.1400	0.3910	0.0250	-0.4920**	0.0940	0.0560
	(1.1640)	(0.3260)	(0.2700)	(0.2080)	(0.1830)	(0.1070)	(0.1440)
c_3	0.0620	0.0650	1.5920**	0.3020	1.0440**	-0.2610	-0.2170
	7.3200)	(1.0900)	(0.8040)	(0.5270)	(0.4160)	(0.2530)	(0.4850)
WLT							
c_p	Agriculture	Distribution	Finance	Industry	Public	Transport	Other sectors
c_1	1.7350**	0.3350**	0.4050**	0.3930**	0.3630**	0.3810**	0.4770**
	(0.2360)	(0.0470)	(0.0640)	(0.0450)	(0.0400)	(0.0410)	(0.0440)
c_2	0.0260	-0.0470**	-0.0820**	-0.0150	-0.1970**	-0.0730**	-0.1460**
	(1.0410)	(0.0240)	(0.0290)	(0.0270)	(0.0420)	(0.0240)	(0.0300)
c_3	0.0350	0.0040	0.0030	0.0060	0.2340**	0.0010	-0.0090
	(6.6580)	(0.0160)	(0.0150)	(0.0180)	(0.0850)	(0.0100)	(0.0220)

Notes: Standard errors are in parentheses, and (*), (**) and (***) indicate significance at the 10%, 5% and 1% levels, respectively.

Table 4. T-test results of the null hypothesis for $H(q=2)=\frac{1}{2}$.

Method	Agriculture	Distribution	Finance	Industry	Public	Transport	Other sectors
DWT	0.0201	2.5454**	3.2805**	1.6936*	3.8791**	2.6161**	7.2352**
WLT	0.0246	6.1795**	3.8548**	1.4286*	9.2903**	3.8636**	6.8511**

Notes: (*), (**) and (***) indicate significance at the 10%, 5% and 1% levels, respectively.

tors show that there is a (short or long) memory in the evolution of returns. Thus, these persistent and antipersistent processes have memory and therefore can be used to predict the future evolution of return series. For some decades, (Mandelbrot, 1971) has argued that the presence of long memory implies less-than-perfect arbitrage and that the resulting prices do not follow a random walk process. This stipulation is contradictory to the efficient market hypothesis, which states that in an efficient market, the prices of securities fully reflect all available information and that, therefore, it is impossible to predict future prices based on past price history.

These results have implications for various economic actors. Investors cannot generate abnormal returns by forecasting future returns. The WAEMU stock exchange has certainly made enormous progress, but such efforts must continue. The low level of capitalization of the stock market and the low number of individual shares, liquidity and trading volume on the WAEMU stock exchange may explain the rejection of the weak form market efficiency. Therefore, policymakers should strengthen supervision and establish laws and reforms that improve the local financial institutions that play an important role in promoting and financing enterprises. To this end, joint efforts must be made to mobilize funds to improve the level of market development and increase liquidity (Ivory Coast hosts 35 of the 44 listed companies (www.brvm.org)). Additionally, the low level of integration of the WAEMU stock exchange should be seen as an asset because it allows diversification of foreign portfolios that often face crises. The implementation of these recommendations could be an essential incentive to attract additional investors.

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