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The equivalence of minimum entropy production and maximum thermal efficiency in endoreversible heat engines

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Abstract

The objective of this study is to investigate the thermal efficiency and power production of typical models of endoreversible heat engines at the regime of minimum entropy generation rate. The study considers the Curzon-Ahlborn engine, the Novikov's engine, and the Carnot vapor cycle. The operational regimes at maximum thermal efficiency, maximum power output and minimum entropy production rate are compared for each of these engines. The results reveal that in an endoreversible heat engine, a reduction in entropy production corresponds to an increase in thermal efficiency. The three criteria of minimum entropy production, the maximum thermal efficiency, and the maximum power may become equivalent at the condition of fixed heat input.

Keyword: Engineering

1. Introduction

The Kelvin-Planck statement of the Second Law of Thermodynamics says that it is impossible to construct a heat engine to receive heat and to convert it completely into work. However, it does not indicate how much of heat is convertible into a useful form of energy; i.e. work. The first mathematical formulation of the second law is credited to Clausius, who showed that in a reversible Carnot cycle operating

with an ideal gas Q_L/Q_H equals T_L/T_H , where Q_H is the amount of heat transfer from a high temperature reservoir maintained at temperature T_H to the cycle, and Q_L is the amount of heat rejected from the cycle to a low temperature reservoir whose temperature is T_L . This conclusion led him to invent the thermodynamic property *Entropy* defined as $S = Q/T$. Clausius implied that if the Carnot engine is the most efficient engine among all engines operating between the same thermal reservoirs, Q_L/Q_H would be greater than the temperature ratio T_L/T_H in other types of engine, hence expressing his well-known inequality; i.e., $\oint dQ/T \leq 0$. Clausius therefore concluded that the real heat engines would result in production of entropy.

It is natural to question whether there is any relationship between the entropy produced by a heat engine and its thermal efficiency or power output. In 1975, Leff and Jones [1] discussed by means of an analytical argument that an increase in the thermal efficiency of an irreversible heat engine would not necessarily result in a decrease in its entropy production. Salamon et al. [2] showed that the maximum work and the minimum entropy production in heat engines might become equivalent under certain design conditions. Haseli et al. [3] analyzed entropy production of an integrated gas turbine and solid oxide fuel cell by accounting for the component inefficiencies. Their results showed that the entropy generation rate of the hybrid cycle did not correlate with the cycle efficiency.

Numerous articles have appeared in the literature claiming a direct relationship between the entropy produced by a power producing system with its thermal efficiency. However, there is a limited number of articles [1, 4, 5] which investigate the possibility of correlation between the entropy production and the thermal efficiency of a heat engine. In some cases, a relation between maximum work production and minimum entropy has been observed [4, 6, 7, 8]. Haseli [4] showed that in *irreversible* Otto, Diesel and Brayton cycles, minimum entropy production neither correlates with maximum thermal efficiency design nor with maximum work output criterion. An earlier attempt on this subject had revealed a consistent relation between the entropy generation and the thermal efficiency of an *endoreversible* Brayton cycle operating with or without a regenerative heat exchanger [5].

The objective of the present article is to further investigate the equivalence of maximum thermal efficiency and minimum entropy production in typical endoreversible engines including the models of Curzon-Ahlborn [9], Novikov [10], and Carnot vapor cycle. It is aimed to show that the thermal efficiency of a heat engine may correlate with the entropy generation associated with the operation of that engine if the engine is endoreversible. The thermal efficiency and power output of the above mentioned engines at the condition of minimum entropy generation rate will also be examined. Unlike many articles appeared in the

literature which interpret entropy production as a measure of lost work, it will be shown that this notion is invalid.

2. Analysis

The Curzon-Ahlborn engine [9] is depicted in Fig. 1 on a temperature-specific entropy (T-s) diagram. It is a Carnot engine which experiences external irreversibility due to finite heat exchange between the engine and the hot and cold thermal reservoirs. Entropy is produced due to the finite time heat exchange between the endoreversible engine (the green rectangle in Fig. 1) and the thermal reservoirs. Hence, the rate of entropy generation is

$$\dot{S}_{Gen} = \left(\frac{\dot{Q}_H}{T_{EH}} - \frac{\dot{Q}_H}{T_H} \right) + \left(\frac{\dot{Q}_L}{T_L} - \frac{\dot{Q}_L}{T_{EL}} \right) \tag{1}$$

where \dot{Q}_H is the heat rate received by the engine from the high temperature thermal reservoir, and \dot{Q}_L is the heat rate rejected by the engine to the low temperature thermal reservoir.

$$\dot{Q}_H = K_h(T_H - T_{EH}) \tag{2}$$

$$\dot{Q}_L = K_l(T_{EL} - T_L) \tag{3}$$

K_h and K_l are thermal conductances (assumed to be constant) at hot-end and cold-end sides, respectively. Also, T_{EH} and T_{EL} denote the highest and the lowest temperatures of the engine; see Fig. 1.

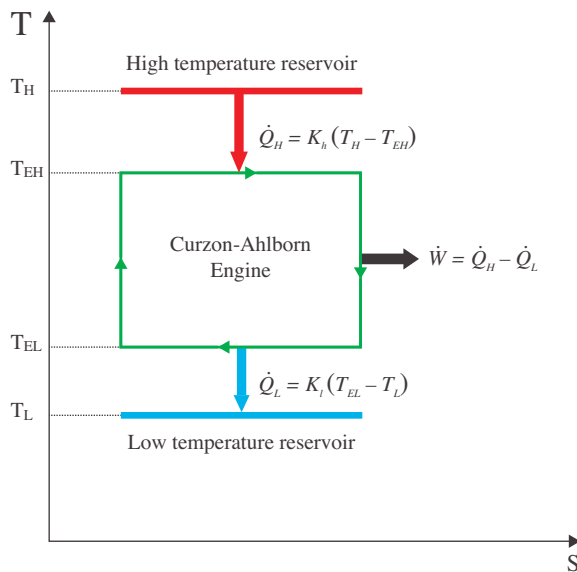


Fig. 1. The heat engine model of Curzon-Ahlborn on a T-s diagram.

As the engine is endoreversible, we conclude that

$$\frac{T_{EL}}{T_{EH}} = \frac{\dot{Q}_L}{\dot{Q}_H} = \frac{K_l(T_{EL} - T_L)}{K_h(T_H - T_{EH})} \tag{4}$$

Solving for T_{EL} gives

$$T_{EL} = \frac{T_{EH}T_LK_l}{T_{EH}(K_h + K_l) - T_HK_h} \tag{5}$$

Using Eq. (4), Eq. (1) reduces to

$$\dot{S}_{Gen} = \frac{\dot{Q}_L}{T_L} - \frac{\dot{Q}_H}{T_H} \tag{6}$$

A combination of Eqs. (2), (3), (5) and (6) allows us to express Eq. (6), after some algebra, as

$$\dot{S}_{Gen} = K_h(T_H - T_{EH}) \left[\frac{1}{\left(1 + \frac{K_h}{K_l}\right)T_{EH} - \frac{K_h}{K_l}T_H} - \frac{1}{T_H} \right] \tag{7}$$

Solving $\partial\dot{S}_{Gen}/\partial T_{EH} = 0$ leads to $(T_{EH})_{opt} = T_H$. Substituting this result into Eq. (5), we also find $(T_{EL})_{opt} = T_L$. The minimization of the entropy generation rate associated with Curzon-Ahlborn model suggests that any irreversibility between the thermal reservoirs and the engine should be removed, which will consequently lead to zero *finite* time power. On the other hand, the power produced by the engine is $\dot{W} = \dot{Q}_H - \dot{Q}_L$. Using Eqs. (2), (3) and (5), it can be shown that

$$\dot{W} = K_h(T_H - T_{EH}) - K_l \left[\frac{T_{EH}T_LK_l}{T_{EH}(K_h + K_l) - T_HK_h} - T_L \right] \tag{8}$$

Applying $\partial\dot{W}/\partial T_{EH} = 0$ leads to the optimum value of T_{EH} for maximizing the power.

$$(T_{EH})_{opt} = \frac{\sqrt{T_H T_L K_l} + T_H K_h}{K_l + K_h} \tag{9}$$

The corresponding maximum power production of the engine is given by

$$\dot{W}_{max} = \frac{K_l K_h}{K_l + K_h} (\sqrt{T_H} - \sqrt{T_L})^2 \tag{10}$$

Curzon and Ahlborn [9] showed that the engine efficiency at maximum power is $(\eta_{CA})_{\dot{W}_{max}} = 1 - \sqrt{T_L/T_H}$, where the subscript CA refers to Curzon-Ahlborn engine. To find out whether there is any relationship between the entropy production of the Curzon-Ahlborn cycle and its thermal efficiency, we represent Eq. (7) in a dimensionless form by dividing it by K_l .

$$S^* = r_K(r_T - T_{EH}^*) \left[\frac{1}{(1 + r_K)T_{EH}^* - r_T r_K} - \frac{1}{r_T} \right] \tag{11}$$

where

$$S^* = \frac{\dot{S}_{Gen}}{K_l}; \quad r_K = \frac{K_h}{K_l}; \quad r_T = \frac{T_H}{T_L}; \quad T_{EH}^* = \frac{T_{EH}}{T_L}$$

Using Eq. (5), the efficiency of the engine may be represented as

$$\eta_{CA} = 1 - \frac{T_{EL}}{T_{EH}} = 1 - \frac{1}{T_{EH}^*(r_K + 1) - r_T r_K} \tag{12}$$

We may also rewrite Eq. (8) in a normalized form as

$$W^* = 1 + r_K(r_T - T_{EH}^*) - \frac{T_{EH}^*}{T_{EH}^*(r_K + 1) - r_T r_K} \tag{13}$$

where $W^* = \dot{W}/(K_l T_L)$.

Fig. 2 shows the variation of the thermal efficiency, the normalized power output and the normalized entropy production of the Curzon-Ahlborn engine with T_{EH}^* for typical values of $r_T = 6$ and $r_K = 2$. It is seen that the entropy production monotonically decreases by increasing T_{EH}^* , whereas the thermal efficiency increases. On the other hand, the power output peaks at $T_{EH}^* = 4.82$. Note that as $T_{EH}^* \rightarrow r_T$, the efficiency approaches η_C ; i.e., the Carnot efficiency, and the rate of entropy produced by the engine approaches zero. However, at this condition, the power output of the engine reaches zero. From this analysis, we conclude that for the Curzon-Ahlborn engine, minimization of the entropy production rate is equivalent to maximization of the thermal efficiency, but not to maximization of power output.

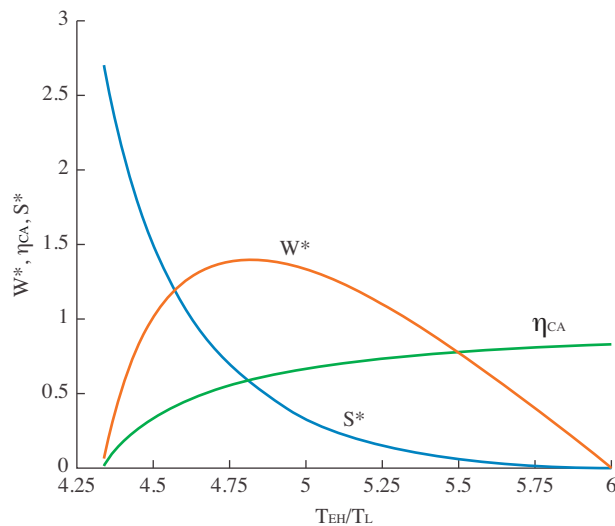


Fig. 2. Variation of the thermal efficiency, normalized power output and normalized entropy production rate of the Curzon-Ahlborn engine with $T_{EH}^* = T_{EH}/T_L$ ($r_T = 6$, $r_K = 2$).

Note that the varying parameter in our thermodynamic optimization is T_{EH}^* , and we assumed r_K is constant. One may treat T_{EH}^* as a fixed parameter and optimize the system by varying r_K . This is also a possible situation; however, our conclusion mentioned in the previous paragraph is still correct. In other words, even with varying r_K and fixed values of T_{EH}^* and r_T , it can be shown with a similar analysis presented above that minimum entropy production correlates with only maximum thermal efficiency, not with maximum power output. This is graphically demonstrated in Fig. 3.

The Novikov engine [10] is depicted on a T-s diagram in Fig. 4. In this model, the temperature at the cold-end side of the engine is the same as the low temperature reservoir's temperature, T_L . In other words, it is a Carnot engine which experiences external irreversibility due to finite heat exchange between the engine and the hot thermal reservoir. As the engine is internally reversible, its efficiency is $\eta_N = 1 - T_L/T_{EH}$, and the power produced by the engine is $\dot{W} = \dot{Q}_H(1 - T_L/T_{EH})$. Using Eq. (2), we find

$$\dot{W} = K_h(T_H - T_{EH}) \left(1 - \frac{T_L}{T_{EH}} \right) \tag{14}$$

Assuming a constant K_h and fixed T_H and T_L , Eq. (14) has an optimal value with respect to T_{EH} . Applying $\partial \dot{W} / \partial T_{EH} = 0$ yields an equation whose solution gives

$$(T_{EH})_{opt} = \sqrt{T_H T_L} \tag{15}$$

Eq. (15) allows us to find the efficiency and the power output of the Novikov engine at maximum power condition.

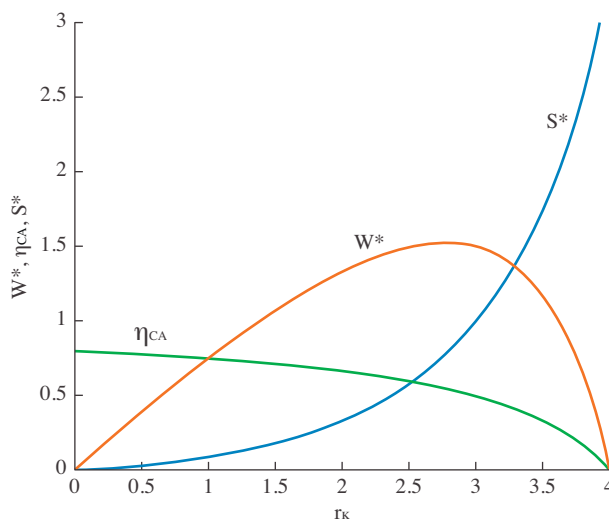


Fig. 3. Variation of the thermal efficiency, normalized power output and normalized entropy production rate of the Curzon-Ahlborn engine with r_K ($r_T = 6$, $T_{EH}^* = 5$).

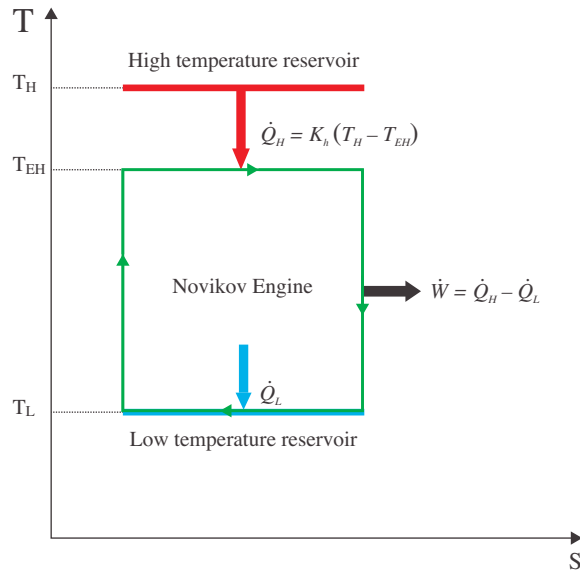


Fig. 4. The heat engine model of Novikov on a T-s diagram.

$$(\eta_N)\dot{W}_{\max} = 1 - \sqrt{\frac{T_L}{T_H}} \tag{16}$$

$$\dot{W}_{\max} = K_h(\sqrt{T_H} - \sqrt{T_L})^2 \tag{17}$$

An interesting observation is that the efficiencies of the engine models shown in Fig. 1 and Fig. 4 at maximum power output are the same. However, comparing Eqs. (17) and (10), one may notice that the maximum power produced by the Novikov engine is larger than that of the Curzon-Ahlborn engine. This is because at the condition of maximum power, the highest temperature of the engine, T_{EH} , of the Curzon-Ahlborn engine is higher than that of the Novikov engine; compare Eqs. (9) and (15), meaning that the input heat requirement of the former engine is less than that of the latter one. As the efficiency of both engines at maximum power is the same, it can be implied that the maximum power of the Curzon-Ahlborn engine is less than that of the Novikov engine.

In the next step, we calculate the entropy generation rate associated with the operation of the Novikov’s model.

$$\dot{S}_{Gen} = \dot{Q}_H \left(\frac{1}{T_{EH}} - \frac{1}{T_H} \right) \tag{18}$$

Notice that the cold-end side temperature of the engine is the same as the low temperature thermal reservoir’s temperature. Inserting Eq. (2) into Eq. (18) yields

$$\dot{S}_{Gen} = K_h(T_H - T_{EH}) \left(\frac{1}{T_{EH}} - \frac{1}{T_H} \right) \tag{19}$$

Solving $\partial \dot{S}_{Gen} / \partial T_{EH} = 0$ leads to $(T_{EH})_{opt} = T_H$. This result reveals that the minimum entropy generation rate takes place when $T_{EH} \rightarrow T_H$, which would give a zero finite time power.

Let us now consider a modified model of Novikov, in which the finite time heat exchange only takes place at the cold-end side of the engine (see Fig. 5). Thus, the efficiency and the power output of the engine are given by $\eta_{MN} = 1 - T_{EL}/T_H$ and $\dot{W} = \dot{Q}_H \eta_{MN} = (\dot{W} + \dot{Q}_L)(1 - T_{EL}/T_H)$, respectively. Note that the subscript “MN” denotes *modified Novikov*. Using Eq. (3), we have

$$\dot{W} = K_l \left(1 - \frac{T_L}{T_{EL}} \right) (T_H - T_{EL}) \tag{20}$$

Maximization of the power output given in Eq. (20) with respect to T_{EL} yields $(T_{EL})_{opt} = \sqrt{T_H T_L}$, which is the same as $(T_{EH})_{opt}$ of the Novikov engine. The efficiency and the power output of the modified Novikov engine at maximum power production are obtained by

$$(\eta_{MN})_{\dot{W}_{max}} = 1 - \sqrt{\frac{T_L}{T_H}} \tag{21}$$

$$\dot{W}_{max} = K_l (\sqrt{T_H} - \sqrt{T_L})^2 \tag{22}$$

Comparing Eqs. (22) and (10), it can be inferred that when the thermal conductance at the hot-end side of the Curzon-Ahlborn model tends to infinity $K_h \rightarrow \infty$, the model of Curzon-Ahlborn reduces to the modified engine model of Novikov. A further important observation is that the efficiency of all three engines that we have examined so far is $1 - \sqrt{T_L/T_H}$ at maximum power output, whereas

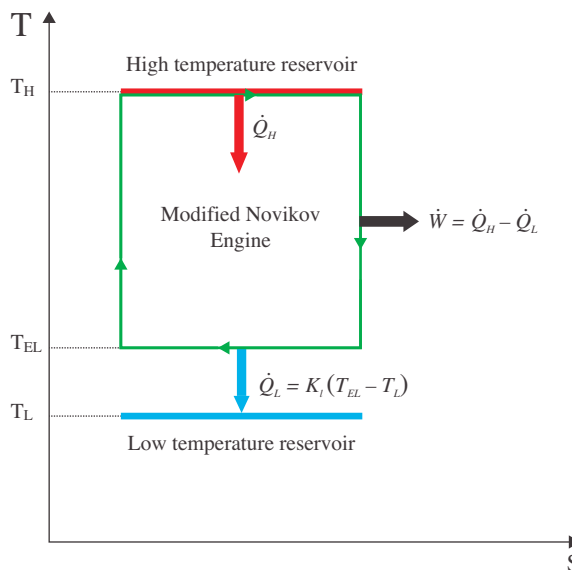


Fig. 5. Modified Novikov engine on a T-s diagram.

the maximum power of the Novikov engine is the highest, and that of the modified Novikov engine is the lowest, and that of the Curzon-Ahlborn engine is in between.

To evaluate the production rate of entropy, we note that there is merely one source of entropy generation at the cold-end side of the engine due to the finite time heat exchange. The heat transfer at the hot-end side of the engine takes place reversibly. So, $\dot{S}_{Gen} = \dot{Q}_L(1/T_L - 1/T_{EL})$ and using Eq. (2), we have

$$\dot{S}_{Gen} = K_I(T_{EL} - T_L) \left(\frac{1}{T_L} - \frac{1}{T_{EL}} \right) \tag{23}$$

Solving $\partial \dot{S}_{Gen} / \partial T_{EL} = 0$ results in $(T_{EL})_{opt} = T_L$, and substituting it into Eq. (20) leads to $\dot{W} = 0$. Therefore, we conclude that the minimum entropy generation and maximum power output are two different operational regimes in the modified Novikov engine model.

Similar to the case of Curzon-Ahlborn model, the relationship between the entropy production and the thermal efficiency is graphically demonstrated in Fig. 6 for the

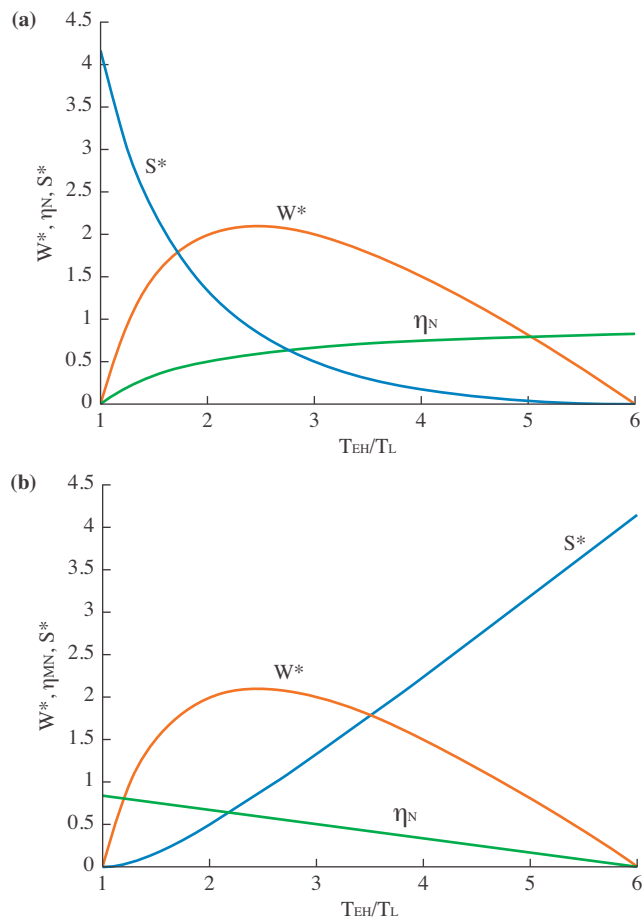


Fig. 6. Variation of thermal efficiency, normalized power output and normalized entropy production rate of (a) Novikov's engine, and (b) modified Novikov's engine ($r_T = 6$).

Novikov engine and the modified Novikov engine. For the case of Novikov engine (Fig. 6a), the normalized power output W^* and the normalized entropy production rate S^* are defined the same way as before. However, because $K_l \rightarrow \infty$ for the case of modified Novikov engine (Fig. 6b), these two parameters are defined as $W^* = \dot{W}/(K_h T_L)$ and $S^* = \dot{S}/K_h$. Also, note that the results for the Novikov's model are presented as a function of T_{EH}/T_L , whereas those for the modified Novikov model are versus T_{EL}/T_L .

Fig. 6 reveals that an increase in the entropy production is equivalent to a decrease in the thermal efficiency for both cases. For the case of Novikov's engine, the thermal efficiency monotonically increases with T_{EH}/T_L and the entropy production consistently decreases with T_{EH}/T_L . The power output of the engine peaks at $T_{EH}/T_L = 2.1$. For the case of modified Novikov's model, with an increase in T_{EL}/T_L , the thermal efficiency decreases and the entropy production increases. The power output attains its maximum at $T_{EL}/T_L = 2.1$. From this discussion, we arrive at the conclusion that for the engine models of Novikov and modified Novikov, the minimization of the entropy production is equivalent to the maximization of the thermal efficiency, a similar conclusion that we reached for the case of Curzon-Ahlborn engine.

A schematic of the Carnot vapor cycle is shown on a T-s diagram in Fig. 7. Unlike the heat engine models examined in above, which are interacting with fixed temperature thermal reservoirs, the model of Fig. 7 exchanges heat with a hot stream whose specific heat is $c_{p,h}$, and a cold stream with a specific heat of $c_{p,l}$. The cycle undergoes an isothermal evaporation process at temperature T_{EH} , and an isothermal condensation process at temperature T_{EL} . The evaporation process takes

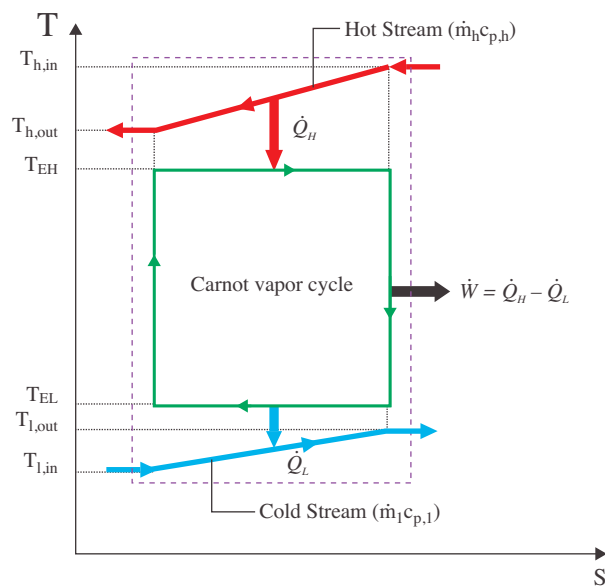


Fig. 7. Schematic representation of a Carnot vapor heat engine on a T-s diagram.

place by receiving heat from the hot stream through a heat exchanger. The inlet and outlet temperatures of the hot stream are $T_{h,in}$ and $T_{h,out}$ ($< T_{h,in}$), respectively. At the cold-end side of the engine, the condensation heat is rejected through another heat exchanger, to the cold stream which enters the heat exchanger at temperature $T_{l,in}$ and exits at temperature $T_{l,out}$ ($> T_{l,in}$).

Let us now assume that both heat exchangers at the hot-end and the cold-end sides are operating ideally with 100 percent effectiveness. In other words, the temperature of the hot stream leaving the hot-end side of the engine is equal to the evaporation temperature; i.e., $T_{h,out} = T_{EH}$, and the exit temperature of the cold stream is the same as the condensation temperature; i.e., $T_{l,out} = T_{EL}$. In this case, the heat rate supplied from the hot stream for evaporation of the working fluid is

$$\dot{Q}_H = (\dot{m}c_p)_h(T_{h,in} - T_{h,out}) = \dot{C}_h(T_{h,in} - T_{EH}) \tag{24}$$

where $\dot{C} = \dot{m}c_p$. The heat rate rejected by the engine at the cold-end side heat exchanger is absorbed by the cold stream; whose temperature rises from $T_{l,in}$ to $T_{l,out}$. Hence,

$$\dot{Q}_L = (\dot{m}c_p)_l(T_{l,out} - T_{l,in}) = \dot{C}_l(T_{EL} - T_{l,in}) \tag{25}$$

The power producing compartment (the green rectangle in Fig. 7) is internally reversible, so $\dot{Q}_L/\dot{Q}_H = T_{EL}/T_{EH}$. Eqs. (24) and (25) allow us to establish a relationship between T_{EL} and T_{EH} .

$$T_{EL} = \frac{\dot{C}_l T_{EH} T_{l,in}}{(\dot{C}_l + \dot{C}_h) T_{EH} - \dot{C}_h T_{h,in}} \tag{26}$$

The power output of the engine is obtained as

$$\dot{W} = \dot{Q}_H - \dot{Q}_L = \dot{C}_h(T_{h,in} - T_{EH}) - \dot{C}_l(T_{EL} - T_{l,in}) \tag{27}$$

Substituting Eq. (26) into Eq. (27) gives

$$\dot{W} = \dot{C}_h(T_{h,in} - T_{EH}) - \dot{C}_l \left[\frac{\dot{C}_l T_{EH} T_{l,in}}{(\dot{C}_l + \dot{C}_h) T_{EH} - \dot{C}_h T_{h,in}} - T_{l,in} \right] \tag{28}$$

For given values of the hot and the cold streams inlet temperatures; i. e., $T_{h,in}$ and $T_{l,in}$, and fixed heat capacitances of the hot and cold streams, the power produced by the engine has only one degree of freedom, T_{EH} . Maximization of the power with respect to T_{EH} yields

$$(T_{EH})_{opt} = \frac{\dot{C}_h T_{h,in} + \dot{C}_l \sqrt{T_{h,in} T_{l,in}}}{\dot{C}_l + \dot{C}_h} \tag{29}$$

The maximum power output of the Carnot vapor cycle is obtained by substituting Eq. (29) into Eq. (28).

$$\dot{W}_{\max} = \left(\frac{\dot{C}_l \dot{C}_h}{\dot{C}_l + \dot{C}_h} \right) (\sqrt{T_{h,in}} - \sqrt{T_{l,in}})^2 \tag{30}$$

An important observation in the model of Fig. 7 is that unlike the models of Fig. 1, Fig. 4 and Fig. 5 in which all amount of heat supplied from the thermal reservoir is transferred to the engine, here only a fraction of the heat, \dot{Q}_H , is transferred to the power producing compartment. If the hot stream is supplied from the ambient (e.g. air) as in most steam engines, it is first heated (for instance, in the furnace of a steam power plant) to the desired temperature $T_{h,in}$, and eventually it is exhausted to the atmosphere characterized with temperature $T_{l,in}$ (see Fig. 8). So, the thermal efficiency of the entire plant is given by $\eta_{th} = \dot{W} / \dot{Q}_{in}$, where \dot{Q}_{in} denotes the rate of heat transferred from the high temperature reservoir to the hot stream.

$$\dot{Q}_{in} = \dot{Q}_H + \dot{C}_h (T_{EH} - T_{l,in}) = \dot{C}_h (T_{h,in} - T_{l,in}) \tag{31}$$

Using Eqs. (30) and (31), we find an expression for the thermal efficiency of the entire plant at maximum power as follows.

$$(\eta_{th}) \dot{W}_{\max} = \left(\frac{\dot{C}_l}{\dot{C}_l + \dot{C}_h} \right) \left(\frac{\sqrt{T_{h,in}} - \sqrt{T_{l,in}}}{\sqrt{T_{h,in}} + \sqrt{T_{l,in}}} \right) \tag{32}$$

Note that the efficiency of the power producing compartment at maximum power is $1 - \sqrt{T_{l,in}/T_{h,in}}$. Also, as the heat input \dot{Q}_{in} is constant, the maximum efficiency occurs at exactly the same optimum T_{EH} that the power output is maximized; i.e., Eq. (29).

The entropy production rate due to the heat exchange between the power producing compartment and the hot and cold streams is $\dot{S}_{Gen} = \Delta\dot{S}_l - \Delta\dot{S}_h$, where $\Delta\dot{S}_l$ and $\Delta\dot{S}_h$

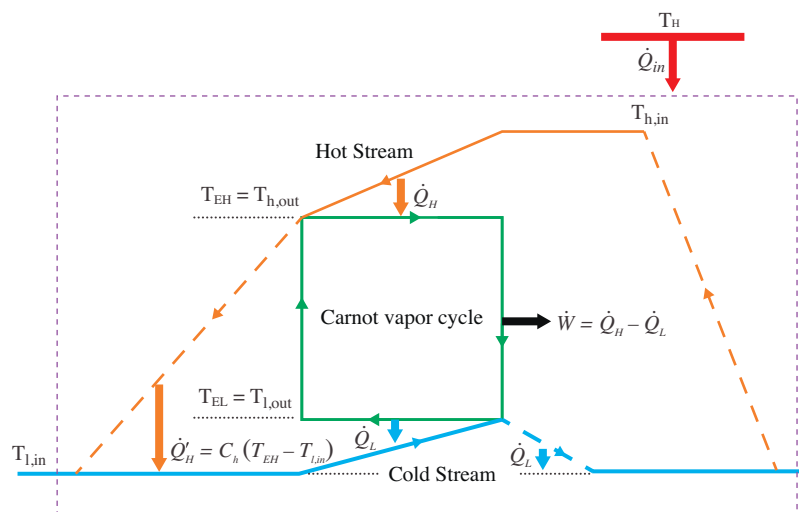


Fig. 8. Illustration of evaluating the total entropy generation rate related to the operation of the heat engine model depicted in Fig. 7.

denote the net change in the entropies of the cold and hot streams, respectively, due to the heat exchange with the working fluid of the cycle.

To evaluate the total entropy generation rate associated with the operation of the engine model of Fig. 7 and based on the arguments of Ref. [8], we need to account for additional sources of entropy generation due to (1) the transfer of heat from the exhaust of the hot stream to the surrounding, (2) the rejection of heat from the exhaust of the cold stream to the surrounding, and (3) heating the hot stream to increase its temperature from $T_{l,in}$ to $T_{h,in}$; see Fig. 8.

As the hot stream is provided from the ambient, its temperature first rises from $T_{l,in}$ to $T_{h,in}$, which then reduces to $T_{h,out}$ within the hot-end side heat exchanger where it loses part of its energy to evaporate the working substance of the engine, and finally it cools down to $T_{l,in}$ after it returns back to the ambient. Thus, the net change in the entropy of the hot stream gas is zero. Likewise, the net change in the entropy of the cold stream is zero as it is supplied from the ambient and eventually discharged to the ambient. So, a more accurate way to determine the total entropy generation rate of the system designated with the dashed-rectangle in Fig. 8, is to evaluate the total heat transferred to the hot stream as well as the total heat rejected from the cycle to the ambient. Thus, by accounting for all possible sources related to the operation of the model of engine under study, the total entropy production rate is

$$\dot{S}_{Gen,tot} = \frac{\dot{Q}_{out}}{T_{l,in}} - \frac{\dot{Q}_{in}}{T_H} = \dot{Q}_{in} \left(\frac{1}{T_{l,in}} - \frac{1}{T_H} \right) - \frac{\dot{W}}{T_{l,in}} \quad (33)$$

where T_H is the combustion temperature. In Eq. (33), \dot{Q}_{out} was eliminated using $\dot{W} = \dot{Q}_{in} - \dot{Q}_{out}$.

As the rate of heat input \dot{Q}_{in} is constant; see Eq. (31), the minimum total entropy generation rate occurs at exactly the same optimum T_{EH} that the power output is maximized; i.e., Eq. (29). Thus, we conclude that maximum power, maximum thermal efficiency, and minimum entropy production rate become coincident for the Carnot vapor engine when the heat input is constant. As reasoned in a previous article [4], for the specific case of constant heat input, optimization of any model of engine that interacts with two thermal sources, based on maximum thermal efficiency, maximum power output and minimum entropy production would result in an identical design.

Nevertheless, from practical viewpoint, optimization of an engine with a fixed heat input is irrelevant. A given heat input is equivalent to a fixed mass of fuel to be burnt in furnace. Once the problem is reduced to a fixed amount of burning fuel, an important ability of varying mass of fuel at other operational conditions (such as at part-load operation) is taken away. So, it would be rational to treat this parameter

as a variable together with many other parameters which may directly or indirectly influence the thermal efficiency of an engine.

Let us now consider a Carnot vapor cycle with varying heat input. For this, we assume that \dot{C}_h is the design parameter, and all other parameters including T_{EH} are constant. Similar to the case with varying T_{EH} , it can be inferred from Eq. (28) that there is an optimum \dot{C}_h given by Eq. (34), which maximizes the power output.

$$(\dot{C}_h)_{opt} = \dot{C}_l \frac{T_{EH} - \sqrt{T_{EH}T_{l,in}}}{T_{h,in} - T_{EH}} \quad (34)$$

Substituting Eq. (34) into Eq. (28), the maximum power output at optimum heat capacity of the hot stream is obtained as

$$\dot{W}_{max} = \dot{C}_l (\sqrt{T_{EH}} - \sqrt{T_{l,in}})^2 \quad (35)$$

Eq. (35) reveals that when the power optimization is performed with \dot{C}_h , the maximum power depends only on three parameters: the heat capacity and the inlet temperature of the cold stream, and the highest temperature of the engine T_{EH} . On the other hand, the thermal efficiency of the cycle at maximum power is

$$(\eta_{th})_{\dot{W}_{max}} = \left(1 - \sqrt{\frac{T_{l,in}}{T_{EH}}}\right) \left(\frac{T_{h,in} - T_{EH}}{T_{h,in} - T_{l,in}}\right) \quad (36)$$

It can be implied from Eq. (36) that the thermal efficiency at maximum power is independent of the heat capacities of the cold and hot streams, but it depends on their inlet temperatures as well as the highest temperature of the engine.

To illustrate the dependence of efficiency, power output and the entropy generation rate of the Carnot vapor cycle with our design parameter, \dot{C}_h , it is convenient to define normalized power output W^* and normalized entropy generation rate S^* as $W^* = \dot{W}/(\dot{C}_l T_{l,in})$ and $S^* = \dot{S}_{Gen,tot}/\dot{C}_l$, where \dot{W} and $\dot{S}_{Gen,tot}$ are given in Eqs. (28) and (33), respectively. One may realize from Eqs. (28) and (33) that both W^* and S^* are functions of \dot{C}_h/\dot{C}_l , $T_{EH}/T_{l,in}$, $T_{h,in}/T_{l,in}$, and $T_H/T_{l,in}$.

The variation of the thermal efficiency, normalized power output and normalized entropy production rate of the Carnot vapor cycle versus the ratio of \dot{C}_h/\dot{C}_l is illustrated in Fig. 9 for typical values of $T_{EH}/T_{l,in} = 5$, $T_{h,in}/T_{l,in} = 6$, and $T_H/T_{l,in} = 7$. The results in Fig. 9 demonstrate that the thermal efficiency correlates with S^* . The thermal efficiency of the engine decreases monotonically with \dot{C}_h/\dot{C}_l , whereas S^* consistently increases with \dot{C}_h/\dot{C}_l . On the other hand, the power produced by the cycle peaks at $\dot{C}_h/\dot{C}_l = 2.8$. Thus, when the heat input is a varying parameter, the regimes of maximum thermal efficiency and minimum entropy generation rate are equivalent, but they are different from the regime of maximum power.

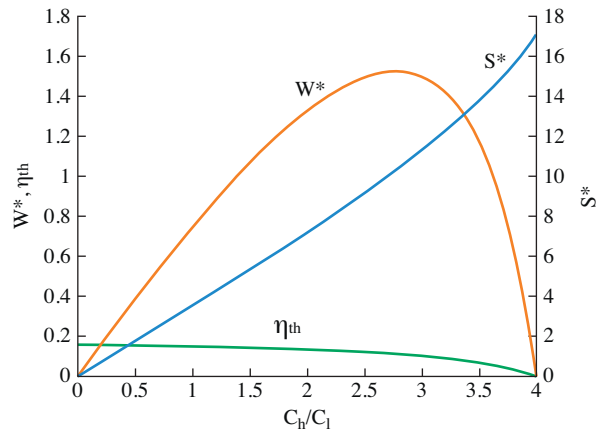


Fig. 9. Variation of the thermal efficiency, normalized power output and normalized entropy production rate of the Carnot vapor cycle with \dot{C}_h/\dot{C}_l . S^* correspond to the total entropy production term given in Eq. (33) divided by \dot{C}_l ($T_{EH}/T_{l,in} = 5$, $T_{h,in}/T_{l,in} = 6$, $T_H/T_{l,in} = 7$).

3. Conclusion

The efficiency of typical endoreversible heat engines including Curzon-Ahlborn, Novikov and modified Novikov models, and Carnot vapor cycle at minimum entropy production is examined. The unique and common feature of these engines is that the compression and the expansion processes take place isentropically. The key conclusion is that the production of entropy is an indication of a reduction in the thermal efficiency of an endoreversible heat engine. This is consistent with the previous findings on endoreversible Brayton cycles [5]. Furthermore, for the case of fixed heat input, an optimization based on minimum entropy production rate, maximum power output, or maximum thermal efficiency results in an identical design.

Declarations

Author contribution statement

Yousef Haseli: Conceived and designed the analysis; Analyzed and interpreted the data; Wrote the paper.

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Additional information

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