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Research article

Particle filtering supported probability density estimation of mobility patterns

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A R T I C L E I N F O

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A B S T R A C T

This paper presents a methodology that aims to enhance the accuracy of probability density estimation in mobility pattern analysis by integrating prior knowledge of system dynamics and contextual information into the particle filter algorithm. The quality of the data used for density estimation is often inadequate due to measurement noise, which significantly influences the distribution of the measurement data. Thus, it is crucial to augment the information content of the input data by incorporating additional sources of information beyond the measured position data. These other sources can include the dynamic model of movement and the spatial model of the environment, which influences motion patterns. To effectively combine the information provided by positional measurements with system and environment models, the particle filter algorithm is employed, which generates discrete probability distributions. By subjecting these discrete distributions to exploratory techniques, it becomes possible to extract more certain information compared to using raw measurement data alone. Consequently, this study proposes a methodology in which probability density estimation is not solely based on raw positional data but rather on probability-weighted samples generated through the particle filter. This approach yields more compact and precise modeling distributions. Specifically, the method is applied to process position measurement data using a nonparametric density estimator known as kernel density estimation. The proposed methodology is thoroughly tested and validated using information-theoretic and probability metrics. The applicability of the methodology is demonstrated through a practical example of mobility pattern analysis based on forklift data in a warehouse environment.

1. Introduction

Mobility pattern analysis can be a complex problem, particularly when dealing with uncertain and noisy position data. Consequently, various methods and techniques must be employed to address these challenges [1]. This paper addresses the question of how to increase the reliability of exploring the probability distribution patterns of location data.

Raw position data presents inherent challenges when attempting to draw reliable conclusions. On the one hand, data sparsity poses a problem [2], since observations are less likely to cover events that occur infrequently in space and time. On the other hand,

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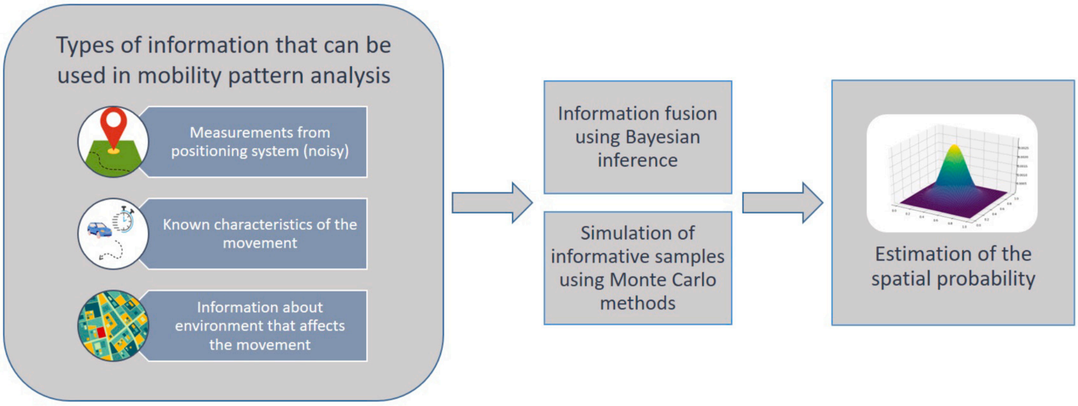

Fig. 1. Schematic diagram of a proposed framework to explore the spatial probability of occurrence. The position measurements are combined with other available background knowledge and samples representative of the mobility pattern are simulated. More reliable information can be extracted from the resulting dataset.

insufficient data quality introduces difficulties due to the reliance on sensor measurements for position observations. Sensors can introduce data quality issues such as noise, offset, and outliers [3].

Building a density model based on limited data can lead to the incorporation of incorrect information, distorting the model, and causing a discrepancy between the approximation and the true underlying distribution. However, since positional measurements from a dynamic system form a coherent time series, knowledge of their characteristics can provide additional information that aids in the density estimation process. Although we may not have complete knowledge about the underlying movement process from which position data originate, we often have assumptions about its basic kinematics or dynamics. In many cases, this knowledge is available in the form of mathematical models. For example, vehicle models are used to estimate the state of a car [4]. Additionally, awareness of related physical constraints, such as considering joint angle limits in human motion tracking, can be valuable [5]. The external environment in which movement occurs also influences the evolution of the process, and knowledge of these factors can be beneficial during mobility analysis. Using such prior knowledge is common in anomaly detection methods, such as considering the maximum speed a vehicle can achieve in a given environment [6] or exclusion zones that restrict vessel entry [7]. By combining raw position measurements with the aforementioned background knowledge, it is possible to reduce uncertainty and develop more informative models.

In our work, we propose to incorporate the dynamic model of the system and other contextual information into the probability estimation for the analysis of mobility patterns, in addition to using raw position data. For this purpose, a Bayesian state estimation is employed, which recursively applies Bayesian inference to combine measurements of how the observed process evolves with prior knowledge about the process's characteristics. By combining Monte Carlo methods [8] with Bayesian estimation, the number of samples can be increased, resulting in a more valuable input for density estimation. Fig. 1 shows a framework for exploring spatial probability using available information and the above techniques.

This study was motivated by the need to analyze the measurements obtained from indoor positioning systems. These measurements are utilized to calculate the occurrence probability of objects in different locations to identify the pathways of tracked objects and characterize their typical places of occurrence and mobility patterns. However, the data collected from these systems are often affected by significant noise, making it difficult to extract valuable information. To address this issue, the paper proposes a novel method that takes advantage of the PF algorithm to generate mobility data with higher information content. By incorporating background knowledge on the moving process and environmental constraints, the method improves the accuracy of probability models and reduces uncertainty.

The key contributions of this work can be summarized as follows.

- Introduction of a methodology for processing mobility data, specifically addressing the challenges posed by noisy measurements.
- Application of the PF algorithm to generate probability-weighted position measurement samples that contain richer information. These samples serve as the basis for more accurate estimation of the probability density.
- Incorporation of spatial environment information into the update formula of the PF, allowing the model to account for environmental constraints during the estimation process.
- Adoption of Kernel Density Estimation (KDE) to model the probability distribution of mobility data based on the information-enriched dataset rather than relying solely on raw position data or expected position estimations.
- Demonstration of the applicability of the proposed methodology through a practical example involving the tracking of forklifts using an indoor positioning system in a warehouse setting.
- Evaluation and validation of the effectiveness of the methodology using probability and information-theoretic measures.

The article is structured as follows. Section 2 presents the relevant studies. Section 3 provides a detailed explanation of the proposed methodology. Section 3.1 presents a formal description of the problem, while Section 3.2 discusses the representation of prior knowledge through mathematical models. Subsection 3.3 describes the application of the PF algorithm to generate informative measurement samples, and Subsection 3.4 explains how contextual information is integrated into the estimation process. Section 3.5 focuses on the Kernel Density Estimation technique. Finally, Section 3.6 presents the metrics used to evaluate the proposed methodology.

Section 4 presents the application of the methodology. Subsection 4.1 illustrates the industrial application potential of the methodology through an example in the field of intralogistics, highlighting its capacity to solve real-world problems. Section 4.2 describes the studies conducted that measure the effectiveness of the method and presents the results obtained. The article concludes with a summary of the findings and their implications.

2. Works related to probability model-based processing of location data

Mobility pattern analysis involves different tasks, such as characterizing specific locations or trajectories [9,10]. In trajectory modeling, transitions between sets of different locations are characterized. If semantic information is available, supervised techniques such as graph neural networks [11] or SVM [12] can be used. In the context-independent case, unsupervised techniques such as clustering [13] or frequent item mining [14] approaches can be used for this purpose. In location characterization, one of the tasks is to characterize locations in terms of user activity using semantic information, e.g. prediction of the number of visitors entering a shop [15]. In many cases, only raw, noisy position data are available, in which case the task is to identify meaningful locations from them. Clustering techniques [16] are applied for grouping location data with similar characteristics to identifying places where data is concentrated. Probability density estimation methods, such as kernel density estimation (KDE) [17] and Gaussian mixture modeling (GMM) [18], can be utilized to infer the unknown spatial probability density function based on position data distributed according to this function, and to identify places with high occurrence probability of objects.

Mobility patterns encompass the movement of objects on various spatial scales and over time. Tracked objects generally transition between different locations and spend varying durations at each location. As a result, position data tend to concentrate to different extents at specific spatial points, forming regions with varying levels of density. The use of single unimodal parametric distributions, such as the normal distribution, does not capture the multimodality present in the position data distribution and tends to oversmooth local regions [19]. A nonparametric estimator, the kernel density estimate (KDE) is suitable for handling data locality [20]. KDE is commonly used for the estimation of spatial probability density in the detection of urban mobility patterns [21–23] and location prediction [24]. In modeling spatial density based on social network location data from people, adaptive KDE outperformed traditional KDE as well as single or mixture Gaussian density modeling [2]. On the other hand, when estimating the probability density function of position and velocity from vessel traffic data, both adaptive KDE and GMM yielded comparable performance [25]. It should be noted that there is no universally optimal model applicable in all cases, as evidenced by a study in which different techniques have been applied to model the spatial probability of animals with varying effectiveness. Depending on the spatial habits of the animals, KDE was found to be superior in some cases, while simple Gaussian or GMM distributions were more suitable in others [26].

Density estimation techniques are effective in addressing the issue of data sparsity. However, in scenarios involving low-probability events, simulation of samples becomes necessary. Monte Carlo methods offer a solution by allowing the generation of random samples from a specific probability distribution and the estimation of that distribution based on the generated samples [27]. Importance sampling (IS) is a Monte Carlo method designed to sample from low-probability regions of distributions that are typically difficult to sample from. Instead of directly sampling from the target distribution, IS samples from a “proposal distribution” that has sufficient overlap with the target distribution but higher probability density in the region of interest. The samples obtained through IS are then weighted by a factor that depends on the ratio of the target distribution to the proposal distribution at each point, preventing an overrepresentation of the target distribution in that region [28]. In a study that focused on estimating the probability distribution of vehicle-pedestrian interactions at intersections, a truncated GMM was used [29]. Given that events with adverse outcomes have a lower probability and limited data are available, IS was used to simulate data for high-risk situations. Similarly, when estimating the probability of airplane collisions with limited available data, IS was applied to simulate trajectories that lead to conflicts [30]. The task of estimating the “trajectories” of successive states in dynamic systems poses a high-dimensional problem where the basic IS approach becomes inefficient. In such cases, it is more reasonable to estimate the state distributions at each instant of time recursively using IS [31]. Sequential importance sampling (SIS) is a technique that approximates the posterior density of states at each time step by applying IS and assuming the Markov property for the dependency between consecutive states [32]. It is advantageous to choose the state transition density as the proposal distribution in SIS, not only for its simplicity and generality but also because it allows the utilization of a dynamic model of the process for sample generation. The obtained samples are assigned importance weights based on their likelihood in the measurement density, thus incorporating Bayesian inference. Bayesian inference-based state estimation techniques [33] are the most elaborated and widely employed techniques for integrating raw sensor measurement into the dynamic model [34,35], including the classic Kalman filter, whose principle of operation can also be derived from Bayesian state estimation [36]. In addition, fuzzy logic and neural networks can also be used for information fusion [37,38]. If several different models are available to describe the dynamics of the movement, then ensemble methods enable the fusion of information from the predictions of different models [39].

This approach enables the generation of informative samples by integrating knowledge of the dynamic process and observations. A modified version of SIS is sequential importance resampling, commonly known as particle filter (PF) [40]. PF incorporates a resampling procedure within the recursion to address issues arising from the insufficient overlap between the state transition and measurement densities. PF is widely employed for state estimation, particularly in handling complex, non-linear, and non-Gaussian systems. Localization is a common application of PF in state estimation [41,42].

PF also provides a straightforward approach to taking physical objects in the environment into account when performing localization. By leveraging map information and state transitions, the weights of particles or samples can be assigned values that reflect the probability of the corresponding displacements in the physical environment. For example, the weights of the position particles can be influenced by factors such as proximity to objects [43] or the sequence of road segments [44]. Furthermore, the weights can be set to zero or close to zero in cases where certain displacements are not feasible, such as crossing a wall [45–47]. Incorporating knowledge about both the dynamics of the movement process and the environmental factors that affect movement enables a more accurate localization. However, the utilization of this knowledge goes beyond position estimation in our case; it extends to the generation of additional data. By retaining the particles of the filter, a multitude of information-enriched samples can be generated. Instead of relying solely on raw position measurements or the expected values of these particles, probability density estimation is performed directly on these samples. Consequently, the resulting approximation distributions exhibit reduced uncertainty.

3. Methodology of probability density estimation on information-enriched data

This research presents a methodology for exploring the underlying unknown probability distribution of position data by incorporating prior knowledge into density estimation, in addition to considering noisy measurements. The PF algorithm is used to generate mobility data with higher content of information leveraging background knowledge. The PF produces multiple samples, called "particles," at each time instant, with each particle weighted on the basis of the probability of representing a possible position. These weights provide additional information that facilitates the construction of more accurate probability density models.

3.1. Problem formulation

The proposed method is implemented in a system in which motion sequences of tracked objects are analyzed, focusing specifically on the measured position data. In the estimation of probability density, observed samples from a random variable of d_y $\mathbf{y} = [y_1, ..., y_{d_y}]$ are typically utilized to estimate the probability density function $\hat{p}(\mathbf{y})$, which represents the likelihood of various outcomes for the random variable $\mathbf{y}$. Taking into account the dynamic nature of the system, the measurement sequences can be interpreted as time series. Let S denote the number of different measurement sequences, where s represents a coherent sequence of movements of a tracked object, and L_s represents the length of the sequence s^{th}. Therefore, the raw observations consist of $S \times L_s$ data points.

In the presented method, density estimation is performed using not only raw measurements but also incorporating prior knowledge about the dynamic characteristics of the system and available contextual information. This fusion of information is achieved through the application of the particle filter algorithm, which generates a set of simulated measurement samples $\{\mathbf{y}_i^{(t)} ... \mathbf{y}_{N_s}^{(t)}\}_s$. These samples are weighted according to the probabilities that they represent a measurement that aligns with the true state $\{w_i^{(t)} ... w_{N_s}^{(t)}\}_s$. The probability-weighted samples are then used to estimate the probability density $\hat{p}(\mathbf{y})$ of the distribution of $\mathbf{y}$, by using Kernel Density Estimation (KDE). The resulting distribution exhibits reduced uncertainty compared to those based solely on raw measurement sequences, and its properties are evaluated using probabilistic and information-theoretic measures. The methodology is summarized in the flow diagram depicted in Fig. 2.

3.2. Modeling prior knowledge about system dynamics

Prior knowledge about the system dynamics is incorporated through a state-space model. Let $\mathbf{x}^{(t)}$ represent a vector of dimensions d_x that denotes the state of the system at time t. The dynamics of the system is described by the state transition equation (or process) (Equation (1)).

$$\mathbf{x}^{(t)} = f(\mathbf{x}^{(t-1)}, \mathbf{u}^{(t-1)}, \mu^{(t-1)}) \tag{1}$$

Here, $\mathbf{u}^{(t-1)}$ represents the d_u-dimensional control input vector, $\mu^{(t-1)}$ denotes the process noise vector with appropriate dimensions, and f is the function that describes the system's evolution over time. In general cases, the state of the system is not directly observable but is observed instead as a combination of uncertain measurements [48]. The measurement (or output) equation (Equation (2)) establishes a relationship between state information and noisy measurements.

$$\mathbf{y}^{(t)} = h(\mathbf{x}^{(t)}, \nu^{(t)}) \tag{2}$$

Here, $\mathbf{y}^{(t)}$ is the d_y dimensional measurement vector at time t, and $\nu^{(t)}$ represents the measurement noise vector with appropriate dimensions in that time instant. The function h describes how a measurement can be predicted given the state.

For our specific example of mobility, the internal states of the system are represented by the two-dimensional coordinates and velocities $\mathbf{x} = [x_1, x_2, x_3, x_4]^T$.

The state-transition model is based on kinematic equations of displacement and velocity, with the velocity being perturbed by noise. Additionally, we assume some inertia, which is represented as the control input $\mathbf{u}^{(t)} = [u_1, u_2, u_3, u_4]^T$ in the system. This control input has a regularization effect similar to the Ornstein-Uhlenbeck process, where a mean-reverting effect pulls the process variables back to an asymptotic value with a force proportional to the deviation from it. The expected velocity components $\bar{\mathbf{x}}^{(t)} = [\bar{x}_3^{(t)}, \bar{x}_4^{(t)}]^T$ are derived from the original position measurements smoothed with a moving average. Let $\mathbf{e} = [\bar{x}_3^{(t)}, \bar{x}_4^{(t)}]^T - [x_3^{(t)}, x_4^{(t)}]^T$ denote the deviation of the estimated velocity components from the expected velocity components. The time interval between two consecutive measurements is denoted by Δt.

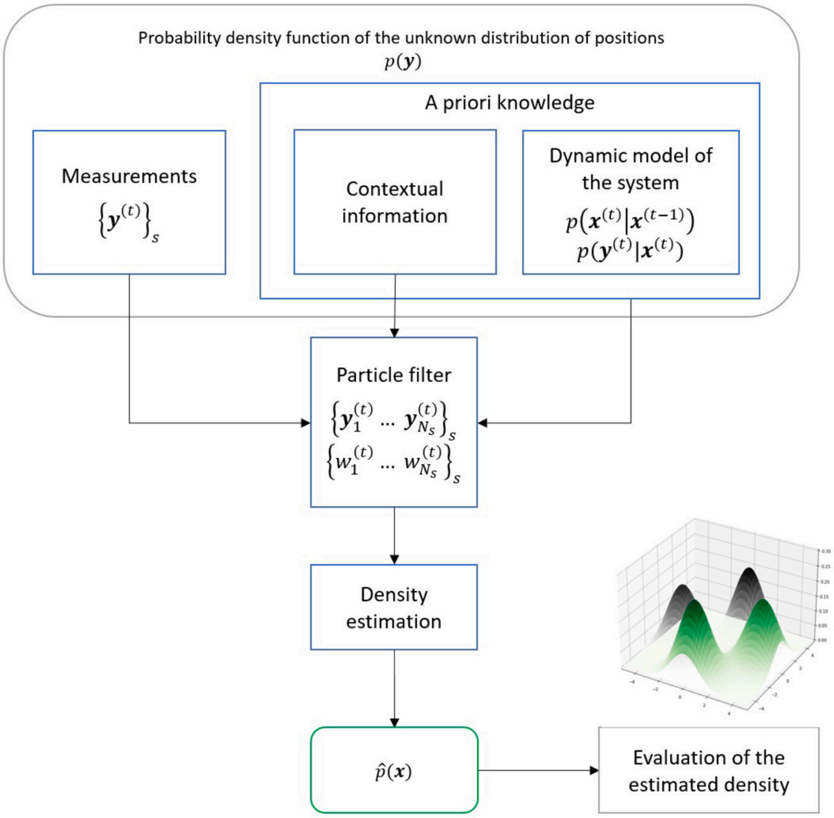

Fig. 2. Flowchart illustrating the proposed methodology. In the traditional case, density estimation relies solely on measurement sequences $\mathbf{y}^{(t)}{}_s$. However, in the proposed method, measurements are combined with prior knowledge about the system's dynamics, represented by the state-space model, and contextual information to improve the accuracy of the probability density estimation.

Let the proportional gain be $K = \Theta\Delta t$, where the parameter Θ adjusts the strength of the regulatory effect. Θ takes a value between 0 and 1, where a value of 0 means that no inertial effect applies, while a value of 1 means that the velocity of the object is the same as the expected velocity if $\Delta t = 1$. Its appropriate value always depends on the dynamics of the given movement and can be easily estimated using the trial-and-error method. This schema can be interpreted as proportional feedback control for velocity components: $[u_3^{(t)}, u_4^{(t)}] = K\mathbf{e}$. The control input for the position components is set to zero.

Hence, the state transition model (Equation (1)) can be described as follows:

$$\mathbf{x}^{(t)} = f(\mathbf{x}^{(t-1)}, \mathbf{u}^{(t-1)}, \mu^{(t-1)}) =$$
$$\begin{bmatrix} x_1^{(t)} \\ x_2^{(t)} \\ x_3^{(t)} \\ x_4^{(t)} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1^{(t-1)} \\ x_2^{(t-1)} \\ x_3^{(t-1)} \\ x_4^{(t-1)} \end{bmatrix} + \Theta\Delta t \begin{bmatrix} 0 \\ 0 \\ (\bar{x}_3^{(t-1)} - x_3^{(t-1)}) \\ (\bar{x}_4^{(t-1)} - x_4^{(t-1)}) \end{bmatrix} + \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{bmatrix} \tag{3}$$

The measurement vector is a subvector of the state vector. Therefore, the output equation (Equation (2)) can be described as:

$$\mathbf{y}^{(t)} = h(\mathbf{x}^{(t)}, \nu^{(t)}) =$$
$$\begin{bmatrix} y_1^{(t)} \\ y_2^{(t)} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1^{(t)} \\ x_2^{(t)} \\ x_3^{(t)} \\ x_4^{(t)} \end{bmatrix} + \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{bmatrix} \tag{4}$$

3.3. Data generation by particle filter

The state transition model can be represented as a probability distribution (Equation (5)). This representation signifies that the state $\mathbf{x}^{(t)}$ is drawn from a distribution of possible current states conditioned on the previous state $\mathbf{x}^{(t-1)}$ and the control input $\mathbf{u}^{(t-1)}$.

$$\mathbf{x}^{(t)} \sim p(\mathbf{x}^{(t)}|\mathbf{x}^{(t-1)}, \mathbf{u}^{(t-1)}) \tag{5}$$

Similarly, due to the uncertainty associated with the measurement model, it can also be represented as a probability distribution (Equation (6)). This representation indicates that the measurement at time t is a sample of the distribution of all possible measurements given the state $\mathbf{x}^{(t)}$, with a specific probability.

$$\mathbf{y}^{(t)} \sim p(\mathbf{y}^{(t)}|\mathbf{x}^{(t)}) \tag{6}$$

The question arises as to how we can use these probability density functions to estimate the distribution of $\mathbf{y}$ and its density function $\hat{p}(\mathbf{y})$. The solution is provided by Bayesian state estimation [33], which recursively applies Bayesian inference to combine the measurements with the probability distributions that describe the process's temporal evolution and the observable information about it.

One technique for implementing this recursive estimation is PF [49]. The PF approximates the probability distributions using a crowd of elementary particles (samples), enabling the handling of nonnormal and arbitrarily shaped distributions. As a result of the filter, the discrete probabilistic distribution of the tracked object's state at each time instant in any given sequence is obtained as a set of N_s simulated samples along with their corresponding weights $(\mathbf{x}_i^{(t)}, w_i^{(t)}{}_s)$. The associated weights $w_i^{(t)}$ represent the probability that a particle represents a real state or a possible measurement since they are proportional to the probability of measurement. These weights are recursively obtained using the *importance sampling* technique. The state transition equation (Equation (5)) is chosen as the proposal distribution from which the particles are sampled, and the weights are updated using the following formula, where the new weight is expressed as the weight from the previous instant multiplied by the probability of measurement (Equation (6)):

$$\tilde{w}_i^{(t)} \propto w_i^{(t-1)} p(\mathbf{y}^{(t)}|\mathbf{x}_i^{(t)}) \tag{7}$$

These weights need to be normalized:

$$w_i^{(t)} = \frac{\tilde{w}_i^{(t)}}{\sum_{j=1}^{N_s} \tilde{w}_j^{(t)}} \tag{8}$$

To overcome sample degeneracy, which is a limitation of the PF technique, resampling should be applied (the different resampling procedures are well summarized in reference [50]).

Finally, the posterior density at time instant t is approximated as a mass of discrete weighted samples:

$$p(\mathbf{x}^{(t)}|\mathbf{y}^{(t)}) \approx \sum_{i=1}^{N_s} w_i^{(t)} \delta(\mathbf{x}^{(t)} - \mathbf{x}_i^{(t)}) \tag{9}$$

The operation of the filter can be summarized as follows: In the prediction step, the particles are propagated according to the kinematic model (Equation (3)), and then their weights are updated based on the likelihood of the received position measurements in the correction step (Equation (7) and (8)). After updating, the particles are resampled with a probability corresponding to their weights. This process iterates at each time step. More details about the PF technique can be found in reference [51].

We obtain a set of samples with appropriate probability weights as the posterior distribution (Equation (9)) of the object's state. These "state particles" $\mathbf{x}_i^{(t)}$ can be transformed into "measurement" or "position particles" $\mathbf{y}_i^{(t)}$ using the output equation (Equation (4)). These particles serve as representative samples for the overall distribution of the measurements, and their weights $w_i^{(t)}$ reflect their probabilities within this distribution $\hat{p}(\mathbf{y})$.

The filter operates independently on each sequence. Consequently, the cardinality of the resulting enriched dataset is $\sum_{s=1}^{S} L_s |\mathbf{y}_i^{(t)}{}_s| \quad \Rightarrow \quad S L_s N_s$.

3.4. Incorporating environmental model in the particle filter

One significant advantage of the PF is its ability to integrate contextual information, particularly with respect to environmental constraints. Real-world environments often contain obstacles that restrict the movement of objects. For example, a vehicle cannot pass through a solid wall or occupy the same space as a storage rack, as such events are physically impossible. Therefore, the movement of particles in the PF should comply with these constraints. An approach to achieving this is to modify the weight update formula to reflect the probability of physical events. Each obstacle corresponds to a specific region in space, which can be represented as forbidden area. These areas are defined by predefined polygons $\mathbf{O} = [o_1, ..., o_K]$ with known corner coordinates, and the possible location of the tracked object within these areas is excluded. Consequently, the likelihood that particles fall within these forbidden areas should be set to zero.

However, it is not only about assigning zero probability to the object being located in a forbidden area, but also considering the probability of the object being in proximity to that area as low. For example, a forklift does not move directly adjacent to a wall. Therefore, during the particle weighting process, a more gradual transition in probability is needed, which can be achieved by penalizing proximity. To implement this, the particle weights should be multiplied by a probability density that allows distant particles to have higher weights while reducing the weights of particles in close proximity. To do this, the distance d_{ik} between each particle $\mathbf{y}_i^{(t)}$ and the closest point of each polygon o_k is calculated during the weight update process. The unnormalized density $\omega_i^{(t-1)}$ by which the weights are multiplied is determined by the sigmoid function of the distance d_{ik}. The sigmoid function offers several beneficial properties. It has a lower limit of 0, and if the curve is shifted in the positive direction, it approaches zero near the origin. In addition, the slope of the curve is low in that domain, allowing a stronger penalty for particles in close proximity to obstacles. On the other hand, the upper limit is 1, so beyond a certain threshold, the value of the sigmoid function no longer increases significantly with distance. Consequently, the "distance rewarding" effect becomes negligible when no forbidden area is in close proximity.

Mathematically, Equation (7) is modified as follows:

$$\tilde{w}_i^{(t)} = w_i^{(t-1)} p(\mathbf{y}^{(t)}|\mathbf{x}_i^{(t)}) \omega_i^{(t)} \tag{10}$$

Where $\omega_i^{(t-1)}$ is a sigmoid function of the distance:

$$\omega_i^{(t)} = \frac{1}{1+e^{(-ad_{ik}+b)}} \tag{11}$$

Here, a and b are constants that allow the sigmoid function to be moved and compressed along the horizontal axis. With the help of these two parameters, it can be set at what distance and to what degree the spatial limiting effect is applied from the boundary of objects. Their value depends on the context and is subject to expert judgment, nevertheless, $a = 1$ and $b = 0$ can be used as default values for them.

This modification facilitates smoother movement of particles towards permissible areas, ensuring that the uncertainty in position estimation is not represented by a truncated distribution. Additionally, particles can explore their surroundings more effectively, leading to more accurate estimations.

3.5. Utilizing particle filter output for probability density estimation

As discussed in previous sections, the weights assigned to particles in the PF encode valuable information that can contribute to a more accurate estimation of probability density. Therefore, it is necessary to incorporate these weights when estimating probability densities. In this method, nonparametric kernel density estimation (KDE) is used to estimate the probability density of the weighted data set.

KDE is a nonparametric density estimator that does not rely on assumptions about the underlying parametric density function. Instead, it automatically learns and explores the shape of the underlying density based on the available samples. This makes it well-suited for analyzing data from complex non-parametric distributions. In our case, the particles serve as samples in the estimation process, and their weights must be taken into account.

Let $\mathbf{y}$ represent the d_y-dimensional output of the state space being examined, which is measured and has multiple possible values. The weighted KDE of this output can be calculated as follows:

$$\hat{p}_K(\mathbf{y}) = \frac{1}{S}\sum_{s=1}^{S}\frac{1}{N_s L_s h^{d_y}}\sum_{t=1}^{L_s}\sum_{i=1}^{N_s} w_i^{(t)} \mathcal{K}\left(\frac{\mathbf{y}-\mathbf{y}_i^{(t)}}{h}\right) \tag{12}$$

Here, $\mathcal{K}$ denotes the kernel function, which plays a smoothing role, and h represents the bandwidth that determines the level of smoothing. The KDE process can be explained as follows: Each elementary particle is smoothed to a certain extent determined by h, resulting in continuous distributions characterized by the shape of the kernel function $\mathcal{K}(\mathbf{y})$. The weights of the particles emphasize the density of these distributions. Regions with a higher concentration of data points will be covered by more distributions, and the densities will be summed for each point in space. Consequently, regions with a larger number of weighty data points will exhibit higher KDE values.

In this approach, the Gaussian kernel is chosen, with $\mathcal{K}$ representing the multivariate normal density function. Using a Gaussian kernel, the density estimate (Equation (12)) can be expressed as:

$$\hat{p}_K(\mathbf{y}) = \frac{1}{S}\sum_{s=1}^{S}\frac{1}{N_s L_s (2\pi)^{d_y/2} h^{d_y}}\sum_{t=1}^{L_s}\sum_{i=1}^{N_s} w_i^{(t)} \exp\left[-\frac{1}{2}\left(\frac{\mathbf{y}-\mathbf{y}_i^{(t)}}{h}\right)^T\left(\frac{\mathbf{y}-\mathbf{y}_i^{(t)}}{h}\right)\right] \tag{13}$$

In this equation, the weighted Gaussian KDE considers the weights of particles and incorporates them into the density estimation process, resulting in a more accurate representation of the underlying probability density.

3.6. Evaluation of probability distributions

The purpose of this method is to reduce the uncertainty associated with modeling distributions. To validate the results, the obtained distributions are compared with those based on raw measurements, assessing their information content. The evaluation of distributions is carried out using a regular rectangular grid that spans the entire distribution space.

The intersections of the grid form a sample set that is independent of the samples used to fit the distributions, and this sample set remains the same for all evaluations. At each point on the grid, the density is estimated using the density function $\hat{p}(\mathbf{y})$ of the model distributions obtained. Various information-theoretic and probabilistic measures are employed to evaluate the results and compare different distributions. Specifically, Shannon entropy, log-likelihood function, Jensen-Shannon divergence (JSD), and root mean squared error (RMSE) are utilized.

Shannon's entropy serves as a commonly used measure for quantifying the amount of uncertainty in a probability distribution. Given the N number of points on the grid $\mathbf{y}_n$ and their estimated probability densities $\hat{p}(\mathbf{y}_n)$ in the modeling distribution $\hat{P}(\mathbf{y})$, the entropy formula is expressed as:

$$H(\hat{P}(\mathbf{y})) = -\sum_{n=1}^{N} \hat{p}(\mathbf{y}_n) \log_2 \hat{p}(\mathbf{y}_n) \tag{14}$$

The log-likelihood function provides a general measure of the goodness of a probabilistic model by expressing the sum of the probability densities of the samples.

$$\ell(\hat{P}(\mathbf{y})) = \sum_{n=1}^{N} \log(\hat{p}(\mathbf{y}_n)) \tag{15}$$

JSD is a dissimilarity measure between two distributions, which can be calculated as the difference between the entropy of the mixture of the two distributions and the average of the entropies of the two distributions. For distributions $\hat{P}_1(\mathbf{y})$ and $\hat{P}_2(\mathbf{y})$, JSD is expressed as:

$$D_{JS}(\hat{P}_1(\mathbf{y})||\hat{P}_2(\mathbf{y})) = H\left(\frac{\hat{P}_1(\mathbf{y}) + \hat{P}_2(\mathbf{y})}{2}\right) - \frac{H(\hat{P}_1(\mathbf{y})) + H(\hat{P}_2(\mathbf{y}))}{2} \tag{16}$$

RMSE is used to compare distributions based on the differences between outcomes in different distributions. The RMSE between $\hat{P}_1(\mathbf{y})$ and $\hat{P}_2(\mathbf{y})$ distributions is expressed as:

$$RMSE = \sqrt{\frac{\sum_{n=1}^{N} (\hat{p}_1(\mathbf{y}_n) - \hat{p}_2(\mathbf{y}_n))^2}{N}} \tag{17}$$

These evaluation metrics provide a comprehensive assessment of the obtained distributions, enabling the comparison and validation of different models based on their information content and dissimilarity.

4. Application example: occurrence pattern detection from indoor position data

In this section, we demonstrate the application of the proposed methodology using position data from forklift movement sequences. The method is employed to create a probability heat map of the tracked forklift locations based on the position data obtained from an indoor positioning system.

Analyzing the probability distribution of forklift truck locations in a warehouse environment has significant managerial benefits. By understanding the probability distribution of the positions of the forklift truck, managers can gain insight into the frequency of access to different warehouse areas. They can identify areas that require additional forklift trucks or resources due to high demand or congestion, and pinpoint peak periods of activity or workflow bottlenecks. This knowledge allows efficient resource allocation, optimized scheduling, and improved warehouse layout. In addition, spatial probability information can help identify areas prone to congestion, collision risks, or safety hazards.

To evaluate the effectiveness and reliability of the proposed method, probability densities were estimated from both raw position data and weighted particles using weighted KDE (Equation (13)). Subsequently, the obtained distributions are compared using probabilistic and information-theoretic measures discussed in the previous subsection.

4.1. Demonstration of the method's application using forklift data

To demonstrate the applicability of the method, we performed estimations using forklift data collected from a warehouse environment. In this operating environment, forklifts are tracked using an indoor positioning system. Each forklift is equipped with a tracking tag that emits a radio signal at regular intervals. Receiver units with known fixed coordinates receive the signal, allowing the system to determine the vehicle coordinates. The system uses the Time Difference of Arrival-based lateration technique to calculate the position. The system accuracy is approximately 0.5 meters.

Due to the presence of obstacles and signal reflections, there is often no direct line of sight between the tags and receivers, leading to signal reflections and absorption, further reducing the system's accuracy. As a result, the distribution of measurement noise is unknown and asymmetric, justifying the use of a particle filter in our methodology. The information provided by the system includes two-dimensional coordinates and timestamps. Therefore, the data set used for the analysis comprises 2D coordinate data recorded with time stamping with a sampling frequency of 0.5 seconds. The data set includes data from 8 forklifts, spanning a period of 10 hours.

The analysis was performed in a Python environment. Before analysis, the data were preprocessed to reduce complexity. Specifically, we selected sequences in which the forklift trucks were in motion. The selection process involved dividing the dataset into data sequences based on the calculated velocity derived from the moving average filtered position data. The smoothed velocity was then compared to a threshold, and sequences in which the velocity exceeded the threshold were selected. These selected sequences were further examined formation, and only those lasting at least 10 seconds were retained. As a result of this preprocessing step, we obtained a dataset consisting of 208 sequences containing a total of 14, 123 data points.

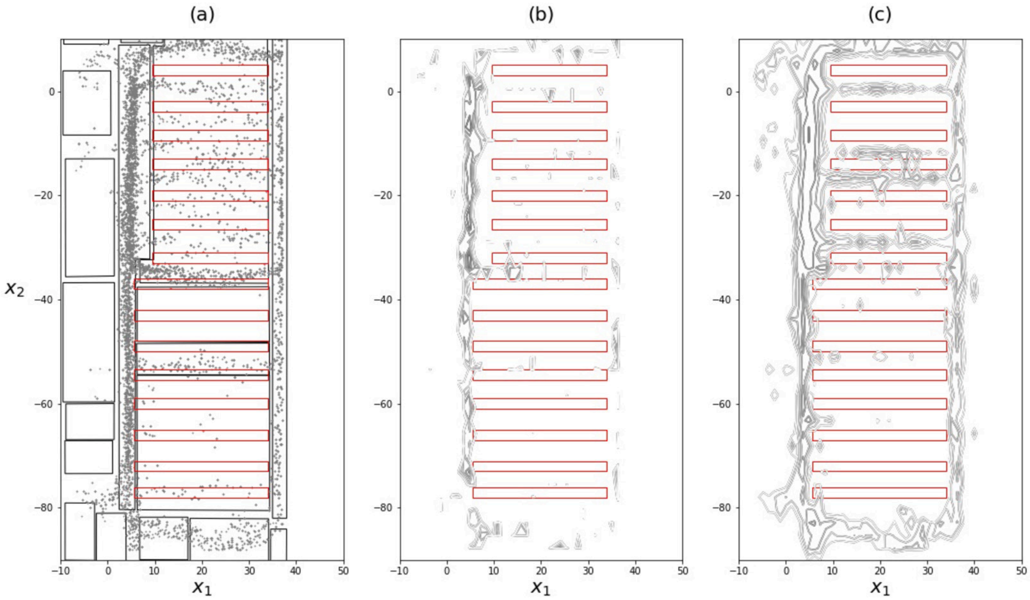

Fig. 3. The raw position measurements(a), distribution obtained by KDE on the raw position data (b), and distribution obtained by KDE on particles (c). The red rectangles indicate the areas of the storage racks.

The warehouse shop floor is divided into lines by storage racks between which forklifts operate. Utilization of a particular line can be understood as the probability of encountering a forklift within the area defined by the two storage racks. The probability densities calculated for these lines can provide valuable information for management. However, due to the inherent uncertainty in the data, it is not always possible to accurately determine in which row a forklift was located. As a result, a significant portion of the measurements is scattered within the storage rack area, as shown in Fig. 3 (a), which does not correspond to the actual positions of the forklifts.

To address this issue, noisy data was filtered using PF, with each sequence processed independently. A sample size of $N_s = 300$ was used. In addition to the dynamics of movement, information about the environment, specifically storage racks, was incorporated into the state estimation process. The storage racks were represented as polygons with known corner coordinates. The distance between each pair of particles and polygons was calculated, and the weights were updated accordingly using the formulas presented in Equations (10) and (11). The process took 712 seconds for computing, in case of PC with Intel(R) Core(TM) $i7-10700$ CPU @ $2.90GHz$ $2.90GHz$ CPU and $32GB$ RAM specifications.

It is important to note that the filter parameter settings were not tuned for each sequence, but rather the same parameters were used for all sequences. Consequently, the filter did not perform well on some sequences, leading to failures in the estimation process. During such instances, the likelihood of certain particles decreased to such a small value that it effectively became zero, resulting in the computer interpreting them as missing values. This occurred in a total of 31 sequences, and the corresponding $670,200$ particles from these sequences were not stored, representing 15

Therefore, a total of $3,566,700$ particles were generated and stored during the filtering processes. KDE was then applied to both the original position data and the stored particles using a bandwidth parameter of 0.1. Figs. 3 (b) and (c) depict the results of this estimation. The particle-based distribution exhibits more characteristic features and displays more uniform density values in regions where forklifts are present. This distribution provides two valuable pieces of information that are not apparent in distributions based solely on raw data. First, the frequently used lines stand out, indicating areas of higher activity. Second, frequently used locations can be identified as certain points exhibit a higher probability density compared to their surroundings. These points suggest specific positions within the lines where faster-moving inventory is often placed, resulting in more frequent loading activities. Identifying specific positions for faster-moving inventory in a warehouse offers several advantages. It facilitates faster loading activities, as commonly requested items can be retrieved more quickly, streamlining the loading process, and expediting order fulfillment. Second, it helps reduce congestion in the warehouse by separating faster-moving inventory from slower-moving stock, promoting smoother movement and minimizing delays.

Furthermore, it is worth noting that the density extends into the forbidden zones at certain points. This occurs because particles tend to concentrate in these locations near the boundaries of the forbidden zones, and the smoothing effect of KDE expands their probability density beyond the boundaries.

It is important to note that the calculation requirements of the algorithm are minimal. The necessary calculation time can be linked to the simulation of the model used, i.e. the calculation complexity primarily stems from the evaluation of the dynamic model. An important aspect is that this simulation must be carried out in amounts corresponding to the number of particles. In the present case, using a simple model, this is not critical, but it is important to emphasize that in the case of critical applications, the PF can be parallelized.

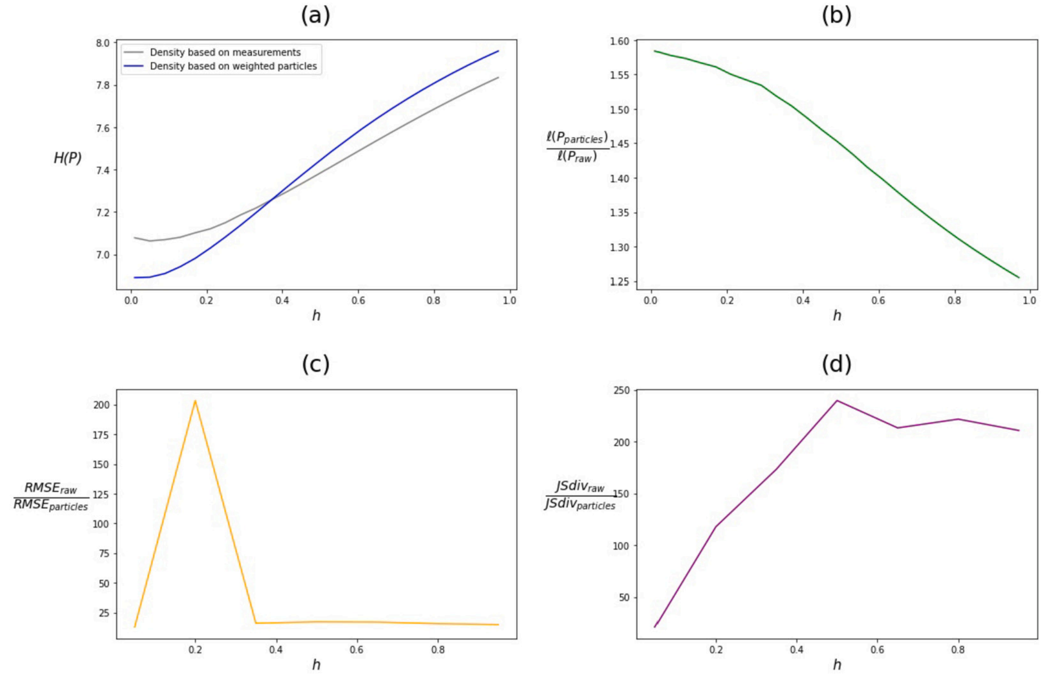

Fig. 4. The result of the validation of the method. Figure part (a) shows the entropy of distributions as a function of bandwidth. Figure part (b) shows the log-likelihood ratio of the particle and raw data-based distributions. Figure part (c) shows the RMSE ratios between distributions based on particle data fragments and raw data fragments. Figure part (d) shows the JS divergence ratios between distributions based on particle data fragments and raw data fragments.

4.2. Test and validation of the method

To assess the effectiveness of the proposed methodology, a series of tests were conducted comparing the raw data distributions with the particle-based distributions. KDE was performed on raw data and particles using different bandwidth values ranging from 0.01 to 1 with a step size of 0.04. Lower bandwidth values resulted in less smoothing, allowing the underlying pattern and irregular fluctuations in the original data to be better represented in the resulting model. On the other hand, particle-based density estimation concentrated the center of mass of the particles around the true position, giving less weight to noisy outliers and resulting in a more compact distribution. Increasing the amount of smoothing diminished this advantage above a certain bandwidth value. The uncertainty of all the distributions obtained was quantified using Shannon's entropy (Equation (14)), as shown in Fig. 4 (a). The figure confirms that the particle-based distributions exhibited lower uncertainty in the lower bandwidth ranges.

The logarithmic likelihood (Equation (15)) of the models was also evaluated. Fig. 4 (b) presents the ratio of logarithmic probability between the distributions based on particles and the distributions based on raw data. The graph reveals conclusions similar to those of the entropy analysis. At low bandwidth values, the particle-based model outperformed the raw data-based model, resulting in a higher ratio. However, as bandwidth increased, this advantage diminished. These findings are summarized in Table 1, which demonstrates the ability of the method to mitigate the impact of measurement uncertainty and achieve a more precise estimation of probability density.

To further examine the reliability of the results, cross-validation was performed. The validation was based on the assumption that if both the noisy measurement data and the particles were randomly split into two sets and the distributions were fitted to each set, the distributions fitted to the raw data would exhibit greater differences due to random noise compared to the distributions fitted to the particles. Density estimations were carried out on both types of partial data sets, and fitted models were compared using two indicators, JSD (Equation (16)) and RMSE (Equation (17)). The results are summarized in Table 2. In all cases, the RMSE between the distributions fitted to the raw data was at least one order of magnitude larger than the RMSE between the distributions fitted to the particles (Fig. 4 (c)). Similarly, the JSD between the distributions fitted to the raw data was higher compared to the divergence between the particle-based distributions (Fig. 4 (d)). These validation results demonstrate that the proposed method not only provides more accurate density estimates but also ensures greater consistency and reliability.

5. Conclusion

The analysis of mobility patterns based on inaccurate position data presents challenges in extracting meaningful information. To address this issue, the proposed methodology takes advantage of the knowledge of movement characteristics and environmental factors to generate informative data, enabling more accurate and reliable probabilistic modeling. By incorporating a particle filter approach, which utilizes the dynamic model of movement and spatial information, the methodology approximates the posterior

Table 1
Summary of goodness indicators in case of distributions that are based on different data.

Bandwidth	Entropy		Log-likelihood	
	Raw Data	Particles	Raw Data	Particles
0.01	7.08	6.89	-898481463.48	-567234573.14
0.05	7.06	6.89	-35795349.71	-22688968.79
0.09	7.07	6.91	-11042705.47	-7019239.86
0.13	7.08	6.94	-5304801.15	-3385676.30
0.17	7.10	6.98	-3118919.38	-1998106.92
0.21	7.12	7.03	-2053638.98	-1324842.55
0.25	7.15	7.08	-1461939.02	-948039.66
0.29	7.19	7.14	-1099184.60	-716437.77
0.33	7.22	7.20	-857740.24	-564898.83
0.37	7.26	7.25	-691369.10	-459614.14
0.41	7.29	7.31	-570869.47	-383850.24
0.45	7.33	7.37	-481204.85	-327472.10
0.49	7.37	7.43	-412660.53	-284047.32
0.53	7.41	7.49	-357821.21	-249746.42
0.57	7.45	7.53	-312160.78	-221400.82
0.61	7.50	7.59	-273858.51	-197016.80
0.65	7.54	7.64	-241174.58	-175762.55
0.69	7.58	7.69	-212465.75	-156948.79
0.73	7.62	7.73	-187184.93	-140022.99

Table 2
Summary of difference indicators between distributions that are based on different partial datasets.

Bandwidth	RMSE		JS divergence	
	Raw Data	Particles	Raw Data	Particles
0.1	0.00061	4.82e-05	0.0500	0.00239
0.2	0.00239	1.18e-05	0.0258	0.00022
0.3	0.00011	6.93e-06	0.0145	8.36e-05
0.4	8.12e-05	4.73e-06	0.0096	4.02e-05
0.5	6.22e-05	3.67e-06	0.0061	2.88e-05

distribution through weighted samples. Subsequently, a weighted kernel density estimation is performed directly on these samples, resulting in more compact and characteristic distributions compared to those based solely on raw measurements.

The application of the method was demonstrated using indoor position data from forklifts operating in a warehouse environment. Through the evaluation of KDE models based on the samples generated by the proposed methodology and raw position data using Shannon entropy and log-likelihood metrics, it was observed that the proposed approach yielded superior results. The reliability of the methodology was further assessed through cross-validation, employing root mean squared error and Jensen-Shannon divergence measures to compare distributions estimated in validation datasets. The results indicated that the estimation using the proposed method provided more consistent results compared to the traditional approach. A limitation of the study is that only KDE models were used for probability density estimation. It is therefore worth considering other density estimation techniques, such as Gaussian mixture modeling, in future research.

In conclusion, the particle filter used in this study proves to be a valuable tool to support probability density estimation in mobility analysis by enriching data and reducing uncertainty. The methodology contributes to improving the understanding of mobility patterns, enabling better decision-making and resource allocation in various domains, such as warehouse management.

CRediT authorship contribution statement

András Darányi: Writing – original draft, Methodology, Conceptualization. **Tamás Ruppert:** Writing – review & editing, Validation, Conceptualization. **János Abonyi:** Writing – review & editing, Validation, Supervision, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Publicly available datasets were analyzed in this study. This data can be found here: https://github.com/abonyilab/.

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