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Research article

Bipolar picture fuzzy hypersoft set-based performance analysis of abrasive textiles for enhanced quality control

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ABSTRACT

Abrasive textiles have widespread industrial applications in the fields of polishing, finishing, deburring, and cleaning of various surfaces. Effective decision making and performance analysis are crucial in the development and manufacturing of abrasive textiles, as it enables manufacturers to evaluate and optimize the performance of these materials for specific applications and to make informed decisions about their production processes. For that purpose, this work aims to introduce an innovative bipolar picture fuzzy hypersoft set (BPFHSS) which is composed of two picture fuzzy hyper soft sets; one of them gives us the positive information, and the other gives us the negative information, for each membership degree, neutral membership, and non-membership degree. The properties of the designed structure and discussed alongside a thorough discussion on the De-Morgan's laws. Also, the bipolar picture fuzzy hypersoft weighted geometric (BPFHSWG) operator is defined for the BPFHSS framework to aggregate bipolar picture fuzzy hypersoft numbers (BPFHSN) information. This research highlights the importance of considering inconsistent, bipolar, and multiple sub-attribute information in decision-making processes by using the defined operators to develop an algorithm for a multi-attribute analysis for quality control of manufacture of abrasive textiles.

1. Introduction

Computer science and mathematics play a crucial role in engineering new materials, providing essential tools and methodologies for analysis, modeling, simulation, and optimization, enabling advancements in scientific design, properties, and performance. These methods demonstrate how computer science is applied to enhance data modeling and image processing [1–3]. These methods have been used in the domains of prediction of traffic flows and transportation related issues [4,5]. These diverse algorithms have also found applications in social sciences and geology [6–8]. In addition to these advancements in diverse domains, computer science has also made significant contributions to the field of abrasive textiles [9]. Abrasive textiles find extensive use in industries such as automotive, aerospace, electronics, and metalworking, where finishing, polishing, and cleaning surfaces are critical. Manufacturers rely on advanced computational methods for quality control and performance analysis to ensure the high performance and quality

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of abrasive textiles. Fuzzy set theory is particularly useful in this scenario, as it provides a framework for handling uncertainty and imprecision in the data generated by such analyses. By leveraging fuzzy set theory, manufacturers can accurately assess their abrasive textiles' performance, identify improvement areas, and optimize their production processes to meet the specific needs of their customers and end-users. Using fuzzy set theory in decision-making and performance analysis can help manufacturers ensure that their abrasive textiles are of the highest quality and perform optimally in a wide range of industrial applications. Managing uncertainty poses a significant challenge in various domains, spanning real-life scenarios and scientific disciplines such as basic sciences, management academia, behavioral sciences, and information sciences. Scholars and decision-makers have devoted considerable effort to tackling this issue. In this regard, notable contributions have been made by influential researchers.

Zadeh made one pioneering effort in this area [10] when he introduced the concept of fuzzy sets, providing a concise approach to address uncertainty. Another significant development came from Pawlak [11], who proposed the idea of rough sets. To tackle the complex nature of uncertainty, Molodtsov [12] put forth the theory of soft sets, which encompasses the merits of both fuzzy and rough sets. Expanding on the work of Molodtsov, Maji [13] contributed by introducing several operations for soft sets. Recognizing limitations in these operations, Ali [14] further enhanced soft sets by introducing additional operations such as extended union, restricted union, restricted intersection, and restricted difference of two soft sets. Subsequently, Ali delved into exploring various algebraic structures [15] associated with these newly specified operations on soft sets. Moreover, Aktas [16] introduced the concept of group theory in both fuzzy and rough sets, adding a valuable perspective to the understanding and managing of uncertainty. These research endeavors have collectively contributed to advancing the field and addressing the challenges posed by uncertainty across various disciplines.

Maji [17] made significant advancements by developing the concept of fuzzy soft sets. In the realm of predicting unknown data within incomplete fuzzy soft sets, Deng and Wang [18] proposed using the object parameter technique. Naz [19] initiated a comprehensive study exploring the algebraic structures associated with fuzzy soft sets. Addressing decision-making challenges, Roy [20] extensively worked on fuzzy soft set theory. Combining a fuzzy set and a soft set, Maji [21] constructed an intuitionistic fuzzy soft set, presenting a comprehensive framework for handling uncertainty. Yolcu [22] introduced the concept of Picture Fuzzy Soft Sets (PFSS), which combines fuzzy images and soft sets to provide a powerful model. Through the employment of PFSS, uncertainties in a picture-fuzzy environment can be effectively visualized from a parametrization perspective. This approach allows each element or alternative in the universal set X to be observed through various parameters or attributes. Dubois [23] emphasized the importance of considering both positive and negative information in human decision-making processes.

In situations where a set of parameters yields positive data, it is imperative to acknowledge the corresponding set of parameters that convey the opposite meaning, commonly referred to as the “not set of parameters.” For example, if the parameter “a” represents the “tall” trait, then “not a” represents the “not tall” feature. Understanding that being “not tall” does not necessarily equate to being “short.” Thus, the universal set encompasses more than just tall and short members, showcasing the intricacies of parameterization within this context. Shabir [24] made notable contributions by introducing the concept of bipolar soft sets, particularly in the realm of decision-making challenges. Bipolar soft sets offer the capability to simultaneously present both the positive and negative aspects of data, which has garnered increasing attention among researchers.

Karaaslan [25] redefined bipolar soft sets by utilizing a bijective map between a collection of parameters and its negative. They also provided an illustrative example of a decision-making method using bipolar soft sets. Furthermore, Shabir [26] explored various algebraic structures associated with bipolar soft sets, enhancing our understanding of this area. Building upon this work, Karaaslan [27] focused on studying the group structure of bipolar soft sets, further expanding the theoretical foundations in this domain. In 2018, Smarandache [28] introduced the concept of hypersoft sets by transforming the argument mapping F into a multi-argument mapping, effectively converting generalized soft sets. Hypersoft sets offer increased flexibility and are well-suited for problems requiring judgment. The notion of hypersoft sets has garnered attention as a generalization of soft sets, and researchers have explored potential extensions in various mathematical areas. Saeed [29,30] and Abbas [31] introduced and examined several operations on hypersoft sets, contributing to the advancement of this field.

Yolcu made significant contributions by introducing two essential mathematical theories: fuzzy hypersoft sets [32] and intuitionistic fuzzy hypersoft sets [33]. These theories have found application in numerous ongoing studies and provide valuable tools for addressing complex problems. Saeed [34] initially proposed the notion of picture fuzzy hypersoft sets and explored their applications in multi-criteria decision-making (MCDM). This extension of the hypersoft set framework offers a visual representation of variables, enabling a more comprehensive analysis. Expanding on the concept of picture fuzzy hypersoft sets, Saeed [35] introduced the idea of a picture fuzzy hypersoft graph, which provides a fresh perspective on product sale risk analysis by visually representing contributing variables. Musa [36] made notable advancements by developing the concept of bipolar hypersoft sets, combining the notions of bipolarity and hypersoft sets, thereby enriching the theoretical framework in this area.

With an application to a textile firm, Altinoz [37] examined the issues and solved them using a rule-based supplier selection(SS) methodology that employs fuzzy logic concepts. For the purpose of choosing providers, Dalalah [38] used modified fuzzy DEMATEL and TOPSIS. Liao [39] suggested integrating multi-choice goal programming and fuzzy TOPSIS techniques for SS. Yucel [40] employs a fuzzy multi-objective linear model and a weighted additive fuzzy programming approach to choose a supplier for a textile company. For green SS, Cao [41] employ the Intuitionistic Fuzzy judgment matrix and TOPSIS. Wood [42] utilizes TOPSIS, Fuzzy TOPSIS, and Intuitionistic Fuzzy TOPSIS, along with three additional approaches to address the SS problem in the petroleum industry facilities. Fallahpour [43] used the fuzzy TOPSIS method to rate potential suppliers for an Iranian textile company. Yu use interval-valued Pythagorean fuzzy sets and extended TOPSIS [44] to facilitate group decision-making and create sustainable SS in the textile industry. Darjan [45] used the bipolar fuzzy MULTIMOORA method in the textile industry for better decision-making.

Irfan [46] introduced Intuitionistic Fuzzy incidence graphs to find the maximum percentage of progress and minimum percentage of non-progress in different departments of the textile industry.

1.1. Motivation

In decision-making problems, the significance of attributes cannot be disregarded as they contribute to the overall evaluation and influence the desirability of alternatives. When attributes can be further divided into subcategories or when more detailed information is required, HSS becomes particularly useful. Fuzzy hypersoft sets offer a framework to handle uncertain or imprecise information associated with both attributes and their subattributes. Intuitionistic fuzzy hypersoft sets provide a comprehensive framework for decision-making problems involving attributes and subattributes, accommodating fuzzy, uncertain, and incomplete information. Picture fuzzy hypersoft sets are a powerful tool for dealing with attributes and subattributes, even in the presence of inconsistent information. While these theories play dynamic roles, they do not allow decision-makers to comprehensively evaluate alternatives when considering both favorable and unfavorable attributes in the presence of inconsistent information. This limitation hampers a balanced and informed decision-making process. Thus, the main objective of this study is to address this research gap by developing a hybrid structure called BPFHSS.

The innovative idea behind BPFHSS is an extensive structure that considers membership degree (MD), non-membership degree (NMD), and abstinence degree (AD) values from both positive and negative viewpoints. It enables the solution of multivariable issues with subparameters by incorporating both positive degrees (+ve) representing favorable aspects and negative degrees (-ve) representing unfavorable aspects. BPFHSS combines the BPFS and HSS structures, utilizing parametric values to determine the optimum option for allocating various values. Its ability to handle large arrangements while considering positive and negative consequences of efficacy, neutrality, and ineffectiveness makes BPFHSS an effective approach for addressing decision-making challenges. These advantages when applied to the concept of quality control in industrial settings can lead to more detailed analysis due to the dynamic and versatile nature of the structure.

The paper is structured as follows: An analysis of previously established structures is provided in Section 2. The novel idea of BPFHSS is introduced in Section 3, along with a discussion of its core principles and distinguishing characteristics. The Section 8 provides the development of some geometric operators of the designed structure for decision making applications. Section 5 provides a complete layout of the decision-support system that was developed from the designed hypersoft structure and that algorithm is applied for performance analysis of abrasive textiles in Section 6. Section 7 provides a comparative analysis of the developed structure to the ones already available in literature highlighting the key factors why the designed structure superior. The final section concludes the article by providing an overview of the study while highlighting the future applications of the study.

2. Preliminaries

In this section, we discuss some previous fuzzy algebra structures are explained that are employed to develop the BPFHSS.

Definition 2.1. ([10]): Let $\mho^{\kappa}_{\varsigma}$ be a universe of discourse, $\mathbb{P}(\mho^{\kappa}_{\varsigma})$ the power set of $\mho^{\kappa}_{\varsigma}$ and E a set of attributes. Then, the pair $(\mathbb{M}, E)$, where $\mathbb{M} : E \longrightarrow \mathbb{P}(\mho^{\kappa}_{\varsigma})$ is called a soft set over $\mho^{\kappa}_{\varsigma}$.

Definition 2.2. ([32]): Let $\mho^{\kappa}_{\varsigma}$ be a universe of discourse, $\mathbb{P}(\mho^{\kappa}_{\varsigma})$ the power set of $\mho^{\kappa}_{\varsigma}$. Let $E = \{\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3, \mathbf{f}_4........\mathbf{f}_n\}$ be n disjoint parameters set whose corresponding attribute values are $\mathbf{Q}_1, \mathbf{Q}_2, \mathbf{Q}_3, \mathbf{Q}_4........\mathbf{Q}_n$. Suppose $\mathbf{Q} = \mathbf{Q}_1 \times \mathbf{Q}_2 \times \mathbf{Q}_3 \times .... \times \mathbf{Q}_n$, with $\mathbf{Q}_t \cap \mathbf{Q}_s = \phi$, $t \neq s$, and t, s $\in \{1, 2, ..., n\}$. The pair $(\mathbb{M}, \mathbf{Q})$, where $\mathbb{M}$: $\mathbf{Q} \rightarrow \mathbb{P}(\mho^{\kappa}_{\varsigma})$ is called a hyper soft set over $\mho^{\kappa}_{\varsigma}$.

Definition 2.3. ([12]): A picture fuzzy set on universe of discourse $\mho^{\kappa}_{\varsigma} = \{\mathbf{u}^{\kappa}_{\varsigma 1}, \mathbf{u}^{\kappa}_{\varsigma 2}, ..., \mathbf{u}^{\kappa}_{\varsigma n}\}$ is defined as

$$\mathrm{V} = \{\langle \mathbf{m}^{\kappa}_{\varsigma \mathrm{V}}(\mathbf{u}^{\kappa}_{\varsigma i}), \mathbf{t}^{\kappa}_{\varsigma \mathrm{V}}(\mathbf{u}^{\kappa}_{\varsigma i}), \mathbf{n}^{\kappa}_{\varsigma \mathrm{V}}(\mathbf{u}^{\kappa}_{\varsigma i})\rangle | \mathbf{u}^{\kappa}_{\varsigma i} \in \mho^{\kappa}_{\varsigma}\}$$

where $\mathbf{m}^{\kappa}_{\varsigma \mathrm{V}}(\mathbf{u}^{\kappa}_{\varsigma i}) : \mho^{\kappa}_{\varsigma} \longrightarrow [0,1]$, $\mathbf{t}^{\kappa}_{\varsigma \mathrm{V}}(\mathbf{u}^{\kappa}_{\varsigma i}) : \mho^{\kappa}_{\varsigma} \longrightarrow [0,1]$ and $\mathbf{n}^{\kappa}_{\varsigma \mathrm{V}}(\mathbf{u}^{\kappa}_{\varsigma i}) : \mho^{\kappa}_{\varsigma} \longrightarrow [0,1]$ represent degree of membership, neutral and non membership function. Also $0 \leq \mathbf{m}^{\kappa}_{\varsigma \mathrm{V}}(\mathbf{u}^{\kappa}_{\varsigma i}) + \mathbf{t}^{\kappa}_{\varsigma \mathrm{V}}(\mathbf{u}^{\kappa}_{\varsigma i}) + \mathbf{n}^{\kappa}_{\varsigma \mathrm{V}}(\mathbf{u}^{\kappa}_{\varsigma i}) \leq 1$.
$\mathbf{r}^{\kappa}_{\varsigma \mathrm{V}}(\mathbf{u}^{\kappa}_{\varsigma i}) = 1 - \mathbf{m}^{\kappa}_{\varsigma \mathrm{V}}(\mathbf{u}^{\kappa}_{\varsigma i}) - \mathbf{t}^{\kappa}_{\varsigma \mathrm{V}}(\mathbf{u}^{\kappa}_{\varsigma i}) - \mathbf{n}^{\kappa}_{\varsigma \mathrm{V}}(\mathbf{u}^{\kappa}_{\varsigma i})$ represent refusal membership degree of $\mathbf{u}^{\kappa}_{\varsigma i}$ in V, $\forall\ \mathbf{u}^{\kappa}_{\varsigma i} \in \mho^{\kappa}_{\varsigma}$.

Definition 2.4. ([11]): Suppose $\mho^{\kappa}_{\varsigma}$ is universal set and $\mathbb{P}(\mho^{\kappa}_{\varsigma})$ be power set of $\mho^{\kappa}_{\varsigma}$. Let $E = \{\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3, \mathbf{f}_4........\mathbf{f}_n\}$ be n disjoint parameters set, $\mathbb{S} \subseteq E$. The triple $(\mathtt{K}^{\kappa}_{\varsigma}, \mathtt{X}^{\kappa}_{\varsigma}, \mathbb{S})$ is said to be bipolar soft set over $\mho^{\kappa}_{\varsigma}$, where $\mathtt{K}^{\kappa}_{\varsigma}$ and $\mathtt{X}^{\kappa}_{\varsigma}$ are functions given by $\mathtt{K}^{\kappa}_{\varsigma} : \mathbb{S} \rightarrow \mathbb{P}(\mho^{\kappa}_{\varsigma})$ and $\mathtt{X}^{\kappa}_{\varsigma} : \mathbb{S} \rightarrow \mathbb{P}(\mho^{\kappa}_{\varsigma})$ such that $\mathtt{K}^{\kappa}_{\varsigma}(\mathtt{q}^{\kappa}_{\varsigma s}) \cap \mathtt{X}^{\kappa}_{\varsigma}(\mathtt{q}^{\kappa}_{\varsigma s}) = \phi$ for all $\mathtt{q}^{\kappa}_{\varsigma s} \in \mathbb{S}$. A bipolar soft set can be represented as following,

$$(\mathtt{K}^{\kappa}_{\varsigma}, \mathtt{X}^{\kappa}_{\varsigma}, \mathbb{S}) = \{(\mathtt{q}^{\kappa}_{\varsigma s}, \mathtt{K}^{\kappa}_{\varsigma}(\mathtt{q}^{\kappa}_{\varsigma s}), \mathtt{X}^{\kappa}_{\varsigma}(\mathtt{q}^{\kappa}_{\varsigma s})) : \mathtt{q}^{\kappa}_{\varsigma s} \in \mathbb{S} \wedge \mathtt{K}^{\kappa}_{\varsigma}(\mathtt{q}^{\kappa}_{\varsigma s}) \cap \mathtt{X}^{\kappa}_{\varsigma}(\mathtt{q}^{\kappa}_{\varsigma s}) = \phi\}.$$

Definition 2.5. ([36]): Suppose $\mho^{\kappa}_{\varsigma}$ is universal set and $\mathbb{P}(\mho^{\kappa}_{\varsigma})$ be powerset of $\mho^{\kappa}_{\varsigma}$. Let $E = \{\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3, \mathbf{f}_4........\mathbf{f}_n\}$ be n disjoint parameters set whose corresponding attribute values are $\mathbf{Q}_1, \mathbf{Q}_2, \mathbf{Q}_3, \mathbf{Q}_4........\mathbf{Q}_n$. Suppose $\mathbf{Q} = \mathbf{Q}_1 \times \mathbf{Q}_2 \times \mathbf{Q}_3 \times .... \times \mathbf{Q}_n$. The triple $(\mathtt{K}^{\kappa}_{\varsigma}, \mathtt{X}^{\kappa}_{\varsigma}, \mathbf{Q})$ is said to be bipolar hypersoft set over $\mho^{\kappa}_{\varsigma}$, where $\mathtt{K}^{\kappa}_{\varsigma}$ and $\mathtt{X}^{\kappa}_{\varsigma}$ are functions given by $\mathtt{K}^{\kappa}_{\varsigma} : \mathbf{Q} \rightarrow \mathbb{P}(\mho^{\kappa}_{\varsigma})$ and $\mathtt{X}^{\kappa}_{\varsigma} : \mathbf{Q} \rightarrow \mathbb{P}(\mho^{\kappa}_{\varsigma})$ such that $\mathtt{K}^{\kappa}_{\varsigma}(\mathtt{q}^{\kappa}_{\varsigma s}) \cap \mathtt{X}^{\kappa}_{\varsigma}(\mathtt{q}^{\kappa}_{\varsigma s}) = \phi$ for all $\mathtt{q}^{\kappa}_{\varsigma s} \in \mathbf{Q}$. A bipolar hypersoft set can be represented as following,

$$(\mathtt{K}^{\kappa}_{\varsigma}, \mathtt{X}^{\kappa}_{\varsigma}, \mathbf{Q}) = \{(\mathtt{q}^{\kappa}_{\varsigma s}, \mathtt{K}^{\kappa}_{\varsigma}(\mathtt{q}^{\kappa}_{\varsigma s}), \mathtt{X}^{\kappa}_{\varsigma}(\mathtt{q}^{\kappa}_{\varsigma s})) : \mathtt{q}^{\kappa}_{\varsigma s} \in \mathbf{Q} \wedge \mathtt{K}^{\kappa}_{\varsigma}(\mathtt{q}^{\kappa}_{\varsigma s}) \cap \mathtt{X}^{\kappa}_{\varsigma}(\mathtt{q}^{\kappa}_{\varsigma s}) = \phi\}.$$

Definition 2.6. ([34]): Let $\mho_\varsigma^\kappa$ be a universe of discourse, $\mathbb{P}(PF(\mho_\varsigma^\kappa))$ the power set of picture fuzzy set over $\mho_\varsigma^\kappa$. Let $E = \{\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3, \mathbf{f}_4 \ldots\ldots \mathbf{f}_n\}$ be n disjoint parameters set whose corresponding attribute values are $\mathbf{Q}_1, \mathbf{Q}_2, \mathbf{Q}_3, \mathbf{Q}_4 \ldots\ldots \mathbf{Q}_n$. Suppose $\mathbf{Q} = \mathbf{Q}_1 \times \mathbf{Q}_2 \times \mathbf{Q}_3 \times \ldots\ldots \times \mathbf{Q}_n$, with $\mathbf{Q}_t \cap \mathbf{Q}_s = \phi$, $t \neq s$, and t, s $\in \{1, 2, ..., n\}$. The pair $(\mathbb{M}, \mathbf{Q})$, where $\mathbb{M}$: $\mathbf{Q} \to \mathbb{P}(PF(\mho_\varsigma^\kappa))$ is called a picture fuzzy hyper soft set over $\mho_\varsigma^\kappa$. It is represented as follows;

$$(\mathbb{M}, \mathbf{Q}) = \mathbb{M}(\mathbf{w}_{\varsigma s}^\kappa) = \{\langle \mathtt{m}_\varsigma^\kappa(\mathbf{u}_{\varsigma i}^\kappa), \mathtt{t}_\varsigma^\kappa(\mathbf{u}_{\varsigma i}^\kappa), \mathbf{n}_\varsigma^\kappa(\mathbf{u}_{\varsigma i}^\kappa)\rangle | \mathbf{u}_{\varsigma i}^\kappa \in \mho_\varsigma^\kappa\} \ \forall \mathbf{w}_{\varsigma s}^\kappa \in \mathbf{Q}$$

3. Bipolar picture fuzzy hypersoft sets

In this section, we explored the original concept of BPFHSS, a novel combination of two distinct mathematical structures: picture fuzzy sets and bipolar hypersoft sets.

Definition 3.1. The triplet $(\mathbf{F}_\varsigma^\kappa, \mathbf{G}_\varsigma^\kappa, \mathbf{R}_\varsigma^\kappa)$ is called a picture fuzzy bipolar hypersoft set (BPFHSS) over $\mho_\varsigma^\kappa$, where $\mathbf{F}_\varsigma^\kappa$ and $\mathbf{G}_\varsigma^\kappa$ are functions define as $\mathbf{F}_\varsigma^\kappa : \mathbf{R}_\varsigma^\kappa \to \mathbb{P}(\mho_\varsigma^\kappa)$ and $\mathbf{G}_\varsigma^\kappa : -\mathbf{R}_\varsigma^\kappa \to \mathbb{P}(\mho_\varsigma^\kappa)$ such that $\mathbf{F}_\varsigma^\kappa(\mathbf{r}_\varsigma^\kappa) \cap \mathbf{G}_\varsigma^\kappa(-\mathbf{r}_\varsigma^\kappa) = \phi$ for all $\mathbf{r}_\varsigma^\kappa \in \mathbf{R}_\varsigma^\kappa$, where $\mathbf{F}_\varsigma^\kappa(\mathbf{r}_\varsigma^\kappa) : \mathbf{r}_\varsigma^\kappa \to [0,1]$ and $\mathbf{G}_\varsigma^\kappa(-\mathbf{r}_\varsigma^\kappa) : -\mathbf{r}_\varsigma^\kappa \to [-1,0]$ is indicated as the bipolar picture fuzzy value of the established parameter range $\mathbf{R}_\varsigma^\kappa$ with $0 \leqslant \mathbf{r}_\varsigma^\kappa - (-\mathbf{r}_\varsigma^\kappa) \leqslant 2$. A BPFHSS can be represented as follows,

$$\begin{aligned}&(\mathbf{F}_\varsigma^\kappa, \mathbf{G}_\varsigma^\kappa, \mathbf{R}_\varsigma^\kappa)\\ &= \{(\mathbf{r}_\varsigma^\kappa, \mathbf{F}_\varsigma^\kappa(\mathbf{r}_\varsigma^\kappa), \mathbf{G}_\varsigma^\kappa(-\mathbf{r}_\varsigma^\kappa)) : \mathbf{r}_\varsigma^\kappa \in \mathbf{R}_\varsigma^\kappa \text{ and } \mathbf{F}_\varsigma^\kappa(\mathbf{r}_\varsigma^\kappa) \cap \mathbf{G}_\varsigma^\kappa(-\mathbf{r}_\varsigma^\kappa) = \phi\}.\end{aligned}$$

Definition 3.2. Let $(\mathbf{F}_{\varsigma 1}^\kappa, \mathbf{G}_{\varsigma 1}^\kappa, \mathbf{R}_{\varsigma 1}^\kappa)$ and $(\mathbf{F}_{\varsigma 2}^\kappa, \mathbf{G}_{\varsigma 2}^\kappa, \mathbf{R}_{\varsigma 2}^\kappa)$ be two BPFHSS over $\mho_\varsigma^\kappa$, $(\mathbf{F}_{\varsigma 1}^\kappa, \mathbf{G}_{\varsigma 1}^\kappa, \mathbf{R}_{\varsigma 1}^\kappa)$ is a BPFHSS subset of $(\mathbf{F}_{\varsigma 2}^\kappa, \mathbf{G}_{\varsigma 2}^\kappa, \mathbf{R}_{\varsigma 2}^\kappa)$ if

(i) $\mathbf{R}_{\varsigma 1}^\kappa \subseteq \mathbf{R}_{\varsigma 2}^\kappa$
(ii) $\mathbf{F}_{\varsigma 1}^\kappa(\mathbf{r}_\varsigma^\kappa) \subseteq \mathbf{F}_{\varsigma 2}^\kappa(\mathbf{r}_\varsigma^\kappa)$ and $\mathbf{G}_{\varsigma 1}^\kappa(-\mathbf{r}_\varsigma^\kappa) \subseteq \mathbf{G}_{\varsigma 2}^\kappa(-\mathbf{r}_\varsigma^\kappa)$ for all $\mathbf{r}_\varsigma^\kappa \in \mathbf{R}_\varsigma^\kappa$

Example 3.3. Suppose $\mho_\varsigma^\kappa = \{\mathbf{c}_{\varsigma 1}^\kappa, \mathbf{c}_{\varsigma 2}^\kappa, \mathbf{c}_{\varsigma 3}^\kappa, \mathbf{c}_{\varsigma 4}^\kappa\}$ be universal set. Consider $(\mathbf{F}_{\varsigma 1}^\kappa, \mathbf{G}_{\varsigma 1}^\kappa, \mathbf{R}_{\varsigma 1}^\kappa)$ and $(\mathbf{F}_{\varsigma 2}^\kappa, \mathbf{G}_{\varsigma 2}^\kappa, \mathbf{R}_{\varsigma 2}^\kappa)$ be two BPFHSS over $\mathbf{c}_\varsigma^\kappa$

$$(\mathbf{F}_{\varsigma 1}^\kappa, \mathbf{G}_{\varsigma 1}^\kappa, \mathbf{R}_{\varsigma 1}^\kappa) = \left\{\begin{array}{l}(\mathbf{F}_{\varsigma 1}^\kappa, \mathbf{G}_{\varsigma 1}^\kappa)(\mathbf{r}_{\varsigma 1}^\kappa) =\\ \{(0.2, 0.3, 0.1, -0.5, -0.1, -0.2)/\mathbf{c}_{\varsigma 1}^\kappa, (0.5, 0.3, 0.2, -0.3, -0.1, -0.4)/\mathbf{c}_{\varsigma 2}^\kappa,\\ (0.3, 0.3, 0.1, -0.1, -0.2, -0.5)/\mathbf{c}_{\varsigma 3}^\kappa, (0.1, 0.4, 0.2, -0.2, -0.1, -0.4)/\mathbf{c}_{\varsigma 4}^\kappa\},\\ (\mathbf{F}_{\varsigma 1}^\kappa, \mathbf{G}_{\varsigma 1}^\kappa)(\mathbf{r}_{\varsigma 2}^\kappa) =\\ \{((0.2, 0.3, 0.2, -0.3, -0.2, -0.3)/\mathbf{c}_{\varsigma 1}^\kappa, (0.4, 0.3, 0.2, -0.4, -0.1, -0.3)/\mathbf{c}_{\varsigma 2}^\kappa,\\ (0.2, 0.3, 0.1, -0.7, -0.0, -0.2)/\mathbf{c}_{\varsigma 3}^\kappa, (0.5, 0.2, 0.2, -0.4, -0.3, -0.3)/\mathbf{c}_{\varsigma 4}^\kappa)\}\end{array}\right\}$$

and

$$(\mathbf{F}_{\varsigma 2}^\kappa, \mathbf{G}_{\varsigma 2}^\kappa, \mathbf{R}_{\varsigma 2}^\kappa) = \left\{\begin{array}{l}(\mathbf{F}_{\varsigma 2}^\kappa, \mathbf{G}_{\varsigma 2}^\kappa)(\mathbf{r}_{\varsigma 1}^\kappa) =\\ \{((0.4, 0.4, 0.1, -0.6, -0.2, -0.1)/\mathbf{c}_{\varsigma 1}^\kappa, (0.4, 0.4, 0.2, -0.4, -0.2, -0.3)/\mathbf{c}_{\varsigma 2}^\kappa,\\ (0.4, 0.5, 0.0, -0.2, -0.3, -0.4)/\mathbf{c}_{\varsigma 3}^\kappa, (0.2, 0.5, 0.1, -0.3, -0.4, -0.3)/\mathbf{c}_{\varsigma 4}^\kappa)\},\\ (\mathbf{F}_{\varsigma 2}^\kappa, \mathbf{G}_{\varsigma 2}^\kappa)(\mathbf{r}_{\varsigma 2}^\kappa) =\\ \{((0.3, 0.4, 0.1, -0.4, -0.3, -0.2)/\mathbf{c}_{\varsigma 1}^\kappa, (0.5, 0.4, 0.1, -0.5, -0.0, -0.2)/\mathbf{c}_{\varsigma 2}^\kappa,\\ (0.3, 0.4, 0.1, -0.8, -0.1, -0.1)/\mathbf{c}_{\varsigma 3}^\kappa, (0.6, 0.3, 0.1, -0.5, -0.4, -0.1)/\mathbf{c}_{\varsigma 4}^\kappa)\}\end{array}\right\}$$

then by Definition 3.2 $(\mathbf{F}_{\varsigma 1}^\kappa, \mathbf{G}_{\varsigma 1}^\kappa, \mathbf{R}_{\varsigma 1}^\kappa) \subseteq (\mathbf{F}_{\varsigma 2}^\kappa, \mathbf{G}_{\varsigma 2}^\kappa, \mathbf{R}_{\varsigma 2}^\kappa)$.

Definition 3.4. The compliment of BPFHSS $(\mathbf{F}_\varsigma^\kappa, \mathbf{G}_\varsigma^\kappa, \mathbf{R}_\varsigma^\kappa)$ is defined as: $(\mathbf{F}_\varsigma^\kappa, \mathbf{G}_\varsigma^\kappa, \mathbf{R}_\varsigma^\kappa)^c = (\mathbf{F}_\varsigma^{\kappa c}, \mathbf{G}_\varsigma^{\kappa c}, \mathbf{R}_\varsigma^\kappa)$.

Example 3.5. Suppose $(\mathbf{F}_\varsigma^\kappa, \mathbf{G}_\varsigma^\kappa, \mathbf{R}_\varsigma^\kappa)$ be BPFHSS

$$(\mathbf{F}_\varsigma^\kappa, \mathbf{G}_\varsigma^\kappa, \mathbf{R}_\varsigma^\kappa) = \left\{\begin{array}{l}(\mathbf{F}_\varsigma^\kappa, \mathbf{G}_\varsigma^\kappa)(\mathbf{r}_\varsigma^\kappa) =\\ \{(0.4, 0.4, 0.1, -0.5, -0.1, -0.2)/\mathbf{c}_{\varsigma 1}^\kappa, (0.4, 0.4, 0.2, -0.3, -0.1, -0.4)/\mathbf{c}_{\varsigma 2}^\kappa,\\ (0.4, 0.5, 0.0, -0.1, -0.2, -0.5)/\mathbf{c}_{\varsigma 3}^\kappa, (0.2, 0.5, 0.1, -0.2, -0.1, -0.4)/\mathbf{c}_{\varsigma 4}^\kappa)\}\end{array}\right\}$$

then the compliment of BPFHSS will be

$$(\mathbf{F}_\varsigma^\kappa, \mathbf{G}_\varsigma^\kappa, \mathbf{R}_\varsigma^\kappa)^c = \left\{\begin{array}{l}(\mathbf{F}_\varsigma^{\kappa c}, \mathbf{G}_\varsigma^{\kappa c})(\mathbf{r}_\varsigma^\kappa) =\\ \{(0.1, 0.4, 0.4, -0.2, -0.1, -0.5)/\mathbf{c}_{\varsigma 1}^\kappa, (0.2, 0.4, 0.4, -0.4, -0.1, -0.3)/\mathbf{c}_{\varsigma 2}^\kappa,\\ (0.0, 0.5, 0.4, -0.5, -0.2, -0.1)/\mathbf{c}_{\varsigma 3}^\kappa, (0.1, 0.5, 0.2, -0.4, -0.1, -0.2)/\mathbf{c}_{\varsigma 4}^\kappa\end{array}\right\}.$$

Definition 3.6. The extended union of $(\mathbf{F}_{\varsigma 1}^\kappa, \mathbf{G}_{\varsigma 1}^\kappa, \mathbf{R}_{\varsigma 1}^\kappa)$ and $(\mathbf{F}_{\varsigma 2}^\kappa, \mathbf{G}_{\varsigma 2}^\kappa, \mathbf{R}_{\varsigma 2}^\kappa)$ defined as $(\mathbf{F}_{\varsigma 3}^\kappa, \mathbf{G}_{\varsigma 3}^\kappa, \mathbf{R}_{\varsigma 3}^\kappa) = (\mathbf{F}_{\varsigma 1}^\kappa, \mathbf{G}_{\varsigma 1}^\kappa, \mathbf{R}_{\varsigma 1}^\kappa) \cup_e (\mathbf{F}_{\varsigma 2}^\kappa, \mathbf{G}_{\varsigma 2}^\kappa, \mathbf{R}_{\varsigma 2}^\kappa)$, where $\mathbf{R}_{\varsigma 3}^\kappa = \mathbf{R}_{\varsigma 1}^\kappa \cup \mathbf{R}_{\varsigma 2}^\kappa$ and for all $\mathbf{r}_\varsigma^\kappa \in \mathbf{R}_{\varsigma 3}^\kappa$,

$$\mathbf{F}^{\kappa}_{\varsigma 3}(\mathbf{r}^{\kappa}_{\varsigma})=\left\{\begin{array}{ll}\mathbf{F}^{\kappa}_{\varsigma 1}(\mathbf{r}^{\kappa}_{\varsigma}), & if\ \mathbf{r}^{\kappa}_{\varsigma}\in\mathbf{R}^{\kappa}_{\varsigma 1}\setminus\mathbf{R}^{\kappa}_{\varsigma 2}\\ \mathbf{F}^{\kappa}_{\varsigma 2}(\mathbf{r}^{\kappa}_{\varsigma}), & if\ \mathbf{r}^{\kappa}_{\varsigma}\in\mathbf{R}^{\kappa}_{\varsigma 2}\setminus\mathbf{R}^{\kappa}_{\varsigma 1}\\ \mathbf{F}^{\kappa}_{\varsigma 1}(\mathbf{r}^{\kappa}_{\varsigma})\cup\mathbf{F}^{\kappa}_{\varsigma 2}(\mathbf{r}^{\kappa}_{\varsigma}), & if\ \mathbf{r}^{\kappa}_{\varsigma}\in\mathbf{R}^{\kappa}_{\varsigma 1}\cap\mathbf{R}^{\kappa}_{\varsigma 2}\end{array}\right\}$$

$$\mathbf{G}^{\kappa}_{\varsigma 3}(\mathbf{r}^{\kappa}_{\varsigma})=\left\{\begin{array}{ll}\mathbf{G}^{\kappa}_{\varsigma 1}(-q), & if\ q\in-\mathbf{R}^{\kappa}_{\varsigma 1}\setminus-\mathbf{R}^{\kappa}_{\varsigma 2}\\ \mathbf{G}^{\kappa}_{\varsigma 2}(-q), & if\ q\in-\mathbf{R}^{\kappa}_{\varsigma 2}\setminus-\mathbf{R}^{\kappa}_{\varsigma 1}\\ \mathbf{G}^{\kappa}_{\varsigma 1}(-q)\cap\mathbf{G}^{\kappa}_{\varsigma 2}(-q) & if\ \mathbf{r}^{\kappa}_{\varsigma}\in-\mathbf{R}^{\kappa}_{\varsigma 1}\cap-\mathbf{R}^{\kappa}_{\varsigma 2}\end{array}\right\}$$

Example 3.7. Considering Example 3.3 extended union of $(\mathbf{F}^{\kappa}_{\varsigma 1},\mathbf{G}^{\kappa}_{\varsigma 1},\mathbf{R}^{\kappa}_{\varsigma 1})$ and $(\mathbf{F}^{\kappa}_{\varsigma 2},\mathbf{G}^{\kappa}_{\varsigma 2},\mathbf{R}^{\kappa}_{\varsigma 2})$ is given by:

$$(\mathbf{F}^{\kappa}_{\varsigma 3},\mathbf{G}^{\kappa}_{\varsigma 3},\mathbf{R}^{\kappa}_{\varsigma 3})=\left\{\begin{array}{l}(\mathbf{F}^{\kappa}_{\varsigma 2},\mathbf{G}^{\kappa}_{\varsigma 2})(\mathbf{r}^{\kappa}_{\varsigma 1})=\\ \{(0.4,0.3,0.1,-0.6,-0.1,-0.1)/\mathbf{c}^{\kappa}_{\varsigma 1},(0.5,0.3,0.2,-0.4,-0.1,-0.3)/)\mathbf{c}^{\kappa}_{\varsigma 2},\\ (0.4,0.3,0.0,-0.2,-0.2,-0.4)/\mathbf{c}^{\kappa}_{\varsigma 3},(0.2,0.4,0.1,-0.3,-0.1,-0.3)/\mathbf{c}^{\kappa}_{\varsigma 4}\}\\ (\mathbf{F}^{\kappa}_{\varsigma 2},\mathbf{G}^{\kappa}_{\varsigma 2})(\mathbf{r}^{\kappa}_{\varsigma 2})=\\ \{(0.3,0.3,0.1,-0.4,-0.2,-0.2)/\mathbf{c}^{\kappa}_{\varsigma 1},(0.5,0.3,0.1,-0.4,-0.1,-0.3)/\mathbf{c}^{\kappa}_{\varsigma 2},\\ (0.3,0.4,0.1,-0.7,-0.0,-0.2)/\mathbf{c}^{\kappa}_{\varsigma 3},(0.6,0.2,0.1,-0.5,-0.3,-0.1)/\mathbf{c}^{\kappa}_{\varsigma 4}\}\end{array}\right\}.$$

Definition 3.8. The extended intersection of $(\mathbf{F}^{\kappa}_{\varsigma 1},\mathbf{G}^{\kappa}_{\varsigma 1},\mathbf{R}^{\kappa}_{\varsigma 1})$ and $(\mathbf{F}^{\kappa}_{\varsigma 2},\mathbf{G}^{\kappa}_{\varsigma 2},\mathbf{R}^{\kappa}_{\varsigma 2})$ defined as: $(\mathbf{F}^{\kappa}_{\varsigma 4},\mathbf{G}^{\kappa}_{\varsigma 4},\mathbf{G}^{\kappa}_{\varsigma 4}) = (\mathbf{F}^{\kappa}_{\varsigma 1},\mathbf{G}^{\kappa}_{\varsigma 1},\mathbf{R}^{\kappa}_{\varsigma 1})\cap_e(\mathbf{F}^{\kappa}_{\varsigma 2},\mathbf{G}^{\kappa}_{\varsigma 2},\mathbf{R}^{\kappa}_{\varsigma 2})$, where $\mathbf{R}^{\kappa}_{\varsigma 4}=\mathbf{R}^{\kappa}_{\varsigma 1}\cup\mathbf{R}^{\kappa}_{\varsigma 2}$ and for all $\mathbf{r}^{\kappa}_{\varsigma}\in\mathbf{R}^{\kappa}_{\varsigma 4}$,

$$\mathbf{F}^{\kappa}_{\varsigma 4}(\mathbf{r}^{\kappa}_{\varsigma})=\left\{\begin{array}{ll}\mathbf{F}^{\kappa}_{\varsigma 1}(\mathbf{r}^{\kappa}_{\varsigma}), & if\ \mathbf{r}^{\kappa}_{\varsigma}\in\mathbf{R}^{\kappa}_{\varsigma 1}\setminus\mathbf{R}^{\kappa}_{\varsigma 2}\\ \mathbf{F}^{\kappa}_{\varsigma 2}(\mathbf{r}^{\kappa}_{\varsigma}), & if\ \mathbf{r}^{\kappa}_{\varsigma}\in\mathbf{R}^{\kappa}_{\varsigma 2}\setminus\mathbf{R}^{\kappa}_{\varsigma 1}\\ \mathbf{F}^{\kappa}_{\varsigma 1}(\mathbf{r}^{\kappa}_{\varsigma})\cap\mathbf{F}^{\kappa}_{\varsigma 2}(\mathbf{r}^{\kappa}_{\varsigma}) & if\ \mathbf{r}^{\kappa}_{\varsigma}\in\mathbf{R}^{\kappa}_{\varsigma 1}\cap\mathbf{R}^{\kappa}_{\varsigma 2}\end{array}\right\}$$

$$\mathbf{G}^{\kappa}_{\varsigma 4}(\mathbf{r}^{\kappa}_{\varsigma})=\left\{\begin{array}{ll}\mathbf{G}^{\kappa}_{\varsigma 1}(-\mathbf{r}^{\kappa}_{\varsigma}), & if\ q\in-\mathbf{R}^{\kappa}_{\varsigma 1}\setminus-\mathbf{R}^{\kappa}_{\varsigma 2}\\ \mathbf{G}^{\kappa}_{\varsigma 2}(-\mathbf{r}^{\kappa}_{\varsigma}), & if\ q\in-\mathbf{R}^{\kappa}_{\varsigma 2}\setminus-\mathbf{R}^{\kappa}_{\varsigma 1}\\ \mathbf{G}^{\kappa}_{\varsigma 1}(-\mathbf{r}^{\kappa}_{\varsigma})\cup\mathbf{G}^{\kappa}_{\varsigma 2}(\mathbf{r}^{\kappa}_{\varsigma}) & if\ \mathbf{r}^{\kappa}_{\varsigma}\in-\mathbf{R}^{\kappa}_{\varsigma 1}\cap-\mathbf{R}^{\kappa}_{\varsigma 2}\end{array}\right\}$$

Example 3.9. Considering Definition 3.8 extended intersection of $(\mathbf{F}^{\kappa}_{\varsigma 1},\mathbf{G}^{\kappa}_{\varsigma 1},\mathbf{R}^{\kappa}_{\varsigma 1})$ and $(\mathbf{F}^{\kappa}_{\varsigma 2},\mathbf{G}^{\kappa}_{\varsigma 2},\mathbf{R}^{\kappa}_{\varsigma 2})$ is given by:

$$(\mathbf{F}^{\kappa}_{\varsigma 4},\mathbf{G}^{\kappa}_{\varsigma 4},\mathbf{R}^{\kappa}_{\varsigma 4})=\left\{\begin{array}{l}(\mathbf{F}^{\kappa}_{\varsigma 2},\mathbf{G}^{\kappa}_{\varsigma 2})(\mathbf{r}^{\kappa}_{\varsigma 1})=\\ \{(0.2,0.4,0.1,-0.5,-0.2,-0.2)/\mathbf{c}^{\kappa}_{\varsigma 1},(0.4,0.4,0.2,-0.3,-0.2,-0.4)/\mathbf{c}^{\kappa}_{\varsigma 2},\\ (0.3,0.5,0.1,-0.1,-0.3,-0.5)/\mathbf{c}^{\kappa}_{\varsigma 3},(0.1,0.5,0.2,-0.2,-0.4,-0.4)/\mathbf{c}^{\kappa}_{\varsigma 4}\},\\ (\mathbf{F}^{\kappa}_{\varsigma 2},\mathbf{G}^{\kappa}_{\varsigma 2})(\mathbf{r}^{\kappa}_{\varsigma 2})=\\ \{(0.2,0.4,0.2,-0.3,-0.3,-0.3)/\mathbf{c}^{\kappa}_{\varsigma 1},(0.4,0.4,0.2,-0.4,-0.1,-0.3)/\mathbf{c}^{\kappa}_{\varsigma 2},\\ (0.2,0.4,0.1,-0.7,-0.1,-0.2)/\mathbf{c}^{\kappa}_{\varsigma 3},(0.5,0.3,0.2,-0.4,-0.4,-0.3)/\mathbf{c}^{\kappa}_{\varsigma 4}\}\end{array}\right\}.$$

Definition 3.10. The OR-operation of $(\mathbf{F}^{\kappa}_{\varsigma 1},\mathbf{G}^{\kappa}_{\varsigma 1},\mathbf{R}^{\kappa}_{\varsigma 1})$ and $(\mathbf{F}^{\kappa}_{\varsigma 2},\mathbf{G}^{\kappa}_{\varsigma 2},\mathbf{R}^{\kappa}_{\varsigma 2})$ defined as $(\mathbf{F}^{\kappa}_{\varsigma 5},\mathbf{G}^{\kappa}_{\varsigma 5},\mathbf{R}^{\kappa}_{\varsigma 5}) = (\mathbf{F}^{\kappa}_{\varsigma 1},\mathbf{G}^{\kappa}_{\varsigma 1},\mathbf{R}^{\kappa}_{\varsigma 1})\vee(\mathbf{F}^{\kappa}_{\varsigma 2},\mathbf{G}^{\kappa}_{\varsigma 2},\mathbf{R}^{\kappa}_{\varsigma 2})$, where
$\mathbf{R}^{\kappa}_{\varsigma 5}=\mathbf{R}^{\kappa}_{\varsigma 1}\times\mathbf{R}^{\kappa}_{\varsigma 2}$ and for all $(\mathbf{r}^{\kappa}_{\varsigma 1},\mathbf{r}^{\kappa}_{\varsigma 2})\in\mathbf{R}^{\kappa}_{\varsigma 5}$ such that $\mathbf{r}^{\kappa}_{\varsigma 1}\in\mathbf{R}^{\kappa}_{\varsigma 1}$ and $\mathbf{r}^{\kappa}_{\varsigma 2}\in\mathbf{R}^{\kappa}_{\varsigma 2}$. $\mathbf{F}^{\kappa}_{\varsigma 5}(\mathbf{r}^{\kappa}_{\varsigma 1},\mathbf{r}^{\kappa}_{\varsigma 2})=\mathbf{F}^{\kappa}_{\varsigma 1}(\mathbf{r}^{\kappa}_{\varsigma 1})\cup\mathbf{F}^{\kappa}_{\varsigma 2}(\mathbf{r}^{\kappa}_{\varsigma 2})$ and $\mathbf{G}^{\kappa}_{\varsigma 5}(-\mathbf{r}^{\kappa}_{\varsigma 1},-\mathbf{r}^{\kappa}_{\varsigma 2})=\mathbf{G}^{\kappa}_{\varsigma 1}(-\mathbf{r}^{\kappa}_{\varsigma 1})\cap\mathbf{G}^{\kappa}_{\varsigma 2}(-\mathbf{r}^{\kappa}_{\varsigma 2})$.

Example 3.11. Considering Example 3.3 $(\mathbf{F}^{\kappa}_{\varsigma 1},\mathbf{G}^{\kappa}_{\varsigma 1},\mathbf{R}^{\kappa}_{\varsigma 1})$ and $(\mathbf{F}^{\kappa}_{\varsigma 2},\mathbf{G}^{\kappa}_{\varsigma 2},\mathbf{R}^{\kappa}_{\varsigma 2})$ be two BPFHSS. Then:
$(\mathbf{F}^{\kappa}_{\varsigma 5},\mathbf{G}^{\kappa}_{\varsigma 5},\mathbf{R}^{\kappa}_{\varsigma 5}) = (\mathbf{F}^{\kappa}_{\varsigma 1},\mathbf{G}^{\kappa}_{\varsigma 1},\mathbf{R}^{\kappa}_{\varsigma 1})\vee(\mathbf{F}^{\kappa}_{\varsigma 2},\mathbf{G}^{\kappa}_{\varsigma 2},\mathbf{R}^{\kappa}_{\varsigma 2})$

$$(\mathbf{F}^{\kappa}_{\varsigma 5},\mathbf{G}^{\kappa}_{\varsigma 5},\mathbf{R}^{\kappa}_{\varsigma 5})=\left\{\begin{array}{l}(\mathbf{F}^{\kappa}_{\varsigma 5},\mathbf{G}^{\kappa}_{\varsigma 5})(\mathbf{r}^{\kappa}_{\varsigma 1},\mathbf{r}^{\kappa}_{\varsigma 1})=\\ \{(0.4,0.3,0.1,-0.5,-0.2,-0.2)/\mathbf{c}^{\kappa}_{\varsigma 1},(0.5,0.3,0.2,-0.3,-0.2,-0.4)/\mathbf{c}^{\kappa}_{\varsigma 2},\\ (0.4,0.3,0.0,-0.1,-0.3,-0.5)/\mathbf{c}^{\kappa}_{\varsigma 3},(0.2,0.4,0.1,-0.2,-0.4,-0.4)/\mathbf{c}^{\kappa}_{\varsigma 4})\},\\ (\mathbf{F}^{\kappa}_{\varsigma 5},\mathbf{G}^{\kappa}_{\varsigma 5})(\mathbf{r}^{\kappa}_{\varsigma 1},\mathbf{r}^{\kappa}_{\varsigma 2})=\\ \{(0.3,0.3,0.1,-0.4,-0.3,-0.2)/\mathbf{c}^{\kappa}_{\varsigma 1},(0.5,0.3,0.1,-0.3,-0.1,-0.4)\mathbf{c}^{\kappa}_{\varsigma 2},\\ (0.3,0.3,0.1,-0.1,-0.2,-0.5)/\mathbf{c}^{\kappa}_{\varsigma 3},(0.6,0.3,0.1,-0.2,-0.4,-0.4)/\mathbf{c}^{\kappa}_{\varsigma 4}),\\ (\mathbf{F}^{\kappa}_{\varsigma 5},\mathbf{G}^{\kappa}_{\varsigma 5})(\mathbf{r}^{\kappa}_{\varsigma 2},\mathbf{r}^{\kappa}_{\varsigma 1})=\\ \{(0.4,0.3,0.1,-0.3,-0.2,-0.3)/\mathbf{c}^{\kappa}_{\varsigma 1},(0.4,0.3,0.2,-0.4,-0.2,-0.3)/\mathbf{c}^{\kappa}_{\varsigma 2},\\ (0.4,0.3,0.0,-0.2,-0.3,-0.4)/\mathbf{c}^{\kappa}_{\varsigma 3},(0.5,0.2,0.1,-0.3,-0.4,-0.3)/\mathbf{c}^{\kappa}_{\varsigma 4})\}\\ (\mathbf{F}^{\kappa}_{\varsigma 5},\mathbf{G}^{\kappa}_{\varsigma 5})(\mathbf{r}^{\kappa}_{\varsigma 2},\mathbf{r}^{\kappa}_{\varsigma 2})=\\ \{(0.3,0.3,0.1,-0.3,-0.3,-0.3)/\mathbf{c}^{\kappa}_{\varsigma 1},(0.5,0.3,0.1,-0.4,-0.1,-0.3)/\mathbf{c}^{\kappa}_{\varsigma 2},\\ (0.3,0.3,0.1,-0.7,-0.1,-0.2)/\mathbf{c}^{\kappa}_{\varsigma 3},(0.6,0.2,0.1,-0.4,-0.4,-0.3)/\mathbf{c}^{\kappa}_{\varsigma 4})\},\end{array}\right\}.$$

Definition 3.12. The AND- operation of $(\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1}, \mathbf{R}^{\kappa}_{\varsigma 1})$ and $(\mathbf{F}^{\kappa}_{\varsigma 2}, \mathbf{G}^{\kappa}_{\varsigma 2}, \mathbf{R}^{\kappa}_{\varsigma 2})$ defined as:
$(\mathbf{F}^{\kappa}_{\varsigma 6}, \mathbf{G}^{\kappa}_{\varsigma 6}, \mathbf{R}^{\kappa}_{\varsigma 6}) = (\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1}, \mathbf{R}^{\kappa}_{\varsigma 1}) \wedge (\mathbf{F}^{\kappa}_{\varsigma 2}, \mathbf{G}^{\kappa}_{\varsigma 2}, \mathbf{R}^{\kappa}_{\varsigma 2})$, where $\mathbf{R}^{\kappa}_{\varsigma 6} = \mathbf{R}^{\kappa}_{\varsigma 1} \times \mathbf{R}^{\kappa}_{\varsigma 2}$ and for all $(\mathbf{r}^{\kappa}_{\varsigma 1}, \mathbf{r}^{\kappa}_{\varsigma 2}) \in (\mathbf{R}^{\kappa}_{\varsigma 6}$ such that $\mathbf{r}^{\kappa}_{\varsigma 1} \in \mathbf{R}^{\kappa}_{\varsigma 1}$ and $\mathbf{r}^{\kappa}_{\varsigma 2} \in \mathbf{R}^{\kappa}_{\varsigma 2}$.

$\mathbf{F}^{\kappa}_{\varsigma 6}(\mathbf{r}^{\kappa}_{\varsigma 1}, \mathbf{r}^{\kappa}_{\varsigma 2}) = \mathbf{F}^{\kappa}_{\varsigma 1}(\mathbf{r}^{\kappa}_{\varsigma 1}) \cap \mathbf{F}^{\kappa}_{\varsigma 2}(\mathbf{r}^{\kappa}_{\varsigma 2})$ and $\mathbf{G}^{\kappa}_{\varsigma 6}(-\mathbf{r}^{\kappa}_{\varsigma 1}, -\mathbf{r}^{\kappa}_{\varsigma 2}) = \mathbf{G}^{\kappa}_{\varsigma 1}(-\mathbf{r}^{\kappa}_{\varsigma 1}) \cup \mathbf{G}^{\kappa}_{\varsigma 2}(-\mathbf{r}^{\kappa}_{\varsigma 2})$.

Example 3.13. Considering Example 3.3 $(\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1}, \mathbf{R}^{\kappa}_{\varsigma 1})$ and $(\mathbf{F}^{\kappa}_{\varsigma 2}, \mathbf{G}^{\kappa}_{\varsigma 2}, \mathbf{R}^{\kappa}_{\varsigma 2})$ be two BPFHSS then
$(\mathbf{F}^{\kappa}_{\varsigma 6}, \mathbf{G}^{\kappa}_{\varsigma 6}, \mathbf{R}^{\kappa}_{\varsigma 6}) = (\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1}, \mathbf{R}^{\kappa}_{\varsigma 1}) \wedge (\mathbf{F}^{\kappa}_{\varsigma 2}, \mathbf{G}^{\kappa}_{\varsigma 2}, \mathbf{R}^{\kappa}_{\varsigma 2})$

$$(\mathbf{F}^{\kappa}_{\varsigma 6}, \mathbf{G}^{\kappa}_{\varsigma 6}, \mathbf{R}^{\kappa}_{\varsigma 6}) = \left\{\begin{array}{l} (\mathbf{F}^{\kappa}_{\varsigma 6}, \mathbf{G}^{\kappa}_{\varsigma 6})(\mathbf{r}^{\kappa}_{\varsigma 1}, \mathbf{r}^{\kappa}_{\varsigma 1}) = \\ \{(0.2, 0.4, 0.1, -0.6, -0.1, -0.1)/\mathbf{c}^{\kappa}_{\varsigma 1}, (0.4, 0.4, 0.2, -0.4, -0.1, -0.3)/\mathbf{c}^{\kappa}_{\varsigma 2}, \\ (0.3, 0.5, 0.1, -0.2, -0.2, -0.4)/\mathbf{c}^{\kappa}_{\varsigma 3}, (0.1, 0.5, 0.2, -0.3, -0.1, -0.3)/\mathbf{c}^{\kappa}_{\varsigma 4}\}, \\ (\mathbf{F}^{\kappa}_{\varsigma 6}, \mathbf{G}^{\kappa}_{\varsigma 6})(\mathbf{r}^{\kappa}_{\varsigma 1}, \mathbf{r}^{\kappa}_{\varsigma 2}) = \\ \{(0.2, 0.4, 0.1, -0.5, -0.1, -0.2)/\mathbf{c}^{\kappa}_{\varsigma 1}, (0.5, 0.4, 0.2, -0.5, -0.0, -0.2)/\mathbf{c}^{\kappa}_{\varsigma 2}, \\ (0.3, 0.4, 0.1, -0.8, -0.1, -0.1)/\mathbf{c}^{\kappa}_{\varsigma 3}, (0.1, 0.4, 0.2, -0.5, -0.1, -0.1)/c_4\}, \\ (\mathbf{F}^{\kappa}_{\varsigma 6}, \mathbf{G}^{\kappa}_{\varsigma 6})(\mathbf{r}^{\kappa}_{\varsigma 2}, \mathbf{r}^{\kappa}_{\varsigma 1}) = \\ \{(0.2, 0.4, 0.2, -0.6, -0.2, -0.1)/\mathbf{c}^{\kappa}_{\varsigma 1}, (0.4, 0.4, 0.2, -0.4, -0.1, -0.3)/\mathbf{c}^{\kappa}_{\varsigma 2}, \\ (0.2, 0.5, 0.1, -0.7, -0.0, -0.2)/\mathbf{c}^{\kappa}_{\varsigma 3}, (0.2, 0.5, 0.1, -0.4, -0.3, -0.3)/\mathbf{c}^{\kappa}_{\varsigma 4}), \\ (\mathbf{F}^{\kappa}_{\varsigma 6}, \mathbf{G}^{\kappa}_{\varsigma 6})(\mathbf{r}^{\kappa}_{\varsigma 2}, \mathbf{r}^{\kappa}_{\varsigma 2}) = \\ \{(0.2, 0.4, 0.2, -0.4, -0.2, -0.2)/\mathbf{c}^{\kappa}_{\varsigma 1}, (0.4, 0.4, 0.2, -0.5, -0.0, -0.2)/\mathbf{c}^{\kappa}_{\varsigma 2}, \\ (0.2, 0.4, 0.1, -0.8, -0.0, -0.1)/\mathbf{c}^{\kappa}_{\varsigma 3}, (0.5, 0.3, 0.2, -0.5, -0.3, -0.1)/\mathbf{c}^{\kappa}_{\varsigma 4}\} \end{array}\right\}.$$

3.1. De Morgan's laws for BPFHSS

In this section, the extended union and extended intersection in BPFHSS satisfy De Morgan's laws.

Theorem 3.14. *If* $(\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1}, \mathbf{R}^{\kappa}_{\varsigma 1})$ *and* $(\mathbf{F}^{\kappa}_{\varsigma 2}, \mathbf{G}^{\kappa}_{\varsigma 2}, \mathbf{R}^{\kappa}_{\varsigma 2})$ *be two BPFHSS over* $\mho^{\kappa}_{\varsigma}$*. then*

(i) $((\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1}, \mathbf{R}^{\kappa}_{\varsigma 1}) \cup_e (\mathbf{F}^{\kappa}_{\varsigma 2}, \mathbf{G}^{\kappa}_{\varsigma 2}, \mathbf{R}^{\kappa}_{\varsigma 2}))' = (\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1}, \mathbf{R}^{\kappa}_{\varsigma 1})' \cap_e (\mathbf{F}^{\kappa}_{\varsigma 2}, \mathbf{G}^{\kappa}_{\varsigma 2} \mathbf{R}^{\kappa}_{\varsigma 2})'$
(ii) $((\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1}, \mathbf{R}^{\kappa}_{\varsigma 1}) \cap_e (\mathbf{F}^{\kappa}_{\varsigma 2}, \mathbf{G}^{\kappa}_{\varsigma 2}, \mathbf{R}^{\kappa}_{\varsigma 2}))' = (\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1}, \mathbf{R}^{\kappa}_{\varsigma 1})' \cup_e (\mathbf{F}^{\kappa}_{\varsigma 2}, \mathbf{G}^{\kappa}_{\varsigma 2} \mathbf{R}^{\kappa}_{\varsigma 2})'$

Proof. (i) Since
$(\mathbf{F}^{\kappa}_{\varsigma 3}, \mathbf{G}^{\kappa}_{\varsigma 3}, \mathbf{R}^{\kappa}_{\varsigma 3}) = (\mathbf{F}^{\kappa}_{\varsigma 1}, Re_1, \mathbf{R}^{\kappa}_{\varsigma 1}) \cup_e (\mathbf{F}^{\kappa}_{\varsigma 2}, \mathbf{G}^{\kappa}_{\varsigma 2}, \mathbf{R}^{\kappa}_{\varsigma 2})$,
where $\mathbf{R}^{\kappa}_{\varsigma 3} = \mathbf{R}^{\kappa}_{\varsigma 1} \cup \mathbf{R}^{\kappa}_{\varsigma 2}$

$$\mathbf{F}^{\kappa}_{\varsigma 3}(\mathbf{r}^{\kappa}_{\varsigma}) = \left\{\begin{array}{ll} \mathbf{F}^{\kappa}_{\varsigma 1}(\mathbf{r}^{\kappa}_{\varsigma}), & if\ \mathbf{r}^{\kappa}_{\varsigma} \in \mathbf{R}^{\kappa}_{\varsigma 1} \setminus \mathbf{R}^{\kappa}_{\varsigma 2} \\ \mathbf{F}^{\kappa}_{\varsigma 2}(\mathbf{r}^{\kappa}_{\varsigma}), & if\ \mathbf{r}^{\kappa}_{\varsigma} \in \mathbf{R}^{\kappa}_{\varsigma 2} \setminus \mathbf{R}^{\kappa}_{\varsigma 1} \\ \mathbf{F}^{\kappa}_{\varsigma 1}, \cup \mathbf{F}^{\kappa}_{\varsigma 2}(\mathbf{r}^{\kappa}_{\varsigma}) & if\ \mathbf{r}^{\kappa}_{\varsigma} \in \mathbf{R}^{\kappa}_{\varsigma 1} \cap \mathbf{R}^{\kappa}_{\varsigma 2} \end{array}\right\}$$

$$\mathbf{G}^{\kappa}_{\varsigma 3}(-\mathbf{r}) = \left\{\begin{array}{l} \mathbf{G}^{\kappa}_{\varsigma 1}(-\mathbf{r}^{\kappa}_{\varsigma}),\ if\ \mathbf{r}^{\kappa}_{\varsigma} \in -\mathbf{R}^{\kappa}_{\varsigma 1} \setminus -\mathbf{R}^{\kappa}_{\varsigma 2} \\ \mathbf{G}^{\kappa}_{\varsigma 2}(-\mathbf{r}^{\kappa}_{\varsigma}),\ if\ \mathbf{r}^{\kappa}_{\varsigma} \in -\mathbf{R}^{\kappa}_{\varsigma 2} \setminus -\mathbf{R}^{\kappa}_{\varsigma 1} \\ \mathbf{G}^{\kappa}_{\varsigma 1}(-\mathbf{r}^{\kappa}_{\varsigma}) \cap \mathbf{G}^{\kappa}_{\varsigma 2}(\mathbf{r}^{\kappa}_{\varsigma})\ if\ \mathbf{r}^{\kappa}_{\varsigma} \in -\mathbf{R}^{\kappa}_{\varsigma 1} \cap -\mathbf{R}^{\kappa}_{\varsigma 2} \end{array}\right\}$$

for all $\mathbf{r}^{\kappa}_{\varsigma} \in \mathbf{R}^{\kappa}_{\varsigma 3}$ then
$(\mathbf{F}^{\kappa}_{\varsigma 3}, \mathbf{G}^{\kappa}_{\varsigma 3}, \mathbf{R}^{\kappa}_{\varsigma 3})' = ((\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1}) \cup_e (\mathbf{F}^{\kappa}_{\varsigma 2}, \mathbf{G}^{\kappa}_{\varsigma 2}, \mathbf{R}^{\kappa}_{\varsigma 2}))' =$

$$(\mathbf{F}^{\kappa}_{\varsigma 3}(\mathbf{r}^{\kappa}_{\varsigma}))' = \left\{\begin{array}{ll} \mathbf{G}^{\kappa}_{\varsigma 1}(-\mathbf{r}^{\kappa}_{\varsigma}), & if\ \mathbf{r}^{\kappa}_{\varsigma} \in -\mathbf{R}^{\kappa}_{\varsigma 1} \setminus -\mathbf{R}^{\kappa}_{\varsigma 2} \\ \mathbf{G}^{\kappa}_{\varsigma 2}(-\mathbf{r}^{\kappa}_{\varsigma}), & if\ \mathbf{r}^{\kappa}_{\varsigma} \in -\mathbf{R}^{\kappa}_{\varsigma 2} \setminus -\mathbf{R}^{\kappa}_{\varsigma 1} \\ \mathbf{G}^{\kappa}_{\varsigma 1}(-\mathbf{r}^{\kappa}_{\varsigma}) \cap \mathbf{G}^{\kappa}_{\varsigma 2}(-\mathbf{r}^{\kappa}_{\varsigma}) & if\ \mathbf{r}^{\kappa}_{\varsigma} \in -\mathbf{R}^{\kappa}_{\varsigma 1} \cap -\mathbf{R}^{\kappa}_{\varsigma 2} \end{array}\right\}$$

$$(\mathbf{G}^{\kappa}_{\varsigma 3}(\mathbf{r}^{\kappa}_{\varsigma}))' = \left\{\begin{array}{ll} \mathbf{F}^{\kappa}_{\varsigma 1}(\mathbf{r}^{\kappa}_{\varsigma}) & if\ \mathbf{r}^{\kappa}_{\varsigma} \in \mathbf{R}^{\kappa}_{\varsigma 1} \setminus \mathbf{R}^{\kappa}_{\varsigma 2} \\ \mathbf{F}^{\kappa}_{\varsigma 2}(\mathbf{r}^{\kappa}_{\varsigma}) & if\ \mathbf{r}^{\kappa}_{\varsigma} \in \mathbf{R}^{\kappa}_{\varsigma 2} \setminus \mathbf{R}^{\kappa}_{\varsigma 1} \\ \mathbf{F}^{\kappa}_{\varsigma 1}(\mathbf{r}^{\kappa}_{\varsigma}) \cup \mathbf{F}^{\kappa}_{\varsigma 2}(\mathbf{r}^{\kappa}_{\varsigma}) & if\ \mathbf{r}^{\kappa}_{\varsigma} \in \mathbf{R}^{\kappa}_{\varsigma 1} \cap \mathbf{R}^{\kappa}_{\varsigma 2} \end{array}\right\}$$

$(\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1}, \mathbf{R}^{\kappa}_{\varsigma 1})' \cap_e (\mathbf{F}^{\kappa}_{\varsigma 2}, \mathbf{G}^{\kappa}_{\varsigma 2}, \mathbf{R}^{\kappa}_{\varsigma 2})' =$

$$(\mathbf{F}^{\kappa}_{\varsigma 3}(\mathbf{r}^{\kappa}_{\varsigma}))' = \left\{ \begin{array}{l} \mathbf{G}^{\kappa}_{\varsigma 1}(-\mathbf{r}^{\kappa}_{\varsigma}), \quad if \ \mathbf{r}^{\kappa}_{\varsigma} \in -\mathbf{R}^{\kappa}_{\varsigma 1} \setminus -\mathbf{R}^{\kappa}_{\varsigma 2} \\ \mathbf{G}^{\kappa}_{\varsigma 2}(-\mathbf{r}^{\kappa}_{\varsigma}), \quad if \ \mathbf{r}^{\kappa}_{\varsigma} \in -\mathbf{R}^{\kappa}_{\varsigma 2} \setminus -\mathbf{R}^{\kappa}_{\varsigma 1} \\ \mathbf{G}^{\kappa}_{\varsigma 1}(-\mathbf{r}^{\kappa}_{\varsigma}) \cap \mathbf{G}^{\kappa}_{\varsigma 2}(-\mathbf{r}^{\kappa}_{\varsigma}) \quad if \ \mathbf{r}^{\kappa}_{\varsigma} \in -\mathbf{R}^{\kappa}_{\varsigma 1} \cap -\mathbf{R}^{\kappa}_{\varsigma 2} \end{array} \right\}$$

$$(\mathbf{G}^{\kappa}_{\varsigma 3}(\mathbf{r}^{\kappa}_{\varsigma}))' = \left\{ \begin{array}{l} \mathbf{F}^{\kappa}_{\varsigma 1}(\mathbf{r}^{\kappa}_{\varsigma}) \quad if \ \mathbf{r}^{\kappa}_{\varsigma} \in \mathbf{R}^{\kappa}_{\varsigma 1} \setminus \mathbf{R}^{\kappa}_{\varsigma 2} \\ \mathbf{F}^{\kappa}_{\varsigma 2}(\mathbf{r}^{\kappa}_{\varsigma}) \quad if \ \mathbf{r}^{\kappa}_{\varsigma} \in \mathbf{R}^{\kappa}_{\varsigma 2} \setminus \mathbf{R}^{\kappa}_{\varsigma 1} \\ \mathbf{F}^{\kappa}_{\varsigma 1}(\mathbf{r}^{\kappa}_{\varsigma}) \cup \mathbf{F}^{\kappa}_{\varsigma 2}(\mathbf{r}^{\kappa}_{\varsigma}) \quad if \ \mathbf{r}^{\kappa}_{\varsigma} \in \mathbf{R}^{\kappa}_{\varsigma 1} \cap \mathbf{R}^{\kappa}_{\varsigma 2} \end{array} \right\}$$

Hence $((\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1}, \mathbf{R}^{\kappa}_{\varsigma 1}) \cup_e (\mathbf{F}^{\kappa}_{\varsigma 2}, \mathbf{G}^{\kappa}_{\varsigma 2}, \mathbf{R}^{\kappa}_{\varsigma 2}))' = (\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1}, \mathbf{R}^{\kappa}_{\varsigma 1})' \cap_e (\mathbf{F}^{\kappa}_{\varsigma 2}, \mathbf{G}^{\kappa}_{\varsigma 2}, \mathbf{R}^{\kappa}_{\varsigma 2})'$

(ii) Since

$(\mathbf{F}^{\kappa}_{\varsigma 3}, \mathbf{G}^{\kappa}_{\varsigma 3}, \mathbf{R}^{\kappa}_{\varsigma 3}) = (\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1}, \mathbf{R}^{\kappa}_{\varsigma 1}) \cap_e (\mathbf{F}^{\kappa}_{\varsigma 2}, \mathbf{R}^{\kappa}_{\varsigma 2})$,

where $\mathbf{R}^{\kappa}_{\varsigma 3} = \mathbf{R}^{\kappa}_{\varsigma 1} \cup \mathbf{R}^{\kappa}_{\varsigma 2}$

$$\mathbf{F}^{\kappa}_{\varsigma 3}(\mathbf{r}^{\kappa}_{\varsigma}) = \left\{ \begin{array}{l} \mathbf{F}^{\kappa}_{\varsigma 1}(\mathbf{r}^{\kappa}_{\varsigma}), \quad if \ \mathbf{r}^{\kappa}_{\varsigma} \in \mathbf{R}^{\kappa}_{\varsigma 1} \setminus \mathbf{R}^{\kappa}_{\varsigma 2} \\ \mathbf{F}^{\kappa}_{\varsigma 2}(\mathbf{r}^{\kappa}_{\varsigma}), \quad if \ \mathbf{r}^{\kappa}_{\varsigma} \in \mathbf{R}^{\kappa}_{\varsigma 2} \setminus \mathbf{R}^{\kappa}_{\varsigma 1} \\ \mathbf{F}^{\kappa}_{\varsigma 1}(\mathbf{r}^{\kappa}_{\varsigma}) \cap \mathbf{F}^{\kappa}_{\varsigma 2}(\mathbf{r}^{\kappa}_{\varsigma}) \quad if \ \mathbf{r}^{\kappa}_{\varsigma} \in \mathbf{R}^{\kappa}_{\varsigma 1} \cap \mathbf{R}^{\kappa}_{\varsigma 2} \end{array} \right\}$$

$$\mathbf{G}^{\kappa}_{\varsigma 3}(\mathbf{r}^{\kappa}_{\varsigma}) = \left\{ \begin{array}{l} \mathbf{G}^{\kappa}_{\varsigma 1}(-\mathbf{r}^{\kappa}_{\varsigma}), \quad if \ \mathbf{r}^{\kappa}_{\varsigma} \in -\mathbf{R}^{\kappa}_{\varsigma 1} \setminus -\mathbf{R}^{\kappa}_{\varsigma 2} \\ \mathbf{G}^{\kappa}_{\varsigma 2}(-\mathbf{r}^{\kappa}_{\varsigma}), \quad if \ \mathbf{r}^{\kappa}_{\varsigma} \in -\mathbf{R}^{\kappa}_{\varsigma 2} \setminus -\mathbf{R}^{\kappa}_{\varsigma 1} \\ \mathbf{G}^{\kappa}_{\varsigma 1}(-\mathbf{r}^{\kappa}_{\varsigma}) \cup \mathbf{G}^{\kappa}_{\varsigma 2}(\mathbf{r}^{\kappa}_{\varsigma}) \quad if \ \mathbf{r}^{\kappa}_{\varsigma} \in -\mathbf{R}^{\kappa}_{\varsigma 1} \cap -\mathbf{R}^{\kappa}_{\varsigma 2} \end{array} \right\}$$

then $((\mathbf{F}^{\kappa}_{\varsigma 3}, \mathbf{G}^{\kappa}_{\varsigma 3}, \mathbf{R}^{\kappa}_{\varsigma 3}))' =$

$((\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1}, \mathbf{R}^{\kappa}_{\varsigma 1}) \cap_e (\mathbf{F}^{\kappa}_{\varsigma 2}, \mathbf{G}^{\kappa}_{\varsigma 2}, \mathbf{R}^{\kappa}_{\varsigma 2}))' =$

$$(\mathbf{F}^{\kappa}_{\varsigma 3}(\mathbf{r}^{\kappa}_{\varsigma}))' = \left\{ \begin{array}{l} \mathbf{G}^{\kappa}_{\varsigma 1}(-\mathbf{r}^{\kappa}_{\varsigma}), \quad if \ \mathbf{r}^{\kappa}_{\varsigma} \in -\mathbf{R}^{\kappa}_{\varsigma 1} \setminus -\mathbf{R}^{\kappa}_{\varsigma 2} \\ \mathbf{G}^{\kappa}_{\varsigma 2}(-\mathbf{r}^{\kappa}_{\varsigma}), \quad if \ \mathbf{r}^{\kappa}_{\varsigma} \in -\mathbf{R}^{\kappa}_{\varsigma 2} \setminus -\mathbf{R}^{\kappa}_{\varsigma 1} \\ \mathbf{G}^{\kappa}_{\varsigma 1}(-\mathbf{r}^{\kappa}_{\varsigma}) \cup \mathbf{G}^{\kappa}_{\varsigma 2}(-\mathbf{r}^{\kappa}_{\varsigma}) \quad if \ \mathbf{r}^{\kappa}_{\varsigma} \in -\mathbf{R}^{\kappa}_{\varsigma 1} \cap -\mathbf{R}^{\kappa}_{\varsigma 2} \end{array} \right\}$$

$$(\mathbf{G}^{\kappa}_{\varsigma 3}(\mathbf{r}^{\kappa}_{\varsigma}))' = \left\{ \begin{array}{l} \mathbf{F}^{\kappa}_{\varsigma 1}(\mathbf{r}^{\kappa}_{\varsigma}) \quad if \ \mathbf{r}^{\kappa}_{\varsigma} \in \mathbf{R}^{\kappa}_{\varsigma 1} \setminus \mathbf{R}^{\kappa}_{\varsigma 2} \\ \mathbf{F}^{\kappa}_{\varsigma 2}(\mathbf{r}^{\kappa}_{\varsigma}) \quad if \ \mathbf{r}^{\kappa}_{\varsigma} \in \mathbf{R}^{\kappa}_{\varsigma 2} \setminus \mathbf{R}^{\kappa}_{\varsigma 1} \\ \mathbf{F}^{\kappa}_{\varsigma 1}(\mathbf{r}^{\kappa}_{\varsigma}) \cap \mathbf{F}^{\kappa}_{\varsigma 2}(\mathbf{r}^{\kappa}_{\varsigma}) \quad if \ \mathbf{r}^{\kappa}_{\varsigma} \in \mathbf{R}^{\kappa}_{\varsigma 1} \cap \mathbf{R}^{\kappa}_{\varsigma 2} \end{array} \right\}$$

$(\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1}, \mathbf{R}^{\kappa}_{\varsigma 1})' \cup_e (\mathbf{F}^{\kappa}_{\varsigma 2}, \mathbf{G}^{\kappa}_{\varsigma 2}, \mathbf{R}^{\kappa}_{\varsigma 2})' =$

$$(\mathbf{F}^{\kappa}_{\varsigma 3}(\mathbf{r}^{\kappa}_{\varsigma}))' = \left\{ \begin{array}{l} \mathbf{G}^{\kappa}_{\varsigma 1}(-\mathbf{r}^{\kappa}_{\varsigma}), \quad if \ \mathbf{r}^{\kappa}_{\varsigma} \in -\mathbf{R}^{\kappa}_{\varsigma 1} \setminus -\mathbf{R}^{\kappa}_{\varsigma 2} \\ \mathbf{G}^{\kappa}_{\varsigma 2}(-\mathbf{r}^{\kappa}_{\varsigma}), \quad if \ \mathbf{r}^{\kappa}_{\varsigma} \in -\mathbf{R}^{\kappa}_{\varsigma 2} \setminus -\mathbf{R}^{\kappa}_{\varsigma 1} \\ \mathbf{G}^{\kappa}_{\varsigma 1}(-\mathbf{r}^{\kappa}_{\varsigma}) \cup \mathbf{G}^{\kappa}_{\varsigma 2}(-\mathbf{r}^{\kappa}_{\varsigma}) \quad if \ \mathbf{r}^{\kappa}_{\varsigma} \in -\mathbf{R}^{\kappa}_{\varsigma 1} \cap -\mathbf{R}^{\kappa}_{\varsigma 2} \end{array} \right\}$$

$$(\mathbf{G}^{\kappa}_{\varsigma 3}(\mathbf{r}^{\kappa}_{\varsigma}))' = \left\{ \begin{array}{l} \mathbf{F}^{\kappa}_{\varsigma 1}(\mathbf{r}^{\kappa}_{\varsigma}) \quad if \ \mathbf{r}^{\kappa}_{\varsigma} \in \mathbf{R}^{\kappa}_{\varsigma 1} \setminus \mathbf{R}^{\kappa}_{\varsigma 2} \\ \mathbf{F}^{\kappa}_{\varsigma 2}(\mathbf{r}^{\kappa}_{\varsigma}) \quad if \ \mathbf{r}^{\kappa}_{\varsigma} \in \mathbf{R}^{\kappa}_{\varsigma 2} \setminus \mathbf{R}^{\kappa}_{\varsigma 1} \\ \mathbf{F}^{\kappa}_{\varsigma 1}(\mathbf{r}^{\kappa}_{\varsigma}) \cap \mathbf{F}^{\kappa}_{\varsigma 2}(\mathbf{r}^{\kappa}_{\varsigma}) \quad if \ \mathbf{r}^{\kappa}_{\varsigma} \in \mathbf{R}^{\kappa}_{\varsigma 1} \cap \mathbf{R}^{\kappa}_{\varsigma 2} \end{array} \right\}$$

Hence $((\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1}, \mathbf{R}^{\kappa}_{\varsigma 1}) \cap_e (\mathbf{F}^{\kappa}_{\varsigma 2}, \mathbf{G}^{\kappa}_{\varsigma 2}, \mathbf{R}^{\kappa}_{\varsigma 2}))' = (\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1}, \mathbf{R}^{\kappa}_{\varsigma 1})' \cup_e (\mathbf{F}^{\kappa}_{\varsigma 2}, \mathbf{G}^{\kappa}_{\varsigma 2}, \mathbf{R}^{\kappa}_{\varsigma 2})'$.

4. Some bipolar picture fuzzy hypersoft geometric operators

This section focuses on defining the scoring, certainty, and accuracy function for bipolar picture fuzzy hypersoft numbers (BPFHSNs).

Definition 4.1. Let $(\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1})(\mathbf{r}^{\kappa}_{\varsigma 1})$ be a BPFHSN. Then the score function $\Psi((\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1})(\mathbf{r}^{\kappa}_{\varsigma 1}))$, accuracy function $\Omega((\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1})(\mathbf{r}^{\kappa}_{\varsigma 1}))$ and certainty function $\Phi((\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1})(\mathbf{r}^{\kappa}_{\varsigma 1}))$ of BPFHSN are defined as follows:

(i) $\Psi((\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1})(\mathbf{r}^{\kappa}_{\varsigma 1})) = (\frac{\ddot{\mu}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 1}} - \frac{\ddot{\eta}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 1}} - \frac{\ddot{\nu}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 1}} + \frac{\ddot{\mu}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 1}} - \frac{\ddot{\eta}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 1}} - \frac{\ddot{\nu}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 1}})/2$

(ii) $\Omega((\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1})(\mathbf{r}^{\kappa}_{\varsigma 1})) = (\frac{\ddot{\mu}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 1}} + \frac{\ddot{\nu}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 1}} - \frac{\ddot{\mu}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 1}} - \frac{\ddot{\nu}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 1}})/2$

(iii) $\Phi((\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1})(\mathbf{r}^{\kappa}_{\varsigma 1})) = (\frac{\ddot{\mu}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 1}} - \frac{\ddot{\nu}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 1}})/2.$

The range of score function $\Psi((\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1})(\mathbf{r}^{\kappa}_{\varsigma 1}))$ is [-1,1], range of accuracy function $\Omega((\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1})(\mathbf{r}^{\kappa}_{\varsigma 1}))$ is [0,1] and range of certainty fruition $\Phi((\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1})(\mathbf{r}^{\kappa}_{\varsigma 1}))$ of BPFHSSN is [0,1].

Definition 4.2. Let $(\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1})(\mathbf{r}^{\kappa}_{\varsigma 1})$ and $(\mathbf{F}^{\kappa}_{\varsigma 2}, \mathbf{G}^{\kappa}_{\varsigma 2})(\mathbf{r}^{\kappa}_{\varsigma 2})$ be two BPFHSSNs. The method to compare the two sets is defined as follows:

(i) If $\Psi((\mathbf{F}^{\kappa}_{\varsigma_1},\mathbf{G}^{\kappa}_{\varsigma_1})(\mathbf{r}^{\kappa}_{\varsigma_1}))>\Psi((\mathbf{F}^{\kappa}_{\varsigma_2},\mathbf{G}^{\kappa}_{\varsigma_2})(\mathbf{r}^{\kappa}_{\varsigma_2}))$ then $(\mathbf{F}^{\kappa}_{\varsigma_1},\mathbf{G}^{\kappa}_{\varsigma_1})(\mathbf{r}^{\kappa}_{\varsigma_1})>(\mathbf{F}^{\kappa}_{\varsigma_2},\mathbf{G}^{\kappa}_{\varsigma_2})(\mathbf{r}^{\kappa}_{\varsigma_2})$

(ii) $\Psi((\mathbf{F}^{\kappa}_{\varsigma_1},\mathbf{G}^{\kappa}_{\varsigma_1})(\mathbf{r}^{\kappa}_{\varsigma_1}))=\Psi((\mathbf{F}^{\kappa}_{\varsigma_2},\mathbf{G}^{\kappa}_{\varsigma_2})(\mathbf{r}^{\kappa}_{\varsigma_2}))$ and $\Omega((\mathbf{F}^{\kappa}_{\varsigma_1},\mathbf{G}^{\kappa}_{\varsigma_1})(\mathbf{r}^{\kappa}_{\varsigma_1}))>\Omega((\mathbf{F}^{\kappa}_{\varsigma_2},\mathbf{G}^{\kappa}_{\varsigma_2})(\mathbf{r}^{\kappa}_{\varsigma_2}))$ then $(\mathbf{F}^{\kappa}_{\varsigma_1},\mathbf{G}^{\kappa}_{\varsigma_1})(\mathbf{r}^{\kappa}_{\varsigma_1})>(\mathbf{F}^{\kappa}_{\varsigma_2},\mathbf{G}^{\kappa}_{\varsigma_2})(\mathbf{r}^{\kappa}_{\varsigma_2})$

(iii) $\Psi((\mathbf{F}^{\kappa}_{\varsigma_1},\mathbf{G}^{\kappa}_{\varsigma_1})(\mathbf{r}^{\kappa}_{\varsigma_1}))=\Psi((\mathbf{F}^{\kappa}_{\varsigma_2},\mathbf{G}^{\kappa}_{\varsigma_2})(\mathbf{r}^{\kappa}_{\varsigma_2}))$, $\Omega((\mathbf{F}^{\kappa}_{\varsigma_1},\mathbf{G}^{\kappa}_{\varsigma_1})(\mathbf{r}^{\kappa}_{\varsigma_1}))=\Omega((\mathbf{F}^{\kappa}_{\varsigma_2},\mathbf{G}^{\kappa}_{\varsigma_2})(\mathbf{r}^{\kappa}_{\varsigma_2}))$ and $\Phi((\mathbf{F}^{\kappa}_{\varsigma_1},\mathbf{G}^{\kappa}_{\varsigma_1})(\mathbf{r}^{\kappa}_{\varsigma_1}))>\Phi((\mathbf{F}^{\kappa}_{\varsigma_2},\mathbf{G}^{\kappa}_{\varsigma_2})(\mathbf{r}^{\kappa}_{\varsigma_2}))$ then $(\mathbf{F}^{\kappa}_{\varsigma_1},\mathbf{G}^{\kappa}_{\varsigma_1})(\mathbf{r}^{\kappa}_{\varsigma_1})>(\mathbf{F}^{\kappa}_{\varsigma_2},\mathbf{G}^{\kappa}_{\varsigma_2})(\mathbf{r}^{\kappa}_{\varsigma_2})$

(iv) $\Psi((\mathbf{F}^{\kappa}_{\varsigma_1},\mathbf{G}^{\kappa}_{\varsigma_1})(\mathbf{r}^{\kappa}_{\varsigma_1}))=\Psi((\mathbf{F}^{\kappa}_{\varsigma_2},\mathbf{G}^{\kappa}_{\varsigma_2})(\mathbf{r}^{\kappa}_{\varsigma_2}))$, $\Omega((\mathbf{F}^{\kappa}_{\varsigma_1},\mathbf{G}^{\kappa}_{\varsigma_1})(\mathbf{r}^{\kappa}_{\varsigma_1}))=\Omega((\mathbf{F}^{\kappa}_{\varsigma_2},\mathbf{G}^{\kappa}_{\varsigma_2})(\mathbf{r}^{\kappa}_{\varsigma_2}))$ and $\Phi((\mathbf{F}^{\kappa}_{\varsigma_1},\mathbf{G}^{\kappa}_{\varsigma_1})(\mathbf{r}^{\kappa}_{\varsigma_1}))=\Phi((\mathbf{F}^{\kappa}_{\varsigma_2},\mathbf{G}^{\kappa}_{\varsigma_2})(\mathbf{r}^{\kappa}_{\varsigma_2}))$ then $(\mathbf{F}^{\kappa}_{\varsigma_1},\mathbf{G}^{\kappa}_{\varsigma_1})(\mathbf{r}^{\kappa}_{\varsigma_1})=(\mathbf{F}^{\kappa}_{\varsigma_2},\mathbf{G}^{\kappa}_{\varsigma_2})(\mathbf{r}^{\kappa}_{\varsigma_2})$.

Definition 4.3. If $(\mathbf{F}^{\kappa}_{\varsigma_i},\mathbf{G}^{\kappa}_{\varsigma_i})(\mathbf{r}^{\kappa}_{\varsigma_i})=(\frac{\ddot{\mu}^+}{\mathbf{r}^{\kappa}_{\varsigma_i}},\frac{\ddot{\eta}^+}{\mathbf{r}^{\kappa}_{\varsigma_i}},\frac{\ddot{v}^+}{\mathbf{r}^{\kappa}_{\varsigma_i}},\frac{\ddot{\mu}^-}{\mathbf{r}^{\kappa}_{\varsigma_i}},\frac{\ddot{\eta}^-}{\mathbf{r}^{\kappa}_{\varsigma_i}},\frac{\ddot{v}^-}{\mathbf{r}^{\kappa}_{\varsigma_i}})$, $(i=1,2,3,........n)$ be a congregation of BPFHSN.
A function from BPFHSWG: $\xi_p\to\xi$ is called a bipolar picture fuzzy hypersoft weighted geometric (BPFHSWG) operator.

$$BPFHSWG\{(\mathbf{F}^{\kappa}_{\varsigma_1},\mathbf{G}^{\kappa}_{\varsigma_1})(\mathbf{r}^{\kappa}_{\varsigma_1}),(\mathbf{F}^{\kappa}_{\varsigma_2},\mathbf{G}^{\kappa}_{\varsigma_2})(\mathbf{r}^{\kappa}_{\varsigma_2}),(\mathbf{F}^{\kappa}_{\varsigma_3},\mathbf{G}^{\kappa}_{\varsigma_3})(\mathbf{r}^{\kappa}_{\varsigma_3}),.......(\mathbf{F}^{\kappa}_{\varsigma_n},\mathbf{G}^{\kappa}_{\varsigma_n})(\mathbf{r}^{\kappa}_{\varsigma_n})\}=\sum_{i=1}^{n}[(\mathbf{F}^{\kappa}_{\varsigma_i},\mathbf{G}^{\kappa}_{\varsigma_i})(\mathbf{r}^{\kappa}_{\varsigma_i})]^{T_i}=\{(\mathbf{F}^{\kappa}_{\varsigma_1},\mathbf{G}^{\kappa}_{\varsigma_1})(\mathbf{r}^{\kappa}_{\varsigma_1})\}^{T_1}\bigotimes\{(\mathbf{F}^{\kappa}_{\varsigma_2},\mathbf{G}^{\kappa}_{\varsigma_2})(\mathbf{r}^{\kappa}_{\varsigma_2})\}^{T_2}\bigotimes\{(\mathbf{F}^{\kappa}_{\varsigma_3},\mathbf{G}^{\kappa}_{\varsigma_3})(\mathbf{r}^{\kappa}_{\varsigma_3})\}^{T_3}\bigotimes.......\bigotimes\{(\mathbf{F}^{\kappa}_{\varsigma_n},\mathbf{G}^{\kappa}_{\varsigma_n})(\mathbf{r}^{\kappa}_{\varsigma_n})\}^{T_n}$$

where T_i is the weight vector (WV) of $[(\mathbf{F}^{\kappa}_{\varsigma_i},\mathbf{G}^{\kappa}_{\varsigma_i})(\mathbf{r}^{\kappa}_{\varsigma_i})]$, $T_i\ \in[0,1]$ and $\sum_{i=1}^{n}T_i=1$.

Theorem 4.4. $(\mathbf{F}^{\kappa}_{\varsigma_i},\mathbf{G}^{\kappa}_{\varsigma_i})(\mathbf{r}^{\kappa}_{\varsigma_i})=(\frac{\ddot{\mu}^+}{\mathbf{r}^{\kappa}_{\varsigma_i}},\frac{\ddot{\eta}^+}{\mathbf{r}^{\kappa}_{\varsigma_i}},\frac{\ddot{v}^+}{\mathbf{r}^{\kappa}_{\varsigma_i}},\frac{\ddot{\mu}^-}{\mathbf{r}^{\kappa}_{\varsigma_i}},\frac{\ddot{\eta}^-}{\mathbf{r}^{\kappa}_{\varsigma_i}},\frac{\ddot{v}^-}{\mathbf{r}^{\kappa}_{\varsigma_i}}),(i=1,2,3,........n)$ *be a congregation of BPFHSN, then*

$$BPFHSWG\ \{(\mathbf{F}^{\kappa}_{\varsigma_1},\mathbf{G}^{\kappa}_{\varsigma_1})(\mathbf{r}^{\kappa}_{\varsigma_1}),(\mathbf{F}^{\kappa}_{\varsigma_2},\mathbf{G}^{\kappa}_{\varsigma_2})(\mathbf{r}^{\kappa}_{\varsigma_2}),(\mathbf{F}^{\kappa}_{\varsigma_3},\mathbf{G}^{\kappa}_{\varsigma_3})(\mathbf{r}^{\kappa}_{\varsigma_3}),.......(\mathbf{F}^{\kappa}_{\varsigma_n},\mathbf{G}^{\kappa}_{\varsigma_n})(\mathbf{r}^{\kappa}_{\varsigma_n})\}=$$
$$\left\{\begin{array}{l}\prod_{i=1}^{n}[\frac{\ddot{\mu}^+}{\mathbf{r}^{\kappa}_{\varsigma_i}}+\frac{\ddot{\eta}^+}{\mathbf{r}^{\kappa}_{\varsigma_i}}]^{T_i}-\prod_{i=1}^{n}[\frac{\ddot{\eta}^+}{\mathbf{r}^{\kappa}_{\varsigma_i}}]^{T_i},\prod_{i=1}^{n}[\frac{\ddot{\eta}^+}{\mathbf{r}^{\kappa}_{\varsigma_i}}]^{T_i},1-\prod_{i=1}^{n}[1-\frac{\ddot{v}^+}{\mathbf{r}^{\kappa}_{\varsigma_i}}]^{T_i},\\ \prod_{i=1}^{n}[\frac{(-\ddot{\mu}^-)}{\mathbf{r}^{\kappa}_{\varsigma_i}}-\frac{(-\ddot{\eta}^-)}{\mathbf{r}^{\kappa}_{\varsigma_i}}]^{T_i}-\prod_{i=1}^{n}[\frac{(-\ddot{\eta}^-)}{\mathbf{r}^{\kappa}_{\varsigma_i}}]^{T_i},\prod_{i=1}^{n}[\frac{(-\ddot{\eta}^-)}{\mathbf{r}^{\kappa}_{\varsigma_i}}]^{T_i},1-\prod_{i=1}^{n}[1-\frac{(-\ddot{v}^-)}{\mathbf{r}^{\kappa}_{\varsigma_i}}]^{T_i}\end{array}\right\}.$$

Proof. This theorem is demonstrated through mathematical induction.
For n = 2

$$[(\mathbf{F}^{\kappa}_{\varsigma_1},\mathbf{G}^{\kappa}_{\varsigma_1})(\mathbf{r}^{\kappa}_{\varsigma_1})]^{T_1}=$$
$$\left\{\begin{array}{l}[\frac{\ddot{\mu}^+}{\mathbf{r}^{\kappa}_{\varsigma_1}}+\frac{\ddot{\eta}^+}{\mathbf{r}^{\kappa}_{\varsigma_1}}]^{T_1}-[\frac{\ddot{\eta}^+}{\mathbf{r}^{\kappa}_{\varsigma_1}}]^{T_1},[\frac{\ddot{\eta}^+}{\mathbf{r}^{\kappa}_{\varsigma_1}}]^{T_1},1-[1-\frac{\ddot{v}^+}{\mathbf{r}^{\kappa}_{\varsigma_1}}]^{T_1},\\ [\frac{(-\ddot{\mu}^-)}{\mathbf{r}^{\kappa}_{\varsigma_1}}-\frac{(-\ddot{\eta}^-)}{\mathbf{r}^{\kappa}_{\varsigma_1}}]^{T_1}-[\frac{(-\ddot{\eta}^-)}{\mathbf{r}^{\kappa}_{\varsigma_1}}]^{T_1},[\frac{(-\ddot{\eta}^-)}{\mathbf{r}^{\kappa}_{\varsigma_1}}]^{T_1},1-[1-\frac{(-\ddot{v}^-)}{\mathbf{r}^{\kappa}_{\varsigma_1}}]^{T_1}\end{array}\right\}$$

$$[(\mathbf{F}^{\kappa}_{\varsigma_2},\mathbf{G}^{\kappa}_{\varsigma_2})(\mathbf{r}^{\kappa}_{\varsigma_2})]^{T_2}=$$
$$\left\{\begin{array}{l}[\frac{\ddot{\mu}^+}{\mathbf{r}^{\kappa}_{\varsigma_2}}+\frac{\ddot{\eta}^+}{\mathbf{r}^{\kappa}_{\varsigma_2}}]^{T_2}-[\frac{\ddot{\eta}^+}{\mathbf{r}^{\kappa}_{\varsigma_2}}]^{T_2},[\frac{\ddot{\eta}^+}{\mathbf{r}^{\kappa}_{\varsigma_2}}]^{T_2},1-[1-\frac{\ddot{v}^+}{\mathbf{r}^{\kappa}_{\varsigma_2}}]^{T_2},\\ [\frac{(-\ddot{\mu}^-)}{\mathbf{r}^{\kappa}_{\varsigma_2}}-\frac{(-\ddot{\eta}^-)}{\mathbf{r}^{\kappa}_{\varsigma_2}}]^{T_2}-[\frac{(-\ddot{\eta}^-)}{\mathbf{r}^{\kappa}_{\varsigma_2}}]^{T_2},[\frac{(-\ddot{\eta}^-)}{\mathbf{r}^{\kappa}_{\varsigma_2}}]^{T_2},1-[1-\frac{(-\ddot{v}^-)}{\mathbf{r}^{\kappa}_{\varsigma_2}}]^{T_2}\end{array}\right\}$$

Then, it follows that

$$[(\mathbf{F}^{\kappa}_{\varsigma_1},\mathbf{G}^{\kappa}_{\varsigma_1})(\mathbf{r}^{\kappa}_{\varsigma_1})]^{T_1}\bigotimes[(\mathbf{F}^{\kappa}_{\varsigma_2},\mathbf{G}^{\kappa}_{\varsigma_2})(\mathbf{r}^{\kappa}_{\varsigma_2})]^{T_2}$$
$$=\left\{\begin{array}{l}\{[\frac{\ddot{\mu}^+}{\mathbf{r}^{\kappa}_{\varsigma_1}}+\frac{\ddot{\eta}^+}{\mathbf{r}^{\kappa}_{\varsigma_1}}]^{T_1}\}\{[\frac{\ddot{\mu}^+}{\mathbf{r}^{\kappa}_{\varsigma_2}}+\frac{\ddot{\eta}^+}{\mathbf{r}^{\kappa}_{\varsigma_2}}]^{T_2}\}-\{[\frac{\ddot{\eta}^+}{\mathbf{r}^{\kappa}_{\varsigma_1}}]^{T_1}\}\{[\frac{\ddot{\eta}^+}{\mathbf{r}^{\kappa}_{\varsigma_2}}]^{T_2}\},\{[\frac{\ddot{\eta}^+}{\mathbf{r}^{\kappa}_{\varsigma_1}}]^{T_1}\}\{[\frac{\ddot{\eta}^+}{\mathbf{r}^{\kappa}_{\varsigma_2}}]^{T_2}\},\\ \{1-[1-\frac{\ddot{v}^+}{\mathbf{r}^{\kappa}_{\varsigma_1}}]^{T_1}\}\{1-[1-\frac{\ddot{v}^+}{\mathbf{r}^{\kappa}_{\varsigma_2}}]^{T_2}\},\\ \{[\frac{(-\ddot{\mu}^-)}{\mathbf{r}^{\kappa}_{\varsigma_1}}-\frac{(-\ddot{\eta}^-)}{\mathbf{r}^{\kappa}_{\varsigma_1}}]^{T_1}\}\{[\frac{(-\ddot{\mu}^-)}{\mathbf{r}^{\kappa}_{\varsigma_2}}-\frac{(-\ddot{\eta}^-)}{\mathbf{r}^{\kappa}_{\varsigma_2}}]^{T_2}\}-\{[\frac{(-\ddot{\eta}^-)}{\mathbf{r}^{\kappa}_{\varsigma_1}}]^{T_1}\}\{[\frac{(-\ddot{\eta}^-)}{\mathbf{r}^{\kappa}_{\varsigma_2}}]^{T_2}\},\\ \{[\frac{(-\ddot{\eta}^-)}{\mathbf{r}^{\kappa}_{\varsigma_1}}]^{T_1}\}\{[\frac{(-\ddot{\eta}^-)}{\mathbf{r}^{\kappa}_{\varsigma_2}}]^{T_2}\},\{1-[1-\frac{(-\ddot{v}^-)}{\mathbf{r}^{\kappa}_{\varsigma_1}}]^{T_1}\}\{1-[1-\frac{(-\ddot{v}^-)}{\mathbf{r}^{\kappa}_{\varsigma_2}}]^{T_2}\}\end{array}\right\}$$

$$=\left\{\begin{array}{l}\prod_{i=1}^{2}[\frac{\ddot{\mu}^{+}}{\mathbf{r}_{\varsigma i}^{\kappa}}+\frac{\ddot{\eta}^{+}}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i}-\prod_{i=1}^{2}[\frac{\ddot{\eta}^{+}}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i},\prod_{i=1}^{2}[\frac{\ddot{\eta}^{+}}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i},1-\prod_{i=1}^{2}[1-\frac{\ddot{v}^{+}}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i},\\ \prod_{i=1}^{2}[\frac{(-\ddot{\mu}^{-})}{\mathbf{r}_{\varsigma i}^{\kappa}}-\frac{(-\ddot{\eta}^{-})}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i}-\prod_{i=1}^{2}[\frac{(-\ddot{\eta}^{-})}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i},\prod_{i=1}^{2}[\frac{(-\ddot{\eta}^{-})}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i},1-\prod_{i=1}^{2}[1-\frac{(-\ddot{v}^{-})}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i}\end{array}\right\}$$

This shows that it is true for $n=2$, now let that it holds

for $n=k$, i.e.,

$\{(\mathbf{F}_{\varsigma 1}^{\kappa},\mathbf{G}_{\varsigma 1}^{\kappa})(\mathbf{r}_{\varsigma 1}^{\kappa}),(\mathbf{F}_{\varsigma 2}^{\kappa},\mathbf{G}_{\varsigma 2}^{\kappa})(\mathbf{r}_{\varsigma 2}^{\kappa}),(\mathbf{F}_{\varsigma 3}^{\kappa},\mathbf{G}_{\varsigma 3}^{\kappa})(\mathbf{r}_{\varsigma 3}^{\kappa}),\ldots\ldots,(\mathbf{F}_{\varsigma k}^{\kappa},\mathbf{G}_{\varsigma k}^{\kappa})(\mathbf{r}_{\varsigma k}^{\kappa})\}=$

$$\left\{\begin{array}{l}\{\prod_{i=1}^{k}[\frac{\ddot{\mu}^{+}}{\mathbf{r}_{\varsigma i}^{\kappa}}+\frac{\ddot{\eta}^{+}}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i}-\prod_{i=1}^{k}[\frac{\ddot{\eta}^{+}}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i},\prod_{i=1}^{k}[\frac{\ddot{\eta}^{+}}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i},1-\prod_{i=1}^{k}[1-\frac{\ddot{v}^{+}}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i},\\ \prod_{i=1}^{k}[\frac{(-\ddot{\mu}^{-})}{\mathbf{r}_{\varsigma i}^{\kappa}}-\frac{(-\ddot{\eta}^{-})}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i}-\prod_{i=1}^{k}[\frac{(-\ddot{\eta}^{-})}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i},\prod_{i=1}^{k}[\frac{(-\ddot{\eta}^{-})}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i},1-\prod_{i=1}^{k}[1-\frac{(-\ddot{v}^{-})}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i}\end{array}\right\}$$

Now $n=k+1$, by operational laws of BPFHSNs, we have

$\{(\mathbf{F}_{\varsigma 1}^{\kappa},\mathbf{G}_{\varsigma 1}^{\kappa})(\mathbf{r}_{\varsigma 1}^{\kappa}),(\mathbf{F}_{\varsigma 2}^{\kappa},\mathbf{G}_{\varsigma 2}^{\kappa})(\mathbf{r}_{\varsigma 2}^{\kappa}),(\mathbf{F}_{\varsigma 3}^{\kappa},\mathbf{G}_{\varsigma 3}^{\kappa})(\mathbf{r}_{\varsigma 3}^{\kappa}),\ldots,(\mathbf{F}_{\varsigma n}^{\kappa},\mathbf{G}_{\varsigma k}^{\kappa}+1)(\mathbf{r}_{\varsigma k}^{\kappa}+1)\}$

$$=\left\{\begin{array}{l}\prod_{i=1}^{k}[\frac{\ddot{\mu}^{+}}{\mathbf{r}_{\varsigma i}^{\kappa}}+\frac{\ddot{\eta}^{+}}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i}-\prod_{i=1}^{k}[\frac{\ddot{\eta}^{+}}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i},\prod_{i=1}^{k}[\frac{\ddot{\eta}^{+}}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i},1-\prod_{i=1}^{k}[1-\frac{\ddot{v}^{+}}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i},\\ \prod_{i=1}^{k}[\frac{(-\ddot{\mu}^{-})}{\mathbf{r}_{\varsigma i}^{\kappa}}-\frac{(-\ddot{\eta}^{-})}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i}-\prod_{i=1}^{k}[\frac{(-\ddot{\eta}^{-})}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i},\prod_{i=1}^{k}[\frac{(-\ddot{\eta}^{-})}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i},1-\prod_{i=1}^{k}[1-\frac{(-\ddot{v}^{-})}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i}\end{array}\right\}$$

$$\bigotimes\left\{\begin{array}{l}[\frac{\ddot{\mu}^{+}}{\mathbf{r}_{\varsigma k}^{\kappa}+1}+\frac{\ddot{\eta}^{+}}{\mathbf{r}_{\varsigma k}^{\kappa}+1}]^{T_{k+1}}-[\frac{\ddot{\eta}^{+}}{\mathbf{r}_{\varsigma k}^{\kappa}+1}]^{T_{k+1}},[\frac{\ddot{\eta}^{+}}{\mathbf{r}_{\varsigma k}^{\kappa}+1}]^{T_{k+1}},1-[1-\frac{\ddot{v}^{+}}{\mathbf{r}_{\varsigma k}^{\kappa}+1}]^{T_{k+1}},\\ [\frac{(-\ddot{\mu}^{-})}{\mathbf{r}_{\varsigma k}^{\kappa}+1}-\frac{(-\ddot{\eta}^{-})}{\mathbf{r}_{\varsigma k}^{\kappa}+1}]^{T_{k+1}}-[\frac{(-\ddot{\eta}^{-})}{\mathbf{r}_{\varsigma k}^{\kappa}+1}]^{T_{k+1}},[\frac{(-\ddot{\eta}^{-})}{\mathbf{r}_{\varsigma k}^{\kappa}+1}]^{T_{k+1}},1-[1-\frac{(-\ddot{v}^{-})}{\mathbf{r}_{\varsigma k}^{\kappa}+1}]^{T_{k+1}}\end{array}\right\}$$

$$=\left\{\begin{array}{l}\prod_{i=1}^{k+1}[\frac{\ddot{\mu}^{+}}{\mathbf{r}_{\varsigma i}^{\kappa}}+\frac{\ddot{\eta}^{+}}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i}-\prod_{i=1}^{k+1}[\frac{\ddot{\eta}^{+}}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i},\prod_{i=1}^{k+1}[\frac{\ddot{\eta}^{+}}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i},1-\prod_{i=1}^{k+1}[1-\frac{\ddot{v}^{+}}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i},\\ \prod_{i=1}^{k+1}[\frac{(-\ddot{\mu}^{-})}{\mathbf{r}_{\varsigma i}^{\kappa}}-\frac{(-\ddot{\eta}^{-})}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i}-\prod_{i=1}^{k+1}[\frac{(-\ddot{\eta}^{-})}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i},\prod_{i=1}^{k+1}[\frac{(-\ddot{\eta}^{-})}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i},1-\prod_{i=1}^{k+1}[1-\frac{(-\ddot{v}^{-})}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i}\end{array}\right\}$$

The above expression proves that $n=K+1$ meaning that Theorem 4.4 holds by the principle of mathematical induction for all values of n.

$$BPFHSWG\{(\mathbf{F}_{\varsigma 1}^{\kappa},\mathbf{G}_{\varsigma 1}^{\kappa})(\mathbf{r}_{\varsigma 1}^{\kappa}),(\mathbf{F}_{\varsigma 2}^{\kappa},\mathbf{G}_{\varsigma 2}^{\kappa})(\mathbf{r}_{\varsigma 2}^{\kappa}),(\mathbf{F}_{\varsigma 3}^{\kappa},\mathbf{G}_{\varsigma 3}^{\kappa})(\mathbf{r}_{\varsigma 3}^{\kappa}),\ldots(\mathbf{F}_{\varsigma n}^{\kappa},\mathbf{G}_{\varsigma n}^{\kappa})(\mathbf{r}_{\varsigma n}^{\kappa})\}$$

$$=\left\{\begin{array}{l}\prod_{i=1}^{n}[\frac{\ddot{\mu}^{+}}{\mathbf{r}_{\varsigma i}^{\kappa}}+\frac{\ddot{\eta}^{+}}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i}-\prod_{i=1}^{n}[\frac{\ddot{\eta}^{+}}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i},\prod_{i=1}^{n}[\frac{\ddot{\eta}^{+}}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i},1-\prod_{i=1}^{n}[1-\frac{\ddot{v}^{+}}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i},\\ \prod_{i=1}^{n}[\frac{(-\ddot{\mu}^{-})}{\mathbf{r}_{\varsigma i}^{\kappa}}-\frac{(-\ddot{\eta}^{-})}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i}-\prod_{i=1}^{n}[\frac{(-\ddot{\eta}^{-})}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i},\prod_{i=1}^{n}[\frac{(-\ddot{\eta}^{-})}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i},1-\prod_{i=1}^{n}[1-\frac{(-\ddot{v}^{-})}{\mathbf{r}_{\varsigma i}^{\kappa}}]^{T_i}\end{array}\right\}\quad\square$$

5. MCDM based on some bipolar picture fuzzy hypersoft geometric operators

Suppose that $\mho_{\varsigma}^{\kappa}=\{\mathbf{u}_{\varsigma 1}^{\kappa},\mathbf{u}_{\varsigma 2}^{\kappa},\ldots,\mathbf{u}_{\varsigma r}^{\kappa}\}$ is the set of alternatives and $\mathbf{R}_{\varsigma}^{\kappa}=\{\mathbf{r}_{\varsigma 1}^{\kappa},\mathbf{r}_{\varsigma 2}^{\kappa},\ldots,\mathbf{r}_{\varsigma q}^{\kappa}\}$ is the set of criterion. Let T be the weight vector, s.t. $T_i\in[0,1]$ and $\sum_{i=1}^{n}T_i=1$ $(i=1,2,\ldots,n)$ and T_i show the weight of $\mathbf{R}_{\varsigma}^{\kappa}$. Alternatives on an attribute are reviewed by the decision-maker (DM) and the assessment measurements has to be in the BPFHSN. Assume that $\beta=(\rho_{ij})r\times q$ is the decision matrix provided by DM. (ρ_{ij}) represent a BPFHSN for alternative $\mathbb{Q}_{\varsigma}^{\kappa}$ associated with the criterion $\mathbb{C}$. Here we have some conditions such that

(i) $[\ddot{\mu}_{ij}^{+},\ddot{\eta}_{ij}^{+},\ddot{v}_{ij}^{+},\ddot{\mu}_{ij}^{-},\ddot{\eta}_{ij}^{-},\ddot{v}_{ij}^{-}]\in[0,1]$

(ii) $0\le[\ddot{\mu}_{ij}^{+}+\ddot{\eta}_{ij}^{+}+\ddot{v}_{ij}^{+}\le 1,\ \textit{and}\ -1\le[\ddot{\mu}_{ij}^{-}+\ddot{\eta}_{ij}^{-}+\ddot{v}_{ij}^{-}\le 0$

An algorithm is developed to discuss Multi-Criteria Decision Making (MCDM). MCDM algorithms are valuable tools for handling decision-making problems that involve multiple criteria or attributes.

5.1. Algorithm

Step 1.
Construct the decision matrix based on the decision-maker's personal opinion in the form of BPFHSNs, $\rho_{ij}=[\ddot{\mu}_{ij}^{+},\ddot{\eta}_{ij}^{+},\ddot{v}_{ij}^{+},\ddot{\mu}_{ij}^{-},\ddot{\eta}_{ij}^{-},\ddot{v}_{ij}^{-}]$

$\beta=(\rho_{ij})r\times q=$

$$\begin{pmatrix} \ddot{\mu}^{+}_{11}, \ddot{\eta}^{+}_{11}, \ddot{v}^{+}_{11}, \ddot{\mu}^{-}_{11}, \ddot{\eta}^{-}_{11}, \ddot{v}^{-}_{11} & \dots & \ddot{\mu}^{+}_{1q}, \ddot{\eta}^{+}_{1q}, \ddot{v}^{+}_{1q}, \ddot{\mu}^{-}_{1q}, \ddot{\eta}^{-}_{1q}, \ddot{v}^{-}_{1q} \\ \dots & \dots & \dots \\ \dots & \dots & \dots \\ \dots & \dots & \dots \\ \ddot{\mu}^{+}_{r1}, \ddot{\eta}^{+}_{r1}, \ddot{v}^{+}_{r1}, \ddot{\mu}^{-}_{r1}, \ddot{\eta}^{-}_{r1}, \ddot{v}^{-}_{r1} & \dots & \ddot{\mu}^{+}_{rq}, \ddot{\eta}^{+}_{rq}, \ddot{v}^{+}_{rq}, \ddot{\mu}^{-}_{rq}, \ddot{\eta}^{-}_{rq}, \ddot{v}^{-}_{rq} \end{pmatrix}$$

Step 2.
We normalized the decision matrix. By normalizing the decision matrix, it is important to ensure that the values from different attributes are comparable and have equal importance in subsequent analysis steps, such as weighting or aggregating the criteria. The formula to normalize the given decision matrix is as follows:

$$p_{ij} = \begin{pmatrix} [\rho_{ij}]^{c} & i \neq j \\ \rho_{ij} & i = j \end{pmatrix}$$

Step 3.
To obtain the collective aggregated value $\mathbf{g}_i$ of each alternative $\mho^{\kappa}_{\varsigma}$ under various parameter $\mathbf{r}^{\kappa}_{\varsigma}$ use the BPFHSWA operators,

$$BPFHSWG\{(\mathbf{F}^{\kappa}_{\varsigma\, 1}, \mathbf{G}^{\kappa}_{\varsigma\, 1})(\mathbf{r}^{\kappa}_{\varsigma\, 1}), (\mathbf{F}^{\kappa}_{\varsigma\, 2}, \mathbf{G}^{\kappa}_{\varsigma\, 2})(\mathbf{r}^{\kappa}_{\varsigma\, 2}), (\mathbf{F}^{\kappa}_{\varsigma\, 3}, \mathbf{G}^{\kappa}_{\varsigma\, 3})(\mathbf{r}^{\kappa}_{\varsigma\, 3}), \dots (\mathbf{F}^{\kappa}_{\varsigma\, n}, \mathbf{G}^{\kappa}_{\varsigma\, n})(\mathbf{r}^{\kappa}_{\varsigma\, n})\}$$
$$= \left\{ \begin{array}{l} \prod_{i=1}^{n} [\frac{\ddot{\mu}^{+}}{\mathbf{r}^{\kappa}_{\varsigma\, i}} + \frac{\ddot{\eta}^{+}}{\mathbf{r}^{\kappa}_{\varsigma\, i}}]^{T_i} - \prod_{i=1}^{n} [\frac{\ddot{\eta}^{+}}{\mathbf{r}^{\kappa}_{\varsigma\, i}}]^{T_i}, \prod_{i=1}^{n} [\frac{\ddot{\eta}^{+}}{\mathbf{r}^{\kappa}_{\varsigma\, i}}]^{T_i}, 1 - \prod_{i=1}^{n} [1 - \frac{\ddot{v}^{+}}{\mathbf{r}^{\kappa}_{\varsigma\, i}}]^{T_i}, \\ \\ \prod_{i=1}^{n} [\frac{(-\ddot{\mu}^{-})}{\mathbf{r}^{\kappa}_{\varsigma\, i}} - \frac{(-\ddot{\eta}^{-})}{\mathbf{r}^{\kappa}_{\varsigma\, i}}]^{T_i} - \prod_{i=1}^{n} [\frac{(-\ddot{\eta}^{-})}{\mathbf{r}^{\kappa}_{\varsigma\, i}}]^{T_i}, \prod_{i=1}^{n} [\frac{(-\ddot{\eta}^{-})}{\mathbf{r}^{\kappa}_{\varsigma\, i}}]^{T_i}, 1 - \prod_{i=1}^{n} [1 - \frac{(-\ddot{v}^{-})}{\mathbf{r}^{\kappa}_{\varsigma\, i}}]^{T_i} \end{array} \right\}$$
$$= \{\mu_j^{ag+}, \eta_j^{ag+}, v_j^{ag+}, \mu_j^{ag-}, \eta_j^{ag-}, v_j^{ag-}\} = \mathbf{g}_j$$

The aggregated value of each alternative $\mho^{\kappa}_{\varsigma\, i}$ $i = (1, 2, \dots, m)$ is determined based on the decision matrix produced from Step 2 and the various parameters $\mho^{\kappa}_{\varsigma\, i}$.

Step 4.
Compute the score functions for all $\mathbf{g}_j$ using the score functions:

$$\Psi(\mathbf{g}_j) = (\mu_j^{ag+} - \eta_j^{ag+} - v_j^{ag+} + \mu_j^{ag-} - \eta_j^{ag-} - v_j^{ag-})/2 \ = \ \mathbf{s}_j.\ j = \{1, 2, 3, \dots n\}.$$

Step 5.
Choose a suitable alternative based on the maximum score obtained from the score functions. By comparing the scores of each alternative identify the alternative with the highest score. The most favorable or preferred option is one who have highest score.

6. Performance analysis of abrasive textiles using BPFHSS

The goal of the science and technology field of pattern recognition is to categorize items into different groups. The identification of data analysis, shapes, pattern classification, network management and regulatory, the processing of natural languages, granitic identifier, biomedical impulses, smell attribution, knowledge of credit fraud tracking, fingerprint scanners, palmprint engineering, and facial recognition software, medical diagnosis, weather prediction, intellectual prowess, informatics, voice to text conversion, and terrorism identification are just a few applications where this technique is frequently used.

Performance analysis and quality control are critical components in ensuring the success and competitiveness of any industry. Analyzing the performance of products or processes and ensuring they meet the desired quality standards is essential in providing customers with products that meet their needs and expectations. Effective performance analysis and quality control can help to identify and correct issues early on, reducing the likelihood of costly recalls or rework and ultimately improving customer satisfaction. Industries prioritizing performance analysis and quality control are more likely to stay ahead of the competition, maintain customer loyalty, and foster a culture of continuous improvement. Therefore, performance analysis and quality control play a vital role in ensuring industries' overall success and sustainability in today's global economy.

In this research, the designed algorithm is designed and applied for the performance analysis of abrasive textiles as have countless applications in metalworking, woodworking, construction, electronics, and automotive industries. Performance analysis of abrasive textiles involves assessing the ability of the material to perform its intended function, which is to remove or smooth a surface by friction. There are several factors that are typically considered when conducting performance analysis of abrasive textiles, including:

1. Abrasive Particle Size: The size of the abrasive particles used in the textile will impact its performance. Smaller particle sizes tend to result in a finer surface finish, while larger particles will remove material more quickly. The size and distribution of the particles in the textile can be analyzed using microscopy or particle size analysis.
2. Bonding Agent: The bonding agent used to hold the abrasive particles in place on the textile can also affect its performance. The bonding agent should be strong enough to hold the particles securely, but not so strong that it interferes with their ability to abrade the surface. Common bonding agents include resin, vitrified clay, and rubber.
3. Textile Density: The density of the abrasive textile can affect its performance. A denser textile will generally be more aggressive

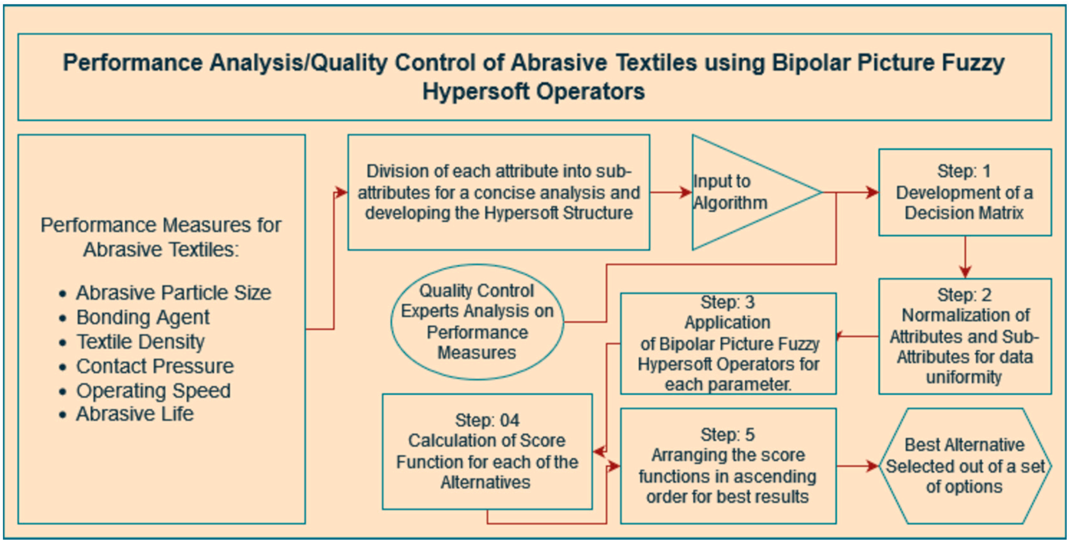

Fig. 1. An Algorithmic overview of Performance Measure/Quality Control of Abrasive Textiles using Bipolar Picture Fuzzy Hypersoft Set Operators.

and remove material more quickly, but it may also produce a rougher surface finish. A less dense textile may produce a smoother surface finish but may take longer to remove material.

4. Contact Pressure: The amount of pressure applied to the textile during use will also impact its performance. Too little pressure may result in ineffective abrasion, while too much pressure may cause excessive wear on the textile or damage to the surface being abraded.
5. Operating Speed: The speed at which the textile is moved over the surface being abraded will also affect its performance. Higher speeds may result in more efficient material removal but may also generate more heat and produce a rougher surface finish.
6. Abrasive Life: The lifespan of the abrasive textile is an important factor to consider as well. The textile should be able to maintain its abrasive properties for a reasonable amount of time before needing to be replaced.

The above parameters can be used for the analysis of the performance of these important textiles. The analysis is done using a few of the BPFS distance measurements. The detailed process of the quality control algorithm is explained below:

In a metal fabrication company, abrasive textiles are used to prepare metal surfaces for painting and coating applications (Fig. 1). To assess the performance of the abrasive textiles, a set of performance attributes was identified, which were further divided into sub-attributes. The quality control experts rely on their human intuition to express their preferences in the form of the presented structure. These preferences are based on the expert's intuition. Consider a batch of four alternatives of abrasive textiles out of which one is to be used for in the metal working industry. Let those four alternatives be represented by $\mho^{\kappa}_{\varsigma\, i}(\mathrm{i} = 1,2,3,4)$. Now, performance analysis is done on the basis of parameters which in this case are represented in a set of the form E = {Material Removal Rate (a_1), Surface Finish Quality (a_2), Abrasive Textile Life (a_3), Surface Coating Adhesion (a_4)}. For a concise and well-defined analysis, each attribute is divided into a set of sub-attributes each with are to be used in the hypersoft set structure. The sub-attribute sets are illustrated as:

$\mathbf{R}^{\kappa}_{\varsigma\,1} = \{b_{11} = \text{Cutting Speed}, b_{12} = \text{Cutting Force}, b_{13} = \text{Cutting Efficiency}\}$

$\mathbf{R}^{\kappa}_{\varsigma\,2} = \{b_{21} = \text{Roughness}, b_{22} = \text{Waviness}, b_{23} = \text{Flatness}\}$

$\mathbf{R}^{\kappa}_{\varsigma\,3} = \{b_{31} = \text{Particle Retention}, b_{32} = \text{Bonding Agent Durability}, b_{33} = \text{Textile Wear Rate}\}$

$\mathbf{R}^{\kappa}_{\varsigma\,4} = \{b_{41} = \text{Adhesion Strength}, b_{42} = \text{Coating Coverage}, b_{43} = \text{Coating Uniformity}, b_{44} = \text{Visual Appeal}\}$.

Now, a selection of experts provide their insight on the quality of the produced batches determining the best outcomes out of the four produced batches by individually examining the attributes and providing information in the form of BPFHSS.

$\mathbf{R}^{\kappa}_{\varsigma} = \mathbf{R}^{\kappa}_{\varsigma\,1} \times \mathbf{R}^{\kappa}_{\varsigma\,2} \times \mathbf{R}^{\kappa}_{\varsigma\,3} \times \mathbf{R}^{\kappa}_{\varsigma\,4}$ there are One Hundred eight possible outcomes but due to computational limitations, a complete step-by-step guide is illustrated for four possible outcomes.

$$\mathbf{R}^{\kappa}_{\varsigma} = \left\{ \begin{array}{ll} \mathbf{r}^{\kappa}_{\varsigma\,1} = (b_{11}, b_{21}, b_{31}, b_{42}), & \mathbf{r}^{\kappa}_{\varsigma\,2} = (b_{12}, b_{23}, b_{33}, b_{44}), \\ \mathbf{r}^{\kappa}_{\varsigma\,3} = (b_{12}, b_{22}, b_{31}, b_{42}), & \mathbf{r}^{\kappa}_{\varsigma\,4} = (b_{11}, b_{23}, b_{33}, b_{43}) \end{array} \right\}.$$

Then BPFHSS is given as: $(\mathbf{F}^{\kappa}_{\varsigma}, \mathbf{G}^{\kappa}_{\varsigma}, \mathbf{R}^{\kappa}_{\varsigma}) =$

Table 1
Tabular Representation of BPFHSS.

$\mho^{\kappa}_{\varsigma i}$	$\mathbf{r}^{\kappa}_{\varsigma 1}$	$\mathbf{r}^{\kappa}_{\varsigma 2}$	$\mathbf{r}^{\kappa}_{\varsigma 3}$	$\mathbf{r}^{\kappa}_{\varsigma 4}$
$\mho^{\kappa}_{\varsigma 1}$	(0.2,0.3,0.1, -0.6,-0.2,-0.1)	(0.5,0.3,0.2, -0.4,-0.2,-0.3)	(0.3,0.3,0.1, -0.2,-0.3,-0.4)	(0.1,0.4,0.2, -0.3,-0.3,-0.3)
$\mho^{\kappa}_{\varsigma 2}$	(0.2,0.3,0.2, -0.5,-0.1,-0.2)	(0.4,0.3,0.2, -0.5,-0.1,-0.2)	(0.2,0.3,0.1, -0.8,-0.1,-0.1)	(0.5,0.2,0.2, -0.4,-0.4,-0.2)
$\mho^{\kappa}_{\varsigma 3}$	(0.4,0.4,0.1, -0.5,-0.1,-0.2)	(0.4,0.4,0.2, -0.1,-0.3,-0.4)	(0.4,0.5,0.0, -0.1,-0.2,-0.5)	(0.2,0.5,0.1, -0.2,-0.2,-0.4)
$\mho^{\kappa}_{\varsigma 4}$	(0.3,0.4,0.1, -0.4,-0.1,-0.3)	(0.5,0.4,0.1, -0.4,-0.1,-0.3)	(0.3,0.4,0.1, -0.7,-0.1,-0.2)	(0.6,0.3,0.1, -0.4,-0.3,-0.3)

$$\left\{\begin{array}{l}
(\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1})(\mathbf{r}^{\kappa}_{\varsigma 1}) = \\
\{((0.2,0.3,0.1,-0.6,-0.2,-0.1)/\mho^{\kappa}_{\varsigma 1}, (0.5,0.3,0.2,-0.4,-0.2,-0.3)/\mho^{\kappa}_{\varsigma 2}, \\
(0.3,0.3,0.1,-0.2,-0.3,-0.4)/\mho^{\kappa}_{\varsigma 3}, (0.1,0.4,0.2,-0.3,-0.4,-0.3)/\mho^{\kappa}_{\varsigma 4})\}, \\
(\mathbf{F}^{\kappa}_{\varsigma 1}, \mathbf{G}^{\kappa}_{\varsigma 1})(\mathbf{r}^{\kappa}_{\varsigma 2}) = \\
\{((0.2,0.3,0.2,-0.4,-0.3,-0.2)/\mho^{\kappa}_{\varsigma 1}, (0.4,0.3,0.2,-0.5,-0.0,-0.2)/\mho^{\kappa}_{\varsigma 2}, \\
(0.2,0.3,0.1,-0.8,-0.1,-0.1)/\mho^{\kappa}_{\varsigma 3}, (0.5,0.2,0.2,-0.5,-0.4,-0.2)/\mho^{\kappa}_{\varsigma 4}, \}, \\
(\mathbf{F}^{\kappa}_{\varsigma 2}, \mathbf{G}^{\kappa}_{\varsigma 2})(\mathbf{r}^{\kappa}_{\varsigma 3}) = \\
\{((0.4,0.4,0.1,-0.5,-0.1,-0.2)/\mho^{\kappa}_{\varsigma 1}, (0.4,0.4,0.2,-0.3,-0.1,-0.4)/)\mho^{\kappa}_{\varsigma 2}, \\
(0.4,0.5,0.0,-0.1,-0.2,-0.5)/\mho^{\kappa}_{\varsigma 3}, (0.2,0.5,0.1,-0.2,-0.1,-0.4)/\mho^{\kappa}_{\varsigma 4})\}, \\
(\mathbf{F}^{\kappa}_{\varsigma 2}, \mathbf{G}^{\kappa}_{\varsigma 2})(\mathbf{r}^{\kappa}_{\varsigma 4}) = \\
\{((0.3,0.4,0.1,-0.3,-0.2,-0.3)/\mho^{\kappa}_{\varsigma 1}, (0.5,0.4,0.1,-0.4,-0.1,-0.3)/\mho^{\kappa}_{\varsigma 2}, \\
(0.3,0.4,0.1,-0.7,-0.0,-0.2)/\mho^{\kappa}_{\varsigma 3}, (0.6,0.3,0.1,-0.4,-0.3,-0.3)/\mho^{\kappa}_{\varsigma 4}), \},
\end{array}\right\}$$

The tabular representation of BPFHSS is shown in Table 1.

Step 1.

Construct the decision matrix given by the decision maker in Table 1 based on BPFHSS information.

$$\beta = \begin{pmatrix}
(0.2,0.3,0.1,-0.6,-0.2,-0.1) & (0.5,0.3,0.2,-0.4,-0.2,-0.3) & (0.3,0.3,0.1,-0.2,-0.3,-0.4) & (0.1,0.4,0.2,-0.3,-0.3,-0.3) \\
(0.2,0.3,0.2,-0.5,-0.1,-0.2) & (0.4,0.3,0.2,-0.5,-0.1,-0.2) & (0.2,0.3,0.1,-0.8,-0.1,-0.1) & (0.5,0.2,0.2,-0.4,-0.4,-0.2) \\
(0.4,0.4,0.1,-0.5,-0.1,-0.2) & (0.4,0.4,0.2,-0.1,-0.3,-0.4) & (0.4,0.5,0.0,-0.1,-0.2,-0.5) & (0.2,0.5,0.1,-0.2,-0.2,-0.4) \\
(0.3,0.4,0.1,-0.4,-0.1,-0.3) & (0.5,0.4,0.1,-0.4,-0.1,-0.3) & (0.3,0.4,0.1,-0.7,-0.1,-0.2) & (0.6,0.3,0.1,-0.4,-0.3,-0.3)
\end{pmatrix}$$

Step 2.

The decision matrix is normalized using the formula given below:

$$N_{\beta} = \begin{pmatrix} [\rho_{ij}]^c & i = j \\ \rho_{ij} & i \neq j \end{pmatrix}$$

A normalized matrix is given below

$$N_{\beta} = \begin{pmatrix}
(0.1,0.3,0.2,-0.1,-0.2,-0.6) & (0.5,0.3,0.2,-0.4,-0.2,-0.3) & (0.3,0.3,0.1,-0.2,-0.3,-0.4) & (0.1,0.4,0.2,-0.3,-0.3,-0.3) \\
(0.2,0.3,0.2,-0.5,-0.1,-0.2) & (0.2,0.3,0.4,-0.2,-0.1,-0.5) & (0.2,0.3,0.1,-0.8,-0.1,-0.1) & (0.5,0.2,0.2,-0.4,-0.4,-0.2) \\
(0.4,0.4,0.1,-0.5,-0.1,-0.2) & (0.4,0.4,0.2,-0.1,-0.3,-0.4) & (0.0,0.5,0.4,-0.5,-0.2,-0.1) & (0.2,0.5,0.1,-0.2,-0.2,-0.4) \\
(0.3,0.4,0.1,-0.4,-0.1,-0.3) & (0.5,0.4,0.1,-0.4,-0.1,-0.3) & (0.3,0.4,0.1,-0.7,-0.1,-0.2) & (0.1,0.3,0.6,-0.3,-0.3,-0.4)
\end{pmatrix}$$

$$
= \begin{pmatrix}
(\frac{\ddot{\mu}_{1}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 1}}, \frac{\ddot{\eta}_{1}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 1}}, \frac{\ddot{v}_{1}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 1}}, \frac{\ddot{\mu}_{1}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 1}}, \frac{\ddot{\eta}_{1}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 1}}, \frac{\ddot{v}_{1}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 1}}) & (\frac{\ddot{\mu}_{1}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 2}}, \frac{\ddot{\eta}_{1}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 2}}, \frac{\ddot{v}_{1}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 2}}, \frac{\ddot{\mu}_{1}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 2}}, \frac{\ddot{\eta}_{1}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 2}}, \frac{\ddot{v}_{1}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 2}}) & (\frac{\ddot{\mu}_{1}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 3}}, \frac{\ddot{\eta}_{1}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 3}}, \frac{\ddot{v}_{1}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 3}}, \frac{\ddot{\mu}_{1}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 3}}, \frac{\ddot{\eta}_{1}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 3}}, \frac{\ddot{v}_{1}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 3}}) & (\frac{\ddot{\mu}_{1}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 4}}, \frac{\ddot{\eta}_{1}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 4}}, \frac{\ddot{v}_{1}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 4}}, \frac{\ddot{\mu}_{1}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 4}}, \frac{\ddot{\eta}_{1}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 4}}, \frac{\ddot{v}_{1}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 4}}) \\
(\frac{\ddot{\mu}_{2}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 1}}, \frac{\ddot{\eta}_{2}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 1}}, \frac{\ddot{v}_{2}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 1}}, \frac{\ddot{\mu}_{2}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 1}}, \frac{\ddot{\eta}_{2}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 1}}, \frac{\ddot{v}_{2}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 1}}) & (\frac{\ddot{\mu}_{2}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 2}}, \frac{\ddot{\eta}_{2}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 2}}, \frac{\ddot{v}_{2}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 2}}, \frac{\ddot{\mu}_{2}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 2}}, \frac{\ddot{\eta}_{2}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 2}}, \frac{\ddot{v}_{2}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 2}}) & (\frac{\ddot{\mu}_{2}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 3}}, \frac{\ddot{\eta}_{2}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 3}}, \frac{\ddot{v}_{2}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 3}}, \frac{\ddot{\mu}_{2}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 3}}, \frac{\ddot{\eta}_{2}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 3}}, \frac{\ddot{v}_{2}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 3}}) & (\frac{\ddot{\mu}_{2}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 4}}, \frac{\ddot{\eta}_{2}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 4}}, \frac{\ddot{v}_{2}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 4}}, \frac{\ddot{\mu}_{2}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 4}}, \frac{\ddot{\eta}_{2}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 4}}, \frac{\ddot{v}_{2}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 4}}) \\
(\frac{\ddot{\mu}_{3}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 1}}, \frac{\ddot{\eta}_{3}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 1}}, \frac{\ddot{v}_{3}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 1}}, \frac{\ddot{\mu}_{3}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 1}}, \frac{\ddot{\eta}_{3}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 1}}, \frac{\ddot{v}_{3}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 1}}) & (\frac{\ddot{\mu}_{3}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 2}}, \frac{\ddot{\eta}_{3}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 2}}, \frac{\ddot{v}_{3}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 2}}, \frac{\ddot{\mu}_{3}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 2}}, \frac{\ddot{\eta}_{3}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 2}}, \frac{\ddot{v}_{3}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 2}}) & (\frac{\ddot{\mu}_{3}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 3}}, \frac{\ddot{\eta}_{3}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 3}}, \frac{\ddot{v}_{3}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 3}}, \frac{\ddot{\mu}_{3}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 3}}, \frac{\ddot{\eta}_{3}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 3}}, \frac{\ddot{v}_{3}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 3}}) & (\frac{\ddot{\mu}_{3}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 4}}, \frac{\ddot{\eta}_{3}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 4}}, \frac{\ddot{v}_{3}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 4}}, \frac{\ddot{\mu}_{3}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 4}}, \frac{\ddot{\eta}_{3}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 4}}, \frac{\ddot{v}_{3}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 4}}) \\
(\frac{\ddot{\mu}_{4}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 1}}, \frac{\ddot{\eta}_{4}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 1}}, \frac{\ddot{v}_{4}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 1}}, \frac{\ddot{\mu}_{4}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 1}}, \frac{\ddot{\eta}_{4}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 1}}, \frac{\ddot{v}_{4}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 1}}) & (\frac{\ddot{\mu}_{4}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 2}}, \frac{\ddot{\eta}_{4}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 2}}, \frac{\ddot{v}_{4}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 2}}, \frac{\ddot{\mu}_{4}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 2}}, \frac{\ddot{\eta}_{4}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 2}}, \frac{\ddot{v}_{4}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 2}}) & (\frac{\ddot{\mu}_{4}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 3}}, \frac{\ddot{\eta}_{4}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 3}}, \frac{\ddot{v}_{4}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 3}}, \frac{\ddot{\mu}_{4}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 3}}, \frac{\ddot{\eta}_{4}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 3}}, \frac{\ddot{v}_{4}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 3}}) & (\frac{\ddot{\mu}_{4}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 4}}, \frac{\ddot{\eta}_{4}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 4}}, \frac{\ddot{v}_{4}^{+}}{\mathbf{r}^{\kappa}_{\varsigma 4}}, \frac{\ddot{\mu}_{4}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 4}}, \frac{\ddot{\eta}_{4}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 4}}, \frac{\ddot{v}_{4}^{-}}{\mathbf{r}^{\kappa}_{\varsigma 4}})
\end{pmatrix}
$$

Step 3.
Obtain the aggregated value of each alternative $\mho^{\kappa}_{\varsigma i}$ using the N_{β} and weight vector $T_i = \{T_1, T_2, T_3, T_4\} = (0.2, 0.3, 0.3, 0.2)$, Apply the BPFHSWA operators,

$$
BPFHSWG\{(\mathbb{F}^{\kappa}_{\varsigma 1}, \mathbb{G}^{\kappa}_{\varsigma 1})(\mathbf{r}^{\kappa}_{\varsigma 1}), (\mathbb{F}^{\kappa}_{\varsigma 2}, \mathbb{G}^{\kappa}_{\varsigma 2})(\mathbf{r}^{\kappa}_{\varsigma 2}), (\mathbb{F}^{\kappa}_{\varsigma 3}, \mathbb{G}^{\kappa}_{\varsigma 3})(\mathbf{r}^{\kappa}_{\varsigma 3}), ...(\mathbb{F}^{\kappa}_{\varsigma n}, \mathbb{G}^{\kappa}_{\varsigma n})(\mathbf{r}^{\kappa}_{\varsigma n})\}
$$

$$
= \left\{ \begin{array}{l} \prod_{i=1}^{4}[\frac{\ddot{\mu}_j^{+}}{\mathbf{r}^{\kappa}_{\varsigma i}} + \frac{\ddot{\eta}_j^{+}}{\mathbf{r}^{\kappa}_{\varsigma i}}]^{T_i} - \prod_{i=1}^{4}[\frac{\ddot{\eta}_j^{+}}{\mathbf{r}^{\kappa}_{\varsigma i}}]^{T_i}, \prod_{i=1}^{4}[\frac{\ddot{\eta}_j^{+}}{\mathbf{r}^{\kappa}_{\varsigma i}}]^{T_i}, 1 - \prod_{i=1}^{4}[1 - \frac{\ddot{v}_j^{+}}{\mathbf{r}^{\kappa}_{\varsigma i}}]^{T_i}, \\ \prod_{i=1}^{4}[\frac{(-\ddot{\mu}_j^{-})}{\mathbf{r}^{\kappa}_{\varsigma i}} - \frac{(-\ddot{\eta}_j^{-})}{\mathbf{r}^{\kappa}_{\varsigma i}}]^{T_i} - \prod_{i=1}^{4}[\frac{(-\ddot{\eta}_j^{-})}{\mathbf{r}^{\kappa}_{\varsigma i}}]^{T_i}, \prod_{i=1}^{4}[\frac{(-\ddot{\eta}_j^{-})}{\mathbf{r}^{\kappa}_{\varsigma i}}]^{T_i}, 1 - \prod_{i=1}^{4}[1 - \frac{(-\ddot{v}_j^{-})}{\mathbf{r}^{\kappa}_{\varsigma i}}]^{T_i} \end{array} \right\}
$$

$= \mathbf{g}_j \ = \ \{\mu_j^{ag+}, \eta_j^{ag+}, v_j^{ag+}, \mu_j^{ag-}, \eta_j^{ag-}, v_j^{ag-}\}, \ j = \{1, 2, 3, 4\}$.
For each of the alternatives, the aggregated value is presented as follows:

$$\mathbf{g}_1 = \{0.1865789862, 0.3365865436, 0.1905216473, -0.1599519266, -0.2449489743, -0.4172370321\}$$

$$\mathbf{g}_2 = \{0.3169482778, 0.2550849001, 02357815965, -0.1741101127, -0.1641101127, -0.2631463439\}$$

$$\mathbf{g}_3 = \{0.2763575003, 0.4472135955, 0.1558825486, -0.1071773463, -0.1761729590, -0.3188511978\}$$

$$\mathbf{g}_4 = \{0.2319324178, 0.3565204916, 0.3493169373, -0.1551845574, -0.15518455739, -0.333011234\}$$

Step 4.
Compute the values of the score function for each of the alternatives using the formula:
$\Psi(\mathbf{g}_j) = (\mu_j^{ag+}, \eta_j^{ag+}, v_j^{ag+}, \mu_j^{ag-}, \eta_j^{ag-}, v_j^{ag-})/2 \ = \ \mathbf{s}_j$.

$$\mathbf{s}_1 = 0.08085243755, \quad \mathbf{s}_2 = 0.03961406255$$

$$\mathbf{s}_3 = 0.03055408335, \quad \mathbf{s}_4 = -0.0704468883$$

Step 5.
The rank of alternatives shows us that

$$\mathbf{s}_4 \leqslant \mathbf{s}_3 \leqslant \mathbf{s}_2 \leqslant \mathbf{s}_1$$

$\mathbf{s}_1$ corresponds to $\mho^{\kappa}_{\varsigma 1}$, so $\mho^{\kappa}_{\varsigma 1}$ is the best choice.

In conclusion, the use of advanced decision-making algorithms is crucial for the performance analysis and quality control of abrasive textiles. By considering multiple attributes and factors, these algorithms can provide accurate and precise evaluations of the performance of different abrasive textiles and help manufacturers optimize their production processes to meet specific industrial needs. Based on the highest score function value computed by the algorithm, it can be concluded that $\mho^{\kappa}_{\varsigma 1}$ performs the best given the circumstances. This underscores the importance of leveraging computational methods and decision-making algorithms to ensure the highest quality and performance of abrasive textiles in a wide range of industrial applications.

7. Comparative analysis

BPFHSS is a novel concept that combines multiple mathematical frameworks to handle decision-making problems with parameterization. It provides a comprehensive structure for analyzing and managing complex decision-making scenarios. BPFHSS is designed to examine all MD, AD, and NMD values from both positive and negative viewpoints. BPFHSS is particularly suitable for handling multivariable problems, where attributes must be further sub-seeded. The structural comparison highlights the unique characteristics of BPFHSS, including its ability to handle parameterization, consideration of both positive and negative viewpoints, and modeling of attributes and sub-attributes at different levels of granularity. These features make BPFHSS a versatile and effective framework for decision-making in situations involving uncertainty, ambiguity, and varying degrees of relevance or importance. The versatility of BPFHSS, through structural comparison based on some key evaluating characteristics, such as DM, NMD, AD, BPLRTY (bipolarity), SAAF (single-argument approximate function), MAAF (multi-argument approximate function), and DFPT (deep focus on parametric tuples) is shown in Table 2.

Table 2
A comprehensive comparison of the proposed structure with hybrid fuzzy structures reported in literature.

Structures	DoM	DoNM	DoNTM	BPLRTY	SAAF	MAAF	DFPT
FS [10]	✓	×	×	×	×	×	×
IFS [47]	✓	✓	×	×	×	×	×
SS [12]	×	×	×	×	×	✓	×
IFSS [21]	✓	✓	×	×	✓	×	×
PFSS [22]	✓	✓	✓	×	✓	×	×
BSS [24]	×	×	×	✓	✓	×	×
FBSS [48]	✓	×	×	✓	✓	×	×
IFBSS [49]	✓	✓	×	✓	✓	×	×
HSS [28]	×	×	×	×	✓	✓	✓
FHSS [32]	✓	×	×	×	✓	✓	✓
IFHSS [33]	✓	✓	×	×	✓	✓	✓
BHSS [36]	×	×	×	✓	✓	✓	✓
PFHSS [34]	✓	✓	✓	×	✓	✓	✓
BPFHSS (proposed)	✓	✓	✓	✓	✓	✓	✓

BPFHSS) can be seen as an upgraded version or an extension of various existing theories and concepts in the field of decision-making and uncertainty modeling. BPFHSS incorporates elements from multiple theories, including Soft Sets (SS), Fuzzy Sets (FS), Intuitionistic Fuzzy Sets (IFS), Bipolar Fuzzy Sets (BFS), Bipolar Fuzzy Soft Sets (BFSS), Bipolar Crisp Sets (BCFS), Intuitionistic Fuzzy soft Sets (IFSS), Intuitionistic Fuzzy Bipolar Soft Sets (IFBSS), Bipolar Intuitionistic Fuzzy Soft Sets (BIFSS), Picture Fuzzy Sets (PFS), Bipolar Picture Fuzzy Sets (BPFS), Bipolar Picture Fuzzy Soft Sets (BPFSS), Picture Fuzzy Hypersoft Sets (BPFHSS), and more. The development of BPFHSS was motivated by the need to integrate three essential elements: parameterization, picture, and bipolarity. These elements were combined to provide a framework that allows for a more comprehensive representation of decision-making situations where all three aspects are present. Following are few benefits of BPFHSS in textile industry which are limited in other existing theories.

- The decision-making process in the textile industry requires a combination of analytical techniques, industry knowledge, and strategic thinking. It involves considering various factors and stakeholders' perspectives to make informed decisions that contribute to the overall success and competitiveness of the textile business. Based on BPFHSS the decision-making in the textile industry allows decision-makers to consider both positive and negative aspects of the criteria. It provides a comprehensive and flexible framework to evaluate alternatives and make informed decisions considering the complex and uncertain nature of the textile industry. BPFHSS in decision-making has ability to decrease uncertainty and enhance decision accuracy as compared to existing theories.
- The textile industry faces various risks, such as supply chain disruptions, changing market demands, and quality issues. BPFHSS facilitates risk management by considering both positive and negative aspects, enabling decision-makers to evaluate the potential risks and drawbacks associated with each alternative. This helps in making informed decisions that mitigate risks and enhance operational resilience.
- The textile industry is characterized by rapid technological advancements, changing consumer preferences, and evolving market dynamics. BPFHSS's parameterization and flexibility enable decision-makers to adapt the analysis to changing conditions, incorporating new information and adjusting the evaluation criteria as needed.
- The textile industry involves various stakeholders, including designers, manufacturers, suppliers, retailers, and customers. BPFHSS facilitates stakeholder involvement by allowing the integration of multiple decision-makers' opinions and preferences. This inclusiveness leads to more balanced and informed decision-making, considering the diverse perspectives and expertise of stakeholders.

8. Conclusions

In conclusion, this research article explored the application of Multi-Criteria Decision Making (MCDM) methods in addressing complex real-world problems characterized by uncertainty, imprecision, and ambiguity due to hazy and insufficient information. While existing MCDM models have relied on fuzzy models and sets to handle ambiguity and uncertainty, they fail to handle situations where conflicting information requires assigning bipolarity to each argument parameter. To overcome these limitations, a novel extension called Bipolar Picture Fuzzy Hypersoft Set (BPFHSS) was proposed in this study, which combines the advantages of Bipolar Fuzzy Hypersoft Sets (BFHS) and Picture Fuzzy Sets (PFS). Various algebraic operations and significant features of BPFHSS were discussed, along with the development of a BPFHSS Weighted Geometric Operator, a new aggregation operator crucial to information aggregation in MCDM. Furthermore, a new MCDM method for ranking objects using BPFHSS was devised based on these operators. The viability of the presented technique were demonstrated using a numerical model. For future research, the proposed aggregation operator can be applied to other MCDM approaches such as VIKOR, AHP, and a variety of ELECTRE and PROMETHEE algorithms while considering additional metrics like Heronian mean, Einstein, Bonferroni mean, and Dombi AOs. The findings of this study are expected to benefit researchers in pattern recognition, medicine, machine learning and design of decision support systems, offering valuable insights into tackling challenging real-world problems.

CRediT authorship contribution statement

Muhammad Imran Harl: Conceived and designed the experiments; Wrote the paper. **Muhammad Saeed:** Conceived and designed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data. **Muhammad Haris Saeed:** Performed the experiments; Wrote the paper. **Talal Alharbi:** Conceived and designed the experiments; Performed the experiments. **Tmader Alballa:** Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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