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Research article

A detailed study on quantification and modeling of drought characteristics using different copula families

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ABSTRACT

This study delves into analyzing drought patterns in Baluchistan by applying copula-based bivariate probabilistic models complemented by Severity Duration Frequency (SDF) curves. The calculation of the Standardized Precipitation Index (SPI) hinges on monthly aggregate precipitation data from ten distinct sites compiled over six-month periods. After evaluating various parametric distributions, the Log-Normal distribution emerges as suitable for modeling drought severity and duration.

A range of bivariate copulas is employed to simulate the characteristics of drought severity and duration, which are then compared against observed data. Remarkably, the Gumbel copula classified as an extreme value copula—outperforms its counterparts according to diverse statistical benchmarks. By utilizing the dependence function, we derive the conditional distribution of drought variables: severity and duration. These conditional distributions subsequently inform the calculation of return periods, forming the basis for constructing SDF diagrams at fixed recurrence levels across the study region. The study's finding indicates that a severe drought could occur over the region with higher return periods for a specific duration.

The implications of this research are significant, showcasing the potential of copula-based joint modeling techniques to generate frequency curves for drought severity and duration. This development holds promise for effective water resource management and the formulation of strategies to mitigate the impact of drought in vulnerable regions.

1. Introduction

Droughts stand out as the most formidable yet least comprehended natural hazards encompassing the entire climate spectrum. Their impact becomes particularly critical in sectors where water-dependent activities render vulnerability to drought situations Ahmed et al. [1]. This concern is especially salient in the developing world, where agricultural systems are not only a vital source of

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sustenance but also play a pivotal role in shaping socio-economic landscapes. In such contexts, the oscillations of rainfall patterns, fluctuations in water availability, and variations in climate dynamics can hold far-reaching consequences.

In the mosaic of global agriculture, Pakistan stands as an agrarian giant, with its rural population intricately woven into the fabric of its fertile land. The province of Baluchistan, representing an integral part of this tapestry, presents a unique case study. Its agricultural sustenance hinges entirely on the delicate balance of precipitation, as its farmlands solely rely on rainwater to nurture crops. However, the canvas of rainfall patterns has undergone noticeable alterations in recent decades, marked by deviations from historical norms. The repercussions of these deviations resonate deeply, triggering an unyielding cycle of water scarcity that permeates through economic, social, and environmental spheres. In a stark illustration of this, the Pakistan Meteorological Department's data for 2020 painted a sobering picture of rainfall deficits of 73.2% in Baluchistan, 72.2% in Sindh, and 12.9% in Khyber Pakhtunkhwa.

Within the intricate web of drought's impacts, perhaps the most insidious aspect is its longevity. The specter of a drought's aftermath lingers, often far beyond the point when meteorological conditions rebound. The scars etched by droughts on communities and economies are enduring, leaving a trail of disrupted livelihoods, agricultural losses, and socio-economic upheavals in their wake. Notably, the annals of Baluchistan's history have been punctuated by instances of devastating droughts that unfolded during pivotal junctures. The years 1967 to 1969, 1971, and the prolonged stretch from 1998 to 2002 bear testament to the recurring vulnerability of this region to the iron grip of drought.

Beyond the immediate and visible effects, the implications of droughts extend into the intangible realms of societal fabric and ecological balance. Addressing the multifaceted challenge of drought mitigation mandates an approach that goes beyond the confines of reactive measures. Instead, a nuanced understanding of the intricate interplay between climate shifts, water availability, and societal vulnerabilities is imperative. This interweaving of complexities demands a comprehensive investigation, one that navigates the interconnected threads of meteorology, hydrology, agriculture, and social dynamics. Moreover, in an era characterized by rapidly shifting global climate patterns, the proactive management of drought's impacts assumes paramount importance. The implications of water scarcity ripple across sectors, transcending geographical boundaries and socioeconomic strata. This urgency lends a distinctive significance to research endeavors that endeavor to unravel the intricacies of drought's behavior, predictability, and the mechanisms through which it manifests.

Numerous studies favoring drought quantification have been done nationally and in different parts of the world. For instance, Durrani et al. [2] and Naz et al. [3] pointed out significantly increasing trends in the frequency of heatwaves in Baluchistan, which was a sign of an upcoming drought. In addition, few other studies have been conducted to model drought behavior in this region. These studies are restricted to assessing the influence of droughts and adaptation schemes for livestock Shafiq and Kakar [4], soil degradation and rangeland efficiency Islam et al. [5], forthcoming drought alleviation policies Ahmed et al. [1], and farmers' coping and adaptation measures Ashraf and Routray[6], seasonal drought characteristics using SPI Ahmed et al. [1], drought monitor using SPI and standardized precipitation evapotranspiration index Qaisrani et al. [7], quantify drought risk to livelihood and mitigation Ashraf et al. [8].

Cancelliere and Salas [9] characterized droughts by duration, severity, and spatial extent. Guttman [10] calculated the Palmer Drought Severity Index (PDSI) of 30 years of historical data to provide drought severity measures and proposed retroactive dry and wet conditions using water balance techniques. McKee et al. [11] developed the SPI, especially for drought monitoring. Yamoah et al. [12] evaluated the SPI values to model droughts for agricultural purposes. Karavitis et al. [13] calculated the SPI to understand better drought duration, amplitude, and spatial extent in semi-arid portions of Greece. Xia et al. [14] estimated the SPIs over the past 60 years of data to analyze drought patterns in China. On the other hand, Joe [15] discussed various multivariate models for capturing the dependence using copula theory. Shiau [16] proposed a bivariate copulas model to model drought variables jointly. Later on, Nelsen [17] explores different copulas families with multivariate functions that integrate one-dimensional marginal distribution functions with uniform one-dimensional margins. Shiau and Modarres [18] developed the copula-based probabilistic approach for deriving drought SDF curves and analyzing the joint occurrences of droughts. Janga Reddy and Ganguli [19] presented copulas-based modeling of SDF concerning drought episodes in western Rajasthan, India.

Mirabbasi et al. [20] applied two-dimensional copulas to analyze meteorological drought characteristics (duration and severity) at the Sharafkhaneh gauge station in northwest Iran. The Galambos copula best fits the observed drought data, offering valuable insights for water resource planning and management. In another study, Nazeri Tahroudi et al. [21] proposed a copula-based new method to analyze meteorological and hydrological drought dynamics in the Zarinehroud basin. The Frank copula showed the best performance, allowing for the estimation of future drought durations and joint probabilities of drought characteristics for water resource management. Yusof et al. [22] characterized the drought variable's severity and duration using bivariate copula functions and calculated joint and conditional return periods for drought occurrences. Rajsekhar et al. [23] derived the drought Severity-Duration Frequency curves at fixed recurrence periods of 5, 10, 25, 50, and 100 years of droughts duration of 3, 6, 9, 12, 18, 24, and 36 months respectively, for the Texas region. Azam et al. [24] applied stochastic simulation through copula functions and measured the risk analysis of droughts using SPIs for different areas of South Korea. Bazrafshan et al. [25] explored the works on the drought events in arid and semi-arid regions of Iran and estimated the copula-based conditional return periods. Zhang et al. [26] practiced a copula-based stochastic multiobjective approach to optimize the irrigation strategy of crops in irrigation districts affected by seasonal agricultural drought in Southwest China. By combining meteorological factors through copula models, Tian et al. [27] constructed a multivariate standardized drought index for drought identification and evaluation over the Xijiang River Basin in South China.

Avsaroglu and Gumus [28] employs Copula functions to assess hydrological drought by considering the joint multivariate distribution for drought characteristics around Tigris River basin, Turkey, and they identified Galambos copula as the best fit. Results indicate a 5-10% difference between univariate and bivariate return periods, highlighting higher drought risk in the central and western parts of the basin. Deger et al. [29] assesses drought duration and severity at Euphrates Basin using Streamflow Drought

Index (SDI) at 3- and 6-month scales, finding strong correlations and selecting Lognormal, Weibull, and Gamma distributions. Gumbel copula proves superior for modeling joint return periods, highlighting varying drought risk across the basin for different return periods. In another research in Turkey Gumus et al. [30] assesses hydrological drought using the SDI method across three historical periods, revealing a significant increase in drought severity in the third period. Galambos copula is found to best represent drought parameters, enabling comparisons between univariate and bivariate return periods.

In the context of these considerations, our research strides forward with a pivotal objective to predict the uncertainty surrounding impending drought events across the expansive canvas of Baluchistan. This pursuit unfurls through the lens of a comprehensive analytical framework, one that harnesses diverse probabilistic models and the dynamic universe of copula families. In embarking on this scientific voyage, we discern the intricate dependencies that weave through drought variables, underscoring their collective behavior and offering insights into their predictive potential. This research foray adopts a meticulous approach, encapsulating the computation of the SPI, the characterization of drought variables, the integration of copula models, the derivation of joint and conditional distributions, and the eventual construction of SDF curves.

The proposed study distinguishes itself through several novel aspects: firstly, it utilizes a six-month timescale for SPI calculation on a moderate timescale, offering a distinctive perspective on drought assessment in the region. Secondly, it uses joint and conditional probability distributions, providing a more comprehensive understanding of the region's drought risk profile. Lastly, the study pioneers applying a conditional copula distribution mechanism for precise forecasting of future drought events, setting it apart from previous research conducted in this region and offering valuable insights for proactive drought management and mitigation.

Structured to provide a coherent narrative, the subsequent sections of this paper assume distinct roles. Section 2 unfurls the methodology, meticulously unveiling the mechanics of SPI computation, the architecture of copula classes, the intricacies of maximum likelihood estimation (MLE) procedures, simulation strategies, and the mathematical expression of joint and conditional distributions. Transitioning into Section 3, we delve into the heart of the matter—the presentation of results and a comprehensive discussion that embraces both the quantitative and qualitative facets of our findings. Finally, Section 4 concludes this scholarly journey by weaving together the strands of exploration, discussion, and analysis, culminating in reflections on the implications of our work and avenues for future inquiry.

2. Methodology

2.1. Study area

The study area encompasses the sprawling region of Baluchistan, spanning from latitude 22°N to 32°N and longitude 66°E to 70°E. This province, constituting the largest landmass in Pakistan, extends across a vast geographical expanse of 347,190 km^2. Characterized by a tapestry of landscapes, including mountains, flatlands, and deserts, Baluchistan boasts diverse climates that underscore its geographical diversity. The plains are subject to scorching heat during summers and mild winters, while the upper plateau experiences the juxtaposition of hot summers and cold winters. The lower plateau, in contrast, endures extreme warmth and aridity during the summer months, and it grapples with the challenge of severe cold during winter. The desert belt maintains a climate marked by its characteristic heat and arid conditions. Annually, the province receives an average precipitation ranging from 200 mm to 350 mm, with temperatures soaring up to 50 °C during the summer months in the plains. The region's heightened rainfall can be attributed to the influx of monsoon winds originating from both the Arabian Sea and the Bay of Bengal. As these monsoons traverse the region, their impact dwindles gradually from east to west due to diminishing air moisture, thereby rendering Baluchistan more susceptible to lower levels of rainfall—a vulnerability that sets the stage for heightened attention toward drought analysis.

The empirical exploration of the study is anchored in an extensive dataset spanning four decades, encompassing daily precipitation records from January 1981 to December 2020. To procure this dataset, meticulous efforts were dedicated to collecting records from ten strategically located meteorological stations across the Baluchistan region. This invaluable dataset was sourced from the reputable National Aeronautics and Space Administration (NASA) Power data access platform. By providing the requisite latitude and longitude coordinates for each precipitation station, the study seamlessly extracted the crucial monthly precipitation data, a cornerstone for the subsequent analytical undertakings. A comprehensive overview of the statistical characteristics of these meticulously selected meteorological stations is furnished in Table 1, and their strategic geographical positions are visually represented in Fig. 1. In addition, we tested the homogeneity, randomness, independence, and stationary assumption of the data by employing the procedure discussed in Naghettini [31, chp. 7] Ahmad et al. [32], Ahmad et al. [33] and Ahmad et al. [34]. To this end, the data of the considering stations meet the fundamental assumptions.

2.2. Standardized precipitation index

A plethora of drought indices exists in the scholarly realm, each designed to quantify and monitor droughts across varying contexts, see, e.g., Zargar et al. [35] and Eslamian et al. [36]. Among these indices, the SPI stands out for its simplicity, flexibility, spatial invariance, and probabilistic nature. The SPI, a powerful analytical instrument for precipitation data, holds the capability to assess diverse forms of drought phenomena—meteorological, agricultural, and hydrological. Unlike other indices that may necessitate multiple parameters, the SPI's elegance lies in its singular parameter setup, enabling seamless implementation and straightforward interpretation.

In its essence, the SPI serves as a mechanism to bestow algebraic values upon precipitation data, facilitating cross-comparisons among regions characterized by starkly distinct climatic conditions. By quantifying the cumulative rainfall within specific intervals

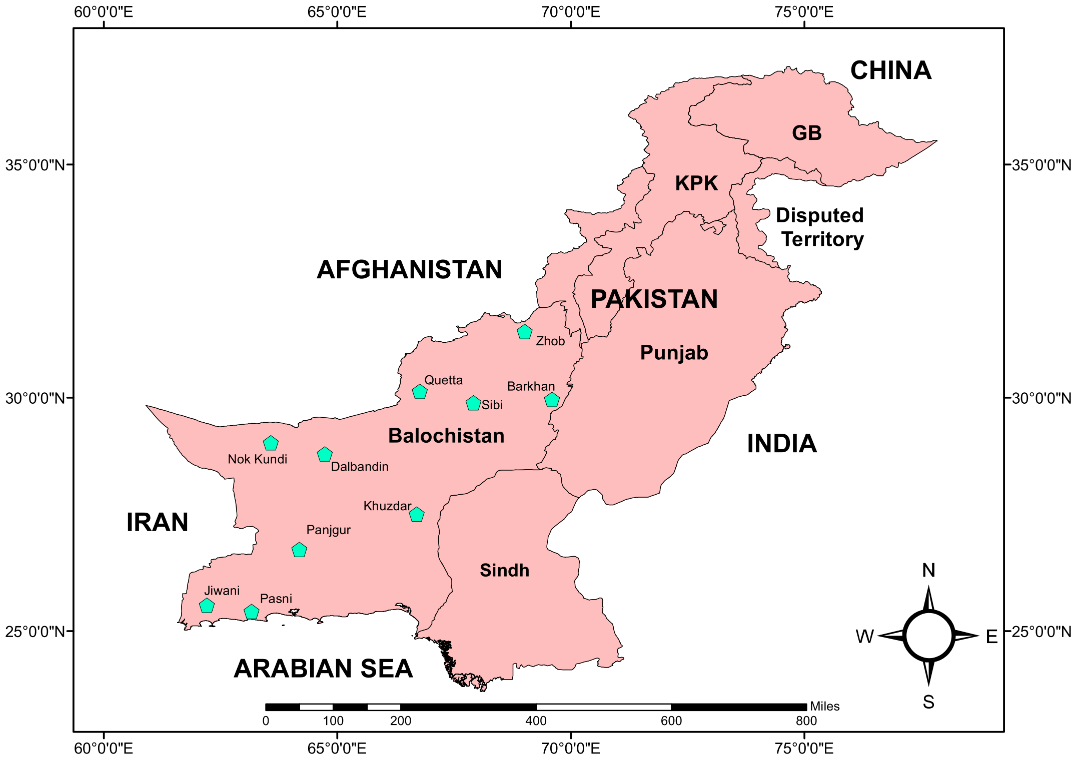


Fig. 1. Geographical spots of considered stations for the study.

Table 1
Summary statistics of the rainfall stations.

Station	Mean	SD	Skewness	Kurtosis
Bar khan	17.34	33.93	7.30	80.25
Quetta	17.12	28.8	3.03	12.15
Dalbandin	5.81	11.51	3.38	13.57
Zhoob	18.83	31.08	6.43	66.29
Jiwani	7.27	17.43	5.85	48.45
Khuzdar	14.8	26.79	4.14	24.54
Nok Kundi	5.18	17.57	14.15	255.45
Panjgur	6.65	14.67	4.89	32.62
Pasni	5.99	14.75	4.79	26.94
Sibbi	15.89	25.06	3.89	23.93

(e.g., 3, 6, 9, 12, 24, and 48 months) and relating it to the average precipitation during analogous periods, the SPI yields insight into deviations from the norm. Conceptually, the SPI quantifies the standard deviation above or below the average record—a measure that unveils the magnitude of precipitation anomalies. Following the established approach outlined by Wu et al. [37], the estimation of the SPI for the Baluchistan region leverages a comprehensive dataset spanning 40 years of recorded precipitation data. Long-term precipitation records are subjected to probability distribution fitting. This process aligns the mean standardized precipitation for the region within any given period to a zero value, while the standard deviation is calibrated to one. In a broader context, the computation of the SPI for a given year i, months j and time scale k is undertaken by following the framework established by Wu et al. [38]. That is,

1. Initially, the cumulative precipitation series is calculated $X_{ij}^{k}, i = 1, \ldots, n$ corresponds to months, and each span is the sum of precipitation of $k-1$ past sequential months.
2. On a monthly precipitation data set with $k = 6$ months, the cumulative probability distribution model (essentially the Gamma distribution) is fitted in the current scenario. The probability density function of the Gamma distribution is

$$f(x, \alpha, \beta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, \tag{1}$$

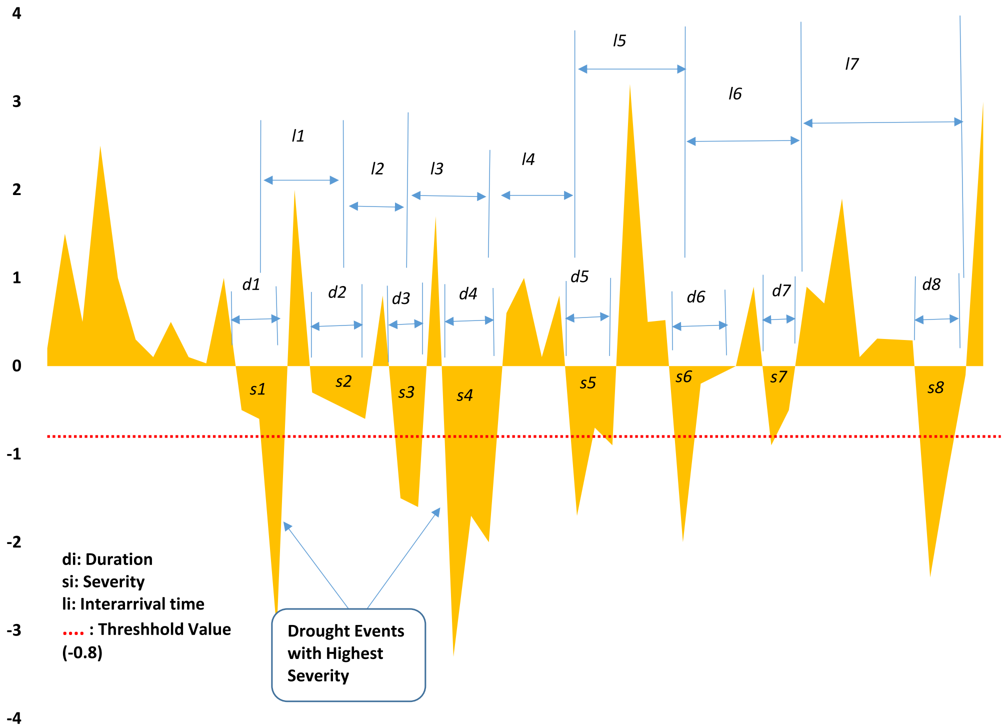


Fig. 2. Droughts characteristics depiction using SPIs.

where $\Gamma(\alpha)$ is the Gamma function and β and α are the scale and shape parameters respectively. The parameters of the model given (1) are estimated through maximum likelihood approach. The Cumulative Distribution Function (CDF) $F(x)$ is obtained using estimated parameters of observed monthly precipitation series at a specified time scale.

3. The precipitation data may have zero values; the Gamma distribution is not fitted for zero values. The mixture distribution function is thus applied for the zero values of precipitation distribution. The CDF of the mixture distribution is defined as

$$G(x) = q + (1 - q)F(x) \tag{2}$$

$F(x)$ is the CDF of Gamma distribution and q represents the probability of zero precipitation based on historical data.

4. Using equiprobability transformation to convert the CDF given in (2) to standard normal distribution.

$$SPI = \phi^{-1}\{G(x)\} \tag{3}$$

The observations produced by (3) are positive and negative, called SPI. We identify the severity and nature of the drought events through SPI runs and signs. For instance, the nature of droughts is classified as (-0.5 to -0.7) unusually drought, (-0.8 to -1.2) moderate drought, (-1.3 to -1.5) severe drought, (-1.6 to -1.9) extreme drought, and -2.0 or less exceptional drought Svoboda et al. [39]. In the present study, the chosen threshold value is -0.8, which labels that all values under the threshold are considered droughts. The severity is determined by using the SPI values that remain under the threshold, while the duration is decided by the number of months that SPI values remain below the threshold. The severity of the drought events i ($S_i, i = 1, 2, 3...$) occurrences is defined in (4) for each value of SPI the i^{th} period as

$$S_i = -\sum_{i=1}^{D} SPI_i \tag{4}$$

where D indicates the length of drought occurrences. Fig. 2 shows the characteristics of the three consecutive drought events based on SPI for a particular time scale. The spikes below the threshold indicate drought severity and duration. For example, the height and width of spikes corresponding to a drought event 1 show the severity and duration of droughts over a certain time scale, respectively.

2.3. Bivariate copula functions

A 2-dimensional copula is a joint CDF on $[0, 1]^2$ with standard uniform CDFs, that is

$$C(u, v) := P(U \leq u, V \leq v)$$

Table 2
Families of Copula functions considered for this study.

Copula	$C(u,v)$	Parameter space	$A(w)$	$\phi(t)$
Clayton	$\left[u^{-\theta}+v^{-\theta}-1\right]^{-1/\theta}$	$\theta\in[-1,\infty)\setminus\{0\}$	NA	$\frac{1}{\theta}(t^{-\theta}-1)$
Frank	$-\frac{1}{\theta}\log\left[1+\frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{(e^{-\theta}-1)}\right]$	$\theta\in(-\infty,\infty)\setminus\{0\}$	NA	$-\ln\frac{e^{-\theta t}-1}{e^{-\theta}-1)}$
Gumbel-Hougard	$\exp\left[-(\bar{u}^{\theta}+\bar{v}^{\theta})^{1/\theta}\right]$	$\theta\in[1,\infty)$	$\left[w^{\theta}+(1-w)^{\theta}\right]^{1/\theta}$	$(-\ln t)^{\theta}$
Galambos	$uv\exp\left[-(\bar{u}^{\theta}+\bar{v}^{\theta})^{-1/\theta}\right]$	$\theta\in[0,\infty)$	$1-\left[w^{-\theta}+(1-w)^{-\theta}\right]^{-1/\theta}$	NA
Plackett	$\frac{1}{2(\theta-1)}(s-q)$	$\theta\in[0,\infty)\setminus\{1\}$	NA	NA
Student's t	$\int_{-\infty}^{t_{\theta}^{-1}(u)}\int_{-\infty}^{t_{\theta}^{-1}(v)}\frac{1}{2\pi\sqrt{(1-r^2)}}\left[1+\frac{x^2-2rxy+y^2}{v(1-r^2)}\right]^{-(v+2)/2}dx\,dy$	$v>2, r\in(0,1)$	NA	NA
	where $t_v(x)=\int_{-\infty}^{x}\frac{\Gamma((v+1)/2)}{\sqrt{\pi v}\Gamma(v/2)}(1+y^2/v)^{-(v+1)/2}dy,\qquad v\neq 0$			

Where $\bar{u}=-\log u$ and $\bar{v}=-\log v$.

The key advantage of the copula in studying bivariate or multivariate distribution functions is summarized by Sklar's theorem Sklar [40], which we precisely recalled. Let $F_{X,Y}(x,y)$ be the joint CDF of X,Y with their marginal CDFs such as $F_X(x)=P(X\leq x)$ and $F_Y(y)=P(Y\leq y)$, respectively. Then, there exists a 2-dimensional copula $C:[0,1]^2\to[0,1],\forall x,y$. That is

$$F_{X,Y}(x,y)=C[F_X(x),F_Y(y)]=C(u,v)\tag{5}$$

If $F_X(x)$ and $F_Y(y)$ are continuous, C is unique; otherwise, the copula C is uniquely determined on $Ran(F)\times Ran(G)$. Furthermore, the scale of copula is under the bounded increasing transformation of random variables X and Y, as u and v, two uniformly distributed random variables on $[0,1]$ are expressed as $u=F_X(x)$ and $v=F_Y(y)$. The bivariate copula distribution function $C(.)$ has the following collective features with marginal CDFs of uniform random variables u and v

- $C(u,0)=0=C(0,v)\qquad\forall u,v\in[0,1]^2$
- $C(u,1)=u;\qquad$ and $\qquad C(1,v)=v\forall u,v\in[0,1]^2$
- If $C(u,v)$ is a joint copula distribution function then, $C(u_2,v_2)-C(u_2,v_1)-C(u_1,v_2)+C(u_1,v_1)>0\quad$ for $\quad 0\leq u_1\leq u_2\leq 1$ and $0\leq v_1\leq v_2\leq 1$.

In a continuous framework, the bivariate copula is differentiable

$$C(u,v)=\partial^2C(u,v))/\partial u\partial v$$

where $c(.)$ is the resulting bivariate copula density. In this paper, we practice different classes of copulas, such as the Archimedean copulas, extreme value copulas, Plackett copulas, and Elliptical copula (Student's- t copula). The description of classes of copulas is given in subsequent sections.

2.4. Archimedean copulas

The derivation of bivariate distributions through Archimedean copulas is more relaxed and straightforward. Archimedean copula has many proficient families for bestowing different patterns of dependence, and various estimations techniques have been established to estimate its parameters. This paper mainly practices two Archimedean copula functions: Clayton and Frank. Their copula functions, along with dependence parameter space and generator function, are given in Table 2

2.5. Extreme value copulas

In extreme value copula, the observations of the extreme events are classified according to the extreme value theory, which defines the tail behavior of the distribution. The extreme value copula function for drought modeling is defined as: let $(X_1,Y_1),\ldots,(X_n,Y_n)$ be independent pair of random variables with identical marginal distributions and joint copula function $C(.)$. Let $C_n(.)$ be a copula function for maximum values, for instance, $X_n=max(X_i)$ and $Y_n=max(Y_i), i=1,\ldots,n$. The copula function for maxima is defined as $C_{(n)}(u,v)=C^{(n)}(u^{1/n},v^{1/n}),\forall u,v\in[0,1]$. The limit sequence $\{C_{(n)}\}$ certainly tends to the notion of extreme value copula. That is

$$C(u,v)=\lim_{n\to\infty}C^{(n)}(u^{1/n},v^{1/n}),\forall u,v\in[0,1]$$

Furthermore, we considered Gumbel and Galambos's extreme value copulas for this study. Their relevant expressions for CDF and dependence functions are pined in Table 2.

In addition, we also consider Plackett's copula and elliptical class of copula in Table 2. The most frequently used elliptical distributions are the multivariate normal and Student-t distributions. The paper examines the bivariate Student's t copula function. The student's t copula has two dependence parameters. The elliptical class of copulas can capture both positive and negative dependency structures. Therefore, both tail dependence lower and upper have the same magnitude.

2.6. Estimation

Maximum likelihood estimation procedure is employed to obtain the copula parameters. Furthermore, the MLE method is merely based on the copula density function. The parameter estimate $\hat{\theta}_n$ is obtained by solving the maximum likelihood function numerically by using sample information, i.e., $U = U_{ij}$ of size n, where $i = 1, 2, \dots$ and $j \in \{1, 2\}$. So, the log-likelihood function is written in (6) as

$$l_U(\theta) = \sum_{i=1}^{n} \log C(U_{i,1}, U_{i,2}; \theta) = \sum_{i=1}^{n} \log \left[C\left(\sum_{i=1}^{n} \frac{R_i}{n+1}, \sum_{i=1}^{n} \frac{S_i}{n+1} \right) \right] \tag{6}$$

where R_i and S_i are the observed data ranks, $\theta \in \Theta$ is the copula dependence parameter, and it lies in $\mathbb{R}^p$ with $(p \geq 1)$. The ML estimator is given by

$$\hat{\theta}_n = \arg\max_{\theta \in \Theta}(l_U(\theta)) \tag{7}$$

We solve (7) numerically for all classes of copulas. Based on AIC and BIC criteria, we identify appropriate copula among many for drought modeling.

2.7. Tail dependence coefficient (TDC)

The most commonly used tail dependence was introduced by Sibuya [41] and later in Joe [15, p. 33] and various other studies. Let (X, Y) be a random pair with joint CDF $F(.)$ and marginals $F_X(x)$ and $F_Y(y)$. If the given limit exists, the upper tail dependence measure is defined as

$$\lambda_U = \lim_{t \to 1^-} \left[F_X(x) > t | F_Y(y) > t \right]$$

Then X and Y are said to be upper tail dependent if $\lambda_U > 0$, otherwise independent. Likewise, the lower tail dependence coefficient is defined as

$$\lambda_L = \lim_{t \to 0^+} \left[F_X(x) \leq t | F_Y(y) \leq t \right]$$

Here, we calculate lower and upper TDC through the joint copula function. For instance, if C is the copula of (X, Y) defined in (5), then the lower and upper TDC are given in (8), respectively. That is

$$\lambda_L = \lim_{t \to 0^+} \frac{C(t,t)}{t} \qquad \text{and} \qquad \lambda_U = \lim_{t \to 1^-} \frac{1 - 2t + C(t,t)}{1 - t} \tag{8}$$

2.8. Joint and conditional probabilities

2.8.1. Joint probability distribution of drought variables

The copula-based joint distribution of drought duration and severity can be used for significant evidence regarding drought supervision Shiau [16]. For instance, the probability of both events over the high thresholds is considered a critical condition for an exact water supply system. It can be used as a trigger for a possible drought policy. However, the joint probability can be easily developed in the form of copulas

$$\begin{aligned} P(D \geq d, S \geq s) &= 1 - F_D(d) - F_S(s) + F_{D,S}(d, s) \\ &= 1 - F_D(d) - F_S(s) + C\left[F_D(d), F_S(s)\right] \end{aligned} \tag{9}$$

where $F_S(s)$ and $F_D(d)$ are the CDF of drought severity and duration distributions, while $C\left[F_D(d), F_S(s)\right]$ is copula-based joint CDF of drought severity and duration distribution. The joint distribution of drought variables can also be obtained by providing the severity and duration values. For example, if the drought severity is recorded at 13 for the 12 months duration, then the resultant quantities are $F_D(d) = 0.976, F_S(s) = 0.953$, and $C\left[F_D(d), F_S(s)\right] = 0.954$, respectively. Using (9), we obtained $P(D \geq d, S \geq s)$ is equal to 0.023.

Further, conditional distribution is needed to explain the probability of specific drought occurrences. For conditional distribution development, we use copula-based bivariate distribution. For example, one can want to assess the drought severity probability given drought duration over a threshold (d'). According to Shiau [16], the conditional distribution is defined as

$$P(S \leq s | D \geq d') = \frac{P(S \leq s, D \geq d')}{P(D \geq d')} = \frac{F_S(s) - F_{D,S}(d', s)}{1 - F_D(d')} = \frac{F_S(s) - C[F_D(d'), F_S(s)]}{1 - F_D(d')} \tag{10}$$

Similarly, the conditional probability distribution of drought duration given severity is given as

$$P(D \leq d | S \geq s') = \frac{P(D \leq d, S \geq s')}{P(S \geq s')} = \frac{F_D(d) - F_{D,S}(d, s')}{1 - F_S(s')} = \frac{F_D(d) - C[F_D(d), F_S(s')]}{1 - F_S(s')} \tag{11}$$

Table 3
Summary statistics of precipitation and droughts variables.

Variable	Characteristics	Statistics	Value
Precipitation		Average annual precipitation	1378.47
		Standard deviation	1001.78
		Coefficient of variation	72.67
Droughts			
		Number of droughts	237
		Interval arrival time (years)	1.74
	Severity	Average	4.26
		Standard deviation	5.21
		Minimum value	0.80
		Maximum value	37.58
		Skewness	3.01
	Duration	Average	3.37
		Standard deviation	3.40
		Minimum value	1.00
		Maximum value	25.00
		Skewness	2.49

To gain wide-range information about droughts in a drought-prone area, conditional probabilities at different levels of thresholds could be helpful for planners and management. The section 3 briefly describes joint and conditional distributions of drought severity and duration.

2.8.2. *Derivation of severity-duration frequency relationship*

The copula-based SDF curve can be used as a gadget to explore the associated drought characteristics. For instance, the severity and duration of every drought episode are branded as bivariate random variables. Therefore, the conditional recurrence interval connected with drought severity, drought duration, and frequency is given by

$$T_{S|D}(s|d) = \frac{1}{\gamma\left[1 - F_{S|D}(s|d)\right]} \tag{12}$$

where $(F_{S|D}(s|d)$ is the conditional CDF of $(S|D = d)$ and γ is showing the frequency of incoming drought events. To represent theoretical SDF relationship, the $T_{S|D}(s|d)$ is given (12) is written as

$$T_{S|D}(s|d) = \frac{1}{\gamma\left[1 - C_{F_S|F_D}(F_S(s)|F_D(d))\right]} \tag{13}$$

$F_D(d)$ and $F_S(s)$ are the CDF of marginal distributions combined through the copula function. For more details, readers can see, for instance, Shiau and Modarres [18]. The drought events occurrence can be defined by practicing (13) to any site having hydrological variables except that $F_D(d), F_S(s), C$ and γ need to be estimated from the observed data.

3. Results and discussion

3.1. *Summary statistics of data and SPI*

In this study, the SPI is computed using monthly total precipitation data of 40 years for 6-month accumulation period. This indicates the medium-term trend that reflects more acute conditions of droughts. The summary statistics of observed data and drought severity and duration for the study region are reported in Table 3. The average precipitation recorded in the Baluchistan region is approximately 1378.47 mm, with a large standard deviation of 1001.78 mm. The standard deviation might be more prominent due to the wide range in yearly precipitation (i.e., 464.81 mm to 6101.06 mm). Table 3 also shows that the 237 droughts occurrences have been identified with a mean inter-arrival period of 1.74 years. The drought severity and duration distribution are positively skewed and have a high standard deviation 5.21 and 3.40, respectively.

Moreover, this fact shows that the drought features fluctuate over the region. In addition, the SPI proposed by McKee et al. [42] is used in this study to define droughts. For instance, the six-month SPI is calculated to observe precipitation data and is called SPI-6. The SPI-6 series for 1992 to 2020 precipitation data of 10 meteorological stations of the Baluchistan region is shown in Fig. 3. The drought is reported once the value of SPI-6 falls under zero in Fig. 3.

It can be noticed that severe droughts occurred from (1991 to 1992) and (2003 to 2004) when SPI-6 approached -4.0 and -3.0, respectively. Furthermore, the consecutive negative SPI-6 period is called drought duration, while the cumulative values of SPI-6 within the drought duration are called drought severity Moreover, Fig. 4 shows a scatter diagram with drought severity at the x-axis and duration on the y-axis. Pearson's correlation coefficient and Kendall's tau measures were estimated to determine the relationship between drought severity and duration. The estimated values of both measures (i.e., 0.97 and 0.85) show that drought variables are highly positively correlated. Also, the values of the measures are significant at 1% level.

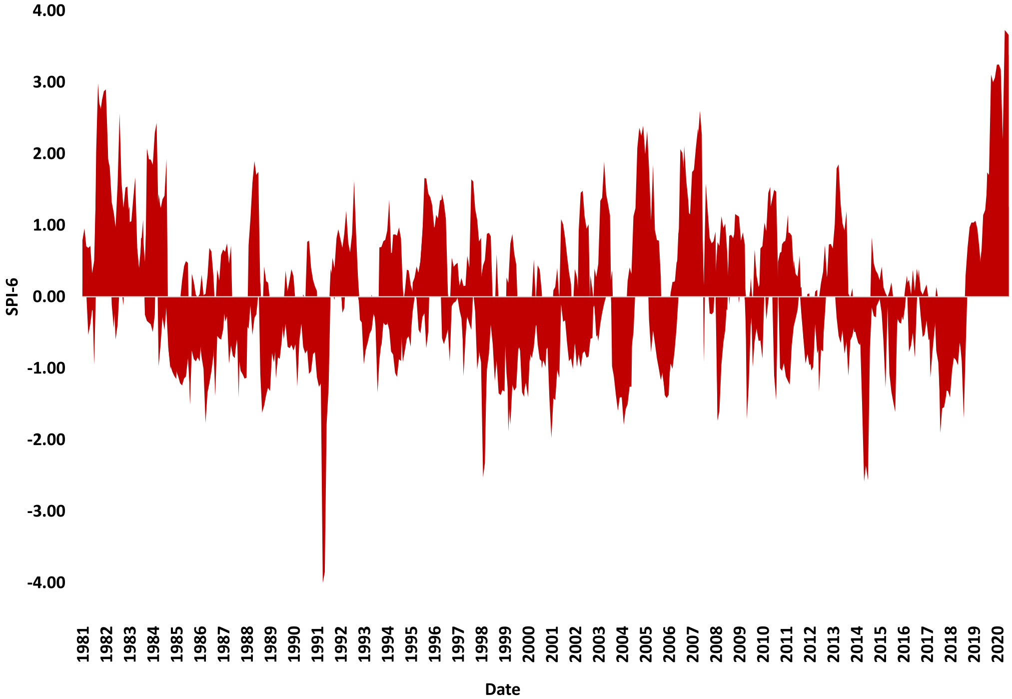


Fig. 3. The six-month SPI series for the period (1981-2020) of Baluchistan region.

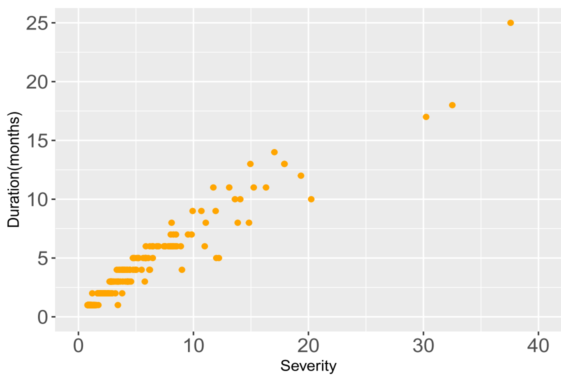


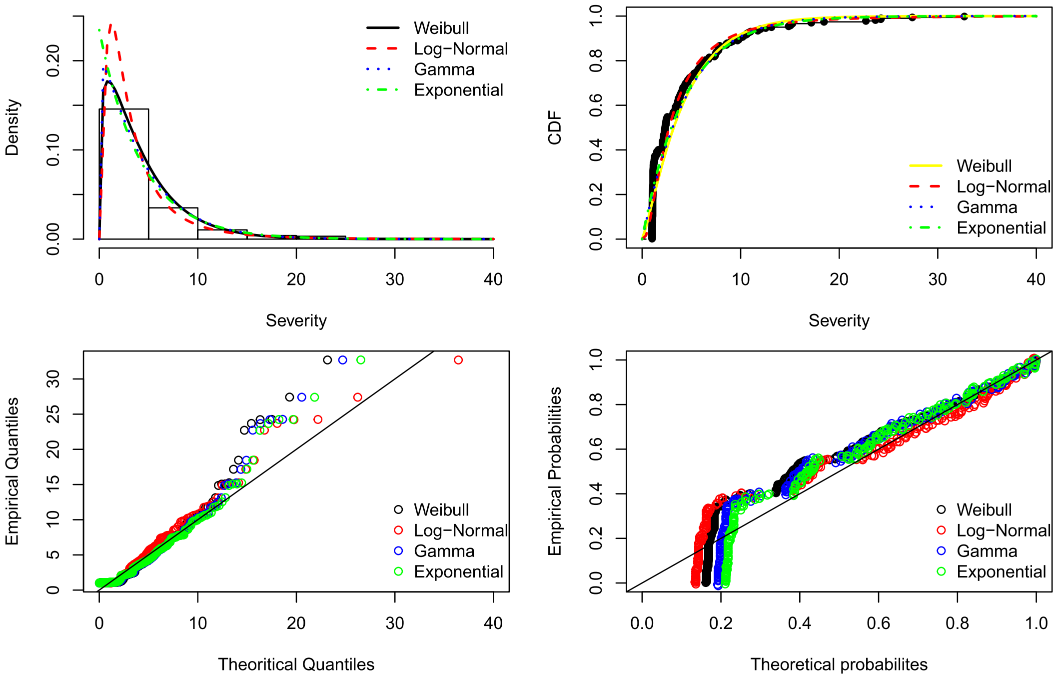
Fig. 4. Scatter diagram of drought severity and duration.

3.2. Fitting marginal distributions

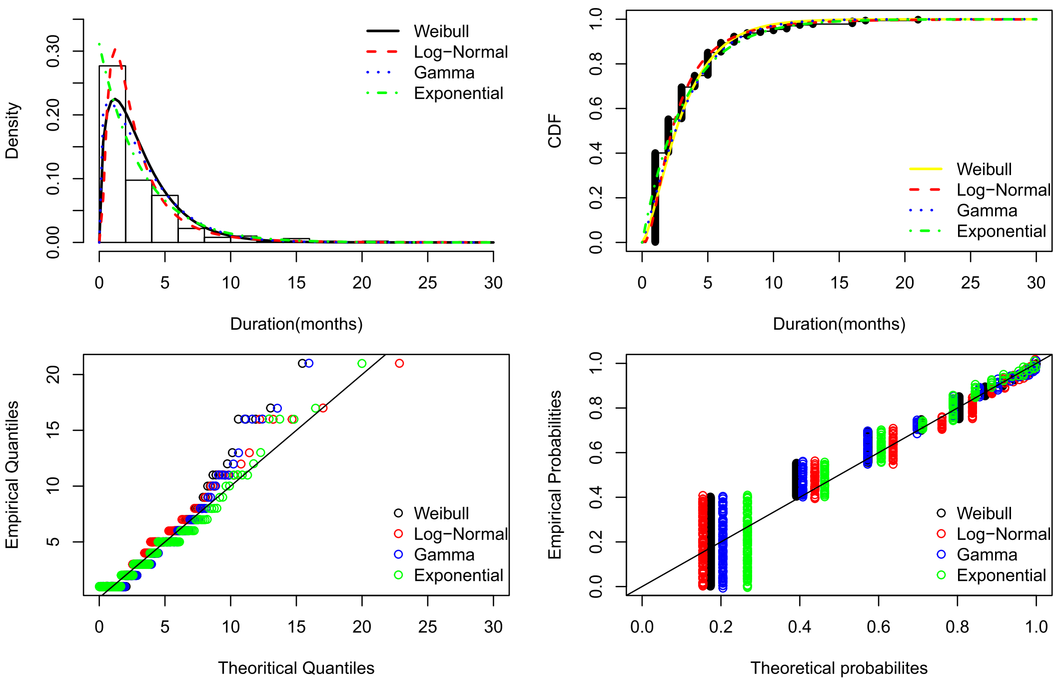
The analysis via copula function requires the marginal CDF for each dependent random variate. The CDFs of Gamma, Lognormal, exponential, and Pearson type III or Weibull were tested to represent drought characteristics. The MLE paradigm was adopted for parameter estimation. The CDF, probability, and QQ plots of considered distributions corresponding to drought variables are depicted in Fig. 5. Fig. 5 shows that the assumed distributions were best fitted to drought variables. Thus, the appropriate one was decided by using AIC and BIC criteria. The numerical results of AIC and BIC are reported in Table 4. The Lognormal distribution with the lowest AIC and BIC (written in bold) is decided to be the best fit for drought severity and drought duration variables.

3.3. Analysis via copula functions

The bivariate copula functions described in Table 2 were used to model the joint behavior of the best-fitted distribution. The parameters of copulas were estimated using the maximum likelihood estimation technique. The ML estimator has been defined in expression (7). Each copula function's parameters were obtained to rank the uniform variates that describe the scale-free dependence property among the droughts variables. The estimated copulas parameters and log-likelihood, AIC, and BIC values are listed in Table 5. Comparing all copula families on the basis of AIC and BIC in Table 5, we found Gumbel extreme value copula is more



(a) Drought severity



(b) Drought duration

Fig. 5. (**a**) Describes density, CDFs, QQ, and PP plots of all four distributions corresponding to drought severity; (**b**) explains density, CDFs, QQ, and PP plots of all four distributions corresponds to drought duration.

appropriate for modeling drought characteristics. Simulation-based visual and tail dependence tests are performed in subsequent for further investigations.

To perform a visual test, 1000 uniform random pairs (u_i, v_i) were generated from each copula family. The simulated pairs were converted into original units using respective marginal distributions to compare (u_i, v_i) with its sample estimates (x_i, y_i). Scatter

Table 4
AIC and BIC of distributions fitted to drought variables.

Variable	Distribution	AIC	BIC
Severity	Gamma	1162.98	1169.91
	Log-normal	**1106.41**	**1113.34**
	Weibull	1165.23	1172.16
	Exponential	1163.23	1166.70
Duration	Gamma	1033.20	1040.14
	Log-normal	**986.73**	**993.67**
	Weibull	1044.37	1051.31
	Exponential	1051.46	1054.93

Table 5
Dependent parameters of fitted copulas along with Log-likelihood, AIC, and BIC values.

Copula family	Member	Copula parameters	Log-likelihood	AIC	BIC
Archimedean	Clayton	$\theta = 2.82$	117.85	-233.20	-230.33
	Frank	$\theta = 19.42$	257.58	-513.15	-509.68
Extreme value	Gumbel	$\theta = 5.03$	**266.76**	**−531.15**	**−528.04**
	Galambos	$\theta = 4.33$	265.90	-529.78	-526.32
Plackett	Plackett	$\theta = 93.47$	250.70	-499.39	-495.92
Elliptical	Student's t	$\theta = 4.33, r = 0.94$	236.22	-468.44	-461.50

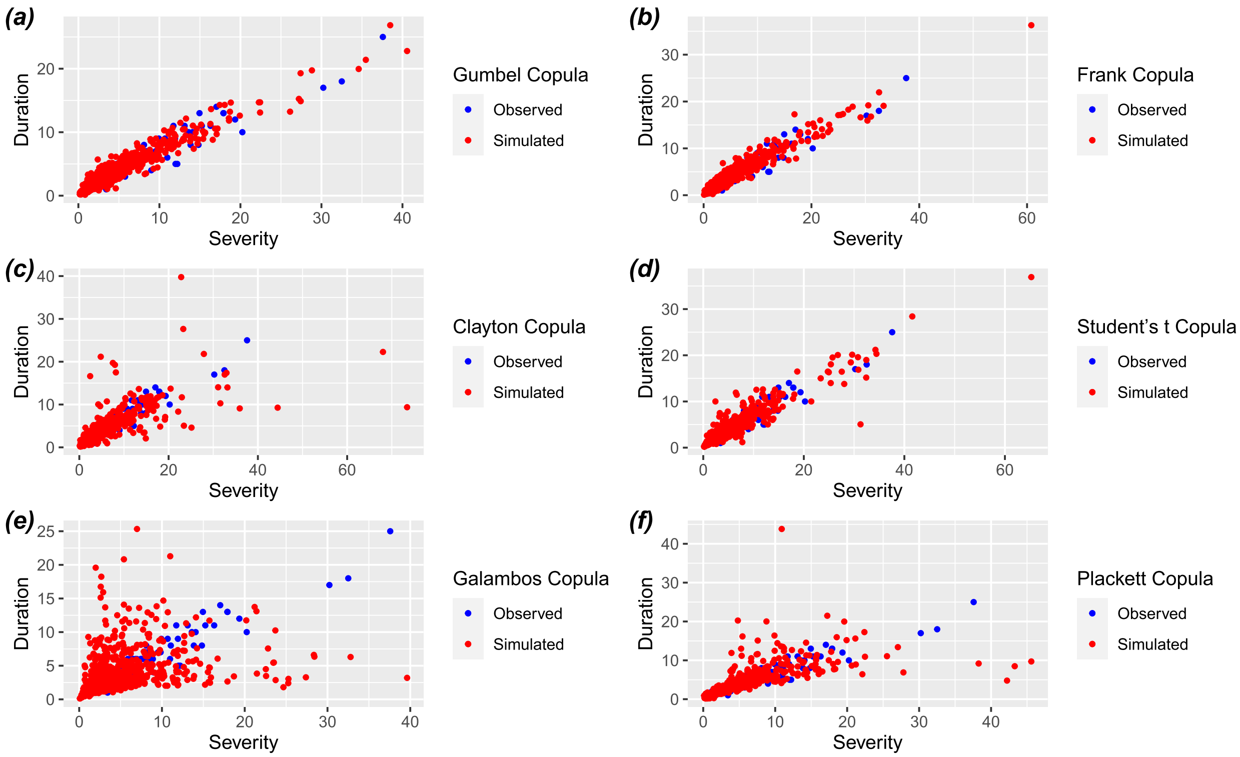


Fig. 6. Scatter plots of the copula-based simulated and observed data (**a**) Gumbel copula, (**b**) Frank copula, (**c**) Clayton copula, (**d**) Student's t copula, (**e**) Galambos copula, and (**f**) Plackett copula.

plots of each family of copulas based on simulated and observed are portrayed in Fig. 6(a-f). It can be realized that the experimental sample dependence structure adequately overlaps extreme value copulas, namely Gumbel and Galambos.

Using the parametric technique, the upper TDC described in section 2.7 is computed for each family of copulas. The expressions and numerical results of parametric TDC involving dependence parameters of copulas are reported in Table 6. Loosely speaking, the numerical results of upper TDC are calculated by providing respective copula dependence parameter values. Based on Table 6, we found that the extreme value Gumbel copula is again more appropriate to capture the dependence at the upper tail. In order to analyze dependence measures further and predict regional drought events, we will use the Gumbel copula function.

Table 6
Expression and numerical results of upper tail dependence coefficients.

Copula family	Member	λ_U expression	$\hat{\lambda}_U$
Archimedean	Clayton	0	0
	Frank	0	0
Extreme value	Gumbel	$2-2^{1/\theta}$	0.86
	Galambos	$2-2^{1/\theta}$	0.82
Plackett	Plackett	0	0
Elliptical	Student's t	$2t_{\theta+1}\left[\sqrt{\frac{(\theta+1)(1-\theta)}{1+\theta}}\right]$	0.76

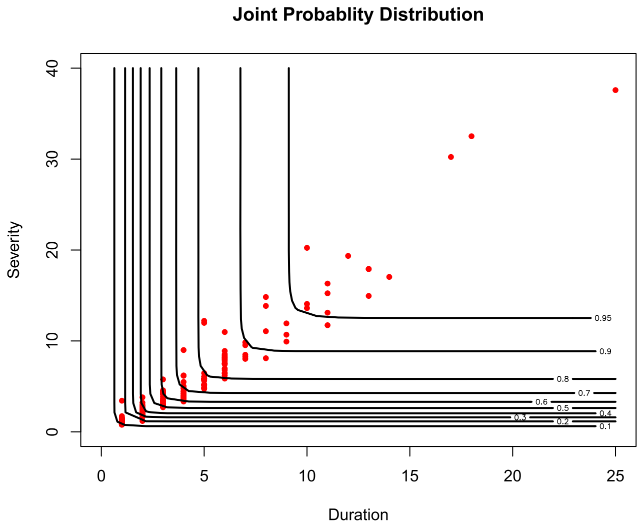


Fig. 7. Contour for the joint probability distribution of drought severity and duration.

3.4. Estimation of joint and conditional distributions

As acknowledged previously, the Gumbel-Hougaard was the best-fitted copula to drought characteristics. So, Gumbel-Hougaard copula-based joint probabilities are calculated for drought variables using the expressions explained in sections 2.8.1 and 2.8.2. Meanwhile, the drought duration and severity are plotted in the same probability, and joint probabilities are established as contour lines. Based on the Gumbel copula, Fig. 7 illustrates the contours of the joint probability for drought duration and severity.

Also, the conditional probabilities of drought severity and duration are obtained using expressions (10) and (11). Calculated probabilities are used to assess drought severity distributions when drought duration exceeds a certain threshold and drought duration distributions when drought severity exceeds a specific threshold. The conditional drought severity distribution given that the drought duration exceeds several threshold levels (i.e., 25th, 50th, 75th, and 95th percentiles), as well as the conditional drought duration distributions, given that the drought severity exceeding the same threshold levels are plotted in Fig. 8 (a, b). Both figures suggest that the conditional drought severity distribution and the conditional drought duration distribution decline with drought duration and severity, respectively. For instance, by looking at Fig. 8 (a), the probabilities for drought severities less than 5 and 10, given a drought duration that surpasses 2 months (50th percentile), are equal to 0.590 and 0.890, respectively. Conversely, according to Fig. 8 (b), the probabilities for drought durations less than 5 and 10 months, given a drought severity exceeding 2.290 (50th percentile), are equal to 0.650 and 0.971, respectively. According to findings, droughts are more likely to occur if drought duration is less than ten months and drought severity is greater than 2.290.

Frequency curves of drought characteristics play an essential role in designing and managing water resource structures. We constructed Gumbel-Hougaard copula SDF curves at 2, 5, 10, 20, 25, 50, and 100-year recurrence levels. The SDF curves are plotted in Fig. 9. It can be noticed from Fig. 9 that a severe drought could occur over the region with higher return periods for a specific duration. For instance, historical drought episodes with a severity of 37.58 were observed in March 1999 and April 2001, with more than 50 years of return periods. The drought was continuously monitored in February 2004 and October 2004, with a severity of 14.83 with a return period of 2.5 years. Using this knowledge, drought mitigation plans such as dam reservoirs or percolation tank conservation structures can help drought-prone areas obtain more water. Furthermore, the SDF curves are considered a device for understanding future temporal traits of drought occurrences. Also, these curves assisted policymakers and engineers in decision-making, risk assessment, and proper water resource planning and management.

In comparison to other studies, we found two more relevant studies that were conducted in this region for the evaluation of drought characteristics using different indices. Ullah and Akbar [43] developed the regional framework by constructing the contours

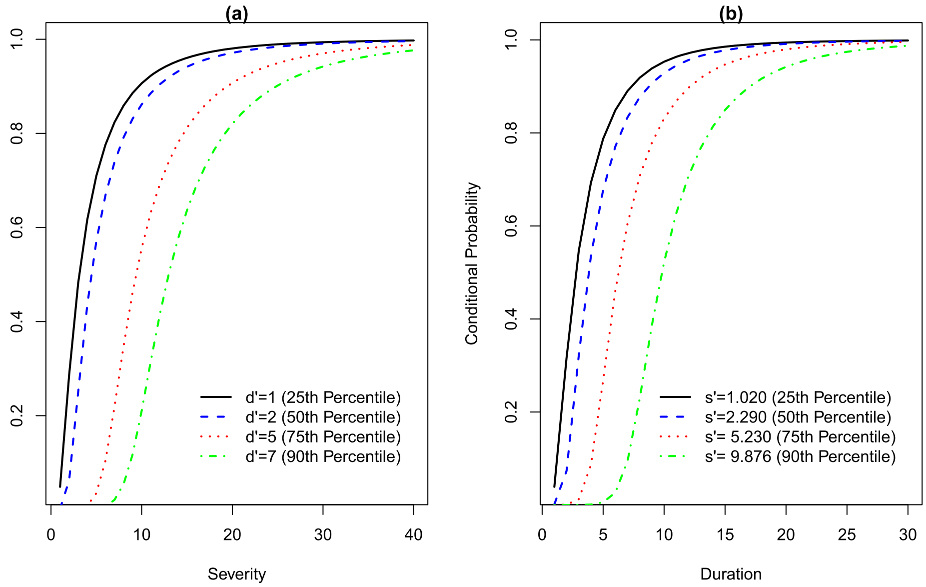


Fig. 8. (a) Conditional probability distribution of drought severity given that drought duration **(b)** conditional probability distribution of drought duration given that drought severity.

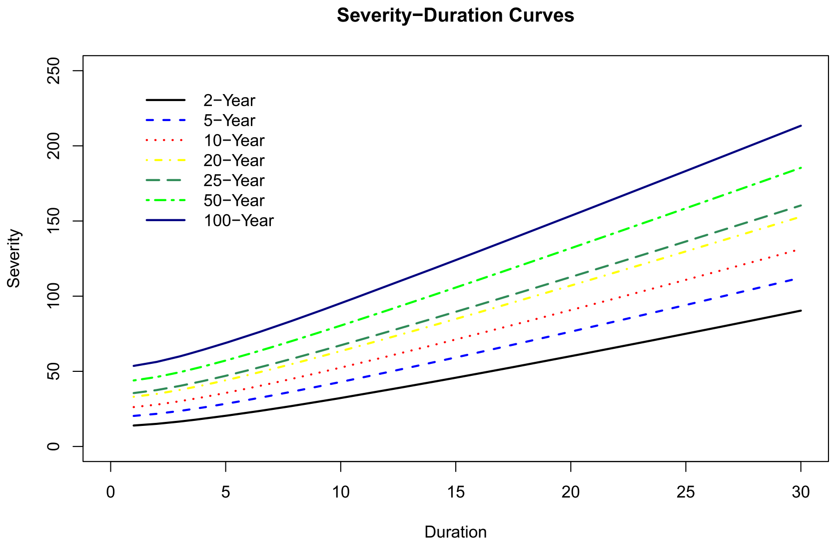


Fig. 9. Drought Severity-Duration-Frequency curves at 2, 5, 10, 20, 25, 50, and 100 years recurrence levels.

of drought severity and duration curves at different fixed reassurance levels. Their results show that the last three regions have more droughts than the other homogeneous regions. In another study, Ullah et al. [44] derive the drought severity and duration curves from different drought indices. They conclude that Baluchistan has a mixed climate with respect to scales and climatic conditions. They argue that severity duration curves rise with the increment of return levels. In the literature, there is no evidence of such type of study that considered meteorological stations of whole region for predicting drought characteristics for the entire region by corporating the collected information from different stations of the region. Our study deviates from a previous study in the following ways: we considered a moderate timescale for calculating SPIs, which accumulate precipitation data at six months. We also derived the joint and conditional probability distributions, which better indicate the chances of droughts in the perspective region. Third, we predict future drought events based on the conditional copula distribution mechanism, which will precisely forecast drought events

3.5. Limitations of the study

The study relies on several simplifying assumptions, such as the choice of copula, parametric distributions, and other model specifications. These assumptions may not fully capture the complexity of the real data processes governing droughts. Different choices of copulas and distributions could yield different results. In addition, the study focuses on the Baluchistan region in Pakistan, and the findings may not be directly applicable to other regions with different climatic and environmental conditions. Drought characteristics and their dependencies may vary across regions, so generalizing the results to other areas should be done cautiously.

Despite these limitations, the study provides valuable insights into drought characteristics and their dependencies in the Baluchistan region, offering a foundation for drought risk assessment and water resource management. Future research could address these limitations by incorporating more diverse data sources, considering non-stationarity, exploring alternative modeling approaches, and validating the models more rigorously.

4. Conclusions and recommendations

In this comprehensive study, we have meticulously employed a robust bivariate copula-based methodology to unravel the intricate fabric of drought characteristics and predict future risks across the Baluchistan region. Droughts, being inherently uncertain, manifest in varying degrees of severity and duration, presenting a complex challenge that demands a proper solution through classy probabilistic models. To articulate droughts, we leveraged the SPI and examined the monthly average precipitation data for the Baluchistan region, spanning a 6-month accumulation period. Our exploration of probabilistic modeling revealed four distinct parametric probability distributions—Gamma, Weibull, Lognormal, and exponential. Each model was evaluated for its suitability in encapsulating the essence of drought events. Among these, the Lognormal distribution emerged as the prime contender for fostering joint dependence modeling of droughts via copulas, ultimately offering a robust foundation for the subsequent analyses.

We relied on a copula framework to model dependence in drought characteristics. Through careful assessment, the Extreme Value Gumbel-Hougaard copula stood out as the most adept choice for effectively capturing the joint behavior of drought characteristics. This rigorous selection paved the way for the derivation of both joint and conditional probability distributions, which in turn provided the groundwork for constructing SDF curves. These curves, associated with various recurrence intervals, offer invaluable insights into constructing vital water harvesting and conservation infrastructure in the region. This study marks a fundamental gait toward comprehending the dynamics of drought characteristics and engineering strategic pathways for managing their impact. The comprehensions harvested and the methodologies employed lay the foundation for a more resilient Baluchistan—a region fortified against the changeable grip of droughts and empowered to forge a sustainable water future. The findings of this study carry critical implications for the water management landscape of Baluchistan. With water scarcity forthcoming, sensible action is needed to prevent permanent shortages of essential necessities. A robust water management system tailored to the unique challenges of the Baluchistan region is crucial. The outline for this scheme can draw upon the insights gathered from this research, ensuring that water resources are harnessed and conserved effectively to alleviate the impact of droughts on everyday life.

We recommend considering spatial extents within copula settings to further refine this study's methodology and insights. Precipitation variability across space and time necessitates an enhanced framework to capture the effects accurately. This spatial refinement would serve as a cornerstone for advancing the accuracy and applicability of our findings. By doing so, we can provide even more precise and detailed insights into drought characteristics and their management in Baluchistan.

CRediT authorship contribution statement

Ishfaq Ahmad: Writing – review & editing, Writing – original draft, Supervision, Project administration. **Touqeer Ahmad:** Writing – review & editing, Writing – original draft, Methodology, Formal analysis, Data curation, Conceptualization. **Shafique Ur Rehman:** Writing – original draft, Software, Formal analysis, Data curation, Conceptualization. **Ibrahim Mufrah Almanjahie:** Writing – review & editing, Writing – original draft, Resources, Funding acquisition. **Fatimah Alshahrani:** Writing – review & editing, Writing – original draft, Resources, Project administration, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The data and code used for this study can be acquired from the corresponding author on reasonable request.

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