

RESEARCH ARTICLE

Effect of Heterogeneous Investments on the Evolution of Cooperation in Spatial Public Goods Game

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Abstract

Understanding the emergence of cooperation in spatial public goods game remains a grand challenge across disciplines. In most previous studies, it is assumed that the investments of all the cooperators are identical, and often equal to 1. However, it is worth mentioning that players are diverse and heterogeneous when choosing actions in the rapidly developing modern society and researchers have shown more interest to the heterogeneity of players recently. For modeling the heterogeneous players without loss of generality, it is assumed in this work that the investment of a cooperator is a random variable with uniform distribution, the mean value of which is equal to 1. The results of extensive numerical simulations convincingly indicate that heterogeneous investments can promote cooperation. Specifically, a large value of the variance of the random variable can decrease the two critical values for the result of behavioral evolution effectively. Moreover, the larger the variance is, the better the promotion effect will be. In addition, this article has discussed the impact of heterogeneous investments when the coevolution of both strategy and investment is taken into account. Comparing the promotion effect of coevolution of strategy and investment with that of strategy imitation only, we can conclude that the coevolution of strategy and investment decreases the asymptotic fraction of cooperators by weakening the heterogeneity of investments, which further demonstrates that heterogeneous investments can promote cooperation in spatial public goods game.

Introduction

Cooperation and defection are two fundamental strategies for individuals confronted with every social dilemma [1–3]. According to Darwin’s theory of the origin of species, cooperation extinction is inevitable [4]. However, in the daily life, cooperation is abundantly observed in animal and human societies. One primary challenge in fields ranging from genetics and cell biology to evolutionary anthropology and behavioral economics is to explain the mechanism for these universal cooperative phenomena. Among the game theoretical approaches to investigating evolution of cooperation, one of the most popular frameworks to study the conflict between individuals is the so-called public goods game (PGG), in which several players decide whether or not to contribute

to the common pool simultaneously [5–7]. In the model of PGG, cooperators contribute a fixed share to the public whereas defectors do not. Subsequently, all contributions are added up and multiplied by an enhancement factor $r(r > 1)$, then the total welfare is equally divided among all players irrespective of their strategies. Consequently, players are faced with the temptation of being free-riders and rational players invest nothing which causes the “Tragedy of the Commons” [8–10].

Aiming at solving the above tragedy and investigating the reason why cooperators can survive in reality, several mechanisms have been proposed, such as reward [11], punishment [12–16], reputation [12, 17, 18], volunteering [9, 19], migration [20–22], social diversity [23–25] and so on. Besides, the enhancement factor evolving with time is identified as a useful mechanism for evolution of cooperative behavior in PGG [26]. Recently, the topology of the game has been intensively studied as a paradigm for illustrating how cooperative behavior evolves in spatial PGG [27–29], such as scale free networks [30–32], interdependent networks [33–39], and they have proved valuable for solving the tragedy to some extent. To summarize, in all the works above it is assumed that the investments of all the cooperators are identical, often equal to 1. However, it is worth mentioning that in the rapidly developing modern society, players are diverse and they often treat the same problem differently [40–42].

In the present work, the diversity of players is taken into consideration. With the aim of modeling the heterogeneous players in spatial PGG without loss of generality, it is assumed that the investment of a cooperator is a random variable with uniform distribution $[1-\sigma, 1+\sigma]$, the mean value of which is equal to 1. Moreover, we will discuss the impact of σ on evolution of cooperative behavior. In order to explore how heterogeneous investments affect the evolution of cooperative behavior, we compare the traditional update rule with the coevolution of strategy and investment, and by statistical analysis we further reveal that heterogeneous investments can promote cooperation in spatial PGG.

Methods

In this article, the public goods game is considered on a two dimensional square lattice of size $L \times L$ with periodic boundary conditions. At the first step, each player can choose a pure strategy cooperation or defection randomly. Suppose that cooperator x contributes Y_x to the public goods while defectors contribute nothing, here Y_x is a random variable, and $Y_x \sim U[1-\sigma, 1+\sigma]$, ($0 \leq \sigma \leq 1$). It is apparent that the mean of Y_x is 1 without loss of generality. Practically, when $\sigma = 0$, the model is identical with the traditional PGG. Suppose that the contribution of x is independent of that of other players.

For the sake of accelerating the simulations but without causing major modifications in the system’s dynamics, it is assumed that the score of player x is determined by a single PGG which involves the focal player and its four nearest neighbors [9]. The sum of all contributions in the group is multiplied by the factor r , and the resulting public goods are distributed among all the group members. Correspondingly, the payoff of player x is

$$P_x = r \frac{\sum_{y=1:N} Y_y}{N} - Y_x = \begin{cases} r \frac{\sum_{s_y=C} Y_y}{N} - Y_x & \text{if } s_x = C \\ r \frac{\sum_{s_y=C} Y_y}{N} & \text{if } s_x = D \end{cases} \quad (1)$$

where s_x denotes the strategy of player x , N denotes the group size, and in this article $N = 5$.

The Monte Carlo (MC) simulations start from a random initial strategy distribution, and the evolution procedure is subsequently controlled by a random sequential strategy update. In

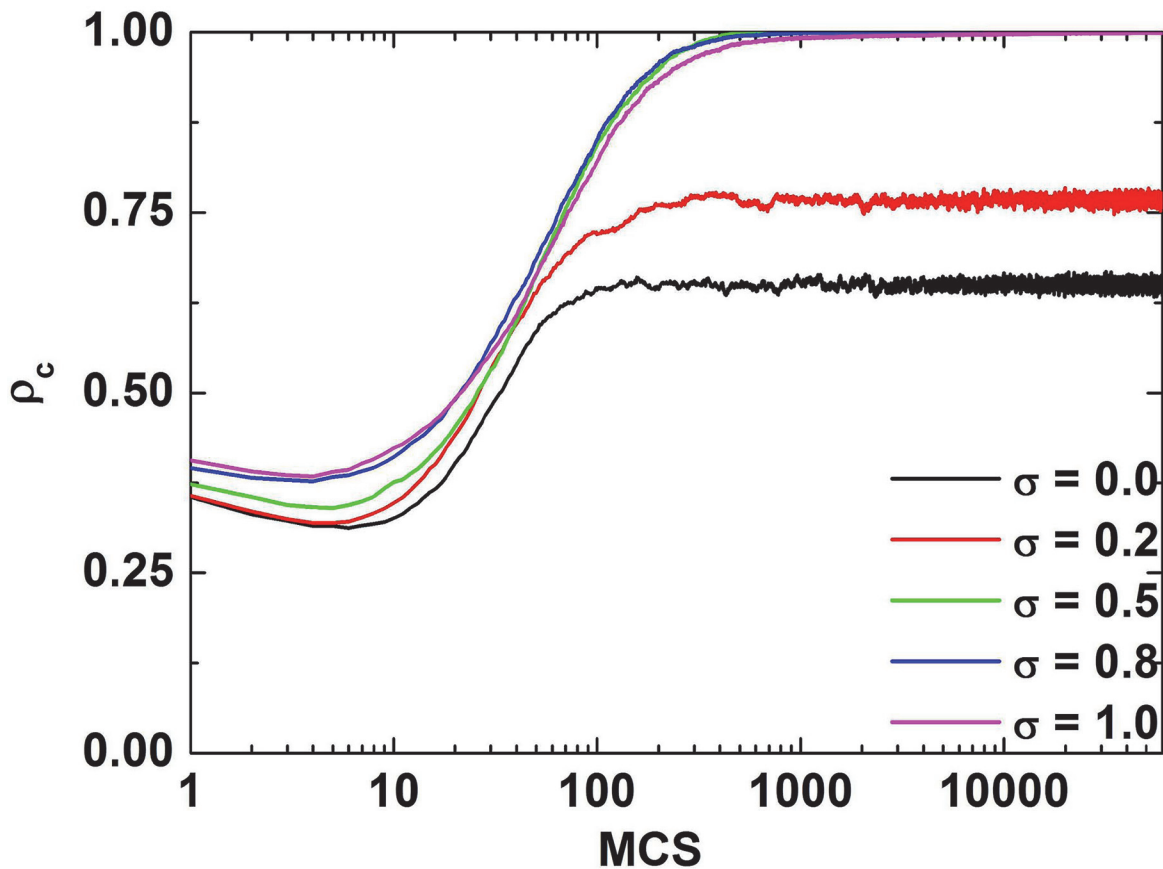


Fig 1. Time series of the fraction of cooperators for different values of σ when r equals to 5. The value of σ is equal to 0.0,0.2,0.5,0.8,1.0 respectively. While $\sigma = 0$, which is exactly the traditional case, the investment of all the cooperators is 1. With the increment of σ , the asymptotic fraction of cooperators will increase.

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detail, player x and one of its neighbors y are chosen randomly. Following the payoffs P_x, P_y , calculated as described above, player x imitates the strategy of player y with a probability

$$p(s_y \rightarrow s_x) = \frac{1}{1 + \exp[(P_x - P_y + \tau)/K]} \tag{2}$$

Where $\tau > 0$ denotes the cost of strategy change and K introduces some noise to allow for irrational decisions, and in this article $\tau = K = 0.1$ [9]. For the sake of comparison, the coevolution of strategy and investment, namely player x imitates also the investment of player y simultaneously, is considered. Furthermore, for the results presented in the next section, the square lattice used is sized $L = 200$ (In order to test whether the main results are robust against population size, this article had verified the results for different population sizes, and it is concluded that the main results are robust to the population size L). The asymptotic fraction of cooperators is determined by averaging over 5000 generations after a transient time of 55000 MCS.

Results

Firstly, it is instructive to examine the time series of the fraction of cooperators. Results presented in Fig. 1 are the time series of the fraction of cooperators for different values of σ when r equals to 5. The figure depicts that the fraction of cooperators decreases at the beginning of the

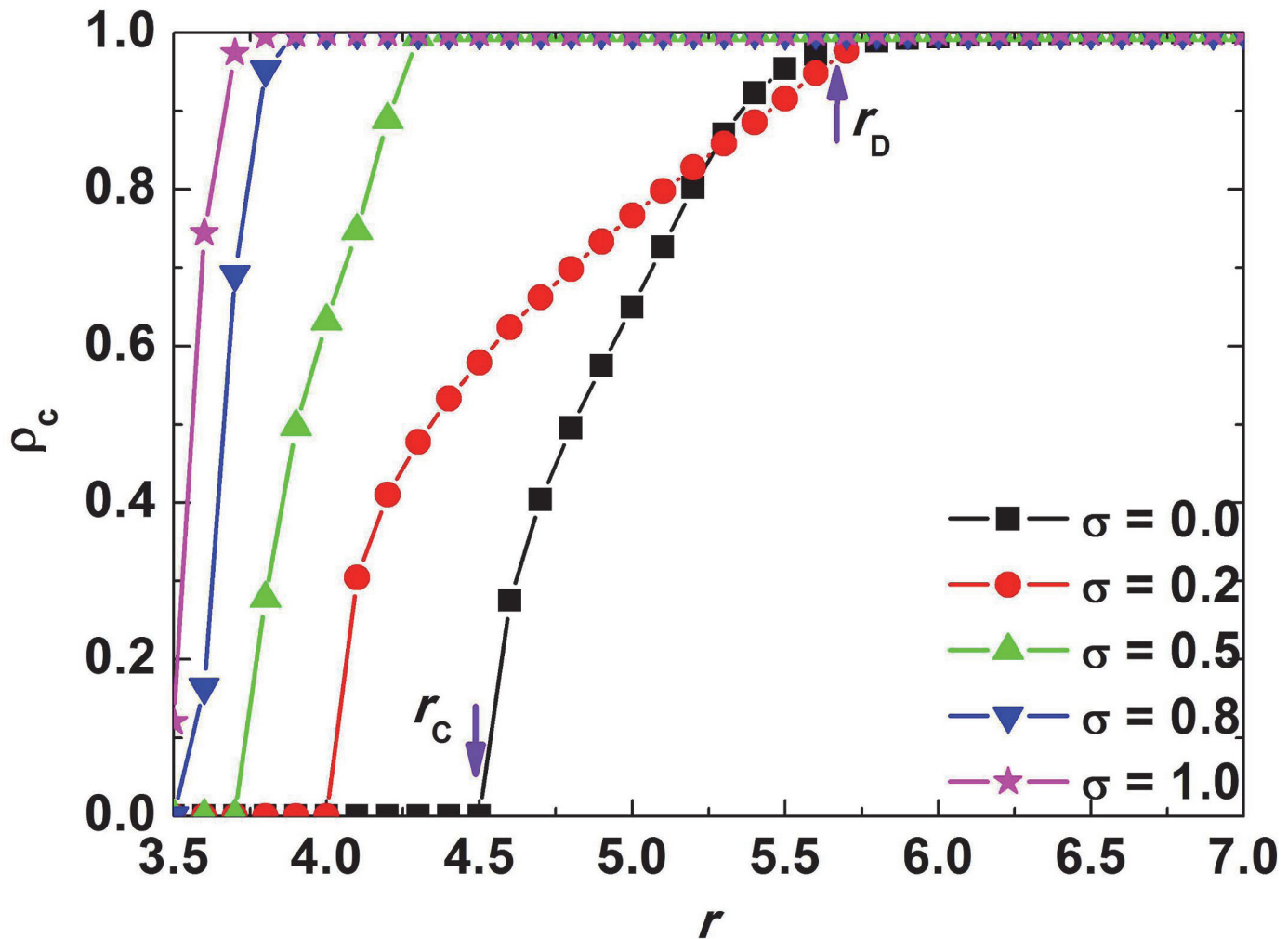


Fig 2. Relationship between asymptotic fraction of cooperators ρ_C and enhancement factor r corresponding to $\sigma = 0, 0.2, 0.5, 0.8, 1.0$ respectively. The curves in the figure show that the larger the value of σ is, the bigger the asymptotic fraction of cooperators ρ_C will be. In addition, larger value of σ can decrease the values of r_C and r_D effectively.

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evolution but increases to the value in the steady state later. The random distribution of the initial strategy gives rise to the decline of cooperation in the beginning, that is, the cooperators are dispersed and the defectors can invade the cooperators easily, which is corresponding to the initial phase in Fig. 1. As the evolution proceeds, the lived cooperators form clusters and the payoffs of them are enhanced, which restrain the invasion of the defectors. Meanwhile, the heterogeneous investments render the manifold payoff difference between cooperators and defectors, which in turn boosts the possibility for defectors to imitate cooperators.

While Fig. 1 shows the time series of fraction of cooperators, the relationship between asymptotic fraction of cooperators ρ_C and enhancement factor r when $\sigma = 0, 0.2, 0.5, 0.8, 1.0$ is presented in Fig. 2. It is worth to note that, the black curve in Fig. 2 corresponding to $\sigma = 0$ is in agreement with the results in reference [9]. The asymptotic fraction of cooperators ρ_C depending on the enhancement factor r reveals that ρ_C is enhanced with increasing of r for different values of σ . Meanwhile, the larger the value of σ is, the bigger the asymptotic fraction of cooperators ρ_C will be. Besides, a large value of σ will guarantee that the cooperators will survive when the enhancement factor r is small. In addition, for each curve in Fig. 2, there exist two thresholds

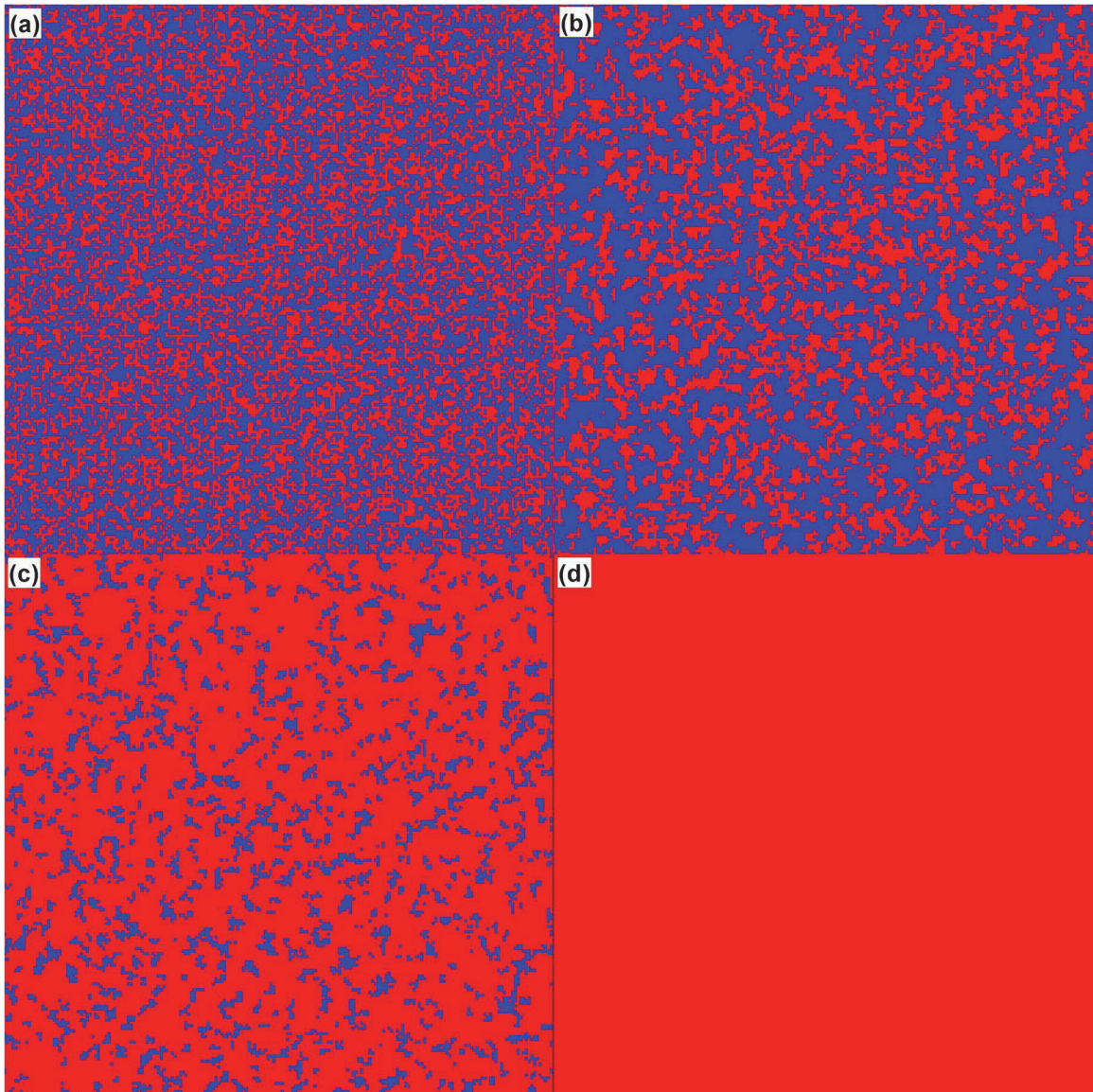


Fig 3. Typical snapshots of strategy distributions on the square lattice when $\sigma = 0.5$ and $r = 5$. Cooperators and defectors are colored red and blue, and the MCS of (a)-(d) is 1, 10, 100, 50000 respectively. The figure shows that the fraction of cooperators decreases at the beginning of the evolution, but as the evolution proceeds, the cooperators form into clusters to restrain the invasion of the defectors, and spread to the defectors reversely. At the end of the evolution all the players in the population hold the cooperation strategy.

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r_C and r_D . In other words, if $r < r_C$ the phase of the population after behavioral evolution is pure defectors, and if $r > r_D$ the phase of the population is pure cooperators, while in case of $r_C < r < r_D$ the population consist of not only cooperators but also defectors. Fig. 2 clearly demonstrates that larger value of σ can decrease the value of r_C and r_D effectively. Therefore, to sum up, when the investment of a cooperator is a random variable with uniform distribution $[1-\sigma, 1+\sigma]$, the asymptotic fraction of cooperators can be promoted. Furthermore, the larger the value of σ is, the better the promotion effect will be.

In order to depict the evolutionary process vividly, the typical snapshots are represented in Fig. 3 when $\sigma = 0.5$ and $r = 5$. From the figure it can be clearly concluded that the fraction of

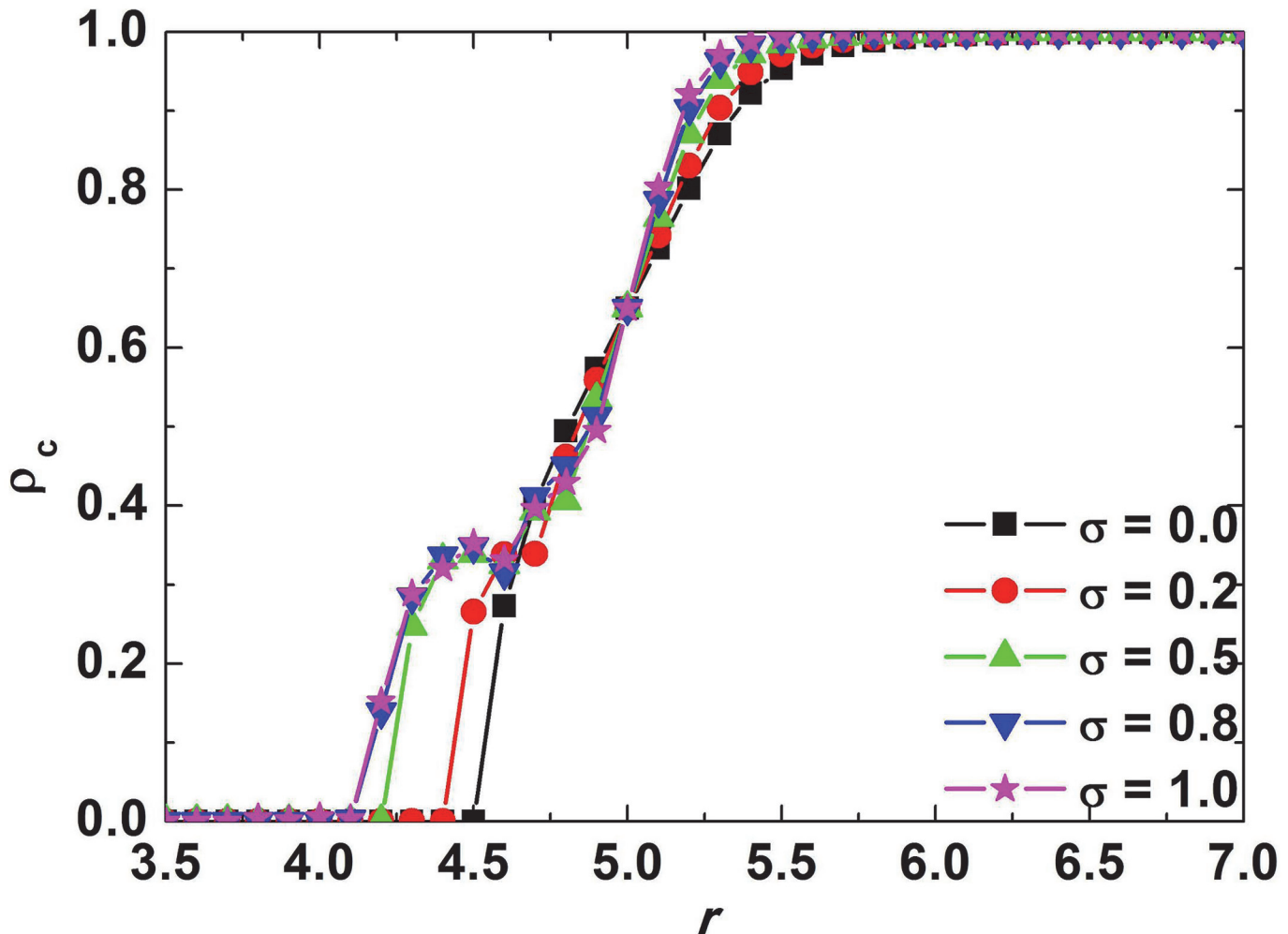


Fig 4. Relationship between asymptotic fraction of cooperators ρ_C and enhancement factor r corresponding to $\sigma = 0, 0.2, 0.5, 0.8, 1.0$ respectively when the coevolution of strategy and investment is taken into account. The curves show that if $\sigma > 0$, the thresholds r_C and r_D decrease compared to the case of $\sigma = 0$, which is exactly the traditional situation. This implies that cooperation can be promoted by introducing the coevolution of strategy and investment.

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cooperators decreases at the beginning of the evolution, but as the evolution proceeds, the cooperators form into clusters to restrain the invasion of the defectors and spread to the defectors reversely. At the end of the evolution all the players in the population hold the cooperation strategy.

Next, the relationship between fraction of cooperators and enhancement factor when the coevolution of strategy and investment is taken into consideration will be discussed. The curves in Fig. 4 show that if $\sigma > 0$, the thresholds r_C and r_D become smaller compared to the case of $\sigma = 0$, which is exactly the traditional situation. This implies that cooperation can be promoted through the coevolution of strategy and investment.

Comparing the curves in Figs. 2 and 4 leads to the interesting conclusion that the promotion effect of coevolution of strategy and investment is inferior to that of strategy imitation only. With the aim to explore the difference between them, we will discuss the investment distribution of cooperators in steady state. Fig. 5 shows the investment distribution in case of coevolution of strategy and investment (Right panel) and the distribution in case of strategy imitation

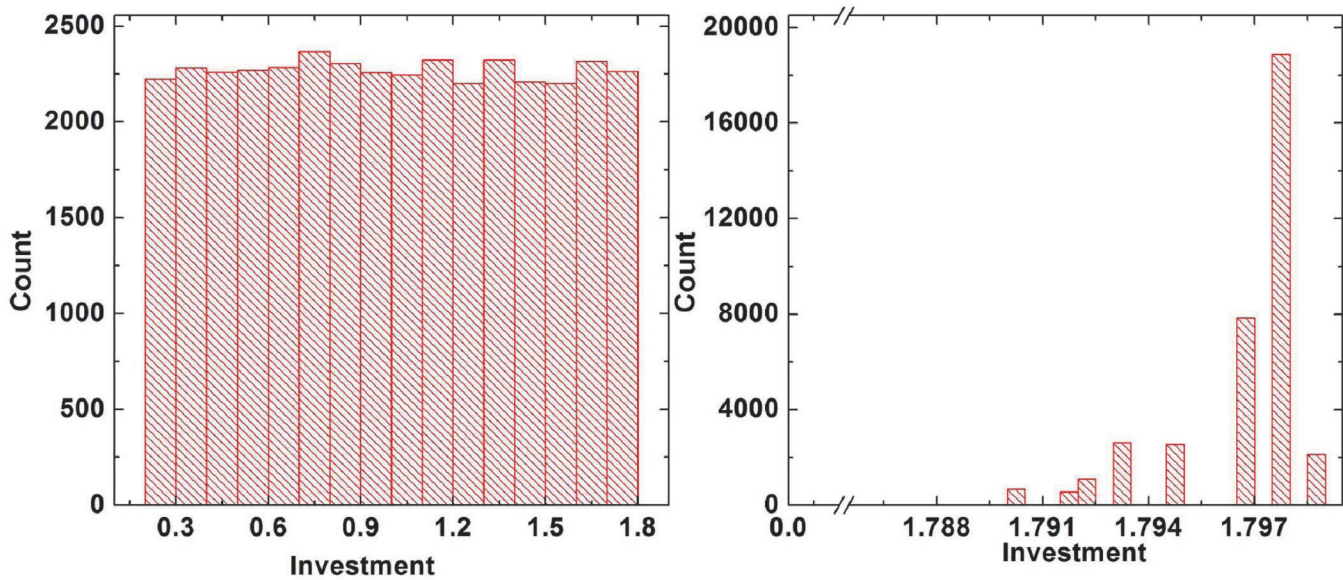


Fig 5. The steady distribution of investment corresponding to strategy evolution only (Left panel) and that corresponding to coevolution of strategy and investment (Right panel), when $\sigma = 0.8$, $r = 5.2$. From the figure, it can be found that when only strategy is updated, the distribution of the investment in steady state is still a uniform distribution [0.2,1.8], however, when the coevolution of strategy and investment is taken into account, the previous uniform distribution is severely distorted, and the steady investment satisfies a discrete distribution valued [1.786,1.790,1.792,1.793,1.795,1.797,1.798,1.799]. In conclusion, compared to the former case, as the heterogeneity of investment is weakened, the asymptotic fraction of cooperators ρ_C decreases in the case of coevolution.

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only (Left panel), and the parameters in the two situations are equal, namely $\sigma = 0.8$, $r = 5.2$. When only strategy evolves with time, the distribution of the investment in steady state is still a uniform distribution [0.2,1.8], which is the same with initial distribution. However, when the coevolution of both strategy and investment is considered, the distribution of investment after evolution comes to an end is severely distorted compared with the uniform distribution at the initial step, and the steady investment satisfies a discrete distribution valued [1.786,1.790,1.792,1.793,1.795,1.797,1.798,1.799]. Therefore, the heterogeneity of investment is weakened in the case of coevolution, leading to the decrement of asymptotic fraction of cooperators ρ_C . In the same way, through statistical analysis it can be concluded that for the same enhancement r , the smaller the value of σ is, the more severe the distortion from previous uniform distribution will be. This implies that the coevolution of strategy and investment decreases the asymptotic fraction of cooperators ρ_C by weakening the heterogeneity of investment once again.

Conclusions

To summarize, this article further explores the impact of heterogeneous investments on the evolution of cooperative behavior in spatial PGG. Above all, the relationship between asymptotic fraction of cooperators ρ_C and enhancement factor r when σ varies has been presented, and it can be concluded that the larger value of σ has the capacity to decrease the values of r_C and r_D effectively, and that the larger the value of σ is, the better the promotion effect will be. In addition, this article has also discussed the influence of heterogeneous investments when the coevolution of strategy and investment is taken into consideration. Comparing the promotion effect of coevolution of strategy and investment with that of strategy imitation only, the conclusion can be naturally drawn that the coevolution of strategy and investment decreases the asymptotic fraction of cooperators ρ_C by weakening the heterogeneity of investment. Therefore,

the results above further demonstrate that heterogeneous investments are capable of promoting cooperation in spatial PGG.

Supporting Information

S1 Dataset. The dataset of [Fig. 1](#).
(XLSX)

S2 Dataset. The dataset of [Fig. 2](#).
(XLSX)

S3 Dataset. The dataset of [Fig. 3](#).
(XLSX)

S4 Dataset. The dataset of [Fig. 4](#).
(XLSX)

S5 Dataset. The dataset of [Fig. 5](#).
(XLSX)

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Author Contributions

Conceived and designed the experiments: KH TW YC XZ. Performed the experiments: KH TW YC XZ. Analyzed the data: KH TW YC XZ. Contributed reagents/materials/analysis tools: KH TW YC XZ. Wrote the paper: KH TW YC XZ.

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