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Research article

Analysis of Hamacher power aggregation operators for circular complex p, q-quasirung orthopair fuzzy 2-tuple linguistic sets and their application in green industry development

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A R T I C L E I N F O

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A B S T R A C T

Green industry development focuses on balancing economic and financial growth with environmental stewardship and ensuring that companies and industries are contributing positively to both environmental sustainability and prosperity. This manuscript aims to develop the novel technique of circular complex p, q-quasirung orthopair fuzzy 2-tuple linguistic (CCp, q-QOF2-TL) set and their operational laws based on algebraic t-norms and Hamacher t-norms, where the algebraic t-norms and Einstein t-norms are the special cases of the Hamacher t-norms for parameter $F\text{Ⅎ}^{s} = 1$ and $F\text{Ⅎ}^{s} = 2$. Further, we derive the Hamacher power aggregation operators based on any finite collection of CCp, q-QOF2-TL numbers (CCp, q-QOF2-TLNs), called CCp, q-QOF2-TL Hamacher power average (CCp, q-QOF2-TLHPA) operator, CCp, q-QOF2-TL Hamacher power weighted average (CCp, q-QOF2-TLHPWA) operator, CCp, q-QOF2-TL Hamacher power geometric (CCp, q-QOF2-TLHPG) operator, CCp, q-QOF2-TL Hamacher power weighted geometric (CCp, q-QOF2-TLHPWG) operator, and described their basic properties, called idempotency, monotonicity, and boundedness. Further, we demonstrate the technique of multi-attribute decision-making (MADM) problem based on the above operators to evaluate the major factor that will be playing in the development of the green industry. Finally, we compare the proposed ranking values with the obtained ranking values of existing techniques to show the supremacy and superiority of the initiated approaches.

1. Introduction

Green industry development [1] explains to talk about the procedure or process of industrialization and fostering economic growth with less environmental impact and promoting sustainability. Further, the decision-making [2],[3] technique is very feasible for evaluating the problem of green industry development, because finding the best optimal among the collection of information is very complex under the consideration of the classical set theory. Anyhow, a lot of scholars have discussed the problem of green industry

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Table 1
Geometrical abstract of the fuzzy sets to Cp, q-ROFSs.

Investigators	Methods	Conditions	Representations
Zadeh [4]	FSs	$0 \le \mathbb{U}^{\mathrm{M}}_{\mathscr{R}} \le \mathfrak{l}$	$\mathbb{U}^{\mathrm{M}}_{\mathscr{R}}$: membership function
Atanassov [6,7]	IFSs	$0 \le \mathbb{U}^{\mathrm{M}}_{\mathscr{R}} + \mathbb{U}^{N}_{\mathscr{R}} \le \mathfrak{l}$	$\mathbb{U}^{\mathrm{M}}_{\mathscr{R}}$: membership function, $\mathbb{U}^{N}_{\mathscr{R}}$: non-membership function
Yager [8]	PFSs	$0 \le (\mathbb{U}^{\mathrm{M}}_{\mathscr{R}})^2 + (\mathbb{U}^{N}_{\mathscr{R}})^2 \le \mathfrak{l}$	PFSs → FSs and IFSs, but PFSs ↚ FSs and IFSs
Yager [9]	q-ROFSs	$0 \le (\mathbb{U}^{\mathrm{M}}_{\mathscr{R}})^{\Psi} + (\mathbb{U}^{N}_{\mathscr{R}})^{\Psi} \le \mathfrak{l}$, $\Psi \ge \mathfrak{l}$	q-ROFSs → FSs, IFSs, and PFSs, but q-ROFSs ↚ FSs, IFSs, and PFSs
Ramot et al. [10]	CFSs	$0 \le \mathbb{U}^{\mathrm{M}}_{\mathscr{R}}, \mathbb{U}^{\mathrm{M}}_{\mathscr{I}} \le \mathfrak{l}$	$(\mathbb{U}^{\mathrm{M}}_{\mathscr{R}}, \mathbb{U}^{\mathrm{M}}_{\mathscr{I}})$: complex-valued membership function
Alkouri and Salleh [11]	CIFSs	$0 \le \mathbb{U}^{\mathrm{M}}_{\mathscr{R}} + \mathbb{U}^{N}_{\mathscr{R}} \le \mathfrak{l}$ and $0 \le \mathbb{U}^{\mathrm{M}}_{\mathscr{I}} + \mathbb{U}^{N}_{\mathscr{I}} \le \mathfrak{l}$	$(\mathbb{U}^{\mathrm{M}}_{\mathscr{R}}, \mathbb{U}^{\mathrm{M}}_{\mathscr{I}})$: complex-valued membership function and $(\mathbb{U}^{N}_{\mathscr{R}}, \mathbb{U}^{N}_{\mathscr{I}})$: complex-valued non-membership function
Ullah et al. [12]	CPFSs	$0 \le (\mathbb{U}^{\mathrm{M}}_{\mathscr{R}})^2 + (\mathbb{U}^{N}_{\mathscr{R}})^2 \le \mathfrak{l}$ and $0 \le (\mathbb{U}^{\mathrm{M}}_{\mathscr{I}})^2 + (\mathbb{U}^{N}_{\mathscr{I}})^2 \le \mathfrak{l}$	CPFSs → CFSs and CIFSs, but CPFSs ↚ CFSs and CIFSs
Liu et al. [13]	Cq-ROFSs	$0 \le (\mathbb{U}^{\mathrm{M}}_{\mathscr{R}})^{\Psi} + (\mathbb{U}^{N}_{\mathscr{R}})^{\Psi} \le \mathfrak{l}$ and $0 \le (\mathbb{U}^{\mathrm{M}}_{\mathscr{I}})^{\Psi} + (\mathbb{U}^{N}_{\mathscr{I}})^{\Psi} \le \mathfrak{l}$, $\Psi \ge \mathfrak{l}$	Cq-ROFSs → CFSs, CIFSs, and CPFSs, but Cq-ROFSs ↚ CFSs, CIFSs, and CPFSs
Atanassov [14]	Cir-IFS	$0 \le \mathbb{U}^{\mathrm{M}}_{\mathscr{R}} + \mathbb{U}^{N}_{\mathscr{R}}, \mathbb{U}^{\mathbb{R}}_{\mathscr{R}} \le \mathfrak{l}$	$\mathbb{U}^{\mathrm{M}}_{\mathscr{R}}$: membership function, $\mathbb{U}^{N}_{\mathscr{R}}$: non-membership function, $\mathbb{U}^{\mathbb{R}}_{\mathscr{R}}$: radius function
Olgun and Unver [15]	Cir-PFSs	$0 \le (\mathbb{U}^{\mathrm{M}}_{\mathscr{R}})^2 + (\mathbb{U}^{N}_{\mathscr{R}})^2, \mathbb{U}^{\mathbb{R}}_{\mathscr{R}} \le \mathfrak{l}$	Cir-PFSs → Circular-FSs and Cir-IFSs, but Cir-PFSs ↚ Circular-FSs and Cir-IFSs
Yusoff et al. [16]	Cirq-ROFSs	$0 \le (\mathbb{U}^{\mathrm{M}}_{\mathscr{R}})^{\Psi} + (\mathbb{U}^{N}_{\mathscr{R}})^{\Psi}, \mathbb{U}^{\mathbb{R}}_{\mathscr{R}} \le \mathfrak{l}$, $\Psi \ge \mathfrak{l}$	Cirq-ROFSs → Circular-FSs, Cir-IFSs, and Cir-PFSs, but Cirq-ROFSs ↚ Circular-FSs, Cir-IFSs, and Cir-PFSs
Ali and Yang [17]	Cirq-ROFSs	$0 \le (\mathbb{U}^{\mathrm{M}}_{\mathscr{R}})^{\Psi} + (\mathbb{U}^{N}_{\mathscr{R}})^{\Psi}, \mathbb{U}^{\mathbb{R}}_{\mathscr{R}} \le \mathfrak{l}$, $\Psi \ge \mathfrak{l}$	Cirq-ROFSs → Circular-FSs, Cir-IFSs, and Cir-PFSs, but Cirq-ROFSs ↚ Circular-FSs, Cir-IFSs, and Cir-PFSs
Ibrahim and Alshammari [18]	p, q-ROFSs	$0 \le (\mathbb{U}^{\mathrm{M}}_{\mathscr{R}})^{\Psi} + (\mathbb{U}^{N}_{\mathscr{R}})^{\psi} \le \mathfrak{l}$, $\Psi, \psi \ge \mathfrak{l}$	p, q-ROFSs → FSs, IFSs, PFSs, and q-ROFSs, but p, q-ROFSs ↚ FSs, IFSs, PFSs, and q-ROFSs
Ibrahim [19]	Cp, q-ROFSs	$0 \le (\mathbb{U}^{\mathrm{M}}_{\mathscr{R}})^{\Psi} + (\mathbb{U}^{N}_{\mathscr{R}})^{\psi} \le \mathfrak{l}$, and $0 \le (\mathbb{U}^{\mathrm{M}}_{\mathscr{I}})^{\Psi} + (\mathbb{U}^{N}_{\mathscr{I}})^{\psi} \le \mathfrak{l}$ $\Psi, \psi \ge \mathfrak{l}$	Cp, q-ROFSs → CFSs, CIFSs, CPFSs, and Cq-ROFSs, but Cp, q-ROFSs ↚ CFSs, CIFSs, CPFSs, and Cq-ROFSs

development based on crisp set theory, but many people have lost their information during decision-making procedures because of limited information. For this, Zadeh [4] computed the technique of fuzzy sets (FSs), where the membership function is the part of FSs. Numerous researchers used FSs in decison maing process [5], owever, the membership function in fuzzy sets is not enough to cope with uncertain and complex problems, and for this, we need two types of information which will be used for yes and no. Therefore, Atanassov [6,7] proposed the intuitionistic fuzzy sets (IFSs), which contained two different functions, called membership function and non-membership function. Further, in 2013, Yager [8] enhanced the condition of IFSs and initiated the technique of Pythagorean fuzzy sets (PFSs), but in many cases, the techniques of IFSs and PFSS have not worked effectively. For this, in 2016, Yager [9] again derived the technique of q-rung orthopair fuzzy sets (q-ROFSs). From the condition of the q-ROFSs, it is clear that the technique of FSs, IFSs, and PFSs are the major cases of the q-ROFSs. Complexity and ambiguity are involved in every field of life, some occur in the shape of one-dimension and some of them in the shape of two-dimension, for coping with the one-dimension problem, the existing technique of FSs, IFSs, PFSs, and q-ROFSs are very reliable tools, but for coping with the two-dimension problem, we need a strong idea and reliable concept. For this, Ramot et al. [10] computed a new idea of complex FSs (CFSs), where the membership function in CFSs is computed in the shape of a complex number, which is a suitable technique for coping with problems is available in the shape of two-dimensional information. Further, Alkouri and Salleh [11] presented the technique of complex IFSs (CIFSs). Moreover, Ullah et al. [12] derived the technique of complex PFSs (CPFSs). Moreover, Liu et al. [13] proposed the theory of complex q-ROFSs (Cq-ROFSs). Noticed that the prevailing technique of FSs, IFSS, PFSs, q-ROFSs, CFSs, CIFSs, and CPFSs are the major parts of the Cq-ROFSs. In 2020, Atanassov [14] introduced a new technique of circular IFSs (Cir-IFSs), which contained the term membership function and non-membership function with radius function. The technique of Cir-IFSs is the dominant technique for depicting vague and uncertain information. Further, the technique of circular PFSs (Cir-PFSs) was proposed by Olgun and Unver [15] in 2023. In 2023, Yusoff et al. [16] derived the technique of circular q-ROFSs (Cirq-ROFSs). From the condition of the Cirq-ROFSs, it is clear that the technique of FSs, IFSs, Cir-IFSs, PFSs, Cir-PFSs, and q-ROFSs are the major cases of the Cirq-ROFSs. Some Dombi operators for Cirq-ROFSs were proposed by Ali and Yang [17]. Different kinds of extensions have been developed by different scholars, but the proposed technique of Ibrahim and Alshammari [18] has received more attention because of their features, called p, q-rung orthopair fuzzy sets (p, q-ROFSs), where the membership and non-membership functions are part of p, q-ROFSs. Further, in 2024, Ibrahim [19] initiated the technique of complex p, q-ROFSs (Cp, q-ROFSs). The limitations and advantages of the existing techniques are described in Table 1.

In Table 1, we briefly explained the limitations and features of the existing technique and based on these limitations, we aim to eliminate the above problems by proposing some new techniques and models.

1.1. Literature review

The model of fuzzy superior Mandelbrot set was derived by Mahmood and Ali [20] in 2022, which is the modified version of FS theory. Further, Mahmood et al. [21] exposed the model of interdependency of complex fuzzy neighborhood operators. The model of aggregation operators based on Dombi norms for IFSs was invented by Seikh and Mandal [22] in 2021. The model of FSs to q-ROFSs are dominant techniques, but the proposed technique of quasirung orthopair fuzzy sets [23] is more reliable and flexible because of their features and verity, whereas the 3, 4-quasirung orthopair fuzzy sets were invented by Seikh and Mandal [24] in 2022. Additionally, the model of average/geometric operators based on frank norms for q-ROFSs was derived by Seikh and Mandal [25]. Recently, Seikh and Chatterjee [26] evaluated the model of a sustainable strategy for electronic waste management based on interval-valued Fermatean fuzzy information. The determination of the most preferable renewable energy source based on interval-valued Fermatean fuzzy information was presented by Seikh and Chatterjee [27]. In 1975, Zadeh [28–30] initiated the theory of linguistic term sets (LTSs), which contained their elements in the shape of linguistic terms. Further, Herrera and Martinez [31] proposed the 2-tuple fuzzy linguistic sets, whereas Herrera and Martinez [32] derived the 2-tuple linguistic sets (2-TLSs). Zhang et al. [33] presented the decision-making technique based on 2-tuple linguistic information based on the cloud technique. In 2001, Yager [34] computed the power average (PA) operators, where the power geometric (PG) operators were initiated by Xu and Yager [35]. Further, Hamacher [36] initiated the technique of Hamacher t-norm (HTN) and Hamacher t-conorm (HTCN), which are the modified versions of the algebraic and Einstein norms. Further, many types of operators based on IFSs to q-ROFSs have been proposed, for instance, power aggregation operators for CIFSs were proposed by Rani and Garg [37]. Moreover, the model of Aczel-Alsina power aggregation operators for CIFSs was evaluated by Mahmood and Ali [38]. Jiang et al. [39] examined the model of power aggregation operators for IFSs. Wei and Lu [40] presented the model of power aggregation operators for PFSs. Wei [41] proposed the model of Hamacher power aggregation operators for PFSs. Yu [42] addressed the technique of Hamacher aggregation operators for complex cubic q-ROFSs. In 2020, Darko and Liang [43] invented the theory of Hamacher aggregation operators for q-ROFSs. Further, Sing et al. [44] proposed the fuzzy difference ideology and Sahoo et al. [45] evaluated the decision-making technique. Moreover, Hussain and Ullah [46] presented the spherical fuzzy Sugeno-Weber operators and Imran et al. [47] discussed the interval-valued intuitionistic fuzzy Aczel-Alsina Bonferroni mean operators.

1.2. Research Gap and Motivation of the proposed theory

From the start of FSs up to Cp, q-ROFSs, many types of applications have been developed by different scholars. During these implementations, all experts observed that the following problems are the major parts of every decision-making procedure, instance: (i) How do we construct the model of circular complex p, q-quasirung orthopair fuzzy 2-tuple linguistic sets? (ii) how do we evaluate the model of Hamacher power operators based on the proposed technique? (iii) how do we address the most preferable optimal among the collection of alternatives? Every decision-making procedure lost a lot of information because of ambiguity and limitations [48]. To evaluate the above problems, we aim to evaluate the model of circular complex p, q-quasirung orthopair fuzzy 2-tuple linguistic sets, because the model of CCp, q-QOF2-TL sets is very dominant because of their features, where the following techniques and model are the special cases of the presented information, such as.

1) FSs (if $\mathbb{U}_{\mathscr{J}}^{\mathrm{M}} = \mathbb{U}_{\mathscr{R}}^{N} = \mathbb{U}_{\mathscr{J}}^{N} = \mathbb{U}_{\mathscr{R}}^{\mathbb{R}} = \mathbb{U}_{\mathscr{J}}^{\mathbb{R}} = (\mathbb{s}_i, \mu_i) = 0$ and $\Psi, \psi = \mathfrak{l}, \mathbb{U}_{\mathscr{R}}^{\mathbb{R}}$).
2) IFSs (if $\mathbb{U}_{\mathscr{J}}^{\mathrm{M}} = \mathbb{U}_{\mathscr{J}}^{N} = \mathbb{U}_{\mathscr{R}}^{\mathbb{R}} = \mathbb{U}_{\mathscr{J}}^{\mathbb{R}} = (\mathbb{s}_i, \mu_i) = 0$ and $\Psi, \psi = \mathfrak{l}$).
3) PFSs (if $\mathbb{U}_{\mathscr{J}}^{\mathrm{M}} = \mathbb{U}_{\mathscr{J}}^{N} = \mathbb{U}_{\mathscr{R}}^{\mathbb{R}} = \mathbb{U}_{\mathscr{J}}^{\mathbb{R}} = (\mathbb{s}_i, \mu_i) = 0$ and $\Psi, \psi = 2$).
4) q-ROFSs (if $\mathbb{U}_{\mathscr{J}}^{\mathrm{M}} = \mathbb{U}_{\mathscr{J}}^{N} = \mathbb{U}_{\mathscr{R}}^{\mathbb{R}} = \mathbb{U}_{\mathscr{J}}^{\mathbb{R}} = (\mathbb{s}_i, \mu_i) = 0$ and $\Psi = \psi$).
5) CFSs (if $\mathbb{U}_{\mathscr{R}}^{N} = \mathbb{U}_{\mathscr{J}}^{N} = \mathbb{U}_{\mathscr{R}}^{\mathbb{R}} = \mathbb{U}_{\mathscr{J}}^{\mathbb{R}} = (\mathbb{s}_i, \mu_i) = 0$ and $\Psi, \psi = \mathfrak{l}$).
6) CIFSs (if $\mathbb{U}_{\mathscr{R}}^{\mathbb{R}} = \mathbb{U}_{\mathscr{J}}^{\mathbb{R}} = (\mathbb{s}_i, \mu_i) = 0$ and $\Psi, \psi = \mathfrak{l}$).
7) CPFSs (if $\mathbb{U}_{\mathscr{R}}^{\mathbb{R}} = \mathbb{U}_{\mathscr{J}}^{\mathbb{R}} = (\mathbb{s}_i, \mu_i) = 0$ and $\Psi, \psi = 2$).
8) Cq-ROFSs (if $\mathbb{U}_{\mathscr{R}}^{\mathbb{R}} = \mathbb{U}_{\mathscr{J}}^{\mathbb{R}} = (\mathbb{s}_i, \mu_i) = 0$ and $\Psi = \psi$).

These all are the special cases of the CCp, q-QOF2-TL sets, where the model of 2-tuple linguistic sets and their extensions, circular fuzzy sets, and their extensions are also some dominant cases of the proposed theory. Further, we aim to derive the model of Hamacher power aggregation operators based on CCp, q-QOF2-TL sets, because the model of Hamacher power operators is more dominant and modified, some special cases of the proposed operators are illustrated below.

1) Algebraic, Einstein, power, and Hamacher operators for FSs to proposed p, q-ROFSs.
2) Algebraic, Einstein, power, and Hamacher operators for CFSs to proposed Cp, q-ROFSs.
3) Algebraic, Einstein, power, and Hamacher operators for circular FSs to proposed circular p, q-ROFSs.
4) Algebraic, Einstein, power, and Hamacher operators for fuzzy linguistic sets to proposed p, q-ROF linguistic sets.
5) Algebraic, Einstein, power, and Hamacher operators for 2-TLSs to proposed CCp, q-QOF2-TL sets.

Similarly, a lot of cases are part of the proposed theory. Keeping the limitations and advantages of the proposed theory, we aim to evaluate the model of Hamacher power operators for CCp, q-QOF2-TL sets.

1.3. The major contribution of the proposed theory

After a long discussion, we observed that the model of CCp, q-QOF2-TL sets is very reliable because of their unlimited features,

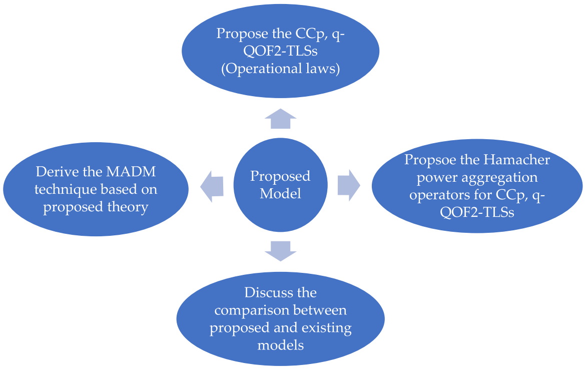

Fig. 1. Geometrical abstract of the proposed information.

therefore, we aim to combine the above-all ideas and compute the technique of circular complex p, q-Quasirung orthopair fuzzy 2-tuple linguistic sets (CCp, q-ROF2-TLSs), because the above-all techniques are the special cases of the proposed theory. Further, we aim to compute some operational laws for the construction of any kind of aggregation operators, called algebraic and Hamacher operational laws. Therefore, the major contribution of this article is listed below.

1) To develop the novel technique of CCp, q-QOF2-TL set and their operational laws based on algebraic t-norms and Hamacher t-norms, where the algebraic t-norms and Einstein t-norms are the special cases of the Hamacher t-norms for parameter $F Ⅎ^{s} = \mathfrak{l}$ and $F Ⅎ^{s} = 2$.
2) To derive the Hamacher power aggregation operators based on any finite collection of CCp, q-QOF2-TLNs, called CCp, q-QOF2-TLHPA operator, CCp, q-QOF2-TLHPWA operator, CCp, q-QOF2-TLHPG operator, CCp, q-QOF2-TLHPWG operator, and described their basic properties, called idempotency, monotonicity, and boundedness.
3) To demonstrate the technique of the MADM problem based on the above operators to evaluate the major factor that will be playing in the development of the green industry.
4) To compare the proposed ranking values with the obtained ranking values of existing techniques to show the supremacy and superiority of the initiated approaches.

1.4. Summary of the proposed theory

This article is summarized in the following form, such as.

In Section 2, we discussed the existing drawbacks of the Cp, q-QOFSs with their properties. Further, we revised to discuss the theory of 2-TLSs, PA operator, PG operator, HTN, and HTCN based on the unit interval.

In Section 3, we developed the novel technique of CCp, q-QOF2-TL set and their operational laws based on algebraic t-norms and Hamacher t-norms, where the algebraic t-norms and Einstein t-norms are the special cases of the Hamacher t-norms for parameter $F Ⅎ^{s} = \mathfrak{l}$ and $F Ⅎ^{s} = 2$.

In Section 4, we derived the CCp, q-QOF2-TLHPA operator, CCp, q-QOF2-TLHPWA operator, CCp, q-QOF2-TLHPG operator, CCp, q-QOF2-TLHPWG operator, and described their basic properties, called idempotency, monotonicity, and boundedness.

In Section 5, we demonstrated the technique of the MADM problem based on the above operators to evaluate the major factor that will be playing in the development of the green industry.

In Section 6, we compared the proposed ranking values with the obtained ranking values of existing techniques to show the supremacy and superiority of the initiated approaches.

Some concluding information is stated in Section 7. Further, the flowchart of the proposed model is described in Fig. 1.

2. Preliminaries

In this section, we discussed the existing drawbacks of the Cp, q-QOFSs with their properties. Further, we revised to discuss the theory of 2-TLSs, PA operator, PG operator, HTN, and HTCN based on the unit interval.

Definition 1. [19] Based on a universal set K, the theory of Cp, q-QOFSs is demonstrated below:

$$\mathfrak{N} = \{(\mathbb{U}^{M}(x), \mathbb{U}^{N}(x)) : x \in X\}$$

where, $\mathbb{U}^{M}(x) = \left(\mathbb{U}^{M}_{\mathscr{R}}(x), \mathbb{U}^{M}_{\mathscr{I}}(x)\right)$ and $\mathbb{U}^{N}(x) = \left(\mathbb{U}^{N}_{\mathscr{R}}(x), \mathbb{U}^{N}_{\mathscr{I}}(x)\right)$ signified the membership function and non-membership function in the form of a complex number with a condition that is $0 \leq \left(\mathbb{U}^{M}_{\mathscr{R}}(x)\right)^{\Psi} + \left(\mathbb{U}^{N}_{\mathscr{R}}(x)\right)^{\psi} \leq \mathfrak{l}$ and $0 \leq \left(\mathbb{U}^{M}_{\mathscr{I}}(x)\right)^{\Psi} + \left(\mathbb{U}^{N}_{\mathscr{I}}(x)\right)^{\psi} \leq \mathfrak{l}$, where the term $\mathbb{U}^{\mathbb{R}}(x) = .\left(\mathbb{U}^{\mathbb{R}}_{\mathscr{R}}(x), \mathbb{U}^{\mathbb{R}}_{\mathscr{I}}(x)\right) = \left(\left(\mathfrak{l} - \left(\left(\mathbb{U}^{M}_{\mathscr{R}}(x)\right)^{\Psi} + \left(\mathbb{U}^{N}_{\mathscr{R}}(x)\right)^{\psi}\right)\right)^{\frac{\mathfrak{l}}{\max(\Psi,\psi)}}, \left(\mathfrak{l} - \left(\left(\mathbb{U}^{M}_{\mathscr{I}}(x)\right)^{\Psi} + \left(\mathbb{U}^{N}_{\mathscr{I}}(x)\right)^{\psi}\right)\right)^{\frac{\mathfrak{l}}{\max(\Psi,\psi)}}\right)$. Further, we described the final form of the Cp, q-QOF number (Cp, q-QOFN), such as $\mathfrak{N}^{j} = \left(\left(\mathbb{U}_{\mathscr{R}}^{M^{j}}, \mathbb{U}_{\mathscr{J}}^{M^{j}}\right), \left(\mathbb{U}_{\mathscr{R}}^{N^{j}}, \mathbb{U}_{\mathscr{J}}^{N^{j}}\right)\right), j = \mathfrak{l}, 2, \ldots, m$.

Definition 2. [19] Based on any two Cp, q-QOFNs $\mathfrak{N}^{j} = \left(\left(\mathbb{U}_{\mathscr{R}}^{M^{j}}, \mathbb{U}_{\mathscr{J}}^{M^{j}}\right), \left(\mathbb{U}_{\mathscr{R}}^{N^{j}}, \mathbb{U}_{\mathscr{J}}^{N^{j}}\right)\right), j = \mathfrak{l}, 2$, we described some operational laws, such as

$$\mathfrak{N}^{\mathfrak{l}} \oplus \mathfrak{N}^{2} = \left(\begin{array}{c} \left(\left(\left(\mathbb{U}_{\mathscr{R}}^{M^{\mathfrak{l}}}\right)^{\Psi} + \left(\mathbb{U}_{\mathscr{R}}^{M^{2}}\right)^{\Psi} - \left(\mathbb{U}_{\mathscr{R}}^{M^{\mathfrak{l}}}\right)^{\Psi}\left(\mathbb{U}_{\mathscr{R}}^{M^{2}}\right)^{\Psi}\right)^{\frac{\mathfrak{l}}{\Psi}}, \left(\left(\mathbb{U}_{\mathscr{J}}^{M^{\mathfrak{l}}}\right)^{\Psi} + \left(\mathbb{U}_{\mathscr{J}}^{M^{2}}\right)^{\Psi} - \left(\mathbb{U}_{\mathscr{J}}^{M^{\mathfrak{l}}}\right)^{\Psi}\left(\mathbb{U}_{\mathscr{J}}^{M^{2}}\right)^{\Psi}\right)^{\frac{\mathfrak{l}}{\Psi}}\right), \\ \left(\left(\mathbb{U}_{\mathscr{R}}^{N^{\mathfrak{l}}}\right)\left(\mathbb{U}_{\mathscr{R}}^{N^{2}}\right), \left(\mathbb{U}_{\mathscr{J}}^{N^{\mathfrak{l}}}\right)\left(\mathbb{U}_{\mathscr{J}}^{N^{2}}\right)\right) \end{array}\right)$$

$$\mathfrak{N}^{\mathfrak{l}} \otimes \mathfrak{N}^{2} = \left(\begin{array}{c} \left(\left(\mathbb{U}_{\mathscr{R}}^{M^{\mathfrak{l}}}\right)\left(\mathbb{U}_{\mathscr{R}}^{M^{2}}\right), \left(\mathbb{U}_{\mathscr{J}}^{M^{\mathfrak{l}}}\right)\left(\mathbb{U}_{\mathscr{J}}^{M^{2}}\right)\right), \\ \left(\left(\left(\mathbb{U}_{\mathscr{R}}^{N^{\mathfrak{l}}}\right)^{\psi} + \left(\mathbb{U}_{\mathscr{R}}^{N^{2}}\right)^{\psi} - \left(\mathbb{U}_{\mathscr{R}}^{N^{\mathfrak{l}}}\right)^{\psi}\left(\mathbb{U}_{\mathscr{R}}^{N^{2}}\right)^{\psi}\right)^{\frac{\mathfrak{l}}{\psi}}, \left(\left(\mathbb{U}_{\mathscr{J}}^{N^{\mathfrak{l}}}\right)^{\psi} + \left(\mathbb{U}_{\mathscr{J}}^{N^{2}}\right)^{\psi} - \left(\mathbb{U}_{\mathscr{J}}^{N^{\mathfrak{l}}}\right)^{\psi}\left(\mathbb{U}_{\mathscr{J}}^{N^{2}}\right)^{\psi}\right)^{\frac{\mathfrak{l}}{\psi}}\right) \end{array}\right)$$

$$\mathrm{T}^{s}\mathfrak{N}^{\mathfrak{l}} = \left(\begin{array}{c} \left(\left(\mathfrak{l} - \left(\mathfrak{l} - \left(\mathbb{U}_{\mathscr{R}}^{M^{\mathfrak{l}}}\right)^{\Psi}\right)^{\mathrm{T}^{s}}\right)^{\frac{\mathfrak{l}}{\Psi}}, \left(\mathfrak{l} - \left(\mathfrak{l} - \left(\mathbb{U}_{\mathscr{J}}^{M^{\mathfrak{l}}}\right)^{\Psi}\right)^{\mathrm{T}^{s}}\right)^{\frac{\mathfrak{l}}{\Psi}}\right), \\ \left(\left(\mathbb{U}_{\mathscr{R}}^{N^{\mathfrak{l}}}\right)^{\mathrm{T}^{s}}\left(\mathbb{U}_{\mathscr{R}}^{N^{2}}\right)^{\mathrm{T}^{s}}, \left(\mathbb{U}_{\mathscr{J}}^{N^{\mathfrak{l}}}\right)^{\mathrm{T}^{s}}\left(\mathbb{U}_{\mathscr{J}}^{N^{2}}\right)^{\mathrm{T}^{s}}\right) \end{array}\right)$$

$$\left(\mathfrak{N}^{\mathfrak{l}}\right)^{\mathrm{T}^{s}} = \left(\begin{array}{c} \left(\left(\mathbb{U}_{\mathscr{R}}^{M^{\mathfrak{l}}}\right)^{\mathrm{T}^{s}}\left(\mathbb{U}_{\mathscr{R}}^{M^{2}}\right)^{\mathrm{T}^{s}}, \left(\mathbb{U}_{\mathscr{J}}^{M^{\mathfrak{l}}}\right)^{\mathrm{T}^{s}}\left(\mathbb{U}_{\mathscr{J}}^{M^{2}}\right)^{\mathrm{T}^{s}}\right), \\ \left(\left(\mathfrak{l} - \left(\mathfrak{l} - \left(\mathbb{U}_{\mathscr{R}}^{N^{\mathfrak{l}}}\right)^{\psi}\right)^{\mathrm{T}^{s}}\right)^{\frac{\mathfrak{l}}{\psi}}, \left(\mathfrak{l} - \left(\mathfrak{l} - \left(\mathbb{U}_{\mathscr{J}}^{N^{\mathfrak{l}}}\right)^{\psi}\right)^{\mathrm{T}^{s}}\right)^{\frac{\mathfrak{l}}{\psi}}\right) \end{array}\right)$$

Definition 3. [19] Based on any Cp, q-QOFN $\mathfrak{N}^{j} = \left(\left(\mathbb{U}_{\mathscr{R}}^{M^{j}}, \mathbb{U}_{\mathscr{J}}^{M^{j}}\right), \left(\mathbb{U}_{\mathscr{R}}^{N^{j}}, \mathbb{U}_{\mathscr{J}}^{N^{j}}\right)\right), j = \mathfrak{l}$, we described the idea of score value and accuracy value, such as

$$SC\left(\mathfrak{N}^{j}\right) = \frac{\mathfrak{l}}{2}\left(\left(\mathbb{U}_{\mathscr{R}}^{M^{j}}\right)^{\Psi} + \left(\mathbb{U}_{\mathscr{J}}^{M^{j}}\right)^{\Psi} - \left(\mathbb{U}_{\mathscr{R}}^{N^{j}}\right)^{\psi} - \left(\mathbb{U}_{\mathscr{J}}^{N^{j}}\right)^{\psi}\right) \in [-\mathfrak{l}, \mathfrak{l}]$$

$$AC\left(\mathfrak{N}^{j}\right) = \frac{\mathfrak{l}}{2}\left(\left(\mathbb{U}_{\mathscr{R}}^{M^{j}}\right)^{\Psi} + \left(\mathbb{U}_{\mathscr{J}}^{M^{j}}\right)^{\Psi} + \left(\mathbb{U}_{\mathscr{R}}^{N^{j}}\right)^{\psi} + \left(\mathbb{U}_{\mathscr{J}}^{N^{j}}\right)^{\psi}\right) \in [0, \mathfrak{l}]$$

For differentiation of any two Cp, q-QORNs, we have the following rules, such as.

1) If $SC\left(\mathfrak{N}^{\mathfrak{l}}\right) > SC\left(\mathfrak{N}^{2}\right)$, thus $\mathfrak{N}^{\mathfrak{l}} > \mathfrak{N}^{2}$.
2) If $SC\left(\mathfrak{N}^{\mathfrak{l}}\right) < SC\left(\mathfrak{N}^{2}\right)$, thus $\mathfrak{N}^{\mathfrak{l}} < \mathfrak{N}^{2}$, but if $SC\left(\mathfrak{N}^{\mathfrak{l}}\right) = SC\left(\mathfrak{N}^{2}\right)$, thus
   i) If $AC\left(\mathfrak{N}^{\mathfrak{l}}\right) > AC\left(\mathfrak{N}^{2}\right)$, thus $\mathfrak{N}^{\mathfrak{l}} > \mathfrak{N}^{2}$.
   ii) If $AC\left(\mathfrak{N}^{\mathfrak{l}}\right) < AC\left(\mathfrak{N}^{2}\right)$, thus $\mathfrak{N}^{\mathfrak{l}} < \mathfrak{N}^{2}$.

Definition 4. [31,32] Based on any LTS $\mathbb{S} = \{\mathbb{s}_0, \mathbb{s}_{\mathfrak{l}}, \ldots, \mathbb{s}_{\mathbb{g}}\}$ with order $\mathbb{g}$ and $\eta \in [0, \mathbb{g}]$, thus

$$\mathrm{T} : [0, \mathbb{g}] \rightarrow \mathbb{S} \times [-0.5, 0.5)$$

$$\mathrm{T}(\eta) = (\mathbb{s}_i, \mu_i) \; with \begin{cases} \mathbb{s}_i & i = round(\eta) \\ \mu_i = \eta - i & \mu_i \in [-0.5, 0.5) \end{cases}$$

Called the 2-TLS, where the inverse function $\mathrm{T}^{-\mathfrak{l}}$ is described below:

$$\mathrm{T}^{-\mathfrak{l}} : \mathbb{S} \times [-0.5, 0.5) \rightarrow [0, \mathbb{g}]$$

$$\mathrm{T}^{-\mathfrak{l}}(\mathbb{s}_i, \mu_i) = i + \mu_i = \eta.$$

Definition 5. [26,27] Based on any positive integers $\mathfrak{N}^{j}, j = \mathfrak{l}, 2, \ldots, m$, we described the idea of PA operator and PG operator, such as

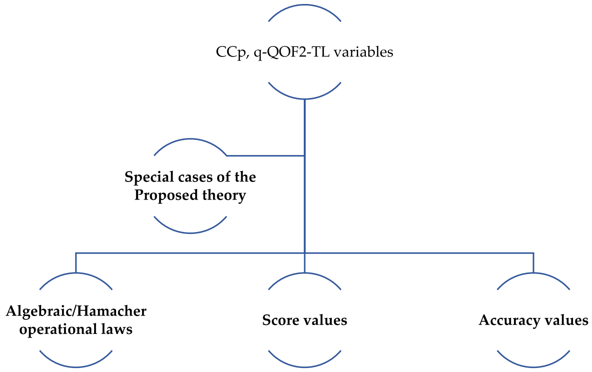

Fig. 2. Geometrical abstract of the proposed operators.

$$PA\left(\mathfrak{N}^{\text{ſ}}, \mathfrak{N}^{2}, \ldots, \mathfrak{N}^{m}\right) = \sum_{j=\text{ſ}}^{m} \frac{\left(\text{ſ} + \#\left(\mathfrak{N}^{j}\right)\right)\mathfrak{N}^{j}}{\sum_{j=\text{ſ}}^{m}\left(\text{ſ} + \#\left(\mathfrak{N}^{j}\right)\right)}$$

$$PG\left(\mathfrak{N}^{\text{ſ}}, \mathfrak{N}^{2}, \ldots, \mathfrak{N}^{m}\right) = \prod_{j=\text{ſ}}^{m} \left(\mathfrak{N}^{j}\right)^{\frac{\left(\text{ſ}+\#\left(\mathfrak{N}^{j}\right)\right)}{\sum_{j=\text{ſ}}^{m}\left(\text{ſ}+\#\left(\mathfrak{N}^{j}\right)\right)}}$$

Further, we explained the term $\#\left(\mathfrak{N}^{j}\right) = \sum_{j\neq k=\text{ſ}}^{m} \sup\left(\mathfrak{N}^{j}, \mathfrak{N}^{k}\right)$ and $\sup\left(\mathfrak{N}^{j}, \mathfrak{N}^{k}\right) = \text{dis}\left(\mathfrak{N}^{j}, \mathfrak{N}^{k}\right)$ with some condition that is: (i) $\sup\left(\mathfrak{N}^{j}, \mathfrak{N}^{k}\right) \in [0, \text{ſ}]$; (ii) $\sup\left(\mathfrak{N}^{j}, \mathfrak{N}^{k}\right) = \sup\left(\mathfrak{N}^{k}, \mathfrak{N}^{j}\right)$; and (iii) if $\sup\left(\mathfrak{N}^{j}, \mathfrak{N}^{k}\right) > \sup\left(\mathfrak{N}^{l}, \mathfrak{N}^{m}\right)$, then $\text{dis}\left(\mathfrak{N}^{j}, \mathfrak{N}^{k}\right) < \text{dis}\left(\mathfrak{N}^{l}, \mathfrak{N}^{m}\right)$.

Definition 6. [36] Based on any two positive integers $\mathfrak{N}^{j} \in [0, \text{ſ}], j = \text{ſ}, 2$, we described the idea of HTN ($\mathfrak{N}^{\text{ſ}} \otimes \mathfrak{N}^{2}$) and HTCN ($\mathfrak{N}^{\text{ſ}} \oplus \mathfrak{N}^{2}$), such as

$$HTN\left(\mathfrak{N}^{\text{ſ}}, \mathfrak{N}^{2}\right) = \mathfrak{N}^{\text{ſ}} \otimes \mathfrak{N}^{2} = \frac{\mathfrak{N}^{\text{ſ}}\mathfrak{N}^{2}}{F\Finv^{s} + \left(\text{ſ} - F\Finv^{s}\right)\left(\mathfrak{N}^{\text{ſ}} + \mathfrak{N}^{2} - \mathfrak{N}^{\text{ſ}}\mathfrak{N}^{2}\right)}, F\Finv^{s} > 0$$

$$HTCN\left(\mathfrak{N}^{\text{ſ}}, \mathfrak{N}^{2}\right) = \mathfrak{N}^{\text{ſ}} \oplus \mathfrak{N}^{2} = \frac{\mathfrak{N}^{\text{ſ}} + \mathfrak{N}^{2} - \mathfrak{N}^{\text{ſ}}\mathfrak{N}^{2} - \left(\text{ſ} - F\Finv^{s}\right)\mathfrak{N}^{\text{ſ}}\mathfrak{N}^{2}}{\text{ſ} - \left(\text{ſ} - F\Finv^{s}\right)\mathfrak{N}^{\text{ſ}}\mathfrak{N}^{2}}, F\Finv^{s} > 0$$

Further, to fix the value of $F\Finv^{s} = \text{ſ}$, thus we have

$$ATN\left(\mathfrak{N}^{\text{ſ}}, \mathfrak{N}^{2}\right) = \mathfrak{N}^{\text{ſ}} \otimes \mathfrak{N}^{2} = \mathfrak{N}^{\text{ſ}}\mathfrak{N}^{2}$$

$$ATCN\left(\mathfrak{N}^{\text{ſ}}, \mathfrak{N}^{2}\right) = \mathfrak{N}^{\text{ſ}} \oplus \mathfrak{N}^{2} = \mathfrak{N}^{\text{ſ}} + \mathfrak{N}^{2} - \mathfrak{N}^{\text{ſ}}\mathfrak{N}^{2}$$

Called the algebraic t-norm (ATN) and algebraic t-conorm (ATCN). Further, to fix the value of $F\Finv^{s} = 2$, thus we have

$$ETN\left(\mathfrak{N}^{\text{ſ}}, \mathfrak{N}^{2}\right) = \mathfrak{N}^{\text{ſ}} \otimes \mathfrak{N}^{2} = \frac{\mathfrak{N}^{\text{ſ}}\mathfrak{N}^{2}}{\text{ſ} + \left(\text{ſ} - \mathfrak{N}^{\text{ſ}}\right)\left(\text{ſ} - \mathfrak{N}^{2}\right)}$$

$$ETCN\left(\mathfrak{N}^{\text{ſ}}, \mathfrak{N}^{2}\right) = \mathfrak{N}^{\text{ſ}} \oplus \mathfrak{N}^{2} = \frac{\mathfrak{N}^{\text{ſ}} + \mathfrak{N}^{2}}{\text{ſ} + \mathfrak{N}^{\text{ſ}}\mathfrak{N}^{2}}$$

Called the Einstein t-norm (ETN) and Einstein t-conorm (ETCN).

3. CCp, q-QOF2-TLSs

This section aims to introduce the novel concept of CCp, q-QOF2-TLSs, which is the modified version of many existing techniques. Further, we derive some basic operational laws and their related results based on any two CCp, q-QOF2-TLNs. The geometrical abstract of the proposed ideas is described in Fig. 2.

Definition 7. Based on a universal set K, the theory of CCp, q-QOF2-TSs is demonstrated below:

$$\mathfrak{N} = \left\{\left(\left(\mathbb{s}_{i}, \mu_{i}\right), \mathbb{U}^{M}(x), \mathbb{U}^{N}(x), \mathbb{U}^{\mathbb{R}}\right) : x \in X\right\}$$

where, $\mathbb{U}^{M}(x) = \left(\mathbb{U}^{M}_{\mathscr{R}}(x), \mathbb{U}^{M}_{\mathscr{I}}(x)\right), \mathbb{U}^{N}(x) = \left(\mathbb{U}^{N}_{\mathscr{R}}(x), \mathbb{U}^{N}_{\mathscr{I}}(x)\right)$, and $\mathbb{U}^{\mathbb{R}}(x) = \left(\mathbb{U}^{\mathbb{R}}_{\mathscr{R}}(x), \mathbb{U}^{\mathbb{R}}_{\mathscr{I}}(x)\right)$ signified the membership function, non-membership function, and radius in the form of a complex number with a condition that is $0 \leq \left(\mathbb{U}^{M}_{\mathscr{R}}(x)\right)^{\Psi} + \left(\mathbb{U}^{N}_{\mathscr{R}}(x)\right)^{\psi} \leq 1$ and $0 \leq \left(\mathbb{U}^{M}_{\mathscr{I}}(x)\right)^{\Psi} + \left(\mathbb{U}^{N}_{\mathscr{I}}(x)\right)^{\psi} \leq 1$ with 2-TLS $(\mathbb{s}_i, \mu_i)$, where the term $\mathbb{U}^{\mathbb{R}}(x) = \left(\mathbb{U}^{\mathbb{R}}_{\mathscr{R}}(x), \mathbb{U}^{\mathbb{R}}_{\mathscr{I}}(x)\right) = \left(\left(1 - \left(\left(\mathbb{U}^{M}_{\mathscr{R}}(x)\right)^{\Psi} + \left(\mathbb{U}^{N}_{\mathscr{R}}(x)\right)^{\psi}\right)\right)^{\frac{1}{\max(\Psi,\psi)}}, \left(1 - \left(\left(\mathbb{U}^{M}_{\mathscr{I}}(x)\right)^{\Psi} + \left(\mathbb{U}^{N}_{\mathscr{I}}(x)\right)^{\psi}\right)\right)^{\frac{1}{\max(\Psi,\psi)}}\right)$. Further, we described the final form of the CCp, q-QOF2-TL number (CCp, q-QOF2-TLN), such as $\mathfrak{N}^{j} = \left(\left(\mathbb{s}^{j}_{i}, \mu^{j}_{i}\right), \left(\mathbb{U}^{M^j}_{\mathscr{R}}, \mathbb{U}^{M^j}_{\mathscr{I}}\right), \left(\mathbb{U}^{N^j}_{\mathscr{R}}, \mathbb{U}^{N^j}_{\mathscr{I}}\right), \left(\mathbb{U}^{\mathbb{R}^j}_{\mathscr{R}}, \mathbb{U}^{\mathbb{R}^j}_{\mathscr{I}}\right)\right), j = 1, 2, \ldots, m$.

Definition 8. Based on any two CCp, q-QOF2-TLNs $\mathfrak{N}^{j} = \left(\left(\mathbb{s}^{j}_{i}, \mu^{j}_{i}\right), \left(\mathbb{U}^{M^j}_{\mathscr{R}}, \mathbb{U}^{M^j}_{\mathscr{I}}\right), \left(\mathbb{U}^{N^j}_{\mathscr{R}}, \mathbb{U}^{N^j}_{\mathscr{I}}\right), \left(\mathbb{U}^{\mathbb{R}^j}_{\mathscr{R}}, \mathbb{U}^{\mathbb{R}^j}_{\mathscr{I}}\right)\right), j = 1, 2$, we described some operational laws, such as

$$\mathfrak{N}^{1} \oplus_{TN} \mathfrak{N}^{2} = \begin{pmatrix} \mathrm{T}\left(\mathbb{g}\left(\left(\frac{\mathrm{T}^{-1}(\mathbb{s}^{1}_{i}, \mu^{1}_{i})}{\mathbb{g}}\right)^{\Psi} + \left(\frac{\mathrm{T}^{-1}(\mathbb{s}^{2}_{i}, \mu^{2}_{i})}{\mathbb{g}}\right)^{\Psi} - \left(\frac{\mathrm{T}^{-1}(\mathbb{s}^{1}_{i}, \mu^{1}_{i})}{\mathbb{g}}\right)^{\Psi}\left(\frac{\mathrm{T}^{-1}(\mathbb{s}^{2}_{i}, \mu^{2}_{i})}{\mathbb{g}}\right)^{\Psi}\right)^{\frac{1}{\Psi}}\right), \\ \left(\left(\left(\mathbb{U}^{M^1}_{\mathscr{R}}\right)^{\Psi} + \left(\mathbb{U}^{M^2}_{\mathscr{R}}\right)^{\Psi} - \left(\mathbb{U}^{M^1}_{\mathscr{R}}\right)^{\Psi}\left(\mathbb{U}^{M^2}_{\mathscr{R}}\right)^{\Psi}\right)^{\frac{1}{\Psi}}, \left(\left(\mathbb{U}^{M^1}_{\mathscr{I}}\right)^{\Psi} + \left(\mathbb{U}^{M^2}_{\mathscr{I}}\right)^{\Psi} - \left(\mathbb{U}^{M^1}_{\mathscr{I}}\right)^{\Psi}\left(\mathbb{U}^{M^2}_{\mathscr{I}}\right)^{\Psi}\right)^{\frac{1}{\Psi}}\right), \\ \left(\left(\mathbb{U}^{N^1}_{\mathscr{R}}\right)\left(\mathbb{U}^{N^2}_{\mathscr{R}}\right), \left(\mathbb{U}^{N^1}_{\mathscr{I}}\right)\left(\mathbb{U}^{N^2}_{\mathscr{I}}\right)\right), \\ \left(\left(\left(\mathbb{U}^{\mathbb{R}^1}_{\mathscr{R}}\right)^{\Psi} + \left(\mathbb{U}^{\mathbb{R}^2}_{\mathscr{R}}\right)^{\Psi} - \left(\mathbb{U}^{\mathbb{R}^1}_{\mathscr{R}}\right)^{\Psi}\left(\mathbb{U}^{\mathbb{R}^2}_{\mathscr{R}}\right)^{\Psi}\right)^{\frac{1}{\Psi}}, \left(\left(\mathbb{U}^{\mathbb{R}^1}_{\mathscr{I}}\right)^{\Psi} + \left(\mathbb{U}^{\mathbb{R}^2}_{\mathscr{I}}\right)^{\Psi} - \left(\mathbb{U}^{\mathbb{R}^1}_{\mathscr{I}}\right)^{\Psi}\left(\mathbb{U}^{\mathbb{R}^2}_{\mathscr{I}}\right)^{\Psi}\right)^{\frac{1}{\Psi}}\right) \end{pmatrix}$$

$$\mathfrak{N}^{1} \oplus_{TCN} \mathfrak{N}^{2} = \begin{pmatrix} \mathrm{T}\left(\mathbb{g}\left(\left(\frac{\mathrm{T}^{-1}(\mathbb{s}^{1}_{i}, \mu^{1}_{i})}{\mathbb{g}}\right)\left(\frac{\mathrm{T}^{-1}(\mathbb{s}^{2}_{i}, \mu^{2}_{i})}{\mathbb{g}}\right)\right)\right), \\ \left(\left(\left(\mathbb{U}^{M^1}_{\mathscr{R}}\right)^{\Psi} + \left(\mathbb{U}^{M^2}_{\mathscr{R}}\right)^{\Psi} - \left(\mathbb{U}^{M^1}_{\mathscr{R}}\right)^{\Psi}\left(\mathbb{U}^{M^2}_{\mathscr{R}}\right)^{\Psi}\right)^{\frac{1}{\Psi}}, \left(\left(\mathbb{U}^{M^1}_{\mathscr{I}}\right)^{\Psi} + \left(\mathbb{U}^{M^2}_{\mathscr{I}}\right)^{\Psi} - \left(\mathbb{U}^{M^1}_{\mathscr{I}}\right)^{\Psi}\left(\mathbb{U}^{M^2}_{\mathscr{I}}\right)^{\Psi}\right)^{\frac{1}{\Psi}}\right), \\ \left(\left(\mathbb{U}^{N^1}_{\mathscr{R}}\right)\left(\mathbb{U}^{N^2}_{\mathscr{R}}\right), \left(\mathbb{U}^{N^1}_{\mathscr{I}}\right)\left(\mathbb{U}^{N^2}_{\mathscr{I}}\right)\right), \\ \left(\left(\mathbb{U}^{\mathbb{R}^1}_{\mathscr{R}}\right)\left(\mathbb{U}^{\mathbb{R}^2}_{\mathscr{R}}\right), \left(\mathbb{U}^{\mathbb{R}^1}_{\mathscr{I}}\right)\left(\mathbb{U}^{\mathbb{R}^2}_{\mathscr{I}}\right)\right) \end{pmatrix}$$

$$\mathfrak{N}^{1} \otimes_{TN} \mathfrak{N}^{2} = \begin{pmatrix} \mathrm{T}\left(\mathbb{g}\left(\left(\frac{\mathrm{T}^{-1}(\mathbb{s}^{1}_{i}, \mu^{1}_{i})}{\mathbb{g}}\right)\left(\frac{\mathrm{T}^{-1}(\mathbb{s}^{2}_{i}, \mu^{2}_{i})}{\mathbb{g}}\right)\right)\right), \\ \left(\left(\mathbb{U}^{M^1}_{\mathscr{R}}\right)\left(\mathbb{U}^{M^2}_{\mathscr{R}}\right), \left(\mathbb{U}^{M^1}_{\mathscr{I}}\right)\left(\mathbb{U}^{M^2}_{\mathscr{I}}\right)\right), \\ \left(\left(\left(\mathbb{U}^{N^1}_{\mathscr{R}}\right)^{\psi} + \left(\mathbb{U}^{N^2}_{\mathscr{R}}\right)^{\psi} - \left(\mathbb{U}^{N^1}_{\mathscr{R}}\right)^{\psi}\left(\mathbb{U}^{N^2}_{\mathscr{R}}\right)^{\psi}\right)^{\frac{1}{\psi}}, \left(\left(\mathbb{U}^{N^1}_{\mathscr{I}}\right)^{\psi} + \left(\mathbb{U}^{N^2}_{\mathscr{I}}\right)^{\psi} - \left(\mathbb{U}^{N^1}_{\mathscr{I}}\right)^{\psi}\left(\mathbb{U}^{N^2}_{\mathscr{I}}\right)^{\psi}\right)^{\frac{1}{\psi}}\right), \\ \left(\left(\mathbb{U}^{\mathbb{R}^1}_{\mathscr{R}}\right)\left(\mathbb{U}^{\mathbb{R}^2}_{\mathscr{R}}\right), \left(\mathbb{U}^{\mathbb{R}^1}_{\mathscr{I}}\right)\left(\mathbb{U}^{\mathbb{R}^2}_{\mathscr{I}}\right)\right) \end{pmatrix}$$

$$\mathfrak{N}^{1} \otimes_{TCN} \mathfrak{N}^{2} = \begin{pmatrix} \mathrm{T}\left(\mathbb{g}\left(\left(\frac{\mathrm{T}^{-1}(\mathbb{s}^{1}_{i}, \mu^{1}_{i})}{\mathbb{g}}\right)^{\psi} + \left(\frac{\mathrm{T}^{-1}(\mathbb{s}^{2}_{i}, \mu^{2}_{i})}{\mathbb{g}}\right)^{\psi} - \left(\frac{\mathrm{T}^{-1}(\mathbb{s}^{1}_{i}, \mu^{1}_{i})}{\mathbb{g}}\right)^{\psi}\left(\frac{\mathrm{T}^{-1}(\mathbb{s}^{2}_{i}, \mu^{2}_{i})}{\mathbb{g}}\right)^{\psi}\right)^{\frac{1}{\psi}}\right), \\ \left(\left(\mathbb{U}^{M^1}_{\mathscr{R}}\right)\left(\mathbb{U}^{M^2}_{\mathscr{R}}\right), \left(\mathbb{U}^{M^1}_{\mathscr{I}}\right)\left(\mathbb{U}^{M^2}_{\mathscr{I}}\right)\right), \\ \left(\left(\left(\mathbb{U}^{N^1}_{\mathscr{R}}\right)^{\psi} + \left(\mathbb{U}^{N^2}_{\mathscr{R}}\right)^{\psi} - \left(\mathbb{U}^{N^1}_{\mathscr{R}}\right)^{\psi}\left(\mathbb{U}^{N^2}_{\mathscr{R}}\right)^{\psi}\right)^{\frac{1}{\psi}}, \left(\left(\mathbb{U}^{N^1}_{\mathscr{I}}\right)^{\psi} + \left(\mathbb{U}^{N^2}_{\mathscr{I}}\right)^{\psi} - \left(\mathbb{U}^{N^1}_{\mathscr{I}}\right)^{\psi}\left(\mathbb{U}^{N^2}_{\mathscr{I}}\right)^{\psi}\right)^{\frac{1}{\psi}}\right), \\ \left(\left(\left(\mathbb{U}^{\mathbb{R}^1}_{\mathscr{R}}\right)^{\psi} + \left(\mathbb{U}^{\mathbb{R}^2}_{\mathscr{R}}\right)^{\psi} - \left(\mathbb{U}^{\mathbb{R}^1}_{\mathscr{R}}\right)^{\psi}\left(\mathbb{U}^{\mathbb{R}^2}_{\mathscr{R}}\right)^{\psi}\right)^{\frac{1}{\psi}}, \left(\left(\mathbb{U}^{\mathbb{R}^1}_{\mathscr{I}}\right)^{\psi} + \left(\mathbb{U}^{\mathbb{R}^2}_{\mathscr{I}}\right)^{\psi} - \left(\mathbb{U}^{\mathbb{R}^1}_{\mathscr{I}}\right)^{\psi}\left(\mathbb{U}^{\mathbb{R}^2}_{\mathscr{I}}\right)^{\psi}\right)^{\frac{1}{\psi}}\right) \end{pmatrix}$$

$$\mathrm{T}^s *_{TN} \mathfrak{N}^1 = \begin{pmatrix} \mathrm{T}\left(\mathbb{g}\left(1-\left(1-\left(\left(\frac{\mathrm{T}^{-1}\left(\mathbb{s}_i^1,\mu_i^1\right)}{\mathbb{g}}\right)\right)^{\Psi}\right)^{\mathrm{T}^s}\right)^{\frac{1}{\Psi}}\right), \\ \left(\left(1-\left(1-\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^1}\right)^{\Psi}\right)^{\mathrm{T}^s}\right)^{\frac{1}{\Psi}},\left(1-\left(1-\left(\mathbb{U}_{\mathscr{I}}^{\mathrm{M}^1}\right)^{\Psi}\right)^{\mathrm{T}^s}\right)^{\frac{1}{\Psi}}\right), \\ \left(\left(\mathbb{U}_{\mathscr{R}}^{N^1}\right)^{\mathrm{T}^s}\left(\mathbb{U}_{\mathscr{R}}^{N^2}\right)^{\mathrm{T}^s},\left(\mathbb{U}_{\mathscr{I}}^{N^1}\right)^{\mathrm{T}^s}\left(\mathbb{U}_{\mathscr{I}}^{N^2}\right)^{\mathrm{T}^s}\right), \\ \left(\left(1-\left(1-\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^1}\right)^{\Psi}\right)^{\mathrm{T}^s}\right)^{\frac{1}{\Psi}},\left(1-\left(1-\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^1}\right)^{\Psi}\right)^{\mathrm{T}^s}\right)^{\frac{1}{\Psi}}\right) \end{pmatrix}$$

$$\mathrm{T}^s *_{TCN} \mathfrak{N}^1 = \begin{pmatrix} \mathrm{T}\left(\mathbb{g}\left(\frac{\mathrm{T}^{-1}\left(\mathbb{s}_i^1,\mu_i^1\right)}{\mathbb{g}}\right)^{\mathrm{T}^s}\right), \\ \left(\left(1-\left(1-\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^1}\right)^{\Psi}\right)^{\mathrm{T}^s}\right)^{\frac{1}{\Psi}},\left(1-\left(1-\left(\mathbb{U}_{\mathscr{I}}^{\mathrm{M}^1}\right)^{\Psi}\right)^{\mathrm{T}^s}\right)^{\frac{1}{\Psi}}\right), \\ \left(\left(\mathbb{U}_{\mathscr{R}}^{N^1}\right)^{\mathrm{T}^s}\left(\mathbb{U}_{\mathscr{R}}^{N^2}\right)^{\mathrm{T}^s},\left(\mathbb{U}_{\mathscr{I}}^{N^1}\right)^{\mathrm{T}^s}\left(\mathbb{U}_{\mathscr{I}}^{N^2}\right)^{\mathrm{T}^s}\right), \\ \left(\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^1}\right)^{\mathrm{T}^s}\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^2}\right)^{\mathrm{T}^s},\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^1}\right)^{\mathrm{T}^s}\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^2}\right)^{\mathrm{T}^s}\right) \end{pmatrix}$$

$$\left(\mathfrak{N}^1\right)^{\mathrm{T}^s_{TN}} = \begin{pmatrix} \mathrm{T}\left(\mathbb{g}\left(\frac{\mathrm{T}^{-1}\left(\mathbb{s}_i^1,\mu_i^1\right)}{\mathbb{g}}\right)^{\mathrm{T}^s}\right), \\ \left(\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^1}\right)^{\mathrm{T}^s}\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^2}\right)^{\mathrm{T}^s},\left(\mathbb{U}_{\mathscr{I}}^{\mathrm{M}^1}\right)^{\mathrm{T}^s}\left(\mathbb{U}_{\mathscr{I}}^{\mathrm{M}^2}\right)^{\mathrm{T}^s}\right), \\ \left(\left(1-\left(1-\left(\mathbb{U}_{\mathscr{R}}^{N^1}\right)^{\psi}\right)^{\mathrm{T}^s}\right)^{\frac{1}{\psi}},\left(1-\left(1-\left(\mathbb{U}_{\mathscr{I}}^{N^1}\right)^{\psi}\right)^{\mathrm{T}^s}\right)^{\frac{1}{\psi}}\right), \\ \left(\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^1}\right)^{\mathrm{T}^s}\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^2}\right)^{\mathrm{T}^s},\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^1}\right)^{\mathrm{T}^s}\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^2}\right)^{\mathrm{T}^s}\right) \end{pmatrix}$$

$$\left(\mathfrak{N}^1\right)^{\mathrm{T}^s_{TCN}} = \begin{pmatrix} \mathrm{T}\left(\mathbb{g}\left(1-\left(1-\left(\left(\frac{\mathrm{T}^{-1}\left(\mathbb{s}_i^1,\mu_i^1\right)}{\mathbb{g}}\right)\right)^{\Psi}\right)^{\mathrm{T}^s}\right)^{\frac{1}{\Psi}}\right), \\ \left(\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^1}\right)^{\mathrm{T}^s}\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^2}\right)^{\mathrm{T}^s},\left(\mathbb{U}_{\mathscr{I}}^{\mathrm{M}^1}\right)^{\mathrm{T}^s}\left(\mathbb{U}_{\mathscr{I}}^{\mathrm{M}^2}\right)^{\mathrm{T}^s}\right), \\ \left(\left(1-\left(1-\left(\mathbb{U}_{\mathscr{R}}^{N^1}\right)^{\psi}\right)^{\mathrm{T}^s}\right)^{\frac{1}{\psi}},\left(1-\left(1-\left(\mathbb{U}_{\mathscr{I}}^{N^1}\right)^{\psi}\right)^{\mathrm{T}^s}\right)^{\frac{1}{\psi}}\right), \\ \left(\left(1-\left(1-\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^1}\right)^{\psi}\right)^{\mathrm{T}^s}\right)^{\frac{1}{\psi}},\left(1-\left(1-\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^1}\right)^{\psi}\right)^{\mathrm{T}^s}\right)^{\frac{1}{\psi}}\right) \end{pmatrix}$$

Definition 9. Based on any CCp, q-QOF2-TLN $\mathfrak{N}^j = \left(\left(\mathbb{s}_i^j,\mu_i^j\right),\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^j},\mathbb{U}_{\mathscr{I}}^{\mathrm{M}^j}\right),\left(\mathbb{U}_{\mathscr{R}}^{N^j},\mathbb{U}_{\mathscr{I}}^{N^j}\right),\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^j},\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^j}\right)\right), j = 1$, we described the idea of score value and accuracy value, such as

$$SC\left(\mathfrak{N}^j\right) = \left(\frac{\mathrm{T}^{-1}\left(\mathbb{s}_i^j,\mu_i^j\right)}{\mathbb{g}}\right) + \left(\frac{1}{2}\left(\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^j}\right)^{\Psi}+\left(\mathbb{U}_{\mathscr{I}}^{\mathrm{M}^j}\right)^{\Psi}-\left(\mathbb{U}_{\mathscr{R}}^{N^j}\right)^{\psi}-\left(\mathbb{U}_{\mathscr{I}}^{N^j}\right)^{\psi}\right)+\frac{1}{2}\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^j}+\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^j}\right)\right) \in [-1,1]$$

$$AC\left(\mathfrak{N}^j\right) = \left(\frac{\mathrm{T}^{-1}\left(\mathbb{s}_i^j,\mu_i^j\right)}{\mathbb{g}}\right) + \left(\frac{1}{2}\left(\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^j}\right)^{\Psi}+\left(\mathbb{U}_{\mathscr{I}}^{\mathrm{M}^j}\right)^{\Psi}+\left(\mathbb{U}_{\mathscr{R}}^{N^j}\right)^{\psi}+\left(\mathbb{U}_{\mathscr{I}}^{N^j}\right)^{\psi}\right)+\frac{1}{2}\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^j}+\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^j}\right)\right) \in [0,1]$$

For differentiation of any two CCp, q-QOR2-TLNs, we have the following rules, such as.

1) If $SC(\mathfrak{N}^{\text{ʅ}}) > SC(\mathfrak{N}^{2})$, thus $\mathfrak{N}^{\text{ʅ}} > \mathfrak{N}^{2}$.
2) If $SC(\mathfrak{N}^{\text{ʅ}}) < SC(\mathfrak{N}^{2})$, thus $\mathfrak{N}^{\text{ʅ}} < \mathfrak{N}^{2}$, but if $SC(\mathfrak{N}^{\text{ʅ}}) = SC(\mathfrak{N}^{2})$, thus
   i) If $AC(\mathfrak{N}^{\text{ʅ}}) > AC(\mathfrak{N}^{2})$, thus $\mathfrak{N}^{\text{ʅ}} > \mathfrak{N}^{2}$.
   ii) If $AC(\mathfrak{N}^{\text{ʅ}}) < AC(\mathfrak{N}^{2})$, thus $\mathfrak{N}^{\text{ʅ}} < \mathfrak{N}^{2}$.

Further, we described Hamacher's operational laws under the consideration of the HTN and HTCN for any collection of CCp, q-QOF2-TLNs.

Definition 10. Based on any two CCp, q-QOF2-TLNs $\mathfrak{N}^{j} = \left(\left(\mathbb{s}_i^{j}, \mu_i^{j}\right), \left(\Cup_{\mathscr{R}}^{M^j}, \Cup_{\mathscr{I}}^{M^j}\right), \left(\Cup_{\mathscr{R}}^{N^j}, \Cup_{\mathscr{I}}^{N^j}\right), \left(\Cup_{\mathscr{R}}^{\mathbb{R}^j}, \Cup_{\mathscr{I}}^{\mathbb{R}^j}\right)\right), j = \text{ʅ}, 2$, we described some operational laws, such as

$$\mathfrak{N}^{\text{ʅ}} \oplus_{TN} \mathfrak{N}^{2} = \left(\begin{array}{c} \mathrm{T}\left(\mathbb{g}\left(\left(\frac{\left(\frac{\mathrm{T}^{-\text{ʅ}}(\mathbb{s}_i^{\text{ʅ}},\mu_i^{\text{ʅ}})}{\mathbb{g}}\right)^{\Psi} + \left(\frac{\mathrm{T}^{-\text{ʅ}}(\mathbb{s}_i^{2},\mu_i^{2})}{\mathbb{g}}\right)^{\Psi} - \left(\frac{\mathrm{T}^{-\text{ʅ}}(\mathbb{s}_i^{\text{ʅ}},\mu_i^{\text{ʅ}})}{\mathbb{g}}\right)^{\Psi}\left(\frac{\mathrm{T}^{-\text{ʅ}}(\mathbb{s}_i^{2},\mu_i^{2})}{\mathbb{g}}\right)^{\Psi} - (\text{ʅ} - F\Finv^{s})\left(\frac{\mathrm{T}^{-\text{ʅ}}(\mathbb{s}_i^{\text{ʅ}},\mu_i^{\text{ʅ}})}{\mathbb{g}}\right)^{\Psi}\left(\frac{\mathrm{T}^{-\text{ʅ}}(\mathbb{s}_i^{2},\mu_i^{2})}{\mathbb{g}}\right)^{\Psi}}{\text{ʅ} - (\text{ʅ} - F\Finv^{s})\left(\frac{\mathrm{T}^{-\text{ʅ}}(\mathbb{s}_i^{\text{ʅ}},\mu_i^{\text{ʅ}})}{\mathbb{g}}\right)^{\Psi}\left(\frac{\mathrm{T}^{-\text{ʅ}}(\mathbb{s}_i^{2},\mu_i^{2})}{\mathbb{g}}\right)^{\Psi}}\right)^{\frac{\text{ʅ}}{\Psi}}\right)\right), \\ \left(\begin{array}{c} \left(\frac{\left(\Cup_{\mathscr{R}}^{M^{\text{ʅ}}}\right)^{\Psi} + \left(\Cup_{\mathscr{R}}^{M^{2}}\right)^{\Psi} - \left(\Cup_{\mathscr{R}}^{M^{\text{ʅ}}}\right)^{\Psi}\left(\Cup_{\mathscr{R}}^{M^{2}}\right)^{\Psi} - (\text{ʅ} - F\Finv^{s})\left(\Cup_{\mathscr{R}}^{M^{\text{ʅ}}}\right)^{\Psi}\left(\Cup_{\mathscr{R}}^{M^{2}}\right)^{\Psi}}{\text{ʅ} - (\text{ʅ} - F\Finv^{s})\left(\Cup_{\mathscr{R}}^{M^{\text{ʅ}}}\right)^{\Psi}\left(\Cup_{\mathscr{R}}^{M^{2}}\right)^{\Psi}}\right)^{\frac{\text{ʅ}}{\Psi}}, \\ \left(\frac{\left(\Cup_{\mathscr{I}}^{M^{\text{ʅ}}}\right)^{\Psi} + \left(\Cup_{\mathscr{I}}^{M^{2}}\right)^{\Psi} - \left(\Cup_{\mathscr{I}}^{M^{\text{ʅ}}}\right)^{\Psi}\left(\Cup_{\mathscr{I}}^{M^{2}}\right)^{\Psi} - (\text{ʅ} - F\Finv^{s})\left(\Cup_{\mathscr{I}}^{M^{\text{ʅ}}}\right)^{\Psi}\left(\Cup_{\mathscr{I}}^{M^{2}}\right)^{\Psi}}{\text{ʅ} - (\text{ʅ} - F\Finv^{s})\left(\Cup_{\mathscr{I}}^{M^{\text{ʅ}}}\right)^{\Psi}\left(\Cup_{\mathscr{I}}^{M^{2}}\right)^{\Psi}}\right)^{\frac{\text{ʅ}}{\Psi}} \end{array}\right), \\ \left(\begin{array}{c} \left(\frac{\left(\Cup_{\mathscr{R}}^{N^{\text{ʅ}}}\right)^{\Psi}\left(\Cup_{\mathscr{R}}^{N^{2}}\right)^{\Psi}}{F\Finv^{s} + (\text{ʅ} - F\Finv^{s})\left(\left(\Cup_{\mathscr{R}}^{N^{\text{ʅ}}}\right)^{\Psi} + \left(\Cup_{\mathscr{R}}^{N^{2}}\right)^{\Psi} - \left(\Cup_{\mathscr{R}}^{N^{\text{ʅ}}}\right)^{\Psi}\left(\Cup_{\mathscr{R}}^{N^{2}}\right)^{\Psi}\right)}\right)^{\frac{\text{ʅ}}{\Psi}}, \\ \left(\frac{\left(\Cup_{\mathscr{I}}^{N^{\text{ʅ}}}\right)^{\Psi}\left(\Cup_{\mathscr{I}}^{N^{2}}\right)^{\Psi}}{F\Finv^{s} + (\text{ʅ} - F\Finv^{s})\left(\left(\Cup_{\mathscr{I}}^{N^{\text{ʅ}}}\right)^{\Psi} + \left(\Cup_{\mathscr{I}}^{N^{2}}\right)^{\Psi} - \left(\Cup_{\mathscr{I}}^{N^{\text{ʅ}}}\right)^{\Psi}\left(\Cup_{\mathscr{I}}^{N^{2}}\right)^{\Psi}\right)}\right)^{\frac{\text{ʅ}}{\Psi}} \end{array}\right), \\ \left(\begin{array}{c} \left(\frac{\left(\Cup_{\mathscr{R}}^{\mathbb{R}^{\text{ʅ}}}\right)^{\Psi} + \left(\Cup_{\mathscr{R}}^{\mathbb{R}^{2}}\right)^{\Psi} - \left(\Cup_{\mathscr{R}}^{\mathbb{R}^{\text{ʅ}}}\right)^{\Psi}\left(\Cup_{\mathscr{R}}^{\mathbb{R}^{2}}\right)^{\Psi} - (\text{ʅ} - F\Finv^{s})\left(\Cup_{\mathscr{R}}^{\mathbb{R}^{\text{ʅ}}}\right)^{\Psi}\left(\Cup_{\mathscr{R}}^{\mathbb{R}^{2}}\right)^{\Psi}}{\text{ʅ} - (\text{ʅ} - F\Finv^{s})\left(\Cup_{\mathscr{R}}^{\mathbb{R}^{\text{ʅ}}}\right)^{\Psi}\left(\Cup_{\mathscr{R}}^{\mathbb{R}^{2}}\right)^{\Psi}}\right)^{\frac{\text{ʅ}}{\Psi}}, \\ \left(\frac{\left(\Cup_{\mathscr{I}}^{\mathbb{R}^{\text{ʅ}}}\right)^{\Psi} + \left(\Cup_{\mathscr{I}}^{\mathbb{R}^{2}}\right)^{\Psi} - \left(\Cup_{\mathscr{I}}^{\mathbb{R}^{\text{ʅ}}}\right)^{\Psi}\left(\Cup_{\mathscr{I}}^{\mathbb{R}^{2}}\right)^{\Psi} - (\text{ʅ} - F\Finv^{s})\left(\Cup_{\mathscr{I}}^{\mathbb{R}^{\text{ʅ}}}\right)^{\Psi}\left(\Cup_{\mathscr{I}}^{\mathbb{R}^{2}}\right)^{\Psi}}{\text{ʅ} - (\text{ʅ} - F\Finv^{s})\left(\Cup_{\mathscr{I}}^{\mathbb{R}^{\text{ʅ}}}\right)^{\Psi}\left(\Cup_{\mathscr{I}}^{\mathbb{R}^{2}}\right)^{\Psi}}\right)^{\frac{\text{ʅ}}{\Psi}} \end{array}\right) \end{array}\right)$$

$$\mathfrak{N}^{\mathfrak{l}} \oplus_{TCN} \mathfrak{N}^{2} = \begin{pmatrix} \mathrm{T}\left(\mathbb{g}\left(\dfrac{\left(\frac{\mathrm{T}^{-\mathfrak{l}}\left(\mathbb{s}_i^{\mathfrak{l}},\mu_i^{\mathfrak{l}}\right)}{\mathbb{g}}\right)^{\psi}\left(\frac{\mathrm{T}^{-\mathfrak{l}}\left(\mathbb{s}_i^{2},\mu_i^{2}\right)}{\mathbb{g}}\right)^{\psi}}{F\Finv^{s}+(\mathfrak{l}-F\Finv^{s})\left(\left(\frac{\mathrm{T}^{-\mathfrak{l}}\left(\mathbb{s}_i^{\mathfrak{l}},\mu_i^{\mathfrak{l}}\right)}{\mathbb{g}}\right)^{\psi}+\left(\frac{\mathrm{T}^{-\mathfrak{l}}\left(\mathbb{s}_i^{2},\mu_i^{2}\right)}{\mathbb{g}}\right)^{\psi}-\left(\frac{\mathrm{T}^{-\mathfrak{l}}\left(\mathbb{s}_i^{\mathfrak{l}},\mu_i^{\mathfrak{l}}\right)}{\mathbb{g}}\right)^{\psi}\left(\frac{\mathrm{T}^{-\mathfrak{l}}\left(\mathbb{s}_i^{2},\mu_i^{2}\right)}{\mathbb{g}}\right)^{\psi}\right)}\right)^{\frac{\mathfrak{l}}{\psi}}\right), \\ \begin{pmatrix} \left(\dfrac{\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{\mathfrak{l}}}\right)^{\Psi}+\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{2}}\right)^{\Psi}-\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{\mathfrak{l}}}\right)^{\Psi}\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{2}}\right)^{\Psi}-(\mathfrak{l}-F\Finv^{s})\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{\mathfrak{l}}}\right)^{\Psi}\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{2}}\right)^{\Psi}}{\mathfrak{l}-(\mathfrak{l}-F\Finv^{s})\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{\mathfrak{l}}}\right)^{\Psi}\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{2}}\right)^{\Psi}}\right)^{\frac{\mathfrak{l}}{\Psi}}, \\ \left(\dfrac{\left(\mathbb{U}_{\mathscr{J}}^{\mathrm{M}^{\mathfrak{l}}}\right)^{\Psi}+\left(\mathbb{U}_{\mathscr{J}}^{\mathrm{M}^{2}}\right)^{\Psi}-\left(\mathbb{U}_{\mathscr{J}}^{\mathrm{M}^{\mathfrak{l}}}\right)^{\Psi}\left(\mathbb{U}_{\mathscr{J}}^{\mathrm{M}^{2}}\right)^{\Psi}-(\mathfrak{l}-F\Finv^{s})\left(\mathbb{U}_{\mathscr{J}}^{\mathrm{M}^{\mathfrak{l}}}\right)^{\Psi}\left(\mathbb{U}_{\mathscr{J}}^{\mathrm{M}^{2}}\right)^{\Psi}}{\mathfrak{l}-(\mathfrak{l}-F\Finv^{s})\left(\mathbb{U}_{\mathscr{J}}^{\mathrm{M}^{\mathfrak{l}}}\right)^{\Psi}\left(\mathbb{U}_{\mathscr{J}}^{\mathrm{M}^{2}}\right)^{\Psi}}\right)^{\frac{\mathfrak{l}}{\Psi}} \end{pmatrix}, \\ \begin{pmatrix} \left(\dfrac{\left(\mathbb{U}_{\mathscr{R}}^{N^{\mathfrak{l}}}\right)^{\psi}\left(\mathbb{U}_{\mathscr{R}}^{N^{2}}\right)^{\psi}}{F\Finv^{s}+(\mathfrak{l}-F\Finv^{s})\left(\left(\mathbb{U}_{\mathscr{R}}^{N^{\mathfrak{l}}}\right)^{\psi}+\left(\mathbb{U}_{\mathscr{R}}^{N^{2}}\right)^{\psi}-\left(\mathbb{U}_{\mathscr{R}}^{N^{\mathfrak{l}}}\right)^{\psi}\left(\mathbb{U}_{\mathscr{R}}^{N^{2}}\right)^{\psi}\right)}\right)^{\frac{\mathfrak{l}}{\psi}}, \\ \left(\dfrac{\left(\mathbb{U}_{\mathscr{J}}^{N^{\mathfrak{l}}}\right)^{\psi}\left(\mathbb{U}_{\mathscr{J}}^{N^{2}}\right)^{\psi}}{F\Finv^{s}+(\mathfrak{l}-F\Finv^{s})\left(\left(\mathbb{U}_{\mathscr{J}}^{N^{\mathfrak{l}}}\right)^{\psi}+\left(\mathbb{U}_{\mathscr{J}}^{N^{2}}\right)^{\psi}-\left(\mathbb{U}_{\mathscr{J}}^{N^{\mathfrak{l}}}\right)^{\psi}\left(\mathbb{U}_{\mathscr{J}}^{N^{2}}\right)^{\psi}\right)}\right)^{\frac{\mathfrak{l}}{\psi}} \end{pmatrix}, \\ \begin{pmatrix} \left(\dfrac{\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{\mathfrak{l}}}\right)^{\psi}\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{2}}\right)^{\psi}}{F\Finv^{s}+(\mathfrak{l}-F\Finv^{s})\left(\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{\mathfrak{l}}}\right)^{\psi}+\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{2}}\right)^{\psi}-\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{\mathfrak{l}}}\right)^{\psi}\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{2}}\right)^{\psi}\right)}\right)^{\frac{\mathfrak{l}}{\psi}}, \\ \left(\dfrac{\left(\mathbb{U}_{\mathscr{J}}^{\mathbb{R}^{\mathfrak{l}}}\right)^{\psi}\left(\mathbb{U}_{\mathscr{J}}^{\mathbb{R}^{2}}\right)^{\psi}}{F\Finv^{s}+(\mathfrak{l}-F\Finv^{s})\left(\left(\mathbb{U}_{\mathscr{J}}^{\mathbb{R}^{\mathfrak{l}}}\right)^{\psi}+\left(\mathbb{U}_{\mathscr{J}}^{\mathbb{R}^{2}}\right)^{\psi}-\left(\mathbb{U}_{\mathscr{J}}^{\mathbb{R}^{\mathfrak{l}}}\right)^{\psi}\left(\mathbb{U}_{\mathscr{J}}^{\mathbb{R}^{2}}\right)^{\psi}\right)}\right)^{\frac{\mathfrak{l}}{\psi}} \end{pmatrix} \end{pmatrix}$$

$$\mathfrak{N}^{\text{ɭ}}\otimes_{TN}\mathfrak{N}^{2}=\left(\begin{array}{c}
\mathrm{T}\left(\mathbb{g}\left(\frac{\left(\frac{\mathrm{T}^{-\text{ɭ}}\left(\mathbb{s}_i^{\text{ɭ}},\mu_i^{\text{ɭ}}\right)}{\mathbb{g}}\right)^{\Psi}\left(\frac{\mathrm{T}^{-\text{ɭ}}\left(\mathbb{s}_i^{2},\mu_i^{2}\right)}{\mathbb{g}}\right)^{\Psi}}{F\text{Ⅎ}^{s}+(\text{ɭ}-F\text{Ⅎ}^{s})\left(\left(\frac{\mathrm{T}^{-\text{ɭ}}\left(\mathbb{s}_i^{\text{ɭ}},\mu_i^{\text{ɭ}}\right)}{\mathbb{g}}\right)^{\Psi}+\left(\frac{\mathrm{T}^{-\text{ɭ}}\left(\mathbb{s}_i^{2},\mu_i^{2}\right)}{\mathbb{g}}\right)^{\Psi}-\left(\frac{\mathrm{T}^{-\text{ɭ}}\left(\mathbb{s}_i^{\text{ɭ}},\mu_i^{\text{ɭ}}\right)}{\mathbb{g}}\right)^{\Psi}\left(\frac{\mathrm{T}^{-\text{ɭ}}\left(\mathbb{s}_i^{2},\mu_i^{2}\right)}{\mathbb{g}}\right)^{\Psi}\right)}\right)^{\frac{\text{ɭ}}{\Psi}}\right), \\
\left(\begin{array}{c}
\left(\frac{\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{\text{ɭ}}}\right)^{\Psi}\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{2}}\right)^{\Psi}}{F\text{Ⅎ}^{s}+(\text{ɭ}-F\text{Ⅎ}^{s})\left(\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{\text{ɭ}}}\right)^{\Psi}+\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{2}}\right)^{\Psi}-\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{\text{ɭ}}}\right)^{\Psi}\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{2}}\right)^{\Psi}\right)}\right)^{\frac{\text{ɭ}}{\Psi}}, \\
\left(\frac{\left(\mathbb{U}_{\mathscr{J}}^{\mathrm{M}^{\text{ɭ}}}\right)^{\Psi}\left(\mathbb{U}_{\mathscr{J}}^{\mathrm{M}^{2}}\right)^{\Psi}}{F\text{Ⅎ}^{s}+(\text{ɭ}-F\text{Ⅎ}^{s})\left(\left(\mathbb{U}_{\mathscr{J}}^{\mathrm{M}^{\text{ɭ}}}\right)^{\Psi}+\left(\mathbb{U}_{\mathscr{J}}^{\mathrm{M}^{2}}\right)^{\Psi}-\left(\mathbb{U}_{\mathscr{J}}^{\mathrm{M}^{\text{ɭ}}}\right)^{\Psi}\left(\mathbb{U}_{\mathscr{J}}^{\mathrm{M}^{2}}\right)^{\Psi}\right)}\right)^{\frac{\text{ɭ}}{\Psi}}
\end{array}\right), \\
\left(\begin{array}{c}
\left(\frac{\left(\mathbb{U}_{\mathscr{R}}^{N^{\text{ɭ}}}\right)^{\psi}+\left(\mathbb{U}_{\mathscr{R}}^{N^{2}}\right)^{\psi}-\left(\mathbb{U}_{\mathscr{R}}^{N^{\text{ɭ}}}\right)^{\psi}\left(\mathbb{U}_{\mathscr{R}}^{N^{2}}\right)^{\psi}-(\text{ɭ}-F\text{Ⅎ}^{s})\left(\mathbb{U}_{\mathscr{R}}^{N^{\text{ɭ}}}\right)^{\psi}\left(\mathbb{U}_{\mathscr{R}}^{N^{2}}\right)^{\psi}}{\text{ɭ}-(\text{ɭ}-F\text{Ⅎ}^{s})\left(\mathbb{U}_{\mathscr{R}}^{N^{\text{ɭ}}}\right)^{\psi}\left(\mathbb{U}_{\mathscr{R}}^{N^{2}}\right)^{\psi}}\right)^{\frac{\text{ɭ}}{\psi}}, \\
\left(\frac{\left(\mathbb{U}_{\mathscr{J}}^{N^{\text{ɭ}}}\right)^{\psi}+\left(\mathbb{U}_{\mathscr{J}}^{N^{2}}\right)^{\psi}-\left(\mathbb{U}_{\mathscr{J}}^{N^{\text{ɭ}}}\right)^{\psi}\left(\mathbb{U}_{\mathscr{J}}^{N^{2}}\right)^{\psi}-(\text{ɭ}-F\text{Ⅎ}^{s})\left(\mathbb{U}_{\mathscr{J}}^{N^{\text{ɭ}}}\right)^{\psi}\left(\mathbb{U}_{\mathscr{J}}^{N^{2}}\right)^{\psi}}{\text{ɭ}-(\text{ɭ}-F\text{Ⅎ}^{s})\left(\mathbb{U}_{\mathscr{J}}^{N^{\text{ɭ}}}\right)^{\psi}\left(\mathbb{U}_{\mathscr{J}}^{N^{2}}\right)^{\psi}}\right)^{\frac{\text{ɭ}}{\psi}}
\end{array}\right), \\
\left(\begin{array}{c}
\left(\frac{\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{\text{ɭ}}}\right)^{\Psi}\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{2}}\right)^{\Psi}}{F\text{Ⅎ}^{s}+(\text{ɭ}-F\text{Ⅎ}^{s})\left(\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{\text{ɭ}}}\right)^{\psi}+\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{2}}\right)^{\Psi}-\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{\text{ɭ}}}\right)^{\Psi}\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{2}}\right)^{\Psi}\right)}\right)^{\frac{\text{ɭ}}{\Psi}}, \\
\left(\frac{\left(\mathbb{U}_{\mathscr{J}}^{\mathbb{R}^{\text{ɭ}}}\right)^{\Psi}\left(\mathbb{U}_{\mathscr{J}}^{\mathbb{R}^{2}}\right)^{\Psi}}{F\text{Ⅎ}^{s}+(\text{ɭ}-F\text{Ⅎ}^{s})\left(\left(\mathbb{U}_{\mathscr{J}}^{\mathbb{R}^{\text{ɭ}}}\right)^{\Psi}+\left(\mathbb{U}_{\mathscr{J}}^{\mathbb{R}^{2}}\right)^{\Psi}-\left(\mathbb{U}_{\mathscr{J}}^{\mathbb{R}^{\text{ɭ}}}\right)^{\Psi}\left(\mathbb{U}_{\mathscr{J}}^{\mathbb{R}^{2}}\right)^{\Psi}\right)}\right)^{\frac{\text{ɭ}}{\Psi}}
\end{array}\right)
\end{array}\right)$$

$$\mathfrak{N}^{\text{ſ}} \otimes_{TCN} \mathfrak{N}^{2} = \left(\begin{array}{c} \mathrm{T}\left(\mathbb{g}\left(\left(\frac{ \left(\begin{array}{c} \left(\frac{\mathrm{T}^{-\text{ſ}}\left(\mathbb{s}_i^{\text{ſ}}, \mu_i^{\text{ſ}}\right)}{\mathbb{g}}\right)^{\psi} + \left(\frac{\mathrm{T}^{-\text{ſ}}\left(\mathbb{s}_i^{2}, \mu_i^{2}\right)}{\mathbb{g}}\right)^{\psi} - \left(\frac{\mathrm{T}^{-\text{ſ}}\left(\mathbb{s}_i^{\text{ſ}}, \mu_i^{\text{ſ}}\right)}{\mathbb{g}}\right)^{\psi} \left(\frac{\mathrm{T}^{-\text{ſ}}\left(\mathbb{s}_i^{2}, \mu_i^{2}\right)}{\mathbb{g}}\right)^{\psi} - \\ \left(\text{ſ} - F\text{Ⅎ}^{s}\right) \left(\frac{\mathrm{T}^{-\text{ſ}}\left(\mathbb{s}_i^{\text{ſ}}, \mu_i^{\text{ſ}}\right)}{\mathbb{g}}\right)^{\psi} \left(\frac{\mathrm{T}^{-\text{ſ}}\left(\mathbb{s}_i^{2}, \mu_i^{2}\right)}{\mathbb{g}}\right)^{\psi} \end{array} \right) }{ \text{ſ} - \left(\text{ſ} - F\text{Ⅎ}^{s}\right) \left(\frac{\mathrm{T}^{-\text{ſ}}\left(\mathbb{s}_i^{\text{ſ}}, \mu_i^{\text{ſ}}\right)}{\mathbb{g}}\right)^{\psi} \left(\frac{\mathrm{T}^{-\text{ſ}}\left(\mathbb{s}_i^{2}, \mu_i^{2}\right)}{\mathbb{g}}\right)^{\psi} } \right)^{\frac{\text{ſ}}{\psi}} \right) \right), \\ \left(\begin{array}{c} \left(\frac{ \left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{\text{ſ}}}\right)^{\Psi} \left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{2}}\right)^{\Psi} }{ F\text{Ⅎ}^{s} + \left(\text{ſ} - F\text{Ⅎ}^{s}\right)\left(\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{\text{ſ}}}\right)^{\Psi} + \left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{2}}\right)^{\Psi} - \left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{\text{ſ}}}\right)^{\Psi} \left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{2}}\right)^{\Psi} \right) } \right)^{\frac{\text{ſ}}{\Psi}}, \\ \left(\frac{ \left(\mathbb{U}_{\mathscr{J}}^{\mathrm{M}^{\text{ſ}}}\right)^{\Psi} \left(\mathbb{U}_{\mathscr{J}}^{\mathrm{M}^{2}}\right)^{\Psi} }{ F\text{Ⅎ}^{s} + \left(\text{ſ} - F\text{Ⅎ}^{s}\right)\left(\left(\mathbb{U}_{\mathscr{J}}^{\mathrm{M}^{\text{ſ}}}\right)^{\Psi} + \left(\mathbb{U}_{\mathscr{J}}^{\mathrm{M}^{2}}\right)^{\Psi} - \left(\mathbb{U}_{\mathscr{J}}^{\mathrm{M}^{\text{ſ}}}\right)^{\Psi} \left(\mathbb{U}_{\mathscr{J}}^{\mathrm{M}^{2}}\right)^{\Psi} \right) } \right)^{\frac{\text{ſ}}{\Psi}} \end{array} \right), \\ \left(\begin{array}{c} \left(\frac{ \left(\mathbb{U}_{\mathscr{R}}^{N^{\text{ſ}}}\right)^{\psi} + \left(\mathbb{U}_{\mathscr{R}}^{N^{2}}\right)^{\psi} - \left(\mathbb{U}_{\mathscr{R}}^{N^{\text{ſ}}}\right)^{\psi} \left(\mathbb{U}_{\mathscr{R}}^{N^{2}}\right)^{\psi} - \left(\text{ſ} - F\text{Ⅎ}^{s}\right) \left(\mathbb{U}_{\mathscr{R}}^{N^{\text{ſ}}}\right)^{\psi} \left(\mathbb{U}_{\mathscr{R}}^{N^{2}}\right)^{\psi} }{ \text{ſ} - \left(\text{ſ} - F\text{Ⅎ}^{s}\right) \left(\mathbb{U}_{\mathscr{R}}^{N^{\text{ſ}}}\right)^{\psi} \left(\mathbb{U}_{\mathscr{R}}^{N^{2}}\right)^{\psi} } \right)^{\frac{\text{ſ}}{\psi}}, \\ \left(\frac{ \left(\mathbb{U}_{\mathscr{J}}^{N^{\text{ſ}}}\right)^{\psi} + \left(\mathbb{U}_{\mathscr{J}}^{N^{2}}\right)^{\psi} - \left(\mathbb{U}_{\mathscr{J}}^{N^{\text{ſ}}}\right)^{\psi} \left(\mathbb{U}_{\mathscr{J}}^{N^{2}}\right)^{\psi} - \left(\text{ſ} - F\text{Ⅎ}^{s}\right) \left(\mathbb{U}_{\mathscr{J}}^{N^{\text{ſ}}}\right)^{\psi} \left(\mathbb{U}_{\mathscr{J}}^{N^{2}}\right)^{\psi} }{ \text{ſ} - \left(\text{ſ} - F\text{Ⅎ}^{s}\right) \left(\mathbb{U}_{\mathscr{J}}^{N^{\text{ſ}}}\right)^{\psi} \left(\mathbb{U}_{\mathscr{J}}^{N^{2}}\right)^{\psi} } \right)^{\frac{\text{ſ}}{\psi}} \end{array} \right), \\ \left(\begin{array}{c} \left(\frac{ \left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{\text{ſ}}}\right)^{\psi} + \left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{2}}\right)^{\psi} - \left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{\text{ſ}}}\right)^{\psi} \left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{2}}\right)^{\psi} - \left(\text{ſ} - F\text{Ⅎ}^{s}\right) \left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{\text{ſ}}}\right)^{\psi} \left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{2}}\right)^{\psi} }{ \text{ſ} - \left(\text{ſ} - F\text{Ⅎ}^{s}\right) \left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{\text{ſ}}}\right)^{\psi} \left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{2}}\right)^{\psi} } \right)^{\frac{\text{ſ}}{\psi}}, \\ \left(\frac{ \left(\mathbb{U}_{\mathscr{J}}^{\mathbb{R}^{\text{ſ}}}\right)^{\psi} + \left(\mathbb{U}_{\mathscr{J}}^{\mathbb{R}^{2}}\right)^{\psi} - \left(\mathbb{U}_{\mathscr{J}}^{\mathbb{R}^{\text{ſ}}}\right)^{\psi} \left(\mathbb{U}_{\mathscr{J}}^{\mathbb{R}^{2}}\right)^{\psi} - \left(\text{ſ} - F\text{Ⅎ}^{s}\right) \left(\mathbb{U}_{\mathscr{J}}^{\mathbb{R}^{\text{ſ}}}\right)^{\psi} \left(\mathbb{U}_{\mathscr{J}}^{\mathbb{R}^{2}}\right)^{\psi} }{ \text{ſ} - \left(\text{ſ} - F\text{Ⅎ}^{s}\right) \left(\mathbb{U}_{\mathscr{J}}^{\mathbb{R}^{\text{ſ}}}\right)^{\psi} \left(\mathbb{U}_{\mathscr{J}}^{\mathbb{R}^{2}}\right)^{\psi} } \right)^{\frac{\text{ſ}}{\psi}} \end{array} \right) \end{array} \right)$$

$$\mathrm{T}^s *_{TN} \mathfrak{N}^{\mathfrak{l}} = \left(\begin{array}{c} \mathrm{T}\left(\mathbb{g}\left(\frac{\left(\mathfrak{l}+(F\Finv^s-\mathfrak{l})\left(\frac{\mathrm{T}^{-\mathfrak{l}}\left(\mathbb{s}_i^{\mathfrak{l}},\mu_i^{\mathfrak{l}}\right)}{\mathbb{g}}\right)^{\Psi}\right)^{\mathrm{T}^s}-\left(\mathfrak{l}-\left(\frac{\mathrm{T}^{-\mathfrak{l}}\left(\mathbb{s}_i^{\mathfrak{l}},\mu_i^{\mathfrak{l}}\right)}{\mathbb{g}}\right)^{\Psi}\right)^{\mathrm{T}^s}}{\left(\mathfrak{l}+(F\Finv^s-\mathfrak{l})\left(\frac{\mathrm{T}^{-\mathfrak{l}}\left(\mathbb{s}_i^{\mathfrak{l}},\mu_i^{\mathfrak{l}}\right)}{\mathbb{g}}\right)^{\Psi}\right)^{\mathrm{T}^s}+(F\Finv^s-\mathfrak{l})\left(\mathfrak{l}-\left(\frac{\mathrm{T}^{-\mathfrak{l}}\left(\mathbb{s}_i^{\mathfrak{l}},\mu_i^{\mathfrak{l}}\right)}{\mathbb{g}}\right)^{\Psi}\right)^{\mathrm{T}^s}}\right)^{\frac{\mathfrak{l}}{\Psi}}\right), \\ \left(\begin{array}{c} \left(\frac{\left(\mathfrak{l}+(F\Finv^s-\mathfrak{l})\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{\mathfrak{l}}}\right)^{\Psi}\right)^{\mathrm{T}^s}-\left(\mathfrak{l}-\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{\mathfrak{l}}}\right)^{\Psi}\right)^{\mathrm{T}^s}}{\left(\mathfrak{l}+(F\Finv^s-\mathfrak{l})\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{\mathfrak{l}}}\right)^{\Psi}\right)^{\mathrm{T}^s}+(F\Finv^s-\mathfrak{l})\left(\mathfrak{l}-\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{\mathfrak{l}}}\right)^{\Psi}\right)^{\mathrm{T}^s}}\right)^{\frac{\mathfrak{l}}{\Psi}}, \\ \left(\frac{\left(\mathfrak{l}+(F\Finv^s-\mathfrak{l})\left(\mathbb{U}_{\mathscr{J}}^{\mathrm{M}^{\mathfrak{l}}}\right)^{\Psi}\right)^{\mathrm{T}^s}-\left(\mathfrak{l}-\left(\mathbb{U}_{\mathscr{J}}^{\mathrm{M}^{\mathfrak{l}}}\right)^{\Psi}\right)^{\mathrm{T}^s}}{\left(\mathfrak{l}+(F\Finv^s-\mathfrak{l})\left(\mathbb{U}_{\mathscr{J}}^{\mathrm{M}^{\mathfrak{l}}}\right)^{\Psi}\right)^{\mathrm{T}^s}+(F\Finv^s-\mathfrak{l})\left(\mathfrak{l}-\left(\mathbb{U}_{\mathscr{J}}^{\mathrm{M}^{\mathfrak{l}}}\right)^{\Psi}\right)^{\mathrm{T}^s}}\right)^{\frac{\mathfrak{l}}{\Psi}} \end{array} \right), \\ \left(\begin{array}{c} \frac{\mathrm{T}^{s\frac{\mathfrak{l}}{\psi}}\left(\mathbb{U}_{\mathscr{R}}^{N^{\mathfrak{l}}}\right)^{\mathrm{T}^s}}{\left(\left(\mathfrak{l}+(F\Finv^s-\mathfrak{l})\left(\mathfrak{l}-\left(\mathbb{U}_{\mathscr{R}}^{N^{\mathfrak{l}}}\right)^{\psi}\right)\right)^{\mathrm{T}^s}+(F\Finv^s-\mathfrak{l})\left(\left(\mathbb{U}_{\mathscr{R}}^{N^{\mathfrak{l}}}\right)^{\psi}\right)^{\mathrm{T}^s}\right)^{\frac{\mathfrak{l}}{\psi}}}, \\ \frac{\mathrm{T}^{s\frac{\mathfrak{l}}{\psi}}\left(\mathbb{U}_{\mathscr{J}}^{N^{\mathfrak{l}}}\right)^{\mathrm{T}^s}}{\left(\left(\mathfrak{l}+(F\Finv^s-\mathfrak{l})\left(\mathfrak{l}-\left(\mathbb{U}_{\mathscr{J}}^{N^{\mathfrak{l}}}\right)^{\psi}\right)\right)^{\mathrm{T}^s}+(F\Finv^s-\mathfrak{l})\left(\left(\mathbb{U}_{\mathscr{J}}^{N^{\mathfrak{l}}}\right)^{\psi}\right)^{\mathrm{T}^s}\right)^{\frac{\mathfrak{l}}{\psi}}} \end{array} \right), \\ \left(\begin{array}{c} \left(\frac{\left(\mathfrak{l}+(F\Finv^s-\mathfrak{l})\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{\mathfrak{l}}}\right)^{\Psi}\right)^{\mathrm{T}^s}-\left(\mathfrak{l}-\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{\mathfrak{l}}}\right)^{\Psi}\right)^{\mathrm{T}^s}}{\left(\mathfrak{l}+(F\Finv^s-\mathfrak{l})\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{\mathfrak{l}}}\right)^{\Psi}\right)^{\mathrm{T}^s}+(F\Finv^s-\mathfrak{l})\left(\mathfrak{l}-\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{\mathfrak{l}}}\right)^{\Psi}\right)^{\mathrm{T}^s}}\right)^{\frac{\mathfrak{l}}{\Psi}}, \\ \left(\frac{\left(\mathfrak{l}+(F\Finv^s-\mathfrak{l})\left(\mathbb{U}_{\mathscr{J}}^{\mathbb{R}^{\mathfrak{l}}}\right)^{\Psi}\right)^{\mathrm{T}^s}-\left(\mathfrak{l}-\left(\mathbb{U}_{\mathscr{J}}^{\mathbb{R}^{\mathfrak{l}}}\right)^{\Psi}\right)^{\mathrm{T}^s}}{\left(\mathfrak{l}+(F\Finv^s-\mathfrak{l})\left(\mathbb{U}_{\mathscr{J}}^{\mathbb{R}^{\mathfrak{l}}}\right)^{\Psi}\right)^{\mathrm{T}^s}+(F\Finv^s-\mathfrak{l})\left(\mathfrak{l}-\left(\mathbb{U}_{\mathscr{J}}^{\mathbb{R}^{\mathfrak{l}}}\right)^{\Psi}\right)^{\mathrm{T}^s}}\right)^{\frac{\mathfrak{l}}{\Psi}} \end{array} \right) \end{array} \right)$$

$$\mathrm{T}^{s}*_{TCN}\mathfrak{N}^{\mathfrak{l}}=\begin{pmatrix}\mathrm{T}\left(\mathbb{g}\left(\frac{\mathrm{T}^{s\frac{\mathfrak{l}}{\psi}}\left(\left(\frac{\mathrm{T}^{-\mathfrak{l}}\left(\mathbb{s}_{i}^{\mathfrak{l}},\mu_{i}^{\mathfrak{l}}\right)}{\mathbb{g}}\right)\right)^{\mathrm{T}^{s}}}{\left(\left(\mathfrak{l}+(F\Finv^{s}-\mathfrak{l})\left(\mathfrak{l}-\left(\left(\frac{\mathrm{T}^{-\mathfrak{l}}\left(\mathbb{s}_{i}^{\mathfrak{l}},\mu_{i}^{\mathfrak{l}}\right)}{\mathbb{g}}\right)\right)^{\psi}\right)\right)^{\mathrm{T}^{s}}+(F\Finv^{s}-\mathfrak{l})\left(\left(\left(\frac{\mathrm{T}^{-\mathfrak{l}}\left(\mathbb{s}_{i}^{\mathfrak{l}},\mu_{i}^{\mathfrak{l}}\right)}{\mathbb{g}}\right)\right)^{\psi}\right)^{\mathrm{T}^{s}}\right)^{\frac{\mathfrak{l}}{\psi}}}\right)\right),\\ \left(\begin{array}{c}\left(\frac{\left(\mathfrak{l}+(F\Finv^{s}-\mathfrak{l})\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{\mathfrak{l}}}\right)^{\Psi}\right)^{\mathrm{T}^{s}}-\left(\mathfrak{l}-\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{\mathfrak{l}}}\right)^{\Psi}\right)^{\mathrm{T}^{s}}}{\left(\mathfrak{l}+(F\Finv^{s}-\mathfrak{l})\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{\mathfrak{l}}}\right)^{\Psi}\right)^{\mathrm{T}^{s}}+(F\Finv^{s}-\mathfrak{l})\left(\mathfrak{l}-\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{\mathfrak{l}}}\right)^{\Psi}\right)^{\mathrm{T}^{s}}}\right)^{\frac{\mathfrak{l}}{\Psi}},\\ \left(\frac{\left(\mathfrak{l}+(F\Finv^{s}-\mathfrak{l})\left(\mathbb{U}_{\mathscr{J}}^{\mathrm{M}^{\mathfrak{l}}}\right)^{\Psi}\right)^{\mathrm{T}^{s}}-\left(\mathfrak{l}-\left(\mathbb{U}_{\mathscr{J}}^{\mathrm{M}^{\mathfrak{l}}}\right)^{\Psi}\right)^{\mathrm{T}^{s}}}{\left(\mathfrak{l}+(F\Finv^{s}-\mathfrak{l})\left(\mathbb{U}_{\mathscr{J}}^{\mathrm{M}^{\mathfrak{l}}}\right)^{\Psi}\right)^{\mathrm{T}^{s}}+(F\Finv^{s}-\mathfrak{l})\left(\mathfrak{l}-\left(\mathbb{U}_{\mathscr{J}}^{\mathrm{M}^{\mathfrak{l}}}\right)^{\Psi}\right)^{\mathrm{T}^{s}}}\right)^{\frac{\mathfrak{l}}{\Psi}}\end{array}\right),\\ \left(\begin{array}{c}\frac{\mathrm{T}^{s\frac{\mathfrak{l}}{\psi}}\left(\mathbb{U}_{\mathscr{R}}^{N^{\mathfrak{l}}}\right)^{\mathrm{T}^{s}}}{\left(\left(\mathfrak{l}+(F\Finv^{s}-\mathfrak{l})\left(\mathfrak{l}-\left(\mathbb{U}_{\mathscr{R}}^{N^{\mathfrak{l}}}\right)^{\psi}\right)\right)^{\mathrm{T}^{s}}+(F\Finv^{s}-\mathfrak{l})\left(\left(\mathbb{U}_{\mathscr{R}}^{N^{\mathfrak{l}}}\right)^{\psi}\right)^{\mathrm{T}^{s}}\right)^{\frac{\mathfrak{l}}{\psi}}},\\ \frac{\mathrm{T}^{s\frac{\mathfrak{l}}{\psi}}\left(\mathbb{U}_{\mathscr{J}}^{N^{\mathfrak{l}}}\right)^{\mathrm{T}^{s}}}{\left(\left(\mathfrak{l}+(F\Finv^{s}-\mathfrak{l})\left(\mathfrak{l}-\left(\mathbb{U}_{\mathscr{J}}^{N^{\mathfrak{l}}}\right)^{\psi}\right)\right)^{\mathrm{T}^{s}}+(F\Finv^{s}-\mathfrak{l})\left(\left(\mathbb{U}_{\mathscr{J}}^{N^{\mathfrak{l}}}\right)^{\psi}\right)^{\mathrm{T}^{s}}\right)^{\frac{\mathfrak{l}}{\psi}}}\end{array}\right),\\ \left(\begin{array}{c}\frac{\mathrm{T}^{s\frac{\mathfrak{l}}{\psi}}\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{\mathfrak{l}}}\right)^{\mathrm{T}^{s}}}{\left(\left(\mathfrak{l}+(F\Finv^{s}-\mathfrak{l})\left(\mathfrak{l}-\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{\mathfrak{l}}}\right)^{\psi}\right)\right)^{\mathrm{T}^{s}}+(F\Finv^{s}-\mathfrak{l})\left(\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{\mathfrak{l}}}\right)^{\psi}\right)^{\mathrm{T}^{s}}\right)^{\frac{\mathfrak{l}}{\psi}}},\\ \frac{\mathrm{T}^{s\frac{\mathfrak{l}}{\psi}}\left(\mathbb{U}_{\mathscr{J}}^{\mathbb{R}^{\mathfrak{l}}}\right)^{\mathrm{T}^{s}}}{\left(\left(\mathfrak{l}+(F\Finv^{s}-\mathfrak{l})\left(\mathfrak{l}-\left(\mathbb{U}_{\mathscr{J}}^{\mathbb{R}^{\mathfrak{l}}}\right)^{\psi}\right)\right)^{\mathrm{T}^{s}}+(F\Finv^{s}-\mathfrak{l})\left(\left(\mathbb{U}_{\mathscr{J}}^{\mathbb{R}^{\mathfrak{l}}}\right)^{\psi}\right)^{\mathrm{T}^{s}}\right)^{\frac{\mathfrak{l}}{\psi}}}\end{array}\right)\end{pmatrix}$$

$$
\left(\mathfrak{N}^{\mathfrak{I}}\right)^{\mathrm{T}^{s}_{TN}} = \begin{pmatrix}
\mathrm{T}\left(\mathbb{g}\left(\dfrac{\mathrm{T}^{s\frac{\mathfrak{I}}{\psi}}\left(\left(\frac{\mathrm{T}^{-\mathfrak{I}}\left(\mathbb{s}_{i}^{\mathfrak{I}},\mu_{i}^{\mathfrak{I}}\right)}{\mathbb{g}}\right)\right)^{\mathrm{T}^{s}}}{\left(\left(\mathfrak{I}+\left(F\Finv^{s}-\mathfrak{I}\right)\left(\mathfrak{I}-\left(\left(\frac{\mathrm{T}^{-\mathfrak{I}}\left(\mathbb{s}_{i}^{\mathfrak{I}},\mu_{i}^{\mathfrak{I}}\right)}{\mathbb{g}}\right)\right)^{\Psi}\right)\right)^{\mathrm{T}^{s}}+\left(F\Finv^{s}-\mathfrak{I}\right)\left(\left(\left(\frac{\mathrm{T}^{-\mathfrak{I}}\left(\mathbb{s}_{i}^{\mathfrak{I}},\mu_{i}^{\mathfrak{I}}\right)}{\mathbb{g}}\right)\right)^{\Psi}\right)^{\mathrm{T}^{s}}\right)^{\frac{\mathfrak{I}}{\Psi}}}\right)\right), \\
\begin{pmatrix}
\dfrac{\mathrm{T}^{s\frac{\mathfrak{I}}{\psi}}\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{\mathfrak{I}}}\right)^{\mathrm{T}^{s}}}{\left(\left(\mathfrak{I}+\left(F\Finv^{s}-\mathfrak{I}\right)\left(\mathfrak{I}-\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{\mathfrak{I}}}\right)^{\Psi}\right)\right)^{\mathrm{T}^{s}}+\left(F\Finv^{s}-\mathfrak{I}\right)\left(\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{\mathfrak{I}}}\right)^{\Psi}\right)^{\mathrm{T}^{s}}\right)^{\frac{\mathfrak{I}}{\Psi}}}, \\
\dfrac{\mathrm{T}^{s\frac{\mathfrak{I}}{\psi}}\left(\mathbb{U}_{\mathscr{I}}^{\mathrm{M}^{\mathfrak{I}}}\right)^{\mathrm{T}^{s}}}{\left(\left(\mathfrak{I}+\left(F\Finv^{s}-\mathfrak{I}\right)\left(\mathfrak{I}-\left(\mathbb{U}_{\mathscr{I}}^{\mathrm{M}^{\mathfrak{I}}}\right)^{\Psi}\right)\right)^{\mathrm{T}^{s}}+\left(F\Finv^{s}-\mathfrak{I}\right)\left(\left(\mathbb{U}_{\mathscr{I}}^{\mathrm{M}^{\mathfrak{I}}}\right)^{\Psi}\right)^{\mathrm{T}^{s}}\right)^{\frac{\mathfrak{I}}{\Psi}}}
\end{pmatrix}, \\
\begin{pmatrix}
\left(\dfrac{\left(\mathfrak{I}+\left(F\Finv^{s}-\mathfrak{I}\right)\left(\mathbb{U}_{\mathscr{R}}^{N^{\mathfrak{I}}}\right)^{\psi}\right)^{\mathrm{T}^{s}}-\left(\mathfrak{I}-\left(\mathbb{U}_{\mathscr{R}}^{N^{\mathfrak{I}}}\right)^{\psi}\right)^{\mathrm{T}^{s}}}{\left(\mathfrak{I}+\left(F\Finv^{s}-\mathfrak{I}\right)\left(\mathbb{U}_{\mathscr{R}}^{N^{\mathfrak{I}}}\right)^{\psi}\right)^{\mathrm{T}^{s}}+\left(F\Finv^{s}-\mathfrak{I}\right)\left(\mathfrak{I}-\left(\mathbb{U}_{\mathscr{R}}^{N^{\mathfrak{I}}}\right)^{\psi}\right)^{\mathrm{T}^{s}}}\right)^{\frac{\mathfrak{I}}{\psi}}, \\
\left(\dfrac{\left(\mathfrak{I}+\left(F\Finv^{s}-\mathfrak{I}\right)\left(\mathbb{U}_{\mathscr{I}}^{N^{\mathfrak{I}}}\right)^{\psi}\right)^{\mathrm{T}^{s}}-\left(\mathfrak{I}-\left(\mathbb{U}_{\mathscr{I}}^{N^{\mathfrak{I}}}\right)^{\psi}\right)^{\mathrm{T}^{s}}}{\left(\mathfrak{I}+\left(F\Finv^{s}-\mathfrak{I}\right)\left(\mathbb{U}_{\mathscr{I}}^{N^{\mathfrak{I}}}\right)^{\psi}\right)^{\mathrm{T}^{s}}+\left(F\Finv^{s}-\mathfrak{I}\right)\left(\mathfrak{I}-\left(\mathbb{U}_{\mathscr{I}}^{N^{\mathfrak{I}}}\right)^{\psi}\right)^{\mathrm{T}^{s}}}\right)^{\frac{\mathfrak{I}}{\psi}}
\end{pmatrix}, \\
\begin{pmatrix}
\dfrac{\mathrm{T}^{s\frac{\mathfrak{I}}{\psi}}\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{\mathfrak{I}}}\right)^{\mathrm{T}^{s}}}{\left(\left(\mathfrak{I}+\left(F\Finv^{s}-\mathfrak{I}\right)\left(\mathfrak{I}-\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{\mathfrak{I}}}\right)^{\Psi}\right)\right)^{\mathrm{T}^{s}}+\left(F\Finv^{s}-\mathfrak{I}\right)\left(\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{\mathfrak{I}}}\right)^{\Psi}\right)^{\mathrm{T}^{s}}\right)^{\frac{\mathfrak{I}}{\Psi}}}, \\
\dfrac{\mathrm{T}^{s\frac{\mathfrak{I}}{\psi}}\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^{\mathfrak{I}}}\right)^{\mathrm{T}^{s}}}{\left(\left(\mathfrak{I}+\left(F\Finv^{s}-\mathfrak{I}\right)\left(\mathfrak{I}-\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^{\mathfrak{I}}}\right)^{\Psi}\right)\right)^{\mathrm{T}^{s}}+\left(F\Finv^{s}-\mathfrak{I}\right)\left(\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^{\mathfrak{I}}}\right)^{\Psi}\right)^{\mathrm{T}^{s}}\right)^{\frac{\mathfrak{I}}{\Psi}}}
\end{pmatrix}
\end{pmatrix}
$$

$$
\left(\mathfrak{N}^{\mathfrak{l}}\right)^{\mathrm{T}^s}_{TCN}=\begin{pmatrix}
\mathrm{T}\left(\mathbb{g}\left(\frac{\left(\mathfrak{l}+(F\daleth^s-\mathfrak{l})\left(\frac{\mathrm{T}^{-\mathfrak{l}}\left(\mathbb{s}_i^{\mathfrak{l}},\mu_i^{\mathfrak{l}}\right)}{\mathbb{g}}\right)^{\Psi}\right)^{\mathrm{T}^s}-\left(\mathfrak{l}-\left(\frac{\mathrm{T}^{-\mathfrak{l}}\left(\mathbb{s}_i^{\mathfrak{l}},\mu_i^{\mathfrak{l}}\right)}{\mathbb{g}}\right)^{\Psi}\right)^{\mathrm{T}^s}}{\left(\mathfrak{l}+(F\daleth^s-\mathfrak{l})\left(\frac{\mathrm{T}^{-\mathfrak{l}}\left(\mathbb{s}_i^{\mathfrak{l}},\mu_i^{\mathfrak{l}}\right)}{\mathbb{g}}\right)^{\Psi}\right)^{\mathrm{T}^s}+(F\daleth^s-\mathfrak{l})\left(\mathfrak{l}-\left(\frac{\mathrm{T}^{-\mathfrak{l}}\left(\mathbb{s}_i^{\mathfrak{l}},\mu_i^{\mathfrak{l}}\right)}{\mathbb{g}}\right)^{\Psi}\right)^{\mathrm{T}^s}}\right)^{\frac{\mathfrak{l}}{\Psi}}\right),\\
\left(\begin{matrix}\frac{\mathrm{T}^{s\frac{\mathfrak{l}}{\Psi}}\left(\Cup^{\mathrm{M}^{\mathfrak{l}}}_{\mathscr{R}}\right)^{\mathrm{T}^s}}{\left(\left(\mathfrak{l}+(F\daleth^s-\mathfrak{l})\left(\mathfrak{l}-\left(\Cup^{\mathrm{M}^{\mathfrak{l}}}_{\mathscr{R}}\right)^{\Psi}\right)\right)^{\mathrm{T}^s}+(F\daleth^s-\mathfrak{l})\left(\left(\Cup^{\mathrm{M}^{\mathfrak{l}}}_{\mathscr{R}}\right)^{\Psi}\right)^{\mathrm{T}^s}\right)^{\frac{\mathfrak{l}}{\Psi}}},\\
\frac{\mathrm{T}^{s\frac{\mathfrak{l}}{\Psi}}\left(\Cup^{\mathrm{M}^{\mathfrak{l}}}_{\mathscr{I}}\right)^{\mathrm{T}^s}}{\left(\left(\mathfrak{l}+(F\daleth^s-\mathfrak{l})\left(\mathfrak{l}-\left(\Cup^{\mathrm{M}^{\mathfrak{l}}}_{\mathscr{I}}\right)^{\Psi}\right)\right)^{\mathrm{T}^s}+(F\daleth^s-\mathfrak{l})\left(\left(\Cup^{\mathrm{M}^{\mathfrak{l}}}_{\mathscr{I}}\right)^{\Psi}\right)^{\mathrm{T}^s}\right)^{\frac{\mathfrak{l}}{\Psi}}}\end{matrix}\right),\\
\left(\begin{matrix}\left(\frac{\left(\mathfrak{l}+(F\daleth^s-\mathfrak{l})\left(\Cup^{N^{\mathfrak{l}}}_{\mathscr{R}}\right)^{\Psi}\right)^{\mathrm{T}^s}-\left(\mathfrak{l}-\left(\Cup^{N^{\mathfrak{l}}}_{\mathscr{R}}\right)^{\Psi}\right)^{\mathrm{T}^s}}{\left(\mathfrak{l}+(F\daleth^s-\mathfrak{l})\left(\Cup^{N^{\mathfrak{l}}}_{\mathscr{R}}\right)^{\Psi}\right)^{\mathrm{T}^s}+(F\daleth^s-\mathfrak{l})\left(\mathfrak{l}-\left(\Cup^{N^{\mathfrak{l}}}_{\mathscr{R}}\right)^{\Psi}\right)^{\mathrm{T}^s}}\right)^{\frac{\mathfrak{l}}{\Psi}},\\
\left(\frac{\left(\mathfrak{l}+(F\daleth^s-\mathfrak{l})\left(\Cup^{N^{\mathfrak{l}}}_{\mathscr{I}}\right)^{\Psi}\right)^{\mathrm{T}^s}-\left(\mathfrak{l}-\left(\Cup^{N^{\mathfrak{l}}}_{\mathscr{I}}\right)^{\Psi}\right)^{\mathrm{T}^s}}{\left(\mathfrak{l}+(F\daleth^s-\mathfrak{l})\left(\Cup^{N^{\mathfrak{l}}}_{\mathscr{I}}\right)^{\Psi}\right)^{\mathrm{T}^s}+(F\daleth^s-\mathfrak{l})\left(\mathfrak{l}-\left(\Cup^{N^{\mathfrak{l}}}_{\mathscr{I}}\right)^{\Psi}\right)^{\mathrm{T}^s}}\right)^{\frac{\mathfrak{l}}{\Psi}}\end{matrix}\right),\\
\left(\begin{matrix}\left(\frac{\left(\mathfrak{l}+(F\daleth^s-\mathfrak{l})\left(\Cup^{\mathbb{R}^{\mathfrak{l}}}_{\mathscr{R}}\right)^{\Psi}\right)^{\mathrm{T}^s}-\left(\mathfrak{l}-\left(\Cup^{\mathbb{R}^{\mathfrak{l}}}_{\mathscr{R}}\right)^{\Psi}\right)^{\mathrm{T}^s}}{\left(\mathfrak{l}+(F\daleth^s-\mathfrak{l})\left(\Cup^{\mathbb{R}^{\mathfrak{l}}}_{\mathscr{R}}\right)^{\Psi}\right)^{\mathrm{T}^s}+(F\daleth^s-\mathfrak{l})\left(\mathfrak{l}-\left(\Cup^{\mathbb{R}^{\mathfrak{l}}}_{\mathscr{R}}\right)^{\Psi}\right)^{\mathrm{T}^s}}\right)^{\frac{\mathfrak{l}}{\Psi}},\\
\left(\frac{\left(\mathfrak{l}+(F\daleth^s-\mathfrak{l})\left(\Cup^{\mathbb{R}^{\mathfrak{l}}}_{\mathscr{I}}\right)^{\Psi}\right)^{\mathrm{T}^s}-\left(\mathfrak{l}-\left(\Cup^{\mathbb{R}^{\mathfrak{l}}}_{\mathscr{I}}\right)^{\Psi}\right)^{\mathrm{T}^s}}{\left(\mathfrak{l}+(F\daleth^s-\mathfrak{l})\left(\Cup^{\mathbb{R}^{\mathfrak{l}}}_{\mathscr{I}}\right)^{\Psi}\right)^{\mathrm{T}^s}+(F\daleth^s-\mathfrak{l})\left(\mathfrak{l}-\left(\Cup^{\mathbb{R}^{\mathfrak{l}}}_{\mathscr{I}}\right)^{\Psi}\right)^{\mathrm{T}^s}}\right)^{\frac{\mathfrak{l}}{\Psi}}\end{matrix}\right)
\end{pmatrix}
$$

4. Hamacher power aggregation operators for CCp, q-QOF2-TLSs

This section described the model of CCp, q-QOF2-TLHPA operator, CCp, q-QOF2-TLHPWA operator, CCp, q-QOF2-TLHPG operator, CCp, q-QOF2-TLHPWG operator, and discuss their basic properties, such as idempotency, monotonicity, and boundedness. The geometrical abstract of the proposed operators is described in Fig. 3.

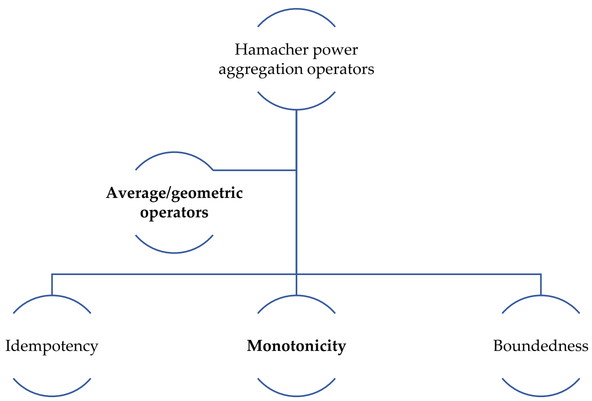

Fig. 3. Geometrical abstract of the proposed operators.

Definition 11. Based on any collection of CCp, q-QOF2-TLNs $\mathfrak{N}^j = \left(\left(\mathbb{s}_i^j, \mu_i^j\right), \left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^j}, \mathbb{U}_{\mathscr{I}}^{\mathrm{M}^j}\right), \left(\mathbb{U}_{\mathscr{R}}^{N^j}, \mathbb{U}_{\mathscr{I}}^{N^j}\right), \left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^j}, \mathbb{U}_{\mathscr{I}}^{\mathbb{R}^j}\right)\right), j = 1, 2, \ldots, m$, we described the technique of CCp, q-QOF2-TLHPA operator for both HTN and HTCN, such as

$$CC\mathrm{p}, \mathrm{q} - QOF2 - TLHPA\left(\mathfrak{N}^1, \mathfrak{N}^2, \ldots, \mathfrak{N}^m\right)_{TN} = \oplus_{TN}{}_{j=1}^{m} \frac{\left(1 + \#\left(\mathfrak{N}^j\right)\right)\mathfrak{N}^j}{\sum_{j=1}^{m}\left(1 + \#\left(\mathfrak{N}^j\right)\right)} = \oplus_{TN}{}_{j=1}^{m} \mathrm{T}_j^s \mathfrak{N}^j$$

$$CC\mathrm{p}, \mathrm{q} - QOF2 - TLHPA\left(\mathfrak{N}^1, \mathfrak{N}^2, \ldots, \mathfrak{N}^m\right)_{TCN} = \oplus_{TCN}{}_{j=1}^{m} \frac{\left(1 + \#\left(\mathfrak{N}^j\right)\right)\mathfrak{N}^j}{\sum_{j=1}^{m}\left(1 + \#\left(\mathfrak{N}^j\right)\right)} = \oplus_{TCN}{}_{j=1}^{m} \mathrm{T}_j^s \mathfrak{N}^j$$

where, $\mathrm{T}_j^s = \frac{\left(1+\#\left(\mathfrak{N}^j\right)\right)}{\sum_{j=1}^{m}\left(1+\#\left(\mathfrak{N}^j\right)\right)}$. Further, we explained the term $\#\left(\mathfrak{N}^j\right) = \sum_{j\neq k=1}^{m}\mathrm{sup}\left(\mathfrak{N}^j, \mathfrak{N}^k\right)$ and $\mathrm{sup}\left(\mathfrak{N}^j, \mathfrak{N}^k\right) = \mathrm{dis}\left(\mathfrak{N}^j, \mathfrak{N}^k\right)$ with some condition that is: (i) $\mathrm{sup}\left(\mathfrak{N}^j, \mathfrak{N}^k\right) \in [0, 1]$; (ii) $\mathrm{sup}\left(\mathfrak{N}^j, \mathfrak{N}^k\right) = \mathrm{sup}\left(\mathfrak{N}^k, \mathfrak{N}^j\right)$; and (iii) if $\mathrm{sup}\left(\mathfrak{N}^j, \mathfrak{N}^k\right) > \mathrm{sup}\left(\mathfrak{N}^l, \mathfrak{N}^m\right)$, then $\mathrm{dis}\left(\mathfrak{N}^j, \mathfrak{N}^k\right) < \mathrm{dis}\left(\mathfrak{N}^l, \mathfrak{N}^m\right)$. Further, we described the technique of distance measures for any CCp, q-QOF2-TLNs, such as

$$\mathrm{dis}\left(\mathfrak{N}^j, \mathfrak{N}^k\right) = (0.5)\left(\begin{array}{c} \left(\frac{\left|\mathrm{T}^{-1}\left(\mathbb{s}_i^j, \mu_i^j\right) - \mathrm{T}^{-1}\left(\mathbb{s}_i^k, \mu_i^k\right)\right|}{\mathbb{g}}\right) + \\ \left(\frac{1}{6}\left(\begin{array}{c} \left|\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^j}\right)^{\Psi} - \left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^k}\right)^{\Psi}\right| + \left|\left(\mathbb{U}_{\mathscr{I}}^{\mathrm{M}^j}\right)^{\Psi} - \left(\mathbb{U}_{\mathscr{I}}^{\mathrm{M}^k}\right)^{\Psi}\right| + \left|\left(\mathbb{U}_{\mathscr{R}}^{N^j}\right)^{\Psi} - \left(\mathbb{U}_{\mathscr{R}}^{N^k}\right)^{\Psi}\right| + \\ \left|\left(\mathbb{U}_{\mathscr{I}}^{N^j}\right)^{\Psi} - \left(\mathbb{U}_{\mathscr{I}}^{N^k}\right)^{\Psi}\right| + \left|\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^j}\right)^{\Psi} - \left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^k}\right)^{\Psi}\right| + \left|\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^j}\right)^{\Psi} - \left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^k}\right)^{\Psi}\right| \end{array}\right)\right) \end{array}\right).$$

Theorem 1. *Based on any collection of CCp, q-QOF2-TLNs* $\mathfrak{N}^j = \left(\left(\mathbb{s}_i^j, \mu_i^j\right), \left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^j}, \mathbb{U}_{\mathscr{I}}^{\mathrm{M}^j}\right), \left(\mathbb{U}_{\mathscr{R}}^{N^j}, \mathbb{U}_{\mathscr{I}}^{N^j}\right), \left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^j}, \mathbb{U}_{\mathscr{I}}^{\mathbb{R}^j}\right)\right), j = 1, 2, \ldots, m$, *we prove that the aggregated theory of CCp, q-QOF2-TLHPA operator is again a CCp, q-QOF2-TLN, such as*

$$
\begin{aligned}
& CC\mathrm{p},\mathrm{q}-QOF2 \\
& -TLHPA\left(\mathfrak{R}^{\mathfrak{l}},\mathfrak{R}^{2},\ldots,\mathfrak{R}^{m}\right)_{TN}= \\
& \left(\begin{array}{c}
\mathrm{T}\left(\mathbb{g}\left(\dfrac{\prod_{j=\mathfrak{l}}^{m}\left(\mathfrak{l}+(F\Finv^{s}-\mathfrak{l})\left(\dfrac{\mathrm{T}^{-\mathfrak{l}}\left(\mathbb{s}_i^j,\mu_i^j\right)}{\mathbb{g}}\right)^{\Psi}\right)^{\mathrm{T}_j^s}-\prod_{j=\mathfrak{l}}^{m}\left(\mathfrak{l}-\left(\dfrac{\mathrm{T}^{-\mathfrak{l}}\left(\mathbb{s}_i^j,\mu_i^j\right)}{\mathbb{g}}\right)^{\Psi}\right)^{\mathrm{T}_j^s}}{\prod_{j=\mathfrak{l}}^{m}\left(\mathfrak{l}+(F\Finv^{s}-\mathfrak{l})\left(\dfrac{\mathrm{T}^{-\mathfrak{l}}\left(\mathbb{s}_i^j,\mu_i^j\right)}{\mathbb{g}}\right)^{\Psi}\right)^{\mathrm{T}_j^s}+(F\Finv^{s}-\mathfrak{l})\prod_{j=\mathfrak{l}}^{m}\left(\mathfrak{l}-\left(\dfrac{\mathrm{T}^{-\mathfrak{l}}\left(\mathbb{s}_i^j,\mu_i^j\right)}{\mathbb{g}}\right)^{\Psi}\right)^{\mathrm{T}_j^s}}\right)^{\frac{\mathfrak{l}}{\Psi}}\right), \\
\left(\begin{array}{c}
\left(\dfrac{\prod_{j=\mathfrak{l}}^{m}\left(\mathfrak{l}+(F\Finv^{s}-\mathfrak{l})\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^j}\right)^{\Psi}\right)^{\mathrm{T}_j^s}-\prod_{j=\mathfrak{l}}^{m}\left(\mathfrak{l}-\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^j}\right)^{\Psi}\right)^{\mathrm{T}_j^s}}{\prod_{j=\mathfrak{l}}^{m}\left(\mathfrak{l}+(F\Finv^{s}-\mathfrak{l})\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^j}\right)^{\Psi}\right)^{\mathrm{T}_j^s}+(F\Finv^{s}-\mathfrak{l})\prod_{j=\mathfrak{l}}^{m}\left(\mathfrak{l}-\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^j}\right)^{\Psi}\right)^{\mathrm{T}_j^s}}\right)^{\frac{\mathfrak{l}}{\Psi}}, \\
\left(\dfrac{\prod_{j=\mathfrak{l}}^{m}\left(\mathfrak{l}+(F\Finv^{s}-\mathfrak{l})\left(\mathbb{U}_{\mathscr{I}}^{\mathrm{M}^j}\right)^{\Psi}\right)^{\mathrm{T}_j^s}-\prod_{j=\mathfrak{l}}^{m}\left(\mathfrak{l}-\left(\mathbb{U}_{\mathscr{I}}^{\mathrm{M}^j}\right)^{\Psi}\right)^{\mathrm{T}_j^s}}{\prod_{j=\mathfrak{l}}^{m}\left(\mathfrak{l}+(F\Finv^{s}-\mathfrak{l})\left(\mathbb{U}_{\mathscr{I}}^{\mathrm{M}^j}\right)^{\Psi}\right)^{\mathrm{T}_j^s}+(F\Finv^{s}-\mathfrak{l})\prod_{j=\mathfrak{l}}^{m}\left(\mathfrak{l}-\left(\mathbb{U}_{\mathscr{I}}^{\mathrm{M}^j}\right)^{\Psi}\right)^{\mathrm{T}_j^s}}\right)^{\frac{\mathfrak{l}}{\Psi}}
\end{array}\right), \\
\left(\begin{array}{c}
\dfrac{\mathrm{T}^{s\frac{\mathfrak{l}}{\psi}}\prod_{j=\mathfrak{l}}^{m}\left(\mathbb{U}_{\mathscr{R}}^{N^j}\right)^{\mathrm{T}_j^s}}{\left(\prod_{j=\mathfrak{l}}^{m}\left(\mathfrak{l}+(F\Finv^{s}-\mathfrak{l})\left(\mathfrak{l}-\left(\mathbb{U}_{\mathscr{R}}^{N^j}\right)^{\psi}\right)\right)^{\mathrm{T}_j^s}+(F\Finv^{s}-\mathfrak{l})\prod_{j=\mathfrak{l}}^{m}\left(\left(\mathbb{U}_{\mathscr{R}}^{N^j}\right)^{\psi}\right)^{\mathrm{T}_j^s}\right)^{\frac{\mathfrak{l}}{\psi}}}, \\
\dfrac{\mathrm{T}^{s\frac{\mathfrak{l}}{\psi}}\prod_{j=\mathfrak{l}}^{m}\left(\mathbb{U}_{\mathscr{I}}^{N^j}\right)^{\mathrm{T}_j^s}}{\left(\prod_{j=\mathfrak{l}}^{m}\left(\mathfrak{l}+(F\Finv^{s}-\mathfrak{l})\left(\mathfrak{l}-\left(\mathbb{U}_{\mathscr{I}}^{N^j}\right)^{\psi}\right)\right)^{\mathrm{T}_j^s}+(F\Finv^{s}-\mathfrak{l})\prod_{j=\mathfrak{l}}^{m}\left(\left(\mathbb{U}_{\mathscr{I}}^{N^j}\right)^{\psi}\right)^{\mathrm{T}_j^s}\right)^{\frac{\mathfrak{l}}{\psi}}}
\end{array}\right), \\
\left(\begin{array}{c}
\left(\dfrac{\prod_{j=\mathfrak{l}}^{m}\left(\mathfrak{l}+(F\Finv^{s}-\mathfrak{l})\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^j}\right)^{\Psi}\right)^{\mathrm{T}_j^s}-\prod_{j=\mathfrak{l}}^{m}\left(\mathfrak{l}-\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^j}\right)^{\Psi}\right)^{\mathrm{T}_j^s}}{\prod_{j=\mathfrak{l}}^{m}\left(\mathfrak{l}+(F\Finv^{s}-\mathfrak{l})\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^j}\right)^{\Psi}\right)^{\mathrm{T}_j^s}+(F\Finv^{s}-\mathfrak{l})\prod_{j=\mathfrak{l}}^{m}\left(\mathfrak{l}-\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^j}\right)^{\Psi}\right)^{\mathrm{T}_j^s}}\right)^{\frac{\mathfrak{l}}{\Psi}}, \\
\left(\dfrac{\prod_{j=\mathfrak{l}}^{m}\left(\mathfrak{l}+(F\Finv^{s}-\mathfrak{l})\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^j}\right)^{\Psi}\right)^{\mathrm{T}_j^s}-\prod_{j=\mathfrak{l}}^{m}\left(\mathfrak{l}-\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^j}\right)^{\Psi}\right)^{\mathrm{T}_j^s}}{\prod_{j=\mathfrak{l}}^{m}\left(\mathfrak{l}+(F\Finv^{s}-\mathfrak{l})\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^j}\right)^{\Psi}\right)^{\mathrm{T}_j^s}+(F\Finv^{s}-\mathfrak{l})\prod_{j=\mathfrak{l}}^{m}\left(\mathfrak{l}-\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^j}\right)^{\Psi}\right)^{\mathrm{T}_j^s}}\right)^{\frac{\mathfrak{l}}{\Psi}}
\end{array}\right)
\end{array}\right)
\end{aligned}
$$

and

$$CCp,q-QOF2-TLHPA\left(\mathfrak{N}^{1},\mathfrak{N}^{2},\ldots,\mathfrak{N}^{m}\right)_{TCN}=$$

$$\left(\begin{array}{c}
T\left(\mathbb{g}\left(\begin{array}{c}
\dfrac{T^{s\frac{1}{\psi}}\prod_{j=1}^{m}\left(\left(\frac{T^{-1}\left(\mathbb{s}_{i}^{j},\mu_{i}^{j}\right)}{\mathbb{g}}\right)\right)^{T_{j}^{s}}}{\left(\prod_{j=1}^{m}\left(1+(F\Finv^{s}-1)\left(1-\left(\left(\frac{T^{-1}\left(\mathbb{s}_{i}^{j},\mu_{i}^{j}\right)}{\mathbb{g}}\right)\right)^{\psi}\right)\right)^{T_{j}^{s}}+(F\Finv^{s}-1)\prod_{j=1}^{m}\left(\left(\left(\frac{T^{-1}\left(\mathbb{s}_{i}^{j},\mu_{i}^{j}\right)}{\mathbb{g}}\right)\right)^{\psi}\right)^{T_{j}^{s}}\right)^{\frac{1}{\psi}}},\\
\dfrac{T^{s\frac{1}{\psi}}\prod_{j=1}^{m}\left(\left(\frac{T^{-1}\left(\mathbb{s}_{i}^{j},\mu_{i}^{j}\right)}{\mathbb{g}}\right)\right)^{T_{j}^{s}}}{\left(\prod_{j=1}^{m}\left(1+(F\Finv^{s}-1)\left(1-\left(\left(\frac{T^{-1}\left(\mathbb{s}_{i}^{j},\mu_{i}^{j}\right)}{\mathbb{g}}\right)\right)^{\psi}\right)\right)^{T_{j}^{s}}+(F\Finv^{s}-1)\prod_{j=1}^{m}\left(\left(\left(\frac{T^{-1}\left(\mathbb{s}_{i}^{j},\mu_{i}^{j}\right)}{\mathbb{g}}\right)\right)^{\psi}\right)^{T_{j}^{s}}\right)^{\frac{1}{\psi}}}
\end{array}\right)\right),\\
\left(\begin{array}{c}
\left(\dfrac{\prod_{j=1}^{m}\left(1+(F\Finv^{s}-1)\left(\mathbb{U}_{\mathscr{R}}^{M^{j}}\right)^{\psi}\right)^{T_{j}^{s}}-\prod_{j=1}^{m}\left(1-\left(\mathbb{U}_{\mathscr{R}}^{M^{j}}\right)^{\psi}\right)^{T_{j}^{s}}}{\prod_{j=1}^{m}\left(1+(F\Finv^{s}-1)\left(\mathbb{U}_{\mathscr{R}}^{M^{j}}\right)^{\psi}\right)^{T_{j}^{s}}+(F\Finv^{s}-1)\prod_{j=1}^{m}\left(1-\left(\mathbb{U}_{\mathscr{R}}^{M^{j}}\right)^{\psi}\right)^{T_{j}^{s}}}\right)^{\frac{1}{\psi}},\\
\left(\dfrac{\prod_{j=1}^{m}\left(1+(F\Finv^{s}-1)\left(\mathbb{U}_{\mathscr{I}}^{M^{j}}\right)^{\psi}\right)^{T_{j}^{s}}-\prod_{j=1}^{m}\left(1-\left(\mathbb{U}_{\mathscr{I}}^{M^{j}}\right)^{\psi}\right)^{T_{j}^{s}}}{\prod_{j=1}^{m}\left(1+(F\Finv^{s}-1)\left(\mathbb{U}_{\mathscr{I}}^{M^{j}}\right)^{\psi}\right)^{T_{j}^{s}}+(F\Finv^{s}-1)\prod_{j=1}^{m}\left(1-\left(\mathbb{U}_{\mathscr{I}}^{M^{j}}\right)^{\psi}\right)^{T_{j}^{s}}}\right)^{\frac{1}{\psi}}
\end{array}\right),\\
\left(\begin{array}{c}
\dfrac{T^{s\frac{1}{\psi}}\prod_{j=1}^{m}\left(\mathbb{U}_{\mathscr{R}}^{N^{j}}\right)^{T_{j}^{s}}}{\left(\prod_{j=1}^{m}\left(1+(F\Finv^{s}-1)\left(1-\left(\mathbb{U}_{\mathscr{R}}^{N^{j}}\right)^{\psi}\right)\right)^{T_{j}^{s}}+(F\Finv^{s}-1)\prod_{j=1}^{m}\left(\left(\mathbb{U}_{\mathscr{R}}^{N^{j}}\right)^{\psi}\right)^{T_{j}^{s}}\right)^{\frac{1}{\psi}}},\\
\dfrac{T^{s\frac{1}{\psi}}\prod_{j=1}^{m}\left(\mathbb{U}_{\mathscr{I}}^{N^{j}}\right)^{T_{j}^{s}}}{\left(\prod_{j=1}^{m}\left(1+(F\Finv^{s}-1)\left(1-\left(\mathbb{U}_{\mathscr{I}}^{N^{j}}\right)^{\psi}\right)\right)^{T_{j}^{s}}+(F\Finv^{s}-1)\prod_{j=1}^{m}\left(\left(\mathbb{U}_{\mathscr{I}}^{N^{j}}\right)^{\psi}\right)^{T_{j}^{s}}\right)^{\frac{1}{\psi}}}
\end{array}\right),\\
\left(\begin{array}{c}
\dfrac{T^{s\frac{1}{\psi}}\prod_{j=1}^{m}\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{j}}\right)^{T_{j}^{s}}}{\left(\prod_{j=1}^{m}\left(1+(F\Finv^{s}-1)\left(1-\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{j}}\right)^{\psi}\right)\right)^{T_{j}^{s}}+(F\Finv^{s}-1)\prod_{j=1}^{m}\left(\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}}\right)^{\psi}\right)^{T_{j}^{s}}\right)^{\frac{1}{\psi}}},\\
\dfrac{T^{s\frac{1}{\psi}}\prod_{j=1}^{m}\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^{j}}\right)^{T_{j}^{s}}}{\left(\prod_{j=1}^{m}\left(1+(F\Finv^{s}-1)\left(1-\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^{j}}\right)^{\psi}\right)\right)^{T_{j}^{s}}+(F\Finv^{s}-1)\prod_{j=1}^{m}\left(\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^{j}}\right)^{\psi}\right)^{T_{j}^{s}}\right)^{\frac{1}{\psi}}}
\end{array}\right)
\end{array}\right)$$

Further, we described some basic properties of the above technique, called idempotency, monotonicity, and boundedness.

Property 1. *Based on any collection of CCp, q-QOF2-TLNs* $\mathfrak{N}^{j}=\left(\left(\mathbb{s}_{i}^{j},\mu_{i}^{j}\right),\left(\mathbb{U}_{\mathscr{R}}^{M^{j}},\mathbb{U}_{\mathscr{I}}^{M^{j}}\right),\left(\mathbb{U}_{\mathscr{R}}^{N^{j}},\mathbb{U}_{\mathscr{I}}^{N^{j}}\right),\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{j}},\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^{j}}\right)\right),j=1,2,\ldots,m$, *then*

1) *When* $\mathfrak{N}^{j}=\mathfrak{N}=\left(\left(\mathbb{s}_{i},\mu_{i}\right),\left(\mathbb{U}_{\mathscr{R}}^{M},\mathbb{U}_{\mathscr{I}}^{M}\right),\left(\mathbb{U}_{\mathscr{R}}^{N},\mathbb{U}_{\mathscr{I}}^{N}\right),\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}},\mathbb{U}_{\mathscr{I}}^{\mathbb{R}}\right)\right),j=1,2,\ldots,m$, *then*

$$CCp,q-QOF2-TLHPA\left(\mathfrak{N}^{1},\mathfrak{N}^{2},\ldots,\mathfrak{N}^{m}\right)_{TN}=\mathfrak{N}$$

$$CC\mathrm{p},\mathrm{q}-QOF2-TLHPA\left(\mathfrak{N}^{1},\mathfrak{N}^{2},\ldots,\mathfrak{N}^{m}\right)_{TCN}=\mathfrak{N}$$

2) *When* $\mathfrak{N}^{j}\leq\mathfrak{N}^{j}_{\#}$, *such as* $\left(\frac{\mathrm{T}^{-1}\left(\mathbb{s}^{j}_{i},\mu^{j}_{i}\right)}{\mathbb{g}}\right)\leq\left(\frac{\mathrm{T}^{-1}\left(\mathbb{s}_{\#}{}^{j}_{i},\mu_{\#}{}^{j}_{i}\right)}{\mathbb{g}}\right)$, $\mathbb{U}^{\mathrm{M}^{j}}_{\mathscr{R}}\leq\mathbb{U}_{\#}{}^{\mathrm{M}^{j}}_{\mathscr{R}}$, $\mathbb{U}^{\mathrm{M}^{j}}_{\mathscr{I}}\leq\mathbb{U}_{\#}{}^{\mathrm{M}^{j}}_{\mathscr{I}}$, $\mathbb{U}^{N^{j}}_{\mathscr{R}}\geq\mathbb{U}_{\#}{}^{N^{j}}_{\mathscr{R}}$, $\mathbb{U}^{N^{j}}_{\mathscr{I}}\geq\mathbb{U}_{\#}{}^{N^{j}}_{\mathscr{I}}$, *and* $\mathbb{U}^{\mathbb{R}^{j}}_{\mathscr{R}}\leq\mathbb{U}_{\#}{}^{\mathbb{R}^{j}}_{\mathscr{R}}$, $\mathbb{U}^{\mathbb{R}^{j}}_{\mathscr{I}}\leq\mathbb{U}_{\#}{}^{\mathbb{R}^{j}}_{\mathscr{I}}$ *then*

$$CC\mathrm{p},\mathrm{q}-QOF2-TLHPA\left(\mathfrak{N}^{1},\mathfrak{N}^{2},\ldots,\mathfrak{N}^{m}\right)_{TN}\leq CC\mathrm{p},\mathrm{q}-QOF2-TLHPA\left(\mathfrak{N}^{1}_{\#},\mathfrak{N}^{2}_{\#},\ldots,\mathfrak{N}^{m}_{\#}\right)_{TN}$$

$$CC\mathrm{p},\mathrm{q}-QOF2-TLHPA\left(\mathfrak{N}^{1},\mathfrak{N}^{2},\ldots,\mathfrak{N}^{m}\right)_{TCN}\leq CC\mathrm{p},\mathrm{q}-QOF2-TLHPA\left(\mathfrak{N}^{1}_{\#},\mathfrak{N}^{2}_{\#},\ldots,\mathfrak{N}^{m}_{\#}\right)_{TCN}$$

3) *When* $\mathfrak{N}^{+}=\max\left\{\mathfrak{N}^{1},\mathfrak{N}^{2},\ldots,\mathfrak{N}^{m}\right\}$ *and* $\mathfrak{N}^{-}=\min\left\{\mathfrak{N}^{1},\mathfrak{N}^{2},\ldots,\mathfrak{N}^{m}\right\}$, *then*

$$\mathfrak{N}^{-}\leq CC\mathrm{p},\mathrm{q}-QOF2-TLHPA\left(\mathfrak{N}^{1},\mathfrak{N}^{2},\ldots,\mathfrak{N}^{m}\right)_{TN}\leq\mathfrak{N}^{+}$$

$$\mathfrak{N}^{-}\leq CC\mathrm{p},\mathrm{q}-QOF2-TLHPA\left(\mathfrak{N}^{1},\mathfrak{N}^{2},\ldots,\mathfrak{N}^{m}\right)_{TCN}\leq\mathfrak{N}^{+}$$

Definition 12. *Based on any collection of CCp, q-QOF2-TLNs* $\mathfrak{N}^{j}=\left(\left(\mathbb{s}^{j}_{i},\mu^{j}_{i}\right),\left(\mathbb{U}^{\mathrm{M}^{j}}_{\mathscr{R}},\mathbb{U}^{\mathrm{M}^{j}}_{\mathscr{I}}\right),\left(\mathbb{U}^{N^{j}}_{\mathscr{R}},\mathbb{U}^{N^{j}}_{\mathscr{I}}\right),\left(\mathbb{U}^{\mathbb{R}^{j}}_{\mathscr{R}},\mathbb{U}^{\mathbb{R}^{j}}_{\mathscr{I}}\right)\right),j=1,2,\ldots,m$, *we described the technique of CCp, q-QOF2-TLHPWA operator for both HTN and HTCN, such as*

$$CC\mathrm{p},\mathrm{q}-QOF2-TLHPWA\left(\mathfrak{N}^{1},\mathfrak{N}^{2},\ldots,\mathfrak{N}^{m}\right)_{TN}=\oplus_{TN}{}^{m}_{j=1}\frac{{}^{\circ}C^{j}\left(1+\#\left(\mathfrak{N}^{j}\right)\right)\mathfrak{N}^{j}}{\sum_{j=1}^{m}{}^{\circ}C^{j}\left(1+\#\left(\mathfrak{N}^{j}\right)\right)}=\oplus_{TN}{}^{m}_{j=1}\mathrm{T}^{s}_{j}\mathfrak{N}^{j}$$

$$CC\mathrm{p},\mathrm{q}-QOF2-TLHPWA\left(\mathfrak{N}^{1},\mathfrak{N}^{2},\ldots,\mathfrak{N}^{m}\right)_{TCN}=\oplus_{TCN}{}^{m}_{j=1}\frac{{}^{\circ}C^{j}\left(1+\#\left(\mathfrak{N}^{j}\right)\right)\mathfrak{N}^{j}}{\sum_{j=1}^{m}{}^{\circ}C^{j}\left(1+\#\left(\mathfrak{N}^{j}\right)\right)}=\oplus_{TCN}{}^{m}_{j=1}\mathrm{T}^{s}_{j}\mathfrak{N}^{j}$$

where, $\mathrm{T}^{s}_{j}=\frac{{}^{\circ}C^{j}\left(1+\#\left(\mathfrak{N}^{j}\right)\right)}{\sum_{j=1}^{m}{}^{\circ}C^{j}\left(1+\#\left(\mathfrak{N}^{j}\right)\right)}$. *Further, we explained the term* $\#\left(\mathfrak{N}^{j}\right)=\sum_{j\neq k=1}^{m}\sup\left(\mathfrak{N}^{j},\mathfrak{N}^{k}\right)$ *and* $\sup\left(\mathfrak{N}^{j},\mathfrak{N}^{k}\right)=\mathrm{dis}\left(\mathfrak{N}^{j},\mathfrak{N}^{k}\right)$ *with some condition that is*: (*i*) $\sup\left(\mathfrak{N}^{j},\mathfrak{N}^{k}\right)\in[0,1]$; (*ii*) $\sup\left(\mathfrak{N}^{j},\mathfrak{N}^{k}\right)=\sup\left(\mathfrak{N}^{k},\mathfrak{N}^{j}\right)$; *and* (*iii*) *if* $\sup\left(\mathfrak{N}^{j},\mathfrak{N}^{k}\right)>\sup\left(\mathfrak{N}^{l},\mathfrak{N}^{m}\right)$, *then* $\mathrm{dis}\left(\mathfrak{N}^{j},\mathfrak{N}^{k}\right)<\mathrm{dis}\left(\mathfrak{N}^{l},\mathfrak{N}^{m}\right)$. *Further, we described the technique of distance measures for any CCp, q-QOF2-TLNs, such as*

$$\mathrm{dis}\left(\mathfrak{N}^{j},\mathfrak{N}^{k}\right)=(0.5)\left(\begin{array}{c}\left(\frac{\left|\mathrm{T}^{-1}\left(\mathbb{s}^{j}_{i},\mu^{j}_{i}\right)-\mathrm{T}^{-1}\left(\mathbb{s}^{k}_{i},\mu^{k}_{i}\right)\right|}{\mathbb{g}}\right)+\\ \left(\frac{1}{6}\left(\begin{array}{c}\left|\left(\mathbb{U}^{\mathrm{M}^{j}}_{\mathscr{R}}\right)^{\Psi}-\left(\mathbb{U}^{\mathrm{M}^{k}}_{\mathscr{R}}\right)^{\Psi}\right|+\left|\left(\mathbb{U}^{\mathrm{M}^{j}}_{\mathscr{I}}\right)^{\Psi}-\left(\mathbb{U}^{\mathrm{M}^{k}}_{\mathscr{I}}\right)^{\Psi}\right|+\left|\left(\mathbb{U}^{N^{j}}_{\mathscr{R}}\right)^{\Psi}-\left(\mathbb{U}^{N^{k}}_{\mathscr{R}}\right)^{\Psi}\right|+\\ \left|\left(\mathbb{U}^{N^{j}}_{\mathscr{I}}\right)^{\Psi}-\left(\mathbb{U}^{N^{k}}_{\mathscr{I}}\right)^{\Psi}\right|+\left|\left(\mathbb{U}^{\mathbb{R}^{j}}_{\mathscr{R}}\right)^{\Psi}-\left(\mathbb{U}^{\mathbb{R}^{k}}_{\mathscr{R}}\right)^{\Psi}\right|+\left|\left(\mathbb{U}^{\mathbb{R}^{j}}_{\mathscr{I}}\right)^{\Psi}-\left(\mathbb{U}^{\mathbb{R}^{k}}_{\mathscr{I}}\right)^{\Psi}\right|\end{array}\right)\right)\end{array}\right)$$

Theorem 2. *Based on any collection of CCp, q-QOF2-TLNs* $\mathfrak{N}^{j}=\left(\left(\mathbb{s}^{j}_{i},\mu^{j}_{i}\right),\left(\mathbb{U}^{\mathrm{M}^{j}}_{\mathscr{R}},\mathbb{U}^{\mathrm{M}^{j}}_{\mathscr{I}}\right),\left(\mathbb{U}^{N^{j}}_{\mathscr{R}},\mathbb{U}^{N^{j}}_{\mathscr{I}}\right),\left(\mathbb{U}^{\mathbb{R}^{j}}_{\mathscr{R}},\mathbb{U}^{\mathbb{R}^{j}}_{\mathscr{I}}\right)\right),j=1,2,\ldots,m$, *we prove that the aggregated theory of CCp, q-QOF2-TLHPWA operator is again a CCp, q-QOF2-TLN, such as*

$$
\begin{aligned}
&CC\mathrm{p},\mathrm{q}-QOF2 \\
&-TLHPWA\left(\mathfrak{R}^{\text{ɩ}},\mathfrak{R}^{2},\ldots,\mathfrak{R}^{m}\right)_{TN}= \\
&\left(\begin{array}{c}
\mathrm{T}\left(\mathbb{g}\left(\dfrac{\prod_{j=\text{ɩ}}^{m}\left(\text{ɩ}+(F\text{Ⅎ}^{s}-\text{ɩ})\left(\dfrac{\mathrm{T}^{-\text{ɩ}}\left(\mathbb{s}_{i}^{j},\mu_{i}^{j}\right)}{\mathbb{g}}\right)^{\Psi}\right)^{\mathrm{T}_{j}^{s}}-\prod_{j=\text{ɩ}}^{m}\left(\text{ɩ}-\left(\dfrac{\mathrm{T}^{-\text{ɩ}}\left(\mathbb{s}_{i}^{j},\mu_{i}^{j}\right)}{\mathbb{g}}\right)^{\Psi}\right)^{\mathrm{T}_{j}^{s}}}{\prod_{j=\text{ɩ}}^{m}\left(\text{ɩ}+(F\text{Ⅎ}^{s}-\text{ɩ})\left(\dfrac{\mathrm{T}^{-\text{ɩ}}\left(\mathbb{s}_{i}^{j},\mu_{i}^{j}\right)}{\mathbb{g}}\right)^{\Psi}\right)^{\mathrm{T}_{j}^{s}}+(F\text{Ⅎ}^{s}-\text{ɩ})\prod_{j=\text{ɩ}}^{m}\left(\text{ɩ}-\left(\dfrac{\mathrm{T}^{-\text{ɩ}}\left(\mathbb{s}_{i}^{j},\mu_{i}^{j}\right)}{\mathbb{g}}\right)^{\Psi}\right)^{\mathrm{T}_{j}^{s}}}\right)^{\frac{\text{ɩ}}{\Psi}}\right), \\
\left(\begin{array}{c}
\left(\dfrac{\prod_{j=\text{ɩ}}^{m}\left(\text{ɩ}+(F\text{Ⅎ}^{s}-\text{ɩ})\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{j}}\right)^{\Psi}\right)^{\mathrm{T}_{j}^{s}}-\prod_{j=\text{ɩ}}^{m}\left(\text{ɩ}-\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{j}}\right)^{\Psi}\right)^{\mathrm{T}_{j}^{s}}}{\prod_{j=\text{ɩ}}^{m}\left(\text{ɩ}+(F\text{Ⅎ}^{s}-\text{ɩ})\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{j}}\right)^{\Psi}\right)^{\mathrm{T}_{j}^{s}}+(F\text{Ⅎ}^{s}-\text{ɩ})\prod_{j=\text{ɩ}}^{m}\left(\text{ɩ}-\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{j}}\right)^{\Psi}\right)^{\mathrm{T}_{j}^{s}}}\right)^{\frac{\text{ɩ}}{\Psi}}, \\
\left(\dfrac{\prod_{j=\text{ɩ}}^{m}\left(\text{ɩ}+(F\text{Ⅎ}^{s}-\text{ɩ})\left(\mathbb{U}_{\mathscr{I}}^{\mathrm{M}^{j}}\right)^{\Psi}\right)^{\mathrm{T}_{j}^{s}}-\prod_{j=\text{ɩ}}^{m}\left(\text{ɩ}-\left(\mathbb{U}_{\mathscr{I}}^{\mathrm{M}^{j}}\right)^{\Psi}\right)^{\mathrm{T}_{j}^{s}}}{\prod_{j=\text{ɩ}}^{m}\left(\text{ɩ}+(F\text{Ⅎ}^{s}-\text{ɩ})\left(\mathbb{U}_{\mathscr{I}}^{\mathrm{M}^{j}}\right)^{\Psi}\right)^{\mathrm{T}_{j}^{s}}+(F\text{Ⅎ}^{s}-\text{ɩ})\prod_{j=\text{ɩ}}^{m}\left(\text{ɩ}-\left(\mathbb{U}_{\mathscr{I}}^{\mathrm{M}^{j}}\right)^{\Psi}\right)^{\mathrm{T}_{j}^{s}}}\right)^{\frac{\text{ɩ}}{\Psi}}
\end{array}\right), \\
\left(\begin{array}{c}
\dfrac{\mathrm{T}^{s\frac{\text{ɩ}}{\Psi}}\prod_{j=\text{ɩ}}^{m}\left(\mathbb{U}_{\mathscr{R}}^{N^{j}}\right)^{\mathrm{T}_{j}^{s}}}{\left(\prod_{j=\text{ɩ}}^{m}\left(\text{ɩ}+(F\text{Ⅎ}^{s}-\text{ɩ})\left(\text{ɩ}-\left(\mathbb{U}_{\mathscr{R}}^{N^{j}}\right)^{\Psi}\right)\right)^{\mathrm{T}_{j}^{s}}+(F\text{Ⅎ}^{s}-\text{ɩ})\prod_{j=\text{ɩ}}^{m}\left(\left(\mathbb{U}_{\mathscr{R}}^{N^{j}}\right)^{\Psi}\right)^{\mathrm{T}_{j}^{s}}\right)^{\frac{\text{ɩ}}{\Psi}}}, \\
\dfrac{\mathrm{T}^{s\frac{\text{ɩ}}{\Psi}}\prod_{j=\text{ɩ}}^{m}\left(\mathbb{U}_{\mathscr{I}}^{N^{j}}\right)^{\mathrm{T}_{j}^{s}}}{\left(\prod_{j=\text{ɩ}}^{m}\left(\text{ɩ}+(F\text{Ⅎ}^{s}-\text{ɩ})\left(\text{ɩ}-\left(\mathbb{U}_{\mathscr{I}}^{N^{j}}\right)^{\Psi}\right)\right)^{\mathrm{T}_{j}^{s}}+(F\text{Ⅎ}^{s}-\text{ɩ})\prod_{j=\text{ɩ}}^{m}\left(\left(\mathbb{U}_{\mathscr{I}}^{N^{j}}\right)^{\Psi}\right)^{\mathrm{T}_{j}^{s}}\right)^{\frac{\text{ɩ}}{\Psi}}}
\end{array}\right), \\
\left(\begin{array}{c}
\left(\dfrac{\prod_{j=\text{ɩ}}^{m}\left(\text{ɩ}+(F\text{Ⅎ}^{s}-\text{ɩ})\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{j}}\right)^{\Psi}\right)^{\mathrm{T}_{j}^{s}}-\prod_{j=\text{ɩ}}^{m}\left(\text{ɩ}-\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{j}}\right)^{\Psi}\right)^{\mathrm{T}_{j}^{s}}}{\prod_{j=\text{ɩ}}^{m}\left(\text{ɩ}+(F\text{Ⅎ}^{s}-\text{ɩ})\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{j}}\right)^{\Psi}\right)^{\mathrm{T}_{j}^{s}}+(F\text{Ⅎ}^{s}-\text{ɩ})\prod_{j=\text{ɩ}}^{m}\left(\text{ɩ}-\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{j}}\right)^{\Psi}\right)^{\mathrm{T}_{j}^{s}}}\right)^{\frac{\text{ɩ}}{\Psi}}, \\
\left(\dfrac{\prod_{j=\text{ɩ}}^{m}\left(\text{ɩ}+(F\text{Ⅎ}^{s}-\text{ɩ})\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^{j}}\right)^{\Psi}\right)^{\mathrm{T}_{j}^{s}}-\prod_{j=\text{ɩ}}^{m}\left(\text{ɩ}-\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^{j}}\right)^{\Psi}\right)^{\mathrm{T}_{j}^{s}}}{\prod_{j=\text{ɩ}}^{m}\left(\text{ɩ}+(F\text{Ⅎ}^{s}-\text{ɩ})\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^{j}}\right)^{\Psi}\right)^{\mathrm{T}_{j}^{s}}+(F\text{Ⅎ}^{s}-\text{ɩ})\prod_{j=\text{ɩ}}^{m}\left(\text{ɩ}-\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^{j}}\right)^{\Psi}\right)^{\mathrm{T}_{j}^{s}}}\right)^{\frac{\text{ɩ}}{\Psi}}
\end{array}\right)
\end{array}\right)
\end{aligned}
$$

and

$$CC\mathrm{p},\mathrm{q}-QOF2-TLHPWA\left(\mathfrak{N}^{ſ},\mathfrak{N}^{2},\ldots,\mathfrak{N}^{m}\right)_{TCN}=$$

$$\left(\begin{array}{c}
Ƭ\left(𝕘\left(\frac{Ƭ^{s\frac{ſ}{\psi}}\prod_{j=ſ}^{m}\left(\left(\frac{Ƭ^{-ſ}\left(s_{i}^{j},\mu_{i}^{j}\right)}{𝕘}\right)\right)^{Ƭ_{j}^{s}}}{\left(\prod_{j=ſ}^{m}\left(ſ+(F⅃^{s}-ſ)\left(ſ-\left(\left(\frac{Ƭ^{-ſ}\left(s_{i}^{j},\mu_{i}^{j}\right)}{𝕘}\right)\right)^{\psi}\right)\right)^{Ƭ_{j}^{s}}+(F⅃^{s}-ſ)\prod_{j=ſ}^{m}\left(\left(\left(\frac{Ƭ^{-ſ}\left(s_{i}^{j},\mu_{i}^{j}\right)}{𝕘}\right)\right)^{\psi}\right)^{Ƭ_{j}^{s}}\right)^{\frac{ſ}{\psi}}},\frac{Ƭ^{s\frac{ſ}{\psi}}\prod_{j=ſ}^{m}\left(\left(\frac{Ƭ^{-ſ}\left(s_{i}^{j},\mu_{i}^{j}\right)}{𝕘}\right)\right)^{Ƭ_{j}^{s}}}{\left(\prod_{j=ſ}^{m}\left(ſ+(F⅃^{s}-ſ)\left(ſ-\left(\left(\frac{Ƭ^{-ſ}\left(s_{i}^{j},\mu_{i}^{j}\right)}{𝕘}\right)\right)^{\psi}\right)\right)^{Ƭ_{j}^{s}}+(F⅃^{s}-ſ)\prod_{j=ſ}^{m}\left(\left(\left(\frac{Ƭ^{-ſ}\left(s_{i}^{j},\mu_{i}^{j}\right)}{𝕘}\right)\right)^{\psi}\right)^{Ƭ_{j}^{s}}\right)^{\frac{ſ}{\psi}}}\right)\right)\right), \\
\left(\left(\frac{\prod_{j=ſ}^{m}\left(ſ+(F⅃^{s}-ſ)\left(𝕌_{\mathscr{R}}^{M^{j}}\right)^{\psi}\right)^{Ƭ_{j}^{s}}-\prod_{j=ſ}^{m}\left(ſ-\left(𝕌_{\mathscr{R}}^{M^{j}}\right)^{\psi}\right)^{Ƭ_{j}^{s}}}{\prod_{j=ſ}^{m}\left(ſ+(F⅃^{s}-ſ)\left(𝕌_{\mathscr{R}}^{M^{j}}\right)^{\psi}\right)^{Ƭ_{j}^{s}}+(F⅃^{s}-ſ)\prod_{j=ſ}^{m}\left(ſ-\left(𝕌_{\mathscr{R}}^{M^{j}}\right)^{\psi}\right)^{Ƭ_{j}^{s}}}\right)^{\frac{ſ}{\psi}},\left(\frac{\prod_{j=ſ}^{m}\left(ſ+(F⅃^{s}-ſ)\left(𝕌_{\mathscr{I}}^{M^{j}}\right)^{\psi}\right)^{Ƭ_{j}^{s}}-\prod_{j=ſ}^{m}\left(ſ-\left(𝕌_{\mathscr{I}}^{M^{j}}\right)^{\psi}\right)^{Ƭ_{j}^{s}}}{\prod_{j=ſ}^{m}\left(ſ+(F⅃^{s}-ſ)\left(𝕌_{\mathscr{I}}^{M^{j}}\right)^{\psi}\right)^{Ƭ_{j}^{s}}+(F⅃^{s}-ſ)\prod_{j=ſ}^{m}\left(ſ-\left(𝕌_{\mathscr{I}}^{M^{j}}\right)^{\psi}\right)^{Ƭ_{j}^{s}}}\right)^{\frac{ſ}{\psi}}\right), \\
\left(\frac{Ƭ^{s\frac{ſ}{\psi}}\prod_{j=ſ}^{m}\left(𝕌_{\mathscr{R}}^{N^{j}}\right)^{Ƭ_{j}^{s}}}{\left(\prod_{j=ſ}^{m}\left(ſ+(F⅃^{s}-ſ)\left(ſ-\left(𝕌_{\mathscr{R}}^{N^{j}}\right)^{\psi}\right)\right)^{Ƭ_{j}^{s}}+(F⅃^{s}-ſ)\prod_{j=ſ}^{m}\left(\left(𝕌_{\mathscr{R}}^{N^{j}}\right)^{\psi}\right)^{Ƭ_{j}^{s}}\right)^{\frac{ſ}{\psi}}},\frac{Ƭ^{s\frac{ſ}{\psi}}\prod_{j=ſ}^{m}\left(𝕌_{\mathscr{I}}^{N^{j}}\right)^{Ƭ_{j}^{s}}}{\left(\prod_{j=ſ}^{m}\left(ſ+(F⅃^{s}-ſ)\left(ſ-\left(𝕌_{\mathscr{I}}^{N^{j}}\right)^{\psi}\right)\right)^{Ƭ_{j}^{s}}+(F⅃^{s}-ſ)\prod_{j=ſ}^{m}\left(\left(𝕌_{\mathscr{I}}^{N^{j}}\right)^{\psi}\right)^{Ƭ_{j}^{s}}\right)^{\frac{ſ}{\psi}}}\right), \\
\left(\frac{Ƭ^{s\frac{ſ}{\psi}}\prod_{j=ſ}^{m}\left(𝕌_{\mathscr{R}}^{\mathbb{R}^{j}}\right)^{Ƭ_{j}^{s}}}{\left(\prod_{j=ſ}^{m}\left(ſ+(F⅃^{s}-ſ)\left(ſ-\left(𝕌_{\mathscr{R}}^{\mathbb{R}^{j}}\right)^{\psi}\right)\right)^{Ƭ_{j}^{s}}+(F⅃^{s}-ſ)\prod_{j=ſ}^{m}\left(\left(𝕌_{\mathscr{R}}^{\mathbb{R}}\right)^{\psi}\right)^{Ƭ_{j}^{s}}\right)^{\frac{ſ}{\psi}}},\frac{Ƭ^{s\frac{ſ}{\psi}}\prod_{j=ſ}^{m}\left(𝕌_{\mathscr{I}}^{\mathbb{R}^{j}}\right)^{Ƭ_{j}^{s}}}{\left(\prod_{j=ſ}^{m}\left(ſ+(F⅃^{s}-ſ)\left(ſ-\left(𝕌_{\mathscr{I}}^{\mathbb{R}^{j}}\right)^{\psi}\right)\right)^{Ƭ_{j}^{s}}+(F⅃^{s}-ſ)\prod_{j=ſ}^{m}\left(\left(𝕌_{\mathscr{I}}^{\mathbb{R}^{j}}\right)^{\psi}\right)^{Ƭ_{j}^{s}}\right)^{\frac{ſ}{\psi}}}\right)
\end{array}\right)$$

Further, we described some basic properties of the above technique, called idempotency, monotonicity, and boundedness.

Property 2. *Based on any collection of CCp, q-QOF2-TLNs* $\mathfrak{N}^{j}=\left(\left(s_{i}^{j},\mu_{i}^{j}\right),\left(𝕌_{\mathscr{R}}^{M^{j}},𝕌_{\mathscr{I}}^{M^{j}}\right),\left(𝕌_{\mathscr{R}}^{N^{j}},𝕌_{\mathscr{I}}^{N^{j}}\right),\left(𝕌_{\mathscr{R}}^{\mathbb{R}^{j}},𝕌_{\mathscr{I}}^{\mathbb{R}^{j}}\right)\right),j=ſ,2,\ldots,m$, *then*

1) *When* $\mathfrak{N}^{j}=\mathfrak{N}=\left((s_{i},\mu_{i}),\left(𝕌_{\mathscr{R}}^{M},𝕌_{\mathscr{I}}^{M}\right),\left(𝕌_{\mathscr{R}}^{N},𝕌_{\mathscr{I}}^{N}\right),\left(𝕌_{\mathscr{R}}^{\mathbb{R}},𝕌_{\mathscr{I}}^{\mathbb{R}}\right)\right),j=ſ,2,\ldots,m$**,** *then*

$$CC\mathrm{p},\mathrm{q}-QOF2-TLHPWA\left(\mathfrak{N}^{ſ},\mathfrak{N}^{2},\ldots,\mathfrak{N}^{m}\right)_{TN}=\mathfrak{N}$$

$$CCp, q - QOF2 - TLHPWA(\mathfrak{R}^1, \mathfrak{R}^2, \ldots, \mathfrak{R}^m)_{TCN} = \mathfrak{R}$$

2) *When* $\mathfrak{R}^j \leq \mathfrak{R}^j_{\#}$, *such as* $\left(\frac{T^{-1}(s_i^j, \mu_i^j)}{g}\right) \leq \left(\frac{T^{-1}(s_{\#i}^j, \mu_{\#i}^j)}{g}\right)$, $\mathbb{U}^{M^j}_{\mathscr{R}} \leq \mathbb{U}_{\#\mathscr{R}}^{M^j}, \mathbb{U}^{M^j}_{\mathscr{I}} \leq \mathbb{U}_{\#\mathscr{I}}^{M^j}, \mathbb{U}^{N^j}_{\mathscr{R}} \geq \mathbb{U}_{\#\mathscr{R}}^{N^j}, \mathbb{U}^{N^j}_{\mathscr{I}} \geq \mathbb{U}_{\#\mathscr{I}}^{N^j}$, *and* $\mathbb{U}^{\mathbb{R}^j}_{\mathscr{R}} \leq \mathbb{U}_{\#\mathscr{R}}^{\mathbb{R}^j}, \mathbb{U}^{\mathbb{R}^j}_{\mathscr{I}} \leq \mathbb{U}_{\#\mathscr{I}}^{\mathbb{R}^j}$ *then*

$$CCp, q - QOF2 - TLHPWA(\mathfrak{R}^1, \mathfrak{R}^2, \ldots, \mathfrak{R}^m)_{TN} \leq CCp, q - QOF2 - TLHPWA(\mathfrak{R}^1_{\#}, \mathfrak{R}^2_{\#}, \ldots, \mathfrak{R}^m_{\#})_{TN}$$

$$CCp, q - QOF2 - TLHPWA(\mathfrak{R}^1, \mathfrak{R}^2, \ldots, \mathfrak{R}^m)_{TCN} \leq CCp, q - QOF2 - TLHPWA(\mathfrak{R}^1_{\#}, \mathfrak{R}^2_{\#}, \ldots, \mathfrak{R}^m_{\#})_{TCN}$$

3) *When* $\mathfrak{R}^+ = \max\{\mathfrak{R}^1, \mathfrak{R}^2, \ldots, \mathfrak{R}^m\}$ *and* $\mathfrak{R}^- = \min\{\mathfrak{R}^1, \mathfrak{R}^2, \ldots, \mathfrak{R}^m\}$, *then*

$$\mathfrak{R}^- \leq CCp, q - QOF2 - TLHPWA(\mathfrak{R}^1, \mathfrak{R}^2, \ldots, \mathfrak{R}^m)_{TN} \leq \mathfrak{R}^+$$

$$\mathfrak{R}^- \leq CCp, q - QOF2 - TLHPWA(\mathfrak{R}^1, \mathfrak{R}^2, \ldots, \mathfrak{R}^m)_{TCN} \leq \mathfrak{R}^+$$

Definition 13. *Based on any collection of CCp, q-QOF2-TLNs* $\mathfrak{R}^j = \left((s_i^j, \mu_i^j), (\mathbb{U}^{M^j}_{\mathscr{R}}, \mathbb{U}^{M^j}_{\mathscr{I}}), (\mathbb{U}^{N^j}_{\mathscr{R}}, \mathbb{U}^{N^j}_{\mathscr{I}}), (\mathbb{U}^{\mathbb{R}^j}_{\mathscr{R}}, \mathbb{U}^{\mathbb{R}^j}_{\mathscr{I}})\right), j = 1, 2, \ldots, m$, *we described the technique of CCp, q-QOF2-TLHPG operator for both HTN and HTCN, such as*

$$CCp, q - QOF2 - TLHPG(\mathfrak{R}^1, \mathfrak{R}^2, \ldots, \mathfrak{R}^m)_{TN} = \otimes_{TN}{}_{j=1}^{m}(\mathfrak{R}^j)^{\frac{(1+\#(\mathfrak{R}^j))}{\sum_{j=1}^{m}(1+\#(\mathfrak{R}^j))}} = \otimes_{TN}{}_{j=1}^{m}(\mathfrak{R}^j)^{T_j^s}$$

$$CCp, q - QOF2 - TLHPG(\mathfrak{R}^1, \mathfrak{R}^2, \ldots, \mathfrak{R}^m)_{TCN} = \otimes_{TCN}{}_{j=1}^{m}(\mathfrak{R}^j)^{\frac{(1+\#(\mathfrak{R}^j))}{\sum_{j=1}^{m}(1+\#(\mathfrak{R}^j))}} = \otimes_{TCN}{}_{j=1}^{m}(\mathfrak{R}^j)^{T_j^s}$$

where, $T_j^s = \frac{(1+\#(\mathfrak{R}^j))}{\sum_{j=1}^{m}(1+\#(\mathfrak{R}^j))}$. *Further, we explained the term* $\#(\mathfrak{R}^j) = \sum_{j\neq k=1}^{m} \sup(\mathfrak{R}^j, \mathfrak{R}^k)$ *and* $\sup(\mathfrak{R}^j, \mathfrak{R}^k) = \mathrm{dis}(\mathfrak{R}^j, \mathfrak{R}^k)$ *with some condition that is*: (*i*) $\sup(\mathfrak{R}^j, \mathfrak{R}^k) \in [0, 1]$; (*ii*) $\sup(\mathfrak{R}^j, \mathfrak{R}^k) = \sup(\mathfrak{R}^k, \mathfrak{R}^j)$; *and* (*iii*) *if* $\sup(\mathfrak{R}^j, \mathfrak{R}^k) > \sup(\mathfrak{R}^l, \mathfrak{R}^m)$, *then* $\mathrm{dis}(\mathfrak{R}^j, \mathfrak{R}^k) < \mathrm{dis}(\mathfrak{R}^l, \mathfrak{R}^m)$. *Further, we described the technique of distance measures for any CCp, q-QOF2-TLNs, such as*

$$\mathrm{dis}(\mathfrak{R}^j, \mathfrak{R}^k) = (0.5)\left(\begin{array}{c}\left(\frac{\left|T^{-1}(s_i^j, \mu_i^j) - T^{-1}(s_i^k, \mu_i^k)\right|}{g}\right) + \\ \left(\frac{1}{6}\left(\begin{array}{c}\left|(\mathbb{U}^{M^j}_{\mathscr{R}})^{\Psi} - (\mathbb{U}^{M^k}_{\mathscr{R}})^{\Psi}\right| + \left|(\mathbb{U}^{M^j}_{\mathscr{I}})^{\Psi} - (\mathbb{U}^{M^k}_{\mathscr{I}})^{\Psi}\right| + \left|(\mathbb{U}^{N^j}_{\mathscr{R}})^{\Psi} - (\mathbb{U}^{N^k}_{\mathscr{R}})^{\Psi}\right| + \\ \left|(\mathbb{U}^{N^j}_{\mathscr{I}})^{\Psi} - (\mathbb{U}^{N^k}_{\mathscr{I}})^{\Psi}\right| + \left|(\mathbb{U}^{\mathbb{R}^j}_{\mathscr{R}})^{\Psi} - (\mathbb{U}^{\mathbb{R}^k}_{\mathscr{R}})^{\Psi}\right| + \left|(\mathbb{U}^{\mathbb{R}^j}_{\mathscr{I}})^{\Psi} - (\mathbb{U}^{\mathbb{R}^k}_{\mathscr{I}})^{\Psi}\right|\end{array}\right)\right)\end{array}\right)$$

Theorem 3. *Based on any collection of CCp, q-QOF2-TLNs* $\mathfrak{R}^j = \left((s_i^j, \mu_i^j), (\mathbb{U}^{M^j}_{\mathscr{R}}, \mathbb{U}^{M^j}_{\mathscr{I}}), (\mathbb{U}^{N^j}_{\mathscr{R}}, \mathbb{U}^{N^j}_{\mathscr{I}}), (\mathbb{U}^{\mathbb{R}^j}_{\mathscr{R}}, \mathbb{U}^{\mathbb{R}^j}_{\mathscr{I}})\right), j = 1, 2, \ldots, m$, *we prove that the aggregated theory of CCp, q-QOF2-TLHPG operator is again a CCp, q-QOF2-TLN, such as*

$$
\begin{aligned}
& CC\mathrm{p},\mathrm{q}-QOF2-TLHPG\left(\mathfrak{N}^{ſ},\mathfrak{N}^{2},\ldots,\mathfrak{N}^{m}\right)_{TN}= \\
& \left(
\begin{array}{c}
\mathrm{T}\left(\mathbb{g}\left(
\dfrac{\mathrm{T}^{s\frac{ſ}{\Psi}}\prod_{j=ſ}^{m}\left(\left(\dfrac{\mathrm{T}^{-ſ}\left(\mathbb{s}_{i}^{j},\mu_{i}^{j}\right)}{\mathbb{g}}\right)\right)^{\mathrm{T}_{j}^{s}}}{\left(\prod_{j=ſ}^{m}\left(ſ+(F\text{Ⅎ}^{s}-ſ)\left(ſ-\left(\left(\dfrac{\mathrm{T}^{-ſ}\left(\mathbb{s}_{i}^{j},\mu_{i}^{j}\right)}{\mathbb{g}}\right)\right)^{\Psi}\right)\right)^{\mathrm{T}_{j}^{s}}+(F\text{Ⅎ}^{s}-ſ)\prod_{j=ſ}^{m}\left(\left(\left(\dfrac{\mathrm{T}^{-ſ}\left(\mathbb{s}_{i}^{j},\mu_{i}^{j}\right)}{\mathbb{g}}\right)\right)^{\Psi}\right)^{\mathrm{T}_{j}^{s}}\right)^{\frac{ſ}{\Psi}}},
\dfrac{\mathrm{T}^{s\frac{ſ}{\Psi}}\prod_{j=ſ}^{m}\left(\left(\dfrac{\mathrm{T}^{-ſ}\left(\mathbb{s}_{i}^{j},\mu_{i}^{j}\right)}{\mathbb{g}}\right)\right)^{\mathrm{T}_{j}^{s}}}{\left(\prod_{j=ſ}^{m}\left(ſ+(F\text{Ⅎ}^{s}-ſ)\left(ſ-\left(\left(\dfrac{\mathrm{T}^{-ſ}\left(\mathbb{s}_{i}^{j},\mu_{i}^{j}\right)}{\mathbb{g}}\right)\right)^{\Psi}\right)\right)^{\mathrm{T}_{j}^{s}}+(F\text{Ⅎ}^{s}-ſ)\prod_{j=ſ}^{m}\left(\left(\left(\dfrac{\mathrm{T}^{-ſ}\left(\mathbb{s}_{i}^{j},\mu_{i}^{j}\right)}{\mathbb{g}}\right)\right)^{\Psi}\right)^{\mathrm{T}_{j}^{s}}\right)^{\frac{ſ}{\Psi}}}
\right)\right), \\[2ex]
\left(
\dfrac{\mathrm{T}^{s\frac{ſ}{\Psi}}\prod_{j=ſ}^{m}\left(\mathbb{U}_{\mathscr{R}}^{M^{j}}\right)^{\mathrm{T}_{j}^{s}}}{\left(\prod_{j=ſ}^{m}\left(ſ+(F\text{Ⅎ}^{s}-ſ)\left(ſ-\left(\mathbb{U}_{\mathscr{R}}^{M^{j}}\right)^{\Psi}\right)\right)^{\mathrm{T}_{j}^{s}}+(F\text{Ⅎ}^{s}-ſ)\prod_{j=ſ}^{m}\left(\left(\mathbb{U}_{\mathscr{R}}^{M^{j}}\right)^{\Psi}\right)^{\mathrm{T}_{j}^{s}}\right)^{\frac{ſ}{\Psi}}},
\dfrac{\mathrm{T}^{s\frac{ſ}{\Psi}}\prod_{j=ſ}^{m}\left(\mathbb{U}_{\mathscr{I}}^{M^{j}}\right)^{\mathrm{T}_{j}^{s}}}{\left(\prod_{j=ſ}^{m}\left(ſ+(F\text{Ⅎ}^{s}-ſ)\left(ſ-\left(\mathbb{U}_{\mathscr{I}}^{M^{j}}\right)^{\Psi}\right)\right)^{\mathrm{T}_{j}^{s}}+(F\text{Ⅎ}^{s}-ſ)\prod_{j=ſ}^{m}\left(\left(\mathbb{U}_{\mathscr{I}}^{M^{j}}\right)^{\Psi}\right)^{\mathrm{T}_{j}^{s}}\right)^{\frac{ſ}{\Psi}}}
\right), \\[2ex]
\left(
\left(\dfrac{\prod_{j=ſ}^{m}\left(ſ+(F\text{Ⅎ}^{s}-ſ)\left(\mathbb{U}_{\mathscr{R}}^{N^{j}}\right)^{\psi}\right)^{\mathrm{T}_{j}^{s}}-\prod_{j=ſ}^{m}\left(ſ-\left(\mathbb{U}_{\mathscr{R}}^{N^{j}}\right)^{\psi}\right)^{\mathrm{T}_{j}^{s}}}{\prod_{j=ſ}^{m}\left(ſ+(F\text{Ⅎ}^{s}-ſ)\left(\mathbb{U}_{\mathscr{R}}^{N^{j}}\right)^{\psi}\right)^{\mathrm{T}_{j}^{s}}+(F\text{Ⅎ}^{s}-ſ)\prod_{j=ſ}^{m}\left(ſ-\left(\mathbb{U}_{\mathscr{R}}^{N^{j}}\right)^{\psi}\right)^{\mathrm{T}_{j}^{s}}}\right)^{\frac{ſ}{\psi}},
\left(\dfrac{\prod_{j=ſ}^{m}\left(ſ+(F\text{Ⅎ}^{s}-ſ)\left(\mathbb{U}_{\mathscr{I}}^{N^{j}}\right)^{\psi}\right)^{\mathrm{T}_{j}^{s}}-\prod_{j=ſ}^{m}\left(ſ-\left(\mathbb{U}_{\mathscr{I}}^{N^{j}}\right)^{\psi}\right)^{\mathrm{T}_{j}^{s}}}{\prod_{j=ſ}^{m}\left(ſ+(F\text{Ⅎ}^{s}-ſ)\left(\mathbb{U}_{\mathscr{I}}^{N^{j}}\right)^{\psi}\right)^{\mathrm{T}_{j}^{s}}+(F\text{Ⅎ}^{s}-ſ)\prod_{j=ſ}^{m}\left(ſ-\left(\mathbb{U}_{\mathscr{I}}^{N^{j}}\right)^{\psi}\right)^{\mathrm{T}_{j}^{s}}}\right)^{\frac{ſ}{\psi}}
\right), \\[2ex]
\left(
\dfrac{\mathrm{T}^{s\frac{ſ}{\Psi}}\prod_{j=ſ}^{m}\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{j}}\right)^{\mathrm{T}_{j}^{s}}}{\left(\prod_{j=ſ}^{m}\left(ſ+(F\text{Ⅎ}^{s}-ſ)\left(ſ-\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{j}}\right)^{\Psi}\right)\right)^{\mathrm{T}_{j}^{s}}+(F\text{Ⅎ}^{s}-ſ)\prod_{j=ſ}^{m}\left(\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}}\right)^{\Psi}\right)^{\mathrm{T}_{j}^{s}}\right)^{\frac{ſ}{\Psi}}},
\dfrac{\mathrm{T}^{s\frac{ſ}{\Psi}}\prod_{j=ſ}^{m}\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^{j}}\right)^{\mathrm{T}_{j}^{s}}}{\left(\prod_{j=ſ}^{m}\left(ſ+(F\text{Ⅎ}^{s}-ſ)\left(ſ-\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^{j}}\right)^{\Psi}\right)\right)^{\mathrm{T}_{j}^{s}}+(F\text{Ⅎ}^{s}-ſ)\prod_{j=ſ}^{m}\left(\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}}\right)^{\Psi}\right)^{\mathrm{T}_{j}^{s}}\right)^{\frac{ſ}{\Psi}}}
\right)
\end{array}
\right)
\end{aligned}
$$

and

$$CCp,q-QOF2-TLHPG\left(\mathfrak{N}^{1},\mathfrak{N}^{2},\ldots,\mathfrak{N}^{m}\right)_{TCN}=$$

$$\left(\begin{array}{c}
T\left(\mathbb{g}\left(\begin{array}{c}\left(\dfrac{\prod_{j=1}^{m}\left(1+(F\text{Ⅎ}^{s}-1)\left(\left(\dfrac{T^{-1}\left(\mathbb{s}_{i}^{j},\mu_{i}^{j}\right)}{\mathbb{g}}\right)\right)^{\psi}\right)^{T_{j}^{s}}-\prod_{j=1}^{m}\left(1-\left(\left(\dfrac{T^{-1}\left(\mathbb{s}_{i}^{j},\mu_{i}^{j}\right)}{\mathbb{g}}\right)\right)^{\psi}\right)^{T_{j}^{s}}}{\prod_{j=1}^{m}\left(1+(F\text{Ⅎ}^{s}-1)\left(\left(\dfrac{T^{-1}\left(\mathbb{s}_{i}^{j},\mu_{i}^{j}\right)}{\mathbb{g}}\right)\right)^{\psi}\right)^{T_{j}^{s}}+(F\text{Ⅎ}^{s}-1)\prod_{j=1}^{m}\left(1-\left(\left(\dfrac{T^{-1}\left(\mathbb{s}_{i}^{j},\mu_{i}^{j}\right)}{\mathbb{g}}\right)\right)^{\psi}\right)^{T_{j}^{s}}}\right)^{\frac{1}{\psi}}, \\
\left(\dfrac{\prod_{j=1}^{m}\left(1+(F\text{Ⅎ}^{s}-1)\left(\left(\dfrac{T^{-1}\left(\mathbb{s}_{i}^{j},\mu_{i}^{j}\right)}{\mathbb{g}}\right)\right)^{\psi}\right)^{T_{j}^{s}}-\prod_{j=1}^{m}\left(1-\left(\left(\dfrac{T^{-1}\left(\mathbb{s}_{i}^{j},\mu_{i}^{j}\right)}{\mathbb{g}}\right)\right)^{\psi}\right)^{T_{j}^{s}}}{\prod_{j=1}^{m}\left(1+(F\text{Ⅎ}^{s}-1)\left(\left(\dfrac{T^{-1}\left(\mathbb{s}_{i}^{j},\mu_{i}^{j}\right)}{\mathbb{g}}\right)\right)^{\psi}\right)^{T_{j}^{s}}+(F\text{Ⅎ}^{s}-1)\prod_{j=1}^{m}\left(1-\left(\left(\dfrac{T^{-1}\left(\mathbb{s}_{i}^{j},\mu_{i}^{j}\right)}{\mathbb{g}}\right)\right)^{\psi}\right)^{T_{j}^{s}}}\right)^{\frac{1}{\psi}}\end{array}\right)\right), \\
\left(\begin{array}{c}\dfrac{T^{s\frac{1}{\Psi}}\prod_{j=1}^{m}\left(\mathbb{U}_{\mathscr{R}}^{M^{j}}\right)^{T_{j}^{s}}}{\left(\prod_{j=1}^{m}\left(1+(F\text{Ⅎ}^{s}-1)\left(1-\left(\mathbb{U}_{\mathscr{R}}^{M^{j}}\right)^{\Psi}\right)\right)^{T_{j}^{s}}+(F\text{Ⅎ}^{s}-1)\prod_{j=1}^{m}\left(\left(\mathbb{U}_{\mathscr{R}}^{M^{j}}\right)^{\Psi}\right)^{T_{j}^{s}}\right)^{\frac{1}{\Psi}}}, \\
\dfrac{T^{s\frac{1}{\Psi}}\prod_{j=1}^{m}\left(\mathbb{U}_{\mathscr{I}}^{M^{j}}\right)^{T_{j}^{s}}}{\left(\prod_{j=1}^{m}\left(1+(F\text{Ⅎ}^{s}-1)\left(1-\left(\mathbb{U}_{\mathscr{I}}^{M^{j}}\right)^{\Psi}\right)\right)^{T_{j}^{s}}+(F\text{Ⅎ}^{s}-1)\prod_{j=1}^{m}\left(\left(\mathbb{U}_{\mathscr{I}}^{M^{j}}\right)^{\Psi}\right)^{T_{j}^{s}}\right)^{\frac{1}{\Psi}}}\end{array}\right), \\
\left(\begin{array}{c}\left(\dfrac{\prod_{j=1}^{m}\left(1+(F\text{Ⅎ}^{s}-1)\left(\mathbb{U}_{\mathscr{R}}^{N^{j}}\right)^{\psi}\right)^{T_{j}^{s}}-\prod_{j=1}^{m}\left(1-\left(\mathbb{U}_{\mathscr{R}}^{N^{j}}\right)^{\psi}\right)^{T_{j}^{s}}}{\prod_{j=1}^{m}\left(1+(F\text{Ⅎ}^{s}-1)\left(\mathbb{U}_{\mathscr{R}}^{N^{j}}\right)^{\psi}\right)^{T_{j}^{s}}+(F\text{Ⅎ}^{s}-1)\prod_{j=1}^{m}\left(1-\left(\mathbb{U}_{\mathscr{R}}^{N^{j}}\right)^{\psi}\right)^{T_{j}^{s}}}\right)^{\frac{1}{\psi}}, \\
\left(\dfrac{\prod_{j=1}^{m}\left(1+(F\text{Ⅎ}^{s}-1)\left(\mathbb{U}_{\mathscr{I}}^{N^{j}}\right)^{\psi}\right)^{T_{j}^{s}}-\prod_{j=1}^{m}\left(1-\left(\mathbb{U}_{\mathscr{I}}^{N^{j}}\right)^{\psi}\right)^{T_{j}^{s}}}{\prod_{j=1}^{m}\left(1+(F\text{Ⅎ}^{s}-1)\left(\mathbb{U}_{\mathscr{I}}^{N^{j}}\right)^{\psi}\right)^{T_{j}^{s}}+(F\text{Ⅎ}^{s}-1)\prod_{j=1}^{m}\left(1-\left(\mathbb{U}_{\mathscr{I}}^{N^{j}}\right)^{\psi}\right)^{T_{j}^{s}}}\right)^{\frac{1}{\psi}}\end{array}\right), \\
\left(\begin{array}{c}\left(\dfrac{\prod_{j=1}^{m}\left(1+(F\text{Ⅎ}^{s}-1)\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{j}}\right)^{\psi}\right)^{T_{j}^{s}}-\prod_{j=1}^{m}\left(1-\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{j}}\right)^{\psi}\right)^{T_{j}^{s}}}{\prod_{j=1}^{m}\left(1+(F\text{Ⅎ}^{s}-1)\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{j}}\right)^{\psi}\right)^{T_{j}^{s}}+(F\text{Ⅎ}^{s}-1)\prod_{j=1}^{m}\left(1-\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{j}}\right)^{\psi}\right)^{T_{j}^{s}}}\right)^{\frac{1}{\psi}}, \\
\left(\dfrac{\prod_{j=1}^{m}\left(1+(F\text{Ⅎ}^{s}-1)\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^{j}}\right)^{\psi}\right)^{T_{j}^{s}}-\prod_{j=1}^{m}\left(1-\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^{j}}\right)^{\psi}\right)^{T_{j}^{s}}}{\prod_{j=1}^{m}\left(1+(F\text{Ⅎ}^{s}-1)\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^{j}}\right)^{\psi}\right)^{T_{j}^{s}}+(F\text{Ⅎ}^{s}-1)\prod_{j=1}^{m}\left(1-\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^{j}}\right)^{\psi}\right)^{T_{j}^{s}}}\right)^{\frac{1}{\psi}}\end{array}\right)
\end{array}\right)$$

Further, we described some basic properties of the above technique, called idempotency, monotonicity, and boundedness.

Property 3. *Based on any collection of CCp, q-QOF2-TLNs* $\mathfrak{N}^{j}=\left(\left(\mathbb{s}_{i}^{j},\mu_{i}^{j}\right),\left(\mathbb{U}_{\mathscr{R}}^{M^{j}},\mathbb{U}_{\mathscr{I}}^{M^{j}}\right),\left(\mathbb{U}_{\mathscr{R}}^{N^{j}},\mathbb{U}_{\mathscr{I}}^{N^{j}}\right),\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{j}},\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^{j}}\right)\right),j=1,2,\ldots,m$, *then*

1) *When* $\mathfrak{N}^{j}=\mathfrak{N}=\left(\left(\mathbb{s}_{i},\mu_{i}\right),\left(\mathbb{U}_{\mathscr{R}}^{M},\mathbb{U}_{\mathscr{I}}^{M}\right),\left(\mathbb{U}_{\mathscr{R}}^{N},\mathbb{U}_{\mathscr{I}}^{N}\right),\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}},\mathbb{U}_{\mathscr{I}}^{\mathbb{R}}\right)\right),j=1,2,\ldots,m$, *then*

$$CCp,q-QOF2-TLHPG\left(\mathfrak{N}^{1},\mathfrak{N}^{2},\ldots,\mathfrak{N}^{m}\right)_{TN}=\mathfrak{N}$$

$$CC\mathrm{p},\mathrm{q}-QOF2-TLHPG\left(\mathfrak{R}^{1},\mathfrak{R}^{2},\ldots,\mathfrak{R}^{m}\right)_{TCN}=\mathfrak{R}$$

2) *When* $\mathfrak{R}^{j}\leq\mathfrak{R}^{j}_{\#}$*, such as* $\left(\frac{\mathrm{T}^{-1}\left(\mathbb{s}^{j}_{i},\mu^{j}_{i}\right)}{\mathbb{g}}\right)\leq\left(\frac{\mathrm{T}^{-1}\left(\mathbb{s}_{\#}{}^{j}_{i},\mu_{\#}{}^{j}_{i}\right)}{\mathbb{g}}\right),\mathbb{U}^{\mathrm{M}^{j}}_{\mathscr{R}}\leq\mathbb{U}_{\#}{}^{\mathrm{M}^{j}}_{\mathscr{R}},\mathbb{U}^{\mathrm{M}^{j}}_{\mathscr{I}}\leq\mathbb{U}_{\#}{}^{\mathrm{M}^{j}}_{\mathscr{I}},\mathbb{U}^{N^{j}}_{\mathscr{R}}\geq\mathbb{U}_{\#}{}^{N^{j}}_{\mathscr{R}},\mathbb{U}^{N^{j}}_{\mathscr{I}}\geq\mathbb{U}_{\#}{}^{N^{j}}_{\mathscr{I}}$*, and* $\mathbb{U}^{\mathbb{R}^{j}}_{\mathscr{R}}\leq\mathbb{U}_{\#}{}^{\mathbb{R}^{j}}_{\mathscr{R}},\mathbb{U}^{\mathbb{R}^{j}}_{\mathscr{I}}\leq\mathbb{U}_{\#}{}^{\mathbb{R}^{j}}_{\mathscr{I}}$ *then*

$$CC\mathrm{p},\mathrm{q}-QOF2-TLHPG\left(\mathfrak{R}^{1},\mathfrak{R}^{2},\ldots,\mathfrak{R}^{m}\right)_{TN}\leq CC\mathrm{p},\mathrm{q}-QOF2-TLHPG\left(\mathfrak{R}^{1}_{\#},\mathfrak{R}^{2}_{\#},\ldots,\mathfrak{R}^{m}_{\#}\right)_{TN}$$

$$CC\mathrm{p},\mathrm{q}-QOF2-TLHPG\left(\mathfrak{R}^{1},\mathfrak{R}^{2},\ldots,\mathfrak{R}^{m}\right)_{TCN}\leq CC\mathrm{p},\mathrm{q}-QOF2-TLHPG\left(\mathfrak{R}^{1}_{\#},\mathfrak{R}^{2}_{\#},\ldots,\mathfrak{R}^{m}_{\#}\right)_{TCN}$$

3) *When* $\mathfrak{R}^{+}=\max\left\{\mathfrak{R}^{1},\mathfrak{R}^{2},\ldots,\mathfrak{R}^{m}\right\}$ *and* $\mathfrak{R}^{-}=\min\left\{\mathfrak{R}^{1},\mathfrak{R}^{2},\ldots,\mathfrak{R}^{m}\right\}$*, then*

$$\mathfrak{R}^{-}\leq CC\mathrm{p},\mathrm{q}-QOF2-TLHPG\left(\mathfrak{R}^{1},\mathfrak{R}^{2},\ldots,\mathfrak{R}^{m}\right)_{TN}\leq\mathfrak{R}^{+}$$

$$\mathfrak{R}^{-}\leq CC\mathrm{p},\mathrm{q}-QOF2-TLHPG\left(\mathfrak{R}^{1},\mathfrak{R}^{2},\ldots,\mathfrak{R}^{m}\right)_{TCN}\leq\mathfrak{R}^{+}$$

Definition 14. *Based on any collection of CCp, q-QOF2-TLNs* $\mathfrak{R}^{j}=\left(\left(\mathbb{s}^{j}_{i},\mu^{j}_{i}\right),\left(\mathbb{U}^{\mathrm{M}^{j}}_{\mathscr{R}},\mathbb{U}^{\mathrm{M}^{j}}_{\mathscr{I}}\right),\left(\mathbb{U}^{N^{j}}_{\mathscr{R}},\mathbb{U}^{N^{j}}_{\mathscr{I}}\right),\left(\mathbb{U}^{\mathbb{R}^{j}}_{\mathscr{R}},\mathbb{U}^{\mathbb{R}^{j}}_{\mathscr{I}}\right)\right),j=1,2,\ldots,m$*, we described the technique of CCp, q-QOF2-TLHPWG operator for both HTN and HTCN, such as*

$$CC\mathrm{p},\mathrm{q}-QOF2-TLHPWG\left(\mathfrak{R}^{1},\mathfrak{R}^{2},\ldots,\mathfrak{R}^{m}\right)_{TN}=\otimes_{TN}{}_{j=1}^{m}\left(\mathfrak{R}^{j}\right)^{\frac{\circ\mathcal{O}^{j}\left(1+\#\left(\mathfrak{R}^{j}\right)\right)}{\sum_{j=1}^{m}\circ\mathcal{O}^{j}\left(1+\#\left(\mathfrak{R}^{j}\right)\right)}}=\otimes_{TN}{}_{j=1}^{m}\left(\mathfrak{R}^{j}\right)^{\mathrm{T}^{s}_{j}}$$

$$CC\mathrm{p},\mathrm{q}-QOF2-TLHPWG\left(\mathfrak{R}^{1},\mathfrak{R}^{2},\ldots,\mathfrak{R}^{m}\right)_{TCN}=\otimes_{TCN}{}_{j=1}^{m}\left(\mathfrak{R}^{j}\right)^{\frac{\circ\mathcal{O}^{j}\left(1+\#\left(\mathfrak{R}^{j}\right)\right)}{\sum_{j=1}^{m}\circ\mathcal{O}^{j}\left(1+\#\left(\mathfrak{R}^{j}\right)\right)}}=\otimes_{TCN}{}_{j=1}^{m}\left(\mathfrak{R}^{j}\right)^{\mathrm{T}^{s}_{j}}$$

where, $\mathrm{T}^{s}_{j}=\frac{\circ\mathcal{O}^{j}\left(1+\#\left(\mathfrak{R}^{j}\right)\right)}{\sum_{j=1}^{m}\circ\mathcal{O}^{j}\left(1+\#\left(\mathfrak{R}^{j}\right)\right)}$*. Further, we explained the term* $\#\left(\mathfrak{R}^{j}\right)=\sum_{j\neq k=1}^{m}\sup\left(\mathfrak{R}^{j},\mathfrak{R}^{k}\right)$ *and* $\sup\left(\mathfrak{R}^{j},\mathfrak{R}^{k}\right)=\mathrm{dis}\left(\mathfrak{R}^{j},\mathfrak{R}^{k}\right)$ *with some condition that is*: (*i*) $\sup\left(\mathfrak{R}^{j},\mathfrak{R}^{k}\right)\in[0,1]$; (*ii*) $\sup\left(\mathfrak{R}^{j},\mathfrak{R}^{k}\right)=\sup\left(\mathfrak{R}^{k},\mathfrak{R}^{j}\right)$; *and* (*iii*) *if* $\sup\left(\mathfrak{R}^{j},\mathfrak{R}^{k}\right)>\sup\left(\mathfrak{R}^{l},\mathfrak{R}^{m}\right)$*, then* $\mathrm{dis}\left(\mathfrak{R}^{j},\mathfrak{R}^{k}\right)<\mathrm{dis}\left(\mathfrak{R}^{l},\mathfrak{R}^{m}\right)$*. Further, we described the technique of distance measures for any CCp, q-QOF2-TLNs, such as*

$$\mathrm{dis}\left(\mathfrak{R}^{j},\mathfrak{R}^{k}\right)=(0.5)\left(\begin{array}{c}\left(\frac{\left|\mathrm{T}^{-1}\left(\mathbb{s}^{j}_{i},\mu^{j}_{i}\right)-\mathrm{T}^{-1}\left(\mathbb{s}^{k}_{i},\mu^{k}_{i}\right)\right|}{\mathbb{g}}\right)+\\ \left(\frac{1}{6}\left(\begin{array}{c}\left|\left(\mathbb{U}^{\mathrm{M}^{j}}_{\mathscr{R}}\right)^{\Psi}-\left(\mathbb{U}^{\mathrm{M}^{k}}_{\mathscr{R}}\right)^{\Psi}\right|+\left|\left(\mathbb{U}^{\mathrm{M}^{j}}_{\mathscr{I}}\right)^{\Psi}-\left(\mathbb{U}^{\mathrm{M}^{k}}_{\mathscr{I}}\right)^{\Psi}\right|+\left|\left(\mathbb{U}^{N^{j}}_{\mathscr{R}}\right)^{\Psi}-\left(\mathbb{U}^{N^{k}}_{\mathscr{R}}\right)^{\Psi}\right|+\\ \left|\left(\mathbb{U}^{N^{j}}_{\mathscr{I}}\right)^{\Psi}-\left(\mathbb{U}^{N^{k}}_{\mathscr{I}}\right)^{\Psi}\right|+\left|\left(\mathbb{U}^{\mathbb{R}^{j}}_{\mathscr{R}}\right)^{\Psi}-\left(\mathbb{U}^{\mathbb{R}^{k}}_{\mathscr{R}}\right)^{\Psi}\right|+\left|\left(\mathbb{U}^{\mathbb{R}^{j}}_{\mathscr{I}}\right)^{\Psi}-\left(\mathbb{U}^{\mathbb{R}^{k}}_{\mathscr{I}}\right)^{\Psi}\right|\end{array}\right)\right)\end{array}\right)$$

Theorem 4. *Based on any collection of CCp, q-QOF2-TLNs* $\mathfrak{R}^{j}=\left(\left(\mathbb{s}^{j}_{i},\mu^{j}_{i}\right),\left(\mathbb{U}^{\mathrm{M}^{j}}_{\mathscr{R}},\mathbb{U}^{\mathrm{M}^{j}}_{\mathscr{I}}\right),\left(\mathbb{U}^{N^{j}}_{\mathscr{R}},\mathbb{U}^{N^{j}}_{\mathscr{I}}\right),\left(\mathbb{U}^{\mathbb{R}^{j}}_{\mathscr{R}},\mathbb{U}^{\mathbb{R}^{j}}_{\mathscr{I}}\right)\right),j=1,2,\ldots,m$*, we prove that the aggregated theory of CCp, q-QOF2-TLHPWG operator is again a CCp, q-QOF2-TLN, such as*

$$CC\mathrm{p},\mathrm{q}-QOF2-TLHPWG\left(\mathfrak{N}^{\text{ſ}},\mathfrak{N}^{2},\ldots,\mathfrak{N}^{m}\right)_{TN}=$$

$$\left(\begin{array}{c}
\mathrm{T}\left(\mathbb{g}\left(\begin{array}{c}
\dfrac{\mathrm{T}^{s\frac{\text{ſ}}{\Psi}}\prod_{j=\text{ſ}}^{m}\left(\left(\dfrac{\mathrm{T}^{-\text{ſ}}\left(\mathbb{s}_i^j,\mu_i^j\right)}{\mathbb{g}}\right)\right)^{\mathrm{T}_j^s}}{\left(\prod_{j=\text{ſ}}^{m}\left(\text{ſ}+(F\text{Ⅎ}^s-\text{ſ})\left(\text{ſ}-\left(\left(\dfrac{\mathrm{T}^{-\text{ſ}}\left(\mathbb{s}_i^j,\mu_i^j\right)}{\mathbb{g}}\right)\right)^{\Psi}\right)\right)^{\mathrm{T}_j^s}+(F\text{Ⅎ}^s-\text{ſ})\prod_{j=\text{ſ}}^{m}\left(\left(\left(\dfrac{\mathrm{T}^{-\text{ſ}}\left(\mathbb{s}_i^j,\mu_i^j\right)}{\mathbb{g}}\right)\right)^{\Psi}\right)^{\mathrm{T}_j^s}\right)^{\frac{\text{ſ}}{\Psi}}},\\
\dfrac{\mathrm{T}^{s\frac{\text{ſ}}{\Psi}}\prod_{j=\text{ſ}}^{m}\left(\left(\dfrac{\mathrm{T}^{-\text{ſ}}\left(\mathbb{s}_i^j,\mu_i^j\right)}{\mathbb{g}}\right)\right)^{\mathrm{T}_j^s}}{\left(\prod_{j=\text{ſ}}^{m}\left(\text{ſ}+(F\text{Ⅎ}^s-\text{ſ})\left(\text{ſ}-\left(\left(\dfrac{\mathrm{T}^{-\text{ſ}}\left(\mathbb{s}_i^j,\mu_i^j\right)}{\mathbb{g}}\right)\right)^{\Psi}\right)\right)^{\mathrm{T}_j^s}+(F\text{Ⅎ}^s-\text{ſ})\prod_{j=\text{ſ}}^{m}\left(\left(\left(\dfrac{\mathrm{T}^{-\text{ſ}}\left(\mathbb{s}_i^j,\mu_i^j\right)}{\mathbb{g}}\right)\right)^{\Psi}\right)^{\mathrm{T}_j^s}\right)^{\frac{\text{ſ}}{\Psi}}}
\end{array}\right)\right),\\
\left(\begin{array}{c}
\dfrac{\mathrm{T}^{s\frac{\text{ſ}}{\Psi}}\prod_{j=\text{ſ}}^{m}\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^j}\right)^{\mathrm{T}_j^s}}{\left(\prod_{j=\text{ſ}}^{m}\left(\text{ſ}+(F\text{Ⅎ}^s-\text{ſ})\left(\text{ſ}-\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^j}\right)^{\Psi}\right)\right)^{\mathrm{T}_j^s}+(F\text{Ⅎ}^s-\text{ſ})\prod_{j=\text{ſ}}^{m}\left(\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^j}\right)^{\Psi}\right)^{\mathrm{T}_j^s}\right)^{\frac{\text{ſ}}{\Psi}}},\\
\dfrac{\mathrm{T}^{s\frac{\text{ſ}}{\Psi}}\prod_{j=\text{ſ}}^{m}\left(\mathbb{U}_{\mathscr{I}}^{\mathrm{M}^j}\right)^{\mathrm{T}_j^s}}{\left(\prod_{j=\text{ſ}}^{m}\left(\text{ſ}+(F\text{Ⅎ}^s-\text{ſ})\left(\text{ſ}-\left(\mathbb{U}_{\mathscr{I}}^{\mathrm{M}^j}\right)^{\Psi}\right)\right)^{\mathrm{T}_j^s}+(F\text{Ⅎ}^s-\text{ſ})\prod_{j=\text{ſ}}^{m}\left(\left(\mathbb{U}_{\mathscr{I}}^{\mathrm{M}^j}\right)^{\Psi}\right)^{\mathrm{T}_j^s}\right)^{\frac{\text{ſ}}{\Psi}}}
\end{array}\right),\\
\left(\begin{array}{c}
\left(\dfrac{\prod_{j=\text{ſ}}^{m}\left(\text{ſ}+(F\text{Ⅎ}^s-\text{ſ})\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{N}^j}\right)^{\psi}\right)^{\mathrm{T}_j^s}-\prod_{j=\text{ſ}}^{m}\left(\text{ſ}-\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{N}^j}\right)^{\psi}\right)^{\mathrm{T}_j^s}}{\prod_{j=\text{ſ}}^{m}\left(\text{ſ}+(F\text{Ⅎ}^s-\text{ſ})\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{N}^j}\right)^{\psi}\right)^{\mathrm{T}_j^s}+(F\text{Ⅎ}^s-\text{ſ})\prod_{j=\text{ſ}}^{m}\left(\text{ſ}-\left(\mathbb{U}_{\mathscr{R}}^{\mathrm{N}^j}\right)^{\psi}\right)^{\mathrm{T}_j^s}}\right)^{\frac{\text{ſ}}{\psi}},\\
\left(\dfrac{\prod_{j=\text{ſ}}^{m}\left(\text{ſ}+(F\text{Ⅎ}^s-\text{ſ})\left(\mathbb{U}_{\mathscr{I}}^{\mathrm{N}^j}\right)^{\psi}\right)^{\mathrm{T}_j^s}-\prod_{j=\text{ſ}}^{m}\left(\text{ſ}-\left(\mathbb{U}_{\mathscr{I}}^{\mathrm{N}^j}\right)^{\psi}\right)^{\mathrm{T}_j^s}}{\prod_{j=\text{ſ}}^{m}\left(\text{ſ}+(F\text{Ⅎ}^s-\text{ſ})\left(\mathbb{U}_{\mathscr{I}}^{\mathrm{N}^j}\right)^{\psi}\right)^{\mathrm{T}_j^s}+(F\text{Ⅎ}^s-\text{ſ})\prod_{j=\text{ſ}}^{m}\left(\text{ſ}-\left(\mathbb{U}_{\mathscr{I}}^{\mathrm{N}^j}\right)^{\psi}\right)^{\mathrm{T}_j^s}}\right)^{\frac{\text{ſ}}{\psi}}
\end{array}\right),\\
\left(\begin{array}{c}
\dfrac{\mathrm{T}^{s\frac{\text{ſ}}{\Psi}}\prod_{j=\text{ſ}}^{m}\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^j}\right)^{\mathrm{T}_j^s}}{\left(\prod_{j=\text{ſ}}^{m}\left(\text{ſ}+(F\text{Ⅎ}^s-\text{ſ})\left(\text{ſ}-\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^j}\right)^{\Psi}\right)\right)^{\mathrm{T}_j^s}+(F\text{Ⅎ}^s-\text{ſ})\prod_{j=\text{ſ}}^{m}\left(\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}}\right)^{\Psi}\right)^{\mathrm{T}_j^s}\right)^{\frac{\text{ſ}}{\Psi}}},\\
\dfrac{\mathrm{T}^{s\frac{\text{ſ}}{\Psi}}\prod_{j=\text{ſ}}^{m}\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^j}\right)^{\mathrm{T}_j^s}}{\left(\prod_{j=\text{ſ}}^{m}\left(\text{ſ}+(F\text{Ⅎ}^s-\text{ſ})\left(\text{ſ}-\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^j}\right)^{\Psi}\right)\right)^{\mathrm{T}_j^s}+(F\text{Ⅎ}^s-\text{ſ})\prod_{j=\text{ſ}}^{m}\left(\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^j}\right)^{\Psi}\right)^{\mathrm{T}_j^s}\right)^{\frac{\text{ſ}}{\Psi}}}
\end{array}\right)
\end{array}\right)$$

and

$$CCp,q-QOF2-TLHPWG\left(\mathfrak{R}^{1},\mathfrak{R}^{2},\ldots,\mathfrak{R}^{m}\right)_{TCN}=$$

$$\left(\begin{array}{c}
\mathrm{T}\left(\mathfrak{g}\left(\begin{array}{c}
\left(\frac{\prod_{j=1}^{m}\left(1+(F\daleth^{s}-1)\left(\left(\frac{\mathrm{T}^{-1}\left(\mathbb{s}_i^j,\mu_i^j\right)}{\mathfrak{g}}\right)\right)^{\psi}\right)^{\mathrm{T}_j^s}-\prod_{j=1}^{m}\left(1-\left(\left(\frac{\mathrm{T}^{-1}\left(\mathbb{s}_i^j,\mu_i^j\right)}{\mathfrak{g}}\right)\right)^{\psi}\right)^{\mathrm{T}_j^s}}{\prod_{j=1}^{m}\left(1+(F\daleth^{s}-1)\left(\left(\frac{\mathrm{T}^{-1}\left(\mathbb{s}_i^j,\mu_i^j\right)}{\mathfrak{g}}\right)\right)^{\psi}\right)^{\mathrm{T}_j^s}+(F\daleth^{s}-1)\prod_{j=1}^{m}\left(1-\left(\left(\frac{\mathrm{T}^{-1}\left(\mathbb{s}_i^j,\mu_i^j\right)}{\mathfrak{g}}\right)\right)^{\psi}\right)^{\mathrm{T}_j^s}}\right)^{\frac{1}{\psi}}, \\
\left(\frac{\prod_{j=1}^{m}\left(1+(F\daleth^{s}-1)\left(\left(\frac{\mathrm{T}^{-1}\left(\mathbb{s}_i^j,\mu_i^j\right)}{\mathfrak{g}}\right)\right)^{\psi}\right)^{\mathrm{T}_j^s}-\prod_{j=1}^{m}\left(1-\left(\left(\frac{\mathrm{T}^{-1}\left(\mathbb{s}_i^j,\mu_i^j\right)}{\mathfrak{g}}\right)\right)^{\psi}\right)^{\mathrm{T}_j^s}}{\prod_{j=1}^{m}\left(1+(F\daleth^{s}-1)\left(\left(\frac{\mathrm{T}^{-1}\left(\mathbb{s}_i^j,\mu_i^j\right)}{\mathfrak{g}}\right)\right)^{\psi}\right)^{\mathrm{T}_j^s}+(F\daleth^{s}-1)\prod_{j=1}^{m}\left(1-\left(\left(\frac{\mathrm{T}^{-1}\left(\mathbb{s}_i^j,\mu_i^j\right)}{\mathfrak{g}}\right)\right)^{\psi}\right)^{\mathrm{T}_j^s}}\right)^{\frac{1}{\psi}}
\end{array}\right)\right), \\
\left(\begin{array}{c}
\frac{\mathrm{T}^{s\frac{1}{\Psi}}\prod_{j=1}^{m}\left(\mathbb{U}_{\mathscr{R}}^{M^j}\right)^{\mathrm{T}_j^s}}{\left(\prod_{j=1}^{m}\left(1+(F\daleth^{s}-1)\left(1-\left(\mathbb{U}_{\mathscr{R}}^{M^j}\right)^{\Psi}\right)\right)^{\mathrm{T}_j^s}+(F\daleth^{s}-1)\prod_{j=1}^{m}\left(\left(\mathbb{U}_{\mathscr{R}}^{M^j}\right)^{\Psi}\right)^{\mathrm{T}_j^s}\right)^{\frac{1}{\Psi}}}, \\
\frac{\mathrm{T}^{s\frac{1}{\Psi}}\prod_{j=1}^{m}\left(\mathbb{U}_{\mathscr{I}}^{M^j}\right)^{\mathrm{T}_j^s}}{\left(\prod_{j=1}^{m}\left(1+(F\daleth^{s}-1)\left(1-\left(\mathbb{U}_{\mathscr{I}}^{M^j}\right)^{\Psi}\right)\right)^{\mathrm{T}_j^s}+(F\daleth^{s}-1)\prod_{j=1}^{m}\left(\left(\mathbb{U}_{\mathscr{I}}^{M^j}\right)^{\Psi}\right)^{\mathrm{T}_j^s}\right)^{\frac{1}{\Psi}}}
\end{array}\right), \\
\left(\begin{array}{c}
\left(\frac{\prod_{j=1}^{m}\left(1+(F\daleth^{s}-1)\left(\mathbb{U}_{\mathscr{R}}^{N^j}\right)^{\psi}\right)^{\mathrm{T}_j^s}-\prod_{j=1}^{m}\left(1-\left(\mathbb{U}_{\mathscr{R}}^{N^j}\right)^{\psi}\right)^{\mathrm{T}_j^s}}{\prod_{j=1}^{m}\left(1+(F\daleth^{s}-1)\left(\mathbb{U}_{\mathscr{R}}^{N^j}\right)^{\psi}\right)^{\mathrm{T}_j^s}+(F\daleth^{s}-1)\prod_{j=1}^{m}\left(1-\left(\mathbb{U}_{\mathscr{R}}^{N^j}\right)^{\psi}\right)^{\mathrm{T}_j^s}}\right)^{\frac{1}{\psi}}, \\
\left(\frac{\prod_{j=1}^{m}\left(1+(F\daleth^{s}-1)\left(\mathbb{U}_{\mathscr{I}}^{N^j}\right)^{\psi}\right)^{\mathrm{T}_j^s}-\prod_{j=1}^{m}\left(1-\left(\mathbb{U}_{\mathscr{I}}^{N^j}\right)^{\psi}\right)^{\mathrm{T}_j^s}}{\prod_{j=1}^{m}\left(1+(F\daleth^{s}-1)\left(\mathbb{U}_{\mathscr{I}}^{N^j}\right)^{\psi}\right)^{\mathrm{T}_j^s}+(F\daleth^{s}-1)\prod_{j=1}^{m}\left(1-\left(\mathbb{U}_{\mathscr{I}}^{N^j}\right)^{\psi}\right)^{\mathrm{T}_j^s}}\right)^{\frac{1}{\psi}}
\end{array}\right), \\
\left(\begin{array}{c}
\left(\frac{\prod_{j=1}^{m}\left(1+(F\daleth^{s}-1)\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^j}\right)^{\psi}\right)^{\mathrm{T}_j^s}-\prod_{j=1}^{m}\left(1-\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^j}\right)^{\psi}\right)^{\mathrm{T}_j^s}}{\prod_{j=1}^{m}\left(1+(F\daleth^{s}-1)\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^j}\right)^{\psi}\right)^{\mathrm{T}_j^s}+(F\daleth^{s}-1)\prod_{j=1}^{m}\left(1-\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^j}\right)^{\psi}\right)^{\mathrm{T}_j^s}}\right)^{\frac{1}{\psi}}, \\
\left(\frac{\prod_{j=1}^{m}\left(1+(F\daleth^{s}-1)\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^j}\right)^{\psi}\right)^{\mathrm{T}_j^s}-\prod_{j=1}^{m}\left(1-\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^j}\right)^{\psi}\right)^{\mathrm{T}_j^s}}{\prod_{j=1}^{m}\left(1+(F\daleth^{s}-1)\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^j}\right)^{\psi}\right)^{\mathrm{T}_j^s}+(F\daleth^{s}-1)\prod_{j=1}^{m}\left(1-\left(\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^j}\right)^{\psi}\right)^{\mathrm{T}_j^s}}\right)^{\frac{1}{\psi}}
\end{array}\right)
\end{array}\right)$$

Further, we described some basic properties of the above technique, called idempotency, monotonicity, and boundedness.

Property 4. *Based on any collection of CCp, q-QOF2-TLNs* $\mathfrak{R}^j=\left(\left(\mathbb{s}_i^j,\mu_i^j\right),\left(\mathbb{U}_{\mathscr{R}}^{M^j},\mathbb{U}_{\mathscr{I}}^{M^j}\right),\left(\mathbb{U}_{\mathscr{R}}^{N^j},\mathbb{U}_{\mathscr{I}}^{N^j}\right),\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^j},\mathbb{U}_{\mathscr{I}}^{\mathbb{R}^j}\right)\right),j=1,2,\ldots,m$, *then*

1) *When* $\mathfrak{R}^j=\mathfrak{R}=\left(\left(\mathbb{s}_i,\mu_i\right),\left(\mathbb{U}_{\mathscr{R}}^{M},\mathbb{U}_{\mathscr{I}}^{M}\right),\left(\mathbb{U}_{\mathscr{R}}^{N},\mathbb{U}_{\mathscr{I}}^{N}\right),\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}},\mathbb{U}_{\mathscr{I}}^{\mathbb{R}}\right)\right),j=1,2,\ldots,m$, *then*

$$CCp,q-QOF2-TLHPWG\left(\mathfrak{R}^{1},\mathfrak{R}^{2},\ldots,\mathfrak{R}^{m}\right)_{TN}=\mathfrak{R}$$

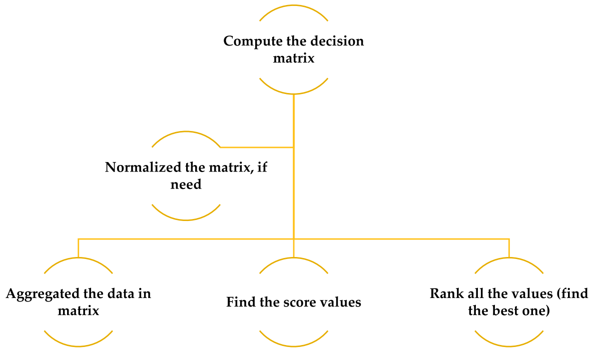

Fig. 4. Geometrical representation of the proposed algorithm.

$$CCp, q - QOF2 - TLHPWG\left(\mathfrak{N}^{ſ}, \mathfrak{N}^{2}, \ldots, \mathfrak{N}^{m}\right)_{TCN} = \mathfrak{N}$$

2) *When* $\mathfrak{N}^{j} \leq \mathfrak{N}^{j}_{\#}$, *such as* $\left(\frac{\mathrm{T}^{-ſ}\left(\mathbb{s}_{i}^{j}, \mu_{i}^{j}\right)}{\mathrm{g}}\right) \leq \left(\frac{\mathrm{T}^{-ſ}\left(\mathbb{s}_{\#_{i}}^{j}, \mu_{\#_{i}}^{j}\right)}{\mathrm{g}}\right), \mathbb{U}_{\mathscr{R}}^{\mathrm{M}^{j}} \leq \mathbb{U}_{\#_{\mathscr{R}}}^{\mathrm{M}^{j}}, \mathbb{U}_{\mathscr{I}}^{\mathrm{M}^{j}} \leq \mathbb{U}_{\#_{\mathscr{I}}}^{\mathrm{M}^{j}}, \mathbb{U}_{\mathscr{R}}^{N^{j}} \geq \mathbb{U}_{\#_{\mathscr{R}}}^{N^{j}}, \mathbb{U}_{\mathscr{I}}^{N^{j}} \geq \mathbb{U}_{\#_{\mathscr{I}}}^{N^{j}}$, *and* $\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^{j}} \leq \mathbb{U}_{\#_{\mathscr{R}}}^{\mathbb{R}^{j}}, \mathbb{U}_{\mathscr{I}}^{\mathbb{R}^{j}} \leq \mathbb{U}_{\#_{\mathscr{I}}}^{\mathbb{R}^{j}}$ *then*

$$CCp, q - QOF2 - TLHPWG\left(\mathfrak{N}^{ſ}, \mathfrak{N}^{2}, \ldots, \mathfrak{N}^{m}\right)_{TN} \leq CCp, q - QOF2 - TLHPWG\left(\mathfrak{N}^{ſ}_{\#}, \mathfrak{N}^{2}_{\#}, \ldots, \mathfrak{N}^{m}_{\#}\right)_{TN}$$

$$CCp, q - QOF2 - TLHPWG\left(\mathfrak{N}^{ſ}, \mathfrak{N}^{2}, \ldots, \mathfrak{N}^{m}\right)_{TCN} \leq CCp, q - QOF2 - TLHPWG\left(\mathfrak{N}^{ſ}_{\#}, \mathfrak{N}^{2}_{\#}, \ldots, \mathfrak{N}^{m}_{\#}\right)_{TCN}$$

Table 2
CCp, q-QOF2-TL decision matrix.

	$\mathfrak{N}^{ſ}_{AT}$	$\mathfrak{N}^{2}_{AT}$	$\mathfrak{N}^{3}_{AT}$
$\mathfrak{N}^{ſ}$	$\begin{pmatrix}(\mathbb{s}_{ſ}, 0.2), (0.5, 0.4),\\(0.2, 0.3), (0.3, 0.3)\end{pmatrix}$	$\begin{pmatrix}(\mathbb{s}_{ſ}, 0.2ſ), (0.5ſ, 0.4ſ),\\(0.2ſ, 0.3ſ), (0.3ſ, 0.3ſ)\end{pmatrix}$	$\begin{pmatrix}(\mathbb{s}_{ſ}, 0.22), (0.52, 0.42),\\(0.22, 0.32), (0.32, 0.32)\end{pmatrix}$
$\mathfrak{N}^{2}$	$\begin{pmatrix}(\mathbb{s}_{2}, 0.3), (0.6, 0.5),\\(0.3, 0.2), (0.4, 0.3)\end{pmatrix}$	$\begin{pmatrix}(\mathbb{s}_{2}, 0.3ſ), (0.6ſ, 0.5ſ),\\(0.3ſ, 0.2ſ), (0.4ſ, 0.3ſ)\end{pmatrix}$	$\begin{pmatrix}(\mathbb{s}_{2}, 0.32), (0.62, 0.52),\\(0.32, 0.22), (0.42, 0.32)\end{pmatrix}$
$\mathfrak{N}^{3}$	$\begin{pmatrix}(\mathbb{s}_{3}, 0.ſ), (0.8, 0.6),\\(0.4, 0.3), (0.2, 0.3)\end{pmatrix}$	$\begin{pmatrix}(\mathbb{s}_{3}, 0.ſſ), (0.8ſ, 0.6ſ),\\(0.4ſ, 0.3ſ), (0.2ſ, 0.3ſ)\end{pmatrix}$	$\begin{pmatrix}(\mathbb{s}_{3}, 0.ſ2), (0.82, 0.62),\\(0.42, 0.32), (0.22, 0.32)\end{pmatrix}$
$\mathfrak{N}^{4}$	$\begin{pmatrix}(\mathbb{s}_{4}, 0.5), (0.7, 0.8),\\(0.6, 0.5), (0.6, 0.7)\end{pmatrix}$	$\begin{pmatrix}(\mathbb{s}_{4}, 0.5ſ), (0.7ſ, 0.8ſ),\\(0.6ſ, 0.5ſ), (0.6ſ, 0.7ſ)\end{pmatrix}$	$\begin{pmatrix}(\mathbb{s}_{4}, 0.52), (0.72, 0.82),\\(0.62, 0.52), (0.62, 0.72)\end{pmatrix}$
$\mathfrak{N}^{5}$	$\begin{pmatrix}(\mathbb{s}_{5}, 0.3), (0.9, 0.4),\\(0.4, 0.3), (0.6, 0.5)\end{pmatrix}$	$\begin{pmatrix}(\mathbb{s}_{5}, 0.3ſ), (0.9ſ, 0.4ſ),\\(0.4ſ, 0.3ſ), (0.6ſ, 0.5ſ)\end{pmatrix}$	$\begin{pmatrix}(\mathbb{s}_{5}, 0.32), (0.92, 0.42),\\(0.42, 0.32), (0.62, 0.52)\end{pmatrix}$
	$\mathfrak{N}^{4}_{AT}$	$\mathfrak{N}^{5}_{AT}$	
$\mathfrak{N}^{ſ}$	$\begin{pmatrix}(\mathbb{s}_{ſ}, 0.23), (0.53, 0.43),\\(0.23, 0.33), (0.33, 0.33)\end{pmatrix}$	$\begin{pmatrix}(\mathbb{s}_{ſ}, 0.24), (0.54, 0.44),\\(0.24, 0.34), (0.34, 0.34)\end{pmatrix}$	
$\mathfrak{N}^{2}$	$\begin{pmatrix}(\mathbb{s}_{2}, 0.33), (0.63, 0.53),\\(0.33, 0.23), (0.43, 0.33)\end{pmatrix}$	$\begin{pmatrix}(\mathbb{s}_{2}, 0.34), (0.64, 0.54),\\(0.34, 0.24), (0.44, 0.34)\end{pmatrix}$	
$\mathfrak{N}^{3}$	$\begin{pmatrix}(\mathbb{s}_{3}, 0.ſ3), (0.83, 0.63),\\(0.43, 0.33), (0.23, 0.33)\end{pmatrix}$	$\begin{pmatrix}(\mathbb{s}_{3}, 0.ſ4), (0.84, 0.64),\\(0.44, 0.34), (0.24, 0.34)\end{pmatrix}$	
$\mathfrak{N}^{4}$	$\begin{pmatrix}(\mathbb{s}_{4}, 0.53), (0.73, 0.83),\\(0.63, 0.53), (0.63, 0.73)\end{pmatrix}$	$\begin{pmatrix}(\mathbb{s}_{4}, 0.54), (0.74, 0.84),\\(0.64, 0.54), (0.64, 0.74)\end{pmatrix}$	
$\mathfrak{N}^{5}$	$\begin{pmatrix}(\mathbb{s}_{5}, 0.33), (0.93, 0.43),\\(0.43, 0.33), (0.63, 0.53)\end{pmatrix}$	$\begin{pmatrix}(\mathbb{s}_{5}, 0.34), (0.94, 0.44),\\(0.44, 0.34), (0.64, 0.54)\end{pmatrix}$	

Table 3
CCp, q-QOF2-TL aggregated decision matrix.

$\psi = 1$	CCp, q-QOF2-TLHPA operator	CCp, q-QOF2-TLHPG operator
$\mathfrak{N}^1$	$\begin{pmatrix} (\mathbb{s}_1, 0.2203), (0.5208, 0.4209), \\ (0.0033, 0.0033), (0.3212, 0.3212) \end{pmatrix}$	$\begin{pmatrix} (\mathbb{s}_0, 0.00065), (0.0379, 0.0130), \\ (0.2218, 0.3212), (0.0033, 0.0033) \end{pmatrix}$
$\mathfrak{N}^2$	$\begin{pmatrix} (\mathbb{s}_2, 0.32017), (0.6207, 0.5208), \\ (0.0130, 0.0033), (0.4209, 0.3212) \end{pmatrix}$	$\begin{pmatrix} (\mathbb{s}_0, 0.0164), (0.0914, 0.0279), \\ (0.3212, 0.2217), (0.0130, 0.0033) \end{pmatrix}$
$\mathfrak{N}^3$	$\begin{pmatrix} (\mathbb{s}_3, 0.12012), (0.8208, 0.6207), \\ (0.0005, 0.0033), (0.2217, 0.3212) \end{pmatrix}$	$\begin{pmatrix} (\mathbb{s}_0, 0.0721), (0.3704, 0.0914), \\ (0.4209, 0.3212), (0.0005, 0.0033) \end{pmatrix}$
$\mathfrak{N}^4$	$\begin{pmatrix} (\mathbb{s}_4, 0.52), (0.72072, 0.82084), \\ (0.0914, 0.1933), (0.6207, 0.7207) \end{pmatrix}$	$\begin{pmatrix} (\mathbb{s}_0, 0.4606), (0.1933, 0.3704), \\ (0.62072, 0.5208), (0.0914, 0.1933) \end{pmatrix}$
$\mathfrak{N}^5$	$\begin{pmatrix} (\mathbb{s}_5, 0.32), (0.9215, 0.4209), \\ (0.0914, 0.0379), (0.6207, 0.5208) \end{pmatrix}$	$\begin{pmatrix} (\mathbb{s}_1, 0.0403), (0.6586, 0.0130), \\ (0.4209, 0.3212), (0.0914, 0.0379) \end{pmatrix}$

3) *When* $\mathfrak{N}^+ = \max\{\mathfrak{N}^1, \mathfrak{N}^2, \ldots, \mathfrak{N}^m\}$ *and* $\mathfrak{N}^- = \min\{\mathfrak{N}^1, \mathfrak{N}^2, \ldots, \mathfrak{N}^m\}$, *then*

$$\mathfrak{N}^- \leq CC\text{p}, \text{q} - QOF2 - TLHPWG(\mathfrak{N}^1, \mathfrak{N}^2, \ldots, \mathfrak{N}^m)_{TN} \leq \mathfrak{N}^+ \mathfrak{N}^- \leq CC\text{p}, \text{q} - QOF2 - TLHPWG(\mathfrak{N}^1, \mathfrak{N}^2, \ldots, \mathfrak{N}^m)_{TCN} \leq \mathfrak{N}^+$$

Further, we utilize the proposed techniques in the environment of MADM problems to enhance the worth of the initiated information.

5. MADM method for presented information

This section briefly explains the utilization of the proposed operators in the environment of the decision-making technique to show the supremacy and validity of the derived theory. For this, we compute the procedure of the MADM technique based on initiated operators for evaluating the problem of green industry development.

5.1. Proposed decision-making procedure

In the presence of the initiated operators based on HTN and HTCN, we aim to design the procedure of the MADM technique for evaluating the application of green industry development. For this, we will use the following steps, such as.

Step 1: First, we arrange the CCp, q-QOF2-TLNs in the form of a decision matrix. Further, when we have cost type of information, then we will go to normalization, such as

$$N = \begin{cases} \mathfrak{N}^j = \left(\left(\mathbb{s}_i^j, \mu_i^j\right), \left(\mathbb{U}_{\mathscr{R}}^{M^j}, \mathbb{U}_{\mathscr{I}}^{M^j}\right), \left(\mathbb{U}_{\mathscr{R}}^{N^j}, \mathbb{U}_{\mathscr{I}}^{N^j}\right), \left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^j}, \mathbb{U}_{\mathscr{I}}^{\mathbb{R}^j}\right)\right) & \textit{for benefit} \\ \left(\mathfrak{N}^j\right)^c = \left(\left(\mathbb{s}_i^j, \mu_i^j\right), \left(\mathbb{U}_{\mathscr{R}}^{N^j}, \mathbb{U}_{\mathscr{I}}^{N^j}\right), \left(\mathbb{U}_{\mathscr{R}}^{M^j}, \mathbb{U}_{\mathscr{I}}^{M^j}\right), \left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^j}, \mathbb{U}_{\mathscr{I}}^{\mathbb{R}^j}\right)\right) & \textit{for cost} \end{cases}$$

where $\left(\mathfrak{N}^j\right)^c$ represents the compliment. Further, when we have a benefit type of information, then we will go to the next step without normalization.

Step 2: Second, we aim to aggregate the select information after normalization (if needed) by using the technique of CCp, q-QOF2-TLHPA operator, and CCp, q-QOF2-TLHPG operator.

Step 3: Third, we describe the Score values of the above-aggregated values by using the theory of score function, such as

Table 4
CCp, q-QOF2-TL decision matrix of score values.

	CCp, q-QOF2-TLHPA operator	CCp, q-QOF2-TLHPG operator
$\mathfrak{N}^1$	0.4995	0.0014
$\mathfrak{N}^2$	0.7263	0.0082
$\mathfrak{N}^3$	0.8939	0.0061
$\mathfrak{N}^4$	1.52	0.14
$\mathfrak{N}^5$	1.57	0.25

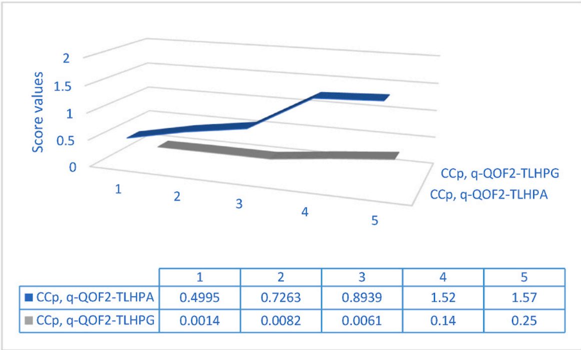

Fig. 5. Geometrical form of data in Table 4.

Table 5
Representation of ranking matrix.

Methods	Ranking values	Best Optimal
CCp, q-QOF2-TLHPA operator	$\mathfrak{N}^5 > \mathfrak{N}^4 > \mathfrak{N}^3 > \mathfrak{N}^2 > \mathfrak{N}^1$	$\mathfrak{N}^5$
CCp, q-QOF2-TLHPG operator	$\mathfrak{N}^5 > \mathfrak{N}^4 > \mathfrak{N}^2 > \mathfrak{N}^3 > \mathfrak{N}^1$	$\mathfrak{N}^5$

Table 6
Score values with ranking information for different weight vectors.

Methods	Score values	Ranking values	Best optimal
CCp, q-QOF2-TLHPA operator	0.5000,0.7269,0.8946,1.521,1.575	$\mathfrak{N}^5 > \mathfrak{N}^4 > \mathfrak{N}^3 > \mathfrak{N}^2 > \mathfrak{N}^1$	$\mathfrak{N}^5$
CCp, q-QOF2-TLHPG operator	0.0014,0.0082,0.0060,0.138,0.248	$\mathfrak{N}^5 > \mathfrak{N}^4 > \mathfrak{N}^2 > \mathfrak{N}^3 > \mathfrak{N}^1$	$\mathfrak{N}^5$

Table 7
Score values with ranking information for different weight vectors.

Methods	Score values	Ranking values	Best optimal
CCp, q-QOF2-TLHPA operator	0.4939,0.7197,0.8855,1.504,1.569	$\mathfrak{N}^5 > \mathfrak{N}^4 > \mathfrak{N}^3 > \mathfrak{N}^2 > \mathfrak{N}^1$	$\mathfrak{N}^5$
CCp, q-QOF2-TLHPG operator	0.016,0.0083,0.0063,0.14,0.249	$\mathfrak{N}^5 > \mathfrak{N}^4 > \mathfrak{N}^2 > \mathfrak{N}^3 > \mathfrak{N}^1$	$\mathfrak{N}^5$

Table 8
Ranking values for CCp, q-QOF2-TLHPA operators.

Methods	Score values	Ranking values
$F\exists^{s} = 1$	0.4995,0.7263,0.8939,1.52,1.57	$\mathfrak{N}^5 > \mathfrak{N}^4 > \mathfrak{N}^3 > \mathfrak{N}^2 > \mathfrak{N}^1$
$F\exists^{s} = 2$	3.0248,3.058,3.1253,3.18,3.1	$\mathfrak{N}^4 > \mathfrak{N}^3 > \mathfrak{N}^5 > \mathfrak{N}^2 > \mathfrak{N}^1$
$F\exists^{s} = 3$	4.3189,4.3380,4.2703,4.27,4.08	$\mathfrak{N}^2 > \mathfrak{N}^1 > \mathfrak{N}^3 > \mathfrak{N}^4 > \mathfrak{N}^5$
$F\exists^{s} = 4$	5.5085,5.5094,5.2908,5.24,4.94	$\mathfrak{N}^2 > \mathfrak{N}^1 > \mathfrak{N}^3 > \mathfrak{N}^4 > \mathfrak{N}^5$
$F\exists^{s} = 5$	6.665,6.62,6.25,6.14,5.76	$\mathfrak{N}^1 > \mathfrak{N}^2 > \mathfrak{N}^3 > \mathfrak{N}^4 > \mathfrak{N}^5$

$$SC(\mathfrak{N}^j) = \left(\frac{T^{-1}\left(\mathbb{s}_i^j, \mu_i^j\right)}{\mathbb{g}}\right) + \left(\frac{1}{2}\left(\left(\mathbb{U}_{\mathscr{R}}^{M^j}\right)^{\Psi} + \left(\mathbb{U}_{\mathscr{I}}^{M^j}\right)^{\Psi} - \left(\mathbb{U}_{\mathscr{R}}^{N^j}\right)^{\Psi} - \left(\mathbb{U}_{\mathscr{I}}^{N^j}\right)^{\Psi}\right) + \frac{1}{2}\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^j} + \mathbb{U}_{\mathscr{I}}^{\mathbb{R}^j}\right)\right) \in [-\mathbb{g}, \mathbb{g}]$$

When we obtain the same values for all aggregated values, then we will use the accuracy values, such as

$$AC(\mathfrak{N}^j) = \left(\frac{T^{-1}\left(\mathbb{s}_i^j, \mu_i^j\right)}{\mathbb{g}}\right) + \left(\frac{1}{2}\left(\left(\mathbb{U}_{\mathscr{R}}^{M^j}\right)^{\Psi} + \left(\mathbb{U}_{\mathscr{I}}^{M^j}\right)^{\Psi} + \left(\mathbb{U}_{\mathscr{R}}^{N^j}\right)^{\Psi} + \left(\mathbb{U}_{\mathscr{I}}^{N^j}\right)^{\Psi}\right) + \frac{1}{2}\left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^j} + \mathbb{U}_{\mathscr{I}}^{\mathbb{R}^j}\right)\right) \in [0, \mathbb{g}]$$

Step 4: Finally, we compute the ranking values for evaluating the best optimal according to their score values. The graphical abstract of the proposed algorithm is listed in Fig. 4.

Further, we employ the above procedure in the environment of the application in green industry development because, with the help of the above procedure, we aim to find the ranking values and also check the stability of the proposed theory.

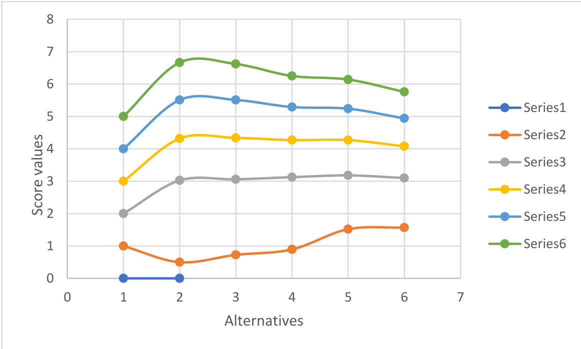

Fig. 6. The geometrical form of the data is in Table 8.

Table 9
Ranking values for CCp, q-QOF2-TLHPG operators.

Methods	Score values	Ranking values
$F⅃^{s} = 1$	0.0014,0.0082,0.0061,0.14,0.25	$\mathfrak{N}^{5} > \mathfrak{N}^{4} > \mathfrak{N}^{3} > \mathfrak{N}^{2} > \mathfrak{N}^{1}$
$F⅃^{s} = 2$	-1.0019,-1.0018,-1.0080,-1.03,-0.98	$\mathfrak{N}^{2} > \mathfrak{N}^{1} > \mathfrak{N}^{3} > \mathfrak{N}^{4} > \mathfrak{N}^{5}$
$F⅃^{s} = 3$	−2.0019424,-2.0019413,-2.0077,-2.04,-2	$\mathfrak{N}^{2} > \mathfrak{N}^{1} > \mathfrak{N}^{3} > \mathfrak{N}^{4} > \mathfrak{N}^{5}$
$F⅃^{s} = 4$	−3.0019061,-3.001906,-3.0072,-3.02,-3.01	$\mathfrak{N}^{2} > \mathfrak{N}^{1} > \mathfrak{N}^{3} > \mathfrak{N}^{4} > \mathfrak{N}^{5}$
$F⅃^{s} = 5$	−4.0018579,-4.0018578,-4.0065,-3.99,-4.01	$\mathfrak{N}^{2} > \mathfrak{N}^{1} > \mathfrak{N}^{3} > \mathfrak{N}^{4} > \mathfrak{N}^{5}$

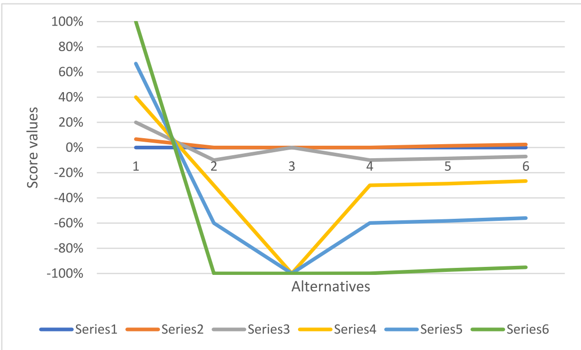

Fig. 7. The geometrical form of the data is in Table 9.

Table 10
Comparative analysis for the data in Table 2.

Methods	Score values	Ranking values
Garg and Rani [37]	Limited (Not possible)	No
Mahmood and Ali [38]	Limited (Not possible)	No
Jiang et al. [39]	Limited (Not possible)	No
Wei and Lu [40]	Limited (Not possible)	No
Wei [41]	Limited (Not possible)	No
Yu [42]	Limited (Not possible)	No
Darko and Liang [43]	Limited (Not possible)	No
CCp, q-QOF2-TLHPA operator	0.4995,0.7263,0.8939,1.52,1.57	$\mathfrak{N}^{5} > \mathfrak{N}^{4} > \mathfrak{N}^{3} > \mathfrak{N}^{2} > \mathfrak{N}^{1}$
CCp, q-QOF2-TLHPG operator	0.0014,0.0082,0.0061,0.14,0.25	$\mathfrak{N}^{5} > \mathfrak{N}^{4} > \mathfrak{N}^{3} > \mathfrak{N}^{2} > \mathfrak{N}^{1}$

5.2. Application in green industry development [1,2]

In this section, we describe the application of green industry development [[1,2]] under the consideration of the proposed information. Green industry development can help in the procedure of industrialization and fostering economic growth in the presence of minimum environmental impact and encouraging sustainability. Green industry development contains the assumption of technologies, policies, and practices that theme to decrease pollution, mitigate climate change, and conserve resources. The major theme of

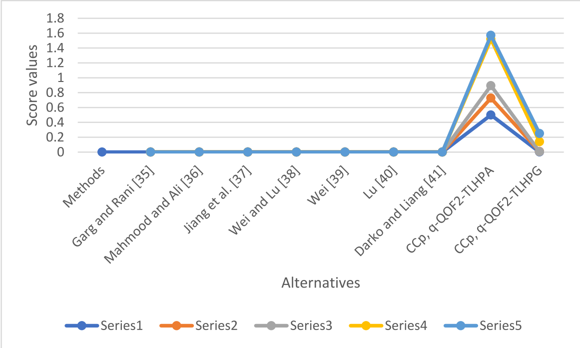

Fig. 8. The geometrical form of the data is in Table 10.

this application is to find the best and worst aspects of green industry development among the following five alternatives, such as.

1) Renewable Energy ($\Re^{1}$).
2) Resources Efficiency ($\Re^{2}$).
3) Waste Management ($\Re^{3}$).
4) Environmental Standards ($\Re^{4}$).
5) Green Innovations ($\Re^{5}$).

To evaluate the above problems, we have considered the following five attributes, such as social impact, growth analysis, political impact, environmental impact, and problem-related resources with weighted vectors which are obtained by using power operators, such as $(0.1999, 0.2, 0.2001, 0.2, 0.1999)^T$. For this, we will use the following steps, such as

Step 1: First, we arrange the CCp, q-QOF2-TLNs in the form of a decision matrix in Table 2. Further, when we have cost type of information, then we will go to normalization, such as

Table 11
Theoretical comparison between proposed and existing methods.

Methods	Membership function	Non-membership function	Radius function	Periodic function	Power ψ and Ψ	2-tuple linguistic information
Zadeh [4]	√	×	×	×	×	×
Atanassov [6,7]	√	√	×	×	×	×
Yager [8]	√	√	×	×	×	×
Yager [9]	√	√	×	×	×	×
Ramot et al. [10]	√		×	√	×	×
Alkouri and Salleh [11]	√	√	×	√	×	×
Ullah et al. [12]	√	√	×	√	×	×
Liu et al. [13]	√	√	×	√	×	×
Atanassov [14]	√	√	√	×	×	×
Olgun and Unver [15]	√	√	√	×	×	×
Yusoff et al. [16]	√	√	√	×	×	×
Ali and Yang [17]	√	√	√	×	×	×
Ibrahim and Alshammari [18]	√	√	×	×	√	×
Ibrahim [19]	√	√	×	√	√	×
Garg and Rani [37]	√	√	×	×	×	×
Mahmood and Ali [38]	√	√	√	×	×	×
Jiang et al. [39]	√	√	×	×	×	×
Wei and Lu [40]	√	√	×	×	×	×
Wei [41]	√	√	×	×	×	×
Yu [42]	√	√	×	×	×	×
Darko and Liang [43]	√	√	×	×	×	×
CCp, q-QOF2-TLHPA operator	√	√	√	√	√	√
CCp, q-QOF2-TLHPG operator	√	√	√	√	√	√

$$N = \begin{cases} \mathfrak{N}^j = \left(\left(\mathbb{s}_i^j, \mu_i^j\right), \left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^j}, \mathbb{U}_{\mathscr{I}}^{\mathrm{M}^j}\right), \left(\mathbb{U}_{\mathscr{R}}^{N^j}, \mathbb{U}_{\mathscr{I}}^{N^j}\right), \left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^j}, \mathbb{U}_{\mathscr{I}}^{\mathbb{R}^j}\right)\right) & \textit{for benefit} \\ \left(\mathfrak{N}^j\right)^c = \left(\left(\mathbb{s}_i^j, \mu_i^j\right), \left(\mathbb{U}_{\mathscr{R}}^{N^j}, \mathbb{U}_{\mathscr{I}}^{N^j}\right), \left(\mathbb{U}_{\mathscr{R}}^{\mathrm{M}^j}, \mathbb{U}_{\mathscr{I}}^{\mathrm{M}^j}\right), \left(\mathbb{U}_{\mathscr{R}}^{\mathbb{R}^j}, \mathbb{U}_{\mathscr{I}}^{\mathbb{R}^j}\right)\right) & \textit{for cost} \end{cases}$$

where $\left(\mathfrak{N}^j\right)^c$ represents the compliment. Further, when we have a benefit type of information, then we will go to the next step without normalization. However, the information in Table 2 is not required to be normalized.

Step 2: Second, we aim to aggregate the select information after normalization (if needed) by using the technique of CCp, q-QOF2-TLHPA operator and CCp, q-QOF2-TLHPG operator for t-norm, see Table 3 for $\psi = 1$.

Step 3: Third, we describe the Score values of the above-aggregated values by using the theory of score function, see Table 4. The geometrical form of information in Table 4 is listed in the form of Fig. 5.

Step 4: Finally, we compute the ranking values for evaluating the best optimal according to their score values, see Table 5.

After a long analysis, we concluded that the best optimal is $\mathfrak{N}^5$ (represented the Green Innovation) according to the theory of CCp, q-QOF2-TLHPA operator and CCp, q-QOF2-TLHPA operator for $\psi = \Psi = 5$ and $F Ⅎ^s = 1$. Moreover, we described the ranking values of the information in Table 2 by using the proposed data for the following weight vectors $(0.24, 0.158, 0.122, 0.25, 0.23)^T$, thus the score values and ranking values are listed in Table 6.

Thus, we concluded that the best optimal is $\mathfrak{N}^5$ (represented the Green Innovation) according to the theory of CCp, q-QOF2-TLHPA operator and CCp, q-QOF2-TLHPA operator for $\psi = \Psi = 5$ and $F Ⅎ^s = 1$. Moreover, we described the ranking values of the information in Table 2 by using the proposed data for the following weight vectors $(0.38, 0.43, 0.02, 0.1, 0.07)^T$, thus the score values and ranking values are listed in Table 7.

Thus, we concluded that the best optimal is $\mathfrak{N}^5$ (represented the Green Innovation) according to the theory of CCp, q-QOF2-TLHPA operator and CCp, q-QOF2-TLHPA operator for $\psi = \Psi = 5$ and $F Ⅎ^s = 1$. Further, we discussed the impact or influence of the parameters $F Ⅎ^s$ for different values and also discussed the stability of the Ψ and ψ.

5.3. *Influence of parameters*

In this section, we analyze the stability and influence of the parameter $F Ⅎ^s$ by using the information in the above subsection with the help of aggregation operators. The major theme of this subsection is to find the stability or effectiveness in the proposed ranking results. The ranking values for different values of a parameter $F Ⅎ^s$ is given in Table 8 using the technique of CCp, q-QOF2-TLHPA operator.

After a long analysis, we concluded that the best optimal is $\mathfrak{N}^5$ (represented the Green Innovation) for $F Ⅎ^s = 1$, the best optimal is $\mathfrak{N}^4$ for $F Ⅎ^s = 2$, the best optimal is $\mathfrak{N}^2$ for $F Ⅎ^s = 3, 4$, and the best optimal is $\mathfrak{N}^1$ for $F Ⅎ^s = 5$ according to the theory of CCp, q-QOF2-TLHPA operator for $\psi = \Psi = 5$. The geometrical form of the data in Table 8 is given in Fig. 6.

The ranking values for different values of the parameter $F Ⅎ^s$ is given in Table 9 using the technique of CCp, q-QOF2-TLHPG operator.

After a long analysis, we concluded that the best optimal is $\mathfrak{N}^5$ (represented the Green Innovation) for $F Ⅎ^s = 1$, and the best optimal is $\mathfrak{N}^2$ for $F Ⅎ^s = 2, 3, 4, 5$ according to the theory of CCp, q-QOF2-TLHPG operator for $\psi = \Psi = 5$. Further, we analyze the comparison between proposed ranking values and existing ranking values to enhance the worth of the proposed theory. The geometrical form of the data in Table 9 is given in Fig. 7.

6. Comparative analysis

In this section, we describe the validity and supremacy of the proposed operators by comparing their ranking values with the ranking values of some prevailing techniques to enhance the worth of the proposed theory. During comparative analysis, we were required to collect some existing techniques, for this, we arranged the following prevailing ideas, for instance, power aggregation operators for CIFSs [37], Aczel-Alsina power aggregation operators for CIFSs [38], power aggregation operators for IFSs [39], power aggregation operators for PFSs [40], Hamacher power aggregation operators for PFSs [41], Hamacher aggregation operators for complex cubic q-ROFSs [42], and Hamacher aggregation operators for q-ROFSs [43]. The comparative analysis for the information in Table 2 is listed in Table 10.

After a long analysis, we concluded that the best optimal is $\mathfrak{N}^5$ (represented the Green Innovation) according to the theory of CCp, q-QOF2-TLHPA operator and CCp, q-QOF2-TLHPA operator for $\psi = \Psi = 5$ and $F Ⅎ^s = 1$. The geometrical form of the data in Table 10 is given in Fig. 8.

Further, we discussed the impact or influence of the parameters $F Ⅎ^s$ for different values and also discussed the stability of the Ψ and ψ. However, the existing techniques have limited features because of their specification and shape of the existing technique. Further, we described the theoretical comparison of the proposed methods with some existing models to enhance the worth of the proposed theory, see Table 11.

Information in Table 11 represents two types of symbols, called "$\surd$" which is used for "yes", and "$\times$" which is used for "no". Therefore, for the data in Table 11, we concluded that the existing techniques have a lot of problems and limitations because of their features and structures. Anyhow, the proposed model is more reliable and more effective compared to existing models. The advantages of the proposed methodologies are listed below:

1) FSs (if $\mathbb{U}^{\mathrm{M}}_{\mathscr{I}} = \mathbb{U}^{N}_{\mathscr{R}} = \mathbb{U}^{N}_{\mathscr{I}} = \mathbb{U}^{\mathbb{R}}_{\mathscr{R}} = \mathbb{U}^{\mathbb{R}}_{\mathscr{I}} = (\mathbb{s}_i, \mu_i) = 0$ and $\Psi, \psi = 1, \mathbb{U}^{\mathbb{R}}_{\mathscr{R}}$).
2) IFSs (if $\mathbb{U}^{\mathrm{M}}_{\mathscr{I}} = \mathbb{U}^{N}_{\mathscr{I}} = \mathbb{U}^{\mathbb{R}}_{\mathscr{R}} = \mathbb{U}^{\mathbb{R}}_{\mathscr{I}} = (\mathbb{s}_i, \mu_i) = 0$ and $\Psi, \psi = 1$).
3) PFSs (if $\mathbb{U}^{\mathrm{M}}_{\mathscr{I}} = \mathbb{U}^{N}_{\mathscr{I}} = \mathbb{U}^{\mathbb{R}}_{\mathscr{R}} = \mathbb{U}^{\mathbb{R}}_{\mathscr{I}} = (\mathbb{s}_i, \mu_i) = 0$ and $\Psi, \psi = 2$).
4) q-ROFSs (if $\mathbb{U}^{\mathrm{M}}_{\mathscr{I}} = \mathbb{U}^{N}_{\mathscr{I}} = \mathbb{U}^{\mathbb{R}}_{\mathscr{R}} = \mathbb{U}^{\mathbb{R}}_{\mathscr{I}} = (\mathbb{s}_i, \mu_i) = 0$ and $\Psi = \psi$).
5) CFSs (if $\mathbb{U}^{N}_{\mathscr{R}} = \mathbb{U}^{N}_{\mathscr{I}} = \mathbb{U}^{\mathbb{R}}_{\mathscr{R}} = \mathbb{U}^{\mathbb{R}}_{\mathscr{I}} = (\mathbb{s}_i, \mu_i) = 0$ and $\Psi, \psi = 1$).
6) CIFSs (if $\mathbb{U}^{\mathbb{R}}_{\mathscr{R}} = \mathbb{U}^{\mathbb{R}}_{\mathscr{I}} = (\mathbb{s}_i, \mu_i) = 0$ and $\Psi, \psi = 1$).
7) CPFSs (if $\mathbb{U}^{\mathbb{R}}_{\mathscr{R}} = \mathbb{U}^{\mathbb{R}}_{\mathscr{I}} = (\mathbb{s}_i, \mu_i) = 0$ and $\Psi, \psi = 2$).
8) Cq-ROFSs (if $\mathbb{U}^{\mathbb{R}}_{\mathscr{R}} = \mathbb{U}^{\mathbb{R}}_{\mathscr{I}} = (\mathbb{s}_i, \mu_i) = 0$ and $\Psi = \psi$).

These all are the special cases of the CCp, q-QOF2-TL sets, where the model of 2-tuple linguistic sets and their extensions, circular fuzzy sets, and their extensions are also some dominant cases of the proposed theory. Hence, the proposed model is novel and up to date no one can derive it yet and because of this reason, they will be receiving more attention from scholars in the future.

7. Conclusion

Cq-ROFSs, Cirq-ROFSs, and 2-TLSs are very reliable and flexible because of their features, inspired by this information, the major contribution of this article is described in the following form, such: we developed the novel technique of CCp, q-QOF2-TL set and their operational laws based on algebraic t-norms and Hamacher t-norms, where the algebraic t-norms and Einstein t-norms are the special cases of the Hamacher t-norms for parameter $F\daleth^{s} = 1$ and $F\daleth^{s} = 2$. Further, we derived the Hamacher power aggregation operators based on any finite collection of CCp, q-QOF2-TLNs, called CCp, q-QOF2-TLHPA operator, CCp, q-QOF2-TLHPWA operator, CCp, q-QOF2-TLHPG operator, CCp, q-QOF2-TLHPWG operator, and described their basic properties, called idempotency, monotonicity, and boundedness. Additionally, we demonstrated the technique of the MADM problem based on the above operators to evaluate the major factor that will be playing in the development of the green industry. Lastly, we compared the proposed ranking values with the obtained ranking values of existing techniques to show the supremacy and superiority of the initiated approaches.

7.1. Limitations of the proposed theory

The model of CCp, q-QOF2-TLSs is very effective, but in the case of positive, abstinence, negative, radius, and 2-tuple linguistic information, the technique of CCp, q-QOF2-TLSs has failed, because the technique of CCp, q-QOF2-TLSs is contained a lot of limitations and problems, for this, we need to construct the model of circular complex p, q-spherical fuzzy 2-tuple linguistic sets and their extensions.

7.2. Future Direction

In the upcoming times, we will develop some new ideas such as circular complex picture fuzzy 2-tuple linguistic sets and their generalizations. Further, we will derive some new techniques, methods, operators, and measures based on the proposed techniques. Finally, we will utilize the proposed techniques in the environment of artificial intelligence, machine learning, game theory, and neural networks to enhance the worth of the proposed theory.

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Data Availability Statement

No data was used for the research described in the article.

CRediT authorship contribution statement

Zeeshan Ali: Writing – review & editing, Validation, Supervision, Methodology, Data curation. **Khizar Hayat:** Writing – review & editing, Visualization, Validation, Supervision, Methodology. **Dragan Pamucar:** Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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