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A generalized class one static solution

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ABSTRACT

In literature, there are three simplest methods of solving Einstein's field equations, namely, (a) assuming conformally flat spacetime, (b) using conformal killing vector and (c) using Karmarkar conditions. In all these approaches the two metric functions g_{tt} and g_{rr} are link via a bridge. However, the first two approaches are facing a critical failure while determining central red-shift while the last method always yields well-behaved solution. Therefore, we are adopting the last method and discover a generalized class one solution. It is found that the maximum mass and radius of the compact star describe by the solution strongly depends on the parameter n. As n increases the maximum mass and radius also increases. For $n = 3.3$, $M_{max} = 1.459M_\odot$ and $R_{max} = 9.52$ km, and for $n = 4.8$ have $M_{max} = 1.766M_\odot$ with $R_{max} = 10.31$ km. For $n = 4.8$ the equation of state is behaving linearly as the speed of sound is almost constant at 0.333. In overall the presented solution is well-behaved in all respects.

1. Introduction

After the discovery of general theory of relativity by Albert Einstein in 1915, Schwarzschild [1] obtained the solution of Einstein's field equation that describes the neighborhood of a compact objects. The obtained solution was spherically symmetric and static with vanishing pressure and density. In later, by using isotropy pressure, Tolman [2] obtained the model of static and spherically symmetry compact object. When the pressure inside a compact object is not equal at each and every point then a situation occurs which is called the pressure anisotropy in the terms of astrophysics. Pressure anisotropy is the difference between transverse pressure and radial pressure and is denoted by Δ. Symbolically, $\Delta = p_t - p_r$. p_t and p_r are transverse and radial pressure respectively and these are the two components of the pressure and p_t acts in the orthogonal direction to p_r. The pressure anisotropy creates an force which is known as anisotropy force ($F_a = \frac{2\Delta}{r}$) and a positive anisotropy force (when $\Delta > 0$, i.e., $p_t > p_r$) protects the compact object from gravitational collapsing by creating a force towards the boundary as stated by Gokhroo & Mehra [3]. They explained the behavior of a neutron star using pressure anisotropy and conclude that the maximum mass of a neutron star does not exceed two solar masses, which confirms the Chandrasekhar limit. Pressure anisotropy can be found in the compact objects like, X-ray pulsar, Her-X-1, X-ray buster 4U 1820-30, the millisecond pulsar SAX J 1804.4-3658, PSR J1614-2230, LMC X-4 etc. whose density of the core exceeds the nuclear density (~10^{15} gm/cc). Ruderman [4] and Canuto [5] first proposed the idea of anisotropy. After that several works have been done by assuming pressure anisotropy with uniform density, Maharaj and Maartens [6] obtained a solution for anisotropic sphere though, most of the researchers proposed stellar model with variable density. From previous investigations found in literature it is proved that the pressure anisotropy is occurred by the presence of $3A$ superfluid [7], different kinds of phase transitions [8]. In the context of Newtonian gravity, anisotropies in spherical galaxies have been studied by Binney & Tremaine [9]. Weber [10] pointed out that strong magnetic fields can generate an anisotropic pressure component inside a compact sphere. The effects of slow rotation in a star was examined by Herrera & Santos [11]. Mixture of two gases such as ionized hydrogen and electrons also caused pressure anisotropy as proposed by Letelier [12].

To solve the EFEs, some researchers used the metric co-efficient along with radial pressure p_r [13, 14, 15], but a common approach is to choose metric coefficients along with equations of state (EoS) (a relation between the pressure and density). Several works can be found in these directions [16, 17, 18, 19, 20, 21, 22]. At that stage when the matter

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density inside the star is much greater than the usual nuclear density, it is really a very difficult task to predict a complete equation of state for the superdense objects. To overcome this situation some researchers used an alternative approach called core-envelope models [23, 24, 25]. To obtain a core-envelope model, a relativistic star is assumed to consist of a central core region surrounded by an outer envelope region, and the matter distribution in these regions exhibit different physical features.

If a space-time can be represented as a hypersurface embedded in a flat space of 5-dimensions then it is called the space-time of embedding class one. A necessary as well as sufficient condition which a space-time should satisfy to be of class one is that there exist a symmetric tensor b_{ij} satisfying the following conditions:

$$R_{\mu\nu\alpha\beta} = e(b_{\mu\alpha} b_{\nu\beta} - b_{\mu\beta} b_{\nu\alpha})$$

$$b_{\mu\nu;\alpha} - b_{\mu\alpha;\nu} = 0$$

where ';' represents covariant derivatives and e takes the value +1 or −1 according to the normal to the manifold being space-like or time-like. In case of embedding class one spacetime, the two metric co-efficients are correlated by an equation. In 2008, Herrera et al. [26] proposed an algorithm to obtain static spherically symmetric anisotropic solutions of Einstein's field equations and they showed that this can be generated from EFEs by two generating functions. So the main advantage of using embedding class one spacetime is that, if one chooses one metric coefficients the whole system can be generated by it (in case of uncharged fluid sphere). Instead two generating functions we need only one. The other advantage is that we can solve the system of highly non-linear EFEs without any help of any EoS. Both charged and uncharged compact star models in embedding class one spacetime can be found in [27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37]. Mirzazadeh and his co-authors have present various perturbation method of solving systems of equations [38, 39, 40, 41, 42, 43].

The purpose of the current investigation is to obtain a new solution of the Einstein field equations (EFEs) in presence of pressure anisotropy in embedding class one spacetime. We have studied in details the different physical properties of the present model and demonstrated the effect of dimensionless quantity n through the graphical representation of the model parameters. The paper is planned as follows: in next section we have discussed about the interior space-time and field equations, in sect. 3 the field equations are solved by choosing a physically reasonable metric coefficient for g_{rr}. The physical analysis of the model parameters are obtained in sect. 4, in the next section we matched our interior solution to the exterior solution to fix different constants and we have also obtained the values of the different constants for some well known compact objects in tabular form. The energy conditions and the stability analysis have been discussed in sect. 4 and finally some concluding remarks are given in sect. 5.

2. Theory

The interior space-time line element assumed as,

$$ds^2 = e^{\nu(r)} dt^2 - e^{\lambda(r)} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{1}$$

where ν and λ are functions of the radial coordinate 'r' only.

Further, assuming an anisotropic fluid distribution the Einstein's field equations can be written as

$$R^{\mu}_{\nu} - \frac{1}{2} g^{\mu}_{\nu} R = -8\pi \left[(p_t + \rho c^2) \upsilon^{\mu} \upsilon_{\nu} - p_t g^{\mu}_{\nu} + (p_r - p_t)\chi_{\nu}\chi^{\mu}\right] \tag{2}$$

where the symbols have their usual meanings.

For the space-time (1), the field equations reduces to

$$\frac{1-e^{-\lambda}}{r^2} + \frac{e^{-\lambda}\lambda'}{r} = 8\pi\rho \tag{3}$$

$$\frac{e^{-\lambda}-1}{r^2} + \frac{e^{-\lambda}\nu'}{r} = 8\pi p_r \tag{4}$$

$$e^{-\lambda}\left(\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\nu'\lambda'}{4} + \frac{\nu'-\lambda'}{2r}\right) = 8\pi p_t. \tag{5}$$

We have also defined that the measure of anisotropy $\Delta = 8\pi(p_t - p_r)$.

If the space-time (1) satisfies the Karmarkar [44]

$$R_{1414} R_{2323} = R_{1212} R_{3434} + R_{1224} R_{1334}. \tag{6}$$

then the two metric functions $\nu(r)$ and $\lambda(r)$ can be link via

$$\frac{\lambda'\nu'}{1-e^{\lambda}} = \lambda'\nu' - 2\nu'' - \nu'^2. \tag{7}$$

This kind of solutions of Einstein's field equations are referred as "embedding class one" i.e. they can be embedded in five dimensional pseudo-Euclidean space. However, the Pandey and Sharma condition [45] is needed to satisfy for a class one solution i.e. $R_{2323} \neq 0$.

On integration (7) we get

$$e^{\nu} = \left(A + B \int \sqrt{e^{\lambda} - 1}\, dr\right)^2 \tag{8}$$

where A and B are constants of integration.

By using (8) we can express the anisotropy as [46] as

$$\Delta = \frac{\nu'}{4e^{\lambda}}\left[\frac{2}{r} - \frac{\lambda'}{e^{\lambda}-1}\right]\left[\frac{\nu' e^{\nu}}{2rB^2} - 1\right]. \tag{9}$$

For isotropic case $\Delta = 0$ and there are three possible solutions when

$$\text{(a)}\ \nu' = 0\ ; \quad \text{(b)}\ \frac{2}{r} - \frac{\lambda'}{e^{\lambda}-1} = 0\ ; \quad \text{(c)}\ \frac{\nu' e^{\nu}}{2rB^2} - 1 = 0. \tag{10}$$

The solution [47] (a) leads to $e^{\nu} = C$ and $e^{\lambda} = 1$ and therefore a configuration with zero density is obtained, hence, not a physical solution. The solution of (b) yields the well known Schwarzschild interior solution. Again this solution is not physical as the density is constant leading to un-physical velocity of sound, adiabatic index etc. The last solution from (c) is also the well celebrated Kohler-Chao solution. This solution is only physical for cosmological model as the pressure vanishes at $r \to \infty$.

3. Calculations

To construct the model, we have assumed the generalized metric function [48] along g_{rr} as

$$e^{\lambda} = 1 + ar^2(1 + br^n)^m \tag{11}$$

where a and b are parameters related to central pressure and density, when n and m are kept as tuning parameters. On integrating (8) we get

$$e^{\nu} = \left(A + \frac{1}{2}\sqrt{a} B r^2 f(r)\right)^2 \tag{12}$$

where $f(r) = {}_2F_1\left[-\frac{m}{2}, \frac{2}{n}; \frac{n+2}{n}; -br^n\right]$. The behavior for metric functions are shown in Fig. 1.

Now we can find the density, radial and transverse pressure as

$$8\pi\rho(r) = \frac{a\,(br^n+1)^{m-1}}{\left[ar^2\,(br^n+1)^m + 1\right]^2}\Big[ar^2\,(br^n+1)^m + br^n \left\{ar^2\,(br^n+1)^m + mn + 3\right\} + 3\Big] \tag{13}$$

$$8\pi p_r(r) = \frac{\sqrt{a}\,(br^n+1)^m}{ar^2\,(br^n+1)^m + 1}\Big[4B - 2A\sqrt{a\,(br^n+1)^m} - aBr^2\,(br^n+1)^{m/2} f(r)\Big]\Big[Brf(r)\sqrt{ar^2\,(br^n+1)^m} + 2A\,(br^n+1)^{m/2}\Big]^{-1} \tag{14}$$

$$\Delta = \frac{2ar^2\,(br^n+1)^m - br^n\left[mn - 2ar^2\,(br^n+1)^m\right]}{2r\,(br^n+1)^{1-\frac{m}{2}}\left[ar^2\,(br^n+1)^m + 1\right]^2}\Big[2aAr\,(br^n+1)^m + Bf(r)\,(br^n+1)^{-\frac{m}{2}}$$

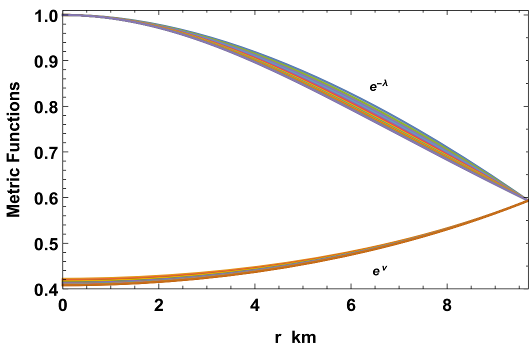

Fig. 1. Variation of metric functions with r for PSR J1614-2230 assuming $M = 1.97M_\odot$, $R = 9.69$ km and $b = 0.01^{n-1}$ in the range $3.3 < n < 5$.

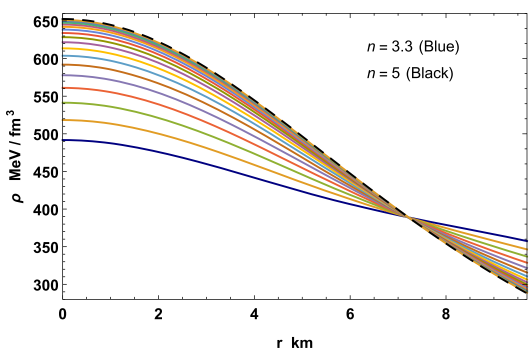

Fig. 2. Variation of density with r for PSR J1614-2230 assuming $M = 1.97M_\odot$, $R = 9.69$ km and $b = 0.01^{n-1}$ in the range $3.3 < n < 5$.

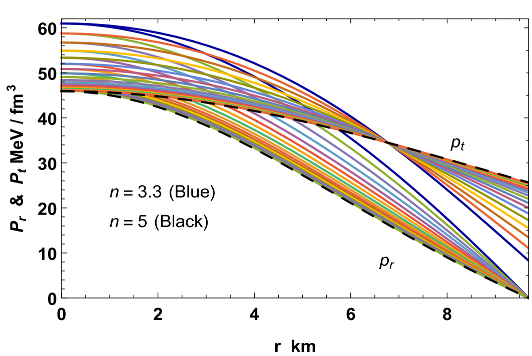

Fig. 3. Variation of pressures with r for PSR J1614-2230 assuming $M = 1.97M_\odot$, $R = 9.69$ km and $b = 0.01^{n-1}$ in the range $3.3 < n < 5$.

$$\left\{ar^2\left(br^n+1\right)^m\right\}^{3/2} - 2B\sqrt{ar^2\left(br^n+1\right)^m}\Big]$$

$$\left[Brf(r)\sqrt{ar^2\left(br^n+1\right)^m} + \left(br^n+1\right)^{m/2}2A\right]^{-1} \tag{15}$$

$$8\pi p_t(r) = 8\pi p_r + \Delta. \tag{16}$$

The trends of the above physical quantities are given in Figs. 2, 3 and 4.

The mass, compactness parameter, equation of state parameter (EOSP) and red-shift can be determined as

$$\mu(r) = 4\pi \int r^2\rho\, dr = \frac{ar^3\left(br^n+1\right)^m}{2ar^2\left(br^n+1\right)^m+2} \tag{17}$$

$$u(r) = \frac{2\mu(r)}{r} = \frac{ar^2\left(br^n+1\right)^m}{ar^2\left(br^n+1\right)^m+1} \tag{18}$$

$$\omega_r = \frac{p_r}{\rho} \;\; ; \;\; \omega_t = \frac{p_t}{\rho} \tag{19}$$

$$z(r) = e^{-\nu/2} - 1. \tag{20}$$

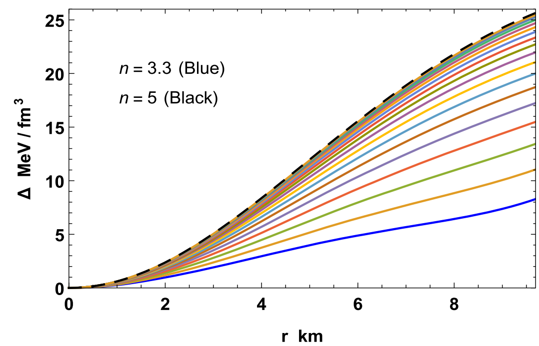

Fig. 4. Variation of anisotropy with r for PSR J1614-2230 assuming $M = 1.97M_\odot$, $R = 9.69$ km and $b = 0.01^{n-1}$ in the range $3.3 < n < 5$.

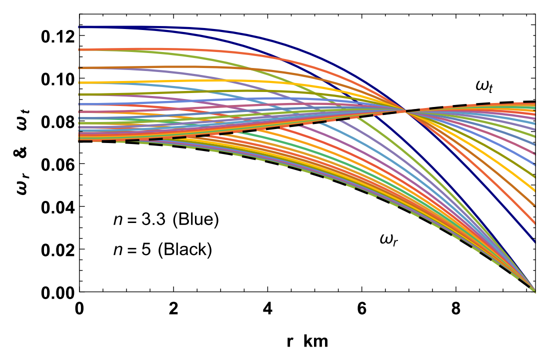

Fig. 5. Variation of equation of state parameters with r for PSR J1614-2230 assuming $M = 1.97M_\odot$, $R = 9.69$ km and $b = 0.01^{n-1}$ in the range $3.3 < n < 5$.

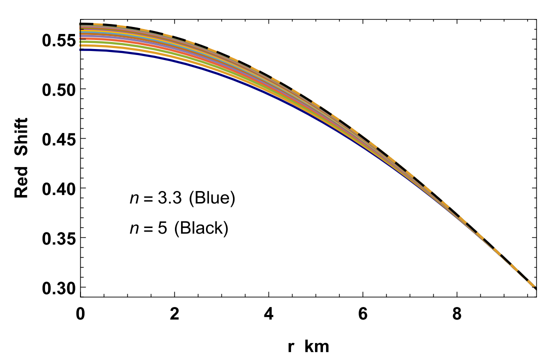

Fig. 6. Variation of red-shift with r for PSR J1614-2230 assuming $M = 1.97M_\odot$, $R = 9.69$ km and $b = 0.01^{n-1}$ in the range $3.3 < n < 5$.

For a realistic equation of state, the equation state parameters must be less than unity. The nature of EOSP and red-shift are shown in Figs. 5 and 6.

4. Analysis

For a physical solution, the central values of density, pressure etc. must be finite. The central density and pressure can be found as

$$\rho_c = 3a > 0, \tag{21}$$

$$p_{rc} = p_{tc} = \frac{\sqrt{a}\left(2B - \sqrt{a}A\right)}{A} > 0. \tag{22}$$

The Zeldovich's criterion i.e. $p_{rc}/\rho_c \le 1$ is also needed so that a solution can represent physical matters. For the solution it is given as

$$\frac{p_{rc}}{\rho_c} = \frac{2B - \sqrt{a}A}{3\sqrt{a}A} \le 1. \tag{23}$$

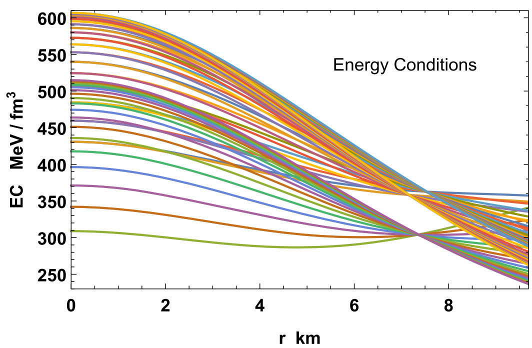

Fig. 7. Variation of energy conditions with r for PSR J1614-2230 assuming $M = 1.97M_\odot$, $R = 9.69$ km and $b = 0.01^{n-1}$ in the range $3.3 < n < 5$.

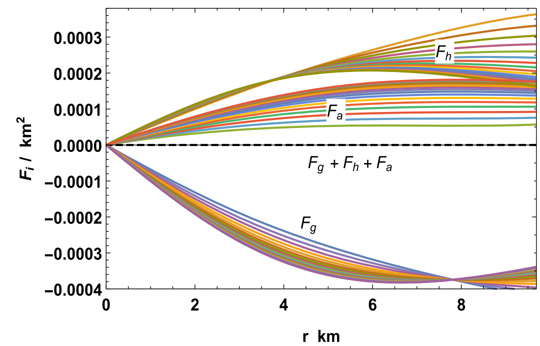

Fig. 8. Variation of various forces with r for PSR J1614-2230 assuming $M = 1.97M_\odot$, $R = 9.69$ km and $b = 0.01^{n-1}$ in the range $3.3 < n < 5$.

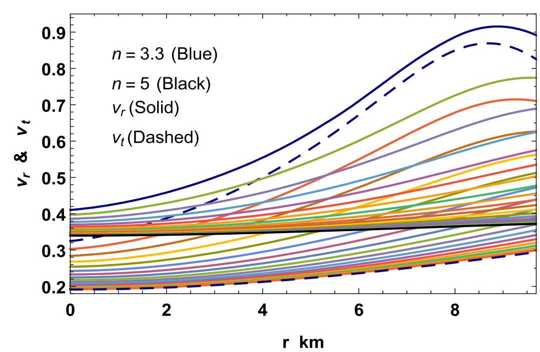

Fig. 9. Variation of sound speeds with r for PSR J1614-2230 assuming $M = 1.97M_\odot$, $R = 9.69$ km and $b = 0.01^{n-1}$ in the range $3.3 < n < 5$.

A constraint arises on B/A due to (22) and (23) as

$$\frac{\sqrt{2}}{a} < \frac{B}{A} \leq 2\sqrt{a}. \tag{24}$$

Assuming the exterior spacetime to be *Schwarzschild*'s which is given as

$$ds^2 = \left(1 - \frac{2\mu}{r}\right) dt^2 - \left(1 - \frac{2\mu}{r}\right)^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right). \tag{25}$$

The continuity of the metric coefficients at the boundary $r = R$ yields

$$1 - \frac{2M}{R} = e^{\nu_s} = e^{-\lambda_s} \tag{26}$$

$$p_r(r = R) = 0 \tag{27}$$

that leads to

$$a = \frac{2M\,(bR^n + 1)^{-m}}{R^2(R - 2M)} \tag{28}$$

$$A = \sqrt{1 - \frac{2M}{R}} - \frac{1}{2}\sqrt{a}BR^2 f(R) \tag{29}$$

$$B = \frac{1}{2}\sqrt{a\,(bR^n + 1)^m}\sqrt{1 - \frac{2M}{R}}. \tag{30}$$

The parameters M and R are mass and radius of the chosen compact star while b is kept as free parameter.

Satisfaction of energy conditions is needed for all physically possible matters. The energy conditions are null energy condition (NEC), dominant energy condition (DEC), strong energy condition (SEC) and weak energy condition (WEC). These are represented as below:

$$\text{WEC}: T_{\mu\nu}t^\mu t^\nu \geq 0 \text{ or } \rho \geq 0,\ \rho - p_i \geq 0 \tag{31}$$

$$\text{NEC}: T_{\mu\nu}l^\mu l^\nu \geq 0 \text{ or } \rho - p_i \geq 0 \tag{32}$$

$$\text{DEC}: T_{\mu\nu}t^\mu t^\nu \geq 0 \text{ or } \rho \geq |p_i| \tag{33}$$

where $T^{\mu\nu}t_\mu \in$ nonspace-like vector

$$\text{SEC}: T_{\mu\nu}t^\mu t^\nu - \frac{1}{2}T^\lambda_\lambda t^\sigma t_\sigma \geq 0 \text{ or } \rho - \sum_i p_i \geq 0, \tag{34}$$

where $i \equiv (radial\ r, transverse\ t)$, t^μ and l^μ are time-like vector and null vector respectively. Fig. 7 verify the satisfaction of energy conditions.

The generalized Tolman-Oppenheimer-Volkoff (TOV) equation determine whether a relativistic stellar system is in equilibrium or not. Mathematically, it is given by

$$-\frac{M_g(r)(\rho + p_r)}{r}e^{(\nu-\lambda)/2} - \frac{dp_r}{dr} + \frac{2}{r}(p_t - p_r) = 0, \tag{35}$$

where $M_g(r)$ is the gravitational mass and is calculated using the Tolman-Whittaker formula and the Einstein field equations. The expression is given as

$$M_g(r) = 4\pi \int_0^r \left(T^t_t - T^r_r - T^\theta_\theta - T^\phi_\phi\right) r^2 e^{(\nu+\lambda)/2} dr. \tag{36}$$

For the Eqs. (3)-(5), the above Eq. (36) reduced to

$$M_g(r) = \frac{1}{2} r e^{(\lambda-\nu)/2}\ \nu'. \tag{37}$$

On using the expression of $M_g(r)$ in (35), we get

$$-\frac{\nu'}{2}(\rho + p_r) - \frac{dp_r}{dr} + \frac{2}{r}(p_t - p_r) = 0 \tag{38}$$

which as also be written as

$$F_g + F_h + F_a = 0, \tag{39}$$

where F_g, F_h and F_a are the gravitational, hydrostatics and anisotropic forces respectively i.e.

$$F_g = -\frac{\nu'}{2}(\rho + p_r) \tag{40}$$

$$F_h = -\frac{dp_r}{dr} \tag{41}$$

$$F_a = \frac{2\Delta}{r}. \tag{42}$$

Fig. 8 easily convinced that the solution is in equilibrium.

Since the general relativity obeyed the causation solution i.e. the sound velocity must be subliminal which is also linked with stability of the system. The radial velocity (v_r^2) and transverse velocity (v_t^2) of sound are determined as

$$v_r^2 = \frac{dp_r}{d\rho}, \quad v_t^2 = \frac{dp_t}{d\rho}. \tag{43}$$

Fig. 9 verify the subliminal sound speed at the interior. The solution also satisfy the Abreu's stability criterion [49] i.e. $0 \leq |v_t^2 - v_r^2| \leq 1$ (see Fig. 10).

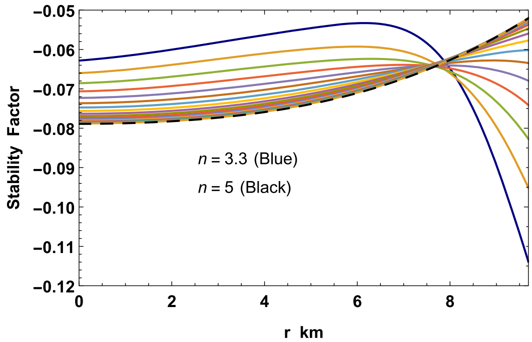

Fig. 10. Variation of stability factor with r for PSR J1614-2230 assuming $M = 1.97M_\odot$, $R = 9.69$ km and $b = 0.01^{n-1}$ in the range $3.3 < n < 5$.

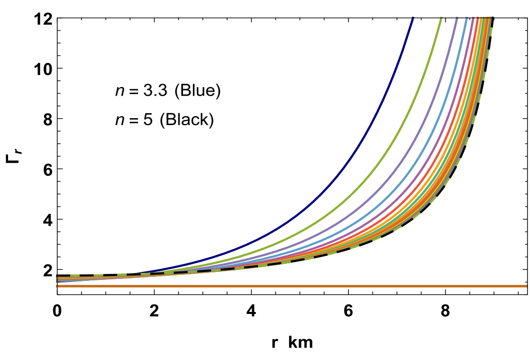

Fig. 11. Variation of stability factor with r for PSR J1614-2230 assuming $M = 1.97M_\odot$, $R = 9.69$ km and $b = 0.01^{n-1}$ in the range $3.3 < n < 5$.

The adiabatic index for an anisotropic fluid distribution is defined as [50],

$$\Gamma_r = \frac{\rho + p_r}{p_r}\frac{dp_r}{d\rho}. \tag{44}$$

The Bondi condition [51] $\Gamma_r > 4/3$ gives a stable Newtonian system whereas $\Gamma = 4/3$ being the condition for a neutral equilibrium. This condition is partially valid for anisotropic case since it depends on nature of anisotropy. This condition modifies to [50],

$$\Gamma > \frac{4}{3} + \left[\frac{4}{3}\frac{(p_{ti} - p_{ri})}{|p'_{ri}|r} + \frac{8\pi}{3}\frac{\rho_i p_{ri}}{|p'_{ri}|}r\right]_{max}, \tag{45}$$

where, p_{ri}, p_{ti}, and ρ_i are the initial radial, tangential pressures and energy density in static equilibrium. Within the square bracket, first term given the anisotropic modification and last term is relativistic correction Γ [50, 52]. If the anisotropy is positive, then a system with $\Gamma_r > 4/3$ may not be in stable and vice versa. For this solution the adiabatic index is more than 4/3 including the extra correcting terms (see Fig. 11) and therefore stable.

The stability analysis proposed by Harrison et al. [53] and Zeldovich-Novikov [54] are much simpler than the Chandrasekhar's method. The static stability criterion states that any stellar system are in stable configuration only if the mass increasing with central density i.e. $d\mu/d\rho_c > 0$ and unstable if $d\mu/d\rho_c < 0$.

For the solution the mass as a function of central density is given below:

$$\mu(\rho_c) = \frac{4\pi\rho_c R^3 (bR^n + 1)^m}{8\pi\rho_c R^2 (bR^n + 1)^m + 3} \tag{46}$$

$$\frac{\partial\mu(\rho_c)}{\partial\rho_c} = \frac{12\pi R^3 (bR^n + 1)^m}{\left[8\pi\rho_c R^2 (bR^n + 1)^m + 3\right]^2} > 0 \tag{47}$$

This solution also fulfilled the static stability criterion (see Fig. 12).

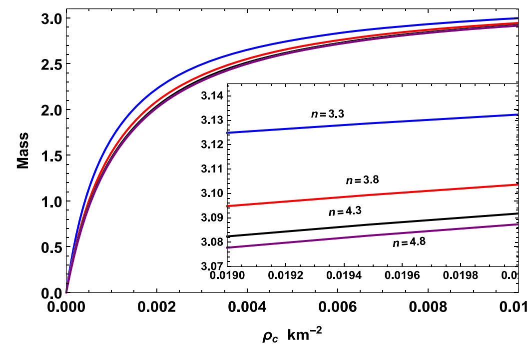

Fig. 12. Variation of stability factor with r for PSR J1614-2230 assuming $M = 1.97M_\odot$, $R = 9.69$ km and $b = 0.01^{n-1}$ in the range $3.3 < n < 5$.

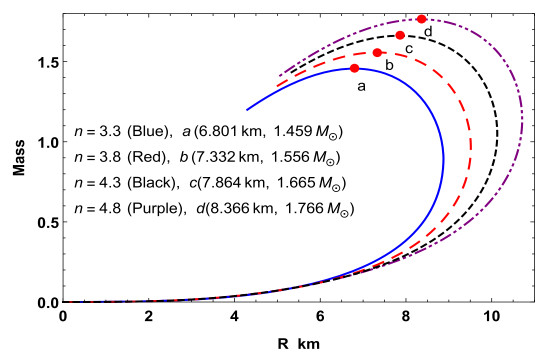

Fig. 13. Variation of mass with R assuming $a = 0.004$ and $b = 0.01^{n-1}$ for the range $3.3 < n < 5$.

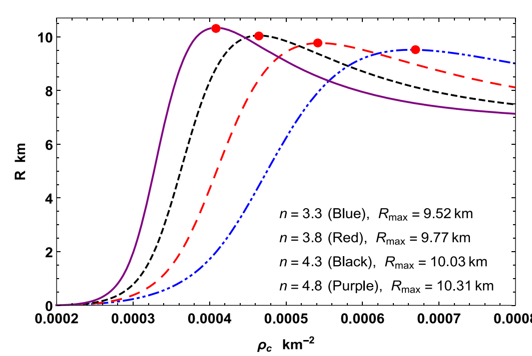

Fig. 14. Variation of radius with ρ_c assuming $b = 0.01^{n-1}$ for the range $3.3 < n < 5$.

5. Discussion & conclusion

We have explored a generalized class one solution satisfying field equations which can be embedded in five dimensional pseudo-Euclidean space. The solution is analyzed rigorously through various physical constraints and shows physically possible only for $3.3 \le n \le 5$ and $6.6 \le m \le 10$. Any solutions outside these ranges either violates of the causality condition or un-physical sound speed. This solution within the above mentioned range satisfy causality condition, equilibrium (TOV-equation), static and stable (stable static criterion). The maximum mass yield from the solution depends on the parameter n. For $n = 3.3$ can attain $M_{max} = 1.459M_\odot$ with $R_{max} = 9.52$ km and $n = 4.8$ gives $M_{max} = 1.766M_\odot$ with $R_{max} = 10.31$ km, Figs. 13 and 14. This implies that the equation of state is slightly stiff for $n = 4.8$ as compare to $n = 3.3$ since the adiabatic index is slightly higher in the first case. Again, from the behavior of sound speed (Fig. 9) it can be seen that for $n = 4.8$ the radial sound speed v_r is almost constant at about 0.33 equivalent similar to constant sound speed in MIT-bag model. This may

Table 1
Variation of central density, central pressure and surface density with parameter n choosing $m = 2n$.

n	ρ_c ($\times 10^{14}$ g/cc)	p_{rc} ($\times 10^{34}$ dyne/cm^2)	ρ_s ($\times 10^{14}$ g/cc)
3.3	8.766	9.766	6.372
3.6	10.002	8.802	5.858
3.9	10.076	8.148	5.537
4.2	11.203	7.754	5.343
4.5	11.448	7.530	5.228
4.8	11.582	7.407	5.162
5.0	11.634	7.358	5.134

signify that the equation of state for $n = 4.8$ is similar with MIT-bag model i.e. the compact star is more likely composed of quarks. It can also observe that as the parameter n increases the central density increases however the central pressure and surface density decreases (see Table 1).

It may be noted that there are three simplest approaches while solving field equations where the metric functions are link via a bridge equation. The first method is the embedding class problem where the bridge equation is Karmarkar condition i.e.

$$e^{\nu} = \left(A + B \int \sqrt{e^{\lambda} - 1}\, dr \right)^2. \tag{48}$$

Second method is based on conformally flat geometry adopted by Ivanov [55] where the bridge equation is

$$e^{\nu} = C_1^2 r^2 \cosh^2 \left(\int \frac{e^{\lambda/2} dr}{r} + C \right) \tag{49}$$

where C_1 and C are constants of integration. This method yields a critical failure in finding the variation of red-shift at the interior especially at $r = 0$. The third method is using conformal killing vector. Here the bridge equation is [56, 57, 58, 59]

$$e^{\nu} = c^2 r^2 \exp \left(-\frac{2k}{B} \int \frac{e^{\lambda/2}}{r} dr \right) \tag{50}$$

where A, B, C and k are constants. We can see that the second and third method are very much similar except one is with $\cosh^2$ and other with exponential function. Again, the third method have the similar problem in finding central red-shift. However, the first method i.e. the class one category always yield physically possible solutions and used by many authors (see refs. in sec. 1) provided the solution must incorporate either anisotropy or charge or both. The property of class one space-time is that the four dimensional space-time is embedded in five dimensional pseudo-Euclidean hyperspace. In other words, a curved four dimensional space-time behaves as flat space in five dimensions. This method simplifies to solve the field equations which can be obtained physically inspired solutions.

Declarations

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Author contribution statement

Ksh. Newton Singh: Analyzed and interpreted the data; Contributed analysis tools or data; Wrote the paper.

Piyali Bhar: Conceived and designed the analysis.

Modhuchandra Laishram: Contributed analysis tools or data.

Farook Rahaman: Analyzed and interpreted the data.

Competing interest statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

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