

## Research article

# The recovery strategy of interdependent networks under targeted attacks

Li Liang

318 Liuhe Road, Xihu District, Hangzhou, Zhejiang University of Science and Technology, School of Science, China

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## ABSTRACT

To effectively mitigate failures in interdependent systems during targeted-attack scenarios, a common approach is to pre-store repair resources. The question arises: what constitutes an appropriate quantity of these pre-stored repair resources? The paper introduces a novel recovery strategy aimed at providing guidance for this issue. Current recovery strategies frequently emphasize the dynamic interplay between cascading failures and recovery processes, indicating that interventions during the recovery phase are permissible. In this context, the recovery strategy focus on recovering a predetermined number of failed nodes that are adjacent to the largest connected component of each individual network, along with their dependent nodes, at each recovery stage. Simulation results demonstrate that this strategy significantly enhances the capacity to prevent system breakdowns for interdependent networks subjected to targeted attacks. Therefore, by determining the necessary recovery steps to prevent system failures and the appropriate repair resources required for each step, this novel strategy can serve as a valuable reference for the pre-storage of repair resources. Significantly, the strategy can be effectively applied to interdependent networks associated with critical infrastructure, such as power grids and communication networks.

## 1. Introduction

In real-world scenarios, there is a notable rise in the interconnected nature of diverse infrastructural systems, encompassing water, electricity, gas, transportation, and communication networks [1]. This network of interconnected systems is commonly referred to as interdependent networks. Within interdependent networks, nodes in one network rely on nodes in another network, and vice versa. Consequently, this initial node failure can set off a recursive chain of cascading failures, potentially resulting in the complete breakdown of both networks. In 2010, Buldyrev et al. [2] investigated the phenomenon of cascading failures in interdependent networks and established a relevant theoretical framework. They observed that interdependent networks under random attacks displayed a first-order discontinuous phase transition, which differs significantly from a single network. This findings indicate that the presence of interdependent networks holds a dual significance. On one hand, network interdependence can improve operational efficiency [3], while on the other hand, it can introduce unforeseen vulnerabilities and risks through cascading failures [4].

The vulnerability and risk inherent in interdependent networks have led to a growing interest among scholars in investigating strategies for preventing or recovering from cascading failures. Proactive prevention strategies are designed to be implemented before failures occur in interdependent networks, typically involving fault detection, pre-protection of critical nodes [5–7], node backup

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E-mail address: [HITLiangLi@163.com](mailto:HITLiangLi@163.com).

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[8,9], and modification of coupling mechanisms [10–12]. It is noteworthy that most studies depict prevention strategies as static processes of non-interference, while the dynamic nature of the real world necessitates more timely and effective emergency responses to swiftly address emerging challenges. Consequently, research into recovery strategies [13–15] is deemed crucial in this context.

In the context of enhancing robustness against failures or attacks within single-layer networks, various recovery strategies have been documented, such as localized recovery [16] and self-healing [17], among others. Building upon this foundation, recovery strategies for interdependent networks are progressively being developed. In 2015, Majdandzic et al. [18] conducted a study on the spontaneous recovery process in interdependent networks, taking into account the potential for fault propagation through the dependent edges connecting two single-layer networks. In 2016, Di Muro [13] et al. introduced the boundary-node recovery strategy for interdependent networks as a means to protect the unaffected portion of the network during cascading failures. This strategy involves the dynamic alternation between cascading failures and recovery processes, with a focus on identifying mutual boundary nodes within interdependent networks. Subsequently, Wu et al. [19] enhanced this strategy by introducing a preferred recovery algorithm based on boundary edges within and outside the giant component, as random repair of mutual boundary nodes was considered suboptimal. In 2018, Rocca et al. [20] proposed the idle link-edge recovery strategy to repair the network with lower recovery cost in interdependent networks. Notably, both strategies emphasize leveraging the network's giant components to mitigate cascade effects. In 2019, in the context of real-world scenarios involving localized damage in complex networks [21], where nodes and their neighbors are sequentially affected, Gong et al. [22] proposed a recovery strategy that prioritizes nodes with minimum degrees based on the failure network model of interdependent lattice networks [23]. In addition, focusing on multilayer networks consisting of a control layer network A and non-control layers B, C, etc., Liu et al. [24] introduced the self-adaption recovery strategy by integrating adaptive edges into multilayer networks. The incorporation of adaptive edges in the control layer network A reduces the risk of node failure, thereby improving the resilience of the control layer network and the overall network robustness.

In practical situations, considering that implementing repairs promptly can minimize losses, the strategy commonly employed is pre-storing repair resources (human and material resources). However, current recovery strategies, such as boundary-node recovery, idle link-edge recovery and recovery under localized attacks, determine the number of nodes repaired based on the proportion of failed nodes at each recovery stage. Additionally, other strategies related to self-healing capabilities emphasize that each failed node or link has the potential to be repaired in order to facilitate the recovery process. Under these recovery strategies, the quantities of pre-stored repair resources cannot be properly determined, which may lead to either insufficient resources or significant waste. Consequently, it is essential to develop novel recovery strategies for interdependent networks that provide adequate guidance for pre-storing repair resources. Particularly, our focus is on interdependent networks under targeted attacks, as these networks are particularly vulnerable to breakdowns when initial failures are targeted. Significantly, the topic of targeted attacks holds a prominent position within the domain of complex network analysis, encompassing interdependent networks [25,26] and higher-order networks [27–29].

The difficulty in developing the recovery strategy is how to determine the appropriate amount of pre-stored repair resources to prevent cascading failures efficiently while minimizing resource wastage. What is the appropriate amount of pre-stored repair resources? According to the recovery strategies discussed emphasize the dynamic alternation between cascading failures and recovery processes, the aforementioned difficulty can be addressed by identifying the necessary recovery steps to prevent system breakdown and determining the appropriate repair resources for each recovery step. In the paper, relying on the cascading failure model introduced by Buldyrev et al. [2], we make some modifications on the boundary-node recovery strategy [13,19] and propose a novel strategy of recovering a fixed number of mutual boundary nodes for interdependent networks. In the target of preventing the complete breakdown of interdependent networks under targeted attacks, simulations show that a smaller amount of repair resources can significantly enhance the robustness of interdependent networks and then the robustness of interdependent networks will not experience a significant improvement despite an increase in repair resources. Therefore, this strategy can provide guidance for pre-storing repair resources in the interdependent networks and help prevent significant waste of resources.

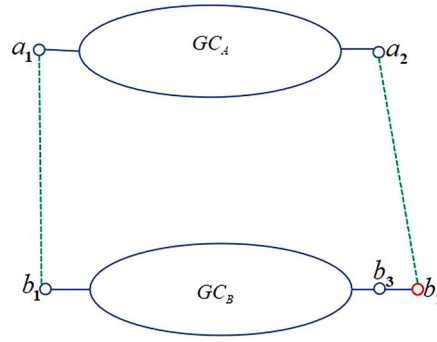
Furthermore, if the pre-stored repair resources are not fully utilized during a recovery phase, the strategy can be adjusted to provide additional protections for essential nodes, thereby preventing waste of resources. Notably, the repair process of failed nodes in the mutual boundary is a passive measure for cascading failure, whereas the revised approach involves more proactive measures and has the potential to significantly improve recovery effectiveness for interdependent networks in targeted-attack problems.

The remainder of the paper is structured as follows. Section 2 outlines the novel strategy of recovering a fixed number of mutual boundary nodes at each recovery stage. Section 3 applies the strategy to the interdependent networks of the power grid and the communication system. Section 4 discusses a modification of the aforementioned strategy, which involves protecting important nodes and recovering a fixed number of mutual boundary nodes at each recovery stage. Section 5 provides the conclusion for the entire paper.

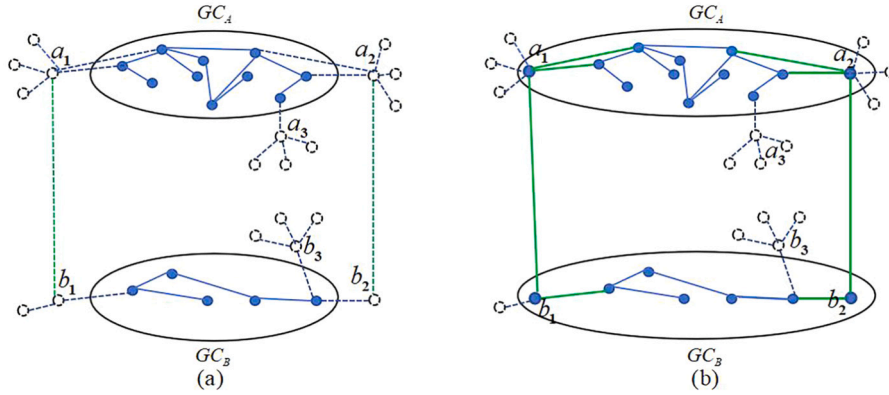
## 2. The strategy of recovering a fixed number of mutual boundary nodes

### 2.1. Model

To examine interdependent networks, we consider for simplicity, two single-layer networks, A and B. In 2016, Muro et al. pioneered a recovery model applicable to interdependent networks by defining mutual boundary nodes, which can refer to pairs of nodes  $(a_i, b_j)$  where node  $a_i$  in network A has a topological distance  $l = 1$  from the giant connected network of A ( $GC_A$ ), and the corresponding coupling node  $b_j$  also has a topological distance  $l = 1$  from the giant connected network of B ( $GC_B$ ). See Fig. 1.



**Fig. 1.** Schematic demonstration of the mutual boundary of  $GC_A$  and  $GC_B$ . Since the topological distance between  $b_2$  and  $GC_B$  is equal to 2, the point of  $(a_2, b_2)$  is not mutual boundary nodes.



**Fig. 2.** (a) Three pairs of mutual boundary nodes  $((a_1, b_1), (a_2, b_2), (a_3, b_3))$  are shown; (b) by calculating the index  $I_{v_i}$  with  $\beta = \frac{2}{3}$ , the  $I_{a_1} = \frac{7}{3}$ ,  $I_{a_2} = \frac{8}{3}$  and  $I_{a_3} = \frac{5}{3}$ . Therefore, if two pairs of mutual boundary nodes at each recovering stage are appointed to repair, interdependent nodes  $(a_1, b_1)$  and  $(a_2, b_2)$  are preferential.

To better reflect real-world scenarios, Wu et al. [19] suggested a preferential recovery algorithm by incorporating the significance index  $I_{v_i}$  of boundary nodes:

$$I_{v_i} = \beta k_i^{gc} + (1 - \beta) k_i^{ngc}$$

where  $v_i$  is a boundary node,  $k_i^{gc}$  is the number of original links between  $v_i$  and  $GC_A$ ,  $k_i^{ngc}$  is the number of original links between  $v_i$  and nodes in  $A - GC_A$  and  $\beta$  denotes the weight of importance of two kinds of links.

However, these strategies are solely based on the number of mutual boundary nodes post-attack and cannot be used as a reference for pre-storing repair resources. Therefore, for highly vulnerable interdependent networks in targeted-attack scenarios, we make adjustments to the recovery strategy by focusing on recovering a fixed number of mutual boundary nodes at each recovery stage.

In detail, the novel recovery strategy consists of two phases: initial phase, cascading failure and recovery orderly alternating phase:

### I. Initial phase

The foundational networks under consideration are interdependent networks A-B, where network A and network B has the same number of nodes,  $N$ . Importantly, network A and network B are interconnected through bidirectional coupling links, whereby the high-degree nodes in one network are contingent upon the corresponding high-degree nodes in the other network. We assume that a fraction,  $1 - p$ , of the nodes of network A fail due to a targeted attack, where  $p$  represents the initial proportion of survival nodes and the term “fail” is defined as the process of removing a faulty node from the network, including its connected edge and coupling links.

### II. Cascading failure and recovery orderly alternating phase

We denote by  $n = 1, 2, \dots$  the time steps of the phase. From the survival fraction  $p$  of nodes, only those within the giant component ( $GC$ ) of a network are considered as functional while the remaining nodes are deemed dysfunctional and classified as failures. After the initial failure, the damage in A transmitted to network B through interdependent links, following the conventional process of cascading failures [2], but we introduce the recovery strategy before spreading the failures back to network A. The rules governing the phase at any stage  $n$  are provided by:

- Stage  $n$  in A
  1. Functional nodes fail if they lose support from their counterpart nodes in B at stage  $n - 1$ .
  2. From the survivors, those nodes that belong to the  $GC_A$  remain functional while the other fail.

- Stage  $n$  in B
  3. Functional nodes become dysfunctional if they lose support from network A due to the cascade of failures at stage  $n$ .
  4. The remaining nodes fail if they do not belong to the  $GC_B$ .
- Recovering
  5. Sort the nodes in mutual boundary of  $GC_A$  in descending order of  $I_{v_i}$ , and then recovery the top  $R$  nodes and their dependent nodes in mutual boundary nodes of  $GC_B$ . Significantly, all their connections with  $GC_A$  and  $GC_B$  are reactivated and also the links between restored boundary nodes if they were connected in the initial phase. See Fig. 2.
  6. Repeat 1-5. Once the current network reaches steady state (no more new failure nodes), the dynamic process of the recovering model ends.

## 2.2. Simulation

Scale-Free (SF) networks exhibit heightened vulnerability to targeted attacks [30]. Consequently, the simulations focus on isomorphic interdependent networks, SF-SF, as well as heterogeneous interdependent networks, SF-ER (Erdős-Rényi random network) and SF-RR (random regular network). In these simulations, the two single-layer networks are interconnected through bidirectional coupling links, adhering to the principle that the  $i$ th node in one network is dependent on the  $i$ th node in the other network based on the descending order of degree,  $i = 1, 2, \dots, N$ . During the initial phase, the top  $(1-p)N$  nodes of network A, i.e., SF network, are subjected to failure in accordance with their descending degree order. The effectiveness of the model is evaluated by measuring the probability of the existence of  $GC$  of the interdependent networks at steady state [2], denoted by  $P_\infty$ , where  $GC$  is considered to be present if the pairs number of nodes within the remaining  $GC$  exceeds two. Additionally, to provide references for pre-storing repair resources, the average number of recovery steps, denoted by  $NOI_{ave}$ , is also taken into account when the system reaches steady state.

Fig. 3 and 4 illustrates the  $P_\infty$  and  $NOI_{ave}$  as a function of  $p$  and  $R$ , where  $p$  represents the percentage of initially survival nodes and  $R$  denotes the fixed pair number of repaired mutual boundary nodes at each recovery stage.

In Fig. 3, the left figures illustrate a significant increase in  $P_\infty$  when  $R$  is small, followed by a period of negligible changes or slight increases as  $R$  becomes larger. Then, the right figures indicate that the recovery process finished in just a few steps. Furthermore, Figs. 3(a), (b) and (c) demonstrate that the quantity of pre-stored repair resources experiences only a modest increase, despite a substantial growth in  $N$ . Specifically, the strategy's recommendations for SF-SF suggest that when  $N = 100$ , it is advisable to pre-store repair resources of approximately 4 pairs of mutual boundary nodes for each of about 3 recovery stages. For  $N = 500$ , the recommendation increases to approximately 7 pairs of mutual boundary nodes for each of about 3 or 4 recovery stages. Finally, when  $N = 1000$ , it is suggested to pre-store repair resources of approximately 10 pairs of mutual boundary nodes for each of about 4 recovery stages.

A comparison of Fig. 3(c) and Fig. 4 reveals that the status of  $P_\infty$  in the heterogeneous interdependent networks SF-ER and SF-RR is analogous to that observed in the isomorphic interdependent networks SF-SF. Significantly, while SF-ER and SF-RR exhibit lower robustness compared to SF-SF, it remains advisable to pre-store repair resources of approximately 10 pairs of mutual boundary nodes for each recovery stages when  $N = 1000$ . Certainly, given to the  $P_\infty$  in SF-ER and SF-RR is more sensitive to parameter  $p$ , the strategy suggests that pre-storing repair resources for about 4 or 5 recovery stages is better for SF-ER and SF-RR, which is marginally greater than that for SF-SF.

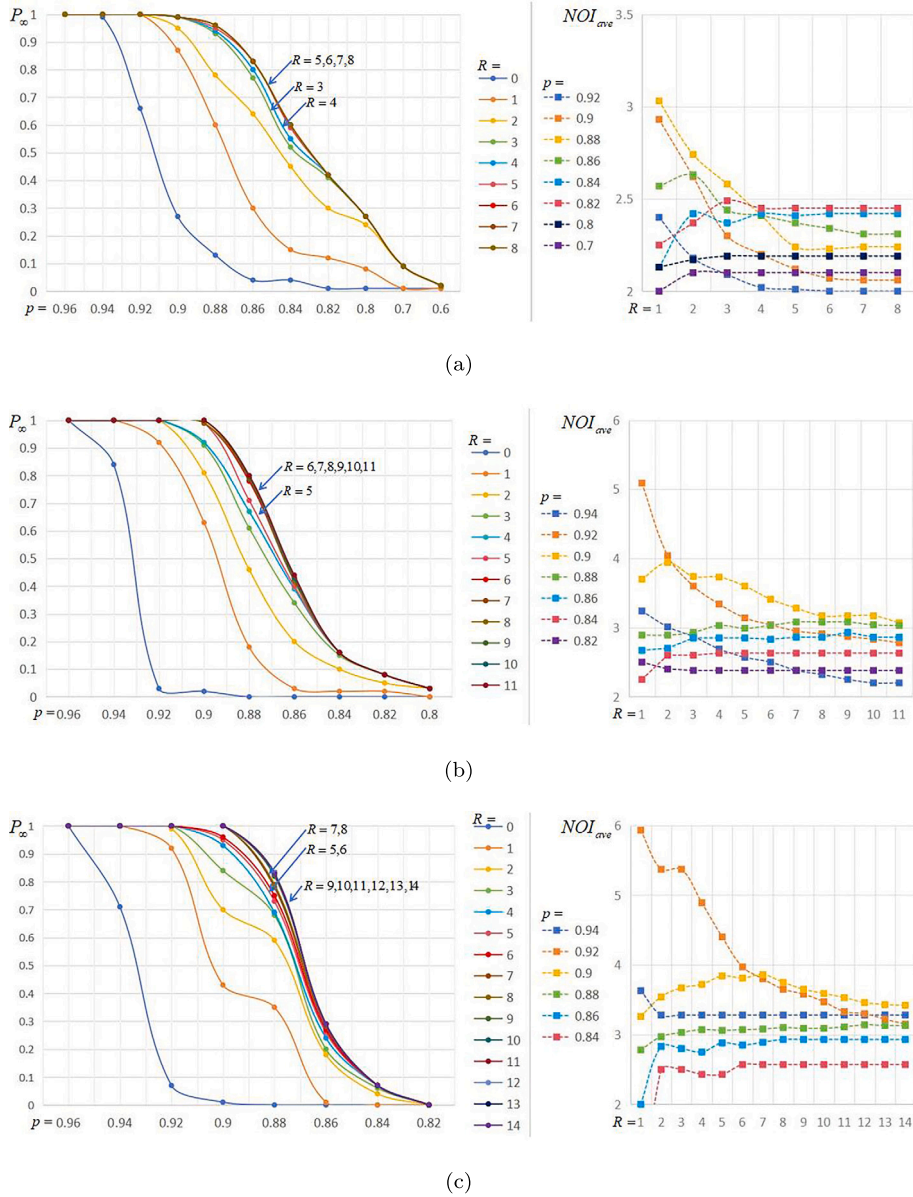
Overall, the strategy prioritizes the repair of mutual boundary nodes based solely on their degree, leading to low computational complexity. Besides, the recovery process does not require adjustments to the existing network structure. Hence, the recovery strategy will result in relatively low technology, labor, and material costs. Furthermore,  $P_\infty$  experiences a significant increase during a few recovery steps when  $R$  is around a relatively small value, followed by a period of negligible changes or slight increases as  $R$  becomes larger. For instance, as shown in Fig. 3(c), 4(a) and 4(b), the probability of network crashes can increase by up to approximately 90% if we pre-store repair resources for about 5% of the nodes. Therefore, the model provides a valuable reference for pre-storing repair resources in interdependent networks facing targeted attack scenarios.

## 3. Application of the strategy

The strategy of recovering a fixed number of mutual boundary nodes aims at providing references for pre-stored repair resources in the interdependent networks under targeted attacks. Significantly, infrastructure networks are intrinsically linked to critical livelihood concerns, thus it is essential to pre-store repair resources to mitigate the risk of network failures. Consequently, it is anticipated that our strategy will be applicable to interdependent networks related to infrastructure systems, including but not limited to interdependent networks of power grids and communication systems, urban transport systems and supply chain networks.

In the section, we should model the interdependent network of the power grid and the communication system. The power grid used is the power-685-bus [31], while the power grid is full of nodes in the power system, without giving its communication control network. Therefore, we established the communication system by SF network with  $\lambda = 3$  and  $N = 685$ . Significantly, a communication node is set for each power node, and then there is a one-to-one correspondence relationship and bidirectional coupling links are interconnected through the dependence between high-degree nodes in two network. According to the principle above, the interdependent network between the power grid and the communication network is established.

During the initial phase, the top  $F$  nodes of network A, i.e., the power grid, are subjected to failure in accordance with their descending degree order, where  $F = -N_{failure}$  = the number of survival nodes during the initial phase  $- N$ . The effectiveness of the strategy is evaluated in Fig. 5. The left of Fig. 5 illustrates a significant increase in  $P_\infty$  when  $R = 7$ , followed by minimal changes



**Fig. 3.** Probability of existence of the giant connected component,  $P_\infty$ , in interdependent network SF-SF characterized by  $\lambda = 3.0$  and  $k \approx 3.9$  is assessed for three different network sizes: (a)  $N = 100$ , (b)  $N = 500$  and (c)  $N = 1000$ . Simulations have been averaged over 100 network realizations.

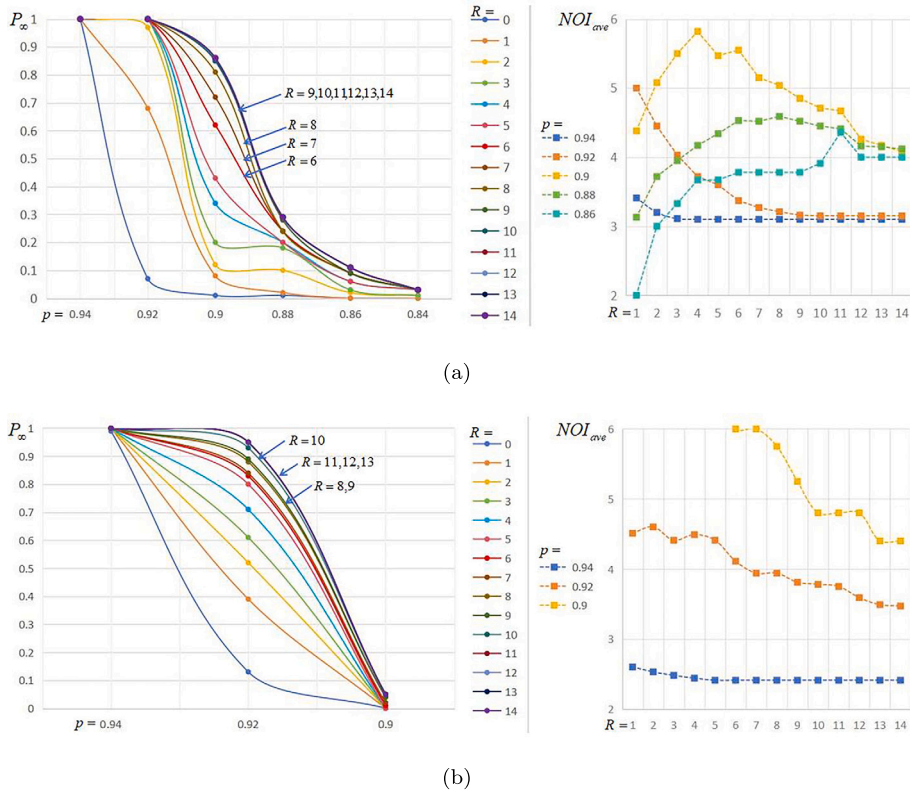
or only slight increases as  $R$  continues to rise. Additionally, the right side of the figure indicates that the recovery process requires only a few steps. Consequently, if pre-stored repair resources are in accordance with the recommendations suggesting the allocation of approximately 7 pairs of mutual boundary nodes for each of the 2 or 3 recovery stages, the adverse effects of targeted attacks can be substantially mitigated.

#### 4. Modification of the strategy

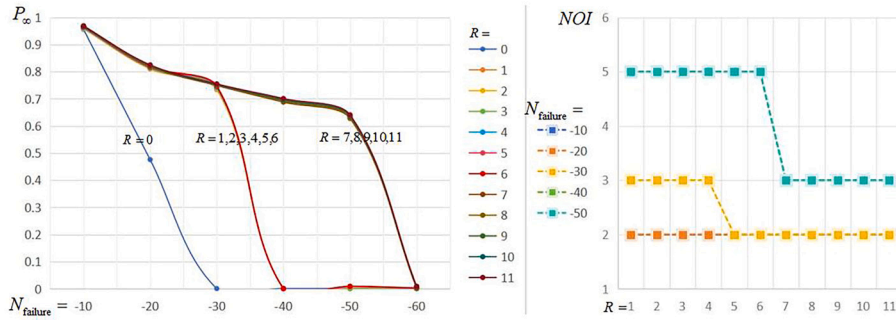
##### 4.1. Model

Pre-stored repair resources have the potential to effectively mitigate the breakdown of interdependent networks in the targeted-attack problems. However, the underutilization of these pre-stored resources may result in resource waste due to fewer mutual boundary nodes in some cases. Consequently, we propose a modification to the strategy of recovering a fixed number of mutual boundary nodes. Significantly, the original strategy predominantly adopts a passive approach in the recovery stage for addressing cascading failures, so the modification will incorporate more proactive measures to achieve improved recovery effectiveness.





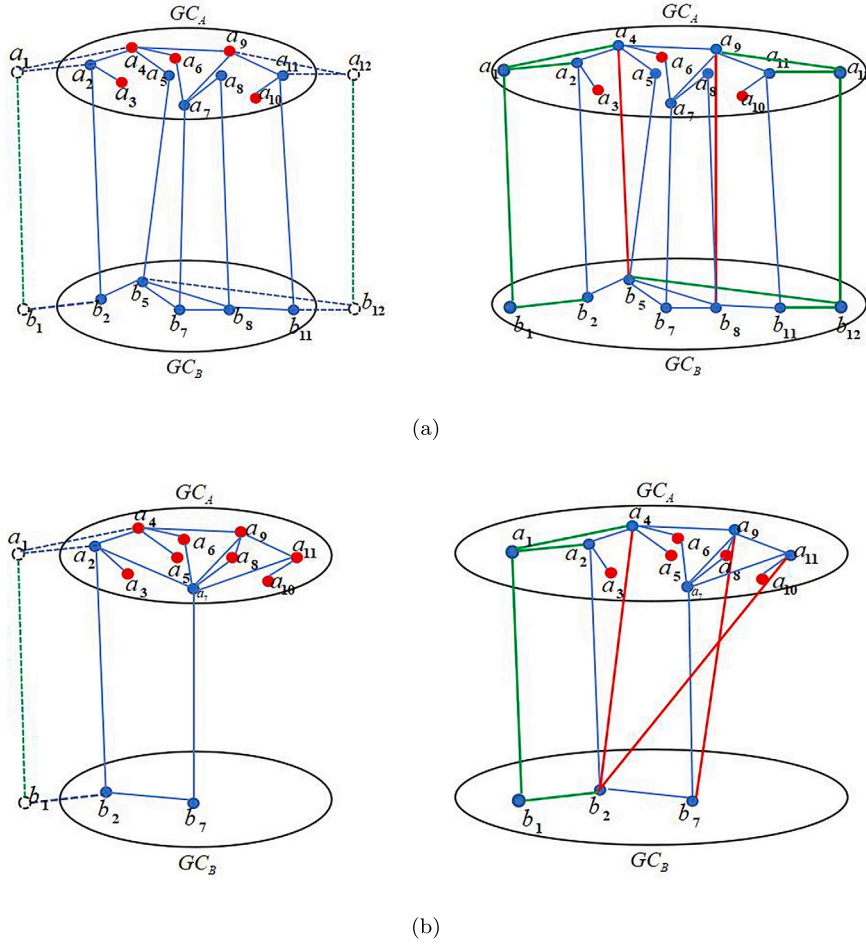
**Fig. 4.** Probability of existence of the giant connected component,  $P_\infty$ , is assessed for different kinds of interdependent network with  $N = 1000$ : (a) SF-ER where SF is characterized by  $\lambda = 3.0$  and  $\langle k \rangle \approx 3.9$  and ER is characterized by  $\langle k \rangle = 3$ , (b) SF-RR where SF is characterized by  $\lambda = 3.0$  and  $\langle k \rangle \approx 3.9$  and RR is characterized by  $\langle k \rangle = 3$ . Simulations have been averaged over 100 network realizations.



**Fig. 5.** Probability of existence of the giant connected component,  $P_\infty$ , is assessed for the power grid-SF, where power grid is the power-685-bus network characterized by  $\langle k \rangle = 3$  and SF is characterized by  $\lambda = 3.0$  and  $\langle k \rangle \approx 3.9$ .

In the section, we modified the original recovery strategy by combining recovering a fixed number of mutual boundary nodes and protecting important nodes. Similarly, the modification also consists of two phases: initial phase, cascading failure and recovery orderly alternating phase. The initial phase is consistent with the original strategy in section 2.1, and after denoting the time steps of the cascading failure and recovery orderly alternating phase by  $n = 1, 2, \dots$ , the rules governing the phase at any stage  $n$  are provided by:

- Stage  $n$  in A
  1. Functional nodes fail if they lose support from their counterpart nodes in B at stage  $n - 1$ .
  2. From the survivors, those nodes that belong to the  $GC_A$  remain functional while the other fail.
- Stage  $n$  in B
  3. Functional nodes become dysfunctional if they lose support from network A due to the cascade of failures at stage  $n$ .
  4. The remaining nodes fail if they do not belong to the  $GC_B$ .
- Recovering and protecting



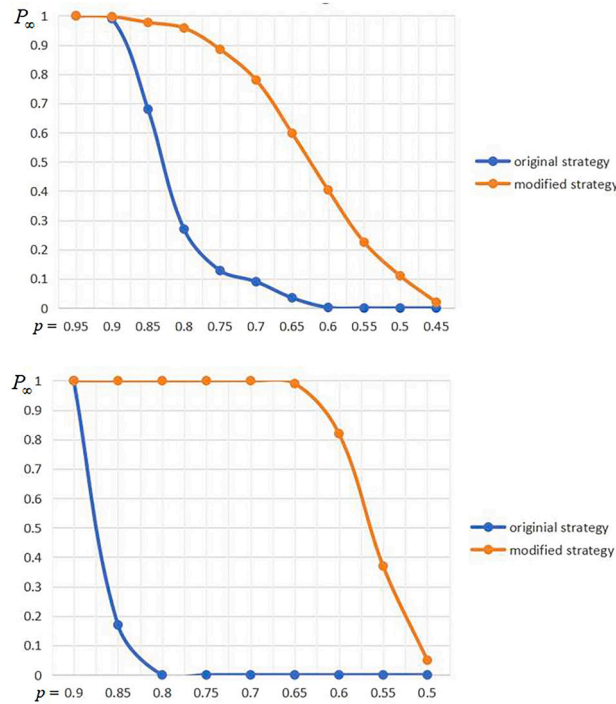
**Fig. 6.** It is appointed that the top 4 pair number of mutual boundary nodes are orderly restored in the descending order based on index  $I_{v_i}$ . Obviously, the number  $d$  of mutual boundary nodes both are less than 4. (a) When  $d = 2$ , by evaluating the degrees of nodes in  $S_{GC_A}$  and the degrees of nodes in the  $GC_B$ , we obtain two sets  $\{a_4, a_9, a_6, a_3, a_{10}\}$  and  $\{b_5, b_8, b_7, b_2, b_{11}, b_{12}, b_1\}$  in descending order of degrees, and then build the coupling links between  $a_4$  and  $b_5$ , as well as  $a_9$  and  $b_8$ . (b) When  $d = 1$ , by evaluating the degrees of nodes in  $S_{GC_A}$  and the degrees of nodes in the  $GC_B$ , we also obtain two sets  $\{a_4, a_9, a_{11}, a_6, a_8, a_5, a_{10}\}$  and  $\{b_2, b_7\}$ , and then build the coupling links between  $a_4$  and  $b_2$ , as well as  $a_9$  and  $b_7$ . Besides, owing to  $N(GC_B) < 3$ , we still build the coupling links between  $a_{11}$  and  $b_2$ .

5. It is appointed that the top  $R$  pair number of mutual boundary nodes are orderly restored in the descending order based on index  $I_{v_i}$ . All their connections with the  $GC_A$  and  $GC_B$  are reactivated and also the links between restored boundary nodes if they were connected before the failure. Actually, there would be the circumstance that the actual pair number  $d$  of mutual boundary nodes in stage  $n$  is smaller than the appointed value  $R$ . At this point, we can proceed to protect important nodes in the subset of  $GC_A$  (denoted as  $S_{GC_A}$ , where the node's dependent node in  $B$  has failed in stage  $n - 1$ ) by rebuilding the coupling links between top  $R - d$  nodes in  $S_{GC_A}$  and top  $R - d$  nodes in  $GC_B$  in the descending order of the degree. See Fig. 6(a). Significantly, if the number of nodes in  $GC_B$  (denoted as  $N(GC_B)$ ) is less than  $R - d$ , top  $N(GC_B)$  nodes in  $S_{GC_A}$  protected orderly build coupling links with nodes in  $GC_B$ , and then the following  $R - d - N(GC_B)$  nodes in  $S_{GC_A}$  also orderly build coupling links with nodes in  $GC_B$  and so forth. See Fig. 6(b).

6. Repeat 1-5. Once the current network reaches steady state(no more new failure nodes), the dynamic process of the recovering model ends.

#### 4.2. Simulation

In the simulations, we will investigate the effectiveness of the modified strategy in preventing system failures. Consistent with the simulations conducted in section 2.2, the two single-layer networks are interconnected through bidirectional coupling links, adhering to the principle that the  $i$ th node in one network is dependent on the  $i$ th node in the other network based on the descending order of degree,  $i = 1, 2, \dots, N$ . During the initial phase, the top  $(1 - p)N$  nodes of network A are subjected to failure in accordance with their descending degree order. The effectiveness of the model is evaluated by measuring  $P_\infty$ . Significantly,  $GC$  of the interdependent networks is considered to be present if the pairs number of nodes within the remaining  $GC$  exceeds two.



**Fig. 7.** Comparison of  $P_\infty$  with the original strategy and its modification of the interdependent networks SF-SF in the targeted-attack problems. (a) Both SF networks with  $N = 100$  are characterized by  $\lambda = 3$  and  $\langle k \rangle \approx 3.9$  and assume that pre-store repair resources about 4 pairs mutual boundary nodes for each of 3 recovery stages. (b) Both SF networks with  $N = 1000$  are characterized by  $\lambda = 3$  and  $\langle k \rangle \approx 3.9$  and assume that pre-store repair resources about 10 pairs mutual boundary nodes for each of 4 recovery stages.

According to the recommendations suggested by the original strategy in section 2.1, we assume to pre-store repair resources of about 4 pairs mutual boundary nodes for each of 3 recovery stages when  $N = 100$  and about 10 pairs mutual boundary nodes for each of 4 recovery stages when  $N = 1000$ . Fig. 7 presents a comparative analysis of  $P_\infty$  under both the original strategy and its modified version. Notably, the incorporation of protective measures for critical nodes results in a substantial increase in  $P_\infty$ , thereby significantly mitigating the effects of targeted attacks on the system.

In summary, the modified strategy combining recovering a fixed number of mutual boundary nodes and protecting important nodes offers several advantages. Particularly, it significantly enhances the effectiveness of preventing system failures and fully utilizes pre-stored resources to mitigate resource wastage.

## 5. Conclusions

We have developed the strategy of recovering a fixed number of mutual boundary nodes, which serves as a valuable reference for pre-stored repair resources in interdependent networks under targeted attacks. Notably, this strategy significantly enhances the system's resilience against complete failure. We anticipate that it can be effectively applied to interdependent networks associated with critical infrastructure, including, but not limited to, interdependent networks of power grids and communication systems, urban transport systems and supply chain networks.

However, it is important to note that the efficacy of the recovery strategy is contingent upon the presence of mutual boundary nodes; its utility diminishes in situations where such nodes are scarce at each recovery stage due to specific dependency relationships. Additionally, the underutilization of pre-stored repair resources may lead to resource waste, thereby the difficulty of implementing the strategy to address the problem and provide valuable references is to explore of supplementary measures tailored to different interdependent networks. The measure can refer to the modification of the original strategy presented in the paper.

In future research, we intend to address the aforementioned difficulties by investigating optimization problems. Furthermore, we aim to conduct theoretical analyses to refine and enhance the proposed strategy.

## CRedit authorship contribution statement

**Li Liang:** Writing – original draft, Methodology.



## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Data availability

Data associated with the study has not been deposited into a publicly available repository, and it will be made available on request.

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