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# **Supervised group Lasso with applications to microarray data analysis**

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#### **Abstract**

**Background:** A tremendous amount of efforts have been devoted to identifying genes for diagnosis and prognosis of diseases using microarray gene expression data. It has been demonstrated that gene expression data have cluster structure, where the clusters consist of coregulated genes which tend to have coordinated functions. However, most available statistical methods for gene selection do not take into consideration the cluster structure.

**Results:** We propose a supervised group Lasso approach that takes into account the cluster structure in gene expression data for gene selection and predictive model building. For gene expression data without biological cluster information, we first divide genes into clusters using the K-means approach and determine the optimal number of clusters using the Gap method. The supervised group Lasso consists of two steps. In the first step, we identify important genes within each cluster using the Lasso method. In the second step, we select important clusters using the group Lasso. Tuning parameters are determined using V-fold cross validation at both steps to allow for further flexibility. Prediction performance is evaluated using leave-one-out cross validation. We apply the proposed method to disease classification and survival analysis with microarray data.

**Conclusion:** We analyze four microarray data sets using the proposed approach: two cancer data sets with binary cancer occurrence as outcomes and two lymphoma data sets with survival outcomes. The results show that the proposed approach is capable of identifying a small number of influential gene clusters and important genes within those clusters, and has better prediction performance than existing methods.

#### **Background**

Development in microarray techniques makes it possible to profile gene expression on a whole genome scale and study associations between gene expression and occurrence or progression of common diseases such as cancer or heart disease. A large amount of efforts have been devoted to identifying genes that have influential effects on diseases. Such studies can lead to better understanding of the genetic causation of diseases and better predictive models. Analysis of microarray data is challenging because of the large number of genes surveyed and small sample sizes, and presence of cluster structure. Here the clusters are composed of co-regulated genes with coordinated functions. Without causing confusion, we use the phrases "clusters" and "gene groups" interchangeably in this article.

Available statistical approaches for gene selection and predictive model building can be roughly classified into two categories. The first type focuses on selection of individual genes. Examples of such studies range from early studies of detecting marginally differentially expressed genes under different experimental settings [1] to selecting important genes for prediction of binary disease occurrence [2,3] and detecting genes associated with patients' survival risks [4,5]. Since the dimension of gene expressions measured  $(\sim 10^{3-4})$  is much larger than the sample size  $(\sim 10^2)$ , variable selection or model reduction are usually needed. Previously employed approaches include the singular value decomposition [6], principal component analysis [7], partial least squares [3] and Lasso [8], among others. These approaches aim at identifying a small subset of genes or linear combinations of genes-often referred as super genes, that can best explain the phenotype variations. A limitation of these approaches is that the cluster structure of gene expression data is not taken into account.

Biologically speaking, complex diseases such as cancer, HIV and heart disease, are caused by mutations in gene pathways, instead of individual genes. Statistically speaking, there exist genes with highly correlated expressions and should be put into clusters [9]. Although functional groups and statistical clusters may not match perfectly, they tend to have certain correspondence [10,11].

The second type of methods focuses on detecting differential gene clusters. Examples include the global test [12], the maxmean approach [13] and the gene set enrichment analysis [14]. In classification and survival analysis, cluster-based approaches have also been considered [5,15]. One approach is to construct gene clusters first, which can be based on statistical measurements (for example Kmeans or Hierarchical methods) or biological knowledge [16] or both. Then the mean expression levels are used as covariates in downstream analysis [17]. With the simple cluster based methods, it is assumed that if a cluster is strongly associated with the outcome, then all genes within that cluster are associated with the outcome, which is not necessarily true. Within cluster gene selection may still be needed.

Lasso [18] is a popular method for variable selection with high-dimensional data, since it is capable of producing sparse models and is computationally feasible. For example, this method has been used for correlating survival with microarray data [8]. Standard Lasso approach carries out variable selection at the individual gene level. A recent development of the Lasso is the group Lasso method [19] (referred as GLasso hereafter). The GLasso is designed for selecting groups of covariates. In a recent study, [20] proposes logistic classification with the GLasso penalty and considers its applications in microarray study. Direct application of the GLasso can identify important gene groups. However, it is not capable of selecting important genes within the selected groups. The fitted model may not be sparse, especially if the clusters are large.

In this article, we propose a supervised group Lasso (SGLasso) approach, which selects both important gene clusters and important genes within clusters. Compared to individual gene based approaches such as Lasso, the SGLasso takes into consideration the cluster structure and can lead to better predictions, as shown in our empirical studies. Compared to cluster based methods such as the GLasso, the within-cluster gene selection aspect of SGLasso leads to more parsimonious models and hence more interpretable gene selection results. The proposed approach is applicable as long as the objective function is well defined and locally differentiable. In this article, we apply the SGLasso to logistic binary classification and Cox survival analysis problems with microarray gene expression data.

# **Results**

# *Binary classification*

#### *Colon data*

In this dataset, expression levels of 40 tumor and 22 normal colon tissues for 6500 human genes are measured using the Affymetrix gene chips. A selection of 2000 genes with the highest minimal intensity across the samples has been made by [2], and these data are publicly available at [21]. The colon data have been analyzed in several previous studies using other statistical approaches, see for example [3,22,23].

# *Nodal data*

This dataset was first presented by [24,25]. It includes expression values of 7129 genes of 49 breast tumor samples. The expression data were obtained using the Affymetrix gene chip technology and are available at [26]. The response describes the lymph node (LN) status, which is an indicator for the metastatic spread of the tumor, a very important risk factor for the disease outcome. Among the 49 samples, 25 are positive (LN+) and 24 are negative (LN-). We threshold the raw data with a floor of 100 and a ceiling of 16000. Genes with max(*expression*)/ min(*expression*) < 10 and/or max(*expression*) – min(*expression*) < 1000 are also excluded [1]. 3332 (46.7%) genes pass the first step screening. A base 2 logarithmic transformation is then applied. The Nodal data have also been studied by [22].

Although there is no limitation on the number of genes that can be used in the proposed approach, we first identify 500 genes for each dataset based on marginal significance to gain further stability as in [23]. Compute the sample standard errors of the *d* biomarkers  $se_{(1)},..., se_{(d)}$ and denote their median as *med.se*. Compute the adjusted standard errors as  $0.5(se_{(1)} + med.se)$ ,...,  $0.5(se_{(d)} + med.se)$ . Then the genes are ranked based on the *t*-statistics computed with the adjusted standard errors. The 500 genes with the largest absolute values of the adjusted *t*-statistics are used for classification. The adjusted *t*-statistic is similar to a simple shrinkage method discussed in [27].

For the Colon and Nodal data, clusters are constructed using the K-means approach and the Gap statistic is used to select the optimal number of clusters. We show in Figure 1 the Gap statistic as a function of the number of clusters. 9 clusters are constructed for the Colon data and 20 clusters are constructed for the Nodal data. Details of the clustering information are available upon request. With the generated clusters, we apply the proposed SGLasso approach. Tuning parameters are chosen using 3-fold cross validation. Summary model features are shown in Table 1. For the Colon data, 22 genes are present in the final model, representing 8 clusters. For the Nodal data, 66 genes are selected, representing 17 clusters. We list the identified genes in Tables 2 and 3.

For the Colon data, gene Has.1039 has also been identified to be associated with Colon cancer in [28,29]. Gene Hsa.42949 is estrogen sulfotransferase. Research show that certain compounds, such as soy, have protective effect for colon cancer. The protective role of these compounds could be due to an ability to inhibit competitively the activation of promutagenic estrogen metabolites into carcinogens by estrogen sulfotransferases. The official symbol of gene Hsa.1454 is CSNK1E. Studies have revealed a negative regulatory function of CK1 in the Wnt signaling pathway, where CK1 acts as a negative regulator of the LEF-1/ beta-catenin transcription complex, thereby protecting







#### **Table 1: Comparison of estimation and prediction performance of different approaches.**

Nonzero: number of genes in the final models. Cluster: number of clusters in the final models. Prediction: for Colon and Nodal, Leave-One-Out prediction error; For Follicular and MCL, the logrank statistic.

#### **Table 2: Colon data: genes with nonzero estimates from SGLasso.**



#### **Table 3: Nodal data: genes with nonzero estimates from SGLasso.**



$-0.094$	AF009674 at	Homo sapiens axin (AXIN) mRNA, partial cds.
$-0.379$	AFFX-BioB-3 at	[04423 E coli bioB gene biotin synthetase]
$-0.184$	AFFX-BioDn-3_at	[04423 E coli bioD gene dethiobiotin synthetase
0.058	HG2465-HT4871_at	Dna-Binding Protein Ap-2, Alt. Splice 3
0.017	D00762_at Human mRNA for proteasome subunit HC8	
$-0.090$	HG1612-HT1612_at	Macmarcks
$-0.062$	U09178 <sub>_S_at</sub>	Human dihydropyrimidine dehydrogenase mRNA, complete cds
0.017	U29175 at	Human transcriptional activator (BRGI) mRNA, complete cds.
$-1.184$	U39817 at	Human Bloom syndrome protein (BLM) mRNA, complete cds
$-0.211$	U41344_at	Human prolargin (PRELP) gene, 5' flanking sequence
$-0.080$	X16832_at	Human mRNA for cathepsin H (EC 3.4.22.16)
0.013	X99226 at	H.sapiens mRNA for FAA protein
0.000	Z49878 at	H.sapiens mRNA for guanidinoacetate N-methyltransferase
$-0.105$	X68560 at	H.sapiens SPR-2 mRNA for GT box binding protein
0.048	HG3998-HT4268 at	L-Glycerol-3-Phosphate:Nad+ Oxidoreductase
0.001	U79285 at	Human clone 23828 mRNA sequence
0.003	X79981 at	H.sapiens VE-cadherin mRNA
0.024	X98176_at	H.sapiens mRNA for MACH-beta-1 protein.
0.001	Z18956_at	H.sapiens mRNA for taurine transporter
0.014	U18548 at	Human GPR12 G protein coupled-receptor gene, complete cds.
$-0.281$	Z22536 at	Homo sapiens ALK-4 mRNA, complete CDS

**Table 3: Nodal data: genes with nonzero estimates from SGLasso.** *(Continued)*

cells from development of cancer. Gene Hsa.8214 has official symbol DCGR6. It has been shown to be associated with mammary cancer and tumor cell proliferation in general. Gene Hsa.462 (official symbol SERPINC1) has been shown to be related to cancer cell proliferation. Gene Hsa.627 is also identified as a Colon cancer biomarker in [28]. Gene Hsa.696 has official symbol BTN1A1. RT-PCR analysis has revealed strongest expression of BTNL3 in small intestine, colon, testis, and leukocytes. Tristetraproline (gene Hsa.1682) has been reported to negatively regulate tumor necrosis factor alpha (TNF-alpha) production by binding the AU-rich element within the 3' noncoding sequences of TNF-alpha mRNA. Gene Hsa.3016 is S100P protein. 100P is expressed in human cancers, including breast, colon, prostate, and lung. In colon cancer cell lines, its expression level was correlated with resistance to chemotherapy.

For the Nodal data, gene U27185\_at has official Symbol RARRES1. Also known as TIG1, the expression of this gene is upregulated by tazarotene as well as by retinoic acid receptors. Silencing of TIG1 promoter by hypermethylation is common in human cancers and may contribute to the loss of retinoic acid responsiveness in some neoplastic cells. The role of the matrilins (gene U69263) in tumorigenesis has not been studied. However, a related family of proteins (fibulins) has been implicated in cancer. Increased fibulin expression is seen in breast cancer, lung adenocarcinoma, colon cancer, and other solid tumors, suggesting that these proteins might play a role in tumor

formation or progression. Gene U07223 is a member of the chimerin family and encodes a protein with a phorbol-ester/DAG-type zinc finger, a Rho-GAP domain and an SH2 domain. Decreased expression of this gene is associated with high-grade gliomas and breast tumors, and increased expression of this gene is associated with lymphomas. Findings suggest that CEACAM1 (gene X16354) participates in immune regulation in physiological conditions and in pathological conditions, such as inflammation, autoimmune disease, and cancer. Gene D87071\_at is a confirmed breast cancer biomarker. Ras (gene J00277) and c-Myc play important roles in the up-regulation of nucleophosmin/B23 during proliferation of cells associated with a high degree of malignancy, thus outlining a signaling cascade involving these factors in the cancer cells. GNA11 (gene M69013) is involved in signaling of gonadotropin-releasing hormone receptor, which negatively regulates cell growth.

Down-regulation is suggested to be involved in human breast cancers. Gene D59532 encodes a member of the Ctype lectin/C-type lectin-like domain (CTL/CTLD) superfamily. Members of this family share a common protein fold and have diverse functions, such as cell adhesion, cell-cell signaling, glycoprotein turnover, and roles in inflammation and immune response. Gene X53587 is human mRNA for integrin beta 4. Colonization of the lungs by human breast cancer cells is correlated with cell surface expression of the alpha(6)beta(4) integrin and adhesion to human CLCA2 (hCLCA2), Tumor cell adhesion to endothelial hCLCA2 is mediated by the beta(4) integrin, establishing for the first time a cell-cell adhesion property for this integrin that involves an entirely new adhesion partner. This adhesion is augmented by an increased surface expression of the alpha $(6)$  beta $(4)$ integrin in breast cancer cells selected in vivo for enhanced lung colonization but abolished by the specific cleavage of the beta(4) integrin with matrilysin. Wnt-5a (gene L20861) has been shown to influence the metastatic behavior of human breast cancer cells, and the loss of Wnt-5a expression is associated with metastatic disease. NFAT1, a transcription factor connected with breast cancer metastasis, is activated by Wnt-5a through a Ca2+ signaling pathway in human breast epithelial cells. Endogenous RXR beta (gene M84820) contributes to ERE binding activity in nuclear extracts of the human breast cancer cell line MCF-7. Detailed microscopic analysis of the morphology of MCF7 breast cancer cells lacking CtBPs (gene U37408) reveals an increase in the number of cells containing abnormal micronucleated cells and dividing cells with lagging chromosomes, indicative of aberrant mitotic chromosomal segregation. Methyl-CpG-binding domain protein-2 (gene X99687) mediates transcriptional repression associated with hypermethylated GSTP1 CpG islands in MCF-7 breast cancer cells. The nmt55/ p54nrb protein (gene U02493) is post-transcriptionally regulated in human breast tumors leading to reduced expression in ER- tumors and the expression of an amino terminal altered isoform in a subset of ER+ tumors. Expression of elastin (gene U77846) in breast carcinoma cells has been demonstrated by immunohistochemistry and in situ hybridization. Cytochrome P450 1B1 (CYP1B1, gene X07618) is active in the metabolism of estrogens to reactive catechols and of different procarcinogens. The CYP1B1 gene polymorphisms do not influence breast cancer risk overall but may modify the risk after long-term menopausal hormone use. Genetic deficiency of DPYD enzyme (gene U09178) results in an error in pyrimidine metabolism associated with thymine-uraciluria and an increased risk of toxicity in cancer patients receiving 5-flourouracil chemotherapy.

We evaluate the prediction performance of the proposed approach via Leave-One-Out (LOO) cross validation. For the Colon and Nodal data, we compute the LOO cross validation errors. In this evaluation process, tuning parameters are computed using 3-fold cross validation for each reduced set. For comparison purposes, we also consider the following alternative approaches.

1. Lasso: we ignore the clustering structures and apply the Lasso directly. This approach has been considered in [18] for Cox survival analysis and [23] for logistic binary classification.

2. GLasso: we ignore the first step supervised selection and apply the GLasso directly. For binary classification, the GLasso has been investigated in [20].

3. Simple clustering: with the generated clusters, we compute the median of the gene expression level for each cluster. The medians are used as covariates. Since the number of "covariates" is less than the sample size, logistic/Cox models can be fit directly. This mimics the approach in [5].

For the alternative approaches, we also compute the LOO cross validation errors. Tuning parameters when presented are also chosen via 3-fold cross validation. Comparison results are shown in Table 1.

We can see from Table 1 that the SGLasso is capable of feature selection at both the cluster level and the within cluster gene level. The number of genes selected is much less than its counterpart from the simple clustering approach and GLasso. The Lasso is also capable of selecting a small number of genes. Especially we note that the number of genes selected by Lasso is smaller than by SGLasso. For simple data sets such as the Colon data, the Lasso prediction error is the same as the SGLasso. However for data sets that are more difficult to classify (Nodal), the SGLasso prediction error is much smaller. Both data sets have also been analyzed by other approaches. For the Colon data, ROC based approach has prediction error 0.14 [23]; LogitBoost has classification errors 0.145, 0.194 and 0.161 [22]; and classification tree has classification error 0.145 [22]. We note that since different sets of genes are used in those studies, Table 1 only provides rough comparisons. For the Nodal data, in [22], LogitBoost yields prediction error 0.184, 0.265 and 0.184, while classification tree has prediction error 0.204 and 1-nearest neighbor has prediction error 0.367.

# *Survival analysis*

#### *Follicular lymphoma data*

Follicular lymphoma is the second most common form of non-Hodgkin's lymphoma, accounting for about 22 percent of all cases. A study was conducted to determine whether the survival probability of patients with follicular lymphoma can be predicted by the gene-expression profiles of the tumors at diagnosis [5]. Fresh-frozen tumorbiopsy specimens and clinical data from 191 untreated patients who had received a diagnosis of follicular lymphoma between 1974 and 2001 were obtained. The median age at diagnosis was 51 years (range 23 to 81), and the median follow up time was 6.6 years (range less than 1.0 to 28.2). The median follow up time among patients alive at last follow up was 8.1 years. Eight records with missing survival information are excluded from the

downstream analysis. Detailed experimental protocol can be found in [5].

Affymetrix U133A and U133B microarray genechips were used to measure gene expression levels from RNA samples. A log2 transformation was applied to the Affymetrix measurements. We first filter the 44928 gene measurements with the following criteria: (1) the max expression value of each gene across 191 samples must be greater than 9.186 (the median of the maximums of all probes). (2) the max-min should be greater than 3.874 (the median of the max-min of all probes). (3) Compute correlation coefficients of the uncensored survival times with gene expressions. Select the genes whose correlation with survival time is greater than 0.2. There are 729 genes that pass this screening process. We normalize genes across samples to have mean 0 and variance 1.

# *Mantel cell lymphoma data*

[4] reported a study using microarray expression analysis of mantle cell lymphoma (MCL). The primary goal of this study was to discover genes that have good predictive power of patient's survival risk. Among 101 untreated patients with no history of previous lymphoma included in this study, 92 were classified as having MCL, based on established morphologic and immunophenotypic criteria. Survival times of 64 patients were available and other 28 patients were censored. The median survival time was 2.8 years (range 0.02 to 14.05 years). Lymphochip DNA microarrays [15] were used to quantify mRNA expression in the lymphoma samples from the 92 patients. The gene expression data that contain expression values of 8810 cDNA elements are available at [30].

We pre-process the data as follows to exclude noises and gain further stability: (1) Compute the variances of all gene expressions; (2) Compute correlation coefficients of the uncensored survival times with gene expressions; and (3) Select the genes with variances larger than the first quartile and with correlation coefficients larger than 0.25. 834 out of 8810 genes (16.5%) pass the above initial screening. We standardize these genes to have zero mean and unit variance.

We use the K-means method and Gap statistic in the cluster analysis. 34 (Follicular) and 30 (MCL) clusters are established. Plot similar to Figure 1 can be obtained and omitted here. Model estimation and prediction features are also provided in Table 1. We show in Tables 4 and 5 the genes included in the final models.

For the Follicular data, gene 23098\_x\_a is associated with tumor protein p53. In transfected cells and KSHV-infected B lymphoma cells, KSHV-encoded latency-associated nuclear antigen (LANA) expression stimulates degradation of tumor suppressors von Hippel-Lindau and p53. In a recent case study with a Japanese girl who had EEC3 and developed diffuse large B-cell type non-Hodgkin lymphoma, researchers identified heterozygosity for a 1079A-G transition in exon 8 of the TP73L gene, resulting in a germline asp312-to-gly (D312G) mutation. Gene 223333\_s\_a is a member of the angiopoietin/angiopoietin-like gene family and encodes a glycosylated, secreted protein with a fibrinogen C-terminal domain. The encoded protein may play a role in several cancers and it also has been shown to prevent the metastatic process by inhibiting vascular activity as well as tumor cell motility and invasiveness. Gene 224357\_s\_a encodes a member of the membrane-spanning 4A gene family. Members of this nascent protein family are characterized by common structural features and similar intron/exon splice boundaries and display unique expression patterns among hematopoietic cells and nonlymphoid tissues. Chemokines (genes 204470\_at, 205114\_s\_a) are a group of small (approximately 8 to 14 kD), mostly basic, structurally related molecules that regulate cell trafficking of various types of leukocytes through interactions with a subset of 7-transmembrane, G protein-coupled receptors. Chemokines also play fundamental roles in the development, homeostasis, and function of the immune system, and they have effects on cells of the central nervous system as well as on endothelial cells involved in angiogenesis or angiostasis. Serpin Al (gene 211429\_s\_a) has an invasionpromoting effect in anaplastic large cell lymphoma. Gene 216950 s a has official symbol FCGR1A. Findings showed that both Fcgamma RIA and FcgammaRIIA mediated enhanced dengue virus immune complex infectivity but that FcgammaRIIA appeared to do so far more effectively. TRPM4-mediated (gene 219360\_s\_a) depolarization modulates Ca2+ oscillations, with downstream effects on cytokine production in T lymphocytes. The protein encoded by gene 210973\_s\_a is a member of the fibroblast growth factor receptor (FGFR) family, where amino acid sequence is highly conserved between members and throughout evolution. Chromosomal aberrations involving this gene are associated with stem cell myeloproliferative disorder and stem cell leukemia lymphoma syndrome. FGFR-1 is expressed in early hematopoietic/endothelial precursor cells, as well as in a subpool of endothelial cells in tumor vessels. Gene 227697\_at encodes a member of the STAT-induced STAT inhibitor (SSI), also known as suppressor of cytokine signaling (SOCS), family. Over expression of suppressor of cytokine signaling 3 is associated with anaplastic large cell lymphoma.

Genes identified in the MCL study also have sound biological basis. When the positive cells are treated with phosphatidylinositol-specific phospholipase C (gene Hs.522568), a significant decrease in both stain intensity

# Estimate Gene ID Gene Description 0.035 227117\_at CDNA FLJ40762 fis, clone TRACH2002847 -0.049 228671\_at hypothetical protein LOC199953 0.070 228776 at gap junction protein, alpha 7, 45 kDa (connexin 45) 0.055 230448\_at hypothetical protein MGC15523 -0.042 230297\_x\_a synaptic Ras GTPase activating protein 1 homolog 0.091 230938\_x\_a activating transcription factor 5 0.053 209863\_s\_a tumor protein p73-like 0.002 224125\_at pleckstrin homology domain containing, family N member 1 0.002 230826\_at monocyte to macrophage differentiation-associated 2 0.001 238605\_at Transcribed locus 0.005 222545\_s\_a chromosome 10 open reading frame 57 0.062 239565 at CDNA FLJ37010 fis, clone BRACE2009732 0.022 242904  $\times$  a 0.026 222015\_at Casein kinase 1, epsilon 0.032 219361\_s\_a interferon stimulated exonuclease gene 20 kDa-like 1 0.046 223333<sub>\_s\_a</sub> angiopoietin-like 4 0.084 224357\_s\_a membrane-spanning 4-domains, subfamily A, member 4 0.046 204470\_at chemokine (C-X-C motif) ligand 1 0.023 205114\_s\_a chemokine (C-C motif) ligand 3 0.118 208470<sub>\_s\_a</sub> haptoglobin 0.058 237542 at Transcribed locus 0.052 202953\_at complement component 1, q subcomponent, B chain 0.022 206214\_at phospholipase A2, group VII 0.085 210321\_at granzyme H (cathepsin G-like 2, protein h-CCPX) 0.054 214038\_at chemokine (C-C motif) ligand 8 -0.074 201841\_s\_a heat shock 27 kDa protein 1 -0.028 211429\_s\_a serpin peptidase inhibitor, clade A, member 1 -0.120 211470<sub>\_S\_a</sub> sulfotransferase family, cytosolic, IC, member 1 0.081 216950<sub>\_s\_a</sub> Fc fragment of IgG, high affnity Ia, receptor (CD64) -0.056 222694\_at hypothetical protein MGC2752 -0.042 232618\_at chromosome Y open reading frame 15A -0.016 232874\_at Dedicator of cytokinesis 9 -0.034 237222\_at 0.024 240105\_at Chromosome 21 open reading frame 66 -0.016 241755\_at Ubiquinol-cytochrome c reductase core protein II -0.032 242306\_at TPA regulated locus -0.040 243705\_at DDHD domain containing 1 0.049 237131\_at hypothetical protein LOC645469 0.042 238359 at -0.045 242601\_at hypothetical protein LOC253012 0.049 243101\_x\_a Chromosome 20 open reading frame 160 0.059 219360<sub>\_S\_a</sub> transient receptor potential cation channel, subfamily M -0.006 226665\_at AHA1, activator of heat shock 90 kDa protein ATPase homolog 2 -0.007 231852\_at -0.009 241946\_at zinc finger, DHHC-type containing 21 -0.009 208067\_x\_a ubiquitously transcribed tetratricopeptide repeat gene -0.009 210973\_s\_a fibroblast growth factor receptor 1 -0.006 220235\_s\_a chromosome 1 open reading frame 103

#### **Table 4: Follicular data: genes with nonzero estimates from SGLasso.**





**Table 4: Follicular data: genes with nonzero estimates from SGLasso.** *(Continued)*

and percentage of positive cells is demonstrated by immunofluorescence. The protein encoded by gene Hs. 120949 is the fourth major glycoprotein of the platelet surface and serves as a receptor for thrombospondin in platelets and various cell lines. Since thrombospondins are widely distributed proteins involved in a variety of adhesive processes, this protein may have important functions as a cell adhesion molecule. Mutations in this gene cause platelet glycoprotein deficiency. The protein encoded by gene Hs.84113 belongs to the dual specificity protein phosphatase family. It was identified as a cyclindependent kinase inhibitor, and has been shown to interact with, and dephosphorylate CDK2 kinase, thus prevent the activation of CDK2 kinase. This gene was reported to be deleted, mutated, or overexpressed in several kinds of cancers. Studies show TOP2A (gene Hs.156346) is a proliferation marker, indicator of drug sensitivity, and prognostic factor in mantle cell lymphoma. This gene encodes a DNA topoisomerase, an enzyme that controls and alters the topologic states of DNA during transcription. The gene encoding this enzyme functions as the target for several

anticancer agents and a variety of mutations in this gene have been associated with the development of drug resistance. Reduced activity of this enzyme may also play a role in ataxia-telangiectasia. Gene Hs.497741 encodes a protein that associates with the centromere-kinetochore complex. Autoantibodies against this protein have been found in patients with cancer or graft versus host disease. DNA Pol theta (gene Hs.241517) has a specialized function in lymphocytes and in tumor progression. Gene Hs.298990 encodes tumor suppressor proteins. The protein encoded by gene Hs. 105956 catalyzes the transfer of galactose to lactosylceramide to form globotriaosylceramide, which has been identified as the  $P(k)$  antigen of the P blood group system. PLG (gene Hs.368912) has the potential to simultaneously regulate calcium signaling pathways and regulate pHi via an association with NHE3 linked to DPP IV, necessary for tumor cell proliferation and invasiveness.In the leave-one-out (LOO) based evaluation, denote  $\hat{\pmb{\beta}}\left(-i\right)$  as the LOO estimate of  $\beta$  based on the reduced data set with the *ith* subject removed. We then compute the pre-

Estimate	Gene ID	<b>Gene Description</b>
0.011	24860	Hs.522568, Phosphatidylinositol-specific phospholipase C
0.005	26556	Hs.173438, Fas apoptotic inhibitory molecule
0.018	28537	Hs.120949, CD36 antigen
0.030	28640	Hs.84113, Cyclin-dependent kinase inhibitor 3
$-0.004$	28679	Hs.469723, RNA, U17D small nucleolar
0.005	30010	Hs.85137, Cyclin A2
0.009	32690	Hs.3104, Kinesin family member 14
0.010	32973	Hs.58992, SMC4 structural maintenance of chromosomes 4-like 1
0.078	27095	Hs.156346, Topoisomerase (DNA) Il alpha 170 kDa
0.094	30157	Hs.497741, Centromere protein F, 350/400 ka
0.100	30898	Hs.532755, Likely ortholog of mouse gene trap locus 3
0.084	31049	Hs.241517, Polymerase (DNA directed), theta
0.080	34771	Hs.524390, Tubulin, alpha, ubiquitous
$-0.067$	16541	Hs.30054, Coagulation factor V
$-0.065$	23972	Hs.431009, Zinc finger protein, multitype 2
$-0.036$	24262	
$-0.061$	24379	Hs.120260, Immunoglobulin superfamily receptor translocation associated 1
$-0.056$	25058	Hs.298990, actin dependent regulator of chromatin
$-0.101$	25171	Hs.21388, Zinc finger, DHHC domain containing 21
$-0.103$	26192	Hs.530274, Aldolase B, fructose-bisphosphate
$-0.037$	27659	Hs.437336, Hypothetical protein MGC61571
$-0.091$	29653	Hs.40758, RAB30, member RAS oncogene family
$-0.053$	31196	Hs.508010, Fibronectin type III domain containing 3
$-0.033$	32497	
$-0.076$	32947	Hs.522863, Chromosome Y open reading frame 15A
$-0.076$	33506	Hs.364045, Hypothetical protein LOC92270
$-0.059$	33892	Hs.105956, Alpha 1,4-galactosyltransferase
$-0.060$	34438	Hs.368912, Dipeptidylpeptidase 4

**Table 5: MCL data: genes with nonzero estimates from SGLasso.**

dicted risk score  $\hat{\bm{\beta}}$  (-*i*)  $\mathbf{Z}_i'$  for the *i<sup>th</sup>* subject. Since the Cox model is a special form of the transformation model, the uncensored survival time depends on β**Z**' via a generalized linear model. So the prediction evaluation can be based on comparing the survival functions of groups composed of different range of β**Z**'. A simple approach is to first dichotomize the predicted risk scores at the median to create two risk groups with equal sizes. We then compare the survival functions of the two generated risk groups. A significant difference (measured by the logrank test statistic with degree of freedom 1) indicates satisfactory prediction performance.

We show the prediction comparison in Table 1. We can again see that the models obtained under SGLasso are much smaller than those from the simple cluster approach and GLasso. However the SGLasso models are larger than their Lasso counterparts. For both data sets, the proposed SGLasso has the largest logrank statistics, indicating the best prediction performance. For the Follicular data, only Lasso and SGLasso have logrank statistics with

p-value less than 0.05. The GLasso and simple approaches cannot properly predict survival based expressions. For the MCL data, all approaches have logrank statistics with p-value less than 0.05, with the largest logrank statistic from the SGLasso.

# **Discussion**

# *Remark: clustering method selection*

Gene expression clustering can be based on many approaches including the K-means, hierarchical, selforganizing map, and model based methods [31], among many others. Without making specific data assumptions, there do not exist optimal clustering method. The proposed K-means approach has been extensively used in microarray study. It is attractive because of its computational simplicity and optimality under the normal distribution assumption. We have also analyzed the four data sets using other clustering schemes including Hierarchical clustering. Our studies show that other approaches generate comparable or worse prediction results than the Kmeans approach. Since the K-means method yields satisfactory estimation and prediction results for the four data sets and other data (results not shown), we focus on the

K-means approach only. A comprehensive comparison of different clustering is interesting but beyond the scope of this paper.

We propose using the Gap statistic for selecting the optimal number of clusters. Empirical studies in [32] and this paper show that it can lead to satisfactory results. We note again that there is no best selection method for number of clusters, unless stronger data assumptions are made. We refer to [33] for comprehensive discussions of gene clustering.

# *Remark: prediction evaluation*

In our examples, we carry out gene screening before analysis. The goal of such screening is to remove noisy genes and obtain more stable models. Gene screening has been employed in almost all microarray studies. We note that such screening may lead to bias in the prediction evaluation, since all records have been used in the screening. However since the number of genes passed screening is still large, the bias in the prediction is expected to be small. Especially all four approaches listed in Table 1 use the same sets of genes. So comparisons in Table 1 should be fair.

#### *Remark: two-step gene selection*

The proposed SGLasso is a two-step approach. Another two-step gene selection approach is the supervised principal component analysis (SPCA, [34]). Significant differences exist between the SGLasso and other two-step approaches like SPCA. In other two-step approaches, the first supervised screening step considers all genes simultaneously. The cluster structure is ignored, whereas the main merit of the SGLasso is the usage of the cluster structure. Moreover, in SPCA, the selected features are the principal components. Although they may have satisfactory prediction performance, biological interpretations may not be clear. As a comparison, clear biological interpretations of gene identification results are available as shown in the Results section.

# **Conclusion**

Gene selection is essential in classification or survival analysis using high dimensional microarray data. Such selection can generate parsimonious, stable models with interpretable estimates. In this article, we propose the SGLasso approach. This approach explicitly takes into account the cluster structure and carries out feature selection at both the gene and cluster levels. Applications of this approach to four data sets show that it can produce parsimonious predictive models with satisfactory prediction performance.

Compared to available approaches, the SGLasso is the first to consider penalized gene selection at both the cluster level and the within cluster level. Compared to individual gene selection methods, the SGLasso is capable of taking cluster information into consideration. This makes it possible to reveal the associations between diseases and gene clusters. With the proposed approach, we can identify coregulated genes which are jointly significantly associated with diseases. Compared to simple cluster based methods, SGLasso carries out the additional within cluster selection. This leads to a small number of genes within each cluster. So beyond identifying influential clusters, the proposed approach can also identify the genes that actually cause the association. From a scientific point of view, identifying important genes (beyond identifying important clusters) is critical.

As we point out, gene clusters can also be constructed based on biological information [16]. We should use such information whenever available. However we also note that such pathway information is far from complete or not available for many genes. In the absence of such information, we can use clustering methods to divide genes into groups. It is of interest to compare results based on statistical clustering with those based on biological clustering, when full pathway information is available. However, such empirical studies is beyond the scope of the current paper and will be pursued in later studies.

# **Methods**

# *Gene clustering*

The proposed SGLasso assumes that the cluster structure has been well defined. When clusters of genes in the same function groups can be constructed based on biological information such as GO [16], such clusters can be used in the analysis. However it is often the case that gene group information may only be partially available or even not available. In this case we propose defining cluster structure based on statistical measurements [9].

We use the K-means approach in this paper. There exist many alternative clustering methods, such as the hierarchical clustering, self-organizing map, tree-truncated vector quantization method, among others. For data sets with unknown data structures, there exists no dominating approach. We use the K-means approach since it is computationally affordable and relatively robust.

We propose using the Gap statistic [32] to determine the optimal number of clusters. With the K-means approach, we first choose *M*-the largest number of clusters. Then for  $m = 1, ..., M$ :

1. Generate *m* clusters using the K-means approach. Denote  $\mathit{rss}_m$  as the total within block sum of squares.

2. Create a new data set by separately permuting each gene expression measurements. Apply the K-means method to the permuted expression data. Let  $\widetilde{rss}_m$  denote the resulting within cluster sum of squares. Repeat this for a number of times and compute the average  $\widehat{ave}(\widetilde{rss}_m)$ .

3. Compute the Gap statistic as  $gap(m) = ave(\widetilde{rss}_m) - rss_m$ .

Choose the value *m* that maximizes *gap*(*m*). We refer to [32] for detailed discussions of the Gap statistic.

# *Data settings*

Let **Z** be a length *d* vector of gene expressions, and let *Y* be the clinical outcome of interest. Assume that *n* i.i.d. copies of (*Y*, **Z**) are available. We generate *m* gene clusters using the K-means approach, where *m* is chosen using the Gap statistic. We assume that the clusters have sizes  $p_1$ ,...,  $p_m$ with  $p_1 + ... + p_m = d$ . We denote  $Z = (Z^1, ..., Z^m)$ , where  $Z^i$ contains the  $p_i$  gene expressions in the  $i<sup>th</sup>$  cluster for  $i = 1, ...,$ *m*. We assume that *Y* is associated with **Z** through a parametric or semiparametric model  $Y \sim \phi(\beta Z')$  with a regression function  $\phi$  and unknown regression coefficient  $\beta$ , where  $\beta = (\beta^1, \ldots, \beta^n)$  and  $\beta = (\beta^1, \ldots, \beta^{ip_i})$  for  $i = 1, \ldots, m$ . In this article, we study the binary classification and censored survival analysis problems because of their wide applications.

# *Binary classification*

For classification problems, *Y* is the categorical variable indicating the disease status, for example occurrence or stage of cancer. We focus on binary classification only. Suppose that  $Y = 1$  denotes the presence and  $Y = 0$  indicates the absence of disease. We assume the commonly used logistic regression model, where the logit of the conditional probability is logit( $P(Y = 1|Z)$ ) =  $\alpha + \beta Z'$  and  $\alpha$  is the unknown intercept. Based on a sample of *n* iid observations  $(Y_1, Z_1)$ ,...,  $(Y_n, Z_n)$ , the maximum likelihood estimator is defined as  $(\tilde{\alpha}, \; \hat{\beta})$  = argmax<sub> $\alpha, \beta$ </sub>R<sub>n</sub>  $(\alpha, \beta)$ , where

$$
R_n(\alpha, \beta) = \sum_{j=1}^n Y_j \log \left( \frac{\exp \left( \alpha + \beta Z_j' \right)}{1 + \exp \left( \alpha + \beta Z_j' \right)} \right) + \left( 1 - Y_j \right) \log \left( \frac{1}{1 + \exp \left( \alpha + \beta Z_j' \right)} \right)
$$

We always keep the intercept  $\alpha$  in the model. For simplicity, we denote  $R_n(\alpha, \beta)$  as  $R_n(\beta)$ .

# *Survival analysis*

For right censored survival data,  $Y = (T, \Delta)$ , where  $T =$  $min(U, V)$  and  $\Delta = I(U \leq V)$ . Here *U* and *V* denote the event time of interest and the random censoring time, respectively. The most widely used model for right censored data is the Cox proportional hazards model [35] which assumes that the conditional hazard function  $\lambda(u|Z) = \lambda_0$ (*u*) exp( $\beta$ **Z**<sup>'</sup>), where  $\lambda_0$  is the unknown baseline function and β is the regression coefficient. Based on a sample of *n* iid observations  $X_j = (Y_j, Z_j)$ ,  $j = 1,..., n$ , the maximum partial likelihood estimator is defined as the value  $\hat{\boldsymbol{\beta}}$  that maximizes

$$
R_n(\boldsymbol{\beta}) = \prod_{j=1}^n \left\{ \frac{\exp(\beta Z_j')}{\sum_{k \in r_j} \exp(\beta Z_j')} \right\}^{\delta_j},
$$

where  $r_j = \{k : T_k \geq T_j\}$  is the risk set at time  $T_j$ .

#### *Supervised group Lasso*

For the logistic regression and Cox proportional hazards models, the SGLasso consists of the following steps.

1. For cluster  $i = 1,..., m$ , compute  $\hat{\boldsymbol{\beta}}$ <sup>*i*</sup>-the cluster-wise Lasso estimate of β*<sup>i</sup>* . Especially,

$$
\hat{\boldsymbol{\beta}}^i
$$
 = argmax  $R_n(\boldsymbol{\beta})$  subject to  $|\boldsymbol{\beta}^i| + ... + |\boldsymbol{\beta}^{i p_i}| \leq u_i$ 

where  $u_i$  is the data-dependent tuning parameter and

$$
R_n\left(\boldsymbol{\beta}^i\right) = \sum_{j=1}^n Y_j \log \left(\frac{\exp\left(\alpha + \boldsymbol{\beta}^i \mathbf{Z}_j^i\right)}{1 + \exp\left(\alpha + \boldsymbol{\beta}^i \mathbf{Z}_j^i\right)}\right) + \left(1 - Y_j\right) \log \left(\frac{1}{1 + \exp\left(\alpha + \boldsymbol{\beta}^i \mathbf{Z}_j^i\right)}\right)
$$
  
for binary classification and

for binary classification and

$$
R_n\left(\boldsymbol{\beta}^i\right) = \prod_{j=1}^n \left\{\frac{\exp\left(\boldsymbol{\beta}^i \mathbf{Z}_j^{i'}\right)}{\sum_{k \in r_j} \exp\left(\boldsymbol{\beta}^i \mathbf{Z}_j^{i'}\right)}\right\}^{\delta_j}
$$

for Cox survival analysis. That is for cluster *i*, we only use genes within that cluster to construct predictive models. Gene selection within that cluster is achieved with the Lasso. Sparse models are achieved when  $u_i \rightarrow 0$ . We propose selection of *ui* using V-fold cross validation [36]. Especially we note that tuning parameters  $u_i$  are selected for each cluster separately. So we allow different tuning parameters, hence different degrees of regularization for different clusters. This flexibility allows us to detect more subtle structures that cannot be detected by applying the Lasso method to all the genes/clusters at the same time.

2. For each cluster, the Lasso models have only a small number of nonzero coefficients. For cluster *i*, denote  $\tilde{\mathbf{Z}}^i$  as the reduced covariate vector composed of covariates with nonzero estimated coefficients in Step 1 cluster-wise models. Denote  $\tilde{\beta}^i$  as the corresponding reduced unknown coefficient. We note that the dimension of the genes may be greatly reduced via Step 1. For example in the examples, a cluster with size  $\sim$  20 may only have 2  $\sim$  5 genes presented in the reduced data.

3. Construct the joint predictive model under the GLasso constraint. Especially,

$$
\hat{\tilde{\boldsymbol{\beta}}} = \arg \max \tilde{R}_n \left( \tilde{\boldsymbol{\beta}} \right) \text{ subject to } \left| \tilde{\boldsymbol{\beta}}^1 \right| + \ldots + \left| \tilde{\boldsymbol{\beta}}^m \right| \leq u,
$$

where  $\tilde{R}_n$  (  $\tilde{\boldsymbol{\beta}}$  ) is  $R_n$  ( $\beta$ ) with  $\beta$  replaced by  $\tilde{\boldsymbol{\beta}}$  and  $\boldsymbol{Z}$ replaced by  $\tilde{\mathbf{Z}}$  .  $u$  is also chosen via V-fold cross validation. With  $u \to 0$ , estimates of some components of  $(\tilde{\beta}^1, \ldots, \tilde{\beta}^m)$  $\tilde{\pmb{\beta}}$  m) can be exactly zero. Selection of important clusters can then be achieved.

In our examples, the objective functions  $R_n$  are continuously differentiable and depend only on data and the unknown regression coefficient  $\beta$ . Other smooth objective functions, for example the log-binomial likelihood for binary classification or the least square type estimating equation for the AFT survival model [37], can also be considered. The SGLasso only needs to assume that the expectation of the objective function has well-separated maximum. However for the proposed computational algorithms to work, we need to assume that the objective function is locally differentiable, i.e, it can be locally approximated by a smooth function.

#### *Computational algorithms*

Since the Lasso constraint is not differentiable, standard derivative based maximization approaches, such as the Newton-Raphson, cannot be used to obtain the Lasso estimate. In most previous studies, the maximization relies on the quadratic programming (QP) or general non-linear programming which are known to be computationally intensive. Moreover, the quadratic programming cannot be applied directly to the settings where the sample size may be smaller than the number of predictors. The *L*1boosting based approach proposed by [38] provides a computationally feasible solution for high dimensional cases.

*Algorithm I:* L*1 boosting Lasso* For the *ith* cluster:

1. Initialize  $\beta = 0$  and  $s = 0$ .

2. With the current estimate of  $\beta = (\beta^1, \dots, \beta^{ip_i})$ , compute  $\phi(\hat{\beta}) = \partial R_n(\hat{\beta})/\partial \hat{\beta}$ . Denote the *k<sup>th</sup>* component of  $\phi$  as  $\phi^k$ .

3. Find  $k^*$  that minimizes min( $\phi^k(\beta)$ ,  $\phi^k(\beta)$ ). If  $\phi^{k^*}(\beta)$ = 0, then stop the iteration.

4. Otherwise denote  $\gamma$  = -sign( $\phi^{k^*}(\beta)$ ). Find  $\hat{\pi}$  that

$$
\hat{\pi} = \operatorname{argmax}_{\pi \in [0,1]} R_n ((1 - \pi) \beta + \pi u_i) \eta^{k^*}),
$$

where  $\eta^{k^*}$  has the  $k^{*th}$  element equals to 1 and the rest equal to 0.

5. Let  $\beta^k = (1 - \hat{\pi}) \beta^k$  for  $k \neq k^*$  and  $\beta^{k^*} = (1 - \hat{\pi}) \beta^{k^*}$  $\gamma u \hat{\pi}$ . Let  $s = s + 1$ .

6. Repeat steps 2–5 until convergence or a fixed number of iterations *S* has been reached.

The β*<sup>i</sup>*at convergence is the Lasso estimate. We conclude convergence if the absolute value of  $\phi^{k^*}$  ( $\beta$ ) computed in step 3 is less than a pre-defined criteria, and/or if  $R_n(\beta)$  is larger than a pre-defined threshold. Alternative algorithm can be LARS based. Since it is not the focus of this study, we omit discussions of other computational algorithms.

For the GLasso, a LARS based approach is proposed in [19]. With high dimensional cases, a computationally more affordable approach is proposed in [39]. This approach shares the same spirit as the  $L<sub>1</sub>$  boosting Lasso and they are both special cases of the gradient-based constraint maximization discussed in [40]. This boosting based algorithm can be summarized into the following iterations.

#### *Algorithm II: boosting group Lasso*

1. Initialize  $\tilde{\beta} = 0$ . Set  $\Delta$  as a sufficiently small positive scalar.

2. With the current estimate of  $\beta$ , calculate the gradient  $\partial\!R_n$  (  $\tilde{{\boldsymbol{\beta}}}\,)$ /ć  $\tilde{{\boldsymbol{\beta}}}$  .

3. Set  $b = \tilde{\beta} - \tilde{c} \tilde{R}_n (\tilde{\beta}) / \tilde{c} \tilde{\beta}$  and  $\tau = \{1,...,m\}$ . Denote the *pth* component of *b* as *bp*.

4. Start Loop.

(a) Calculate the projection  $u_p = I$  ( $p \in \tau$ ) × (||bp|| + {*u* - $\sum_{p \in \tau} ||b^p||$ }/| $\tau$ |) for  $p = 1,..., m$ , where  $\tau$  is the cardinality of τ.

(b) If  $(u_p \ge 0)$  for all  $p$ , then abort the loop.

(c) Else update the active set  $\tau = \{p : u_p > 0\}.$ 

5. End Loop.

6. Get a new estimate  $\tilde{\beta}^p = b^p u_p / ||b^p||$  for  $p = 1,..., m$ .



#### **Figure 2**

Paths of parameter estimates for Lasso, GLasso and SGLasso. Red lines, cluster 1; Blue lines, cluster 2; Green lines, cluster 3. Solid lines,  $\beta_1$ ,  $\beta_4$  and  $\beta_7$ ; Dashed lines,  $\beta_2$ ,  $\beta_5$ , and  $\beta_6$ ; Dashed-Dotted lines,  $\beta_3$ ,  $\beta_6$ , and  $\beta_9$ . The grey lines show the selected tuning parameters. C1, C2 and C3 in the lower-left panel denote clusters 1, 2 and 3, respectively.

7. Repeat steps 2–6 until convergence or a fixed number of iterations has been reached.

#### *A graphic presentation*

We use the following numerical example to graphically demonstrate the parameter path of the proposed approach. For a better resolution, we only consider a small study with nine covariates (genes) and three clusters. Since the proposed approach does not depend on the special format of the objective function, we consider a simple linear regression model and use the least squares loss function.

Consider the linear model  $\gamma = \beta_1 z_1 + ... + \beta_9 z_9 + \varepsilon$ , where  $\beta = (\beta_1, ..., \beta_n)$  is the vector of regression coefficients and  $\varepsilon$ is the random error. We assume that there are three clusters, where  $(z_1, z_2, z_3)$  form cluster 1,  $(z_4, z_5, z_6)$  form cluster 2 and the rest belong to cluster 3. We assume that all *z* are marginally *N* (0, 1) distributed; the pairwise correlation coefficients are 0.4, 0.4 and 0.2 for covariates in clusters 1, 2, and 3, respectively; and different clusters are independent. Moreover, we set  $\beta = (-1, -1, 0, -1, -1, 0, 0, 0,$ 0). In this simulated dataset, we have three clusters, two of which are associated with the outcome. Within the first two clusters, two out of three covariates contribute to the outcome.

We generate 100 random data points from the above model. The regression parameters are estimated using the Lasso, GLasso and SGLasso. Tuning parameters are selected using 3-fold cross validation. In Figure 2, we show the parameter path as a function of the tuning parameter *u*. In the upper panels, we show the parameter paths for Lasso (left) and GLasso (right). In the lower-left panel, we show parameter paths for the first step estimates using the SGLasso. We see that the within-cluster Lasso has paths close to those in the upper-left panel. The parameter paths for the second step SGLasso (lower-right panel) are similar to those in the upper-right panel, with simpler structures due to the reduced number of covariates. The SGLasso selects  $(z_1, z_2, z_4, z_5, z_6)$  with nonzero estimates, while the Lasso selects the covariates  $(z_1,..., z_n)$  $z_8$ ), and the GLasso selects all covariates.

# **Authors' contributions**

SM designed the study and analyzed the data. XS designed the computational algorithms. JH contributed to the development of feature selection at the individual gene and cluster levels. All participated in writing or revising the paper.

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