



## Research article

## Discontinuous rock slope stability analysis under blocky structural sliding by fuzzy key-block analysis method



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## ABSTRACT

This study presents a fuzzy logical decision-making algorithm based on block theory to effectively determine discontinuous rock slope reliability under various wedge and planar slip scenarios. The algorithm was developed to provide rapid response operations without the need for extensive quantitative stability evaluations based on the rock slope sustainability ratio. The fuzzy key-block analysis method utilises a weighted rational decision (multi-criteria decision-making) function to prepare the ‘degree of reliability (degree of stability-instability contingency)’ for slopes as implemented through the Mathematica software package. The central and analyst core of the proposed algorithm is provided as based on discontinuity network geometrical uncertainties and hierarchical decision-making. This algorithm uses block theory principles to proceed to rock block classification, movable blocks and key-block identifications under ambiguous terms which investigates the sustainability ratio with accurate, quick and appropriate decisions especially for novice engineers in the context of discontinuous rock slope stability analysis. The method with very high precision and speed has particular matches with the existing procedures and has the potential to be utilised as a continuous decision-making system for discrete parameters and to minimise the need to apply common practises. In order to justify the algorithm, a number of discontinuous rock mass slopes were considered as examples. In addition, the SWedge, RocPlane softwares and expert assignments (25-member specialist team) were utilised for verification of the applied algorithm which led to a conclusion that the algorithm was successful in providing rational decision-making.

## 1. Introduction

Stability analysis of rock structures is almost always associated with many uncertainties which originate from slope topography, mechanical properties, discontinuity network characteristics, loading conditions, failure features and reliability status (Steffen et al., 2008) which lead to complicated stability assessment and deviation in precise decision-making operations (Rodríguez and Montero, 2003). Some of these uncertainties are partially controlled by various assumptions, modifications and tests (Tiwari and Latha, 2019); some are normalised by probabilistic, statistical and geostatistical functions (Azarafza et al., 2017a, 2017b); and some are inevitably accepted. Nevertheless, logical

decision-making regarding rock structure stability is very complicated and poses many ambiguities. The fuzzy logical concepts are founded on ambiguity considerations and assessment of unclear conditions based on intuitive and qualification analysis (Dadios, 2012). Fuzzy logic is a multiple-valued logic identification of the generalised classical logic which can be expressed in terms of both qualitative and indeterminate issues for the sake of reducing time to increase the accuracy of the decisions in regards to the mechanism of materials with uncertainties which can be used to describe empirical values quantitatively. This preponderance leads to the “expert” approach where the result of engineering judgement can be entered as a special parameter in the analysis. Zadeh (1965) has presented modern fuzzy logic by the introduction of fuzzy sets

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which used a series of truth values continuous sets for logical propositions instead of two alternatives. Fuzzy logic has a weak relation with probability theory. In the probabilistic approach, ambiguous relations are formulated in the Bayesian framework, but in fuzzy logic there is no need for justifying probability where algebraic structural solving is conducted for defining multi-valued logical sets (Zadeh, 1968). Fuzzy set theory and the semantic of fuzzy operators are based on the expert-memberships definition which is expressed by specialists with simple control rules characterised for dynamic systems with less effort (Oh et al., 2009). Thus, application of this inference logical procedure for the conversion of the intuitive inputs to reliable quantitative results aids to rapidly quantify accurate, quick and appropriate decisions especially for novice engineers in the context of experimental problems. Zadeh (1965) ratiocinated that the capacity of the fuzzy sets manipulated concepts should be viewed as a significant asset. The real world complexities defy precise measurement and fuzzy logic defines concepts and techniques that provide a mathematical method that is able to deal with thought processes which are often too imprecise and ambiguous to deal with by classical mathematical techniques (Zadeh, 2004, 2005, 2006, 2011).

Application of the fuzzy theory is extended based on the works of Yager and Zadeh (1992) who have utilised decision processes in complex and intelligent systems which are widely used in various basic sciences, engineering and humanities (Asan and Soyer, 2009; Saabun, 2014; Saabun et al., 2015; Doumposm et al., 2016; Garg, 2017; Faizi et al., 2017; Benítez et al., 2018; Toth and Vacik, 2018; Trivedi and Singh, 2019; Gharanfoli and Valmohammadi, 2019). In geo-engineering and rock engineering, use of fuzzy theory is outreached quickly. Ruigeng (1998) presented one of the first applications of decision-based fuzzy sets for control plans on slope instability assessments. Yuanyou and Ruigeng (2000) presented an intelligent aided decision-making system for the control plan of the unstable rock mass evaluations in Banyan Mountain, Huangshi City, China. Liu and Chen (2007) presented a new method for rock mass classification on rock slope stability based on the analytic hierarchy process, fuzzy logic and the linear discriminant analysis model. Liu et al. (2008) proposed the fuzzy probability procedure to evaluate the stability of depth cutting slope in a rock mass. They have used fuzzy methods to aid to simplify the traditional approaches to achieve stability. Wu et al. (2008) applied the fuzzy sets pair analyses to solve indeterminate problems in discrepancy coefficient evaluations for the stability of loess cutting slopes. Fang and Shen (2011) have utilised fuzzy-based multi-attribute group decision-making for instable rock slope treatment on a large-scale road project. Mikaeil et al. (2013) used fuzzy Delphi and multi-criteria decision-making techniques (TOPSIS) for the evaluation of rock sawability in mining works. Jato-Espino et al. (2014) presented a review for multi-criteria decision-making method application in construction and civil engineering. Su et al. (2014) established the fuzzy optimal recognition theory for slope stability evaluations in hydropower projects. Rafiee et al. (2016) proposed the fuzzy method for rock engineering system improvement, which allowed the consideration of uncertainties based on classic expert semi-quantitative methodology which were used to evaluate rock cavability in mining projects. Khakestar et al. (2016) applied multi-criteria decision-making methods for the assessment of the stability of open-pit mines based on geotechnical parameters. Mirhossein et al. (2016) evaluated the possibility of fuzzy decision-making to appraise the key-group method (a branch of key-block theory) in rock slope stability analysis. Haghshenas et al. (2017) used the fuzzy and multi-criteria decision-making procedure to obtain the most crucial tasks in rock-fill slope stabilisation for reservoir dam projects.

The study performed herein presents a simplified algorithm to investigate discontinuous rock slope instability under various blocky sliding failures, especially wedge and planar slip scenarios, and evaluates the 'degree of reliability (degree of stability-instability contingency)' considering these advantages of the fuzzy-based decision-making system, especially the expert system technique. The advantage of the proposed

method is high accuracy and quick response for appropriate decisions especially for novice engineers in the context of discontinuous rock slope stability analysis. The presented algorithm is to be used for stabilisation, particularly in road/railway projects.

## 2. Rock block stability analysis theorem

Goodman and Shi (1985) introduced the 'block theory' for the analyses of instability in rock masses based on geometrical properties and rock block dimensions resulting from the intersection of discontinuities that are assumed to be planar and persistent regardless of the lateral pressures that allow static analyses of rock masses in two-dimensional and three-dimensional spaces (Kulatilake et al., 2011). Block theory attempts to cover several main uncertainties in rock media which provides a more accurate coupled stability analysis of rock structures such as discontinuous rock slopes where it is possible to graphically and vectorially analyse various rupture conditions (Azarafza et al., 2014, 2015; Zhou and Wang, 2017). In the stability analysis of discontinuous rock slopes, failure that is typically developed along discontinuities with proper overlapping and assumptions that satisfy this theory were assessed by rock block geometrical emplacement which was classified for various block types and different failure conditions that were created by the discontinuity network (Um and Kulatilake, 2001).

The block theory classifies blocks in various types as infinite blocks (V), tapered blocks (IV), stable-even without friction blocks (III), potential key blocks (II) and key blocks (I) which are illustrated in Figure 1. In this classification, the 'V' block is "infinite" and the rest of the blocks are "finite" which have the property of removability (Goodman and Shi, 1985). Although rock blocks in reality are three-dimensional, they are shown in two dimensions for simplicity and for providing ease to the analysis (Zheng et al., 2014, 2015). In the primary stability analyses, the removable rock blocks with different geometries that are problematic are calculated by kinematic or limit state analyses which require detailed input parameters that are costly to obtain and time-consuming. Thus, by considering the removable finite blocks as the leading causes of progressive ruptures in discontinuous rock slopes, failure analysis and its types can be evaluated as initiated by the unstable key block. This study attempts to use these rules for structural configuration in the sustainability analysis of discontinuous rock slopes in case the input values are ambiguous. For this purpose, fuzzy provisions are used in ambiguous-based decisions for stability analysis of rock slopes with uncertainties.

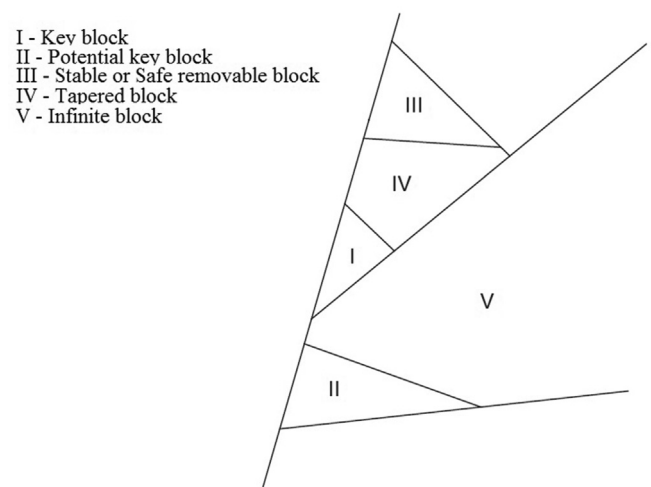


Figure 1. Rock slope classification provided by Block theory (Kulatilake et al., 2011).

### 3. Methodology of study

Referring to the structural condition of the discontinuous rock slopes, application of the appropriate method which investigates the problems, evaluates sustainability ratio with accurate, quick and suitable decisions based on primary field survey especially for novice engineers is always desirable. In this study, utilisation of field evidence and expert system technique was proposed to develop a simplified algorithm for discontinuous rock slope instability assessment. In this regard, a combination of the fuzzy inference logic procedure was used. Application of fuzzy functions to cover the in-situ uncertainties, which are related to rock slope geometry and discontinuity network characteristics, is essential to obtain failure features and a generalised safety factor. In classic fuzzy applications, fuzzy sets are defined by multidimensional alternative sets  $(X = \{x_1, x_2, \dots, x_n\})$ , Universal set  $(U = \{u_1, u_2, \dots, u_n\})$ , fuzzy set's membership function  $(A, \mu_A: U \rightarrow [0, 1])$  and ordered pairs sets  $(X' = \{(x_1, \mu_A(u_1)), (x_2, \mu_A(u_2)), \dots, (x_n, \mu_A(u_n))\})$  where each element  $\mu_A(u_i)$  represents the  $u_i$  degree of membership which has the property  $\{\forall u_i \in U, 0 \leq \mu_A(u_i) \leq 1\}$ . By using the fuzzy sets, the non-comparability and intransitivity alternatives are explained, and the ambiguous features are

considered in computational relations (Zimmermann, 2001). Following the definition of the sets for using the logical decision process, there is a need to explain some framework details. It can assess the condition of the fuzzy status, along with describing and calculating input data to the output(s) which are referred to as fuzzy rules and are presented in Figure 2. The membership functions definition generally implements the fuzzy rules based on the description of the logical conditions. The common form of fuzzy rules is described in Eq. (1) (Zimmermann, 2001):

$$\text{Rule : IF } x \text{ is } M \text{ OR/AND } y \text{ is } M' \text{ THEN } z \text{ is } M'' \quad (1)$$

where,  $x, y, z$  are elements,  $M, M', M''$  are fuzzy sets functions. IF, OR, AND and THEN are fuzzy logical decision-making operator protocols. All rules can have quantitative or qualitative dimensions or values. These rules allow the fuzzy analytical operating core to make the optimal decision of system and environment operation.

Fuzzy controlling system (or controllers) is a regulating system with specified fuzzy rules which are defined in the rule definition unit. Data (inputs) after fuzzification and definition of fuzzy membership functions in fuzzy forms are processed after being entered into the fuzzy inference system (FIS) core, in which they are computed and defuzzified. The Mamdani method is the most common FIS controller, which is utilised as the main fuzzy inference core. In Mamdani's model, the fuzzy implication is modelled by Mamdani's minimum operator (the conjunction operator is *min*), where the *t-norm* from compositional rule is *min* and for the aggregation of rules the maximum operator (the conjunction operator is *max*) is used (Mamdani and Assilian, 1975; Mamdani, 1977).

In fuzzy applications, the multi-criteria decision-making (MCDM) method is mainly used to achieve the best choice between various alternatives and elements (Lin et al., 2007) which is capable of being coupled by uncertainty theory which has been introduced by Bellman and Zadeh (1970) and Zadeh (1983). Integration of the fuzzy and MCDM approaches may be summarised by Eqs. (2) and (3) as presented below which indicates the decision lists for preparing the fuzzified input data:

$$A = \{(x_i, \mu_A(x_i), \nu_A(x_i)) | x \in M\} \text{ and } \mu_A : M \rightarrow [0, 1] ; \nu_A : M \rightarrow [0, 1] \quad (2)$$

$$\text{For all } x \text{ in } M \Rightarrow 0 \leq (\mu_A(x_i) + \nu_A(x_i)) \leq 1 \quad (3)$$

where,  $M$  is defined as the intuitionistic fuzzy sets covered by the Universal set,  $\mu_A(x_i)$  and  $\nu_A(x_i)$  are the grade of memberships. All rules applied are considered as operator sets (intuitionistic fuzzy sets) that can explain all conditions in quantifying the elements/events. By considering

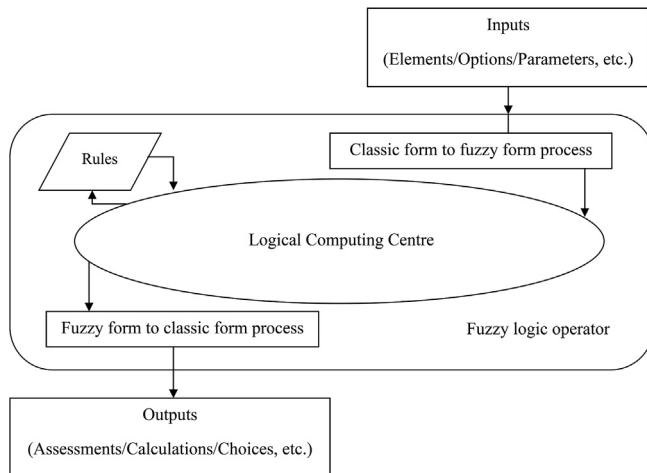


Figure 2. The fuzzy logic system configuration.

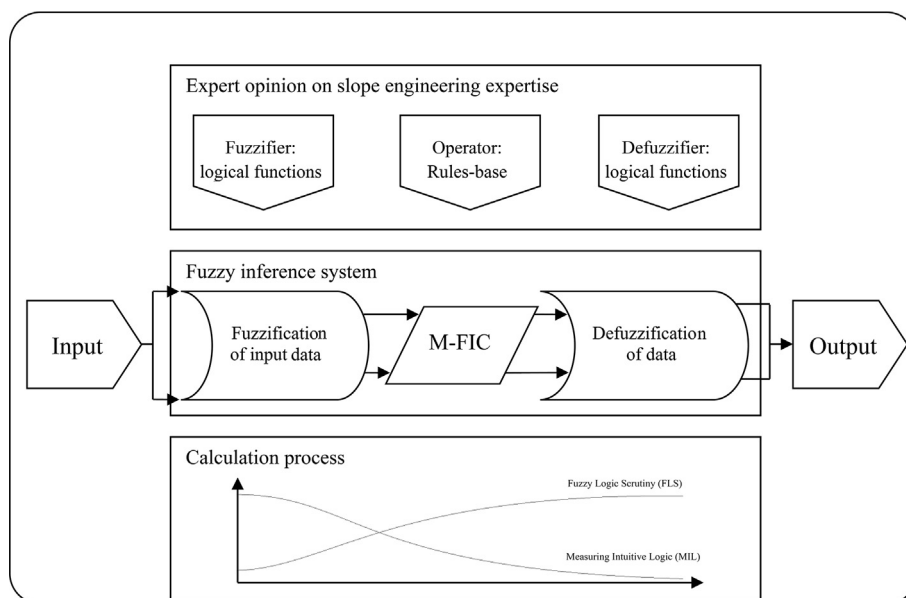


Figure 3. The work process of the fuzzy key-block analysis method.

these features, making an appropriate decision is very important which can cover more uncertainty in rock media and provide rapid response operations without the need for extensive quantitative stability evaluations based on the rock slope sustainability ratio.

**4. Key-block based fuzzy logical decision-making algorithm**

Discontinuous rock slopes have frequently been associated with uncertainties in regards to geometric characteristics which may lead to structural failures. The nature of these uncertainties is a significant issue and should be assessed by the analyst since uncertainty can be due to “fortuity”, “ambiguity”, “lack of sufficient knowledge”, or “inaccuracy and errors”. Thus, the application of approaches to reduce these uncertainties can lead to an appropriate decision in the least time. Although various methods have been proposed to increase the accuracy of the analysis and reduce ambiguity in rock slope stability assessments, there are still quite a few number of uncertainties that have not received convincing attention. There are logical relations between these uncertainties and the durability of the rock structures which are capable of

being used to evaluate the stability factor of the slope and to make the right decision in regards to rock slope stabilisation.

The key-block based fuzzy logical decision-making algorithm is presented to evaluate the degree of stability-instability contingency for discontinuous rock slopes with accurate, quick and appropriate decisions. This algorithm is implemented through the Mathematica software package which is accurately applied to wedge and planar slip scenarios. The advantages of the presented algorithm are quick analysis and a close approximation of discontinuous rock slope reliability which is capable of being used as a primary sustainability assessment. This algorithm has a potential for implementing rapid and correct critical decisions such as for assessing appropriate stabilisation techniques for massive constructions like tunnel portal excavation or road/railway slope cuts in discontinuous rock masses. The other advantages of the used algorithm are:

- Simple field reconnaissance requirement to evaluate the stability of slopes,
- Evaluation of the approximate factor of safety based on lower volume of input data,
- Classification of slope reliability based on the degree of stability-instability,
- Identification of the rock block type and detection of the key-block,
- Evaluation of the importance and prioritisation of slope failure triggering parameters,
- Use of a range of answers instead of one solution that leads to the consideration of more possibility in the analyses,
- Definition of an extensive set of qualitative or intuitive data as input,
- An opportunity for interpretation of the results and the incorporation of engineering judgements.

The presented algorithm uses a fuzzy inference decision core to optimise multi-criteria decision-making reliably from simple inputs obtained from a field survey to perform stability analyses of discontinuous rock slopes regardless of the failure mechanism. For this purpose, a number of assumptions are required which must be considered for a

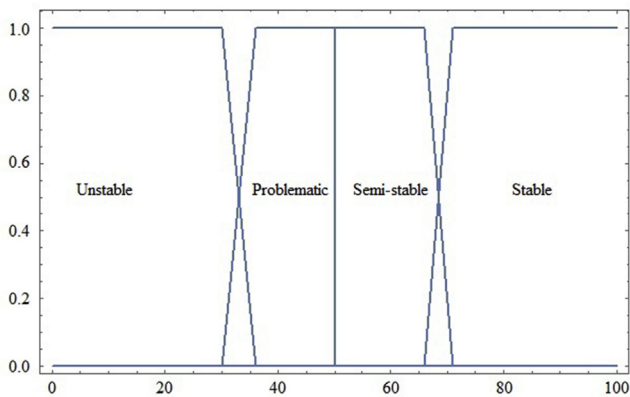


Figure 4. The Fuzzy outlet of the algorithm.

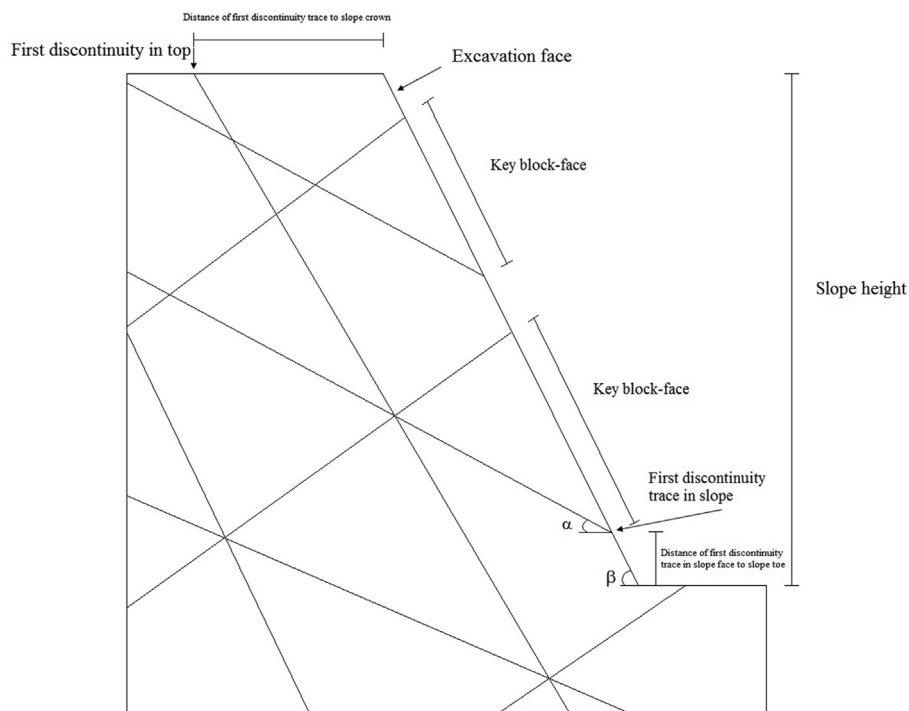


Figure 5. The basic input requirements of the algorithm.

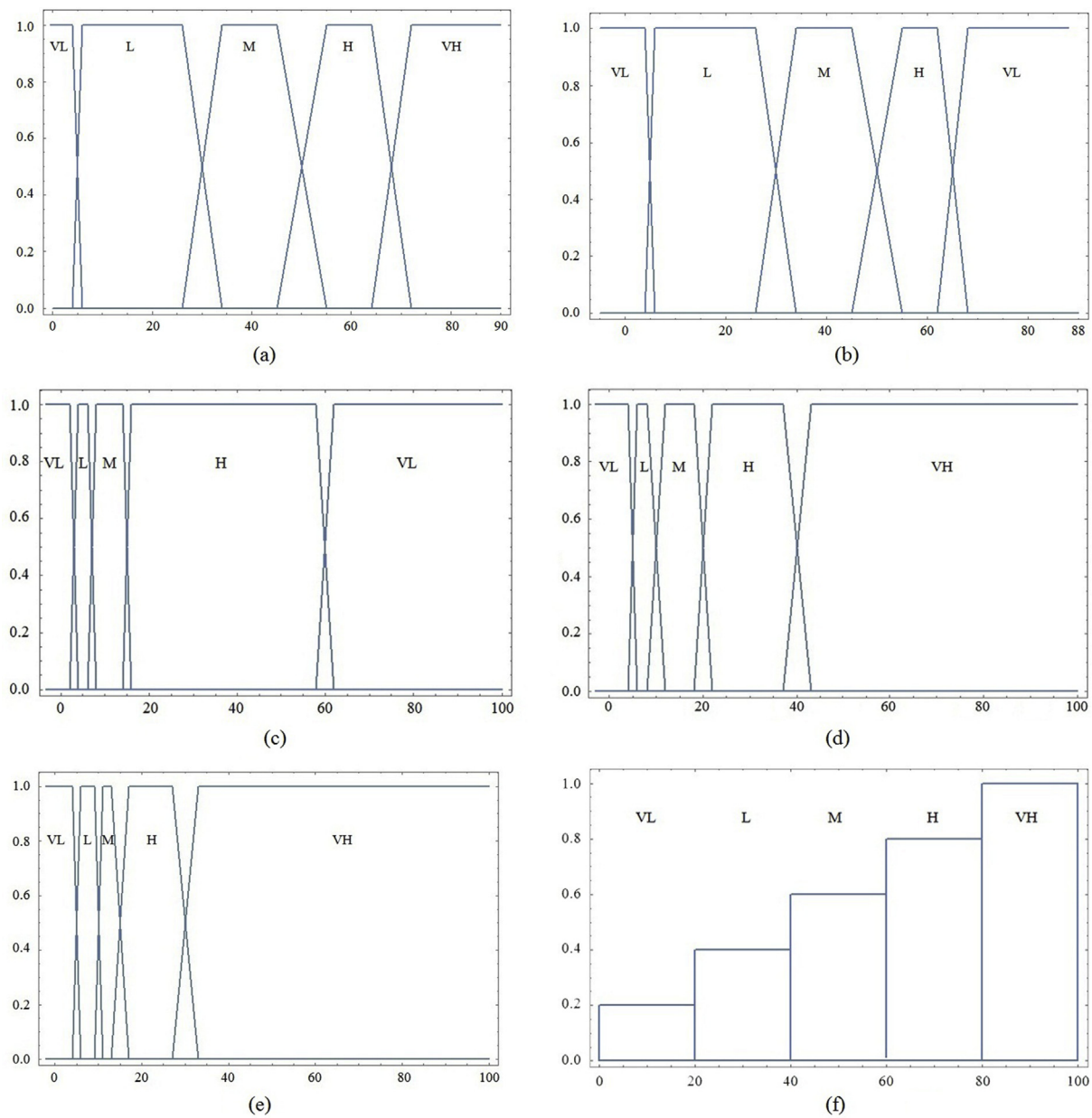


Figure 6. Membership function plot of the input parameters: (a) Slope-dip, (b) Joint-dip, (c) Excavation-face, (d) Key block-face, (e) Riskable-face, (f) Block-class.

Table 1. Ranks and intuitive terms for the consequences of the presented algorithm.

Variables	Riskability intuitive class (value)				
	Very low	Low	Moderate	High	Very high
Slope-dip (degree)	1–5	5–30	30–50	50–68	68–90
Joint-dip (degree)	1–5	5–30	30–50	50–65	65–88
Excavation-face (%)	0–3	3–7	7–15	15–60	60–100
Key block-face (%)	0–5	5–10	10–20	20–40	40–100
Riskable-face (%)	0–5	5–10	10–15	15–30	30–100
Block-class (definition)	Infinite	Finite	Finite	Potential	Key-block

proper algorithm application and hence, for a proper analysis. These assumptions are as follows:

- The principal assumptions of Block theory are valid,
- All discontinuities are considered planar and extend to the mass border,
- Blocks are created based on the discontinuity intersection and their geometry depends on the discontinuity emplacement in the rock mass,
- The blocks are considered rigid, and the internal deformation in the blocks is ignored,



- Displacement is assumed to occur in the direction of the aligned discontinuities (i.e., along the slope dip where interlocking failure is not allowed),
- The displacement is based on gravity under static conditions (i.e., under kinematic conditions).

The procedure is used in the simplified inputs and in the intuitive rules to evaluate slope stability for primary stabilisation. To this end, initially, the input parameters for the slopes are obtained and then are plotted by using the fuzzy inference decision core optimal stability status. The result is a stability-instability analysis number which represents the stability class and indirectly presents the safety factor (F.S). The scoring table of the method used is classified between 0 and 100, which shows the degree of stability-instability contingency for slopes as implemented through the Mathematica software package. If the score is estimated as 50, then the slope status is critical and the F.S is approximately 1.00. If the calculated score is lower than 50, then the slope condition is problematic and local instability in the slope body is observed where the F.S is approximately less than 1.00. On the other hand, if the estimated score is higher than 50, then the slope condition is mainly stable and the F.S is

greater than 1.00. The advantage of this method is the rapid answers it provides for slope stability by utilising optimised decision-making procedures.

The fuzzy logic is built on ambiguity and descriptive/inherent values which can contain a set of values (data lists) for the description instead of a single value which is expressed by fuzzy membership functions. In probabilistic or classical descriptions of F.S, the factor is limited as a number (1.00) which probabilistically must satisfy 99% probability of failure (Pf); however, in fuzzy description, it does not require an extensive statistical/probabilistic evaluation (which is an advantage of this method). The fuzzy-based method expresses the critical status (which is quantitatively introduced as a threshold limit or  $F.S = 1.00$ ) as a range of stability/instability variation. It covers balances between the rock blocks in discontinuous rock slopes as partial and spatial loss in limited locations which are capable of starting main failures in slopes. In conventional safety factors, attention is generally paid to the local failure conditions of the rock slope and it is attempted to provide a safety factor for the most likely probable surface of failure. This in reality it usually causes some operational errors. For example, in a road slope cut or mine, the domain may be generally stable but may seem partially unstable. The result

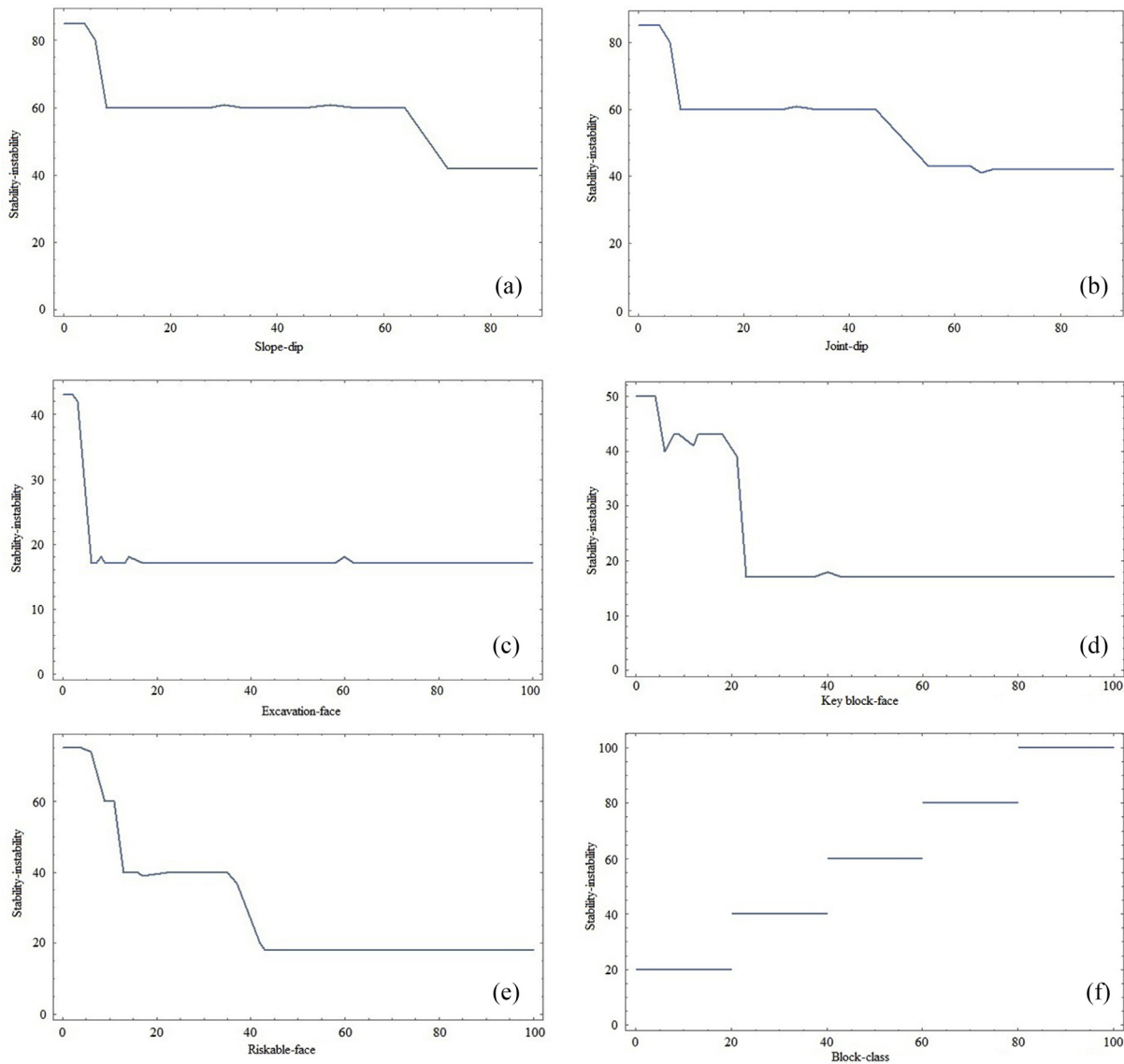


Figure 7. Graphical representation of the fuzzifier elements of the fuzzy key-block analysis method: (a) Slope-dip, (b) Joint-dip, (c) Excavation-face, (d) Key block-face, (e) Riskable-face, (f) Block-class.

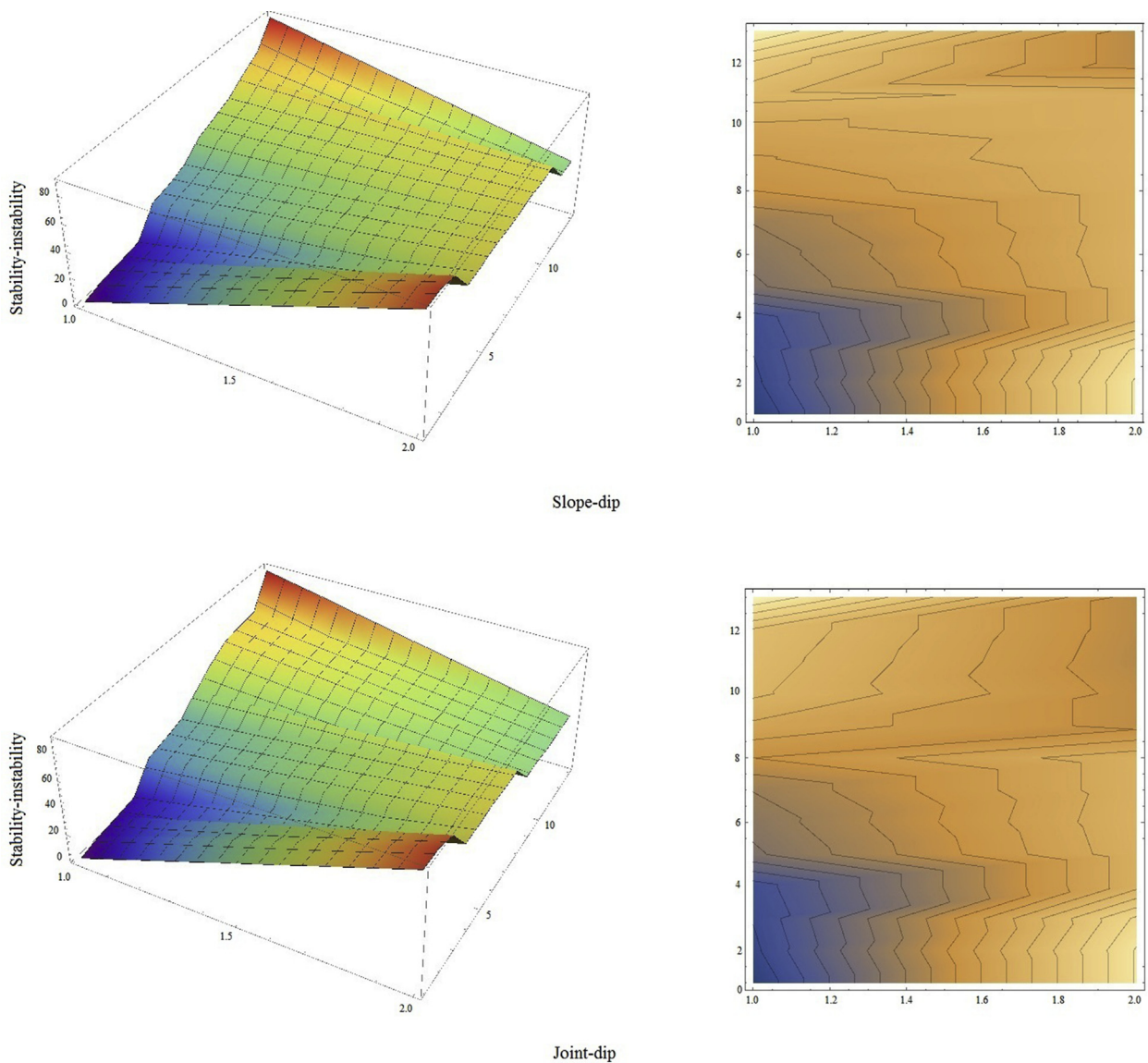
obtained by numerical and equilibrium approaches is in line with the domain instability. This description in traditional methodology is classified as unstable which is not entirely appropriate, because it cannot adequately describe the ground conditions. These errors in addition to increasing assessment time require correction by an expert geo-engineer through applying the presented method. The presented method uses fuzzy functions to describe the stability/instability conditions in a field survey which represents real terms of discontinuous rock slopes. Hence, the F.S is a description of condition terms for slope status. It should be noted that the nature of the F.S coefficient in this method differs from the conventional/classical approaches and represents a numerical quantity for defining a fuzzy decision-making series related to slip events in the domain.

**5. The algorithm modus operandi**

The fuzzy key-block analysis algorithm is developed for fast multi-objective decision-making of stability-instability analysis for discontinuous rock slopes under blocky structural sliding. This algorithm attempts to detect the degree of reliability and to evaluate the safety factor of slopes by a logical decision between stability-instability ambiguous

spectrums through multi-value geometrical characteristics (as input parameters). Given the fuzzy decision features, the intuitive rules based on input parameters and pursuant to block theory are then defined by employing a fuzzy inference operator which consists of membership functions to perform the input variables and to assess the response of the participation rate of the input parameters (e.g., slope-dip, joint-dip, excavation-face, key block-face, riskable-face rate and block-class) and to investigate the stability (degree of stability-instability contingency). The work process of fuzzy key-block analysis algorithm is shown in Figure 3. The algorithm outputs are based on the uncertainty definition related to discontinuous rock slope reliability and fuzzy stability-instability inference decision core. As presented in Figure 4, the algorithm is classified into expressive classes such as stable, semi-stable, problematic and unstable, which are described as follows:

- *Stable*: Refers to the rock mass state which does not require stabilisation ( $F.S < 1.5$ ),
- *Semi-stable*: In general, the rock mass is stable, but this does not imply that the stability should be ignored without any associated concern. So the stability condition could be revoked under various conditions



**Figure 8.** Graphical representation of 'Slope-dip' vs. 'Joint-dip' variations.

such as dynamic loading, seismic shock, degree of saturation, etc. ( $1.00 < F.S \leq 1.5$ ),

- **Critical state:** This state refers to the critical condition or static boundary. Under this condition, the resistance forces equal the mobilised forces, and the safety factor is 1.00 (i.e.,  $F.S = 1.00$ ),
- **Problematic:** This state refers to the existing instability and local failure in the slope. But these local failures are not prerequisites for hazardous global failure in the slope ( $0.95 \leq F.S < 1.00$ ),
- **Unstable:** Hazardous and risky state is considered for the slope and it needs to be stabilised ( $F.S < 0.95$ ).

As mentioned previously, the presented algorithm does not require many ground data which are presented in Figure 5. Hence, the primary inputs are rock slope geometrical parameters, including discontinuity network properties, slope topology, block classification parameters and key-block parameters, which can be obtained during the engineering geological field investigation (Priest, 1993; Hudson and Harrison, 1997). We define the components of each of the above factors as follows:

- Discontinuity network properties (i.e., dip amount and dip direction),
- Slope topology (i.e., slope height, slope surface length, excavation dip, distance from the first discontinuity trace to the slope crown, distance of the first discontinuity trace in the slope face to the slope toe),
- Block classification parameters (i.e., key-block, potential block, tapered block, finite block, infinite block),
- Key-block special parameters (i.e., slope contact surface, riskable-face to total excavation face ratio, removable face to total key-block face).

At first, by defining the Universal set ( $U = \{u_1, u_2, \dots, u_n\}$ ), all processes/output space is considered (Universal set =  $\{1, 2, \dots, 100\}$  in  $m \times m$  for a 2D analysis). Later, to create the fuzzy rules of the proposed procedure based on block theory regulation, the components in five classes are classified as “very low (VL)”, “low (L)”, “moderate (M)”, “high (H)” and “very high (VH)” which is presented in Figure 6. The Mamdani inference operator is utilised to implement the rules in a reasonable, realistic and efficient manner. These rules are classified into six main categories which are illustrated in Table 1:

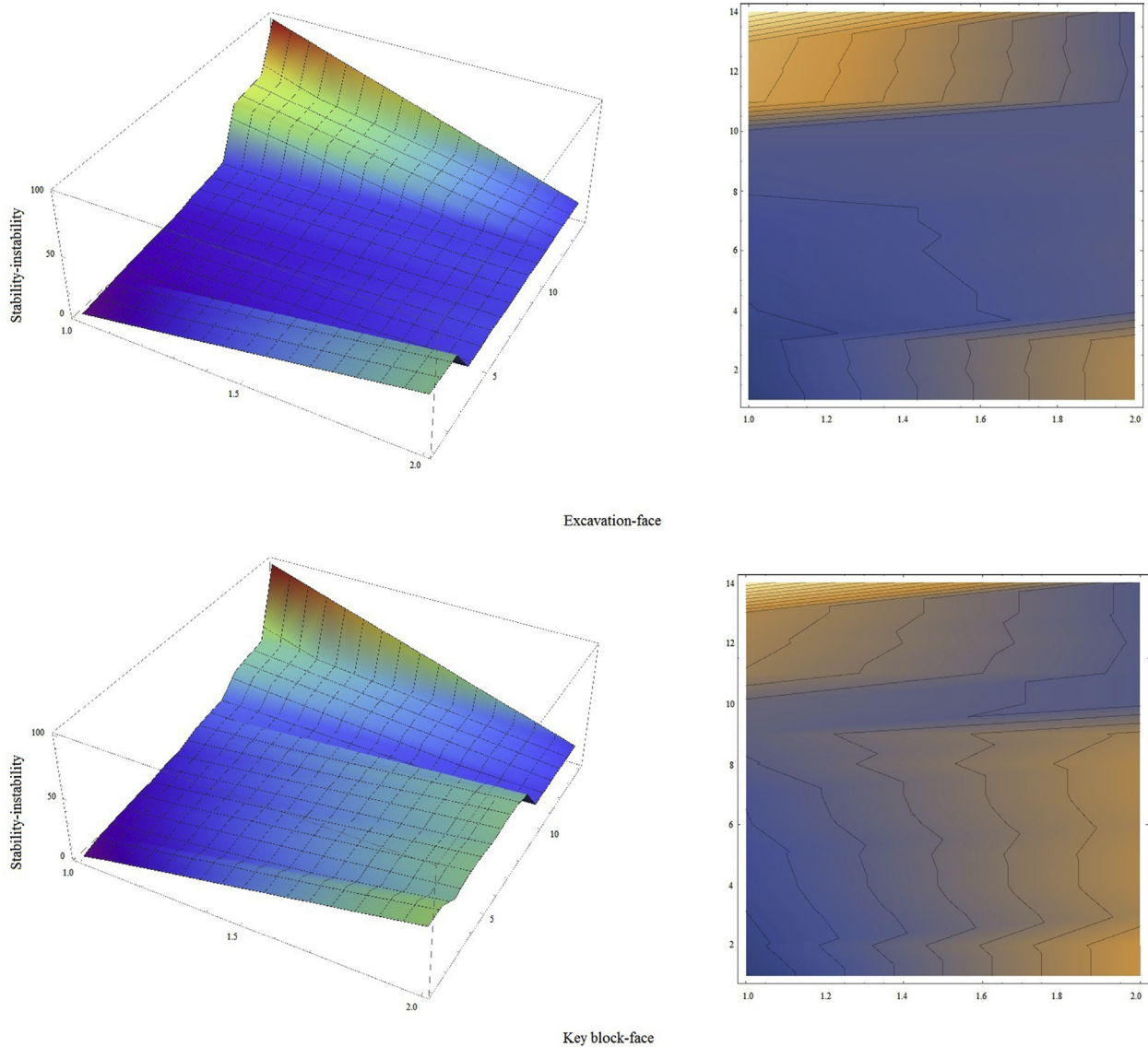


Figure 9. Graphical representation of ‘Excavation-face’ vs. ‘Key block-face’ variations.



- *Slope-dip*: Depends on the slope tilt changes ( $\beta$ ),
- *Joint-dip*: Describes the systematic discontinuity or main split of the alignment with the slope face (sliding surface),
- *Excavation-face*: Describes the discontinuity density in various rock block generations at the excavated surface,
- *Key block-face*: Describes the rate of the key-block area to the total excavation area,
- *Riskable-face*: Describes the rate of the hazardous area (potential unstable area) to the entire area of the slope considered,
- *Block-class*: Classifier related to Block theory (key-block, potential key-block, tapered block, etc.).

The evaluation process is divided into three stages, including; fuzzification, leading method and defuzzification. Figure 6 presents the fuzzifier values of input parameters, which are regularised by expert system identification. According to the degree of priority and the algorithm output, the discontinuous rock slope condition along with the support system and stabilisation methods may be decided upon.

The defined rules for fuzzy sets based on classifications which are stated above consist of 3175 rules that make up the “Basic Rules” of the key-block analysis method.

By using these rules, the logical inference core evaluates the input data and sorts the raw data into optimal decision-making results. Figures 7, 8, 9, 10, and 11 present the settlement process (defuzzifier) of fuzzifier elements which are used for multi-criteria decision-making and plot the sustainability analysis of discontinuous rock slopes under blocky failures.

### 6. Justification of the utilised method

To demonstrate the presented algorithm for actual conditions and to evaluate the application's accuracy, four cases of discontinuous rock slopes, namely, ‘Slope 1’, ‘Slope 2’, ‘Slope 3’ and ‘Slope 4’ were selected for justification. In addition, for comparing the capability of the algorithm process core, the SWedge (Rocscience, 2010a) and RocPlane (Rocscience, 2010b) softwares were considered for protocol assignments. It should be noted that wedge failure assessment is considered as a block sliding along the discontinuity intersection leading to an unstable key-block.

The studied slopes are located at the Kangan - Assalouyeh highway in the South Pars complex (SPC) region which consist of the weathered Aghajari formation (marlstones, limey marls). Figures 12 and 13 present the SPC location and a view of the slopes. To perform a stability analysis

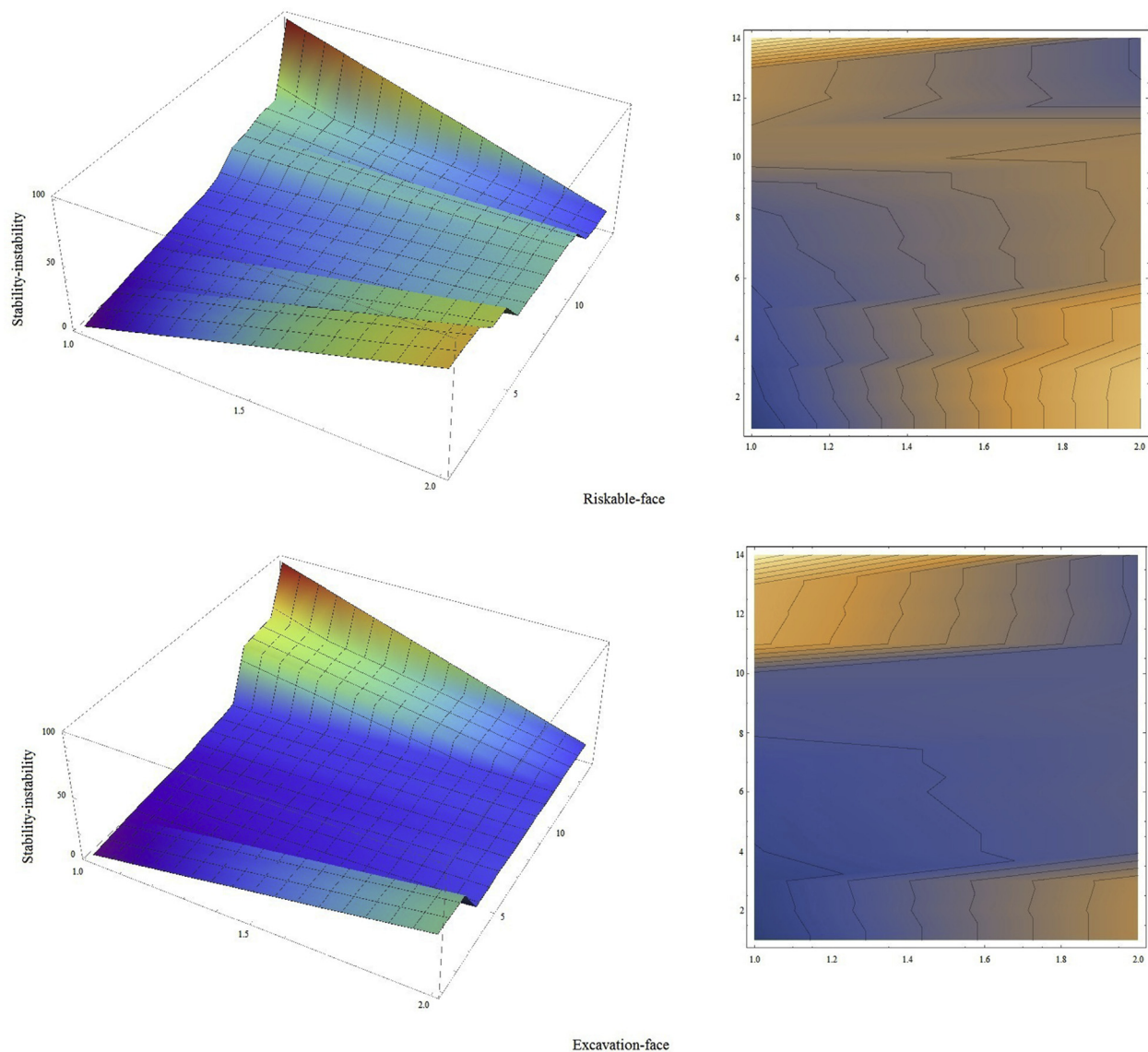


Figure 10. Graphical representation of ‘Riskable-face’ vs. ‘Excavation-face’ variations.

of these slopes, the required parameters were obtained from an engineering geological field survey and geotechnical investigations which are summarised in Table 2. The field survey was conducted to evaluate the geometrical characteristics of the rock slope discontinuity network and to obtain the discontinuity properties based on geomechanical principles (Hudson and Harrison, 1997). In this regard, the discontinuity parameters obtained from the scan-line survey that was performed for each of the studied slopes was entered into the DIPS software (Rocscience, 2010c) for a kinematic evaluation of the slopes, which is presented in Figure 14. According to the results reported in Figure 14, the main discontinuity emplacement in the rock masses were 3 systematic sets for ‘Slope 1’, 4 systematic sets for ‘Slope 2’ and 2 systematic sets for ‘Slopes 3–4’ with unsystematic (random) discontinuity sets. The geomechanical properties were obtained from in-situ and laboratory tests performed which included uniaxial compressive strength testing (ASTM D7012, 2014), point load index testing (ASTM D5731, 2016), direct-shear testing (ASTM D5607, 2016) and Schmidt rebound hammer testing (ASTM D5873, 2014). After obtaining the required parameters for the slopes, the algorithm was performed to determine the degree of stability-instability contingency and then, the results were compared with the results obtained from the SWedge and RocPlane softwares. In addition, for the assessment of the proposed algorithm’s weighted decision functions,

expert judgement that was prepared from a 25-member specialist team study as related to the SPC engineering department was utilised. The expert information was collected by means of a questionnaire and was sorted by the importance and the priority of the input parameters. A hierarchical matrix was provided with the parametric weighted method related to the algorithm as presented in Table 3. Table 4 shows the results of these analyses, which indicates a good agreement between the approaches. As seen in Table 4, safety factors ranging from 50 to 88 were obtained with the presented method. In the description, these values are associated with critical, semi-stable and stable conditions. On the other hand, the results of the conventional methods represent these conditions as stable and partially unstable which indicates that the method presented herein leads to a more conservative approach as compared to the kinematic assessment. The conservative results of this algorithm can be interpreted as taking into account the influence of the specific environmental conditions such as local slope failures, precipitation impact, dynamic loads, etc., which are ignored in static analysis. This is also quite natural given the base study which uses the key block method, because Block theory is principally a conservative method. According to the specialist’s opinions (expert judgement), the presented algorithm is highly effective. As known, the kinematic methodology presents the critical limit equilibrium status (F.S = 1.00) which represents a transition

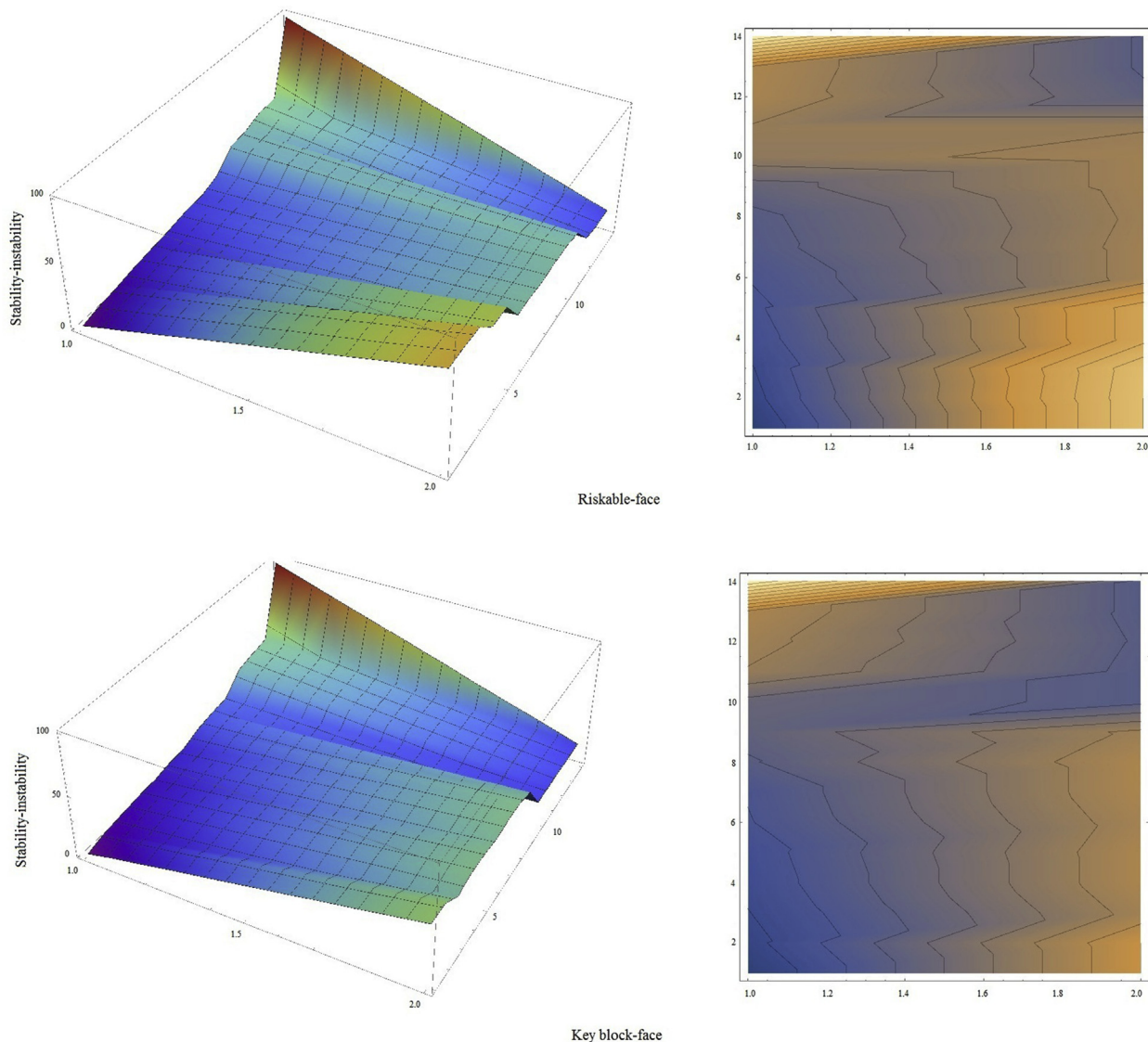


Figure 11. Graphical representation of ‘Riskable-face’ vs. ‘Key block-face’ variations.





Figure 12. The location map of the SPC in Iran.



Figure 13. A view of the rock slopes analysed in the case studies which are named as 'Slope 1', 'Slope 2', 'Slope 3' and 'Slope 4'.

**Table 2.** A summary of the field survey data for the studied cases.

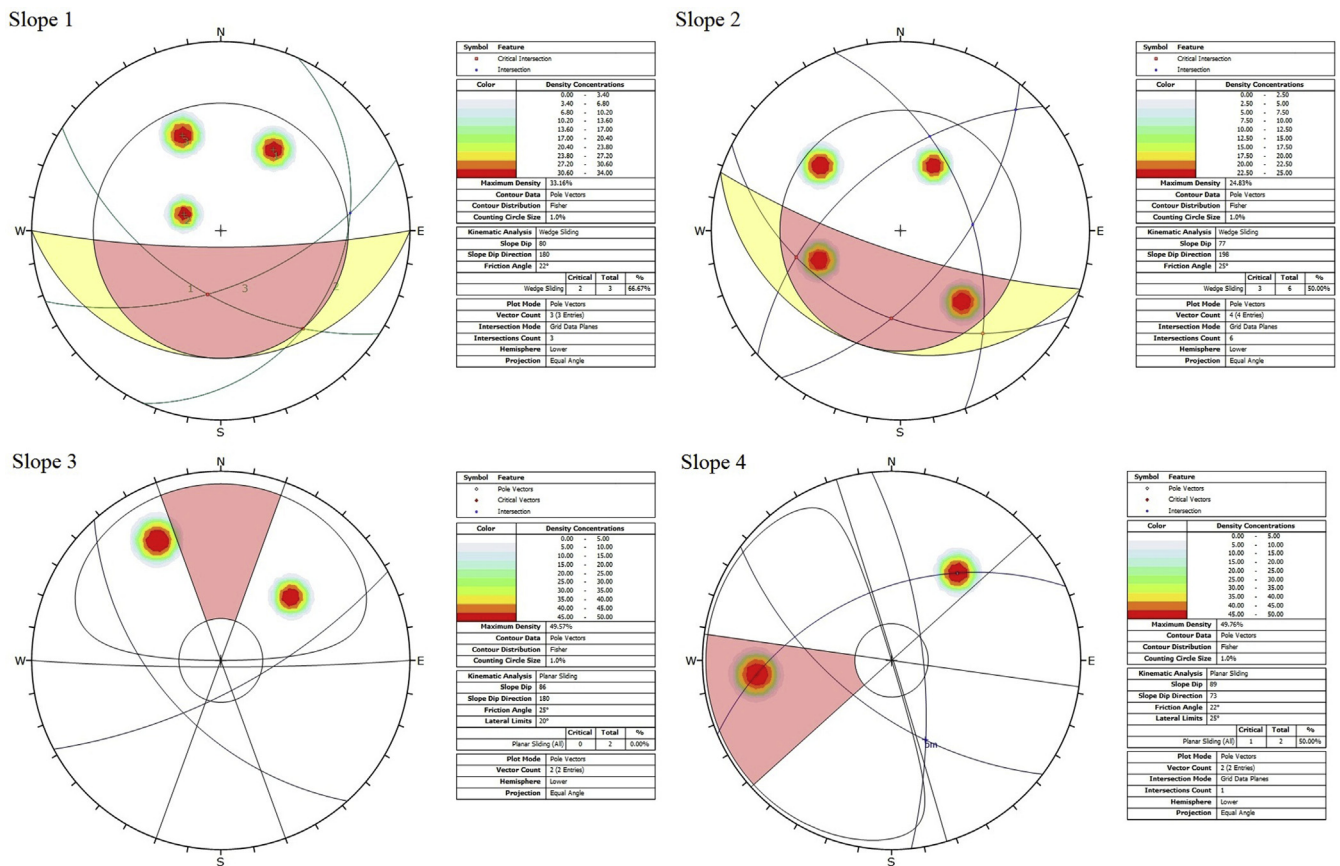
Parameters	Specifications	Cases			
		Slope 1	Slope 2	Slope 3	Slope 4
Topography	Slope height (m)	65	72	32	102
	Morphology	Natural	Trenches	Trenches	Natural
	Slope topology	Rough	Smooth	Smooth	Rough
Discontinuity network (dip direction)	Discontinuity set 1	54/213	42/207	71/152	60/217
	Discontinuity set 2	24/114	49/070	53/228	71/084
	Discontinuity set 3	57/158	53/319	-	-
	Discontinuity set 4	-	57/129	-	-
Algorithm requirements	Slope dip ( $\beta$ )	80°	77°	86°	89°
	Discontinuity dip ( $\alpha$ )	45°	65°	78°	68°
	Excavation face (m)	66	73.9	18	100
	DDC* (m)	27	1.2	15	5
	DDT** (m)	0.2	0.2	0.3	0.7
Geo-material property	UCS (MPa)	25	25	25	22
	Cohesion (kPa)	75	80	80	72
	Friction angle	22°	25°	25°	22°
Empirical engineering classification	RQD	65	63	70	47
	SMR	IIIa	IIIb	IIb	IVb
	Q-slope	Uncertain	Uncertain	Stable	Unstable

\* Distance between the first discontinuity and slope crown.

\*\* Distance between the first discontinuity trace in slope face.

from stable to unstable conditions and vice-versa which is considered as a ‘critical status’. The algorithm is capable of approximating the equivalent F.S as stable ( $1.5 < F.S$ ), semi-stable ( $1.00 < F.S < 1.5$ ), critical state ( $F.S$

= 1.00), problematic ( $0.90 < F.S < 1.00$ ) and unstable ( $F.S < 0.90$ ). Hence, the unstable status of the kinematic assessment is classified in the problematic and unstable categories which aid novice geo-engineers to



**Figure 14.** The DIPS results for discontinuity network projection (Note: each of the DIPS results outlines the studied slopes named as ‘Slope 1’, ‘Slope 2’, ‘Slope 3’ and ‘Slope 4’ as shown in Figure 13).



**Table 3.** The results of hierarchical parametric weighing.

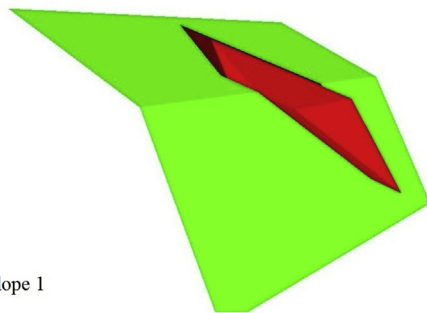
Parameter	Weighted value	Normalised	Variability	Fuzzy weighted
Slope height	0.135	0.429	0.463	0.446
Slope-dip	0.127	0.317	0.377	0.347
Joint-dip	0.134	0.558	0.592	0.575
Excavation-face	0.107	0.498	0.499	0.498
Key block-face	0.189	0.634	0.655	0.644
Riskable-face	0.122	0.780	0.785	0.782
Block-class	Discrete	Discrete	Discrete	Discrete
Distance between the first discontinuity and slope crown	0.085	0.471	0.492	0.481
Distance between the first discontinuity trace in slope face	0.101	0.523	0.563	0.543

**Table 4.** The results of the stability analysis and an evaluation of the studied cases (the cases are shown in Figure 13).

Case	Stability analysis	Factor of safety	Description
Slope 1	SWedge	1.39	Wedge is stable
	RocPlane	-	-
	Expert Judgement	1.52	Stable but some loss under saturation
	Presented algorithm	57	Semi-stable
Slope 2	SWedge	1.14	Wedge is stable
	RocPlane	-	-
	Expert Judgement	1.33	Stable but localised loss under saturation
	Presented algorithm	63	Semi-stable
Slope 3	SWedge	-	-
	RocPlane	2.18	Plane is stable
	Expert Judgement	2.50	Stable
	Presented algorithm	88	Stable
Slope 4	SWedge	-	-
	RocPlane	1.00	Partially unstable
	Expert Judgement	1.25	Partially unstable
	Presented algorithm	50	Critical

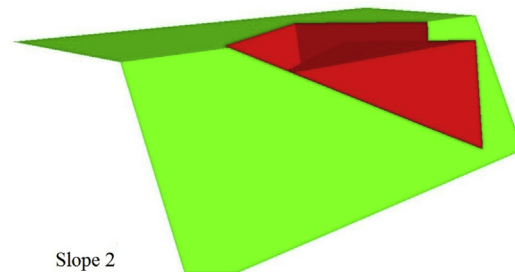
**Swedge Analysis**

Document name: Slope 01  
View: PRESPECTIVE  
Analysis type: Deterministic  
Safety Factor: 1.39369



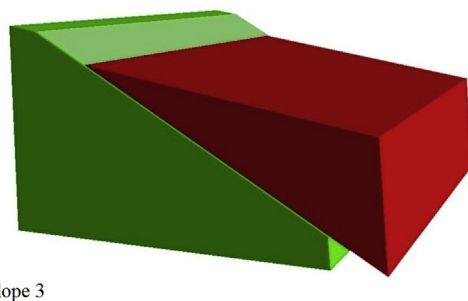
**Swedge Analysis**

Document name: Slope 02  
View: PRESPECTIVE  
Analysis type: Deterministic  
Safety Factor: 1.14703



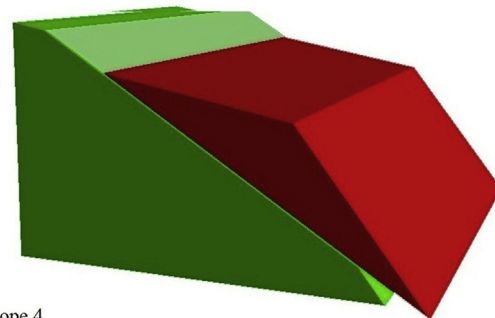
**RocPlane Analysis**

Document name: Plane 01  
View: PRESPECTIVE  
Analysis type: Deterministic  
Safety Factor: 2.88721



**RocPlane Analysis**

Document name: Plane 02  
View: PRESPECTIVE  
Analysis type: Deterministic  
Safety Factor: 1.008073



**Figure 15.** The results of the stability analysis for case studies based on the SWedge (Slopes 1–2) and RocPlane (Slopes 3–4) softwares.

separate critical conditions that involve local failure conditions. It should be noted that in classical limit equilibrium analysis, statistical analysis is usually not included. Thus, in estimating the reliability of slopes by these conventional methods, probabilistic expansions are not considered. The results in Table 3 imply that the presented algorithm for analysing critical condition is performed in a shorter time than those of the softwares and has a close approximation to those of the expert assessments. The focus of this study is a fast and accurate estimation of the reliability coefficient and slope condition description. Thus, by using the proposed algorithm, it is possible to analyse the stability and to obtain the slope conditions as well as to describe the slope status for early stabilisation decisions. Figure 15 illustrates the graphical results of the SWedge and RocPlane softwares. The graphical results represent the main failure mechanisms which are used for proper description related decision results. The main failure mechanism in Slope 1 is wedge failure which is estimated as stable in SWedge ( $F.S = 1.39$ ). On the other hand, expert state of the condition of the slope is stable but there is some loss under saturation which is not similar to the SWedge result. So, the appropriate method must cover both quantitative and qualitative tasks which are addressed appropriately by the proposed method. This fact will be valid for all slopes in this study and future tasks that are analysed by this proposed algorithm. This advantage can serve as a framework for all concepts of discontinuous rock slope stability and stabilisation works.

## 7. Conclusions

The application of methods that reduce or consider computational uncertainties and provide logical answers has always been one of the most important demands of engineering geologists and geotechnical engineers. The use of artificial intelligence in making logical decisions to identify such situations can be useful in the effective implementation of the stabilisation methods. The fuzzy-based key-block analysis algorithm by using fuzzy logic attempts to conveniently and intuitively estimate the stability-instability status (degree of stability-instability contingency) of discontinuous rock slopes under blocky structural failures based on wedge and planar slip scenarios. When the instability condition and the confidence degree of the rock slope are determined, it is easy to implement and apply a suitable stabilisation method. Regarding the control approaches, the presented algorithm has excellent compatibility and provides intuitive and experimental satisfaction. As compared to the existing analysis methods (SWedge and RocPlane softwares), the presented algorithm is relatively simple and requires low CPU time. In addition, the expert assignment methodology for weighted decision functions of the presented algorithm is properly satisfied. Another feature of the block theory related method is to reduce the weight of the engineering judgement on the primary stability analysis and to provide a default in choosing the stabilisation method so that it is possible to be used by students and less experienced people. Accordingly, logical results with proper accuracy and precision could be extracted from the presented algorithm.

## Declarations

### Author contribution statement

M. Azarafza, H. Akgün, M.R. Feizi-Derakhshi, M. Azarafza, J. Rahnamarad, R. Derakhshani: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

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## Competing interest statement

The authors declare no conflict of interest.

## Additional information

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