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Asymmetric dependence of intraday frequency components in the Brazilian stock market

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Abstract

The multivariate dependence plays an important role in financial instrument management. Due to the inherent characteristics in the financial market, such as heavy tails in the returns unconditional distribution and asymmetry between gain and loss, we obtained the asymmetric dependence structure in different short-term variation scales based on the wavelet technique MODWT. The study sought to capture the relations between financial returns represented by its frequency components. Intraday returns series was used in the 15-min sampling interval from stocks and applied the D-Vine pair-copula to decompose in trade frequencies of 15 min, 1 h, 1 day, and 1 week with margin adjustments of ARIMA-APARCH class and BB7 copula function, responsible for measuring the dependence on tails. The results indicated the prevalence of a high dependence during market upturns, rising over the analyzed frequencies. Being an important tool in financial management and allowing short-term strategies of diversification.

Keywords Multivariate dependence · Financial returns · Copulas · Wavelets · High frequency

JEL Classification C22 · C58 · G10

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SN Business & Economics
A **SPRINGER NATURE** journal

Introduction

The behavior of the multivariate dependence structure of financial markets configures a relevant point on funding instruments management. Since Modern Portfolio Theory (Markowitz 1952) a general discussion of this topic, including other aspects as risk and return, expected return, measures of risk and volatility, and diversification, has been in the current literature. Several studies such as Ergen (2014), Jondeau (2016) and Caldeira et al. (2017) have shown that the measuring of dependence existing among the returns of a portfolio is essential to investment strategy development, mainly in the diversification context, which consists of the efficient allocation of distinct assets to minimize risks.

Specifically, in risk management, portfolio selecting, and asset pricing, there are important aspects such as non-linearity, asymmetrical dependence, and also heavy tails of the marginal and joint probability distribution (Wang and Xie 2016). To deal with these questions, inferences based on tail multivariate probabilities are necessary. Tail dependence refers to asset returns that exhibit greater dependence during market downturns or during market upturns and has long been an issue of interest to academics, fund managers, and traders, as it has important implications for portfolio allocation and asset pricing. Patton (2004), Malevergne and Sornette (2006) and Hatherley and Alcock (2007) demonstrated that incorporate the effects of asymmetric asymmetrical dependence in asset allocation proved to be better for protect portfolios and minimize risk. Besides that, Cherubini et al. (2004) and Chollete et al. (2011) showed that most economic policies of systemic risk involve tail dependence.

The evidences raised above have been widely reported over the years, principally the marginal distribution skewness and dependence structure asymmetry. According to Peng and Ng (2012) and Patton (2001), an inappropriate dependence model can lead to inefficient portfolios and imprecise evaluations of risks expositions. To deal with these problems, the application of the copulas approach is proposed. Copulas are functions that connect multivariate distribution functions to their marginal distributions (Cherubini et al. 2004). They contain all the relevant information about the dependence structure among the variables, for both symmetric and asymmetric correlation structures. In financial data, the copulas form an ideal tool for analyzing extreme dependence movements without the restrictions imposed by the classic multivariate models, reflecting the dependence between assets (Embrechts et al. 2003).

There are different copulas approach applications for the optimization of returns of assets seen on Righi and Ceretta (2013), Kakouris and Rustem (2014), Bartels and Ziegelmann (2016) and Abbara and Zevallos (2017). One of these methods is the multivariate pair-copula models of Joe (1997), extended by Bedford and Cooke (2001) and Bedford and Cooke (2002) with a hierarchic graphic construction of bivariate copulas called *regular vines* copulas. According to Joe et al. (2010), the modeling of dependence with multivariate copulas, such as the Vine approach, enables to development of appropriate parametric families for multivariate financial data with different dependence structures. Joe and

Kurowicka (2011) provides an extensive review of Vine copulas, including applications of this methodology in financial.

The purpose of the present paper is to provide the behavior of asymmetrical relation between financial returns in the domain time–frequency. Frequency is a relevant factor in assets analysis, relating to the changes in the investment horizons of the players of the market, ranging from short-run to long-run. The significance of this analysis lies in considering the impact of the time horizons of investment rules on the portfolio analysis, measuring asymmetric dependence in different timescales. The time horizons of economic decisions are related to the stock price changes, then different temporal frequencies (scales) of a returns series are useful to capture subjacent financial information from these data, as seen by Jammazi and Reboredo (2016), Shah et al. (2018), Biage (2019) and Berger and Gençay (2019). This multiscale financial behavior can be captured applying the wavelet decomposition, which enables to identify the trend in different periods of time and to locate the relevant oscillation moments (Crowley 2007; Gallegati 2014).

The portfolio analyzed in this study was composed of stocks traded in the Brazilian financial market (B3). We consider the 15-min sampling interval as the regularly spaced time for the 7 h of continuous negotiation in the B3 from February 17th to May 8th of 2020 of six relevant stocks: PETR4 (Petrobras), AZUL4 (Azul), USIM5 (Usiminas), BBDC4 (Bradesco), WEG3 (Weg) e MGLU3 (Magazine Luiza). The period analyzed reflects the negative effects of the COVID-19 pandemic on the financial markets, reinforcing the importance of modeling this event, providing tools for decision making. The stock choice was based on different economic segments to generate a diverse portfolio, and Chang et al. (2008) showed that high-frequency horizons are important to investigate the effects of short market trade activities, which reflect changes in an asset trajectory at many different scale levels (Crowley 2007).

The decomposed series for each intraday stock returns sample was obtained, applying the wavelet technique by Percival and Walden (2000) using the Daubechies wavelet filter of length 2 (two null moments) by Daubechies (1992). The short-term trade frequencies are the variations scale series regarding 15 min, 1 h, 1 day and 1 week. To carry out the analysis of asymmetric multivariate dependence analysis, we applied the D-Vine pair-copula constructions according to Joe (1997) and Bedford and Cooke (2002) in the decomposed series. The marginal distributions were specified as the process from ARIMA and ARIMA-APARCH classes by Ding et al. (1993), to capture important characteristics evidenced in these series. The pair-copula analysis proceeds with the standardized residuals. The BB7 copula function is estimated for its property of capturing asymmetrical dependence in financial data as shown by Nikoloulopoulos et al. (2012).

Background

Wavelet analysis

The maximal overlap discrete wavelet transformation (MODWT) is a modification of discrete wavelet transformation (DWT), proposed by Percival and Walden

(2000). Both the DWT and the MODWT draw on multiresolution analysis to decompose a signal into different levels of resolution. At each level, weighted moving average values (smooths) and the information to reconstruct the signal (details) from the averages are obtained describing the original signal at coarser and coarser levels of resolution.

In contrast to the DWT, the MODWT not is characterized by a data reduction (to the half) by each decomposition, keeping the data length constant. Actually, the MODWT presents essential proprieties in the time series decomposition: the translation is non-orthogonal and invariant, conserving the original series variation. This enables the impact of any event to be analyzed over specific timescales, so this method will be used in this paper. Percival and Walden (2000) presented an extensive revision of the MODWT characteristics in time series.

The MODWT follows the same pyramid algorithm (Mallat 1989) as the DWT (see Percival and Walden 2000). Letting $j = 1, \ldots, J$ be the scale numbers and the initial series entrance $s_{0,t} = X_{t=0}^{N-1}$. The decomposition process occurs with the successive filtering of a time series X_t with low-pass filters $\{\widetilde{g}_{j,l}\}$ and high-pass $\{\widetilde{h}_{j,l}\}$ given by

$$\tilde{s}_{j,t} \equiv \sum_{l=0}^{L_j-1} \tilde{g}_{j,l} X_{t-l \bmod N} \tag{1}$$

and

$$\tilde{d}_{j,t} \equiv \sum_{l=0}^{L_j-1} \widetilde{h}_{j,l} X_{t-l \bmod N}, \tag{2}$$

where $L_j = (2^j - 1)(L - 1) + 1$ correspond to filter size associated to each scale j and $\bmod N$ is the modulus operator. In Eqs. (1) and (2) the MODWT filters $\tilde{h}_{j,l} = h_{j,l}/2^j$ and $\tilde{g}_{j,l} = g_{j,l}/2^j$, respectively, are expressed in terms of DWT rescaled filters $g_{j,l}$ and $h_{j,l}$ that satisfies useful proprieties in the decomposition of a sign: (1) $\sum_{l=0}^{L-1} h_l = 0$; (2) $\sum_{l=0}^{L-1} h_l^2 = \frac{1}{2}$; (3) $\sum_{i=0}^{L-1} h_i h_{i+2N} = 0$, similarly for the $g_{j,l}$.

Several wavelet filters do exist and the choice of the adequate filter heavily depends on the purpose of its application. Thus, considering that intraday series present non-stationary and drastic fluctuations the Daubechies wavelet filter by Daubechies (1992) was employed in this paper. With the compact support advantage and orthogonality the general form of Daubechies filters is given by $h_{l,j} = (-1)^{l-L_j} g_{L_j-1-l}$. Applications with Daubechies filters in multiscale analyzes as intraday financial series can be seen in Sun et al. (2011), Xue et al. (2014), and Xu (2018).

At each scale, the MODWT coefficients $\tilde{s}_{j,t}$ and $\tilde{d}_{j,t}$ constitute a time series describing X_t in non-aggregated over time way, such that $X_t = \sum_{j=1}^{J} (\tilde{d}_{j,t} + \tilde{s}_{J,t})$. At the levels $j = 1, \ldots, J$ and in the time t, the scale coefficients $\tilde{s}_{j,t}$ represent the smooth coefficients that capture the trend of X_t, while the detail coefficients $\tilde{d}_{j,t}$ capture the short oscillations, as structural changes, representing the detailing of X_t. Taking into account these brief description of properties of the MODWT, we

estimated the asymmetric dependence of portfolios using the detail series obtained from the decomposition.

Copulas and pair-copulas

The concept of copula is introduced in the statistical literature by Sklar (1959). Let the random variables $X_1, \ldots, X_d$ with joint distribution function H, such as $(x_1, \ldots, x_d) \in [-\infty, \infty]^d$, where $X_1, \ldots, X_d$, $d = 1, \ldots, 6$, represent the details series obtained from the MODWT decomposition in the intraday log-returns. The dependence between $X_1 \ldots, X_d$ can be completely described by a d-dimensional copula function C, such as $H(x_1, \ldots, x_d) = C(F_1(x_1), \ldots, F_d(x_d))$. Conversely, $C(u_1, \ldots, u_d) = H(F_1^-(u_1), \ldots, F_d^-(u_d))$, where F_i^- correspond to the inverse generalized of F_i, $i = 1, \ldots, d$.

Then, according to Sklar (1959), a C is defined as a function of joint distribution in $[0, 1]^d$ with Uniform marginals. Assuming C is absolutely continuous, and by taking the partial derivatives, one obtains:

$$h(x_1, \ldots, x_d) = c(u_1, \ldots, u_d) \prod_{i=1}^{d} f_i(x_i), \qquad (3)$$

where c represents the copula density.

For the multivariate case modeling, Aas et al. (2009) explained that a pair-copula decomposition is a flexible alternative and easily implemented. The pair-copulas is a hierarchical construction, based on bivariate copulas chosen between any parametric family. The variables are sequentially incorporated into the conditioning sets as one moves from the first modeling level d until the last level $d - 1$. The pair-copula factorization, according to Joe (1997), is obtained from the following decomposition of h:

$$h(x_1, \ldots, x_d) = f_d(x_d) \cdot f(x_{d-1}|x_d) \cdot f(x_{d-2}|x_{d-1}, x_d) \cdot f(x_1|x_2, \ldots, x_d), \qquad (4)$$

where for d variables at T time points, assumed that the observations of each variable are independent over time.

Based on the joint density in Eq. (3), all conditional densities in Eq. (4) can be expressed from only univariate marginal distributions and bivariate copulas by means:

$$h(x|\mathbf{v}) = c_{xv_j|\mathbf{v}_{-j}} \left\{ F(x|\mathbf{v}_{-j}), F(v_j|\mathbf{v}_{-j}) \right\} \cdot f(x|\mathbf{v}_{-j}), \qquad (5)$$

where $c_{xv_j|\mathbf{v}_{-j}(.)}$ corresponds to the density of a bivariate copula, and $\mathbf{v}_{-j}$ denotes the vector $\mathbf{v}$ excluding the jth component.

For the representation of Eq. (5) there is different pair-copulas construction (PCC). Then Bedford and Cooke (2001) and Bedford and Cooke (2002) introduced the systematic model called *regular vines* that involves the construction of hierarchic graphic models. Each of these models provides a specific way of decomposing

the d-dimensional h density. The main types are the hierarchical canonical vines (C-vines) and the drawable vines (D-Vines).

In this paper, the h density was estimated from the D-Vine PCC, which is written as

$$h(x_1, \dots, x_d) = \prod_{k=1}^{d} f(x_k) \prod_{j=1}^{d-1} \prod_{i=1}^{d-j} c_{i,i+j|i+1,\dots,i+j-1}(F(x_i|x_{i+1},\dots x_{i,j-1}) \cdot F(x_{i,j}|x_{i+1},\dots x_{ij-1})),$$

(6)

where index j identifies the trees, while i runs over the edges in each tree. In a D-vine, no node in any tree T_j is connected to more than two edges. There are $d(d-1)/2$ bivariate copulas density in the $d-1$ trees. The tree T_j of the D-vine has $d-j$ bivariate copulas, $j = 1,\dots, (d-1)$. Those in tree 1 are unconditional, and all others are conditional (Aas et al. 2009).

In the inference process of the D-Vine PCC, it is necessary to obtain the respective functions of conditional distribution $F(x|\mathbf{v})$ in a sequential way, this is

$$h(x|\mathbf{v}, \boldsymbol{\theta}) = F(x|\mathbf{v}) = \frac{\partial C_{x,v_j|\mathbf{v}_{-j}}\left(F(x|\mathbf{v}_{-j}), F(v_j|\mathbf{v}_{-j})|\boldsymbol{\theta}\right)}{\partial F(v_j|\mathbf{v}_{-j})},$$

(7)

where $\boldsymbol{\theta}$ is the vector parameters of the $C_{x,v_j|\mathbf{v}_{-j}}$ specified in the j tree.

The bivariate copulas involved can belong to different families in a way of reflecting various ways of dependence, including tail dependence (see Joe 1997). The concept of tail dependence refers to the amount of dependence on the right higher quadrant tail or on the left lower quadrant tail of a bivariate distribution (Embrechts et al. 2003).

This feature enables construct h estimating different margins independently. In the presence of temporal dependence, univariate time series models for the conditional mean and the conditional variance can be fitted to the margins and the analysis could henceforth proceed with the residuals standardized. The standardized residual vectors are converted to uniform variables using the empirical distribution functions before further modeling (Nikoloulopoulos et al. 2012).

Methodological procedure

The data and context

The high-frequency data used were the log-return stocks of B3 and covers the 55 working days from February 17th of 2020 to May 8th of 2020, presented in Fig. 1. In the analyzed period, it is needed to emphasize the expressive influence of COVID-2019 in the worldwide financial markets. According to Laurini and Chaim (2020), the COVID-19 pandemic drop in prices in March 2020 has spurred volatility increases with levels faster. Along with the phenomenon, the Brazilian stock market has been suffering an impact on internal political instability.

(a)

(b)
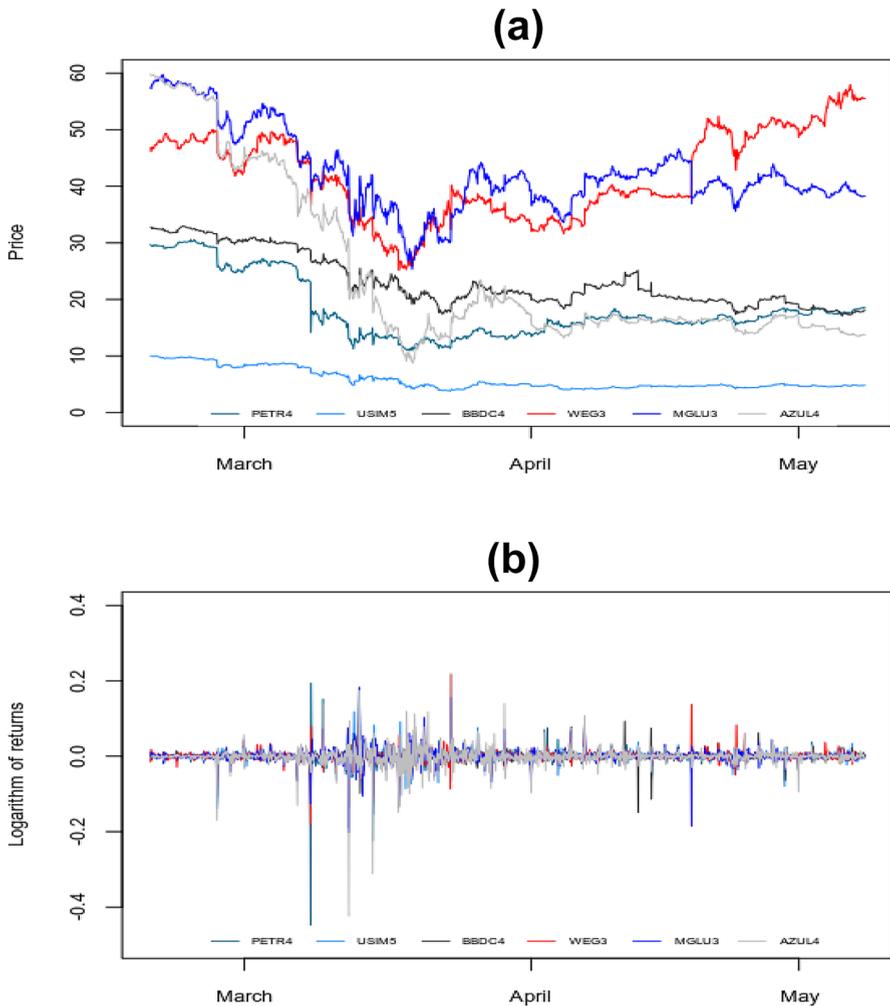
Fig. 1 Series behavior in the analyzed period for the series: **a** quotes and **b** log-returns

The sampling interval regarded was of $\Delta = 15$ min as the spaced time for 7 h of continuous trading. The number of sampled observations per trading session is $m = 28$ interval/day with a total of $N = 1497$ observations. The data filtering process was made according to Morettin (2017), keeping the circuit-breakers in the final sample. Plots in Fig. 1a, b indicate the intraday prices and log-returns behavior in the period.

General information about the data classification with the base on the sector and action segment is presented in Table 1.

The MODWT constructed by the Daubechies wavelet filter with length 2 (two moments null) $D2$ was applied to the intraday series using the methodology

Table 1 Classification data

Company	Stock's code	Sector/business segment
Azul	AZUL4	Air transport
Bradesco	BBDC4	Financial
Magazine Luiza	MGLU3	Cyclic consumption
Petrobras	PETR4	Oil, gas and biofuels
Usiminas	USIM5	Steel industry
Weg	WEGE3	Industrial

Table 2 Impact and interpretation of the variances of the decomposition components MODWT on scales $j = 1,3,6$ e 8 of the intraday log-returns (Δ = 15 min)

Frequency	$\tilde{d}_j$	Memory
15 min	$\tilde{d}_1$	Stochastic noise
60 min	$\tilde{d}_3$	Stochastic noise
480 min $\approx$ 1 day	$\tilde{d}_6$	Short-term
1920 min $\approx$ 5 day	$\tilde{d}_8$	Trend of short-term

submitted in Sect. 2.1, obtaining $J = 10$ decomposition levels. Once the variance of the original returns series is preserved, we can measure the dependence using the series from the decomposition. Thus, the D-Vine PCC was obtained four details series for each original series in four dyadic scales of variation: 15 min, 1 h, 1 day and 1 week. The frequencies are measured according to Table 2, conform to the 7 h of B3 trading, and Fig. 2 illustrate the series generated by the MODWT decomposition in levels $j = 1, 3, 6, 8$.

Dependence estimation

Since we are mainly interested in the dependence structure between wavelet series obtained, the estimation process of copulas was made through the methods of maximum likelihood in two steps according to the inference function for margins approach by Joe and Xu (1996), they are (1) univariate adjustment of margins and (2) adjustment of the copula with the standardized residues of margins under pseudo-observations.

As stated in Sect. 2.2, the observations of each variable must be independent over time. Hence, in the first stage, the margin distributions were estimated by models of the conditional mean and variance. The ARIMA(p, d, q)-APARCH(1, 1) process by Ding et al. (1993) was used. That is, for details series j in time $t = 1, \ldots, N$, we have the following model:
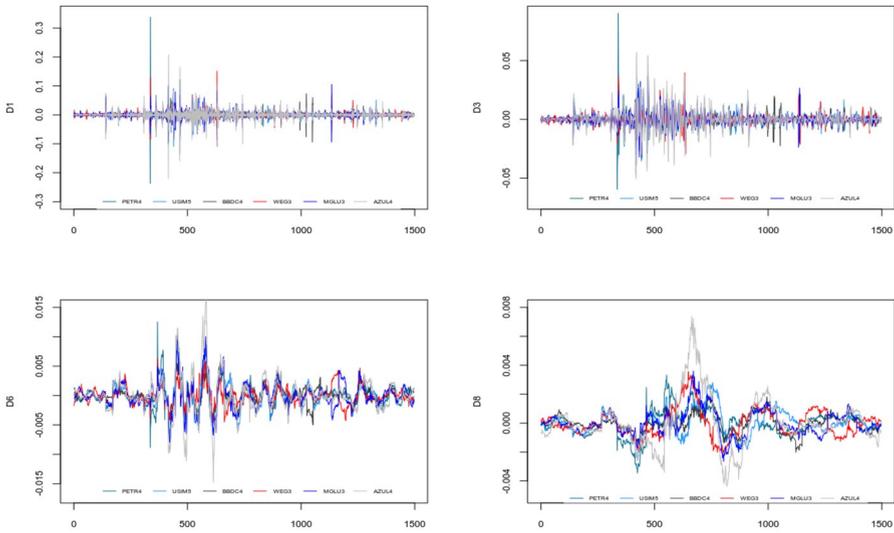
Fig. 2 Decomposition intraday log-returns series at the levels: **a** $\tilde{d}_1$, **b** $\tilde{d}_3$, **c** $\tilde{d}_6$ and **d** $\tilde{d}_8$

$$\tilde{d}_{j,t} = \sum_{i=1}^{p} \phi \tilde{d}_{j,t-i} + \sum_{i=1}^{q} \theta \varepsilon_{j,t-i} + \varepsilon_{j,t}$$
$$\varepsilon_{j,t} = \sigma_{j,t} z_{j,t}$$
$$\sigma_{j,t}^{\delta} = \omega + \alpha_1 \left(\left| \varepsilon_{j,t-1} \right| - \gamma_1 \varepsilon_{j,t-1} \right) + \beta_1 \sigma_{j,t-1}^{\delta}, \tag{8}$$

where the standardized residual $z_{j,t} \sim t - \text{skewed}(\nu_j, \xi_j)$ to consider the conditional heteroscedastic heavy-tailed behavior of the financial assets. For the mean equation, ϕ represents the p autoregressive components and θ the q moving average components. In the variance equation, ω corresponds to the unconditional variance, δ allows to estimate of other powers to the standard conditional deviation, through a Box–Cox transformation, γ_1 captures the leverage effects, α_1 and β_1 together depict the volatility persistence. We use the modified Q-statistic (Ljung and Box 1979) to validate the modeling.

For the second adjustment stage, initially defined the PCC order estimation. The originals series were ordered by the non-linear dependence, measured through Kendall's tau. After, adjustment the D-Vine PCC with the BB7 copula function performed on the standardized residuals of margins ($z_{j,t}$). The standardized residual vectors are converted to uniform variables u_1 and u_2 using the empirical distribution functions before the adjust. The BB7 bivariate copula captures the tail dependence and has representation given by Joe (1997):

$$C_{BB7}\left(u_1, u_2 | \tau_U, \tau_L\right) = 1 - \left(1 - \left\{ \left[1 - \left(1 - u_1 \right)^{\varepsilon} \right]^{-\gamma} + \left[1 - \left(1 - u_2 \right)^{\varepsilon} \right]^{-\gamma} - 1 \right\}^{-1/\gamma} \right)^{-1/\varepsilon}, \tag{9}$$

with $\kappa = 1/\log_2\left(2 - \tau_U\right)$ and $\gamma = -1/\log_2\left(\tau_L\right)$ the parameters related to dependence coefficients of the higher and lower tails, respectively $\tau_U, \tau_L \in (0, 1)$.

These measures were used to quantify the asymmetric dependence, i.e, determine if the relationship between the intraday log-returns, in the different timescales, has intensified in periods of market downward (τ_L) or during the market upward (τ_U).

The results were obtained with the software (Team 2019). The data were provided by *alphavantager* package by Dancho and Vaughan (2019). The analysis was performed with the packages *wtmsa* by Constantine and Percival (2017) for the MODWT application, *fgarch* by Wuertz et al. (2019) for margins adjustment, and *Vine-copula* by Nagler et al. (2019) for D-Vine PCC with the BB7 copula.

Results

Modelling of marginal distributions

The univariate margin models were defined with ARIMA(p, 0, q)-APARCH(1, 1) in the 15-min, 1-h and 1-day scales, which presented stochastic noise and short variations. As the 1-week frequency case reflects the trend of the short term, just the conditional mean was adjusted appealed to ARIMA(p, 1, q) class models. The specification of margins is according to the results of Table 3.

The estimate results of the coefficients are found in Tables 4, 5, 6 and 7. In general, the results corroborating statistical characteristics commonly present in financial time series. In the μ equation, it became evident that stock returns price movements to become more persistent (Schulmeister 2009) and the presence of intraday seasonality (Morettin 2017). And, the σ results reflect volatility persistence, heavy-tails, and asymmetry (Patton 2004). In some cases were evidenced significant leverage effects ($\gamma_1 > 0$), the phenomenon that arises when periods of falling prices are followed by significant volatility (Ding et al. 1993).

Copula modelling

Subsequent to this marginal specification, we obtained matrix of dependence through Kendall's tau to select the order in PCC estimation. The criterion adopted was the absolute sum of dependence between each index with all others. The D-vine

Table 3 Margin distributions for levels $j = 1, 3, 6, 8$

Stock	ARIMA(p,d,q) - APARCH(1, 1)			
	$\tilde{d}_1$	$\tilde{d}_3$	$\tilde{d}_6$	$\tilde{d}_8$
PETR4	(2,0,0)(1,1)	(0,0,7)(1,1)	(4,0,3)(1,1)	(2,1,1)(0,0)
MGLU3	(0,0,1)(1,1)	(0,0,7)(1,1)	(4,0,3)(1,1)	(1,1,1)(0,0)
WEGE3	(1,0,1)(1,1)	(0,0,7)(1,1)	(1,0,4)(1,1)	(3,1,5)(0,0)
USIM5	(1,0,1)(1,1)	(0,0,7)(1,1)	(4,0,3)(1,1)	(2,1,4)(0,0)
AZUL4	(0,0,1)(1,1)	(0,0,7)(1,1)	(5,0,2)(1,1)	(4,1,4)(0,0)
BBDC4	(1,0,1)(1,1)	(0,0,7)(1,1)	(4,0,2)(1,1)	(9,1,2)*(0,0)

*model with specific lags.

Table 4 Estimated parameters and diagnostics of residuals of the ARIMA-APARCH models for d_t series

Parameters	PETR4	MGLU3	WEGE3	USIM5	AZUL4	BBDC4
ϕ_1	− 0.6402		− 0.0528	− 0.0982		− 0.0590
	(0.0220)		(0.0211)	(0.0245)		(0.0009)
ϕ_2	− 0.3129					
	(0.0232)					
θ_1		− 0.9840	− 0.9570	− 0.9979	− 0.9743	− 0.9972
		(0.0028)	(0.0064)	(0.0013)	(0.0036)	(0.0016)
ω	0.0006	0.0009	0.0019	0.0019	0.0015	0.0035
	(0.0001)	(0.0004)	(0.0007)	(0.0011)	(0.0007)	(0.0014)
α_1	0.7338	0.2669	0.2684	1.0000	0.3436	0.2920
	(0.1846)	(0.0804)	(0.0615)	(0.3956)	(0.1258)	(0.0860)
γ_1	− 0.0517	0.1499	0.0544	− 0.0161	0.0469	− 0.0325
	(0.0728)	(0.0937)	(0.0905)	(0.1026)	(0.0874)	(0.0994)
β_1	0.5552	0.8342	0.7954	0.8750	0.8164	0.7919
	(0.0457)	(0.0399)	(0.0419)	(0.0558)	(0.0460)	(0.0415)
δ	1.1154	0.7800	0.7090	0.9074	0.7181	0.5482
	(0.1960)	(0.1228)	(0.1179)	(0.1777)	(0.1373)	(0.1024)
ξ	0.9847	0.9990	1.0022	0.9639	0.9582	0.9944
	(0.0252)	(0.0232)	(0.0239)	(0.0216)	(0.0213)	(0.0243)
ν	2.4164	2.2855	2.3715	2.0071	2.1900	2.334
	(0.1733)	(0.1467)	(0.1590)	(0.0015)	(0.1374)	(0.1760)
$Q(20)$	0.83	0.96	0.99	0.96	0.99	0.99

PCC order result was USIM5, PETR4, MGLU3, BBDC4, AZUL4, and WEGE3. The results are verified in Table 8. It is observed a moderate positive association between all stocks analyzed (34% in mean). The result shows that some stocks pairs can move together, emphasizing the diversification question. The greatest magnitude of dependence was related to the pair PETR4 and USIM5, reaching 41%. Moreover, we noted that the stocks with a higher dependency are associated with the sectors that are sensitive to the actual world economic situation due to COVID-19 impacts, for example, the commodity sector.

With order among log-returns established, the D-Vine PCC was adjusted with standardized residuals of the marginal distributions. The dependence parameters of BB7 copula were converted in the measures of the lower tail (τ_L) and upper tail (τ_U) presented in Table 9.

As demonstrated in the literature, the asymmetry pattern is captured in the majority of relationships between the stock's returns, in the different time frequencies analyzed. The general pattern of association between these stocks is more intense during the market upward ($\tau_U > \tau_L$) in all scales. It means that a rising in B3 prices tends to occur simultaneously in the period (de Melo Mendes and Accioly 2012). This may also suggest an asymmetry to the right in the multivariate distribution as indicated (Silva Filho et al. 2014). Some left asymmetry ($\tau_L > \tau_U$) between pairs of stocks has been observed in variations intraday (15 min and 1 h) in the first trees.

Table 5 Estimated parameters and diagnostics of residuals of the ARIMA-APARCH models for d_2 series

Parameters	PETR4	MGLU3	WEGE3	USIM5	AZUL4	BBDC4
θ_1	0.9457	0.9333	0.9306	0.9450	0.9722	0.9170
	(0.0059)	(0.0060)	(0.0112)	(0.0076)	(0.0082)	(0.0089)
θ_2	0.8563	0.9425	0.9121	0.9531	0.8839	0.8293
	(0.0094)	(0.0077)	(0.0110)	(0.0038)	(0.0141)	(0.0095)
θ_3	0.8163	0.8753	0.8215	0.8978	0.8588	0.7573
	(0.0098)	(0.0075)	(0.0136)	(0.0048)	(0.0139)	(0.0087)
θ_4	− 0.9770	− 0.9679	− 0.9876	− 0.9814	− 0.9813	− 0.9705
	(0.0031)	(0.0026)	(0.0027)	(0.0042)	(0.0020)	(0.0039)
θ_5	− 0.9284	− 0.9069	− 0.9271	− 0.9300	− 0.9578	− 0.8984
	(0.0063)	(0.0059)	(0.0116)	(0.0104)	(0.0076)	(0.0069)
θ_6	− 0.8500	− 0.9130	− 0.9095	− 0.9365	− 0.8780	− 0.8196
	(0.0141)	(0.0077)	(0.0115)	(0.0054)	(0.0130)	(0.0088)
θ_7	− 0.8153	− 0.8516	− 0.8237	− 0.8882	− 0.8566	− 0.7602
	(0.0102)	(0.0070)	(0.0133)	(0.0066)	(0.0127)	(0.0075)
ω	0.00008	0.0002	0.00002	0.0004	0.0001	0.0001
	(0.00004)	(0.0001)	(0.00001)	(0.0008)	(0.00005)	(0.00005)
α_1	0.1770	0.1351	0.1761	0.1710	0.1783	0.2069
	(0.0348)	(0.0297)	(0.0335)	(0.0419)	(0.0399)	(0.0442)
γ_1	0.1696	0.2597	0.0832	0.0197	0.0072	0.1002
	(0.1065)	(0.1170)	(0.0922)	(0.2198)	(0.0964)	(0.0992)
β_1	0.8870	0.9038	0.8794	0.8701	0.8801	0.8592
	(0.0203)	(0.0191)	(0.0204)	(0.0714)	(0.0259)	(0.0230)
δ	0.8526	0.7029	1.0730	0.7131	0.8475	0.8290
	(0.1583)	(0.1419)	(0.1856)	(1.1797)	(0.1400)	(0.1568)
ξ	1.0240	1.0472	0.9755	1.0108	0.9926	0.9951
	(0.0277)	(0.0278)	(0.0296)	(0.0262)	(0.0276)	(0.0271)
ν	2.5050	2.5757	2.7570	2.4776	2.6440	2.6420
	(0.1925)	(0.2406)	(0.2167)	(0.1914)	(0.2016)	(0.2723)
$Q(20)$	0.99	0.94	0.71	0.98	0.98	0.95

Note that in all scales, in the trees with USIM5,AZUL4|PETR4,MGLU3,BBDC4 and PETR4,WEGE3|MGLU3,BBDC4,AZUL4 have concerned $\tau_L = 0$, indicating independence of the lower tail. This condition indicates that, in general, the adjusted BB7 D-vine has multivariate dependence of the higher tail (Joe et al. 2010).

The results of the magnitude of dependence in the trees demonstrate that the estimates of τ_U and, mainly, τ_L have presented decreasing behavior due to the nature of the hierarchical construction, as indicated by Joe et al. (2010). These results highlight the importance of asset diversification in the way that (Markowitz 1952) had intended. The decrease in the joint probability obtained in tails indicates that it possibilities to minimize portfolio risk based on asset allocation in these stocks, especially in times of negative innovations, such as the scenario

Table 6 Estimated parameters and diagnostics of residuals of the ARIMA-APARCH models for d_6 series

Parameters	PETR4	MGLU3	WEGE3	USIM5	AZUL4	BBDC4
ϕ_1	− 0.0311	0.1628	0.9855	0.5851	1.0000	0.9412
	(0.0032)	(0.0212)	(0.0060)	(0.0556)	(0.0040)	(0.0085)
ϕ_2	0.8560	− 0.1316		− 0.4684	0.3275	− 0.5431
	(0.0041)	(0.0187)		(0.0530)	(0.0152)	(0.0072)
ϕ_3	0.0928	0.9325		0.3279	− 0.3301	0.5555
	(0.0036)	(0.0199)		(0.0554)	(0.0151)	(0.0120)
ϕ_4	− 0.5389	0.7720		0.4860	0.1012	0.0283
	(0.0035)	(0.0185)		(0.0470)	(0.0044)	(0.0087)
ϕ_5					− 0.1172	
					(0.0045)	
θ_1	1.0000	0.7929	− 0.0240	0.3442	− 0.0216	0.0310
	(0.0019)	(0.0223)	(0.0059)	(0.0062)	(0.0015)	(0.0082)
θ_2	1.137	0.9671	0.0540	0.8551	− 04021	0.5684
	(0.0035)	(0.0108)	(0.0016)	(0.0443)	(0.0168)	(0.0084)
θ_3	− 0.5636	0.7949	− 0.0286	0.4247		0.0957
	(0.0017)	(0.0017)	(0.0033)	(0.0053)		(0.0040)
θ_4	0.01840		0.1383			− 0.0053
	(0.0067)		(0.0012)			(0.0004)
ω	0.000002	0.00002	0.00005	0.0000006	0.000004	0.0001
	(0.0000006)	(0.000001)	(0.000004)	(0.0000004)	(0.000004)	(0.00009)
α_1	0.0486	0.0622	0.0718	0.0747	0.0794	0.1210
	(0.0123)	(0.0128)	(0.0192)	(0.0174)	(0.0227)	(0.0330)
γ_1	0.1437	− 0.1920	− 0.1813	0.1939	− 0.0160	0.0941
	(0.2192)	(0.1465)	(0.1798)	(0.1305)	(0.0160)	(0.1472)
β_1	0.9769	0.9607	0.9585	0.9566	0.9552	0.9326
	(0.0056)	(0.0062)	(0.0098)	(0.0074)	(0.0123)	(0.0171)
δ	1.0470	0.9884	0.9384	1.1450	0.9132	0.5901
	(0.1527)	(0.1001)	(0.1623)	(0.1505)	(0.1271)	(0.1083)
ξ	1.0020	1.0010	1.0220	0.9911	0.9915	1.0140
	(0.0230)	(0.0252)	(0.0244)	(0.0248)	(0.0241)	(0.0215)
v	2.2850	2.6850	2.4070	2.5940	2.4840	2.1750
	(0.1745)	(0.2045)	(0.1706)	(0.0206)	(0.1794)	(0.1238)
$Q(20)$	0.08	0.25	0.43	0.99	0.87	0.99

of the COVID-19 pandemic. Among the scales, increments at the magnitude of dependence measures were noticed in the majority of trees in lower frequencies, which can reflect the effects of continuous changes in the movements of the prices of the assets in time horizons of minutes and hour (Billio et al. 2012; Xu 2018).

Table 7 Estimated parameters and diagnostics of residuals of the ARIMA models for d_8 series

Coefficients	PETR4	MGLU3	WEGE3	USIM5	AZUL4	BBDC4
ϕ_1	0.2058	− 0.5241	0.2139	0.7484	0.7835	0.5515
	(0.0880)	(0.2365)	(0.0165)	(0.1554)	(0.0992)	(0.1024)
ϕ_2	− 0.2000		− 0.2109	− 0.4771	− 1.2786	− 0.6018
	(0.0284)		(0.0138)	(0.1255)	(0.1209)	(0.0868)
ϕ_3			0.9343		0.3726	
			(0.0153)		(0.1159)	
ϕ_4					− 0.5488	
					(0.0712)	
ϕ_5						
ϕ_6						
ϕ_7						
ϕ_8						0.0705
						(0.0210)
ϕ_9						− 0.1189
						(0.0206)
θ_1	− 0.3171	0.5561	− 0.3290	− 0.8574	− 0.8185	− 0.6370
	(0.0886)	(0.2291)	(0.0300)	(0.1542)	(0.1114)	(0.1078)
θ_2			0.1635	0.5126	1.1179	0.5351
			(0.0285)	(0.1376)	(0.1361)	(0.0942)
θ_3			− 0.9714	− 0.1208	− 0.2419	
			(0.0160)	(0.0380)	(0.1297)	
θ_4			0.0890	0.1276	0.3439	
			(0.0276)	(0.1298)	(0.0790)	
θ_5			0.1235		0.3440	
			(0.0259)		(0.0840)	
Q(20)	0.46	0.33	0.03	0.09	0.23	0.07

Table 8 Kendall's tau dependence matrix of the intraday log-returns (Δ = 15minutos)

	PETR4	MGLU3	USIM5	AZUL4	WEGE3	BBDC4
PETR4	1.00	0.35	0.41	0.35	0.31	0.38
MGLU3	0.35	1.00	0.36	0.35	0.34	0.35
USIM5	0.41	0.36	1.00	0.33	0.32	0.38
AZUL4	0.35	0.35	0.33	1.00	0.30	0.31
WEGE3	0.31	0.34	0.32	0.30	1.00	0.29
BBDC4	0.38	0.35	0.38	0.31	0.29	1.00

Final remarks

In the relevance's face of multivariate analysis in the financial area, in this paper, we explored the asymmetric dependence from intraday frequency components of

Table 9 D-Vine PCC results for the levels d_1, d_3, d_6 e d_8

Stocks	d_1		d_3		d_6		d_8	
	τ_L	τ_U	τ_L	τ_U	τ_L	τ_U	τ_L	τ_U
USIM5|PETR4	0.31	0.32	0.40	0.40	0.39	0.44	0.41	0.50
PETR4|MGLU3	0.30	0.31	0.36	0.31	0.31	0.42	0.39	0.45
MGLU3|BBDC4	0.35	0.27	0.30	0.25	0.22	0.40	0.34	0.43
BBDC4|AZUL4	0.32	0.24	0.24	0.37	0.28	0.37	0.33	0.42
AZUL4|WEGE3	0.31	0.33	0.35	0.28	0.25	0.40	0.28	0.41
USIM5.MGLU3|PETR4	0.10	0.26	0.10	0.23	0.15	0.22	0.10	0.17
PETR4.BBDC4|MGLU3	0.11	0.18	0.14	0.35	0.16	0.22	0.07	0.27
MGLU3.AZUL4|BBDC4	0.08	0.34	0.14	0.22	0.17	0.25	0.10	0.24
BBDC4.WEGE3|AZUL4	0.04	0.17	0.08	0.07	0.03	0.17	0.06	0.19
USIM5.BBDC4|PETR4.MGLU3	0.07	0.18	0.05	0.11	0.02	0.20	0.04	0.19
PETR4.AZUL4|MGLU3.BBDC4	0.02	0.10	0.08	0.16	0.02	0.10	0.01	0.13
MGLU3.WEGE3|BBDC4.AZUL4	0.10	0.13	0.09	0.24	0.02	0.21	0.09	0.21
USIM5.AZUL4|PETR4.MGLU3.BBDC4	0.02	0.07	0.01	0.09	0.00	0.12	0.01	0.07
PETR4.WEGE3|MGLU3.BBDC4.AZUL4	0.00	0.09	0.00	0.09	0.00	0.12	0.01	0.11
USIM5.WEGE3|PETR4.MGLU3.BBDC4.AZUL4	0.00	0.01	0.00	0.03	0.00	0.06	0.00	0.08

financial assets. For the portfolios formed by intraday series of log-returns of the Brazilian stock market: PETR4 (Petrobras), AZUL4 (Azul), USIM5 (Usiminas), BBDC4 (Bradesco), WEGE3 (Weg) e MGLU3 (Magazine Luiza), we compute the asymmetric dependence in the domain of time–frequency. The scenario analyzed reflected the COVID-19 pandemic effects and the Brazilian economics policies.

The evidence about financial markets like non-linearity, kurtosis excess, asymmetry dependence structures, and high-frequency was considered. We quantified the higher and lower tail dependence through the D-Vine PCC by Bedford and Cooke (2002) at intraday, daily, and weekly scales. The D-Vine PCC method reflects the dependence on extremes with the construction of a multivariate distribution, estimating different marginal without normality presupposition. For this purpose, the estimation process was based on the MODWT details series which reflects the financial market variations, capturing the effects of the trade activity in the different time horizons. The frequencies analyzed were related to short-term trade: 15 min, 1 h, 1 day and 1 week.

The univariate marginal distributions were specified as ARIMA(p, 0, q)-APARCH(1, 1) and ARIMA(p, 1, q) models by Eq. (8). We can see that all the scales the information passed of series affect the conditional mean and conditional variance of returns, reflecting the dynamic of stock price movements and seasonality intraday. In addition, asymmetric and heavy tails were evidenced for scales related to minutes, hour, and day. The asymmetric dependence was captured based on BB7 copula parameters, present in Eq. (9), that quantified the dependence on extremes with the tail dependence coefficients. The upper tail dependence exceeded the absolute and lower tail ones in many cases, which

indicates the presence of asymmetry in many relationships and a market upward pattern. It was observed also a decreasing magnitude of the dependence in all cases, due to the nature of the D-Vine PCC.

These results reflecting important practical aspects, regarding financial management. First, the importance of skewness and asymmetric dependence in stock returns for asset allocation. We conclude that a portfolio constructed based on the distribution model that allowed for asymmetric dependence can lead to significantly better asset allocation decisions in time horizons analyzed. Based on the traditional mean-variance analysis by Markowitz (1952), studies such as Patton (2004), Hatherley and Alcock (2007), Jondeau (2016) and Wang and Xie (2016), and others have indicated that these benefits are results of more flexibility specifying the dependence structure on the portfolio. The measurement of asymmetric dependence allows diversifying the allocation of resources in portfolios, providing a balance between risks and returns.

A second point is the performance with changes in the investment time horizon. The attention to the horizon to be employed in investment analysis is evidenced by papers as Gunthorpe and Levy (1994). Considering the frequency dynamics enabled us to study the different degrees of behaviors of stock returns of the B3 market and its relations stemming from heterogeneous shocks. We show that the different investment planning horizons can change the portfolio strategies as Ibragimov et al. (2011) and Chakrabarty et al. (2015). A view that focuses on short fluctuations, with the use of dealing strategies in short-term scales, can also result in a portfolio with considerable profits as Zhang et al. (2016), Baralis et al. (2017) and Berger and Gençay (2019) suggests.

When incorporating the effects of asymmetric correlations in asset allocation, in different time frequencies, this study contributes to emphasizes the importance of statistics applications about financial analysis, principally in the short-term. A multiscale multivariate financial analysis through wavelet techniques allows obtaining specific information of certain periods, which jointly with the flexibility of copulas methods for measuring the asymmetrical dependence of non-aggregated way over time has the potential of assisting as in the strategies process of selection/diversification of investment portfolios, as in the control and management of risks.

Declarations

Conflict of interest The authors declare that there is no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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