Research article

Activity theory as a framework for teaching mathematics: An experimental study

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A R T I C L E   I N F O

A B S T R A C T

This article introduces activity theory and how it can be employed to instruct the topic of straight-line equations in a plane - Geometry 10. Using the activity theory approach, we studied and developed a teaching process. The procedure is divided into three stages: Phase 1: Motivation and goal orientation, in which teachers present situations to attract students to the lesson; Phase 2: Knowledge formation, in which students engage in a variety of learning activities to build the knowledge they need to learn; Phase 3: Practice and consolidation, in which many exercises are assigned to students to solve in order to consolidate their knowledge and assist teachers in detecting and correcting students' misconceptions. To examine the effectiveness of applying the proposed three-phase model, we used a two-group pretest-posttest experimental model to determine whether or not teaching with the activity theory approach is more effective than the traditional teaching method by testing four research hypotheses. The experimental teaching took place in the Mo Cay district of Ben Tre province, Vietnam. Both the experimental and control classes began with the same level of mathematics, which was then tested using inferential statistics. After completing the pedagogical experiment, we discovered that students in the experimental class who were taught using activity theory achieved better learning outcomes than students in the control class, who were taught using the traditional teaching method; in the experimental class, the number of weak students decreased in comparison to the original; however, the number of good students did not increase. This is an issue that requires further studies to find ways to influence a wide range of students with different levels of mathematics so that the effectiveness of teaching according to the activity theory approach is improved.

1. Introduction

To improve the general education quality, the general education program announced by the Ministry of Education and Training (2018) has set a demand for teachers in high schools to use methods of activating students' activities, in which they play the role of organizing and guiding activities for students, creating a friendly learning environment and problem situations to encourage students to actively participate in learning activities, discover their own abilities and aspirations, practice habits and self-study abilities. Vietnamese educational reformers are emphasizing innovation in teaching methods as a means of combating the habit of passive learning. According to the Education Law of the Socialist Republic of Vietnam (2019), "Educational methods must promote positivity and self-discipline, initiative, and creative thinking in learners; fostering learners' ability to self-study and cooperate, ability to practice, fervor for learning, and will to rise". When discussing how to improve the quality of teaching activities, we must consider the relationship among students, knowledge to be learned (teaching content), and the impact of teachers. Kim (2017) believes that each teaching content is closely related to specific activities that students engage in while forming and applying that content. As a result, teaching is essentially discovering latent activities in content and mapping out a path for learners to dominate that content. Through activity theory, Vygotsky (1986, 2012) and Engeström (1999) has provided a systematic theoretical framework that clearly shows the relationship between the three constituent elements of an activity, namely subject, object, and tool.

Activity theory was used for varying purposes. For example, in the study of Robert (2012), from a local point of view, the goal of Activity

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Theory is to analyze students' mathematical activities in the classroom. Furthermore, in research by Beatty and Feldman (2012), they explored teachers' pedagogical change through a professional development assessment program. Meanwhile (Hardman, 2007, 2015, 2019), selected object-oriented pedagogical activity as the unit of analysis to investigate pedagogy in a classroom and employed the activity theory approach to surface the pedagogical activity in primary mathematics classrooms in South Africa. In addition, Huang and Lin (2013) modeled a mathematics activity (experiment) for teaching the addition and subtraction of integers in the Taiwan Atayal students' classroom. Based on the Activity theory approach, Liaw and Huang (2016) developed a research model to understand learner attitudes toward e-books. Activity Theory was applied in understanding teachers' pedagogical content knowledge and curriculum development. Naidoo (2017) explored the use of activities for teaching and learning mathematics in the USA. Chizhik and Chizhik (2018) used Activity Theory to examine how teachers' lesson plans meet students' learning needs. Activity theory and its activity system unit of analysis was used by Karen (2019) to describe changes in the professional learning of the teachers, the contradictions or tensions they addressed, and how they transformed the aspects of their professional knowledge and classroom practice. Kirby and Anwar (2020) designed using e-books and digital text documents on an e-reader for academic study, and the results were analyzed with the framework of Activity Theory. From seven constituents of Activity theory review research has been conducted by Lin et al. (2020) to describe the interaction and inter-relationships among components of reading development facilitated through mobile-assisted language learning. It was employed as the guiding theoretical framework to analyze and interpret the data by (Lee et al., 2021). Activity theory was used as a lens to elaborate on the discussion of learners' motivation (see more in (Kim, 2013; Nguyen and Habók, 2021), while from the perspective of Activity Theory, Rosari and Mbato (2021) explored the motivational fluctuation of Indonesian teachers teaching in Thailand. However, there are very few previous studies that systematically analyze the use of Activity Theory to design the teaching process in mathematics education.

One question is whether or not applying this theoretical framework of activities to teaching mathematics is effective. Our initial research goal is to find an answer to the above question through a case study of experimental teaching of a mathematical topic of the equation of a straight line in Geometry 10 – Vietnam.

2. Theoretical background

Activity theory is becoming more and more prevalent in educational research as it offers a way for individuals to understand and react to their activities, thus giving them the power to change various situations and conditions (Jonassen, 2011; Jonassen and Rohrer-Murphy, 1999; Loc, 2016; Roth, 2004). In addition, activity theory can allow the researcher to investigate a variety of situations and problems (Huang and Lin, 2013; Kent et al., 2016; Lou and Moon-Michel, 2018; Marwan and Sweeney, 2019; Wade-Jaimes et al., 2019). Activity theory has its origins in the classical German philosophy of Kant and Hegel, the dialectical materialism of Marx, and the socio-cultural and socio-historical tradition of Russian psychologists such as Lev Vygotsky (the 1930s), Alexey Leont'ev (1974), and Yrjö Engeström (1987).

Activity theory is related to the historical development of human activity. The essence of activity theory is the dialectical transformations of individuals and their communities due to their involvement in an activity (Abboud-Blanchard and Cazes, 2012; Loc, 2016; Vandebrouck et al., 2012). According to Vandebrouck et al. (2012), the essence of activity theory is the dialectical change in the psychology and behavior of individuals during the activity. This is because humans can change the conditions that mediate their activities in addition to responding to them. According to Engeström (1987), participating in group activities improves one's social skills and self-esteem. The distinction between activity interaction and other types of interaction is based on two key factors: (a)

The activity's subject has needs that should be met through interaction with the world, and (b) the activity and the subject define each other: the activity transforms both the subject and the object, and the object's and subject's characteristics affect each other. The problem and the solver's ability, for example, will determine the progress of the solution in problem-solving activities; on the other hand, the problem-solving process also reveals the subject's ability to solve problems and transform the subject.

According to activity theory, an activity (or sometimes an activity system) is motivated by an object (the object of the activity), and it is the object that distinguishes one activity from another. In some texts, this object is referred to as the motive of activity. According to Leont'ev (1974), all activity is motivated, even though the motive may not be explicit. When it is explicit, it is referred to as a motive-goal.

According to Vygotsky (1986, 2012), an activity consists of a subject, an object, and a tool (see Figure 1).

The subject is the person or group whose point of view is used to analyze the activity (Cole and Engeström, 1993). Within the teaching activity triangle, Hardman (2007, 2015) and Beatty and Feldman (2012) identified their subject as the individual teacher while, for Huang and Lin (2013), the students and the teacher are the subjects of the activity system; in the instructional activity, they are the subjects during the teaching and learning process.

An activity is always an activity with an object. The object is the target of the activity within the system that people need to do. The object of the activity can be in physical or mental form, and it is the purpose of the activity. The object of the activity is also the motive that motivates the subject to change the object, turn it into a product, or receive it transferred into his mind, creating a new psychological structure.

According to Loc (2016), an activity comprises a subject and an object, mediated by a tool, where the subject is a learner or learners involved in an activity, while an object is held by the subject. The object motivates the activity, giving it a specific direction. Besides the above notions about the active object, there are many different opinions about this concept. It was defined as the motivating influence behind the subject's participation in the activity (Cole and Engeström, 1993). In Beatty and Feldman's (2012) study, the teacher's teaching and the set of students were the objects. In Hardman (2007, 2015) and Huang and Lin's (2013) study on mathematics teaching, the object of the activity was the teaching and learning of mathematics; the object was to study the development of the mathematics content and conceptual knowledge (Naidoo, 2017) and finding new knowledge (Torrey (2017). Zevenbergen and Lerman (2007) argued that the object is identified within the activity theory lens and cannot specify where it is in the teaching process.

Tools are mediating artifacts used by the subject to act on the object that helps attain the activity's outcomes (Cole and Engeström, 1993). It can be a physical object (for example, an electronic computer, a handheld computer, …) or a non-physical object (for example, thought, language, signs, …). A mediating artifact is transformed into a tool when the subject uses it to solve a task. Tools are created and changed throughout the activity and are influenced by culture and history. In Beatty and Feldman's (2012) study, the tools of the activity system included the many different professional development tactics and resources employed that help teachers reflect upon their practice and improve their skills. For Hardman (2015), tools were the language and the computer in a context where there were different roles. The tools in Huang and Lin's (2013) study were defined as material tools (textbooks and handouts) and psychological tools (language and signs), while Naidoo (2017) defined his tools as visual tools like pictures, charts, technology, chalkboard, and diagrams that were used to teach mathematics. The use of various tools
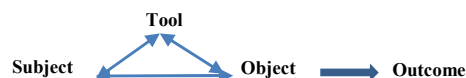

Figure 1. Vygotsky's model of activity (1986).

could be inferred as the researcher's creativity in designing valuable tools to improve mathematics teaching and learning. For instance, in the activity of solving the problem with the help of GeoGebra in the study of Loc (2014), the subject is a problem solver, the tool is GeoGebra, and the object is the problem.

According to Leont'ev (1974), analysis, using activity theory, takes into account three levels: analyzing the activity and its motive, analyzing the action and its goal, and analyzing the operation and its conditions. This is translated into the activity the learners will engage in toward an objective. Three levels of activity are illustrated in Figure 2 below.

The activity model is presented in table form (Albrechtsen et al., 2001) (see Table 1).

Activity theory is a general framework for studying different forms of human activity as development processes (Kuutti, 1996, pp. 17–44). Within this general context, Engeström (1987) suggested a model (Figure 3) that conceptualizes all determined human activity as the interaction of the following elements: subject, object, tools, community, rules, and division of labor.

In this model of an activity system, the community is comprised of one or more people who share the objective with the subject (Cole and Engeström, 1993). The collection of facilitators and participating teachers, such as support staff, counselors, and administrators, was the community in Beatty and Feldman's (2012) study. In Hardman's (2007, 2015) study, the community included only the teacher and the students. The community is defined by all the teachers and subjects of the Atayal School in Huang and Lin's (2013) research, while the community was the learners within the mathematics classroom, the teachers, the staff at each school in Naidoo (2017), and like-minded individuals with the same goals and values in Torrey (2017).

Rules are explicit and implicit regulations, norms, and conventions that constrain actions and interactions within the activity system (Cole and Engeström, 1993). Rules can be understood as principles of control. For Hardman (2015), there were contracts in this activity system, namely the math and behavioral rules. Meanwhile, according to Beatty and Feldman (2012), the rules, which can be explicit or implicit and vary between teachers and classes, specify norms and expectations for behavior.

The *division of labor* is involved of one or more people who share an ordinary object and who use artifacts to act on that object, transforming it (Cole and Engeström, 1993). The division of labor is defined as the roles teachers and students play (Hardman, 2007; Huang and Lin, 2013).

A learning *outcome* states what the young people are working with should achieve, known by the end of the activity. For instance, while the outcome in Huang and Lin (2013) is the result of the teaching and learning activities as an understanding of the addition and subtraction of integers, for Naidoo (2017), the teaching and learning of mathematics
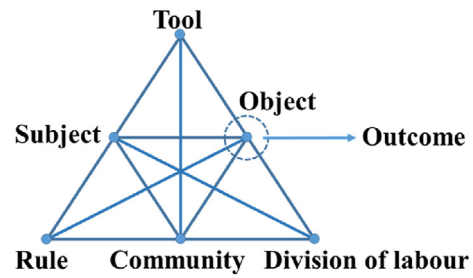

Figure 3. Engeström's structure of human activity system (1987).

was the outcome. According to Torrey (2017), the outcome was the intention to improve student engagement.

2.1. Research questions

On the basis of the primary arguments about the activity, we study and apply it to teaching the topic of straight-line equations with the following two research questions:

Research question 1. How are the teaching activities on the topic "The equation of a straight line" designed if they are based on the theory of activity?

Research question 2: Does teaching with an activity theory approach lead to better student achievement?

3. Method

3.1. How to design a learning activity (answering the research question 1)

For the first research question, in order to design a learning activity for students, we choose Engeström's activity model as the basis to identify the components outlined in the diagram and propose a three-phase process to teach knowledge unit as follows (see Table 2):

3.1.1. Phase 1: Motivation and goal orientation

During this phase, teachers use different methods to connect students' known knowledge with what they are about to learn. Teachers need to take measures to motivate students to learn (Wentzel, 2020; Wolters et al., 1996), and show them what the goal of the lesson is. If students know learning objectives, this will increase their learning achievement (Dreyfus, 2018; Fernández and Morris, 2018; Fry et al., 2003). According to activity theory, the object in the diagram of Engeström and Vygotsky is both an activity target and a driving force.

3.1.2. Phase 2: knowledge formation

In this phase, the teacher has to set up student tasks, often a system of guiding questions or exercises, so that, after completing these tasks, students seem to discover what they need to learn. Teachers need to create a collaborative learning environment where students have many opportunities to discuss and debate (Simon, 2018). We must always keep
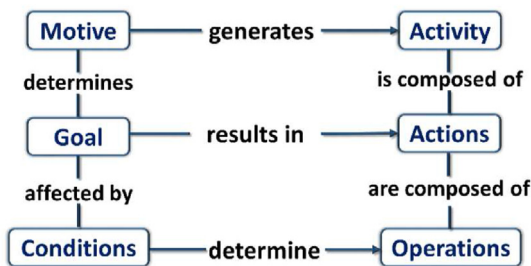

Figure 2. Activity, actions, and operations (Nussbaumer, 2012).

Table 1. Different aspects of the activity structure (Albrechtsen et al., 2001).

Level of activity	Directed at	Analysis
Activity	Motive	Why something takes place
Action	Goals	What takes place
Operation	Conditions	How it is carried out

Table 2. Activity theory-based model for teaching knowledge unit.

Phase 1 *Motivation and goal orientation*	Phase 2 *Knowledge formation*	Phase 3 *Practice and consolidation*
• Creating problematic situation • Connecting known knowledge of students • Creating positive attitude • Creating a positive attitude and willingness to learn for students	• Creating opportunities for students to construct knowledge • Using guided discovery • Using software to support • Encouraging student discussion and debate	• Giving students exercises-based learning objectives • Correcting misunderstandings among students • Using different measures to consolidate or systemize

in mind that students learning in a constructive environment will always produce positive outcomes (Adak, 2017; Birgin and Acar, 2020; Qarareh, 2016).

The second phase is the most crucial. It includes the learning actions taken by students to achieve instructional goals. According to the three-phase model, the effectiveness of the teaching process is primarily determined by the tasks assigned to students in order for them to acquire the knowledge to be taught. The following are two examples to help students form concepts through induction (Table 3) and deduction (Table 4).

According to the scenario above (Table 3), the teacher allows students to perform actions such as analysis, comparison, abstraction, and generalization so that they can finally state the definition of the concept to learn.

Table 4 depicts a deductive concept formation process. The main task of the students is to analyze the concept definition provided by the teacher in order to recognize the cues to recognize the concept.

3.1.3. Phase 3: Practice and consolidation

Practice and consolidating knowledge for students after the end of the lesson is very important (Carmona-Medeiro et al., 2021; Graeff, 2010; Harvey and Averill, 2012). Exercises are designed to help students achieve lesson objectives. Thanks to the students' work results, the teacher knows the level of understanding of the knowledge just acquired by the students and detects students' misunderstandings; thanks to that, teachers promptly correct students' mistakes and misunderstandings. Experienced teachers, after practice, also reinforce students' knowledge in many vivid ways, such as using quizzes, systematizing knowledge with mind maps, and pointing out relationships between newly learned knowledge and students' already known knowledge.

Learning, whether it is studying any topic in Mathematics in particular or other scientific disciplines in general, is a type of activity; thus, the three-phase teaching model described above can be used to teach any subject. In order for the teaching process to be highly effective, all three phases must be performed by the teacher, and no one can be ignored.

Table 3. Process (scenario) of Concept formation through induction.

The teacher's activities	Students' activities
The teacher gives some examples of the concepts to be taught, and Ask students questions: What do these examples have in common?	Students perform actions such as observing, analyzing, and comparing the features in each example to discover their common characteristic
The teacher said that because the examples have such common properties, they are called…(concept name). In general, would you please state the definition of the concept….?	Students state the concept definition
The teacher corrects the concept definition and asks the students to restate the definition	Students state the definition.

Table 4. Process (scenario) of Concept formation through deduction.

Teacher's activity	Students' activity
The teacher introduces the concept definition.	Students perform analytical actions to point out the distinctive features of the concept
Ask students to point out the distinctive features of the concept	
The teacher provides some examples and non-examples and then asks students to identify which are examples and which are non-examples.	Students analyze each case and compare it with the definition to determine which are an example and which are non-examples.
The teacher asks the students to give some more examples and non-examples.	Students rely on concept definitions to complete the teacher's requirements.

3.2. How to conduct a pedagogical experiment (for answering the research question 2)

Hypotheses.
To find the answer to the second research question, we conducted a pedagogical experiment to verify the following pairs of hypotheses.

H0.1. The learning outcome achievement of students in the experimental class and one of the students in the control class is equivalent.

H1.1. The learning outcome achievement of students in the experimental class is better than the one of students in the control class.

H0.2. The learning outcome achievement of students in the experimental class and one of these students before the experiment are equivalent

H1.2. The learning outcome achievement of students in the experimental class is better than one of these students before the experiment.

H0.3. The number of weak students in the experimental class before and after being taught by the activity theory approach was the same.

H1.3. The number of weak students in the experimental class before being taught by the activity theory approach is more than the number of students in this class after teaching by the activity theory approach.

H0.4. The number of good students in the experimental class before and after being taught by the activity theory approach was the same.

H1.4. The number of good students in the experimental class before being taught by the activity theory approach is more than the number of students in this class after teaching by the activity theory approach.

Pedagogical experiments to test the research hypotheses are carried out by using the pretest-posttest two-group model (Table 5), which is described in more detail below.

3.2.1. Teaching content

The teaching content of "the equation of a straight line" consists of a direction vector of a straight line, a parametric equation of a straight line, a normal vector of a straight line and a general equation of a straight line, and relative positions of two straight lines. The experiment's topic is taken from the Vietnamese curriculum's 10th-grade Geometry textbook.

3.2.2. Participants

The research was conducted at Ca Van Thinh high school, Mo Cay district, Ben Tre province, Vietnam, in the 2020–2021 academic year. Two classes were selected as follows: 10C3 class is an experimental class, which consists of 37; 10C1 class is the control class which consists of 38 students. This study was conducted in strict accordance with Can Tho University's regulations on ethics in scientific and technological activities.

The test scores at the end of the semester I - the academic year 2020–2021 of the experimental and control classes are used as the result of the pretest because the experimental time is the beginning of semester II - the school year 2020–2021 (see Table 6). The average points of students in the experimental class and control class were 5.38 and 5.50, respectively; the median marks were 6.00 and 5.00, respectively. Table 6

Table 5. Pretest - posttest two-group model for pedagogical experiment.

Class	Pretest	Treatment	Posttest
10C3 (Treatment class)	O_1	X_1	O_2
10C1 (Control class)	O_3	X_0	O_4

Where
O_1: Pretest in the experimental class; O_2: Posttest in the experimental class;
X_1: Treatment of the activity theory approach in the experimental class;
O_3: Pretest in the control class; O_4: Posttest in the control class;
X_0: Traditional method used in the control class.

Table 6. Descriptive statistics of 10C3 and 10C1.

	10C3	10C1
N	38	37
Mean	5.50	5.38
Median	6.00	5
Standard deviation	1.66	1.46
Minimum	1	3
Maximum	8	9
Shapiro-Wilk W	0.935	0.946
Shapiro-Wilk p	0.029	0.072

Table 7. Mann-Whitney U test before the experiment.

	Statistic	p
Mann-Whitney U	642	0.510

indicates that the scores of the experimental class are not normal distribution ($p = 0.029 < 0.05$ – Shapiro -Wilk test for normality) (Brace et al., 2012).

Using the Mann-Whitney U test, $p = 0.51$ (see Table 7). Hence, the mathematical level of the experimental class is equivalent to the one of the control class.

3.2.3. Instrument and materials

- Teaching method: The teaching method in the experimental class was carried out according to the activity theory approach; specifically, the teacher applied the three-phase process that we have proposed.

The teaching method in the control class was the traditional method. In Vietnam, the lecture method of teaching is regarded as a traditional approach; it is a teacher-centered method in which the instructor is the source of knowledge. The teacher mostly talks and presents in the classroom, while the students passively absorb knowledge. In Vietnamese high schools, this strategy is commonly employed.

- Teaching tool: Worksheet, GeoGebra software, essay questions for posttest.

The essay test items were developed based on the 2000 curriculum (Ministry of Education and Training, 2006) and the mathematics textbooks published in 2015 by the Vietnamese Ministry of Education. Scores of the tests are scored on a scale of 10. The test comprised four essay questions that covered the topic "the equation of a straight line" (see Appendix 1). The content validity of the essay test was validated by four experienced high school teachers and one expert from the postgraduate Program of Secondary Education. The split-haft method and the Spearman-Brown formula were used to examine the reliability of the essay test. The results showed that the reliability of the test was 0.71, which meant that the reliability of the essay test was acceptable.

4. Results

4.1. Designing activities for teaching five concepts in the topic "the equation of a straight line (answering research question 1)

As mentioned in the research methods section, we taught five mathematical concepts in the experiment on the topic of straight-line equations; the steps for making a learning activity are as follows:

1) Determining the elements of Engeström's activity model applicable to each teaching content;
2) Designing a learning activity according to the three-phase process.

4.2. An illustration

As an illustration, in this article, we will only report the outcomes of creating learning activities for the object "The general equation of a straight line."

4.2.1. Elements of the activity to teach the equations of a straight line according to Engeström's model

Elements of the activity to teach the equations of a straight line according to Engeström's model are determined as follows.

Subject: Students.

Tool: Dynamic mathematics software GeoGebra, worksheets.

Object: The general equation of a straight line.

Rule: The class is divided into four groups. Each group includes both good and weak students. It is reasonable to form such a study group because weak students' knowledge is frequently unstable and has many gaping holes; it allows students to help each other discuss and solve assigned learning tasks. The groups must perform well the learning tasks assigned by the teacher and present the results in front of the class if required.

Community: The classroom community consists of teachers and students.

4.2.2. Division of labor

- Students in each group cooperate to complete the learning task; when a representative of one group presents the results in front of the class, the other groups' comment and critique.
- Teachers only have the role of assigning learning tasks to students, advising, controlling, and managing the class, and formalizing knowledge.

4.2.3. Expected learning results

- State the general equation of a straight line;
- Know how to write a straight line, the normal vector of it, and 1 point on it;
- Be able to solve problems related to writing general equations of straight lines at a low application level.

Regarding how to determine elements of the activity as above, we find that:

1. Learning outcomes are specified clearly. They are outlined based on the knowledge and skills standards of the Ministry of Education and Training, Vietnam. Another basis for setting learning goals is that students in remote rural areas have average academic performance; many children are busy with work to earn a living with their families, so they have little time to study at home. Therefore, the initial goal set in this lesson is just to meet the criteria of the knowledge and skills standards, and it must be suitable for students' learning ability; that is, the lesson must be in the zone of proximal development (Vygotsky, 2012). With the support of teachers and classmates, each student can acquire the necessary knowledge and skills.
2. Regarding teaching tools, we use GeoGebra dynamic math software; because it integrates both geometry and algebra, it is very useful for teaching analytic geometry. Teachers can create an intuitive and lively learning environment; It is a means for teachers to organize teaching in an exploratory approach easily. Many studies have shown that GeoGebra software is a perfect support software for teaching mathematics, especially Analytical Geometry, in high schools (Loc, 2016).
3. Students' tasks are specified in an obvious and specific way: cooperate with friends in the same group to complete learning tasks, and participate in debates with other groups when needed. Hence, students have the opportunity to discuss in the study group and argue

with each other in front of the whole class when a representative of a certain group is assigned to report the results of their group. Students who learn in an interactive social environment will have good academic results (Badri et al., 2017; Beatty and Feldman, 2012; Good et al., 1992; Wang et al., 2017). At the same time, students have the opportunity to practice skills such as mathematical communication and mathematical reasoning.

4. The teacher's role is only to assign tasks for students to perform; the teacher is only a consultant and manages the classroom so that the classroom atmosphere is friendly and all students have the opportunity to participate in learning activities. Such teaching is learner-centered teaching (Lankford, 2021; Weimer, 2002; Zita, 2021).

4.2.4. The three-phase process of teaching the general equation of a straight line

Teaching the general equation of a straight line in the experimental class was carried out according to the three-phase process described above.

4.2.5. Phase 1: Motivation and goal orientation

The teacher asked, "Please tell me what the graph of the function $y = ax + b$ is. Write the equation of the straight line through $A(0, 1)$ and $B(1, 0)$?"

Many students could do question (a) by substituting the coordinates of A, B to find a, b in the equation $y = ax + b$.

Next, the teacher created student motivation by posing a problematic situation, "The question is whether any line in the coordinate plane Oxy has the form $y = ax + b$ or not? Today's lesson will help you answer the question the teacher just raised."

4.2.6. Phase 2: knowledge formation

To form the general equation knowledge of a line, the teacher first asks the students to observe the line and its equation displayed on the GeoGebra screen. Each time "Button1 is pressed, a line and corresponding equation appear on the screen, for example, the equation $3x + 4y + 1 = 0$ in Figure 4. After many repetitions, the teacher asks the students to predict what is the equation of a line of general form. Next, the teacher instructs the students to verify what they have just predicted ($ax + by + c = 0$).

The following is the process by which the teacher guides the students to verify their predictions.

The teacher gave the task to the students:

Problem: In the coordinate plane, let a straight line Δ pass through the point $M(x_0; y_0)$ and take $\vec{n}(a; b)$ as a normal vector, find the condition for the point $M(x; y) \in \Delta$ (see Figure 5).

Students discuss in four groups to answer four questions in the worksheets as follows:

Question 1: What do you think about two vectors $\vec{n}(a; b)$ as and vector $\overrightarrow{M_0M}$?

Question 2: The condition that M belongs to the line is whether two vector $\vec{n}(a; b)$ and vector $\overrightarrow{M_0M}$ are perpendicular or not. Explain.
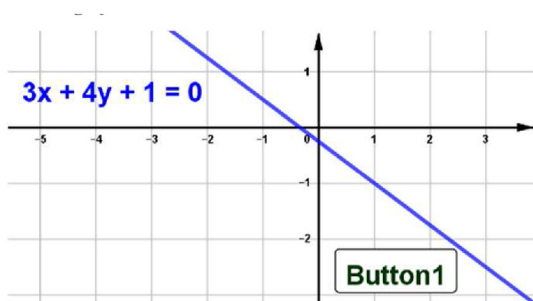

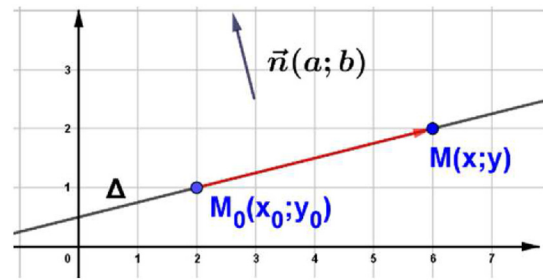**Figure 4.** The GeoGebra-based figure for the general equation of a straight line.

Figure 5. The GeoGebra-based figure of a straight line.

Question 3: Apply the way of calculating vector coordinates to calculate $\vec{n} \cdot \overrightarrow{M_0M} = 0$. How will the result be?

Question 4: From the results obtained, put $c = -ax_0 - by_0$; what is the shortened equation in question 3?

After students in groups finish finding out the answers to the questions, the teacher chooses representatives of groups to report in front of the class. The whole class will comment and debate the solutions presented.

After answering question 4, students obtain the solution as being $ax + by + c = 0$. It is the general equation of a straight line.

It is clear that, during the process of knowledge formation described above, students discovered new knowledge on their own through group discussion and debate in front of the entire class. In line with the current world teaching trend, such instruction has placed the learner at the center.

4.2.7. Phase 3: Practice and consolidation

*Practice.

The teacher asks students to read the questions and discuss in groups to answer the questions in:

Question 1: In the Oxy plane, given a straight line $d : 2x - 3y + 2 = 0$. Find one normal vector and one direction vector of d.

Question 2 In the Oxy plane, given a straight line d passing through point $A(1; 2)$ and taking $\vec{n}(-1; 2)$ as a normal vector.

a) Write down the general equation of straight line d.
b) Write down the general equation of straight line d', where d' passes through $B(1; -1)$ and $d'//d$.

Question 3: In the Oxy plane, given two points, $A(1; -4)$, $B(3; 2)$, write the general equation of the straight line AB.

*Consolidation.

In order to reinforce what students have just learned, we have refined the content knowledge in the lesson and organized the core content into a systematized format for ease of use. Table 8 is a summary of the general equation of a line, as well as the lines that are parallel or perpendicular to it.

By examining the overall process of creating a lesson, as illustrated above, we can see how the model of activity theory helps to determine the roles of teachers and students clearly. Students are motivated to learn; in an interactive learning environment, they complete tasks assigned by the teacher. This method of instruction, in our opinion, is appropriate for students living in remote rural areas who lack the motivation to learn, have limited time for self-study, and have numerous knowledge gaps.

4.3. Results of pedagogical experiment

4.3.1. The result of testing $H_{0.1}$ and $H_{1.1}$

After completing the practical teaching, we gave students in the experimental and control classes a 45-minute test. We collected data and conducted a quantitative analysis based on the test results. The learning results of the teaching experiment are presented in Table 9.

Table 8. Summary of some types of the straight-line equation based on the relation.

D passing through $M(x_0; y_0)$, $\vec{n}(a;b)$ being a normal vector	$D'//D$	$D\perp D$
$D : ax + by + c = 0,$ $D : a(x - x_0) + b(y - y_0) = 0$ $(c = -(ax_0 + by_0))$	$D' : ax + by + c' = 0$	$D' : bx - ay + c' = 0$

Table 9. Statistics of students' scores after posttest.

Score	1	2	3	4	5	6	7	8	9	10	N
Control class (10A3)	1	1	2	7	8	9	8	2	0	0	38
Experimental class (10A1)	0	0	0	3	3	16	13	1	1	0	37

From the data in Table 9, we have the two charts below to compare the scores of the two classes.

Figure 6 shows that the height of the point columns and the distribution of points of the two classes have differences. The experimental class received scores ranging from 4 to 9, with the majority receiving 6-7 marks. In comparison, the control class was graded from 1 to 8, with the majority of scores lying between 4 and 7.

Table 10 shows that the two classes' mean and median differ. In addition, both the mean and median of the experimental class are higher than those of the control class. The question is whether $H_{1.2}$ is correct or not. The answer obtained by the Mann-Whitney U-test is as follows.

Using Shapiro–Wilk test for normality indicates that the experimental class has a p-value of 0.001 (<0.05). That is, the posttest data for this class do not follow a normal distribution (see Table 11). The Mann-Whitney U test was then used to test hypotheses $H_{0.1}$ and $H_{1.1}$.

The Mann-Whitney U test result is presented in Table 11. Because the median number of scores in the experimental and control groups is 6
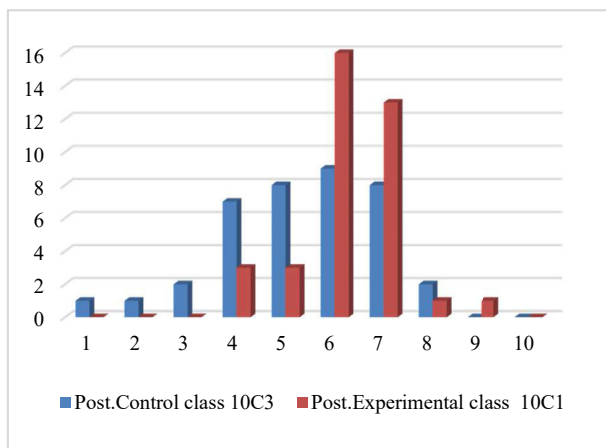

Figure 6. Column chart comparing test scores of experimental and control classes.

Table 10. Descriptive statistics after the experiment.

	Experimental class	Control class
N	37	38
Mean	6.24	5.34
Median	6	5.50
Standard deviation	1.04	1.60
Minimum	4	1
Maximum	9	8
Shapiro-Wilk W	0.879	0.942
Shapiro-Wilk p	<.001	0.049

and 5.5, respectively (Table 8) and p = 0.002 < 0.05 (Table 11), $H_{0.1}$ is rejected. This indicates that the experimental students' learning outcomes are superior to those of the control students. It also demonstrates that using the activity theory to teach the topic "the equation of a straight line" is more effective than using the traditional teaching method.

4.3.2. The results of testing $H_{0.2}$ and $H_{1.2}$

Table 12 shows that the experimental class scores before and after using the activity theory-based teaching method were 6 and 5, respectively.

The posttest class scores are not normally distributed. Using the Wilcoxon W test, p = 0.002 (Table 13). Hypothesis $H_{0.2}$ is thus rejected ($H_{1.2}$ is accepted). This means that the teaching method based on activity theory improves student learning outcomes.

4.3.3. The results of testing $H_{0.3}$ and $H_{1.3}$

Table 14 shows that the number of student scoring below five in the posttest is less than that one in the pretest. Otherwise, the number of student scoring five and above in the posttest is more than that one in the pretest.

By applying the Chi-square test, the results are as follows: Chi-square is 5.63 and p = 0.01 (<0.05). Therefore, hypothesis $H_{0.3}$ is rejected. In other words, the percentage of weak students in the experimental class before the researcher's intervention was greater than that of this class after it was treated ($H_{1.3}$ is correct).

4.3.4. The results of testing $H_{0.4}$ and $H_{1.4}$

Table 15 shows that the number of students who are not good (score below 7) in the posttest is less than that one in the pretest. Otherwise, the number of students who are good (score 7 and above) in the posttest is more than that one in the pretest.

Using the Chi-squared test, we obtain the following results: Chi-square is 3.09, and the p-value is 0.08. The result is not statistically significant at p < 0.05. As a result, hypothesis $H_{0.4}$ is accepted. This means that the proportion of students who achieve good results in the experimental and control groups is not significantly different.

Table 16 summarizes the results of testing the hypotheses. Some points that we need to discuss next are:

- The learning achievement of the experimental class is better than that of the control class;
- The learning achievement of the experimental class was improved;
- The number of weak students in the experimental class decreased, but the number of good students did not increase.

5. Discussion

Based on the findings of the educational experiment – accepting hypotheses $H_{1.1}$ and $H_{1.2}$ – we can conclude that the experimental class,

Table 11. The results of the Mann-Whitney U-Test.

	Statistic	p
Mann-Whitney U	430	0.002

Note. Ha Pre < Post.

Table 12. Descriptive statistics.

	Pretest	Posttest
N	37	37
Median	5	6
Standard deviation	1.46	1.04
Minimum	3	4
Maximum	9	9
Shapiro-Wilk W	0.946	0.879
Shapiro-Wilk p	0.072	<.001

Table 13. Paired samples U-test.

			Statistic		p
D	E	Wilcoxon W	62.0	a	0.002

Note. H_a Measure 1 < Measure 2.
[a] 11 pair(s) of values were tied.

Table 14. The percentage of weak students in the experimental class.

Students in the experimental class	Pretest	Posttest
Number of students scoring below 5 (<5)	11 (29.7%)	3 (8%)
Number of students scoring 5 and above (≥5)	26 (70.3%)	34 (91.9%)

Table 15. Comparing math level of 10C3 from pretest and posttest.

Students in the experimental class (10C3)	Pretest	Posttest
Number of students who are not good (score below 7)	29 (78.4%)	22 (59.4%)
Number of students who are good (score 7 and above)	8 (21.6%)	15 (40.6%)

taught using the activity theory-based teaching method, achieved learning outcomes better than the control class, which was taught using traditional methods. The conclusion can be interpreted as follows: when using the model of activity theory, the teacher first knows clearly and precisely the object of knowledge to be taught and clearly states the expected learning outcome. As a result, teachers can transform knowledge objects into learning motivations for their students. Creating learning motivation for each lesson is critical for an effective lesson (Dreyfus, 2018; Tien, 2019). Another factor that contributes to the success of the teaching process is that teachers assign learning tasks to help students self-discover the knowledge they need to learn; students learn through group and class discussion and debate; and, according to constructivist theory, learning is more effective through social interaction. Furthermore, the teaching follows Engeström's activity model, and class members (including teachers) are assigned obvious jobs. According to Marzano (2007), students who clearly understand their work and know how they should be will create a positive mood when performing learning actions. One point that needs to be addressed further is that, compared to the pre-intervention, the percentage of weak students in

Table 16. Summary of pedagogical experiment.

10C3- experimental class, 10C1- control class	
Before the experiment	Learning achievements of 10C3 and 10C1 are the same
After the experiment	Learning achievements of 10C3 are better than the one of 10C1
	Learning achievements of 10C3 are better than the one of 10C3 before the experiment
	The number of weak students of 10C3 decreases
	The number of good students of 10C3 is the same as before the experiment

the experimental class decreased, but the percentage of quite good students did not increase (see Table 16). This can be explained as follows: the objectives of classroom teaching that teachers set are based on the Ministry of Education and Training of Vietnam's standards of knowledge and skills. It is the teacher's responsibility to ensure that all students meet the common standards. Only in the end-of-chapter or semester-end review sessions include additional advanced exercises for good students. The experimental class was also taught in the above-mentioned spirit. We did not conduct any advanced review during the experimental teaching period. This could be one of the reasons why the number of good students did not increase in the experimental class; more specialized research, in our opinion, is required to determine exactly which reasons.

In summary, teaching based on activity theory will assist teachers in achieving their teaching objectives while also fostering a collaborative learning environment; students will have numerous opportunities for discussion and debate.

Next, we further discuss the issue of what the teacher needs to do in each phase of the three-phase process to make the lesson effective.

5.1. Phase 1: Motivation and goal orientation

According to our teaching experience, in phase 1 - motivational and goal-oriented, teachers need to use measures to give students a need to learn what they are about to learn, such as creating problem situations to stimulate students' thinking and pique their curiosity from the very beginning of the class. The teacher must know how to connect learned knowledge to lead to the knowledge to be learned (Jeong and González-Gómez, 2022). For example, in the case of teaching the general equation of a straight line, we used what they learned about the graph of the function $y = ax + b$ as a straight line to elicit motivation and goal direction.

In another experience, in order to evoke motivation and target direction in teaching the lesson vector pointing the direction of a straight line, we posed the problem at the beginning of the lesson as follows: "Can you tell me the ways to determine a straight line?" After students answered this question from their previous knowledge, the teacher said that, in addition to the ways they mentioned, "today we will learn another way thanks to the concept of a direction vector of a straight line. How is the concept of a vector of a straight line defined? In this lesson, we will study it."

5.2. Phase 2: knowledge formation

- Step by step, lead students to discover knowledge by themselves (Freudenthal, 2006). We often guide students to explore the knowledge to be learned by the system of questions or exercises; in many cases, we must give some guiding suggestions (Vygotsky, 2012).
- Creating a debate situation: When choosing students to report in front of the whole class, we choose the group with the wrong answer. The wrong solution often catches students' attention, stimulates thought, and creates a problem for debate in the classroom (Aderibigbe, 2021; Al-Osaimi and Fawaz, 2022).

> **2.** Do the conditions for two vectors, $\vec{u}$ and $\overrightarrow{M_0 M}$, have the same direction?
>
> *Answer:* Two vectors have the same direction when their corresponding coordinates are equal.

Figure 7. A wrong solution of one student group (see Appendix 2).

For instance, one group gave the wrong answer to the question, "Do the conditions for two vectors, $\vec{u}$ and $\overrightarrow{M_0 M}$, have the same direction?" (see Figure 7) We asked a representative of this group to present this

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Table 17. The intersection of two straight lines.

$D: Ax + By + C = 0, d: ax + by + c = 0$		
$\begin{cases} Ax + By + C = 0 \\ ax + by + c = 0 \end{cases}$	$\begin{cases} Ax + By + C = 0 \\ ax + by + c = 0 \end{cases}$	$\begin{cases} Ax + By + C = 0 \\ ax + by + c = 0 \end{cases}$
No solution	Only 1 solution: $(x_0; y_0)$	the infinite number of solutions
$D \ // \ d$	$D \cap d = A (x_0; y_0)$	$D \equiv d$

result to the class. Many students commented. Finally, the whole class agreed that the two vectors satisfying one of the following three conditions have the same direction.

- The bases of them are parallel or overlap.
- There exists a real number t such that one vector is equal to t times the other.
- Two ratios of corresponding coordinates are equal.

5.3. Phase 3: Practice and consolidation

To ensure the required level of knowledge and skills following the regulations of the math curriculum, we relied on teaching goals and standards of knowledge and skills prescribed by the Ministry of Education and Training to create practice exercises. In Phase 3, we paid particular attention to correcting mistakes and misunderstandings of students when doing practice exercises. In order to help students to remember the core knowledge of the lesson, we always took the time to consolidate or systematize what students had just learned at the end of the lesson. For instance, Table 17 is a summary of the lesson about the intersection of two straight lines.

In short, according to our experience, to effectively apply the three-phase teaching process that we have described, teachers must first study and analyze the content of knowledge (objects of teaching and learning activities) in depth and then map out the learning tasks that students must perform in order to acquire the object. Teachers should focus on creating learning motivation and a positive learning environment in the classroom at the start of each lesson; by doing so, students will enthusiastically and voluntarily perform learning actions with their classmates.

In the school year 2021, there were two teachers at the high school who used the three-phase teaching model to teach math. When asked about students' learning spirit and attitude in the classroom, they stated that using the three-phase teaching model, students work actively in study groups and discuss and answer questions posed by the teacher together. The study group, especially the average and weak students, boldly expressed their opinions. Previously, using the traditional classroom teaching method, average and weak students were passive and always silent; only good students responded to questions raised by the teacher in front of the class.

According to the opinions of two teachers who have used the three-phase model to teach, as mentioned above, the majority of the students have accepted the teaching method that we have proposed.

The challenge is to implement a three-stage teaching model that improves learning outcomes for both average and good students. Teachers, in our opinion, can solve this problem by changing the way they group students according to each learning task; in some cases, each group includes average students, while in others, average students and good students are separated. More research is needed to confirm the above solution.

6. Conclusion

In this paper, we developed a three-stage teaching model based on activity theory. In the experimental class, students, including weak students, actively participated in answering the teacher's questions; the teaching process encouraged social interaction in the classroom; as a result, the class's learning outcomes improved, meeting the knowledge and skill standards set by the Vietnamese Ministry of Education and Training. The goal of mathematics education in Vietnam has recently changed significantly. If the aim of teaching mathematics in the past was to help students grasp knowledge and skills according to prescribed standards, the purpose of teaching math in the coming years would be to develop students' competencies such as thinking and mathematical reasoning, mathematical communication competence, problem-solving, mathematical modeling competence, and the ability to use learning tools and media. We believe we can conduct extensive research based on activity theory to develop appropriate teaching and learning activities for students to develop each of the aforementioned competencies. Aside from activity theory, numerous teaching theories are currently available, including constructivist teaching theory, realistic mathematics education, experiential teaching, and so on. We can approach them to improve mathematics performance.

Appendix 1 (Posttest questions)

STRAIGHT-LINE EQUATIONS.

Question 1 (6 points): In *Oxy* plane, given ΔABC with $A(-4;1)$, $B(2;4)$, $C(2;-2)$.

a) (3 points) Write the general equation of the straight line passing through A and B.
b) (3 points) Write the parametric equation of the perpendicular bisector of AC.

Question 2 (2 points): In *Oxy* plane, given $d_1 : x - y - 2 = 0$ and $d_2 : x + 2y - 2 = 0$. Write the general equation of the straight line that passes through M and has the normal vector $\vec{u}(1;-1)$, where M is the intersection of d_1 and d_2.

Question 3 (2 points): Given ΔABC, the equation of AB and the equation of altitude BH are $AB : x - 3y + 11 = 0$ and $BH : 3x - 5y + 13 = 0$, respectively. Write the equation of the altitude AK.

Appendix 2

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